Credit Market Incompleteness and the Inflation Target - Rate Policy Mix

Abstract

In a neoclassical growth model, I identify a source of price level drifts to be the market incompleteness preventing firms from hiring factors on households' credit, while uncovering a condition for an inflation target to be optimal. Specifically, I find that (i) when firms cannot hire factors on credit from their owners —nor borrow from financial intermediaries for that— prices need to be stable at a steady state, while inflation is indeterminate with financial intermediaries, making financialization a necessary condition for sustained inflation in the model; (ii) there is a steady state positive relation between banks' money creation and inflation, and (iii) financial intermediation partially improves the steady state efficiency, but restores full efficiency only for specific combinations of inflation targets and lending rates, given firms' leverage and discounting. Then, the characterizations of the first-best steady state and market steady state with financial intermediaries show that the former can be sustained by *any* inflation target, provided the adequate lending rate, calling into question the choice of 2% —by the Fed and the ECB among others— as the right target. Although 2% can be optimal for specific lending rates (depending on firms' leverage bounds and discounting), so can other targets for the adequate rates. Lending rates turning 2% optimal run, in the model, from 0.68% to 3.06%, for a firms' leverage of their period output of 1.2 to 4.0 times its value, at a discounting $\delta_f = .98$, with an upper limit —as the leverage bound increases unboundedly— of 4.08% for the given discounting. These values are shown to depend robustly on discounting and leverage conditions. Finally, the lending rate turning an inflation target optimal increases affinely with the latter —as does its upper bound for an unlimited leverage— at a rate depending on the firms' leverage.

Keywords: optimal inflation target, credit market incompleteness, rates policy, firm leverage

1 Introduction

The current consensus about inflation among major central banks —e.g. the US Federal Reserve, the European Central Bank, the Bank of England, the Bank of Japan, the Reserve Bank of New Zealand,...— is to use the instruments at their disposal to implement a target of 2%, but why would the same target be adequate for economies as different as theirs, or why would *this* specific target be optimal for any of them, are questions conspicuously left unanswered.[1] Against this backdrop, I contend here that the usual explanations for inflation (reviewed further below) are not the whole story when it comes to explaining why there tends to be *sustained* inflation—i.e. a constant, moderate, steady drift upwards of the general level of prices— and that the missing piece matters for addressing the unanswered questions above.

Indeed, it follows from the analysis below that any de-anchoring of prices from a no-inflation state is the necessary consequence of missing markets: inflation results, in the model next, from the need for firms to resort to financial intermediaries to pay for inputs in advance of receiving any proceeds from the sale of the output these very inputs produce[2] —i.e. from being unable to operate on credit from the households that supply the capital and labor they use—[3] which will be established in a model departing only minimally from the standard infinite-horizon neoclassical growth model. The ad-

[1] It is well known —see, for an interesting account of how the Fed came up with the 2% target, Wells (2024)— that the current target of 2% adopted by the Fed and the ECB, as well as other major central banks, stem from arguments, that do not pin down any specific figure, such as: aim at an inflation target low enough to have a modicum of price stability, but one that is sufficiently away from zero to avoid risking deflation, while leaving enough room to decrease, if needed, positive nominal interest rates to stimulate demand —on whether nominal policy rates can, or even should, ever be negative see Dávila and Lukmanova (2025).

[2] While it can be argued that labor, for instance, is not paid for in advance but rather after the job is done, it is nonetheless a fact that the lead time from the initiation to the completion of a production process exceeds on average the typical periodicity of labor remuneration —Wen (2005) finds that inventory investment is strongly countercyclical at very high frequencies (e.g. 2–3 quarters per cycle), production is less volatile than sales around the high frequencies, and more volatile than sales only around business-cycle or lower frequencies. This indicates that production planning horizons are quarterly or longer, consistent with lead times exceeding payroll cycles. Should the whole product development process be considered instead —ranging, for instance, from 1–2 years in hardware to 10–15 in pharma— it substantially exceeds too the periodicity of dividend distributions (typically quarterly) and debt service (typically monthly).

[3] There are good reasons for this missing market, starting from households' inability to monitor firms at the heart of Diamond (1984), to the adverse selection and moral hazard problems between firms and households in Stiglitz and Weiss (1981). Firms may or (less realistically) may not have access to credit from financial intermediaries to plug the gap. Both cases are considered below to assess the value of financial intermediation in terms of increased efficiency.

vantage of working with general equilibrium equations with inflation as an endogenous variable —and the possibility of comparing them with those characterizing the first-best allocations instead— is that it will allow to provide a hard answer to whether aiming at an inflation target can deliver the first-best, or at least Pareto-improve the allocation at the steady state and, if yes, for what specific target value.

