Fiscal Constraints on Monetary Policy: How Debt Limits Monetary Effectiveness

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Abstract

Monetary policy invariably leaves a fiscal footprint; not only whether but also when that footprint is eliminated is central to monetary transmission. We provide closed-form solutions for a New Keynesian model with government debt. Debt provides liquidity services. Debt, thus, affects the natural rate of interest. We show how the timing of fiscal adjustment shapes the effects of monetary tightening. As adjustment is deferred, outcomes become reminiscent of active fiscal policy, even if fiscal policy is known eventually to absorb the footprint: Even with finite fiscal delays, a rate hike can become inflationary, and potentially expansionary. The effect is found in data and quantitatively also in a richer quantitative HANK-OLG.

Keywords: Fiscal theory of the price level, fiscal-monetary interaction, natural rate of interest, liquidity services of government debt, HANK, OLG

JEL codes: E52, E62, E63, H63

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1 Introduction

Interest-rate changes invariably leave a fiscal footprint. A long tradition in monetary economics thus holds that monetary policy aimed at price stability requires fiscal backing. Since the seminal contribution of Sargent and Wallace (1981), fiscal backing has typically been understood as a fiscal policy that eventually stabilizes government debt, even if only in the very distant future. Indeed, adjustment at infinity lies at the very heart of the fiscal theory of the price level (FTPL; see Cochrane, 2001, 2023; Leeper, 1991; Sims, 1994; Woodford, 1995, 2001): In the textbook FTPL, public debt constrains monetary policy only when primary surpluses are expected never to stabilize debt. In that case, equilibrium outcomes can be sharply different: a contractionary monetary tightening can, for example, become inflationary. For practical monetary policy making, then, distinguishing between consolidating generations hence or never may seem a risky call to make.

This paper argues that fiscal consolidation in finite time does not rule out FTPL-like outcomes. The cutoff at infinity is relevant only when government debt provides neither utility nor liquidity services. When debt does provide such services (as, for example, in Bayer et al., 2023; Bilbiie, 2025; Kaplan and Violante, 2018; Werning, 2015), monetary transmission depends smoothly on when fiscal policy offsets the fiscal footprint of monetary policy: the longer the delay, the closer the economy to FTPL-like outcomes. The paper presents both theory and empirical evidence.

The mechanism for the theory is straightforward. To fix ideas, consider flexible prices and a one-time rate hike. The hike raises debt-service costs, leaving government debt elevated until fiscal consolidation occurs. In the textbook FTPL, the natural rate is exogenous. Because households' consumption demand rises with financial wealth, goods-market clearing constrains wealth: if the fiscal footprint is never removed—and only then—the initial price level must rise to revalue households' nominal wealth. When debt provides liquidity services, instead, there is an additional, "lingering-footprint channel:" higher debt reduces the convenience yield. A rate hike, thus, raises the short-term natural rate of interest for as long as debt remains elevated. Hence the longer consolidation is delayed, the higher the cumulative (or long-run) natural interest rate response and the stronger the stimulus from higher debt. Goods-market clearing then requires a higher price level than in the absence of liquidity services, even when consolidation is known to occur eventually. The mechanism differs from the FTPL but the limiting outcome is the same even if adjustment is not postponed indefinitely, giving rise to a *passive-fiscal theory of the price level*.

More specifically, the paper proceeds as follows. We first provide a tractable New Keynesian model with government debt. We devise fiscal policy in such a way that equilibrium dynamics of inflation, output, and debt can be solved for analytically, allowing direct comparison with the textbook case. The production side is as in the textbook New

Keynesian model. There is an infinitely-lived representative household. Taxation is lump sum. Next to the fiscal rule, the one assumption we change relative to the textbook model is that the real value of government debt provides direct utility to households, capturing the liquidity services of debt. The household's problem gives rise to the usual dynamic IS equation in which consumption demand depends on the gap between the actual real rate and the natural rate. Marginal propensities to consume out of income are the same as in the textbook model. What differs is that the natural rate now depends on the real value of government debt. This generates the lingering-footprint channel and implies an important result: in this setting monetary interest-rate policy and debt management policy are two sides of the same coin. For conventional interest-rate policy and forward guidance alike, the model's dynamics perfectly coincide with the textbook model whenever primary surpluses prevent any fiscal footprint of monetary policy. Otherwise, delayed fiscal adjustment weakens monetary effectiveness, whereas it gets strengthened by a pre-dated fiscal response in anticipation of the rate change.

Next, we assess the link to the FTPL. We study fundamental equilibria after a one-time increase in the ex-ante real interest rate, with monetary policy keeping the real rate at its steady-state level thereafter. When debt provides no liquidity services, there is a discontinuity at infinity: any finite-time consolidation leaves inflation and output determined by contractionary substitution effects alone. If consolidation never occurs, inflation shifts permanently as does output when the Phillips curve is non-vertical. By contrast, we show that when government debt provides even the slightest liquidity service, FTPL-like outcomes arise even when consolidation is known to occur in finite time. To show our central finding, we devise a tractable, passive fiscal policy. This allows the rate hike to leave a footprint on debt with full consolidation occurring each period with a fixed positive probability; until then, the government services the higher interest burden. Consolidation therefore occurs with probability one, and the debt path is never explosive. Nevertheless, FTPL-type outcomes emerge. This is the case if the probability of consolidation is sufficiently small (but positive). Indeed, the exact active-fiscal equilibrium of the textbook model arises as the limit case when the probability of consolidation converges to zero and the liquidity services subsequently vanish. The mechanism is transparent and entirely conventional: as long as debt provides some liquidity services, the lingering fiscal footprint raises the natural rate, stimulates demand and output, and fuels inflation.

The simple model is parsimoniously parameterized and calibrated based on empirical estimates. A back-of-the-envelope calculation shows that a seven-year delay in fiscal adjustment renders a monetary policy shock ineffective at changing inflation on impact and makes a rate hike (cut) inflationary (deflationary) in the years between the shock and fiscal adjustment. A detailed determinacy analysis is provided in the Appendix. As usual, fiscal policy is passive if it responds strongly enough to higher debt; then, monetary policy satisfying the Taylor principle ensures determinacy. But the required fiscal discipline is tighter: primary surpluses must respond to debt by more than its steady-state interest

cost. They must, instead, cover the full fiscal cost of debt, including changes in outlays driven by changing liquidity premia (see also Bayer et al., 2023).

Next, we show that the data also support the mechanism. We do so with two exercises based on US data. First, we take identified US monetary policy shocks and estimate the impact response of government debt in rolling windows. Then, we run a local projection of output and the price level on the monetary policy shock and an interaction term that measures the debt response. We show that the more positive the debt response, the less contractionary is a monetary tightening, potentially even increasing the price level rather than decreasing it. Second, in a more involved exercise, we estimate the impulse responses of output, debt, and prices to monetary policy, tax policy, and expenditure shocks and construct, using the McKay and Wolf (2023) method, counterfactual policy experiments in which fiscal policy eliminates the fiscal footprint. In line with the theory presented earlier, we find that reducing the fiscal footprint increases the contractionary effect of a monetary tightening on output and prices.

Finally, to gauge the quantitative relevance of the stylized model's findings also from a modeling perspective, we analyze an incomplete-markets OLG model in which households hold assets for both life-cycle and precautionary reasons. The timing effects of fiscal adjustment after a monetary policy shock also arise in this setting. While quantitatively weaker than in the stylized model, the results are qualitatively the same.

Our results closely connect to recent work by Angeletos et al. (2024a,b, 2026) on the interaction of fiscal deficits and monetary policy in economies with non-Ricardian households. Particularly close is Angeletos et al. (2024b), which shows that an OLG structure resolves the discontinuity in the textbook FTPL: an arbitrarily delayed fiscal consolidation is no longer fundamentally different from no consolidation at all. The modeling differs from ours, however, as does the emphasis on channels. Their result hinges on modeling government bonds as Blanchard-Yaari annuities (Blanchard, 1985), so that the private return for surviving households exceeds the fiscal cost of the debt. Building on this mechanism and emphasizing that marginal propensities to consume are higher than with representative households, Angeletos et al. (2026) show that the central bank may prefer slow, or even no, fiscal adjustment, because fiscal inaction supports aggregate demand and mitigates tax distortions. Our focus, instead, is positive in nature. Wolf (2024) and Angeletos and Lian (2023) reinforce this broader message by highlighting, respectively, timing-equivalence in demand management and non-standard determinacy conditions.

While the results are similar, our framework offers analytical advantages and a different economic perspective. In terms of analytical advantages, we provide a variant of the New Keynesian model that is about as tractable as the standard textbook model. The analytical results follow directly. In terms of interpretation, we provide a shift in focus. The model offers a simple natural-rate interpretation, rather than relying on the more involved logic of sequences of marginal propensities to consume and a—potentially

counterfactual—assumption about government bond returns. In our setting, delaying fiscal adjustment keeps debt elevated and thereby raises the natural rate. Putting this into the context of the heterogeneous-agent literature, the mechanism we emphasize operates through improved insurance, not through wedges between MPCs and market interest rates. As fiscal adjustment is delayed further, the cumulative gap between the natural rate and the actual policy rate widens, generating ever stronger demand stimulus. In the limit of infinite delay, this cumulative gap diverges, so prices must respond to restore equilibrium, recovering the textbook FTPL outcome as a special case. Key is that additional liquidity in the form of government bonds changes the natural rate.

The remainder of this paper is structured as follows. Next, we discuss the related literature. Section 2 provides the simple New Keynesian model variant we study and introduces the lingering-footprint channel. Section 3 discusses the passive-fiscal theory of the price level and implications for monetary transmission. Section 4 shows that the fiscal-footprint channel has empirical support. Section 5 introduces the quantitative OLG model, calibrates it, and derives quantitative implications. Section 6 concludes. An appendix collects formal proofs and robustness analysis.

Literature review

In addition to the papers mentioned above, we build on several different strands of literature: on fiscal-monetary interaction, government debt, and transmission in economies with heterogeneous agents.

The fiscal theory of the price level (FTPL), developed by Leeper (1991), Sims (1994), and Woodford (1995, 2001), derives conditions under which government solvency can determine the price level. Under "active" fiscal policy, the price level jumps to ensure solvency; under "passive" fiscal policy, surpluses adjust to debt and monetary policy retains its conventional stabilizing role. Cochrane (2001) extends this logic to long-term debt, Cochrane (2023) offers a synthesis, and Sargent and Wallace (1981) provide the key monetarist precursor. Our contribution moves beyond this binary active-passive distinction: when government debt provides liquidity services, the *timing* of fiscal adjustment becomes central. FTPL-like equilibrium outcomes emerge even under passive fiscal policy.

This timing perspective complements research on richer monetary-fiscal interactions. Bianchi and Ilut (2017) emphasize belief-driven shifts between monetary and fiscal dominance, whereas Davig and Leeper (2007) focus on exogenous shifts. Bianchi et al. (2023) quantify persistent fiscal inflation when some deficits are never fiscally funded, and Sims (2011) stresses the fiscal roots of inflation in the 1970s. We add a transparent analytical mechanism showing how delayed adjustment changes the sign and magnitude of monetary transmission.

Our mechanism relies on debt providing liquidity services, so that debt supply affects

the natural rate. The idea that public debt can serve as private liquidity goes back at least to Woodford (1990) and Aiyagari and McGrattan (1998). This endogeneity of the natural rate arises naturally in heterogeneous-agent New Keynesian (HANK) environments (Acharya and Dogra, 2020, 2022; Alves et al., 2020; Auclert, 2019; Bayer et al., 2024, 2019; Bilbiie, 2025; Gornemann et al., 2021; Kaplan et al., 2018; Kaplan and Violante, 2018), where incomplete markets and borrowing constraints render the quantity of safe public debt a key state variable for aggregate demand. Werning (2015) provides foundational aggregation results for such incomplete-markets economies and clarifies how liquidity provision modifies the aggregate Euler relation, effectively shifting the time preference rate of the as-if representative agent. Challe and Ragot (2011) and Challe et al. (2017) provide tractable heterogeneous-agent analyses of how debt and its liquidity provision interact with precautionary savings and affect aggregate demand. Related reduced-form approaches that give wealth or safe assets a direct role in preferences include Michaillat and Saez (2021) and Rannenberg (2021). Quantitative evidence for the debt-liquidity channel is provided by Bayer et al. (2023), and Campos et al. (2024) analyze (the effectiveness of) monetary policy when the natural rate of interest, r_t^n, is fiscally driven by debt supply.

Our experiments also cover announced future interest rate changes. In this regard, our paper connects to the forward-guidance literature. McKay et al. (2016) and Hagedorn et al. (2019) study how anticipated policy-rate changes affect current demand, while Rannenberg (2024) analyzes forward guidance in a model with preferences over safe assets. We add the fiscal-financing side: the timing of future primary surpluses changes the trajectory of debt, the natural rate, and hence the transmission of anticipated monetary policy.

Related to our quantitative analysis, there is an earlier literature on monetary policy in OLG settings. Bénassy (2005) shows that, in non-Ricardian economies, determinacy requirements under interest-rate rules can differ sharply from the standard Taylor-principle benchmark. In a sticky-price OLG model, Ascari and Rankin (2013) show debt-based demand management to be sensitive to the monetary rule. Dupraz and Rogantini-Picco (2025) study the fiscal conditions under which monetary policy can insulate the price level from fiscal shocks.

Empirically, our paper connects to evidence that the natural rate is not policy-invariant. Laubach and Williams (2003) and Holston et al. (2017) estimate persistent variation in r^*; Del Negro et al. (2017) attribute an important part of long-run rate movements to safety and liquidity premiums; Jordà et al. (2024) and Bauer and Swanson (2022) provide evidence consistent with persistent effects of monetary shocks on real activity and long rates. Our model framework provides a fiscal channel behind these patterns: monetary policy leaves a fiscal footprint, moving liquidity premiums and thereby r^* in the direction of the policy intervention. Through this mechanism, our paper connects to a macro-finance

literature that shows that convenience yields react to the supply of government debt. Laubach (2009) documents that deficits and debt raise long rates, Krishnamurthy and Vissing-Jorgensen (2012) document preferred-habitat demand for Treasuries and quantity effects on prices and yields, and Greenwood et al. (2015) show that debt-supply management affects bond risk premiums. This evidence provides calibration targets for our study and supports the key reduced-form ingredient in our model: higher effective bond supply lowers convenience yields, shifting the natural rate of interest.

Our paper also relates to empirical evidence that public debt shapes monetary transmission. De Luigi and Huber (2018) find weaker monetary transmission in US regimes characterized by rising debt-to-GDP ratios. For the euro area, Johns et al. (2026) find weaker price and inflation-expectation responses to monetary tightening at higher public debt levels, while output contracts at least as strongly; they also document a nonlinear role for debt maturity. Our empirical analysis complements this evidence by examining how monetary transmission varies with the fiscal response to the debt footprint of a monetary shock. Our model isolates a mechanism through which the timing of fiscal adjustment changes the supply of liquidity and the natural rate, even when fiscal policy eventually absorbs the footprint in full.

Next to the papers by Angeletos, Lian, and Wolf discussed earlier, our work is finally also related to Rachel and Ravn (2025) who also analyze how non-Ricardian household behavior changes monetary–fiscal transmission. Their main focus is on the question of determinacy. We isolate fiscal timing in an economy with textbook MPCs and derive closed-form solutions that transparently relate the (dis-)inflationary effect of a monetary shock to the timing of the fiscal response. Our quantitative model assesses the strength of the debt–liquidity mechanism and its interaction with debt maturity.

2 Analytical model: demand side

We start by analyzing an analytically tractable New Keynesian model. The central bank controls the path of the actual real rate of interest. The current section lays out the demand side of the model. We follow the textbook version with one deviation only, namely, we capture the liquidity services that assets generate, particularly government bonds, by including bonds in the representative household's utility function (bonds-in-the-utility, BIU). The section derives the dynamic IS equation and introduces the lingering-footprint channel on aggregate demand. It also clarifies that the channel relies on a shifting natural rate of interest, not on high marginal propensities to consume. Section 3, thereafter, will specify also the supply side and government policies, then already in linearized form, and derive implications for monetary transmission. Appendix A collects proofs for all propositions.

2.1 Household problem

Consider an economy with a unit mass of identical, infinitely lived households. $t = 0, 1, ...$ marks time. There is no idiosyncratic risk. Households discount future felicity at time-preference rate $\rho > 0$. They obtain felicity from consumption c_t and, potentially, from their holdings of nominal one-period liquidity-providing bonds, the real value of which is b_t. The household solves

$$\begin{aligned} \max_{\{c_t, b_t, d_t\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \Big\{ \textstyle\sum_{t=0}^{\infty} \beta^t \, u\,(c_t, b_t) \Big\} \\ \text{s.t.} \quad & b_t + d_t + c_t + \tau_t = y_t + (1 + r_t)\, b_{t-1} + (1 + r_t^d) d_{t-1}, \qquad t = 0, 1, ... \end{aligned} \tag{1}$$

for some concave felicity function u and with $\beta := \frac{1}{1+\rho}$. Here y_t is income from production. Letting i_t mark the nominal interest rate and π_t the inflation rate, $1+r_t := (1+i_{t-1})/(1+\pi_t)$ marks the realized real gross return on the liquidity-providing bonds. Further, d_t and r_t^d are the holdings of non-liquidity-providing bonds and their realized return, respectively. Last, τ_t is the real value of lump-sum taxes that the government uses to service its debt.

To obtain a tractable expression and provide intuition, we specify felicity as $u(c, b) := U(c + \Gamma(b))$, with $U(x) := \frac{x^{1-\sigma}}{1-\sigma}$, $\sigma > 0$, being of the constant relative risk aversion form. Function Γ governs the liquidity services. Assume that Γ is twice continuously differentiable with $\Gamma_b > 0$ and $\Gamma_{bb} < 0$ marking first and second derivatives. Writing U_c as the derivative with respect to consumption, the household's problem (1) gives rise to the conventional consumption Euler equation for non-liquidity-providing assets

$$U_c\,(C_t, b_t) = \beta\, \mathbb{E}_t \big\{ U_c\,(C_{t+1}, b_{t+1})\ (1 + r_{t+1}^d) \big\}.$$

The consumption Euler equation for liquidity providing assets is given by

$$U_c\,(C_t, b_t) = \frac{\beta}{1 - \Gamma_b(b_t)}\, \mathbb{E}_t \big\{ U_c\,(C_{t+1}, b_{t+1})\ (1 + r_{t+1}) \big\},$$

so that $\Gamma_b(b_t)$ can be interpreted as the usual "liquidity premium" or "convenience yield" on government debt. Using the constant relative risk aversion functional form, the latter consumption Euler equation becomes

$$[c_t + \Gamma(b_t)]^{-\sigma} = \frac{\beta}{1 - \Gamma_b(b_t)} \mathbb{E}_t \Big\{ [c_{t+1} + \Gamma(b_{t+1})]^{-\sigma}\, (1 + r_{t+1}) \Big\}. \tag{2}$$

Euler equation (2) shows that real bond holdings, b_t, in effect shift the discount factor that the representative household applies. The feature that debt policy affects the economy's effective discount factor is a cornerstone of incomplete-market models.[1]

[1] The incomplete-markets setup of Werning (2015), for example, does not have bonds in the utility function. Assuming zero liquidity, constant relative risk aversion preferences, and perfect foresight, his

2.2 Liquidity-augmented IS curve

From here on, we rely on the linearized version of Euler equation (2). Toward that end, let a bar over a variable denote its steady-state value. Assume, without loss of generality, that $\bar{c} = \bar{y} = 1$ and $\Gamma(\bar{b}) = 0$, the latter a normalization. Then (2) gives rise to a liquidity-augmented IS equation of the form

$$\widehat{c}_t = \mathbb{E}_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}\left[\mathbb{E}_t\{\widehat{r}_{t+1}\} - \frac{-\Gamma_{bb}(\bar{b})}{1-\overline{\Gamma}_b}\widehat{b}_t\right] + \overline{\Gamma}_b\left[\mathbb{E}_t\{\widehat{b}_{t+1}\} - \widehat{b}_t\right], \tag{3}$$

where $\widehat{c}_t = (c_t - \bar{c})/\bar{c}$, $\widehat{b}_t = (b_t - \bar{b})/\bar{y}$, and $\widehat{r}_{t+1} = (r_{t+1} - \bar{r})/(1+\bar{r})$ are the deviations of consumption, debt, and real interest rates from their steady-state levels. For future reference, define $\widehat{\pi}_t = (\pi_t - \bar{\pi})/(1+\bar{\pi})$ and $\widehat{i}_t = (i_t - \bar{i})/(1+\bar{i})$ as the deviations of inflation and the nominal interest rate from their steady states, too. The term $\overline{\Gamma}_b := \Gamma_b(\bar{b})$ captures the marginal value of additional liquidity and equals the gap between the steady-state interest rate and the household's time preference rate, that is, in steady state $\overline{\Gamma}_b = \frac{\rho - \bar{r}}{1+\rho}$. For ease of exposition, let $\chi/[\bar{b}(1+\bar{r})] := \frac{-\Gamma_{bb}(\bar{b})}{1-\Gamma_b(\bar{b})} > 0$ denote the semi-elasticity of the liquidity premium with respect to debt. That semi-elasticity measures how quickly the returns on government debt rise with rising debt.[2]

Equation (3) can be expressed in terms of the gap between the actual real interest rate and the natural rate of interest

$$\widehat{c}_t = E_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}\left[E_t\{\widehat{r}_{t+1}\} - \widehat{r}^n_t\right], \tag{4}$$

where the natural rate of interest is defined as the real rate of interest that, for a given path of debt, would arise in a flex-price equilibrium

$$\widehat{r}^n_t := \chi\frac{\widehat{b}_t}{\bar{b}(1+\bar{r})} + \sigma\overline{\Gamma}_b\left[\mathbb{E}_t\{\widehat{b}_{t+1}\} - \widehat{b}_t\right]. \tag{5}$$

Equations (4)–(5) are the liquidity-modified analog of the standard IS curve: consumption still responds to the gap between the *ex-ante* real rate of interest and the natural rate of interest. The crucial difference, however, is that now the natural rate itself is no longer exogenous but a function of the outstanding stock of government debt. This is the reason

setting delivers the "aggregate" consumption Euler equation

$$c_t^{-\sigma} = \frac{\beta}{(1-\Gamma_t)}c_{t+1}^{-\sigma}(1+r_{t+1}), \qquad \text{with } \Gamma_t := 1 - \left[\mathbb{E}_t\left\{\max_i\left(\frac{\gamma_{i,t}}{\gamma_{i,t+1}}\right)^{\sigma}\right\}\right]^{-1},$$

where $\gamma_{i,t}$ is the consumption share of household i. An increase in government debt, an increase in the supply of assets, would allow for better consumption smoothing and thus decrease Γ_t, analogously to the decrease in our representative household setting with bonds in the utility.

[2] One important thing to note is that χ need not be constant; at a given local debt–rate semi-elasticity, χ is proportional to the outstanding debt stock. More generally, χ varies with both debt and the local liquidity-demand curvature.

why as far as demand stabilization is concerned monetary policy and government debt management are intimately linked.

2.3 The lingering fiscal footprint and its effect on demand

We will spell out fiscal policy in more detail later. For now, it suffices to only link the liquidity held by households to government debt and to monetary policy. The government budget constraint is given by

$$b_t = b_{t-1}(1 + r_t) - \tau_t. \tag{6}$$

Or in linearized terms,

$$\widehat{b}_t = (1 + \bar{r})\,\widehat{b}_{t-1} + \bar{b}\,(1 + \bar{r})\,\widehat{r}_t - \widehat{\tau}_t, \tag{7}$$

where $\widehat{r}_t = (r_t - \bar{r})/(1+\bar{r})$ and $\widehat{\tau}_t = (\tau_t - \bar{\tau})/\bar{y}$. This notation anticipates that, in equilibrium, non-liquidity bearing debt d_t is in zero net supply so that the representative household holds government debt only. For given taxes, changes in the real rate of interest, thus, not only invariably leave a fiscal footprint but they also affect the liquidity that households have access to. This section argues that monetary transmission, therefore, depends on exactly when fiscal consolidation eliminates the fiscal footprint left by monetary policy. The longer the footprint lingers, the less contractionary a monetary tightening—the "*lingering-footprint channel*" of demand.

Iterate (4) forward, use (5), and use that $\overline{\Gamma}_b = \frac{\rho - \bar{r}}{1+\rho}$ to obtain

$$\widehat{c}_t \;=\; \mathbb{E}_t\left\{\widehat{c}_{t+T+1}\right\} - \frac{1}{\sigma}\left[\mathbb{E}_t\left\{\sum_{s=0}^{T}\widehat{r}_{t+1+s}\right\} - \mathbb{E}_t\left\{\sum_{s=0}^{T}\frac{\chi}{\bar{b}(1+\bar{r})}\widehat{b}_{t+s}\right\}\right] - \frac{\rho - \bar{r}}{1+\rho}\left(\widehat{b}_t - \mathbb{E}_t\{\widehat{b}_{t+T+1}\}\right). \tag{8}$$

Applying the terminal conditions that $\lim_{T\to\infty}\mathbb{E}_t\{\widehat{c}_{t+T+1}\} = \widehat{c}_\infty$ and $\lim_{T\to\infty}\mathbb{E}_t\{\widehat{b}_{t+T+1}\} = \widehat{b}_\infty$ exist, and assuming that monetary policy is such that $\lim_{T\to\infty}\mathbb{E}_t\{\widehat{r}_{t+T+1}\} = 0$, then for any $\chi/\bar{b}(1+\bar{r}) > 0$ the second sum converges only if $\widehat{b}_\infty = 0$. This is important for it constrains equilibrium outcomes: if higher debt reduces the convenience yield ($\chi/\bar{b} > 0$), then

$$\widehat{c}_t \;=\; \widehat{c}_\infty - \frac{1}{\sigma}\left[\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\widehat{r}_{t+1+s}\right\} - \mathbb{E}_t\left\{\sum_{s=0}^{\infty}\frac{\chi}{\bar{b}(1+\bar{r})}\widehat{b}_{t+s}\right\}\right] - \frac{\rho - \bar{r}}{1+\rho}\widehat{b}_t \tag{9}$$

in equilibrium. The term in the square brackets in (9) captures the very essence of the lingering-footprint channel. The key insight is that a multiple of the *cumulative* path of debt (the second sum) is related to aggregate demand much like the actual long real rate (the first sum), except with the opposite sign. For the sake of the argument, focus on a conventional monetary tightening, that is, an increase in the short-term real rate of interest. If that tightening raises debt, this increase in debt works against the monetary tightening by stimulating demand. Importantly, the debt-related stimulus is stronger the

longer the fiscal footprint lingers.[3]

2.4 Marginal propensity to consume vs. liquidity

Why does the lingering-footprint channel matter? The following lemma clarifies the issue:

Lemma 1 (MPC equivalence under GHH-type bonds-in-the-utility (BIU) preferences)**.** *Consider the individual household's problem of Section 2.1 with* $u(c,b) = U(c+\Gamma(b))$*, where* U *and* $\Gamma(b)$ *are concave with* $\lim_{b\to\infty}\Gamma_b(b) < \frac{\bar{r}^d-\bar{r}}{1+\bar{r}^d} < \lim_{b\to-\infty}\Gamma_b(b)$*. Suppose that prices throughout are at their steady-state values, namely,* $(1+\bar{r})/(1-\Gamma_b(\bar{b})) = 1+\bar{r}^d = 1/\beta$*. Indexing individual choices by superscript* i*, suppose the individual household's initial wealth position is consistent with the underlying state,* $b^i_{-1} = \bar{b}$ *and* $d^i_{-1} = 0$*. Define the composite good* $x^i_t = c^i_t + \Gamma(b^i_t)$*. Then*
(i) the marginal propensity to consume (MPC) out of beginning-of-period resources $\mathcal{M}_c := \left\{\frac{\partial c^i_t}{\partial y^i_0}\right\}_{t=0}^{\infty}$ *is given by* $\mathcal{M}_c = \{1-\beta\}_{t=0}^{\infty}$*, and*
(ii) the MPC for composite consumption $\mathcal{M}_x := \left\{\frac{\partial x^i_t}{\partial y^i_0}\right\}_{t=0}^{\infty}$ *is given by* $\mathcal{M}_x = \mathcal{M}_c$*.*
The MPCs for actual and composite consumption, therefore, are identical to the MPC in the textbook representative-agent New Keynesian (RANK) model without bonds in utility.

Proof. See Appendix A.1. □

Households in the BIU model, thus, leave their consumption demand unchanged after net-present-value-neutral changes in their income sequence. In equilibrium, consumption nevertheless responds to higher debt levels because higher debt has to be held. The marginal liquidity services of debt fall, raising the natural rate of interest. In other words, the lingering-footprint channel formulated here relies on changes in liquidity rather than higher-than-textbook MPCs.

3 The fiscal footprint and monetary transmission

This section explores how the fiscal footprint of monetary policy shapes monetary transmission. We derive analytical results to highlight the key mechanisms, using fiscal and monetary specifications that permit closed-form solutions. We proceed in two steps. Section 3.1 studies the role of fiscal timing using the liquidity-augmented IS equation (3) for a given initial inflation rate. The section covers both conventional policy and forward guidance, as well as the effects of anticipation and delay in erasing the footprint. Section 3.2

[3] There also is the final term in (9). Marginal felicity falls in the composite $x_t := c_t + \Gamma(b_t)$. Debt, thus, dampens consumption demand due to intertemporal smoothing. For typical calibrations $\rho - \bar{r}$ will tend to be small. Besides, the rise in debt enters only once and not cumulatively. Expressing (9) in terms of $\widehat{x}_t$ gives $\widehat{x}_t = \widehat{x}_\infty - \frac{1}{\sigma}\left[\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\widehat{r}_{t+1+s}\right\} - \mathbb{E}_t\left\{\sum_{s=0}^{\infty}\frac{\chi}{\bar{b}(1+\bar{r})}\widehat{b}_{t+s}\right\}\right]$. The term in square brackets, thus, captures a natural rate effect of debt in a narrow sense, while the final term in (9), $\frac{\rho-\bar{r}}{1+\rho}\widehat{b}_t$, captures a demand composition effect (liquidity vs. goods). For the case $\rho = \bar{r}$, the composition effect vanishes.

closes the model with a Phillips curve and examines how fiscal timing affects transmission to inflation and output.[4] Throughout, we also provide first quantitative assessments.

3.1 The role of anticipation and delay

The current section aims to clarify the role of *anticipation* (fiscal consolidation before the rate change) and *delay* of fiscal consolidation (fiscal consolidation after the rate change). We assume that all policy changes are announced at time zero. We first spell out specific paths for monetary and fiscal policy. We then clarify what we mean by "fiscal footprint of monetary policy." Thereafter, we discuss how monetary policy's impact effect on consumption depends on the timing of consolidation. All this in a setting aiming for analytical tractability.

Timing The only shocks we consider are one-time changes in fiscal and monetary policy announced in period $t = 0$, with the economy assumed to have been in steady state beforehand. There are three special points in time. At $t = 0$, the government announces the entire paths of fiscal and monetary policy. Monetary policy is chosen so that the real interest rate deviates from its steady-state value only once, at $t = M \geq 1$. Likewise, taxes deviate from their steady-state value only once, at $t = F \geq 0$. Taxes are set so that, by the end of period $T := \max\{M, F\}$, government debt has returned to its steady-state value.

Monetary policy is conducted as follows. The central bank directly controls the real interest rate, $\widehat{r}_t$, in all periods except the initial one.[5] At $t = 0$, the central bank announces the period of the policy deviation, M, and the value of the real interest rate in that period, $\widehat{r}_M \neq 0$. In all other periods the real interest rate remains at its steady-state value, so that $\widehat{r}_t = 0$ for all $t > 0$, $t \neq M$.

Fiscal policy is designed as follows. In steady state, the government maintains a constant real stock of debt, $\overline{b} > 0$, with corresponding tax revenue $\overline{\tau} = \overline{r} \cdot \overline{b}$. At $t = 0$, the government announces the period of time at which taxes adjust to stabilize debt, F. The size of the tax adjustment is chosen so that revenue is sufficient to return debt to its steady-state level for all $t \geq T$, but not earlier.

The following assumes, in addition, that output is demand determined. The arguments here rely only on the household's problem and the government budget constraint. This section takes the initial period's inflation as given and imposes eventual convergence to steady-state consumption so that $\lim_{t\to\infty} \widehat{c}_t = 0$ by assumption.

[4] An analysis of determinacy is provided in Appendix B.

[5] Since i_{-1} is at its steady-state value and predetermined while inflation can adjust, we have $\widehat{r}_0 = -\widehat{\pi}_0$.

3.1.1 The monetary footprint on debt

Next, we provide a precise expression for the "monetary footprint on debt." Toward this end, we conceptually decompose the debt level into three components

$$\widehat{b}_t = \underbrace{\mathbf{b}_{\pi,t} + \mathbf{b}_{r,t}}_{\text{monetary footprint on debt}} + \mathbf{b}_{\tau,t}. \tag{10}$$

The first reflects surprise inflation in the initial period (taken as parametrically given in this section); the second term reflects the change in government financing costs induced by the real-rate change at time M. The sum of the first two components we label the "monetary footprint on debt." The third term reflects the fiscal adjustment at time F. The monetary footprint on debt is "erased" whenever $\mathbf{b}_{\tau,t} = -[\mathbf{b}_{\pi,t} + \mathbf{b}_{r,t}]$. The effect of surprise inflation $\widehat{\pi}_0$ on the path of debt is given by

$$\mathbf{b}_{\pi,t} = -\bar{b}(1+\bar{r})^{t+1} \cdot \widehat{\pi}_0, \tag{11}$$

reflecting the initial debt outstanding, the revaluation of that debt through inflation, and the accumulated interest expenses. The effect on debt of the change in financing costs is given by

$$\mathbf{b}_{r,t} = \begin{cases} 0 & 0 \leq t < M, \\ \bar{b}(1+\bar{r})^{t+1-M} \cdot \widehat{r}_M & t \geq M. \end{cases} \tag{12}$$

Fiscal policy is designed to have erased the monetary footprint on debt in (and after) period T. So as to raise the present value of resources needed for this, it levels taxes on the order of $\widehat{\tau}_F = -(1+\bar{r})^{F-T}\left[\mathbf{b}_{\pi,T} + \mathbf{b}_{r,T}\right]$. With this, $\mathbf{b}_{\tau,t} = -\left[\mathbf{b}_{\pi,t} + \mathbf{b}_{r,t}\right]$ for all $t \geq T$. All considered,

$$\mathbf{b}_{\tau,t} := \begin{cases} 0 & 0 \leq t < F, \\ -(1+\bar{r})^{t-T}(\mathbf{b}_{\pi,T} + \mathbf{b}_{r,T}) & t \geq F. \end{cases} \tag{13}$$

Equations (10) - (13) characterize the evolution of debt.

3.1.2 Consumption response to a monetary shock

The role of fiscal and monetary timing for consumption is most transparent in the impact response to the policy announcement. Using equations (9) - (13), this is given by

$$\begin{aligned} \widehat{c}_0 \;=\; & -\tfrac{1}{\sigma}\widehat{r}_M + \tfrac{1}{\sigma}\chi\left[\tfrac{(1+\bar{r})^{F-M}-1}{\bar{r}}\widehat{r}_M - \tfrac{(1+\bar{r})^{F}-1}{\bar{r}}\widehat{\pi}_0\right] \\ & + \tfrac{\rho-\bar{r}}{1+\rho}\begin{cases} \bar{b}(1+\bar{r})^{-M+1}\widehat{r}_M, F=0, \\ \bar{b}(1+\bar{r})\cdot\widehat{\pi}_0, \qquad F>0. \end{cases} \end{aligned} \tag{14}$$

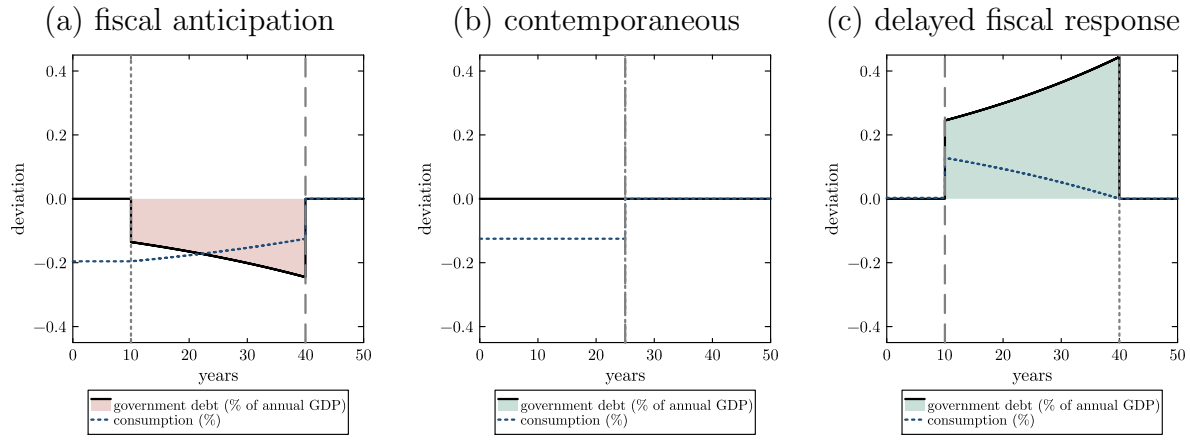
Consider a rate hike in period M, $\widehat{r}_M > 0$. The first row highlights intertemporal substitution. As is conventional, a rate hike lowers consumption through the standard intertemporal substitution effect (first term). When liquidity matters, $\chi > 0$, the timing of the fiscal response also matters for the strength of intertemporal substitution depends on the natural rate as well. A higher real rate raises debt in period M, all else equal. If fiscal policy anticipates the shock, $F < M$, debt falls on impact, lowering the natural rate and further compressing consumption (second term). If the fiscal response is delayed, $F > M$, instead, the effect goes in the opposite direction: back-loaded fiscal adjustment lets debt rise, which raises the natural rate and supports consumption. Initial inflation, captured by the last term in square brackets, also affects debt, so the timing of consolidation matters along this margin as well. For a discussion of the terms in the second row, see Footnote 3 earlier.

3.1.3 A quantitative, graphical illustration

Figure 1 provides a graphical illustration of (14). The figure illustrates how monetary transmission depends on when fiscal policy removes the fiscal footprint of a rate hike. It assumes fixed prices. In each panel, the dashed vertical line marks time M (in years), at which time monetary policy tightens by 25 bps (not annualized) for one month. The y-axis measures the consumption response and debt response in percent or as percent of annual GDP, respectively; the x-axis shows the time dimension in years. A dotted vertical line marks the fiscal response time F. The blue dotted line extending from left to right shows the consumption response, and the solid black line extending from left to right shows the debt response. The area between the horizontal zero line and the debt response measures the cumulative debt response, $\sum_{s=0}^{\infty} \widehat{b}_{t+s}$, which governs the effect on the natural rate; recall its importance from (9).

The fiscal response to the monetary footprint strongly shapes monetary transmission. Panel (b) provides the textbook benchmark: a rate hike in year 25 is matched by a contemporaneous increase in the primary surplus ($M = F = 25$), leaving debt unchanged throughout. Consumption responds exactly as it would if liquidity played no role, falling by a little over 0.1 percent on impact. Panel (a), on the left, instead shows fiscal tightening before monetary tightening: to eliminate the anticipated monetary footprint in year 40, the primary surplus rises already in year 10 ($M = 40, F = 10$). Debt falls persistently, lowering the natural rate of interest (cf. (5)) and, hence, widening the gap between the actual and natural long real rates. In this case, the fall in liquidity significantly amplifies the impact decline in consumption relative to the benchmark. Panel (c) shows the arguably most relevant case: a delayed fiscal response, here occurring 30 years after the rate hike in year 10 ($M = 10, F = 40$). The delay in consolidation allows debt to rise. The natural rate increases. In the simulation shown here, this rise is strong enough to fully offset the contractionary effect of a rate hike: consumption rises rather than falling. Full

Figure 1: Consumption response to an announced rate hike



Notes: Consumption and debt response to a one-time pre-announced rate hike. The model is monthly; the figure plots monthly numbers against time in years. The real rate rises by 25 bps (not annualized) for exactly one month. Parametrization: $\sigma = 2, \chi = 2.5\%, \bar{b} = 98\%, \rho = \bar{r} = 2\%$ (in annual terms where applicable). Panel (a): fiscal surplus adjusts in year 10; monetary shock in year 40. Panel (b): surplus and monetary shock both in year 25. Panel (c): monetary shock in year 10; surplus adjusts in year 40. Blue dotted: consumption response (%). Black solid: debt response (percentage points of annual steady-state GDP). Shading indicates the debt footprint underlying the liquidity effect. Vertical dashed/dotted lines mark monetary/fiscal adjustment dates, respectively.

consolidation is known to occur: Fiscal policy matters for monetary transmission even though it remains passive in the sense of Leeper (1991).

3.2 Determination of the price level – general equilibrium

Amid the lingering-footprint channel, a key factor for monetary transmission is the initial response of inflation; compare, for example, (14). The current section endogenizes this response in general equilibrium. It then documents that the lingering footprint channel leads to equilibria that are reminiscent of those induced by active fiscal policy in the textbook model, even if, in reality, fiscal policy is passive. To simplify the exposition, and with little loss, for the remainder of this section we focus on the case $\rho = \bar{r}$ (that is, $\beta(1 + \bar{r}) = 1$, $\Gamma_b(\bar{b}) = 0$).[6]

3.2.1 Supply side and market clearing

To isolate the role of liquidity, we depart from the textbook case only for the IS equation. We keep the production side unchanged and close the model with a conventional New Keynesian Phillips curve,

$$\widehat{\pi}_t = \beta_\pi \, \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + \kappa \, \widehat{y}_t, \tag{15}$$

[6]In Appendix C, we generalize those results beyond the case when steady-state liquidity is at the bliss point, that is, for $\Gamma_b(\bar{b}) \neq 0$.

where $\beta_\pi \leq 1$ is the discount factor in the Phillips curve and $\kappa > 0$ its slope.[7] Goods-market clearing implies $\widehat{y}_t = \widehat{c}_t$.

3.2.2 Timing and policies

Studying the lingering-footprint channel analytically in general equilibrium while retaining analytical tractability requires that equilibria have a Markov structure. Toward this end, we move from perfect foresight to a probabilistic timing of consolidation. We will look at an immediate monetary tightening and consider two different fiscal policies: one active, one passive in the sense of Leeper (1991).

Monetary policy. In the current section, we exclusively focus on the case of an immediate rate hike, that is, $M = 1$ in the notation of Section 3.1. The central bank announces in $t = 0$ that it will adjust $\widehat{r}_1$ and keep $\widehat{r}_t = 0$ for all subsequent periods. Note, for future reference, that absent a fiscal response the initial fiscal footprint, that is, the footprint of monetary policy on end-of-period-1 debt is, therefore, $\mathbf{b}_{\pi,1} + \mathbf{b}_{r,1} = \bar{b}(1+\bar{r})[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0]$, recall Section 3.1.1.

Fiscal policy - an active version. As one fiscal scenario, fix taxes to their steady-state value, so that $\widehat{\tau}_t = 0$ in all periods. This policy clearly is active in the sense of the FTPL since the expected present value of primary surpluses does not offset the debt revaluation from period-0 inflation.

Fiscal policy - a passive version. As the other fiscal scenario, we consider the following passive fiscal policy. At $t = 0$, taxes remain at their steady-state level. Thereafter, in each period fiscal policy eliminates the monetary footprint with probability $\lambda \in (0, 1]$. Otherwise, it merely services the interest on the footprint while leaving the initial footprint itself in place. Therefore, debt remains bounded, regardless of initial inflation. More formally, let $F \geq 1$ denote the now-random consolidation date. Taxes adjust such that

$$\widehat{b}_t = \begin{cases} -\bar{b} \cdot (1+\bar{r}) \cdot \widehat{\pi}_0, & t = 0, \\ \bar{b} \cdot (1+\bar{r}) \cdot \left[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0\right], & t = 1, \ldots, F-1, \\ 0, & t \geq F. \end{cases} \tag{16}$$

We adopt this fiscal structure for tractability: The induced Markov structure for debt allows closed-form solutions. Note that $1/\lambda$ is the expected number of periods for which the fiscal footprint of monetary policy lingers (including the present period).[8]

[7] We introduce a separate parameter β_π to show how long-run (non-)neutrality affects the results below. A reader not interested in that dimension may just think of the textbook case, $\beta_\pi = \beta$.

[8] In Appendix C, we generalize the results beyond the assumption of a constant arrival rate λ for fiscal consolidation. The qualitative results are unaffected. Quantitatively, of course, the results depend not only on the expected time of consolidation but also on other features of its distribution.

3.2.3 No role for liquidity: the conventional benchmark

The key finding later is that if liquidity plays a role, FTPL-like equilibrium outcomes can arise even when fiscal policy is passive. The current section presents the point of reference for this: benchmark "FTPL-like equilibrium outcomes" absent a role for liquidity, that is, if $\chi = 0$.

Proposition 1 (No liquidity effects: FTPL vs. passive fiscal)**.** *Consider the model introduced in Section 2.1 (demand side and government budget) and Section 3.2.1 (supply side and market clearing). Next, consider the monetary and fiscal policies spelled out in Section 3.2.2. Assume that $\chi = 0$ so that there are no liquidity effects. Then the following statements characterize all the fundamental stable equilibria.*

(a) FTPL under the active fiscal policy:
On impact, in $t = 0$: $\widehat{\pi}_0 = \frac{\widehat{r}_1}{1+\bar{r}}$ and $\widehat{y}_0 = \left[\frac{1}{\kappa}(1-\beta_\pi) - \frac{\beta_\pi}{\sigma}(1+\bar{r})\right]\frac{\widehat{r}_1}{1+\bar{r}}$
Thereafter, for $t > 0$: $\widehat{\pi}_t = \left[1 + \frac{\kappa}{\sigma}(1+\bar{r})\right]\frac{\widehat{r}_1}{1+\bar{r}}$ and $\widehat{y}_t = \frac{1-\beta_\pi}{\kappa}\widehat{\pi}_t$.

(b) FTPL or no FTPL under the passive fiscal policy with $\lambda = 0$. If fiscal policy never fully consolidates, there is a continuum of equilibria indexed by a constant continuation inflation rate $\pi^A \in \mathbb{R}$.
On impact in $t = 0$: $\widehat{\pi}_0 = \pi^A - \frac{\kappa}{\sigma}\widehat{r}_1$, $\widehat{y}_0 = \frac{1-\beta_\pi}{\kappa}\pi^A - \frac{1}{\sigma}\widehat{r}_1$.
For all $t \geq 1$, $\widehat{\pi}_t = \pi^A$, $\widehat{y}_t = \frac{1-\beta_\pi}{\kappa}\pi^A$.

(c) No FTPL under the passive fiscal policy with $\lambda > 0$:
On impact, in $t = 0$: $\widehat{\pi}_0 = -\frac{\kappa}{\sigma}\widehat{r}_1$ and $\widehat{y}_0 = -\frac{1}{\sigma}\widehat{r}_1$. Thereafter, for $t > 0$, $\widehat{y}_t = \widehat{\pi}_t = 0$.
This is true for arbitrarily small positive values of λ.

Note that case (b) is not the limit of case (c) as $\lambda \to 0$.

Proof. For a proof of statements (a), (b), and (c), see Appendix A.2. That case (b) is not the limit of case (c) as $\lambda \to 0$ follows immediately from the fact that case (b) has infinitely many equilibria, each indexed by a different value π^A for inflation. In case (b), the equilibrium of case (c) notably arises only if $\pi^A = 0$. Instead, for example, if $\pi^A = \left[1 + \frac{\kappa}{\sigma}(1+\bar{r})\right]\frac{\widehat{r}_1}{1+\bar{r}}$, case (b)'s equilibrium resembles that of case (a). Either value of π^A is admissible. □

Henceforth, we refer to outcomes of the form (a) as "FTPL-like equilibrium outcomes." Under active fiscal policy, households and firms are certain that fiscal policy will *never* respond to the monetary footprint. On impact, inflation then rises exactly enough to offset the effect of the rate hike on the net present value of surpluses. With rigid prices, this inflation response is consistent with optimizing behavior of households and firms only if output moves in period 0, and if future inflation and output rise. Why do such persistent booms arise, however? The intuition is opaque. The more so since the effect

of the one-time real-rate change on debt is wiped out by the end of period $t = 1$. That is, an inflationary boom in output persists even though real rates and debt have returned to steady state. This is the key point of our paper: we show later that once liquidity is taken seriously, "FTPL-like" equilibria arise even under *passive* fiscal policy. And with a role for liquidity, the channels that induce such equilibria are fully transparent.

Under the passive fiscal policy debt never explodes by design, even if it never is fully consolidated ($\lambda = 0$). This knife-edge case, case (b), gives rise to multiplicity. In that case, the debt level is constant from period 1 onward, as are output and inflation, with the level of all three determined by the long-term inflation response π^A.

Case (c), instead, shows the conventional benchmark with passive fiscal policy in which eventual consolidation is certain ($\lambda > 0$). Then, a monetary tightening affects output and inflation only in period $t = 0$, through the usual intertemporal substitution effects; see item (c) in Proposition 1. In that case, the timing of consolidation is entirely irrelevant. We will show next that this is a knife-edge outcome, too; an outcome that will not arise once liquidity is taken into account.