Specifically, I consider an economy with fully flexible markets in which factors need to be paid for by the firm upon delivery, and hence before having received the proceeds from the sale of the output they produce. Then, I characterize first (i) the complete markets benchmark,[4] —which, as expected, delivers the planner allocations— then (ii) the incomplete markets equilibrium allocations in the absence of financial intermediation,[5] and finally (iii) the equilibrium allocations with access to financial intermediation.

It results from the characterizations obtained that, when firms cannot hire factors on credit from households, nor borrow from financial intermediaries, prices need to be stable at an equilibrium steady state, while introducing financial intermediation leaves steady state inflation undetermined instead, which shows financialization to be a necessary condition for any drift in the level of prices in the model. It is moreover obtained a steady state positive relation between banks' money creation and inflation, along the lines of the quantitative theory of money. Financial intermediation is also shown to partially improve the efficiency of the steady state allocation, but an affine relationship is found between inflation and the lending rate that makes it optimal —because of implementing the first-best steady state— depending on how much can firms leverage their revenue as collateral for loans, for a given discount factor. As a consequence, the market steady state does not necessarily deliver the planner's, even with the help of financial intermediation, unless the inflation target and lending rate satisfy the affine relationship found, for the prevailing leverage and discounting.

I use then the characterizations of the planner steady state and market steady state with financial intermediation to show that the former can be sustained by *any* inflation target, provided that the lending rate is set (through the policy rate) at the adequate level, which strips the choice of 2% by major central banks of its special status. An inflation target of 2% can still be optimal, for specific lending rates (depending on the

[4]That is to say, the same economy without having to advance factors' payments, which amounts to assume complete markets.

[5]To be able to assess next the added value, in terms of efficiency gains, of financial intermediation. Firms can, in this case, use for payments only the proceeds from previous sales.

firms' leverage bounds and discounting), but so can be any other target. In particular, the lending rate that turns the 2% inflation target optimal runs in the model from 0.68% to 3.06% for a range of firms' leverage of their period output of, respectively, 1.2 to 4.0 times its value, at a firms' discounting of $\delta_f = .98$ —which, interestingly, implies negative real rates for low leverages.[6] Also, there turns out to be an upper limit for the lending rate that makes a 2% inflation target optimal, as the bound on leverage increases, which in the model is 4.08% at the given discounting.

The results are driven by the varying size of the wedge introduced by the missing markets between households' optimal marginal rates of substitution and firms' optimal marginal productivities, with or without access to financial intermediation, and do not therefore depend on the way in which the latter is modeled. The only aspects of financial intermediation that are relevant for the matter at hand are that, in its presence, the wedge is modified only by the terms of that access, namely the lending rate and the degree of collateralization, and that financial intermediaries effectively create money by lending, like banks do. As a consequence, the results are robust to a detailed modeling of the financial sector.

This paper is related to several traditions and literatures. Of course price increases play a role in clearing markets and efficiently allocate resources, so that variability in prices *per se* is not to be stymied. The problem is, besides hyperinflation episodes, *sustained* inflation, that long-term, even secular, trend of increasing prices, that has long been associated to "too much money chasing too few goods": any increase in the ability to pay from additional money can only lead —so the argument goes— to higher prices for any given "stock" of goods and services through competing, auction-like demands for it.[7] The implication is that with a constant money supply the level of prices would not exhibit any trend. Thus, barring an "increasing monetary supply", only demand-pull inflation could result from aggregate demand outstripping aggregate supply due to, for instance, population growth, wars, or debt-fueled government spending, among

[6] See Section 7 for the choice of these parameter values for the purpose of illustrating the relationship.

[7] Famously, Cairo's 1324 crash of gold value (amounting to an annualized 70% inflation, approximately) is attributed to Mali's mansa Musa three-month stay during his hajj pilgrimage to Mecca and his largesse in the free distribution of gold in the city, *cf.* Schultz (2006). But more to the point, one can think of the centuries-long Price Revolution induced by the massive injection of gold and, specially, silver (mined in the Americas) into the European economies from the late 15th century through the mid-17th century (see Hamilton (1934)), as well as shorter episodes of hyperinflation resulting from massive printing of fiat money —e.g. Germany 1923, Zimbabwe 2008, or the worst hyperinflation ever recorded (by far) in Hungary 1945-46.