3.2.4 Liquidity matters ($\chi > 0$): a passive-fiscal theory of the price level

We now focus on the case in which liquidity matters. It is obvious that FTPL-like outcomes can arise here, too, if fiscal policy were to be active. This is not the focus of the current section. Instead, the section derives the equilibrium outcomes if fiscal policy is passive and is expected to consolidate in finite time ($\lambda > 0$). It then shows that FTPL-like outcomes arise regardless.

Equilibrium outcomes. It is conceptually straightforward to derive the fundamental equilibrium outcomes. The key observation is that, prior to consolidation but after the monetary impulse—that is, for all periods t with $1 \leq t < F$—the state of the economy is the same. Therefore, in such periods t

$$\mathbb{E}_t\{\widehat{\pi}_{t+1}\} = (1-\lambda)\widehat{\pi}_t, \quad \mathbb{E}_t\{\widehat{y}_{t+1}\} = (1-\lambda)\widehat{y}_t. \tag{17}$$

The same holds true for debt, such that for all such periods t the expected sum of liquidity services from debt in Euler equation (8) is given by

$$\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\frac{\chi}{\bar{b}(1+\bar{r})}\widehat{b}_{t+s}\right\} = \sum_{s=0}^{\infty}(1-\lambda)^s\frac{\chi}{\bar{b}(1+\bar{r})}\bar{b}(1+\bar{r})[\widehat{r}_1-(1+\bar{r})\widehat{\pi}_0] = \frac{\chi}{\lambda}[\widehat{r}_1-(1+\bar{r})\widehat{\pi}_0].$$

Equation (8), together with (17), thus, directly implies that $\widehat{y}_t = \frac{1}{\sigma}\frac{\chi}{\lambda}[\widehat{r}_1-(1+\bar{r})\widehat{\pi}_0]$ for all periods t with $1 \leq t < F$. One can then make use of $\widehat{y}_t = \frac{1-\beta_\pi(1-\lambda)}{\kappa}\widehat{\pi}_t$, and solve the Euler equation and Phillips curve for outcomes in period $t = 0$. Defining $\Lambda := \lambda[1-\beta_\pi(1-\lambda)]$,

inflation and output in the unique stable equilibrium are given by

$$\widehat{\pi}_t = \begin{cases} -\frac{\kappa}{\sigma}\frac{(1+\chi)\Lambda-\chi}{\left(1-\frac{\kappa}{\sigma}\chi\bar{r}\right)\Lambda+\frac{\kappa}{\sigma}\chi(1+\bar{r})}\,\widehat{r}_1, & t=0, \\ \frac{\kappa}{\sigma}\frac{\chi}{\Lambda}[\widehat{r}_1-(1+\bar{r})\widehat{\pi}_0], & 1\le t<F, \\ 0, & t\ge F. \end{cases} \tag{18}$$

and

$$\widehat{y}_t = \begin{cases} -\frac{1}{\sigma}\frac{\lambda-(1-\lambda)\chi}{\lambda}\widehat{r}_1-\frac{1}{\sigma}\chi\frac{1+\bar{r}-\lambda\bar{r}}{\lambda}\widehat{\pi}_0, & t=0, \\ \frac{1}{\sigma}\frac{\chi}{\lambda}[\widehat{r}_1-(1+\bar{r})\widehat{\pi}_0], & 1\le t<F, \\ 0, & t\ge F. \end{cases} \tag{19}$$

With no role for liquidity (if $\chi = 0$) and with $\lambda > 0$, this simplifies to the textbook result under passive fiscal policy, case (c) of Proposition 1. In that case, even if λ is tiny, the equilibrium coincides with the conventional passive benchmark. Instead, with a role for liquidity ($\chi > 0$), these formulas provide a novel perspective on the fiscal theory of the price level, as we will discuss next.

Delaying fiscal adjustment *ad infinitum*. We first study the limit behavior of our economy as fiscal adjustment following a monetary shock is delayed further, while fiscal policy still remains passive. Suppose that consolidation is put off longer and longer. Recall that under the policy specified, from period $t = 1$ onward the level of debt is maintained constant until fiscal policy fully consolidates. Suppose that this level of debt deviates from its steady-state level. Then the cumulative value of liquidity services from this change in debt would also diverge to infinity. In turn, consumption would no longer be stable; recall (9). Therefore, the only equilibrium can be one where an initial jump in the price level renders the initial monetary footprint on debt nil. That is, as $\lambda \to 0$ $\widehat{\pi}_0$ moves so that $\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0 \to 0$ and, thus, $\widehat{b}_t \to 0$ for all $t \ge 1$. This gives the same FTPL-like inflation outcomes as in Proposition 1 case (a). More formally, we have

Proposition 2 (Passive-Fiscal theory of the price level)**.** *Consider the same conditions as in Proposition 1, in particular a single monetary policy shock $\widehat{r}_1$ announced at $t = 0$, but now assume $\chi > 0$, strictly. Focus on the case that $\beta(1+\bar{r}) = 1$. Let F mark the random time of fiscal consolidation. Focus on the passive fiscal policy. As the probability of fiscal consolidation $\lambda \to 0$, we have that*

(a) $\widehat{\pi}_0 \to \frac{\widehat{r}_1}{1+\bar{r}}$, that is, the initial jump in the price level exactly offsets, in present value terms, the effect of the interest rate change on the government's budget constraint. This result is independent of the strength of the liquidity effects, as long as the channel is active at all.

(b) For any period t with $1 \le t < F$, $\widehat{\pi}_t \to \left[1+\frac{\kappa}{\sigma}(1+\bar{r})+\frac{\kappa}{\sigma}\chi\right]\frac{\widehat{r}_1}{1+\bar{r}}$. Relative to case (a) of Proposition 1 the effects arising from liquidity amplify inflation after a monetary

tightening further as the rise in liquidity generates additional demand.

(c) *On impact,* $\widehat{y}_0 \to \left[\frac{1}{\kappa}(1-\beta_\pi) - \frac{\beta_\pi}{\sigma}(1+\bar{r}+\chi)\right]\frac{\widehat{r}_1}{1+\bar{r}}$. *For every* $\lambda > 0$, *output before consolidation satisfies the exact identity* $\widehat{y}_t = \frac{1-\beta_\pi(1-\lambda)}{\kappa}\widehat{\pi}_t$ *for* $1 \leq t < F$. *Output responses thus depend on liquidity, too.*

Proof. See Appendix A.3. □

That is, an equilibrium with the same FTPL-like properties of Proposition 1 case (a) emerges here if consolidation is postponed long enough.[9] However, while the outcomes resemble those under active fiscal policy, the mechanism is fundamentally different.

A natural-rate interpretation of FTPL-like outcomes. FTPL predicts that a rate hike is inflationary only under active fiscal policy. This result is typically interpreted through government solvency: only a commensurate increase in the price level can place government debt on a stable path. Our framework offers a different, and arguably more intuitive, interpretation through the natural rate. When fiscal adjustment is delayed indefinitely, a central-bank rate hike raises the natural rate through monetary policy's fiscal footprint on debt. This creates an expansionary natural-rate gap in all subsequent periods; recall equations (4) and (5). This stimulates consumption demand. The initial price level jumps so that output and consumption in period 0 remain bounded. Importantly, we obtain FTPL-like outcomes even though the government collects enough taxes, in present-value terms, to service any change in debt caused by monetary policy. We also emphasize that the proof of Proposition 2 only requires some role for liquidity ($\chi > 0$), however small. For FTPL, equilibria are discontinuous at $\lambda = 0$; the Passive-Fiscal TPL mechanism described above does not inherit this discontinuity.

3.2.5 Liquidity matters ($\chi > 0$): reversals despite finite-time consolidation

This section provides even broader conditions for an FTPL-like response of inflation or output; namely, it documents reversals with finite consolidation horizons. We start with FTPL-like inflation reversals. The following proposition shows that for any parameter configuration, there is a finite threshold for the expected duration of the fiscal footprint beyond which a monetary tightening becomes altogether inflationary.

Proposition 3 (Threshold for impact inflation to reverse sign)**.** *Consider the same conditions as in Proposition 2, but now let the probability of fiscal policy consolidating in any given period,* λ, *remain fixed and strictly positive. Continue to focus on the passive fiscal policy. Then, for* $t = 0$, *for any value of* $\chi > 0$, *there exists a unique threshold*

[9] The exact equivalence prevails in the limit as the probability of consolidation $\lambda \to 0$, and subsequently we let χ tend to zero. Technically, we require that $\chi, \lambda \to 0$ and $\chi/\Lambda \to \infty$ to obtain the exact FTPL outcomes of Proposition 1 case (a) when liquidity matters.

$\lambda^*(\chi) \in (0,1)$, *such that a rate hike is inflationary iff* $\lambda \in (0, \lambda^*(\chi))$. *The threshold* $\lambda^*(\chi)$ *solves*

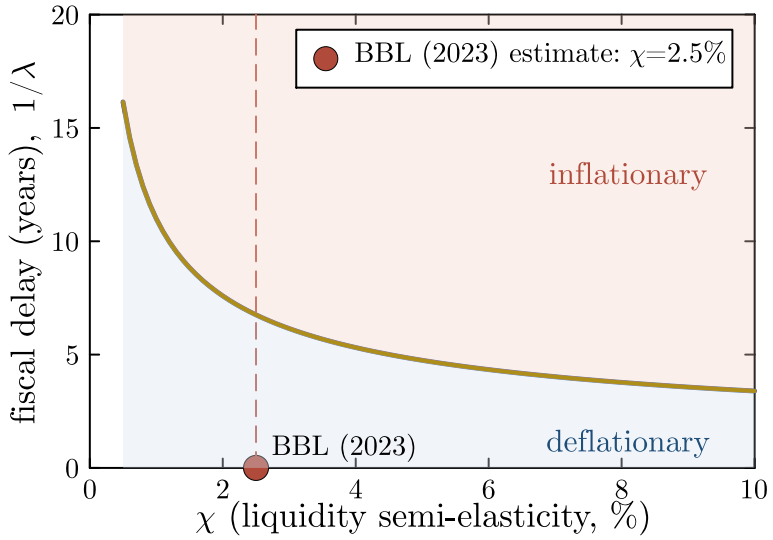
$$\Lambda(\lambda^*(\chi)) := \lambda^*(\chi) \cdot [1 - \beta_\pi(1 - \lambda^*(\chi))] = \frac{\chi}{1+\chi},$$

and is strictly increasing in χ. *For all periods* $t > 0$ *but before fiscal adjustment* ($t < F$), *a rate hike always is both expansionary and inflationary:* $\widehat{y}_t > 0$ *and* $\widehat{\pi}_t > 0$.

Proof. See Appendix A.4. □

The probability of fiscal adjustment λ determines the expected duration of the debt deviation, while the liquidity semi-elasticity χ determines how a debt deviation changes the natural rate at a given date. This explains why, for larger values of χ, the inflation response reverses sign at shorter delays. Figure 2 provides a graphical illustration. The

Figure 2: Critical fiscal delay for inflationary rate hikes



Notes: Critical expected delay of consolidation (in years), $1/\lambda^*$, above which a rate hike is inflationary at $t = 0$, as a function of χ (x-axis truncated at 10%; y-axis at 20 years), solid line. Parameter combinations above the curve come with inflationary rate hikes. The vertical dashed line extending from the red dot on the x-axis marks the estimate of χ from Bayer et al. (2023). The graph is truncated in the upper-left corner for readability. Figure 1 provides the parameter values; $\beta_\pi = \beta$.

solid line plots the critical expected adjustment time, $1/\lambda^*(\chi)$, beyond which a rate hike is inflationary (y-axis) against the liquidity semi-elasticity χ (x-axis). The graph assumes $\beta_\pi = \beta$ and keeps all other parameters as in Figure 1. By way of empirical guidance, the red marker and associated vertical dashed line show an estimate for χ, from Bayer et al. (2023) ($\chi = 2.5\%$ annually).[10] At that value, a rate hike becomes inflationary throughout

[10]Bayer et al. (2023) provide one estimate. Laubach (2009) estimates that a one percentage point increase in the debt-to-GDP ratio raises the long-term interest rate by 3-4 bps. At a debt/GDP ratio of 100%, this would imply a higher χ. Krishnamurthy and Vissing-Jorgensen (2012) argue that an increase of government debt by 19 percent of GDP lowers the yield spread by 0.44 pp., so that a 100%-of-GDP increase in debt reduces the convenience yield by roughly 2.5 percentage points.

if the monetary footprint on debt is expected to linger for roughly 6.75 years or more (values of $1/\lambda$ above the curve).

Output reversals with finite consolidation horizons. Even when a rate hike is inflationary on impact, however, it need not be expansionary, too. We have already established this result for the limit of an infinite delay. Indeed, item (c) of Proposition 2 suggests that more rigid prices (a smaller value of κ) make a rate hike more expansionary, while a higher fiscal cost of debt ($\bar{r} + \chi$) does the opposite. If the Phillips curve were vertical, only the latter effect would prevail. The next proposition characterizes the combination of parameters for which a rate hike is altogether expansionary.

Proposition 4 (Threshold for output impact response to reverse sign)**.** *Consider the same assumptions as in Proposition 3, passive fiscal policy in particular. Consider parameters $\sigma > 0$, $\chi > 0$, $\lambda \in (0,1)$, and $\beta_\pi \in (0,1]$, with $\bar{r}$ the steady-state real rate of interest. Then upon a monetary rate hike, $\widehat{r}_1 > 0$, output in period 0 rises iff the slope of the Phillips curve, κ, satisfies*

$$\kappa < \kappa^*(\lambda, \chi, \bar{r}, \sigma, \beta_\pi),$$

where

$$\kappa^*(\lambda, \chi, \bar{r}, \sigma, \beta_\pi) := \sigma \frac{[1 - \beta_\pi(1-\lambda)]\left[\chi - \frac{\lambda}{1-\lambda}\right]}{\beta_\pi \chi (1 + \bar{r} + \chi)}. \tag{20}$$

A positive value of κ satisfying this condition exists whenever $1/\lambda > 1 + 1/\chi$.

Proof. See Appendix A.5. □

The role of κ in Proposition 4 is best understood through the fiscal footprint. The logic proceeds in two steps. First note that under the conditions of Proposition 4, $\lambda < 1$, so the fiscal footprint is expected to linger. A rate hike, thus, unambiguously raises expected inflation; recall Proposition 3. In addition, a rate hike that raises output on impact raises current marginal costs. Such a rate hike, thus, must come with an immediate inflationary impact; compare the Phillips curve, (15). More formally, note that the condition in the last sentence of Proposition 4 implies $\Lambda(\lambda) < \frac{\chi}{1+\chi}$, precisely the condition Proposition 3 provides for an inflationary impact of a rate hike.

Second, it is important to recall that initial inflation reduces the fiscal footprint and hence liquidity. To see the role of this for initial output, combine the dynamic IS equation (4) with (5) and the debt response, (16), focusing on period 0:

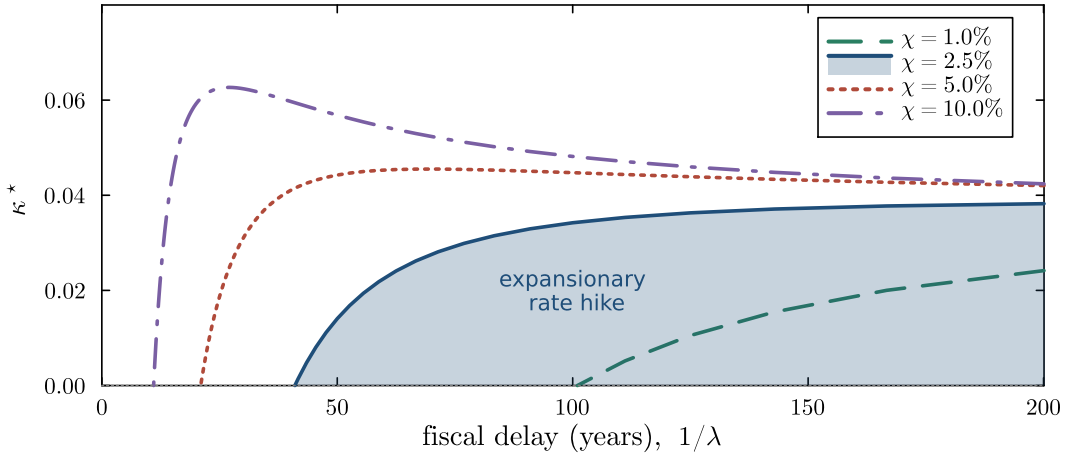
$$\widehat{y}_0 = \mathbb{E}_0\{\widehat{y}_1\} - \frac{1}{\sigma}\left[\widehat{r}_1 + \chi \widehat{\pi}_0\right].$$

Period-0 output depends negatively on period-0 inflation for two reasons. Directly, higher inflation reduces the fiscal footprint in period 0, as captured by the term in square brackets.

Indirectly, it also reduces the future fiscal footprint, dampening the rise in expected future output.

In sum, the response of initial output may reverse after a monetary tightening, provided that initial inflation rises, but not by too much–this requires the Phillips curve to be sufficiently flat (a small-enough value of κ).

Figure 3: Visualizing Proposition 4: critical slope κ^*



Notes: The figure plots values of $\kappa^*(\lambda, \chi, \bar{r}, \sigma, \beta_\pi)$ defined in (20) against the expected time until adjustment $1/\lambda$. Each curve corresponds to a different value of parameter χ. For a value of $\chi = 2.5\%$, the shaded area shows combinations of κ and $1/\lambda$ for which the rate is expansionary. The remaining parameters' values are given in the notes to Figure 1 and Figure 2.

Figure 3 visualizes the cutoff for κ in Proposition 4. Each line corresponds to a different value of χ. Curves for higher χ lie above those for lower χ, reflecting a stronger role for liquidity. The x-axis traces different expected times until adjustment $1/\lambda$. A rate hike is expansionary if the point $(1/\lambda, \kappa)$ lies below the relevant curve. Each curve starts at its critical threshold $1/\lambda = 1 + 1/\chi$, before which a rate rise will never be expansionary. To interpret the magnitudes, recall that the model is calibrated at an annual frequency. Values of κ and λ are, thus, the slope of the Phillips curve at annual frequency and the probability of fiscal adjustment per year. The figure suggests that output reversals on impact in practice might be rare. Namely, for the value of $\chi = 2.5\%$ that Bayer et al. (2023) propose, fiscal consolidation would need to be expected to be delayed by more than 41 years (intercept of the solid line with the x-axis). And even with a delay of 100 years ($\lambda = 0.01$), for an output reversal to occur, the slope of the Phillips curve must not exceed $\kappa = 0.038$ at annual frequency, a small value in light of the literature.

4 Empirical evidence: the fiscal footprint and monetary transmission

Before moving on to the quantitative version of the model that micro-founds the liquidity services, we provide empirical evidence. The evidence assesses if the timing of consolidating the fiscal footprint matters for monetary transmission. We do so in two sets of exercises. First, we estimate the debt response to a monetary policy shock over time using rolling windows. We then estimate local projections interacting the monetary shock with this debt-response measure and evaluate the output and price-level responses at values one standard deviation above and below the response's mean. Second, we perform a counterfactual exercise using the methodology of McKay and Wolf (2023). Here, we use a mix of identified fiscal shocks for expenditures and taxation to compare the average response to a monetary policy shock to a counterfactual response under rapid elimination of the fiscal footprint. In both exercises, we find that a monetary tightening is more contractionary when the fiscal footprint is eliminated quickly. Below, we sketch the two exercises. Appendix D provides details, among others on the data sources and specifications.

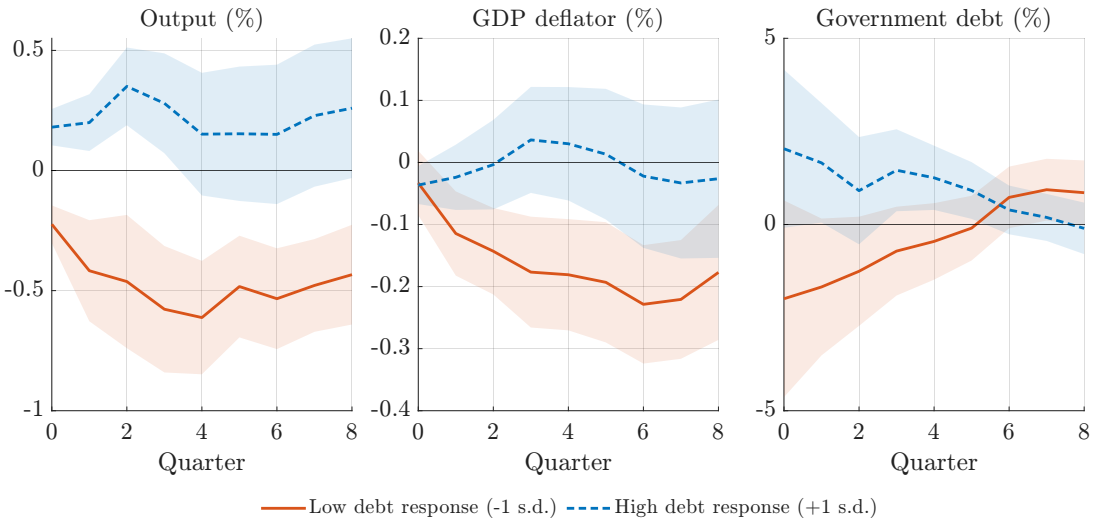
In both sets of exercises, we take the monetary-policy shock identified from the high-frequency, orthogonalized policy surprises of Bauer and Swanson (2022) (sample 1988Q1–2019Q4), and we present it as a monetary tightening that raises the policy rate by about 13 basis points on impact. For the first exercise, we start by estimating the contemporaneous response of government debt $\widehat{gdr}_t$ (short for government debt response) to the monetary policy shock based on a centered 21-quarter window around time t (controlling for lagged real output and lagged price level). Thereafter, we run local projections of output and the price level on the monetary policy shock to estimate state-dependent impulse-response functions as

$$x_{t+h} = \alpha_h + \beta_{0,h}\Delta i_t + \beta_{1,h}\Delta i_t \times \widehat{gdr}_t + \beta_{2,h}\widehat{gdr}_t + \gamma X_t + \epsilon_{t,h}, \quad h = 0 \dots H, \tag{21}$$

where x_{t+h} is output or prices h periods ahead, Δi_t is the monetary shock and X_t contains lags of output and the price level as controls. We standardize $\widehat{gdr}_t$ so that we can interpret $\beta_{0,h} \pm \beta_{1,h}$ as the impulse response at a ± 1 standard deviation higher/lower than average contemporaneous debt response.

Figure 4 shows the results of this exercise. The blue dashed lines and corresponding confidence bands (top in each panel) concern impulse responses when the increase in debt after a monetary tightening is 1 standard deviation larger than average. In this case, government debt expands persistently after a monetary tightening (right panel). The red solid lines and corresponding confidence bands show the case when the debt response is 1 standard deviation weaker than average and when debt falls after the monetary tightening. When government debt expands in response to a monetary tightening, prices move little

Figure 4: State-dependent response to a monetary tightening



Notes: Local-projection responses of GDP, the GDP deflator, and public debt to a monetary tightening as identified by Bauer and Swanson (2022). The monetary policy shock is interacted with its rolling window impact estimate on public debt. The blue dashed line displays the effect under a high debt response ($\widehat{gdr}_t = 1$), $\beta_{0,h} + \beta_{1,h}$; the red solid line reflects the effect under a low debt response ($\widehat{gdr}_t = -1$), $\beta_{0,h} - \beta_{1,h}$. US GDP and GDP deflator data 1988:Q1–2019:Q4. Debt: 1983:Q1–2024:Q4. Shaded areas correspond to 66% confidence bounds. The centered window measures the local fiscal environment. The confidence bands condition on the estimated rolling debt-response measure.

in response to the monetary policy shocks. Only when government debt contracts do we see a significant decline in economic activity (the monetary shock leading to a contraction of activity of roughly half a percent) and a fall in prices (by 0.2 percent at the trough).

This initial evidence is suggestive of effects consistent with the mechanism highlighted in the model. We consider an alternative exercise that is more exacting regarding the state-dependence of the debt response but requires additional assumptions. In this, we first estimate the unconditional impulse responses of the variables of interest to monetary and fiscal shocks; namely, once more the Bauer and Swanson (2022) monetary policy shocks (sample: 1988:Q1–2019:Q4), shocks reflecting unanticipated tax changes (Mertens and Ravn, 2011), or exogenously legislated tax changes (e.g., due to sunset clauses; Romer and Romer, 2010) (sample: 1950Q2–2007Q4), and military spending shocks (Ramey and Zubairy, 2018) (sample: 1947Q1–2015Q4). Appendix D reports the underlying impulse responses, which we then combine to obtain a counterfactual impulse response in the sense of McKay and Wolf (2023).

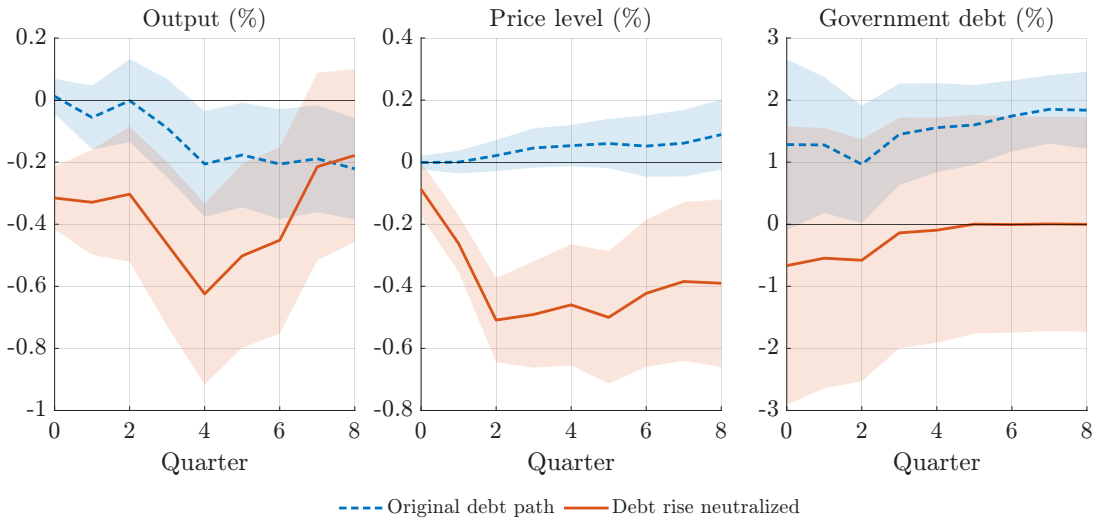
More in detail, let $y^m(h), p^m(h)$ and $b^m(h)$ denote the output, price level, and government-debt responses to the monetary tightening at horizon h, and let $y^j(h), p^j(h)$ and $b^j(h)$ be the corresponding responses to fiscal shock $j = 1, 2, 3$. We then search for the fiscal shock combination $\boldsymbol{\alpha}^* = (\alpha_1^*, \alpha_2^*, \alpha_3^*)$ that, combined with the response to the monetary shock, minimizes the squared deviations of debt from zero in year two after the monetary

policy shock:[11]

$$\boldsymbol{\alpha}^* = \arg\min_{\boldsymbol{\alpha}} \sum_{h=5}^{8} \left[b^m(h) + \sum_{j=1}^{3} \alpha_j \, b^j(h) \right]^2. \tag{22}$$

Figure 5 shows the results of this second exercise.

Figure 5: Monetary transmission under alternative government-debt paths



Notes: The blue dashed lines report the responses of output, the price level, and government debt to a one-standard-deviation monetary tightening using monetary shocks identified by Bauer and Swanson (2022). The red solid line reports counterfactuals constructed from local-projection impulse responses following the methodology of McKay and Wolf (2023). The counterfactual fiscal response to the monetary shock is constructed using Mertens and Ravn (2011) and Romer and Romer (2010) tax shocks, and Ramey and Zubairy (2018) expenditure shocks. The fiscal instruments are rescaled by the vector $\boldsymbol{\alpha}$ in (22) in such a way that the combined government-debt response tracks the target path $b^{\text{counterfactual}}(h) = 0$ in year two after the monetary policy shock ($h = 5, \ldots, 8$). Shaded areas correspond to the respective 66% confidence bounds. All responses are in percent.

The impulse responses to the monetary shock under the baseline fiscal policy mix are shown as blue dashed line (with corresponding confidence bands). A monetary contraction increases government debt persistently (right panel). In line with the theory presented earlier, the price-level response is small and, if anything, positive (center panel), while output falls to some extent (blue dashed line left panel). The solid red line shows responses to the monetary shock under a counterfactual fiscal policy mix that approximately neutralizes its effect on public debt in year two after the shock. The policy interpretation maintains the private-sector relationships and information structure underlying the component impulse responses. With earlier fiscal consolidation, the price level falls sharply—by about

[11] The optimal weights are $\boldsymbol{\alpha}^* = (0.61, -0.84, -1.41)$ for the Mertens and Ravn (2011), Romer and Romer (2010), and Ramey and Zubairy (2018) shocks, respectively. That is, consolidation is brought about by a mixture of unanticipated tax increases (Mertens and Ravn, 2011) and tax cuts (Romer and Romer, 2010) together with less public spending (Ramey and Zubairy, 2018). If we force the debt response to be zero in all periods, the impulse responses for output and inflation are not affected qualitatively but the uncertainty band for debt under the counterfactual policy increases significantly.

0.4 percent—and output contracts much more than under the baseline fiscal policy. At its trough, the output decline is almost three times as large. These results suggest that timely fiscal consolidation underpins the textbook effects of monetary tightening: lower inflation accompanied by a real contraction.

5 Quantitative model

The stylized model has highlighted why the timing of fiscal policy responses matters for monetary transmission, even if taxes are lump-sum and fiscal policy is passive. The data appear consistent with this theory. The key mechanism is that liquidity services from public debt shape the natural rate that enters the aggregate IS equation. The stylized model leaves the source of liquidity services unspecified; they may reflect the liquidity needs of firms, banks, or households. The current section takes a stand on the source of liquidity needs and models them as household demand for self insurance. It then provides a quantitative assessment of how fiscal delays affect monetary transmission in this more specific model of liquidity demand. That is, we build an incomplete-markets life-cycle model that captures liquidity demand in a micro-founded way and ask whether the timing and quantitative logic established earlier carries over to a heterogeneous-agent environment with borrowing constraints. Appendix E provides further exercises within the framework of the model.

5.1 Model setup

The model features the usual New Keynesian core with nominal price rigidities and real wage rigidity. On the household side, there are overlapping generations of households that face idiosyncratic income risk which they insure against through saving in nominal bonds. We study perfect-foresight transitions following an unanticipated aggregate shock. Lowercase letters denote variables linked to individual agents, uppercase letters denote variables at the aggregate level. To the extent possible, we maintain the same assumptions as in Sections 2 and 3.

Households: There is a constant population of overlapping generations of households, each living for 60 periods. The first period of life corresponds to the age of 25. People work for 40 years and spend 20 years in retirement. Without loss of generality, we normalize the mass of households to unity. When working, a household's income at time t is given by

$$y_{it} = l_{it} z_{it} Y_t. \tag{23}$$

Here, l_{it} is a deterministic age-dependent component, and z_{it} is idiosyncratic productivity risk. Y_t is aggregate income. We assume that $\log z_{it}$ follows an AR(1) process, $\log z_{it} = \varrho_z \log z_{it-1} + \epsilon_{it}$, where ϵ_{it} is white noise with standard deviation σ_ϵ, and $\varrho_z \in [0, 1)$. In retirement, income is

$$y_{it} = f(z_i) Y_t, \tag{24}$$

where z_i marks the productivity that household i had at the end of its working life and $f(\cdot)$ is a potentially nonlinear function. Note that, once households retire, they no longer are subject to idiosyncratic risk. The profiles $l_{i,t}$ and z_{it} are normalized such that $\int y_{it} di = Y_t$.

Households have access to the bond market and can save in short-term bonds, but cannot borrow. Households born at time t maximize their expected lifetime utility subject to budget constraints and borrowing constraints:

$$\begin{aligned} &\max_{\{c_{it+h}, b_{it+h}\}_{h=0}^{59}} \mathbb{E}_t \left\{ \sum_{h=0}^{59} \beta^h \frac{c_{it+h}^{1-\sigma}}{1-\sigma} \right\} \\ s.t. \quad & c_{it+h} + b_{it+h} + \tau_{it+h} = y_{it+h} + (1 + r_{t+h}) b_{it+h-1}, \quad h = 0, 1, 2, ..., 59 \\ & b_{it+h} \geq 0, \quad h = 0, 1, 2, ..., 59. \end{aligned} \tag{25}$$

Here β is the subjective discount factor and $1/\sigma$ is the intertemporal elasticity of substitution. Apart from the borrowing constraint, the limited time horizon, and the fact that we no longer introduce liquidity needs by fiat, the household choice problem and budget are in substance identical to the planning problem (1) that underpinned Sections 2 and 3. In the baseline model, newborn households start with zero wealth and there are no bequests. Note that there is also no uncertainty about the time of death so that we do not need to consider the "accidental" bequests that lead to the Blanchard-Yaari interest gap between what the government effectively pays and what households receive.

Labor Supply: Because our focus is on demand transmission, we use a reduced-form labor block. Letting N_t mark hours per worker, following Alves et al. (2020) all workers supply the same number of hours $n_{it} = N_t$ and the aggregate labor supply curve is given by

$$W_t = \overline{W} \left(\frac{N_t}{\overline{N}} \right)^{\epsilon_w}. \tag{26}$$

Here W_t is the real wage. If $\epsilon_w = 0$, wages are perfectly rigid, and employment is determined only by labor demand. If $\epsilon_w > 0$, there is pressure on wages whenever employment deviates from its steady-state level. Wages determine marginal costs; aggregate income is distributed according to (23) and (24).

Firms: A competitive final-good producer aggregates intermediate inputs, each indexed by $j \in [0,1]$, according to $Y_t = \left(\int_0^1 y_{j,t}^{1/\mu} dj\right)^{\mu}$, where $\mu/(\mu-1) > 1$ is the elasticity of substitution. Intermediate firms are monopolistic competitors with linear technology $y_{j,t} = A l_{j,t}$, where $l_{j,t}$ is labor hired by firm j and $A > 0$. By symmetry, aggregate labor demand is $L_t = l_{j,t}$. Prices are set under Rotemberg adjustment costs (Rotemberg, 1982) of the form

$$\Theta_t(p_{j,t}, p_{j,t-1}) = \frac{\mu}{\mu-1}\frac{1}{2\kappa}[\log(p_{j,t}/p_{j,t-1})]^2 Y_t \tag{27}$$

where $\kappa > 0$. Intermediate firms receive a constant proportional sales subsidy s, implying a steady-state gross markup of $\bar{\mu} = \mu/(1+s)$. The resulting Phillips curve[12] (linearized around the zero-inflation steady state) is

$$\widehat{\pi}_t = \beta_\pi \widehat{\pi}_{t+1} + \kappa\left(\frac{W_t}{A} - \frac{1}{\bar{\mu}}\right). \tag{28}$$

This concludes the economy's supply side.

Fiscal Policy: To stabilize the debt, the government collects lump-sum taxes. Taxes τ_{it} are independent of household type i, and their real value is given by τ_t. The tax rule will be specified later. The government budget constraint is

$$B_t + \tau_t = (1+r_t)B_{t-1}, \tag{29}$$

where B_t is the real value of total debt outstanding.

Monetary Policy: Through its choice of the nominal policy rate i_t, the central bank effectively controls real rates from $t = 1$ onward. Ex-post real returns satisfy the Fisher relation

$$1 + r_t = \frac{1+i_{t-1}}{1+\pi_t},$$

where r_t is the net real rate.

Equilibrium Definition: Given real-rate and tax paths, $\{r_t\}_{t=1}^{\infty}$ and $\{\tau_t\}_{t=0}^{\infty}$, and the initial state, an equilibrium consists of aggregate paths $\{Y_t, W_t, N_t, L_t, i_t, \pi_t, B_t\}$ for all periods and household choices $\{c_{it}, b_{it}\}$ for all periods and households such that:

(i) individual optimization: given initial states and aggregate paths $\{r_t, \tau_t, Y_t\}$, households choose $\{c_{it}, b_{it}\}$ so as to maximize utility subject to budget and borrowing

[12] Adjustment costs are zero up to first order. More generally, we treat adjustment costs as virtual: they affect pricing decisions but do not absorb real resources.

constraints; given $\{Y_t, W_t\}$, firms choose $\{l_{j,t}, p_{j,t}\}$ to maximize profits under price adjustment costs;

(ii) the Phillips curve, the labor supply schedule, the government budget constraint, and the Fisher equation hold for all t;

(iii) aggregation and market-clearing: for $t = 0, 1, \ldots$, the labor, goods, and bond markets clear:

$$N_t = L_t, \tag{30}$$

$$C_t = Y_t, \ \textit{where } C_t = \int c_{it} di; \tag{31}$$

$$B_t^d = B_t, \ \textit{where } B_t^d = \int b_{it} di. \tag{32}$$

5.2 Calibration

Table 1: Parameters

Parameter	Value	Description/Target
σ	2.0	Intertemporal elasticity $1/\sigma = 0.5$
$\bar{r}$	2%	Steady-state net real interest rate.
$\overline{B}$	1.4	Steady-state bond supply target (McKay et al., 2016).
β	0.92	Chosen to match $\overline{B} = 1.4$
ϱ_z	0.91	Persistence of idiosyncratic earnings (Panel Study of Income Dynamics, PSID).
σ_ϵ	0.21	Std. dev. of idiosyncratic earnings shocks (PSID)
A	1	Normalizes output to unity for given employment.
$\overline{N}$	1	Normalizes employment to unity in steady state.
κ	0.23	Auclert et al., 2024
ϵ_w	0.5	Christiano et al., 2016
β_π	0.98	$\beta_\pi = 1/(1+\bar{r})$
$\bar{\mu}$	1	Zero steady-state markup

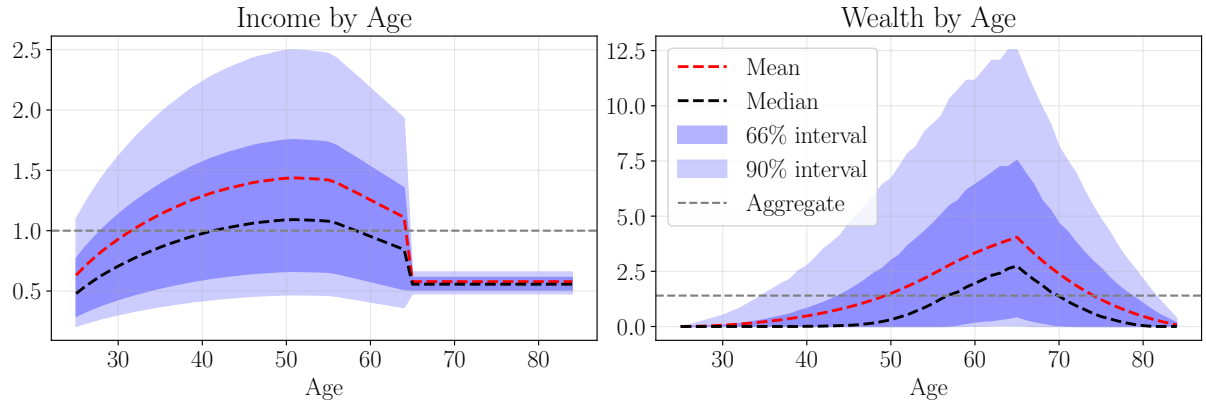
Notes: Aggregate output is normalized to 1 in steady state. The deterministic life-cycle component l_{it} follows Guvenen et al. (2021). The retirement-income schedule $f(z_i)$ follows Guvenen and Smith (2014).

Table 1 summarizes all calibrated parameters. The model is annual. We set σ to 2 and the steady-state net real interest rate to $\bar{r} = 2\%$. By way of normalization, we assume that aggregate output $\overline{Y}$ and employment $\overline{N}$ in steady state are equal to unity, which implies $A = 1$, too. The discount factor β is chosen to hit a target for bond supply in steady state, which is set to $\overline{B} = 1.4$ following McKay et al. (2016). This

gives $\beta = 0.92$. We set $\beta_\pi = 1/(1+\bar{r})$. The deterministic age-dependent income process, l_{it}, is chosen to fit average yearly earnings by age, estimated by Guvenen et al. (2021). The idiosyncratic income parameters $\varrho_z = 0.91$ and $\sigma_\epsilon = 0.21$ are chosen to match the persistence and standard deviation of yearly earnings in the PSID after removing the life-cycle components. We follow Guvenen and Smith (2014) for the shape of the non-linear function $f(z_i)$ that governs income in retirement. The slope of the Phillips curve κ has a value of 0.23, in line with Auclert et al. (2024). We set the elasticity of wages to employment to a value of $\epsilon_w = 0.5$, consistent with the VAR evidence in Christiano et al. (2016). Together, the values of κ and ϵ_w imply a slope of the Phillips curve with respect to output of $\kappa_y = 0.12$. We set $s = \mu - 1$ and $\bar{\mu} = 1$ such that the steady-state net markup is zero, which is without consequence since we fix how aggregate income is distributed.

Figure 6 shows the age-dependent income profile used for calibration and the endogenous wealth profile by age. Households face income risk with a life-cycle profile when of working age. Pension income is less dispersed (left panel). Households accumulate wealth until they retire and then, since the age at death is known with certainty, draw down their wealth until it is exhausted by the time they die.

Figure 6: Income and wealth profiles by age



Notes: Income (left panel) and wealth (right panel) distributions by age (x-axis). y-axes: multiples of average income (both panels). Right panel: average liquid wealth is 1.4 times average income. A dashed line marks average income and wealth, respectively.

5.3 Quantitative results

In this model environment, we mirror the timing experiment in Section 2 and Section 3. Concretely, we assume monetary policy raises the real rate of interest $\widehat{r}_1$ by 25 basis points. For realism, the real rate follows an AR(1) process with persistence 0.5. For fiscal policy, instead of a stochastic or time-based consolidation at time $t = F$ as in Section 3, we opt for a gradual fiscal consolidation with a debt-decay rate formulated such that it matches the expected decay rate under stochastic consolidation with a constant arrival

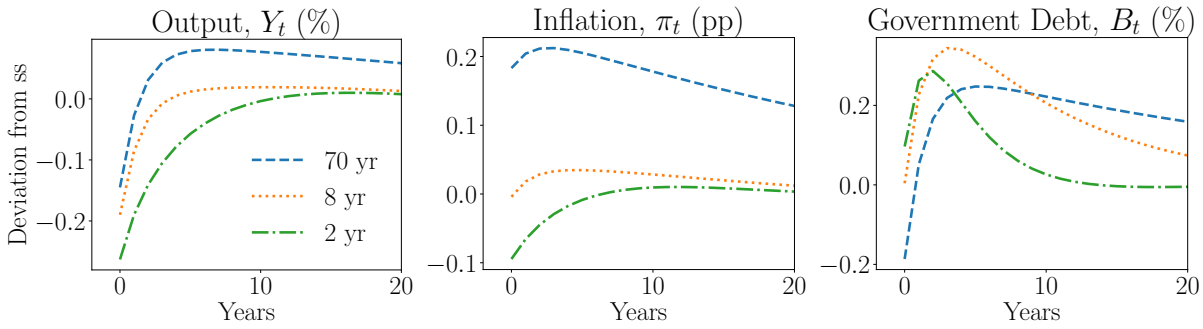
rate in Section 3. That is, we postulate that taxes follow

$$\tau_t = \bar{\tau} + (\lambda + \bar{r}) \cdot (B_{t-1} - \bar{B}), \quad 0 < \lambda < 1. \tag{33}$$

This specification lets taxes respond to the deviation of debt from target at the beginning of the period. As with stochastic adjustment, λ controls the path of the expected deviation of debt from steady state (target). The half-life of deviations of debt from target, for given real rates, is $\psi := \ln(0.5)/\ln(1-\lambda)$. We report this more interpretable statistic later (alongside the raw λ parameter).

Figure 7 shows the response to an immediate rate hike ($M = 1$). We vary the speed of tax adjustment to debt deviations. In particular, we consider values of $\lambda \in \{0.01, 0.08, 0.29\}$, which translate to debt half-lives, ψ, of 70 (blue dashed lines), 8 (orange dotted lines), and 2 years (green dashed-dotted lines), respectively. The real rate

Figure 7: Response to a rate hike

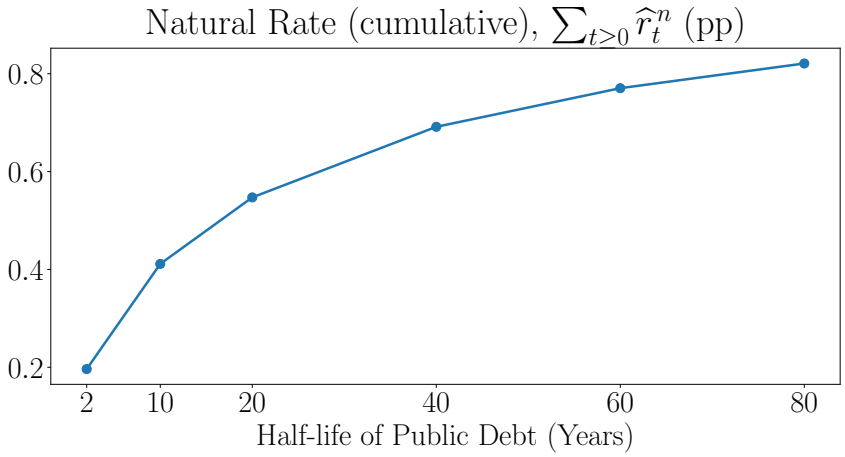


Notes: For the full quantitative model, the figure displays the response of output, inflation, and debt to a 25-basis-point increase in the real rate from period 0 to period 1, $\widehat{r}_1$, announced at $t = 0$. The green dash-dotted lines represent the impulse response for an economy where the fiscal rule implies a half-life of debt deviations from steady state of two years, the orange dotted lines the response when debt half-life is eight years, the blue dashed lines are the responses when debt half-life is 70 years. The model is solved as a nonlinear perfect-foresight transition based on the model's sequence-space representation.

rises persistently. The extent to which this translates into output and inflation aligns with the analytical results earlier in that it depends on how the rising burden of financing existing government debt is dealt with. A fiscal policy with a lower value of ψ (higher λ) implies a faster fiscal adjustment. An early fiscal adjustment (green dash-dotted line) is the most contractionary (left panel). A rapid tax increase (not displayed) rapidly consolidates government debt (right panel). In the simulations shown, output falls by 0.25 percent on impact and remains persistently low. In turn, inflation falls by about 10 basis points on impact (center panel, still the green dash-dotted line). A delayed adjustment (higher half-life of debt, ψ) allows debt to absorb the fiscal footprint of monetary policy for longer. This supports private spending by providing liquidity. If taxes hardly react (blue dashed line, half-life of 70 years), the initial decline in output rapidly turns into a boom and inflation rises throughout instead of falling, much in line with Propositions

3 and 4. Indeed, already at a half-life of debt of 8 years (orange dotted line) the inflation response is approximately zero on impact and becomes positive subsequently. By way of comparison, the simple model variant saw such a reversal at an expected delay of adjustment of roughly 6.75 years, cf. Figure 2.

Figure 8: Interest rate responses to the public debt shock induced by fiscal delays



Notes: Cumulative deviation of the natural rate from steady state, $\sum_{t\geq 0} \widehat{r}_t^n$ (percentage points), against the half-life of debt deviations implied by (33), for the 25 basis point rate hike of Figure 7. We obtain a measure of $\widehat{r}_t^n$ in two steps. First, for a given value of λ that corresponds to the respective half-life of debt, we solve the full model and store the equilibrium path of public debt (the values in the right panel of Figure 7). Second, we feed that path back to the household block as an exogenous supply of liquidity, holding output, employment, and wages at their steady-state values, and let taxes be residually determined from (29). Then we solve for the perfect-foresight path of real rates at which households willingly hold the given stock of bonds at each date, $\int b_{it}\, di = B_t$. This is the real-rate path that would keep output at potential—the quantitative analog of $\widehat{r}_t^n$ in (5).

To understand the role of natural rates in the quantitative model, Figure 8 computes for each speed of fiscal adjustment, the path of long real interest rates that would keep output at potential given the equilibrium path of public debt that the monetary rate hike induces. The resulting measure is the quantitative counterpart of the long natural rate (5) of the analytical model. Here, however, the measure has to be computed numerically because the marginal value of liquidity now depends on the entire joint distribution of age, income, and wealth; the notes to the figure spell out the construction. Figure 8 reports the resulting cumulative deviation of the natural rate from its steady-state value. In the RANK-BIU version, it is this cumulative response that matters for demand, recall (9): the counterpart to the long real rate is the sum of natural rates, not the natural rate's value at any single date. Another statistic may be worth reporting since it provides a concise summary of outcome that can be compared to the implications of the simple model of Sections 2 and 3: Table 2 summarizes the information from Figure 7 and Figure 8 in terms of cumulative responses not only of the natural rate but also of output. Exactly as in the simple model variant, here the cumulative deviation of the natural rate is higher the longer the fiscal footprint lingers. And, here, too, the rate hike become less deflationary

Table 2: Monetary transmission and fiscal adjustment

Debt half-life		Output			Inflation			Cumulative
λ	(years)	$t = 0$	$t \leq 5$	$t \leq 20$	$t = 0$	$t \leq 5$	$t \leq 20$	natural rate
0.29	2	-0.26	-0.83	-0.86	-0.09	-0.26	-0.16	0.19
0.08	8	-0.19	-0.30	-0.04	0.00	0.15	0.49	0.37
0.01	70	-0.15	0.07	1.14	0.18	1.23	3.67	0.79

Notes: Summary statistics for the responses to the 25-basis-point monetary tightening shown in Figure 7. Impact output is in percent and impact inflation in percentage points. Output sums the annual percentage responses over the indicated horizons and is in percent of the annual steady state value. Inflation sums the cumulated price level deviations over the indicated horizon. The final column reports the total cumulative natural-rate deviation shown in Figure 8, in percentage-point-years.

and more expansionary if the monetary footprint on debt is allowed to linger.