other causes. Alternatively, cost-push inflation would explain *sustained* inflation as the result of a (necessarily sustained too) inflation of costs —which begs the question of what causes the "sustained-ness" of the latter in the first place— or, still, inflation would be self-fulfilling, highlighting the role that expectations play in sustaining inflation —whence central banks' self-appointed role in "anchoring expectations" follows.[8] More recently, the literature on the Fiscal Theory of the Price Level explains the determination of the price level through fiscal rather than monetary considerations: the equilibrium price level would adjust so that the real value of outstanding nominal government liabilities equals the present value of future primary fiscal surpluses under an appropriate fiscal regime —see, e.g., Cochrane (2023). From this perspective, inflation is ultimately anchored by the government's inter-temporal budget constraint rather than by monetary aggregates or nominal rigidities. While this approach offers an alternative mechanism for price-level determination, it focuses on the interaction between fiscal and monetary authorities, rather than on the contractual frictions faced by firms in financing production developed here, identifying how incomplete credit markets and financial intermediation generate a steady-state relationship between optimal inflation and lending rates unrelated to fiscal considerations. While all these approaches can rightly claim to capture different bits of what drives the inflation humming in the background unrelated to business cycle conditions, they do not seem to lend themselves nonetheless to provide a specific, hard answer to whether adopting a positive inflation target is welfare improving and, if so, why the target should be precisely 2%.[9]

The paper is related also to the macro-finance literature studying the interaction between financial frictions and aggregate activity. Financial accelerator models (Bernanke et al. (1999)) point to imperfections in credit markets amplifying business-cycle fluctuations through firms' balance sheets and external finance premia, while collateral-cycle models (Kiyotaki and Moore (1997)) show how borrowing constraints tied to asset values generate persistent macroeconomic fluctuations. More recent work integrates financial intermediaries into DSGE models, highlighting the role of banks' balance sheets

[8] See, e.g., Leduc et al. (2007).

[9] For demand-pull inflation, a rates policy is the means to implement an inflation target, not the *reason* to have one. For cost-push inflation, structural reforms, competition policies, and industrial policies seem more adequate to deal with it. If inflation is driven by expectations declaring a target seems reasonable, but it does not escape the question next on the specific target. Finally, if the price level is ultimately determined by fiscal solvency, as in Cochrane (2023), the interaction between fiscal and monetary policy becomes central to price-level determination, yet the theory does not single out a particular inflation target such as the 2% adopted by many central banks.

and endogenous leverage in the transmission of monetary and financial shocks —e.g. Gertler and Kiyotaki (2010), Gertler and Karadi (2011). The objective of the present paper is nonetheless different, I abstract from business-cycle dynamics and focus on the steady-state implications of incomplete contracting between firms and households. Financial intermediation matters here because it partially completes an otherwise missing credit market, modifying the equilibrium wedge between households' marginal rates of substitution and firms' marginal productivities. The resulting characterization identifies the combinations of lending rates and inflation targets that decentralize the planner's steady state, a question that is distinct from the monetary-financial transmission mechanisms emphasized in the financial accelerator and banking literatures.

In their review of theoretical justifications for the observed positive inflation targets, Schmitt-Grohé and Uribe (2010) point that the "two major sources of monetary non-neutrality [that] govern the determination of the optimal long-run rate of inflation [—namely] a nominal friction stemming from a demand for fiat money [and] the assumption of price stickiness"— only imply optimal inflation targets that equal either minus the real interest rate or zero, respectively. As a result, the literature reviewed in Schmitt-Grohé and Uribe (2010) justifies an inflation target distinct from the Friedman rule[10] either introducing into an otherwise standard neoclassical growth model a transaction cost that happens to affect only households' consumption —effectively creating a demand for money— or assuming that firms cannot freely change the price for their output, à la Calvo (1983). I choose instead below not to resort to neither transaction costs nor sticky prices, in order to isolate the pure welfare costs of inflation without the confounding effects of price adjustment frictions or other costs. Schmitt-Grohé and Uribe (2010) point too to other arguments justifying a positive optimal inflation based on (i) a presumed zero lower bound to nominal interest rates —cast under a new light by the negative nominal rates implemented by, e.g., the ECB or the Bank of Japan, in 2014-22; (ii) downward nominal rigidities in wages —reported to lead to a barely above zero optimal inflation; or (iii) a quality bias in measured inflation —concentrated mostly in the information and communications technology sector, leaving out at least 80% of the GDP of the US, see Moulton and Wasshausen (2006). The survey by Adam and Weber (2025) on more recent literature still points to sticky prices, lower bounds on nominal rates, and downward nominal wage rigidities. Yet, for the reason mentioned, I choose rather

[10]Requiring money to have the same real return as the alternative saving asset, implying an optimal deflation of prices at the same rate as the return to capital.

not to impose a lower bound on nominal interest rates, and to let all prices (including wages) to be flexible.

Recent HANK literature has also examined how heterogeneous agents and incomplete markets affect optimal monetary policy —e.g. Acharya et al. (2023); Dávila and Schaab (2023)— focusing on stabilization policy rather than the long-run inflation target level. There are notable differences between the goals of these papers and the paper at hand, starting with what "optimal" means. Beyond the differences in assumptions —advance payments and collateral constraints versus heterogeneous agents and incomplete markets, flexible prices versus nominal rigidities— while Acharya et al. (2023) is concerned with the "trade-off between consumption inequality, productive efficiency, and price stability", with welfare gains of stabilization in mind, I focus here, in a complementary approach, on decentralizing the long run first-best steady state. On the other hand, Dávila and Schaab (2023) are interested too in the optimal long-run inflation, but again the different choices of assumptions mentioned lead the two papers to different conclusions: a zero optimal inflation with nominal rigidities and heterogeneity in Dávila and Schaab (2023), and an indeterminate optimal inflation depending on firms' borrowing rate and leverage here, once more in complementary ways that help understanding which assumptions deliver what.

This paper is also related to the literature on cashless New Keynesian models insofar it similarly shows that monetary policy can be conducted, and inflation determined, without explicit reference to monetary aggregates, see Woodford (2000), Woodford (2003). In these other models, inflation is anchored by nominal rigidities and interest-rate Taylor rules —e.g. Clarida et al. (1999)— typically implying low or zero optimal inflation absent additional frictions —see Woodford (2002) and Schmitt-Grohé and Uribe (2010). But this paper differs by abstracting from nominal rigidities altogether and instead highlighting how advance-payment constraints and financial intermediation shape the equilibrium relationship between inflation and interest rates. In this sense, the paper complements the New Keynesian approach by identifying a distinct, financial mechanism through which inflation targets and policy rates interact, even in a fully flexible-price environment. Indeed, in standard cashless New Keynesian models, inflation is pinned down by the interaction of sticky prices and an interest-rate policy rule, even in the absence of money or money demand. By contrast, in the model below all prices are fully flexible, and inflation emerges as an equilibrium outcome of advance-payment constraints and missing credit markets between firms and households. The

analysis thus provides a complementary perspective to New Keynesian models, showing how inflation targets and interest-rate policy can matter even in the absence of nominal rigidities, through their interaction with financial structure and incomplete contracting rather than through nominal rigidities or monetary-financial transmission mechanisms.

Finally, the present framework shares with the literature on working-capital, credit-in-advance, and collateral-constrained production models the assumption that firms must finance production before receiving sales revenues. Working-capital models typically impose advance financing requirements on firms in order to study the transmission of monetary policy through firms' financing costs and the resulting cost channel of inflation —e.g. Christiano et al. (1997), Ravenna and Walsh (2006). Similarly, credit-in-advance models impose advance payment constraints as a monetary friction linking money, credit, and production decisions —Lucas and Stokey (1987), Fuerst (1992). In collateral-constrained production models, borrowing limits tied to collateral values generate financial amplification and investment distortions —Kiyotaki and Moore (1997), Bernanke et al. (1999), Albuquerque and Hopenhayn (2004), Drechsel (2023). The present paper differs in that none of these financing arrangements constitutes the primitive friction. Instead, the distortion originates from a missing market: firms cannot contract directly with households to hire factors on credit against future output. Financial intermediation therefore appears not as the source of the distortion but as an institution that partially completes the missing market. As a consequence, the paper is concerned neither with monetary transmission nor with business-cycle amplification, but with the steady-state welfare implications of incomplete contracting and, in particular, with the combinations of inflation targets and lending rates that decentralize the planner's allocation. Seen from this perspective, the paper is closer in spirit to the incomplete-markets tradition than to the working-capital literature. The analysis begins from the complete-markets benchmark, in which firms can obtain implicit trade credit from the households supplying factors of production, and then asks how the removal of that market alters equilibrium allocations. The advance-payment constraint is therefore not introduced as an exogenous technological assumption, but as the equilibrium manifestation of a specific contractual incompleteness. This perspective permits a comparison between the complete-markets allocation, the incomplete-markets allocation without financial intermediation, and the allocation with financial intermediation, making it possible to isolate the welfare gains from financial intermediation itself and

to characterize the inflation target–lending rate combinations that decentralize the first-best steady state.