Taken together, these results align with the key insights of the analytical model in Section 2: the timing of fiscal adjustment governs the strength—and potentially the sign—of monetary transmission. The quantitative magnitudes are slightly smaller, though, than in the bonds-in-the-utility model, reflecting that the micro-founded liquidity demand in the OLG economy is spread unevenly across the wealth distribution. Asset-rich older households in the quantitative model are less sensitive to changes in bond supply than the representative agent in Section 2. Nonetheless, the qualitative ranking across regimes of fiscal timing is preserved, and the threshold at which inflation effects change sign remains within a plausible range of fiscal delay and broadly similar to the stylized model.

5.4 Robustness

Primarily there may be two potential concerns one might have with our results. First, our stylized model identifies liquidity as the central mechanism: a lingering fiscal footprint changes the natural rate. The quantitative model, however, not only features changes in liquidity but also redistribution between households with low and high marginal propensities to consume. We address this concern by making taxes depend on idiosyncratic productivity. This way taxation remains nondistortionary, but the alternative assigns a larger share of the additional tax burden to high-income households; that is, households that have lower MPCs. We find that the tax burden quantitatively plays a small role. Section 5.4.1 provides details.

Second, one might wonder whether the one-period-bond assumption is innocuous. The lingering footprint channel rests on the market value of the liquidity in circulation. Debt maturity affects how an interest rate hike affects that value. If government debt has a maturity of more than one year, a short-lived monetary policy shock will move only the refinancing costs of bonds that mature at that time. For all other bonds, the monetary tightening only changes the discount factor and thereby the bonds' prices but

not the nominal stream of payments the government has to make. What is more, the price level and inflation affect the real value of all outstanding nominal bonds. That is, the relative importance of the terms determining the lingering fiscal footprint changes. We find that delayed fiscal adjustment continues to weaken monetary transmission but that quantitatively the effects are less pronounced. At the same time, we unearth a new channel of monetary policy: as the present value of outstanding debt changes, the resulting reduction in liquidity itself moves the natural rate in a direction opposite to the monetary policy impulse. Section 5.4.2 provide details.

As per further robustness, Appendix E.1 provides simulations when monetary shocks are not persistent. Appendix E.2 looks into forward guidance (a pre-announced future monetary tightening, $M > 1$). As above, in both cases, fiscal delay continues to dampen the recessionary effects of a monetary tightening; and if the delay is long enough, the inflation response eventually reverses.

5.4.1 Taxation based on productivity

Government debt has two effects, the economics of which differ sharply. First, it provides liquidity. Second, its financing redistributes resources. So as to separate the roles of liquidity and redistribution of income to low-MPC-households in our results, one would ideally eliminate all income redistribution. This is technically not feasible, however. The reason is that we would need to track not only the initial wealth of each household (so as to determine who gained from the real rate changes who lost), but because all equilibrium price change, also the entire path of the multiplicative idiosyncratic income shocks thereafter.

All we can do is to reduce the role of redistribution (towards high-MPC households) as much as is feasible. Toward this end, we replace the uniform lump-sum taxes employed so far with taxes based on exogenous idiosyncratic productivity. That way we can indirectly probe the role of liquidity,by reducing the incidence of taxation of low-MPC households as an important explanation. To be specific and in order to avoid effects on labor supply, we make the additional taxes (taxation that exceeds the steady-state level) income-dependent, as follows.

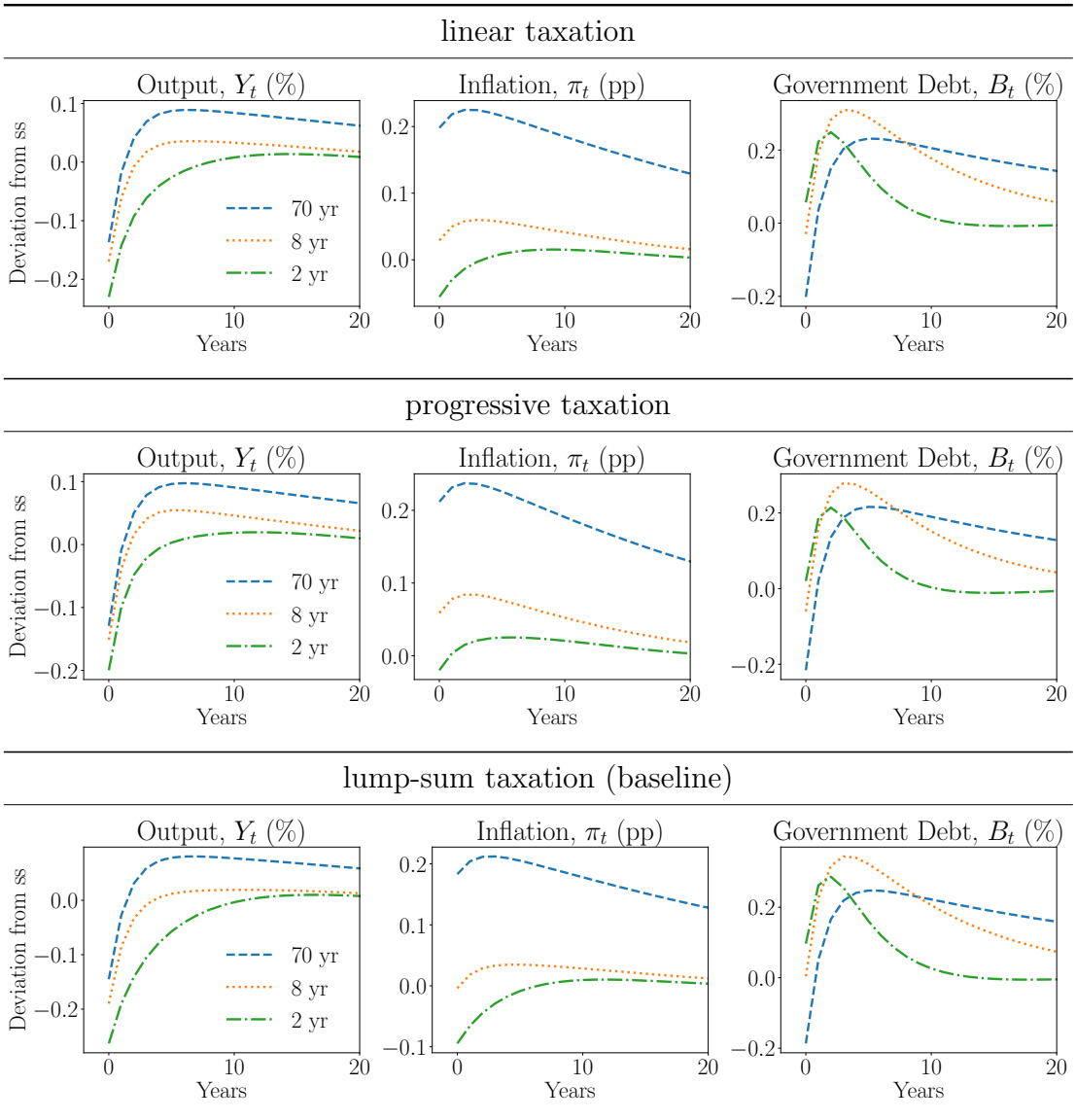
$$\tau_{it} - \overline{\tau} = \frac{e_{it}^{\theta_p}}{\int e_{it}^{\theta_p} di}(\tau_t - \overline{\tau}), \quad e_{it} := \bar{y}_{it}/\overline{Y} \tag{34}$$

where θ_p is a parameter that governs the progressivity of the additional tax and e_{it} is the share of aggregate income that would go to household i in the steady state given its productivity shock realizations.

The more progressive the tax system, the larger the share of the fiscal footprint financed by high-income households. In other words, the higher the progressivity, the more

do households with high current income mechanically "benefit" from the fiscal delay (as their future income and thus tax rate will be lower). Since those high-income households in the model have low and relatively stable MPCs, the experiments above redistributed the tax burden across households with different MPCs, something we wish to avoid here. Figure 9 reports the results of the robustness exercise. Shown are three degrees of progressivity: linear taxation $\theta_p = 1$ (top row), progressive taxation $\theta_p = 2$ (middle row), and for comparison the lump-sum-tax baseline $\theta_p = 0$ studied so far (bottom row).

Figure 9: Response to a rate hike under productivity-dependent taxation



Notes: See Figure 7, but taxes depend on idiosyncratic productivity as explained in the main text.

Comparing the rows, the figure shows that the different specifications of the tax incidence have only minor effects on the results. This is true both for monetary transmission itself and for the effect of fiscal delay on the very transmission of monetary policy. If the fiscal adjustment comes on impact, a more progressive tax scheme dampens the output

and inflation response somewhat. The longer the fiscal footprint lingers, however, the less important tax progressivity becomes. At any rate, the differences across fiscal delay dwarf the differences in responses across degrees of tax progressivity. These results support the interpretation that the lingering footprint channels of monetary policy in the quantitative model, too, operates through liquidity rather than redistribution toward high-MPC households.

5.4.2 Long-term bonds

So as to assess the role of a longer maturity in government debt, we extend the model to long-term nominal debt. Even though we will not go down that route here, this also opens the door to considering the lingering footprint channel of monetary policy in terms of monetary policy's effect on the maturity of government debt instruments held by the public. Following Woodford (2001), we model long-term government debt as a geometrically decaying nominal-coupon bond: one bond issued in t costs $P_t Q_t$ units of currency and pays P_t units of currency in $t+1$, $P_t\delta$ in $t+2$, $P_t\delta^2$ in $t+3$, and so forth. Parameter $\delta \in [0,1)$ governs duration (at one end, with $\delta = 0$ debt is one-period debt; at the other, with $\delta \to 1$ a consol). The realized real gross return on debt held from $t-1$ to t is

$$1 + r_t = \frac{1 + \delta Q_t}{Q_{t-1}} \frac{1}{1 + \pi_t}. \tag{35}$$

With long-term debt ($\delta > 0$), the realized gross real return $(1+r_t)$ depends on the realized bond price Q_t and inflation. Hence, unexpected policy-rate changes create valuation gains or losses not only through current inflation but also by revaluing the flow of future nominal income. The household and the government flow budget constraints, (25) and (29), respectively, remain unchanged, with B_t now marking the period t real market value of outstanding government bonds.

Defining i_t as the net short-term nominal rate and using perfect foresight, for $t = 0, 1, ...$

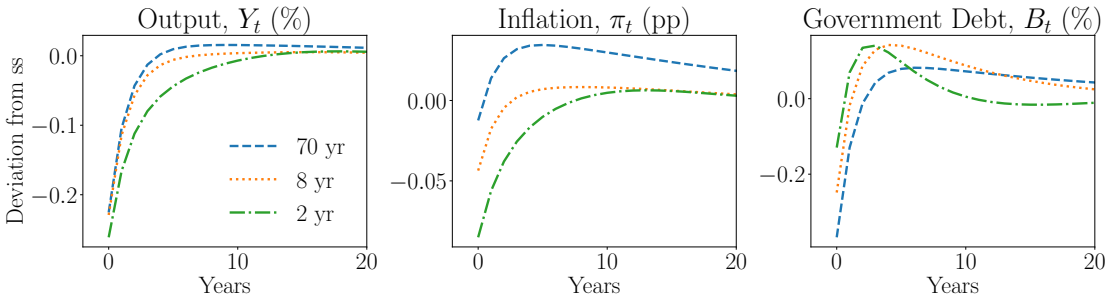
$$(1 + i_t) = \frac{1 + \delta Q_{t+1}}{Q_t}. \tag{36}$$

Taxes follow the same functional form as in (33), but for clarity – and realism – we specify the near-term tax response to exclude valuation effects. Define the units D_t of bonds outstanding (that is, "the face value" of bonds) through $B_t = Q_t D_t$. The fiscal response is

$$\tau_t = \bar{\tau} + (\lambda + \bar{r}) \cdot \bar{Q} \left(D_{t-1} - \bar{D} \right). \tag{37}$$

We use the same calibrations of fiscal persistence as before. In the model, the measure of liquidity is the market value of the government debt that circulates in the household

Figure 10: The economy's response to a rate hike with a four-year bond duration



Notes: Same as Figure 7 but here government bonds have a four-year maturity ($\delta = 0.75$) instead of one year. See the main text for details.

sector. A maturity structure of debt changes both how a monetary tightening affects the market value of existing debt and the interest expenses of the government. Maturity here, thus, affects monetary transmission even though taxes are lump sum. Figure 10 shows the results of this exercise with government bonds of four-year maturity.

First, long-term debt means that the direct fiscal effect of higher real rates on the government's funding costs is less pronounced. Only a fraction of debt is rolled over while the shock lasts, reducing the direct fiscal footprint and the subsequent increase in liquidity. This alone shifts the aggregate responses closer to those of the conventional New Keynesian model. Second, long-term debt means that a temporary rise in the *ex-ante* real rate of interest temporarily reduces the market value of existing debt through changes in discounting. This matters because it now moves the natural rate initially in a direction opposite to the monetary policy shock, whereas in the baseline they move in the same direction. Third, the inflation-related valuation effects of vintage debt change. The reason is that the real value of the nominal debt (and, thus, the amount of liquidity) now is affected by both current inflation and the expected path of future inflation. This matters since in the current setup a monetary tightening tends to be followed by inflationary booms; recall Proposition 2. As a result of all three mechanisms, the market value of public debt falls on impact for all consolidation scenarios, see the rightmost panel of Figure 10.

Consequently, with long-term government debt, a monetary tightening initially drains liquidity from the economy, and this itself has a contractionary effect that goes beyond the standard intertemporal substitution channel. Only with a long fiscal delay does the initial drain of liquidity eventually give way to an increase in liquidity. In fact, with four-year debt the real contraction is larger than with one-year debt under all calibrations of the debt rules. For the price level (center panel) this means that, on impact, the inflation response remains slightly negative even with a very long fiscal delay. Nevertheless, the ranking of outcomes by fiscal delay remains intact: the longer the fiscal tightening is delayed, the smaller the initial fall in inflation and output.

What is more, the basic economics that is behind the effects in Sections 2 and 3 remains the same or even more important for monetary transmission: Monetary policy affects (together with fiscal policy) the liquidity that households can use to self-insure. This has an impact on the natural rate and determines the gap between the natural rate and the policy rate. Together with our baseline results, our maturity robustness exercise also shows that monetary policy transmission becomes weaker and more dependent on fiscal policy if either Treasury debt-management policy or central-bank open-market operations have effectively shortened the maturity of government liabilities held by the public.

6 Conclusion

This paper shows that the *timing* of fiscal responses is a central determinant of monetary transmission when government debt provides liquidity services. In our setup, a monetary policy shock necessarily creates a fiscal footprint. Whether that footprint is removed early or late changes the path of debt. This, in turn, changes the natural rate of interest if households hold government bonds for liquidity services. We analyze this nexus first in a minimal, analytical bonds-in-the-utility framework and then in a richer heterogeneous-agent model with incomplete markets. A delayed fiscal response to a monetary impulse can attenuate or even overturn the impulse's effect on output and inflation.

Fiscal anticipation strengthens the contractionary effect of a rate hike, while delayed fiscal adjustment weakens it and can overturn the contraction. In general equilibrium, sufficiently long expected fiscal delays can make impact inflation rise after a rate hike even though fiscal policy remains passive in present-value terms.

We show that this basic mechanism is borne out by the data. When fiscal policy eliminates the footprint of monetary policy on public debt quickly, monetary policy is more effective.

A quantitative heterogeneous-agent life-cycle model gives a microeconomic foundation for our analytical and empirical results. Delaying fiscal consolidation attenuates recessions and can generate positive inflation responses as well as, at some horizons, positive output responses. This finding is robust to changing the incidence of taxation toward high-income households to steer clear of redistribution channels. With long-term debt, valuation effects dampen magnitudes but do not eliminate the timing pattern: maturity changes the size of the fiscal footprint, not the underlying logic.

The main implication is that monetary policy cannot be assessed independently of the timing of fiscal adjustment. In economies with sizable public debt and liquidity premia, the question is both *whether* and *when* fiscal policy accommodates monetary shocks.

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Appendix

A Proofs

This appendix collects the formal proofs that are omitted from the main text for readability.

A.1 Proof of Lemma 1

Lemma 1 (restated). Consider the individual household's problem of Section 2.1 with $u(c,b) = U(c + \Gamma(b))$, where U and $\Gamma(b)$ are concave with

$$\lim_{b\to\infty} \Gamma_b(b) < \frac{\bar{r}^d - \bar{r}}{1 + \bar{r}^d} < \lim_{b\to-\infty} \Gamma_b(b) \tag{38}$$

. Suppose that prices throughout are at their steady-state values, namely, $(1 + \bar{r})/(1 - \Gamma_b(\bar{b})) = 1 + \bar{r}^d = 1/\beta$. Indexing individual choices by superscript i, suppose the individual household's initial wealth position is consistent with the underlying state, $b^i_{-1} = \bar{b}$ and $d^i_{-1} = 0$. Define the composite good $x^i_t = c^i_t + \Gamma(b^i_t)$. Then
(i) the marginal propensity to consume (MPC) out of beginning-of-period resources $\mathcal{M}_c := \left\{\frac{\partial c^i_t}{\partial y^i_0}\right\}_{t=0}^{\infty}$ is given by $\mathcal{M}_c = \{1-\beta\}_{t=0}^{\infty}$, and
(ii) the MPC for composite consumption $\mathcal{M}_x := \left\{\frac{\partial x^i_t}{\partial y^i_0}\right\}_{t=0}^{\infty}$ is given by $\mathcal{M}_x = \mathcal{M}_c$.
The MPCs for actual and composite consumption, therefore, are identical to the MPC in the textbook representative-agent New Keynesian (RANK) model without bonds in utility.

Proof. Consider household problem (1) under GHH preferences when all aggregate variables are in steady state and $1 + r \leq 1 + r^d = 1/\beta$. Consider a one-time deviation of individual income and no aggregate or idiosyncratic risk. Expressed in terms of the composite good $x^i_t = c^i_t + \Gamma(b^i_t)$, the Lagrangian is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[U(x^i_t) - \lambda^i_t \left(x^i_t - \Gamma(b^i_t) + b^i_t + d^i_t + \bar{\tau} - y^i_t - (1 + \bar{r})\, b^i_{t-1} - (1 + \bar{r}^d) d^i_{t-1} \right) \right],$$

where $y^i_t = \bar{y}$ for all $t > 0$. The first-order conditions are:

$$\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial x^i_t} &= U_x(x^i_t) - \lambda^i_t = 0. \\
\tfrac{\partial \mathcal{L}}{\partial b^i_t} &= -\lambda^i_t \cdot (1 - \Gamma_b(b^i_t)) + \beta \lambda^i_{t+1}(1 + \bar{r}) = 0. \\
\tfrac{\partial \mathcal{L}}{\partial d^i_t} &= -\lambda^i_t + \beta \lambda^i_{t+1}(1 + \bar{r}^d) = 0.
\end{aligned}$$

The above uses that due to the concavity of $\Gamma(b)$ and (38), the household will choose

interior values for b_t^i and hence d_t^i. Combining the last two first-order conditions gives

$$1 - \Gamma_b(b_t^i) = \frac{1+\bar{r}}{1+\bar{r}^d}.$$

Thus, for given prices, regardless of income, the household keeps its savings in government bonds constant at $b_t^i = \bar{b}$. Since $(1+\bar{r}^d) = 1/\beta$, the first-order conditions, thus, give

$$x_t^i = x_{t+1}^i, \tag{39}$$

and the budget constraint collapses to

$$x_t^i + d_t^i + \bar{\tau} = \underbrace{y_t^i + \Gamma(\bar{b}) + \bar{r}\bar{b}}_{\text{given}} + (1+\bar{r}^d) d_{t-1}^i. \tag{40}$$

Note that the income terms for household i are exogenously given. From this and $1+\bar{r}^d = 1/\beta$, we know that the dynamic multiplier is given by $\mathcal{M}_x := \left\{\frac{\partial x_t^i}{\partial y_0^i}\right\}_{t=0}^{\infty} = \{1-\beta\}_{t=0}^{\infty}$. More formally, iterating (40) forward and using the transversality condition for savings in d, we have

$$d_{-1}^i(1+\bar{r}^d) = \sum_{t=0}^{\infty} \beta^t x_t^i - \sum_{t=0}^{\infty} \beta^t [y_t^i + \Gamma(\bar{b}) + \bar{r}\bar{b} - \bar{\tau}],$$

or, combined with (39), and since $d_{-1}^i = \bar{d}$ and $y_t^i = \bar{y}$ for all $t > 0$,

$$x_0^i (= x_t^i) = \bar{y} + \Gamma(\bar{b}) + \bar{r}\bar{b} - \bar{\tau} + (1-\beta)\left[y_0^i - \bar{y} + \bar{d}(1+\bar{r}^d)\right]. \tag{41}$$

From there, the MPC for x $\mathcal{M}_x$ follows immediately. Since $c_t^i = x_t^i - \Gamma(b_t^i)$ and since the household holds $b_t^i = \bar{b}$ constant, $\mathcal{M}_c := \left\{\frac{\partial c_t^i}{\partial y_0^i}\right\}_{t=0}^{\infty} = \mathcal{M}_x$. □

A.2 Proof of Proposition 1

Proposition 1 (restated). Consider the model introduced in Section 2.1 (demand side and government budget) and Section 3.2.1 (supply side and market clearing). Next, consider the monetary and fiscal policies spelled out in Section 3.2.2. Assume that $\chi = 0$ so that there are no liquidity effects. Then the following statements characterize all the fundamental stable equilibria.

(a) FTPL under the active fiscal policy:
On impact, in $t = 0$: $\widehat{\pi}_0 = \frac{\widehat{r}_1}{1+\bar{r}}$ and $\widehat{y}_0 = \left[\frac{1}{\kappa}(1-\beta_\pi) - \frac{\beta_\pi}{\sigma}(1+\bar{r})\right]\frac{\widehat{r}_1}{1+\bar{r}}$
Thereafter, for $t > 0$: $\widehat{\pi}_t = \left[1 + \frac{\kappa}{\sigma}(1+\bar{r})\right]\frac{\widehat{r}_1}{1+\bar{r}}$ and $\widehat{y}_t = \frac{1-\beta_\pi}{\kappa}\widehat{\pi}_t$.

(b) FTPL or no FTPL under the passive fiscal policy with $\lambda = 0$. If fiscal policy never fully consolidates, there is a continuum of equilibria indexed by a constant continuation inflation rate $\pi^A \in \mathbb{R}$.
On impact in $t = 0$: $\widehat{\pi}_0 = \pi^A - \frac{\kappa}{\sigma}\widehat{r}_1$, $\widehat{y}_0 = \frac{1-\beta_\pi}{\kappa}\pi^A - \frac{1}{\sigma}\widehat{r}_1$.
For all $t \geq 1$, $\widehat{\pi}_t = \pi^A$, $\widehat{y}_t = \frac{1-\beta_\pi}{\kappa}\pi^A$.

(c) No FTPL under the passive fiscal policy with $\lambda > 0$:
On impact, in $t = 0$: $\widehat{\pi}_0 = -\frac{\kappa}{\sigma}\widehat{r}_1$ and $\widehat{y}_0 = -\frac{1}{\sigma}\widehat{r}_1$. Thereafter, for $t > 0$, $\widehat{y}_t = \widehat{\pi}_t = 0$.
This is true for arbitrarily small positive values of λ.

Note that case (b) is not the limit of case (c) as $\lambda \to 0$.

Proof. When $\chi = 0$ and $\rho = \bar{r}$, the IS curve and Phillips curve reduce to

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\widehat{r}_{t+1}, \qquad \widehat{\pi}_t = \beta_\pi \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + \kappa \widehat{y}_t. \tag{42}$$

By assumption, the only real-rate deviation is $\widehat{r}_1$; hence $\widehat{r}_{t+1} = 0$ for all $t \geq 1$.

For part (a), taxes remain unchanged, $\widehat{\tau}_t = 0$. The government budget constraint gives

$$\widehat{b}_1 = \bar{b}(1+\bar{r})\left[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0\right], \qquad \widehat{b}_t = (1+\bar{r})^{t-1}\widehat{b}_1 \quad (t \geq 1).$$

Since $\bar{r} > 0$, bounded debt requires $\widehat{b}_1 = 0$. Thus the FTPL closure is

$$[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0]\bar{b} = 0,$$

so

$$\widehat{\pi}_0 = \frac{\widehat{r}_1}{1+\bar{r}}.$$

For $t \geq 1$, there are no further shocks. A bounded fundamental solution to (42) is therefore time-invariant from $t = 1$ onward: let $\widehat{y}_t = \widehat{y}^A$ and $\widehat{\pi}_t = \widehat{\pi}^A$ for all $t \geq 1$. Then (42) implies

$$\widehat{y}^A = \frac{1-\beta_\pi}{\kappa}\widehat{\pi}^A.$$

Evaluating the IS curve at $t = 0$ gives

$$\widehat{y}_0 = \widehat{y}^A - \frac{1}{\sigma}\widehat{r}_1,$$

and evaluating the Phillips curve at $t = 0$ gives

$$\widehat{\pi}_0 = \beta_\pi \widehat{\pi}^A + \kappa \widehat{y}_0.$$

Substituting the expression for $\widehat{y}_0$ and the relation between $\widehat{y}^A$ and $\widehat{\pi}^A$ yields

$$\widehat{\pi}_0 = \beta_\pi \widehat{\pi}^A + \kappa\left(\frac{1-\beta_\pi}{\kappa}\widehat{\pi}^A - \frac{1}{\sigma}\widehat{r}_1\right) = \widehat{\pi}^A - \frac{\kappa}{\sigma}\widehat{r}_1.$$

Hence

$$\widehat{\pi}^A = \widehat{\pi}_0 + \frac{\kappa}{\sigma}\widehat{r}_1 = \left[1 + \frac{\kappa}{\sigma}(1+\bar{r})\right]\frac{\widehat{r}_1}{1+\bar{r}}.$$

Therefore, for all $t > 0$,

$$\widehat{\pi}_t = \widehat{\pi}^A = \left[1 + \frac{\kappa}{\sigma}(1+\bar{r})\right]\frac{\widehat{r}_1}{1+\bar{r}}, \qquad \widehat{y}_t = \widehat{y}^A = \frac{1-\beta_\pi}{\kappa}\widehat{\pi}_t.$$

Using $\widehat{y}_0 = \widehat{y}^A - \widehat{r}_1/\sigma$ gives

$$\widehat{y}_0 = \frac{1-\beta_\pi}{\kappa}\left[1 + \frac{\kappa}{\sigma}(1+\bar{r})\right]\frac{\widehat{r}_1}{1+\bar{r}} - \frac{1}{\sigma}\widehat{r}_1 = \left[\frac{1-\beta_\pi}{\kappa} - \frac{\beta_\pi}{\sigma}(1+\bar{r})\right]\frac{\widehat{r}_1}{1+\bar{r}},$$

which proves part (a).

For part (b), set $\lambda = 0$ in the passive fiscal rule. Then the fiscal footprint is serviced forever but never repaid, so end-of-period debt from $t = 1$ onward is constant at

$$\widehat{b}_t = [\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0]\bar{b}(1+\bar{r}), \qquad t \geq 1.$$

Since $\chi = 0$, this debt path does not feed back into the IS curve. Hence for all $t \geq 1$ the private-sector block is again (42) with $\widehat{r}_{t+1} = 0$. A bounded deterministic continuation can therefore be constant: let $\widehat{\pi}_t = \pi^A$ and $\widehat{y}_t = y^A$ for all $t \geq 1$. Then (42) implies

$$y^A = \frac{1-\beta_\pi}{\kappa}\pi^A.$$

Applying the IS curve at $t = 0$ gives

$$\widehat{y}_0 = y^A - \frac{1}{\sigma}\widehat{r}_1,$$

while the Phillips curve at $t = 0$ gives

$$\widehat{\pi}_0 = \beta_\pi \pi^A + \kappa \widehat{y}_0 = \beta_\pi \pi^A + \kappa \left(\frac{1 - \beta_\pi}{\kappa} \pi^A - \frac{1}{\sigma} \widehat{r}_1 \right) = \pi^A - \frac{\kappa}{\sigma} \widehat{r}_1.$$

Substituting for y^A yields

$$\widehat{y}_0 = \frac{1 - \beta_\pi}{\kappa} \pi^A - \frac{1}{\sigma} \widehat{r}_1.$$

This proves part (b). Setting $\pi^A = 0$ gives the allocation stated in part (c). Choosing

$$\pi^A = \left[1 + \frac{\kappa}{\sigma} (1 + \bar{r}) \right] \frac{\widehat{r}_1}{1 + \bar{r}}$$

implies

$$\widehat{\pi}_0 = \frac{\widehat{r}_1}{1 + \bar{r}},$$

thereby reproducing the FTPL allocation from part (a) as one element of the continuum.

For part (c), passive fiscal policy guarantees that debt remains bounded and is eventually consolidated. Since $\chi = 0$, debt does not feed back into the IS curve, so after $t = 0$ the textbook New Keynesian system has no further exogenous forcing. We select the fundamental solution that returns to the original steady state after consolidation. Before consolidation, the stationary Markov IS relation is $\widehat{y} = (1 - \lambda)\widehat{y}$, so $\widehat{y} = 0$ for $\lambda > 0$. The Phillips curve then gives $\widehat{\pi} = 0$. Hence

$$\widehat{y}_t = \widehat{\pi}_t = 0 \qquad \forall t > 0.$$

Applying (42) at $t = 0$ then yields

$$\widehat{y}_0 = -\frac{1}{\sigma} \widehat{r}_1, \qquad \widehat{\pi}_0 = \kappa \widehat{y}_0 = -\frac{\kappa}{\sigma} \widehat{r}_1,$$

which proves part (c).

□

A.3 Proof of Proposition 2

Proposition 2 (restated). Consider the same conditions as in Proposition 1, in particular a single monetary policy shock $\widehat{r}_1$ announced at $t = 0$, but now assume $\chi > 0$, strictly. Focus on the case that $\beta(1 + \bar{r}) = 1$. Let F mark the random time of fiscal consolidation. Focus on the passive fiscal policy. As the probability of fiscal consolidation $\lambda \to 0$, we have that

(a) $\widehat{\pi}_0 \to \frac{\widehat{r}_1}{1+\bar{r}}$, that is, the initial jump in the price level exactly offsets, in present value terms, the effect of the interest rate change on the government's budget constraint.

This result is independent of the strength of the liquidity effects, as long as the channel is active at all.

(b) For any period t with $1 \leq t < F$, $\widehat{\pi}_t \to \left[1 + \frac{\kappa}{\sigma}(1+\bar{r}) + \frac{\kappa}{\sigma}\chi\right] \frac{\widehat{r}_1}{1+\bar{r}}$. Relative to case (a) of Proposition 1 the effects arising from liquidity amplify inflation after a monetary tightening further as the rise in liquidity generates additional demand.

(c) On impact, $\widehat{y}_0 \to \left[\frac{1}{\kappa}(1-\beta_\pi) - \frac{\beta_\pi}{\sigma}(1+\bar{r}+\chi)\right] \frac{\widehat{r}_1}{1+\bar{r}}$. For every $\lambda > 0$, output before consolidation satisfies the exact identity $\widehat{y}_t = \frac{1-\beta_\pi(1-\lambda)}{\kappa}\widehat{\pi}_t$ for $1 \leq t < F$. Output responses thus depend on liquidity, too.

Proof. (a) follows directly from (18) taking the limit and noting that $\Lambda \to 0$ as $\lambda \to 0$. From (18) we also obtain

$$\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0 = \frac{\Lambda\left[1 + \frac{\kappa}{\sigma}(1+\bar{r}) + \frac{\kappa}{\sigma}\chi\right]}{\Lambda + \frac{\kappa}{\sigma}\chi\left[1 + \bar{r}(1-\Lambda)\right]}\widehat{r}_1.$$

Using the $t > 0$ branch of (18):

$$\widehat{\pi}_t = \frac{\kappa\chi}{\sigma\Lambda}\frac{\Lambda\left[1 + \frac{\kappa}{\sigma}(1+\bar{r}) + \frac{\kappa}{\sigma}\chi\right]}{\Lambda + \frac{\kappa}{\sigma}\chi\left[1 + \bar{r}(1-\Lambda)\right]}\widehat{r}_1 = \frac{\kappa\chi}{\sigma}\frac{\left[1 + \frac{\kappa}{\sigma}(1+\bar{r}) + \frac{\kappa}{\sigma}\chi\right]}{\Lambda + \frac{\kappa}{\sigma}\chi\left[1 + \bar{r}(1-\Lambda)\right]}\widehat{r}_1.$$

Taking $\Lambda \to 0$ then gives

$$\widehat{\pi}_t \to \left[1 + \frac{\kappa}{\sigma}(1+\bar{r}) + \frac{\kappa}{\sigma}\chi\right]\frac{\widehat{r}_1}{1+\bar{r}},$$

which proves (b). For every $\lambda > 0$, the pre-consolidation Phillips curve gives $\widehat{y}_t = [1 - \beta_\pi(1-\lambda)]\widehat{\pi}_t/\kappa$ for $1 \leq t < F$. At date zero, the same curve gives

$$\widehat{y}_0 = \frac{\widehat{\pi}_0 - \beta_\pi(1-\lambda)\widehat{\pi}_1}{\kappa}.$$

Substituting the inflation limits from (a) and (b),

$$\begin{aligned}\lim_{\lambda\to 0}\widehat{y}_0 &= \frac{1}{\kappa}\left\{1 - \beta_\pi\left[1 + \frac{\kappa}{\sigma}(1+\bar{r}+\chi)\right]\right\}\frac{\widehat{r}_1}{1+\bar{r}} \\ &= \left[\frac{1-\beta_\pi}{\kappa} - \frac{\beta_\pi}{\sigma}(1+\bar{r}+\chi)\right]\frac{\widehat{r}_1}{1+\bar{r}},\end{aligned}$$

which proves (c). □

A.4 Proof of Proposition 3

Proposition 3 (restated). Consider the same conditions as in Proposition 2, but now let the probability of fiscal policy consolidating in any given period, λ, remain fixed and

strictly positive. Continue to focus on the passive fiscal policy. Then, for $t = 0$, for any value of $\chi > 0$, there exists a unique threshold $\lambda^*(\chi) \in (0,1)$, such that a rate hike is inflationary iff $\lambda \in (0, \lambda^*(\chi))$. The threshold $\lambda^*(\chi)$ solves

$$\Lambda(\lambda^*(\chi)) := \lambda^*(\chi) \cdot [1 - \beta_\pi(1 - \lambda^*(\chi))] = \frac{\chi}{1+\chi},$$

and is strictly increasing in χ. For all periods $t > 0$ but before fiscal adjustment ($t < F$), a rate hike always is both expansionary and inflationary: $\widehat{y}_t > 0$ and $\widehat{\pi}_t > 0$.

Proof. For a rate hike, the prefactor in (18) is negative and its denominator is positive. Thus $\widehat{\pi}_0$ has the opposite sign to $(1+\chi)\Lambda - \chi$, which vanishes when $\Lambda(\lambda) = \chi/(1+\chi)$. Since $\Lambda(0) = 0$, $\Lambda(1) = 1$, and

$$\Lambda'(\lambda) = 1 - \beta_\pi + 2\beta_\pi\lambda > 0 \qquad (0 < \lambda < 1),$$

there is a unique threshold in $(0,1)$. Since $\chi/(1+\chi)$ is strictly increasing in χ, this threshold is strictly increasing in χ. Impact inflation is positive below the threshold. For $t \geq 1$ before fiscal adjustment, $(1+\bar{r})\widehat{\pi}_0 < \widehat{r}_1$, so $\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0 > 0$ and the claims follow from (18). □

A.5 Proof of Proposition 4

Proposition 4 (restated). Consider the same assumptions as in Proposition 3, passive fiscal policy in particular. Consider parameters $\sigma > 0$, $\chi > 0$, $\lambda \in (0,1)$, and $\beta_\pi \in (0,1]$, with $\bar{r}$ the steady-state real rate of interest. Then upon a monetary rate hike, $\widehat{r}_1 > 0$, output in period 0 rises iff the slope of the Phillips curve, κ, satisfies

$$\kappa < \kappa^*(\lambda, \chi, \bar{r}, \sigma, \beta_\pi),$$

where

$$\kappa^*(\lambda, \chi, \bar{r}, \sigma, \beta_\pi) := \sigma \frac{[1 - \beta_\pi(1-\lambda)]\left[\chi - \frac{\lambda}{1-\lambda}\right]}{\beta_\pi \chi (1 + \bar{r} + \chi)}. \tag{43}$$

A positive value of κ satisfying this condition exists whenever $1/\lambda > 1 + 1/\chi$.

Proof. At $t = 0$, the Phillips curve implies

$$\widehat{y}_0 = \frac{1}{\kappa}\left[\widehat{\pi}_0 - \beta_\pi(1-\lambda)\widehat{\pi}_1\right].$$

Let

$$\Lambda := \lambda[1 - \beta_\pi(1-\lambda)].$$

Using (18) and (19), this gives

$$\frac{\widehat{y}_0}{\widehat{r}_1} = \frac{\chi(1-\Lambda) - \Lambda - \beta_\pi(1-\lambda)\chi - \frac{\kappa}{\sigma}\beta_\pi(1-\lambda)\chi(1+\bar{r}+\chi)}{\sigma\mathcal{D}}, \quad \mathcal{D} := \Lambda + \frac{\kappa}{\sigma}\chi\big[1+\bar{r}(1-\Lambda)\big] > 0.$$

Now

$$\chi(1-\Lambda) - \Lambda - \beta_\pi(1-\lambda)\chi = [1-\beta_\pi(1-\lambda)]\big[(1-\lambda)\chi - \lambda\big].$$

Since $1-\beta_\pi(1-\lambda) > 0$, this term is positive iff

$$(1-\lambda)\chi - \lambda > 0 \iff \chi > \frac{\lambda}{1-\lambda}.$$

Under this condition, solving

$$\chi(1-\Lambda) - \Lambda - \beta_\pi(1-\lambda)\chi - \frac{\kappa}{\sigma}\beta_\pi(1-\lambda)\chi(1+\bar{r}+\chi) > 0$$

for κ yields (20). □

B Determinacy

The main text, when concerned with the RANK-BIU setup, worked with fiscal policy rules that were passive by design. This appendix revisits the question what strength of tax response renders fiscal policy passive or active. In this dimension, too, the model is amenable to tractable results. We continue to focus on the case $\rho = \bar{r}$. To understand determinacy, we give up the assumption that the central bank can directly control the real rate (as this would require determinacy in the first place) and instead focus on the case that the central bank follows a Taylor rule.

B.1 Determinacy with a Taylor rule based on realized inflation

Concretely, we consider a Taylor rule reacting to observed inflation:

$$\widehat{i}_t = \phi_\pi \widehat{\pi}_t, \qquad \phi_\pi > 0. \tag{44}$$

Further, we assume fiscal policy follows a linear tax rule that feeds back on past debt:

$$\widehat{\tau}_t = (\lambda_b + \bar{r})\widehat{b}_{t-1}, \qquad \lambda_b > -\bar{r}. \tag{45}$$

With this rule at steady-state real rates, the expected path of debt is just as in Section 3 (for $\lambda_b = \lambda$) $\widehat{b}_{t+s} = (1-\lambda_b)^s \widehat{b}_t$. The difference is that λ_b here governs a geometric decay where λ in the main text is an adjustment probability.

Define an auxiliary variable $\widehat{\tilde{b}}_t$ as

$$\widehat{\tilde{b}}_t := \widehat{b}_t + \bar{b}(1+\bar{r})\widehat{\pi}_t. \tag{46}$$

Then the system of equations can be expressed as

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} + \frac{1}{\sigma}\mathbb{E}_t\{\widehat{\pi}_{t+1}\} - \frac{\phi_\pi + \chi}{\sigma}\widehat{\pi}_t + \frac{\chi}{\sigma \bar{b}(1+\bar{r})}\widehat{\tilde{b}}_t, \tag{47}$$

$$\widehat{\pi}_t = \kappa \widehat{y}_t + \beta_\pi \mathbb{E}_t \widehat{\pi}_{t+1}, \tag{48}$$

$$\widehat{\tilde{b}}_{t+1} = (1-\lambda_b)\widehat{\tilde{b}}_t + \bar{b}(1+\bar{r})\Big[\phi_\pi - (1-\lambda_b)\Big]\widehat{\pi}_t, \tag{49}$$

where (47) uses market clearing and equations (4), (5), Taylor rule (44), and (46). Equation (49) follows from moving government budget constraint (7) one period forward, substituting the Taylor rule (44), the fiscal rule (45), and definition (46).

The system of equations (47) through (49) has one genuinely predetermined state, $\widehat{\tilde{b}}_t$, and two jump variables, $\widehat{\pi}_t$ and $\widehat{y}_t$. Notice the scaling: debt enters the IS with $\frac{\chi}{\bar{b}(1+\bar{r})}$, while debt accumulation scales with $\bar{b}(1+\bar{r})$; these terms offset in the characteristic polynomial, so the threshold for a passive fiscal response will later be $\lambda_b^\star = \chi$.

Let $s_t := (\widehat{\pi}_t, \widehat{y}_t, \widehat{\tilde{b}}_t)'$. Equations (47)–(49) can be written as

$$A\,\mathbb{E}_t s_{t+1} = B\,s_t, \tag{50}$$

with

$$A = \begin{bmatrix} \beta_\pi & 0 & 0 \\ \frac{1}{\sigma} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -\kappa & 0 \\ \frac{\phi_\pi+\chi}{\sigma} & 1 & -\frac{\chi}{\sigma\bar{b}(1+\bar{r})} \\ \bar{b}(1+\bar{r})[\phi_\pi - (1-\lambda_b)] & 0 & 1-\lambda_b \end{bmatrix}.$$

Lemma B.1 (3-by-3 Eigenvalue Characterization)**.** *The generalized eigenvalues are the roots of*

$$(\beta_\pi\xi - 1)(\xi - 1)[\xi - (1-\lambda_b)] - \frac{\kappa}{\sigma}(\xi - \phi_\pi)\big[\xi - (1-\lambda_b) - \chi\big] = 0. \tag{51}$$

With one predetermined state ($\widehat{\tilde{b}}_t$) and two jump variables ($\widehat{\pi}_t, \widehat{y}_t$), local determinacy requires exactly one stable root and two unstable roots.

Proof. Under the debt redefinition used in Section B, $\widehat{\tilde{b}}_t$ is the predetermined state. From (50), the transition matrix is $M = A^{-1}B$, so eigenvalues solve $\det(\xi I - M) = 0$, equivalently $\det(\xi A - B) = 0$. With the revised matrices,

$$\xi A - B = \begin{bmatrix} \beta_\pi\xi - 1 & \kappa & 0 \\ -\frac{\phi_\pi+\chi}{\sigma} + \frac{\xi}{\sigma} & \xi - 1 & \frac{\chi}{\sigma\bar{b}(1+\bar{r})} \\ -\bar{b}(1+\bar{r})[\phi_\pi - (1-\lambda_b)] & 0 & \xi - (1-\lambda_b) \end{bmatrix}.$$

Expanding this determinant yields

$$\det(\xi A - B) = (\beta_\pi\xi - 1)(\xi - 1)[\xi - (1-\lambda_b)] - \frac{\kappa}{\sigma}(\xi - \phi_\pi)\big[\xi - (1-\lambda_b) - \chi\big],$$

which is (51). The Blanchard-Kahn count then implies the stated root condition. □

Proposition B.1 (Determinacy, Taylor Principle, and Fiscal Cost Threshold)**.** *Consider the BIU economy above with the contemporaneous Taylor rule and debt-feedback tax rule. Assume $\lambda_b < 2$. Define the excess fiscal cost of one additional unit of debt (see also Bayer et al., 2023) as*

$$\lambda_b^\star := \chi,$$

i.e., the cost in excess of the direct carry cost, $\bar{r}$, i.e., the interest semi-elasticity w.r.t. debt. Then, local determinacy requires

$$\phi_\pi \begin{cases} > 1 & \text{if } \lambda_b > \lambda_b^*, \\ < 1 & \text{if } \lambda_b < \lambda_b^*. \end{cases} \tag{52}$$

For $\chi = 0$ (i.e., no liquidity effect), this is the standard Taylor principle and is necessary and sufficient for local determinacy if $0 < \lambda_b < 2$. For $\chi > 0$, the condition above is a side-switching necessary condition; full sufficiency is given by the complete Blanchard-Kahn root count in Lemma B.1.

Proof. For $\chi = 0$ (and $\lambda_b > 0$), (51) factorizes as

$$[\xi - (1 - \lambda_b)] \left[(\beta_\pi \xi - 1)(\xi - 1) - \frac{\kappa}{\sigma}(\xi - \phi_\pi) \right] = 0.$$

The jump-block roots solve $q(\xi) = 0$ with

$$q(\xi) := (\beta_\pi \xi - 1)(\xi - 1) - \frac{\kappa}{\sigma}(\xi - \phi_\pi).$$

Define $p(z) := z^2 q(1/z) = \left(1 + \frac{\kappa \phi_\pi}{\sigma}\right) z^2 - \left(1 + \beta_\pi + \frac{\kappa}{\sigma}\right) z + \beta_\pi$. Schur/Jury conditions imply the only nontrivial inequality is $p(1) = \frac{\kappa}{\sigma}(\phi_\pi - 1) > 0$, i.e. $\phi_\pi > 1$ (the other inequalities hold for $\phi_\pi > 0$, $\beta_\pi < 1$). Together with $|(1 - \lambda_b)| < 1$, this gives exactly one stable root and two unstable roots.

For $\chi > 0$, evaluate (51) at $\xi = 1$:

$$P(1) = -\frac{\kappa}{\sigma}(1 - \phi_\pi)(\lambda_b - \chi).$$

Also,

$$P(-1) = -2(1 + \beta_\pi)(2 - \lambda_b) - \frac{\kappa}{\sigma}(1 + \phi_\pi)(2 - \lambda_b + \chi) < 0.$$

Local determinacy requires exactly one root inside the unit circle and two outside it, with no roots on the circle. Since P has real coefficients, the single stable root must be real and lie in $(-1, 1)$. The sign therefore changes exactly once between -1 and 1, implying $P(1) > 0$. The displayed expression for $P(1)$ gives $(\phi_\pi - 1)(\lambda_b - \chi) > 0$, establishing the necessary condition and threshold $\lambda_b^\star = \chi$. □

The previous proposition gives the side-switching necessary condition. The next result provides a practical sufficient lower bound on monetary aggressiveness for the general $\chi > 0$ case.

Proposition B.2 (Practical Sufficient Condition for General χ)**.** *Assume the augmented fiscal-stability range*

$$\lambda_b^\star < \lambda_b < \min\{2,\, 1 + \chi\},$$

where $\lambda_b^\star = \chi$ and $\Delta_b := \lambda_b - \lambda_b^\star$. Assume also $\phi_\pi > 1$. Away from unit-circle roots, an equilibrium exists but need not be locally determinate. A sufficient condition for local determinacy is

$$\phi_\pi > \max\left\{1, \phi_\pi^{h_1}\right\}, \qquad \phi_\pi^{h_1} := \frac{\sigma}{\kappa} \frac{\beta_\pi + \Delta_b + \chi - 1}{1 - \Delta_b}.$$

Proof. Under the stated range, fiscal stability $0 < \lambda_b < 2$ holds and, additionally, $\Delta_b < 1$. Lemma B.2 in the next subsection, Section B.2, implies that there is at least one stable root. If $h_2 > 0$, determinacy follows immediately from Lemma B.2. We therefore focus on the branch $h_2 < 0$. For $\phi_\pi > 1$ and $\Delta_b > 0$,

$$\phi_\pi(4 - 3\Delta_b) - \Delta_b = 4\phi_\pi(1 - \Delta_b) + \Delta_b(\phi_\pi - 1) > 4\phi_\pi(1 - \Delta_b).$$

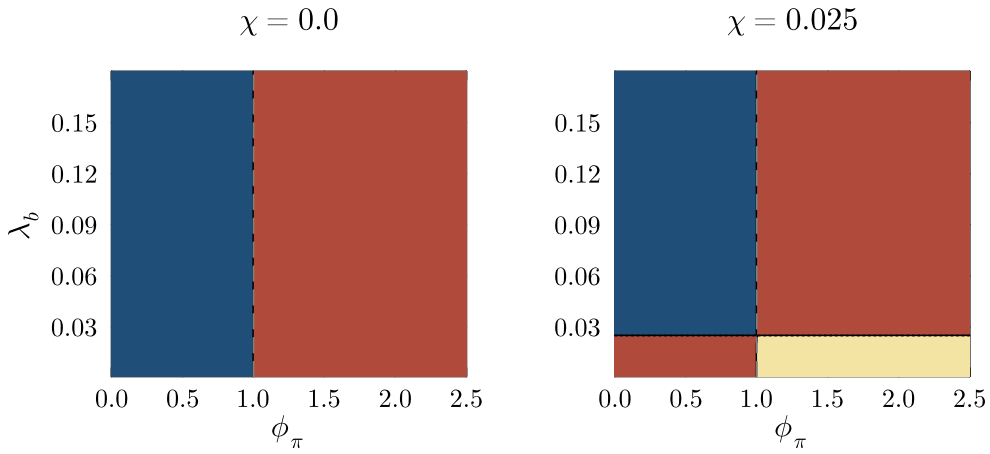
Hence

$$\begin{aligned} h_1 &= 4(\beta_\pi + \lambda_b - 1) - \frac{\kappa}{\sigma}[\phi_\pi(4 - 3\Delta_b) - \Delta_b] \\ &< 4(\beta_\pi + \lambda_b - 1) - 4\frac{\kappa}{\sigma}\phi_\pi(1 - \Delta_b) = 4\left[(\beta_\pi + \Delta_b + \chi - 1) - \frac{\kappa}{\sigma}\phi_\pi(1 - \Delta_b)\right], \end{aligned} \tag{53}$$

where we used $\lambda_b = \Delta_b + \chi$. If $\phi_\pi > \phi_\pi^{h_1}$, the bracketed term is negative, so $h_1 < 0$. Then with $h_2 < 0$, Lemma B.2 implies determinacy. □

Note that the upper-bound restriction $\lambda_b < 1 + \chi$ (i.e., $\Delta_b < 1$) is extremely mild for any empirical calibration; with $\chi \approx 2.5\%$, it allows λ_b below 1.025, far above any estimated fiscal-feedback parameter. The proposition shows that a stronger monetary policy response might be warranted. The inflation coefficient increases in the semi-elasticity of fiscal cost of debt χ and price stickiness κ^{-1}.

Figure B.1: Determinacy region in the 3-by-3 system



Notes: Regions are classified by root counts: blue indicates indeterminacy (multiple equilibria; at least two stable roots), red indicates local determinacy (exactly one stable root), and yellow indicates no bounded equilibrium (no stable root). Left panel: $\chi = 0$; right panel: $\chi = 0.025$. Horizontal dotted lines indicate $\lambda_b^\star = \chi$ in each panel. Calibration: $\beta_\pi = 0.99$, $\sigma = 2$, $\kappa = 0.1$, $\bar{r} = 2\%$, $\bar{b} = 1$. The y-axis range is truncated to $\lambda_b \in [0.001, 0.18]$, corresponding to relatively slow fiscal debt correction (roughly a 4-year or longer half-life of debt deviations).

For the baseline calibration, Figure B.1 partitions (ϕ_π, λ_b) space into determinate, indeterminate, and no-equilibrium regions. The region with the inverted Taylor principle, $\lambda_b < \chi$, links the determinacy analysis to inflationary and potentially expansionary rate

hikes under slow fiscal adjustment in the previous section. There, the necessary condition for a possible impact output reversal was $\lambda < \frac{\chi}{1+\chi}$. Both λ and λ_b measure the expected speed of debt repayment.[13] In the baseline calibration used for Figure B.1, the necessary condition from Proposition B.1 is also numerically sufficient away from the knife-edge lines $\phi_\pi = 1$ and $\lambda_b = \lambda_b^\star$, which is why the determinacy region looks close to rectangular. This coincidence is not generic across parameter configurations. For more details, see Section B.2.

B.2 Exact Determinacy Test for General χ

For general χ, the side-switch condition from $P(1)$ need not be sufficient. The exact Blanchard-Kahn (BK) condition can be written as a cubic root-count test. The idea is simple: determinacy requires a specific number of stable roots in ξ-space; the Cayley transform $\xi = (s-1)/(s+1)$ maps $|\xi| < 1$ to $\Re(s) > 0$, and the Routh array then counts those right-half-plane roots by signs. Operationally, the reader can focus on Lemma B.2: once h_3, h_2, h_1, h_0 are computed, determinacy is decided by sign checks.

From (51), define

$$P(\xi) := (\beta_\pi \xi - 1)(\xi - 1)[\xi - (1 - \lambda_b)] - \frac{\kappa}{\sigma}(\xi - \phi_\pi)[\xi - (1 - \lambda_b) - \chi] .$$

Let

$$\lambda_b^\star := \chi, \qquad \Delta_b := \lambda_b - \lambda_b^\star,$$

and apply the Cayley transform $\xi = (s-1)/(s+1)$. Then

$$H(s) := (s+1)^3 P\left(\frac{s-1}{s+1}\right) = h_3 s^3 + h_2 s^2 + h_1 s + h_0,$$

with

$$\begin{aligned}
h_3 &= \frac{\kappa}{\sigma}(\phi_\pi - 1)\Delta_b, \\
h_2 &= 2(1 - \beta_\pi)(\lambda_b) + \frac{\kappa}{\sigma}[(2 - \Delta_b) + \phi_\pi(3\Delta_b - 2)] , \\
h_1 &= 4(\beta_\pi + \lambda_b - 1) - \frac{\kappa}{\sigma}[\phi_\pi(4 - 3\Delta_b) - \Delta_b] , \\
h_0 &= -2(1 + \beta_\pi)(2 - \lambda_b) - \frac{\kappa}{\sigma}(1 + \phi_\pi)(2 - \Delta_b).
\end{aligned}$$

For a cubic, the number of roots in $\Re(s) > 0$ equals the number of sign changes in the

[13] In terms of the quantitative relevance of the cases, we can compare to the estimates in Bayer et al. (2024), who estimate the direct debt growth response to outstanding debt for the U.S. comparable to λ_b here, between 1.3% and 4.2% and thus in the range where determinacy might fail. However, they use a richer debt rule that also includes inflation, which also helps to generate determinacy.

first Routh column:

$$\begin{array}{c|cc} s^3 & h_3 & h_1 \\ s^2 & h_2 & h_0 \\ s^1 & \dfrac{h_2h_1 - h_3h_0}{h_2} & 0 \\ s^0 & h_0 & 0 \end{array}$$

excluding knife-edge cases (roots on the unit circle and $h_2 = 0$). Hence

$$N_s = \#\text{sign-changes}\left(h_3,\ h_2,\ \frac{h_2h_1 - h_3h_0}{h_2},\ h_0\right),$$

and local determinacy is exactly $N_s = 1$.

Lemma B.2 (Sign Reduction on the Standard Side)**.** *Assume liquidity-augmented fiscal stability $\chi =: \lambda_b^* < \lambda_b < 2$ and an active inflation response in the Taylor rule, $\phi_\pi > 1$. Then $h_3 > 0$ and $h_0 < 0$, and thus $N_s \in \{1, 3\}$. Moreover,*

$$N_s = 1 \iff \big(h_2 > 0\big) \text{ or } \big(h_2h_1 - h_3h_0 > 0\big).$$

In particular, in the $h_2 < 0$ branch, $h_1 < 0$ is sufficient for determinacy.

Proof. Under $\lambda_b > \lambda_b^\star$ and $\phi_\pi > 1$, $h_3 = \frac{\kappa}{\sigma}(\phi_\pi - 1)\Delta_b > 0$. Under $0 < \lambda_b < 2$, we have $2 - \lambda_b > 0$, so $h_0 < 0$. Denote $r_3 := (h_2h_1 - h_3h_0)/h_2$ (the s^1 Routh entry). With first-column signs $(+, \operatorname{sgn} h_2, \operatorname{sgn} r_3, -)$, the sign-change count is one iff not both $(h_2 < 0,\ r_3 > 0)$, i.e. iff

$$\big(h_2 > 0\big) \text{ or } \big(h_2h_1 - h_3h_0 > 0\big).$$

If $h_2 < 0$ and $h_1 < 0$, then $h_2h_1 > 0$ and $-h_3h_0 > 0$, hence $h_2h_1 - h_3h_0 > 0$. □

Intuition. The condition $(\phi_\pi - 1)(\lambda_b - \lambda_b^\star) > 0$ comes from $P(1)$ and captures only crossings at $\xi = 1$ (a local side switch). For larger liquidity feedbacks (high χ), another pair of roots can move across the unit circle away from $+1$ (typically as a complex pair near $|\xi| = 1$), which is why the side-switch condition alone need not be sufficient. The Routh/Jury count above captures all such crossings.

C Generalizations of the fiscal-footprint results

This appendix considers two generalizations of the analytical model. First, we allow the arrival rate of fiscal consolidation to vary over time. The impact responses then depend on the distribution of the consolidation date beyond its expectation. Second, we retain a constant arrival rate but allow steady-state bond holdings to be away from liquidity bliss, $\Gamma_b(\bar{b}) > 0$. Throughout, we retain the normalization $\Gamma(\bar{b}) = 0$, the definition $\chi/[\bar{b}(1+\bar{r})] = -\Gamma_{bb}(\bar{b})/[1-\Gamma_b(\bar{b})]$, and $\sigma, \kappa, \chi, \bar{b} > 0$, $1+\bar{r} > 0$, and $0 < \beta_\pi \leq 1$.

Policy experiment and equilibrium conditions. The economy starts at steady state. At date zero, monetary policy announces a one-time increase in the expected real return between dates zero and one, $\widehat{r}_1 > 0$. Expected real returns in all subsequent periods remain at steady state. Fiscal policy keeps date-zero taxes unchanged and maintains the fiscal footprint until the consolidation date $F \geq 1$, when it removes the footprint completely. Thus,

$$\widehat{b}_t = \begin{cases} -\bar{b}(1+\bar{r})\widehat{\pi}_0, & t = 0, \\ \bar{b}(1+\bar{r})[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0], & 1 \leq t < F, \\ 0, & t \geq F. \end{cases} \tag{54}$$

The distribution of F is known at date zero; thereafter, the only new information is whether consolidation has occurred. Fiscal policy adjusts taxes to implement (54), including the financing effects of unexpected inflation on realized bond returns.

Goods-market clearing gives $\widehat{y}_t = \widehat{c}_t$. The linearized Euler equation and Phillips curve are

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\mathbb{E}_t\{\widehat{r}_{t+1}\} + \frac{\chi}{\sigma\bar{b}(1+\bar{r})}\widehat{b}_t + \Gamma_b(\bar{b})[\mathbb{E}_t\{\widehat{b}_{t+1}\} - \widehat{b}_t], \tag{55}$$

$$\widehat{\pi}_t = \kappa\widehat{y}_t + \beta_\pi\mathbb{E}_t\{\widehat{\pi}_{t+1}\}. \tag{56}$$

We consider the fundamental solution with output and inflation at steady state after consolidation and with the forward terminal conditions $\lim_{T\to\infty}\mathbb{E}_t\{\widehat{y}_{T+1}\} = 0$ and $\lim_{T\to\infty}\beta_\pi^{T+1-t}\mathbb{E}_t\{\widehat{\pi}_{T+1}\} = 0$.

C.1 General consolidation dates

For this subsection, retain $\Gamma_b(\bar{b}) = 0$. Allow any distribution of F on the positive integers satisfying

$$\mathcal{L} := \sum_{t=1}^{\infty}\Pr(F > t)\sum_{j=0}^{t}\beta_\pi^j < \infty. \tag{57}$$

For $\beta_\pi < 1$, a finite $\mathbb{E}_0\{F\}$ suffices; for $\beta_\pi = 1$, a finite $\mathbb{E}_0\{F^2\}$ suffices. The unweighted survival sum satisfies

$$\mathcal{F} := \sum_{t=1}^{\infty} \Pr(F > t) = \mathbb{E}_0\{F\} - 1. \tag{58}$$

Finally, note that

$$\mathcal{L} - \mathcal{F} = \sum_{t=1}^{\infty} \Pr(F > t) \sum_{j=1}^{t} \beta_\pi^j. \tag{59}$$

Proposition C.1 (Monetary transmission under general consolidation dates)**.** *Under the assumptions above, impact inflation and output are*

$$\widehat{\pi}_0 = \frac{\chi\mathcal{L} - 1}{\dfrac{\sigma}{\kappa} + \chi + \chi(1+\bar{r})\mathcal{L}} \widehat{r}_1, \tag{60}$$

$$\widehat{y}_0 = -\frac{1}{\sigma}\widehat{r}_1 + \frac{\chi}{\sigma}\Big(\mathcal{F}\widehat{r}_1 - [1 + (1+\bar{r})\mathcal{F}]\widehat{\pi}_0\Big). \tag{61}$$

In particular:

(a) Impact inflation is positive if and only if

$$\chi\mathcal{L} > 1. \tag{62}$$

Equality gives zero impact inflation; reversing the inequality gives negative impact inflation.

(b) If $\chi\mathcal{F} \le 1$, impact output is negative. If $\chi\mathcal{F} > 1$, impact output is positive if and only if

$$\kappa < \frac{\sigma[\chi\mathcal{F} - 1]}{\chi(1 + \bar{r} + \chi)(\mathcal{L} - \mathcal{F})}. \tag{63}$$

Equality gives zero impact output.

(c) In every period $1 \le t < F$, output and inflation are positive. Both return to steady state when consolidation occurs.

Proof. Step 1: Expected output after the monetary impulse. Iterating (55) from t through T and taking date-zero expectations gives, since $\Gamma_b(\bar{b}) = 0$,

$$\mathbb{E}_0\{\widehat{y}_t\} = \mathbb{E}_0\{\widehat{y}_{T+1}\} - \frac{1}{\sigma}\sum_{s=t}^{T} \mathbb{E}_0\{\widehat{r}_{s+1}\} + \frac{\chi}{\sigma\bar{b}(1+\bar{r})}\sum_{s=t}^{T} \mathbb{E}_0\{\widehat{b}_s\}. \tag{64}$$

For $t \ge 1$, expected future real-rate deviations are zero. Moreover, (54) implies

$$\mathbb{E}_0\{\widehat{b}_s\} = \bar{b}(1+\bar{r})[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0] \Pr(F > s), \qquad s \ge 1.$$

Letting $T \to \infty$ in (64) therefore yields

$$\mathbb{E}_0\{\widehat{y}_t\} = \frac{\chi}{\sigma}[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0] \sum_{s=t}^{\infty} \Pr(F > s), \qquad t \geq 1. \tag{65}$$

Step 2: Impact output. At date zero, (55) and $\widehat{b}_0 = -\bar{b}(1+\bar{r})\widehat{\pi}_0$ imply

$$\sigma\widehat{y}_0 = -\widehat{r}_1 - \chi\widehat{\pi}_0 + \sigma\mathbb{E}_0\{\widehat{y}_1\}.$$

Substitute (65) at $t = 1$ and use (58) to obtain

$$\sigma\widehat{y}_0 = -\widehat{r}_1 - \chi\widehat{\pi}_0 + \chi(\mathbb{E}_0\{F\} - 1)[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0]. \tag{66}$$

Collecting terms gives (61).

Step 3: Impact inflation. Forward iteration of the Phillips curve gives

$$\widehat{\pi}_0 = \kappa\widehat{y}_0 + \kappa \sum_{t=1}^{\infty} \beta_\pi^t \mathbb{E}_0\{\widehat{y}_t\}.$$

The continuation term in (65) involves

$$\sum_{j=1}^{\infty} \beta_\pi^j \sum_{t=j}^{\infty} \Pr(F > t) = \sum_{t=1}^{\infty} \Pr(F > t) \sum_{j=1}^{t} \beta_\pi^j = \mathcal{L} - \mathcal{F}.$$

This interchange is valid under (57). Substituting (66) and adding its unweighted survival sum gives

$$\widehat{\pi}_0 = \frac{\kappa}{\sigma}\left(-\widehat{r}_1 - \chi\widehat{\pi}_0 + \chi[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0]\mathcal{L}\right). \tag{67}$$

Move all terms in $\widehat{\pi}_0$ to the left and divide by their strictly positive coefficient. This gives (60) and the inflation condition in part (a).

Step 4: The output threshold. Substitute (60) into (61) and collect terms over the same positive denominator:

$$\frac{\sigma\widehat{y}_0}{\widehat{r}_1} = \frac{-1 + \chi\left[\mathcal{F} - \frac{\kappa}{\sigma}(1+\bar{r}+\chi)(\mathcal{L}-\mathcal{F})\right]}{1 + \frac{\kappa}{\sigma}\chi + \frac{\kappa}{\sigma}\chi(1+\bar{r})\mathcal{L}}. \tag{68}$$

$\mathcal{L} - \mathcal{F}$ is strictly positive whenever $\Pr(F > 1) > 0$. Thus, if $\chi\mathcal{F} \leq 1$, the numerator is strictly negative. If $\chi\mathcal{F} > 1$, solving for κ gives part (b).

Step 5: Responses before consolidation. Equation (60) also implies

$$\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0 = \frac{1 + \frac{\kappa}{\sigma}(\chi + 1 + \bar{r})}{1 + \frac{\kappa}{\sigma}\chi + \frac{\kappa}{\sigma}\chi(1+\bar{r})\mathcal{L}}\widehat{r}_1 > 0. \tag{69}$$

Conditional on $F > t$, the same forward iteration as in Step 1 gives

$$\widehat{y}_t = \frac{\chi}{\sigma}[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0] \sum_{s=t}^{\infty} \Pr(F > s \mid F > t) > 0.$$

The first term in the survival sum equals one. Forward iteration of (56) then gives $\widehat{\pi}_t = \kappa \sum_{j=0}^{\infty} \beta_\pi^j \mathbb{E}_t\{\widehat{y}_{t+j}\} > 0$. After consolidation, the fundamental solution has $\widehat{y}_t = \widehat{\pi}_t = 0$. □

Relation to the constant-arrival-rate case. For $\Pr(F > t) = (1-\lambda)^t$, the two sums reduce to

$$\mathbb{E}_0\{F\} - 1 = \frac{1-\lambda}{\lambda},$$
$$\sum_{t=1}^{\infty}(1-\lambda)^t \sum_{j=0}^{t} \beta_\pi^j = \frac{1}{\lambda[1-\beta_\pi(1-\lambda)]} - 1 = \frac{1}{\Lambda} - 1,$$

where $\Lambda = \lambda[1-\beta_\pi(1-\lambda)]$, as in the main text. For $\lambda < 1$, it follows that

$$\sum_{t=1}^{\infty}(1-\lambda)^t \sum_{j=1}^{t} \beta_\pi^j = \frac{\beta_\pi(1-\lambda)}{\Lambda}.$$

Consequently, the impact responses and both sign-reversal conditions reduce to the constant-arrival-rate expressions in the main text.

Proposition C.2 (Dispersion in the consolidation date)**.** *Maintain the assumptions of Proposition C.1. A mean-preserving spread of F, for which (57) continues to hold, weakly raises impact inflation and weakly lowers impact output following $\widehat{r}_1 > 0$.*

Proof. Step 1: The weighted survival sum increases with dispersion. Write the sum as an expectation over consolidation dates:

$$\sum_{t=1}^{\infty} \Pr(F > t) \sum_{j=0}^{t} \beta_\pi^j = \mathbb{E}_0 \left\{ \sum_{t=1}^{F-1} \sum_{j=0}^{t} \beta_\pi^j \right\}. \tag{70}$$

Increasing the consolidation date from an integer f to $f+1$ raises the expression inside this expectation by $\sum_{j=0}^{f} \beta_\pi^j$. This increment is strictly increasing in f, because the next increment adds $\beta_\pi^{f+1} > 0$. The expression is therefore convex in the consolidation date, so a mean-preserving spread weakly raises its expectation.

Step 2: Inflation rises. A larger weighted survival sum increases the denominator in (69) while leaving its numerator unchanged. Thus $\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0$ weakly falls, and $\widehat{\pi}_0$ weakly rises.

Step 3: Output falls. The mean-preserving spread leaves $\mathbb{E}_0\{F\}$ unchanged. Equation (61) is strictly decreasing in $\widehat{\pi}_0$ at a given mean consolidation date. The increase in impact inflation therefore weakly lowers impact output. □

For $\beta_\pi = 1$, the role of dispersion is particularly transparent:

$$\sum_{t=1}^{\infty} \Pr(F > t)(t+1) = \frac{\mathbb{E}_0\{F^2\} + \mathbb{E}_0\{F\} - 2}{2}. \tag{71}$$

This follows by evaluating $\sum_{t=1}^{F-1}(t+1) = (F^2+F-2)/2$ inside the expectation. At a given expected consolidation date, a higher variance then raises the weighted survival sum. For $\beta_\pi < 1$, the mean-preserving-spread result still applies, although the variance alone does not summarize the relevant distributional change. Economically, delayed adjustment sustains future demand, which feeds back into current inflation through the Phillips curve. The stronger initial revaluation of nominal debt then lowers impact output, even when the expected consolidation date is unchanged.

C.2 Steady-state bond holdings away from liquidity bliss

Now return to a constant arrival rate $\lambda \in (0,1]$ and allow

$$0 < \Gamma_b(\bar{b}) = \frac{\rho - \bar{r}}{1+\rho} < 1. \tag{72}$$

The debt policy (54) and Phillips curve are unchanged. The additional term in (55) now matters: a change in bond holdings affects the consumption–liquidity composite directly. Continue to write $\Lambda = \lambda[1 - \beta_\pi(1-\lambda)]$.

Proposition C.3 (Monetary transmission away from liquidity bliss)**.** *Under the policy experiment above, the fundamental impact responses are*

$$\widehat{\pi}_0 = -\frac{\kappa}{\sigma} \frac{(1+\chi)\Lambda - \chi + \sigma\Gamma_b(\bar{b})\bar{b}(1+\bar{r})\beta_\pi\lambda(1-\lambda)}{\left(1 - \frac{\kappa}{\sigma}\chi\bar{r}\right)\Lambda + \frac{\kappa}{\sigma}\chi(1+\bar{r}) - \kappa\Gamma_b(\bar{b})\bar{b}(1+\bar{r})\lambda[1+\beta_\pi\bar{r}(1-\lambda)]}\widehat{r}_1, \tag{73}$$

provided its denominator is nonzero, and

$$\begin{aligned}\widehat{y}_0 = {} & \frac{1}{\sigma}\left[\chi\frac{1-\lambda}{\lambda} - 1\right]\widehat{r}_1 \\ & - \frac{1}{\sigma}\left[\chi\frac{1+\bar{r}-\lambda\bar{r}}{\lambda} - \sigma\Gamma_b(\bar{b})\bar{b}(1+\bar{r})\right]\widehat{\pi}_0.\end{aligned} \tag{74}$$

Before consolidation, the continuation responses are

$$\widehat{y}_t = \left[\frac{\chi}{\sigma\lambda} - \Gamma_b(\bar{b})\bar{b}(1+\bar{r})\right][\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0], \tag{75}$$

$$\widehat{\pi}_t = \frac{\kappa}{\sigma}\frac{\chi - \sigma\Gamma_b(\bar{b})\bar{b}(1+\bar{r})\lambda}{\Lambda}[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0], \qquad 1 \le t < F. \tag{76}$$

Both responses are zero after consolidation.

If the denominator in (73) is positive, impact inflation is positive if and only if

$$(1+\chi)\Lambda - \chi + \sigma\Gamma_b(\bar{b})\bar{b}(1+\bar{r})\beta_\pi\lambda(1-\lambda) < 0. \tag{77}$$

The expression on the left has a unique zero $\lambda^* \in (0,1)$. It is negative for $\lambda < \lambda^*$ and positive for $\lambda > \lambda^*$. Holding the other parameters fixed, a higher $\Gamma_b(\bar{b})$ lowers this threshold. Furthermore,

$$\lim_{\lambda \downarrow 0} \widehat{\pi}_0 = \frac{\widehat{r}_1}{1+\bar{r}}. \tag{78}$$

Proof. Step 1: Forward iteration of the Euler equation. The new term telescopes. For any $T \ge t$,

$$\Gamma_b(\bar{b}) \sum_{s=t}^{T} \mathbb{E}_t\{\widehat{b}_{s+1} - \widehat{b}_s\} = \Gamma_b(\bar{b})[\mathbb{E}_t\{\widehat{b}_{T+1}\} - \widehat{b}_t].$$

Debt returns to steady state under eventual consolidation. Consequently, forward iteration of (55) gives

$$\widehat{y}_t = -\frac{1}{\sigma}\mathbb{E}_t\left\{\sum_{s=t}^{\infty}\widehat{r}_{s+1}\right\} + \frac{\chi}{\sigma\bar{b}(1+\bar{r})}\mathbb{E}_t\left\{\sum_{s=t}^{\infty}\widehat{b}_s\right\} - \Gamma_b(\bar{b})\widehat{b}_t. \tag{79}$$

Relative to liquidity bliss, the direct composition effect is therefore $-\Gamma_b(\bar{b})\widehat{b}_t$.

Step 2: Output before consolidation. For $1 \le t < F$, expected future real-rate deviations are zero. Conditional on survival, expected cumulative debt is

$$\mathbb{E}_t\left\{\sum_{s=t}^{\infty}\widehat{b}_s\right\} = \bar{b}(1+\bar{r})[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0]\sum_{j=0}^{\infty}(1-\lambda)^j = \frac{\bar{b}(1+\bar{r})}{\lambda}[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0].$$

Substitute this expression and (54) into (79) to obtain (75).

Step 3: Impact output. At $t = 0$, the expected cumulative real-rate deviation is $\widehat{r}_1$, while expected cumulative debt equals

$$\mathbb{E}_0\left\{\sum_{s=0}^{\infty}\widehat{b}_s\right\} = -\bar{b}(1+\bar{r})\widehat{\pi}_0 + \bar{b}(1+\bar{r})\frac{1-\lambda}{\lambda}[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0].$$

Using this in (79) gives

$$\begin{aligned}\sigma\widehat{y}_0 = & -\widehat{r}_1 - \chi\widehat{\pi}_0 + \chi\frac{1-\lambda}{\lambda}[\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0] \\ & + \sigma\Gamma_b(\bar{b})\bar{b}(1+\bar{r})\widehat{\pi}_0.\end{aligned} \tag{80}$$

Collecting the coefficients of $\widehat{r}_1$ and $\widehat{\pi}_0$ gives (74).

Step 4: Inflation before consolidation. The conditional state is constant before consolidation, and inflation is zero afterwards. Therefore, $\mathbb{E}_t\{\widehat{\pi}_{t+1}\} = (1-\lambda)\widehat{\pi}_t$ for $1 \le t < F$. The Phillips curve becomes

$$[1 - \beta_\pi(1-\lambda)]\widehat{\pi}_t = \kappa\widehat{y}_t.$$

Substitution of (75) gives (76).

Step 5: Impact inflation. At date zero, the continuation term in the Phillips curve is

$$\beta_\pi\mathbb{E}_0\{\widehat{\pi}_1\} = \frac{\kappa}{\sigma}\frac{\beta_\pi(1-\lambda)}{\Lambda}[\chi - \sigma\Gamma_b(\bar{b})\bar{b}(1+\bar{r})\lambda][\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0].$$

Add $\kappa\widehat{y}_0$ from (80) and use

$$\frac{1-\lambda}{\lambda} + \frac{\beta_\pi(1-\lambda)}{\Lambda} = \frac{1-\Lambda}{\Lambda}.$$

After multiplying the Phillips curve by Λ, this yields

$$\begin{aligned}\Lambda\widehat{\pi}_0 = \frac{\kappa}{\sigma}\Big\{ & -\Lambda\widehat{r}_1 - \chi\Lambda\widehat{\pi}_0 + \sigma\Gamma_b(\bar{b})\bar{b}(1+\bar{r})\Lambda\widehat{\pi}_0 \\ & + \big[\chi(1-\Lambda) - \sigma\Gamma_b(\bar{b})\bar{b}(1+\bar{r})\beta_\pi\lambda(1-\lambda)\big] \\ & \times [\widehat{r}_1 - (1+\bar{r})\widehat{\pi}_0]\Big\}.\end{aligned} \tag{81}$$

Collect the terms in $\widehat{\pi}_0$ on the left. Simplify their coefficient using

$$\Lambda + (1+\bar{r})\beta_\pi\lambda(1-\lambda) = \lambda[1 + \beta_\pi\bar{r}(1-\lambda)].$$

Division by this coefficient gives (73). When the denominator is positive, the sign of impact inflation is the opposite of the sign of its displayed numerator, proving (77).

Step 6: The threshold and the long-delay limit. The left-hand side of (77) is a polynomial of degree at most two in λ. It equals $-\chi < 0$ at $\lambda = 0$ and $1 > 0$ at $\lambda = 1$. It therefore has exactly one zero in $(0,1)$, with the stated sign on either side. Raising $\Gamma_b(\bar{b})$ raises this polynomial at every $\lambda \in (0,1)$, so its zero shifts to a lower arrival rate.

Finally, as $\lambda \downarrow 0$, $\Lambda \to 0$ and all terms multiplied by $\Gamma_b(\bar{b})$ in (73) vanish. Its numerator converges to $-\chi$ and its denominator to $(\kappa/\sigma)\chi(1+\bar{r}) > 0$, yielding (78). □

Setting $\Gamma_b(\bar{b}) = 0$ in these expressions recovers the main-text results exactly. Away from liquidity bliss, a positive debt deviation still supplies a cumulative liquidity stimulus,

while the additional composition term lowers goods demand at a given debt path. Holding χ and the other parameters fixed, this term requires a longer expected delay for impact inflation to reverse sign, on the branch with a positive denominator. The long-delay inflation limit remains unchanged. For any fixed $\lambda > 0$, the denominator is positive at $\Gamma_b(\bar{b}) = 0$; hence strict inflation and output sign results at liquidity bliss persist for sufficiently small positive $\Gamma_b(\bar{b})$ by continuity.

D Details and additional results for the empirical evidence

This appendix documents the estimation underlying Section 4. Section D.1 describes the state-dependent local projection, Section D.2 the local projections that the counterfactual is built from, together with the estimated impulse responses, and Section D.3 the construction of the counterfactual.

D.1 State-dependent local projections

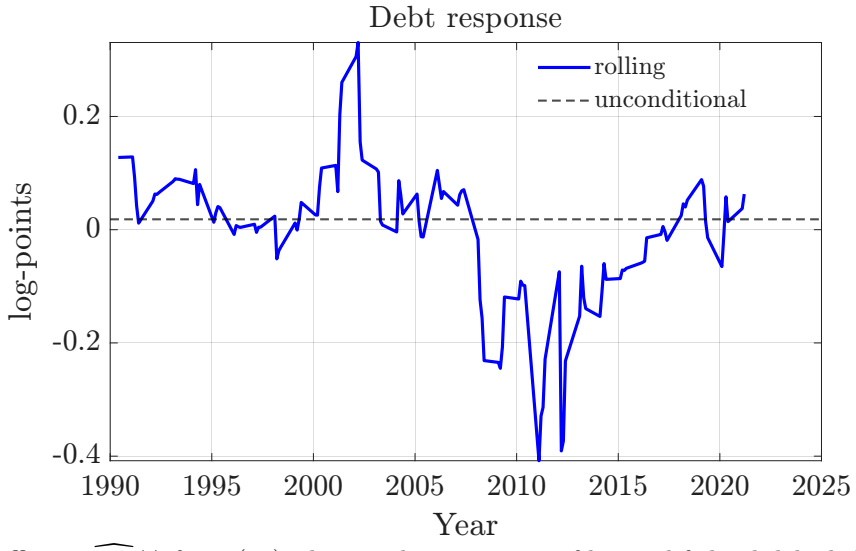
The first exercise splits the transmission of monetary policy by the size of the contemporaneous fiscal response, estimated in two steps.

Step 1: the debt response For every quarter t, we regress the log real value of federal debt held by the public on the monetary surprise, a constant and lagged output and GDP deflator, over a centered window running from $t-10$ to $t+10$,

$$b_s = gdr_t \, \Delta i_s + a_t + c_t \left[GDP_{s-1} P_{s-1}\right] + u_s, \qquad s = t-10, \ldots, t+10. \tag{82}$$

The estimated coefficient $\widehat{gdr}_t$ is the contemporaneous response of debt to a monetary surprise as estimated on the window around t, and it is the state variable used in step 2. Figure D.2 plots the resulting series.

Figure D.2: Estimated contemporaneous response of government debt to a monetary surprise



Notes: The coefficient $\widehat{gdr}(t)$ from (82), that is, the regression of log real federal debt held by the public on the Bauer and Swanson (2022) monetary surprise, a constant and lagged real output and the GDP deflator, over a centered window running from $t-10$ to $t+10$. The dashed line is the same coefficient estimated on the full sample.

Step 2: the interacted projection We then standardize $\widehat{gdr}(t)$ and interact it with the monetary surprise, so that $\beta_{0,h} \pm \beta_{1,h}$ is the impulse response one standard deviation above and below the average debt response. The projection uses a horizon of $H = 8$ quarters, one lag of the controls and of the shock, no deterministic trend, and lagged output and the GDP deflator as controls. The sample is 1988Q2–2019Q4, as in the monetary projection below. The 66% confidence bands use Newey-West standard errors with bandwidth equal to the horizon.

D.2 Local projection estimation

For each response variable x and horizon $h = 0, 1, \ldots, H$ with $H = 8$ quarters, we estimate by ordinary least squares the local projection (Jordà, 2005)

$$x_{t+h} = \beta_h \, s_t + \mu_h + \sum_{\ell=1}^{L} \gamma'_{h,\ell} \, w_{t-\ell} + \varepsilon_{t+h}, \tag{83}$$

where s_t is the identified shock, w_t is a vector of controls and μ_h is an intercept. The impulse response is the sequence of coefficients $\{\beta_h\}_{h=0}^{H}$. Standard errors are Newey–West heteroskedasticity-and-autocorrelation-consistent (HAC) with bandwidth equal to the horizon h; the counterfactual uses the point estimates. The lag length is $L = 1$ throughout.

We estimate four such projections. All of them control for output and the GDP deflator, and the fiscal projections additionally control for the lagged fiscal aggregate affected by the respective shock:

- *Monetary policy*: s_t is the monetary surprise, the sample is 1988Q2–2019Q4 (ending before the COVID period), and the controls are output and the GDP deflator.
- *Government spending*: s_t is the military-news shock, the sample is 1947Q1–2015Q2, and the controls are output, government spending and the GDP deflator.
- *Taxes*: s_t is one of the two narrative tax shocks, the sample is 1950Q1–2007Q4 (bounded by the shock series), and the controls are output, tax revenue and the GDP deflator.

All quantity variables enter in logs; the policy rate enters in percentage points and the unemployment rate as a fraction. In every projection the shock is standardized to a unit standard deviation, and we present the monetary shock as a contractionary tightening (a sign normalization).

Data and shocks The monetary-policy projection uses quarterly U.S. series from the Federal Reserve Bank of St. Louis (FRED): real GDP (`GDPC1`), real personal consumption

expenditures (PCECC96), real gross private domestic investment (GPDIC1), the GDP deflator (GDPDEF), the effective federal funds rate (FEDFUNDS), the civilian unemployment rate (UNRATE), and federal debt held by the public (FYGFDPUN). Real quantities and debt are expressed per capita using the civilian noninstitutional population (CNP16OV); nominal debt is deflated by the GDP deflator. The monetary shock is the orthogonalized high-frequency policy surprise of Bauer and Swanson (2022); monthly surprises are summed to a quarterly series over quarters with all three months available.

The government-spending projection uses the quarterly dataset of Ramey and Zubairy (2018): real GDP, nominal government spending, federal current receipts, federal debt held by the public, the GDP deflator, the unemployment rate, and the military-news series, with real quantities and debt per capita and debt deflated by the GDP deflator. Consumption and investment are the same FRED series as in the monetary projection, also expressed per capita. The fiscal shock is the Ramey and Zubairy (2018) military-news variable, scaled by lagged potential GDP as in their baseline. The price level used throughout is the GDP deflator.

The two tax projections use the narrative record of Romer and Romer (2010), assembled on a common quarterly basis in the replication files of Ramey (2016). The Romer and Romer (2010) series is their *exogenous* component: legislated tax changes that the narrative record attributes to deficit reduction or long-run growth rather than to current macroeconomic conditions, expressed as a ratio to GDP. The Mertens and Ravn (2011) series is the *unanticipated* component of the same record, namely changes implemented within ninety days of becoming law. The distinction matters because much of the Romer–Romer record is legislated well in advance, so that agents may respond before the dated shock; restricting attention to changes implemented promptly removes that anticipation. Real quantities and debt come from the same file, per capita and deflated by the GDP deflator; federal debt held by the public there is the same concept as FYGFDPUN and is numerically almost identical to it over the overlapping sample. A positive shock is a tax increase.

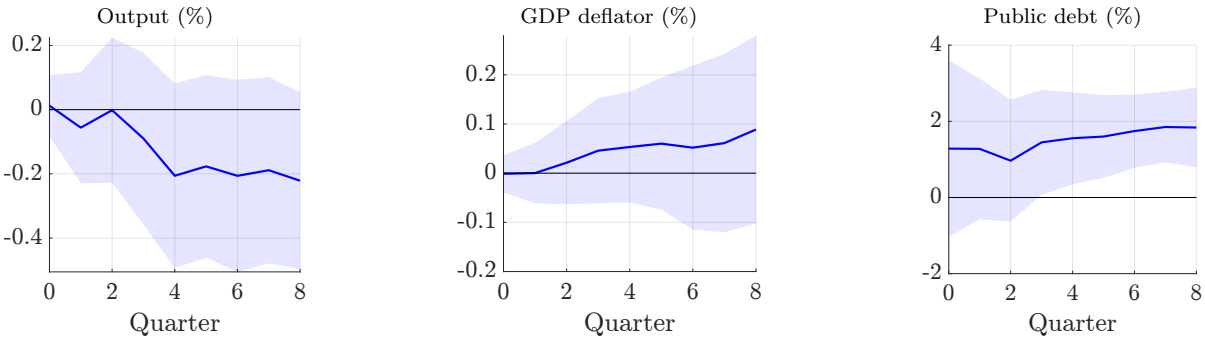
Estimated impulse responses Figure D.3 reports the local-projection impulse responses that the counterfactual is built from, one panel per shock. The monetary and spending shocks are shown in the contractionary direction and the tax shocks as tax increases.

D.3 Constructing the counterfactual

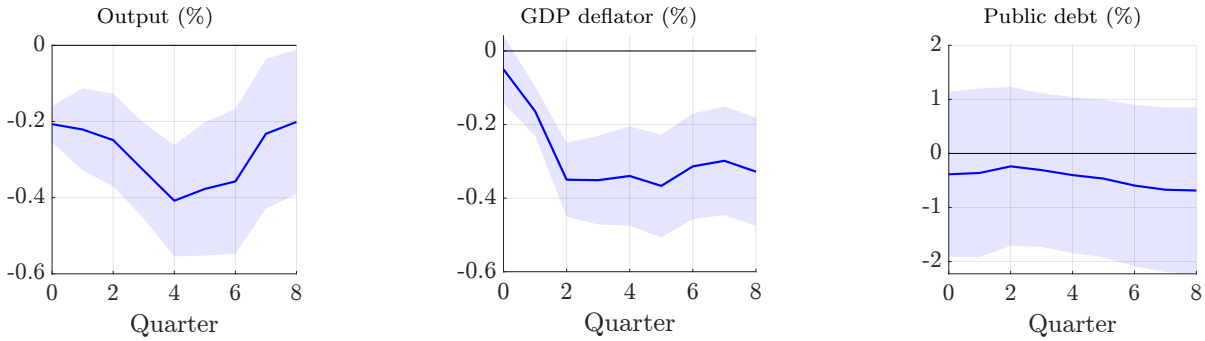
The counterfactual adds a linear combination of the three fiscal impulse responses to the monetary ones, with weights chosen so that the combined debt response is as close to zero as possible over the second year after the shock, as in (22). The minimization runs over

Figure D.3: Local-projection responses to the monetary-policy and fiscal shocks

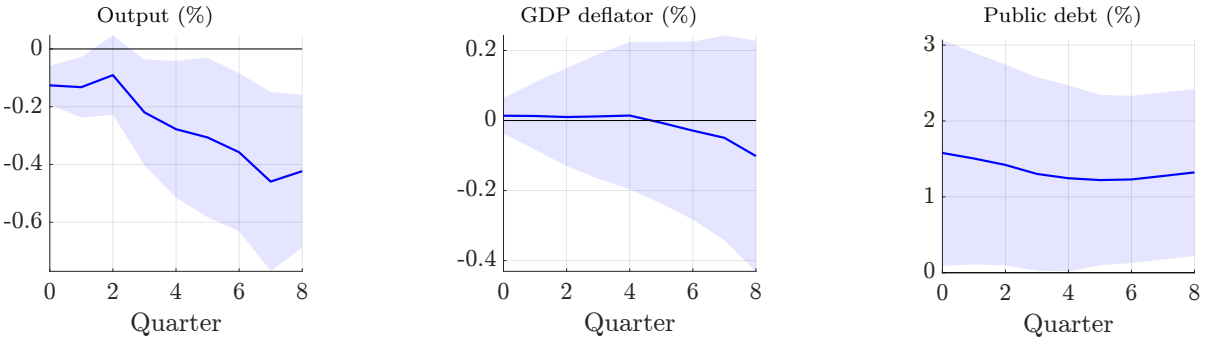
Panel A. Contractionary monetary-policy shock



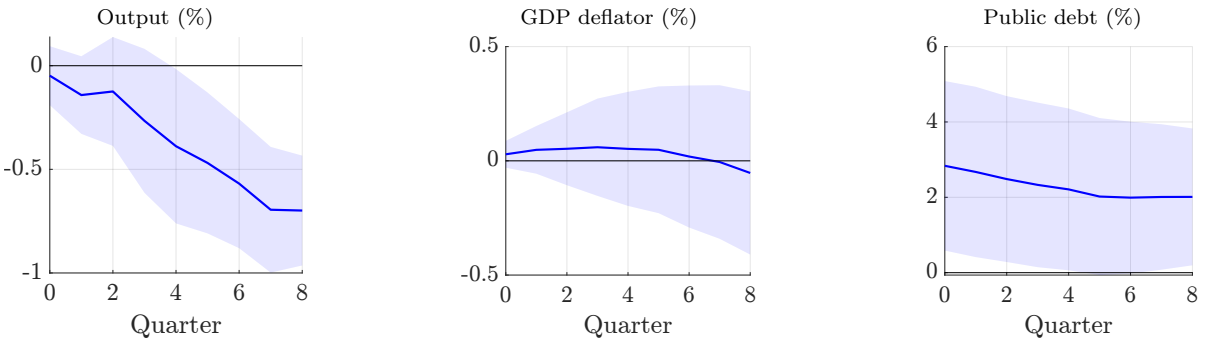
Panel B. Contractionary government-spending shock



Panel C. Unanticipated tax increase (Mertens–Ravn)



Panel D. Exogenous tax increase (Romer–Romer)



Notes: Impulse responses estimated by local projection, using the specification described above. Panel A: contractionary monetary-policy shock (Bauer and Swanson, 2022); Panel B: contractionary government-spending shock (Ramey and Zubairy, 2018); Panel C: unanticipated tax increase (Mertens and Ravn, 2011); Panel D: exogenous tax increase (Romer and Romer, 2010). Each response is in percent, to a one-standard-deviation shock, over a horizon of eight quarters. Shaded regions are 90 percent Newey–West confidence bands with bandwidth equal to the horizon.

horizons $h = 5, \ldots, 8$ and delivers $\alpha^* = (0.61, -0.84, -1.41)$ for the Mertens and Ravn (2011), Romer and Romer (2010) and Ramey and Zubairy (2018) shocks, respectively. The counterfactual response of any variable x is then $x_m(h) + \sum_j \alpha_j^* x_j(h)$, evaluated with the same weights at every horizon.

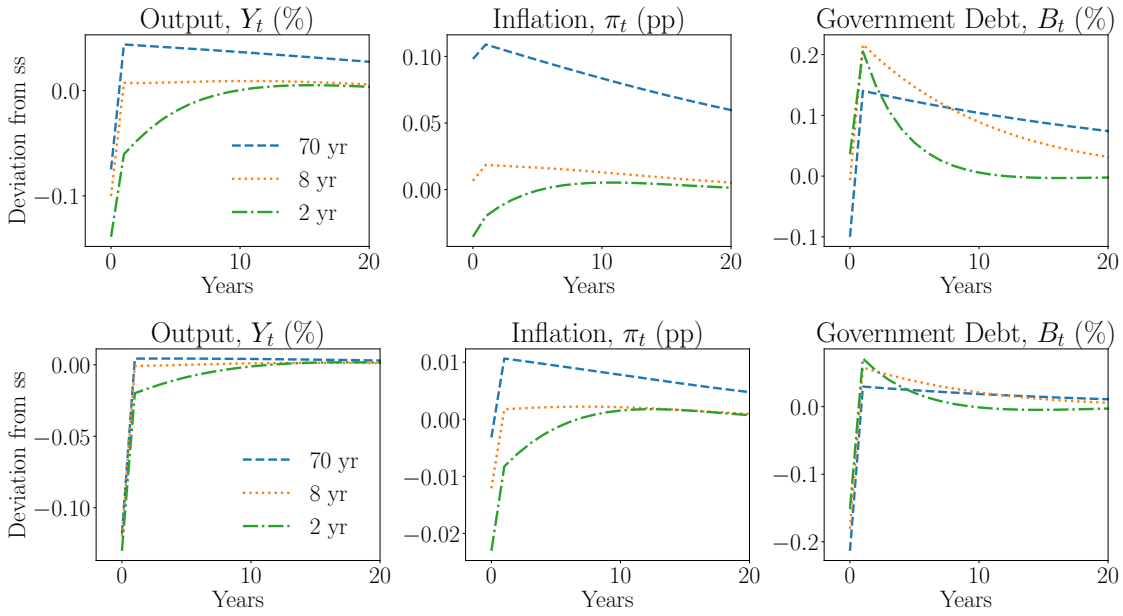
Confidence bands for the counterfactual combine the Newey–West standard errors of the underlying responses under the assumption that the shocks are independent, so that $\mathrm{Var}(x_m + \sum_j \alpha_j x_j) = \mathrm{Var}(x_m) + \sum_j \alpha_j^2 \mathrm{Var}(x_j)$. The monetary and fiscal projections are estimated on separate samples with separately identified shocks, which makes this reasonable across the two groups; the two narrative tax series are two filterings of the same underlying record and are not independent of each other, so the reported bands (66% confidence) are best read as indicative.

E Additional Quantitative Results

This appendix collects the results for additional experiments that we report for completeness. These experiments examine the role of the persistence of the monetary shock, and they look into the effects of forward guidance. Figure E.4 below replicates the timing experiment of Figure 7 in the main text for the case of a purely transitory monetary shock ($\rho_r = 0$). Figure E.5 looks into forward guidance with one-period debt. As regards the timing of fiscal policy, the qualitative ranking across fiscal regimes is the same as in the baseline: earlier fiscal adjustment makes a monetary tightening more contractionary, and sufficient fiscal delay can reverse the sign of the response of inflation.

E.1 No monetary persistence, $\rho_r = 0$

Figure E.4: The economy's response to an immediate rate hike at $M = 1$, $\rho_r = 0$



Notes: Same as Figure 7 and Figure 10 but without persistence of the monetary shock, $\rho_r = 0$. The first row reports the results for one-period debt, and the second row reports the results for long-term debt.

E.2 Forward guidance: change in the future real rate ($M = 15$).

Figure E.5 reports the analogous exercise to Figure 7 in the main text, for an announced future rate hike.

Taxes follow

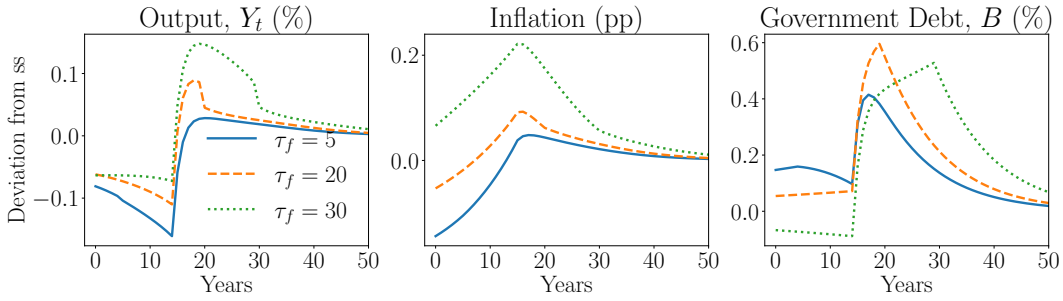
$$\tau_t = \bar{\tau} + \lambda_t(B_{t-1} - \bar{B}).$$

Here λ_t is the total tax-feedback coefficient, rather than the debt-decay rate λ in the baseline rule with coefficient $\lambda + \bar{r}$. At a fixed real rate, debt persistence under the rule here is $1 + \bar{r} - \lambda_t$. We parametrize the timing of fiscal adjustment as

$$\lambda_t = \begin{cases} 0, & \forall \quad 0 \le t < F, \\ 0.3, & \forall \quad t \ge F. \end{cases} \tag{84}$$

For $t < F$, the fiscal authority does not actively stabilize debt, so the fiscal footprint is absorbed by debt issuance. For $t \ge F$, taxes respond and debt is brought back gradually.

Figure E.5: The economy's response to an announced rate hike at $M = 15$, one-period bonds



Notes: The monetary shock and other parameters are as in Figure 7, but the rate hike is announced in period 0 for period 15, and fiscal policy follows the rule specified above: the total tax-feedback coefficient is zero before F and 0.3 thereafter. See the notes to Figure 7 for further details.

For forward guidance (a future rate hike), the qualitative influence of the fiscal timing remains the same as in the main text. The later fiscal policy consolidates, the weaker the recession and the less disinflationary the rate hike. Relative to $M = 1$, the interval between announcement and implementation matters because inflation and bond valuations move at announcement, before the policy-rate realization. This shifts the quantitative threshold at which reversals occur, but not the mechanism: debt-financed fiscal adjustment is less contractionary than immediate tax-financed adjustment.