The rest of the paper is as follows. Section 2 introduces the economy and Section 3 characterizes, for comparison, the equilibrium allocations for the counterfactual case in which markets are complete and therefore firms can hire factors on credit from households, not needing to advance the payment of these factors. Section 4 characterizes the equilibrium allocations when firms cannot hire factors on credit from households and there is no financial intermediaries to address this market incompleteness either. Section 5 characterizes the equilibrium allocations when financial intermediation allows firms to borrow in order to pay for factors beyond their proceeds from past sales. Section 6 characterizes the first-best steady state allocation, which coincides with that of the counter-factual complete markets case. Section 7 characterizes the lending rates that make *any* given inflation target implement the planner's steady state, depending on the degree of leverage of firms. A final section of concluding remarks follows.

2 The economy

The economy consists, in its production and consumption core, of (i) a representative household[11] that consumes, saves, supplies capital and labor, and owns firms,[12] and (ii) a representative firm that produces output —using a neoclassical[13] production function with, possibly, uncertain productivities— out of capital and labor, distributes profits to its household owners, and has as equity an endowment of capital.

Remuneration for the inputs used, at any point in time, to produce output *precedes* the revenue of the latter's sale, which makes the market allocation dependent on additional institutional aspects of the economy. Indeed, the impossibility for households to monitor firms' performance by themselves[14] prevents the latter —because of a market incompleteness— from hiring households' capital and labor *on credit from their*

[11] On extensions allowing for heterogeneity of households, firms and (when present) financial intermediaries, see the remarks in Section 8.

[12] And, when present, financial intermediaires.

[13] That is to say, satisfying standard assumptions (besides enough differentiability) of linear homogeneity and good behaviour at the boundary —as captured by, for instance, Inada conditions— guaranteeing interior solutions.

[14] See, for instance, Diamond (1984) for the delegated monitoring aspect of banking, and Stiglitz and Weiss (1981) for the adverse selection and moral hazard problems between firms and households.

suppliers against future sales. This market incompleteness creates room for financial intermediation between households and firms.

In Section 5, an additional representative financial intermediary is, therefore, assumed to allow firms to borrow against proceeds from future sales of output yet to be produced in order have additional funds —beyond the proceeds from previous output sales— to pay for factors and distribute dividends. I start, nonetheless, for the sake of identifying the first-best benchmark, by characterizing next the complete markets case in which firms are assumed to be able to hire households' capital and labor on credit from these same households against the sales revenue from the resulting output.

3 Case with complete markets

As a benchmark of what the market allocations would be in the absence of any friction, I consider first the case in which firms can rely on revenue from sales of output to pay for the factors producing this very same output —which implicitly presumes the existence of unhindered credit contracts granted to firms by the households supplying the factors of production, capital and labor. As a consequence, there is no role for any financial intermediary to play in this case. The resulting allocations will be the same as those of a planner unconstrained by non-existent property rights.

3.1 Firms

The representative firm, with equity $\kappa > 0$,[15] maximizes the sum of all real profits[16] distributed to households $\frac{\pi_t^f}{p_t}$ discounted by some positive $\delta_f < 1$,[17] at all periods $t \geq$

[15]Without positive equity κ, firms could —once they have access to credit from an financial intermediary in Section 5, secured by output used as collateral— distribute debt-funded dividends without bound, by means of scaling up the linearly homogeneous production. Specifically, κ is the per firm amount of equity resulting from a (uniformly distributed) given aggregate capital endowment of the economy. The linear homogeneity of the production function f turns the number of firms —for both market equilibrium allocations and planner allocations— to be irrelevant even with positive profits and free entry.

[16]The firm's problem is, with complete markets, equivalent to maximizing nominal profits.

[17]Firm discounting is usually interpreted as a reduced-form representation of exit or default risk. Indeed, expected future profits are deemed to be adjusted by a survival probability due to firm exit risks, generating an effective discount factor below unity —see Luttmer (2011)— and, in environments with borrowing constraints (to be introduced in Section 5), firms value future profits only conditional on continuation, which is equivalent to discounting future profits at a rate higher than households —see , e.g., Cooley et al. (2004), Albuquerque and Hopenhayn (2004).

1, under the constraint —written in real terms in (1) below— of affording with each period's stochastic[18] proceeds from output sales $p_t e^{z_t} f(\kappa + k_t, h_t)$ the wage bill $w_t h_t$ of hiring h_t units of labor, the cost $r_t p_{t-1} k_t$ of using as capital, at period t, k_t units of output produced in $t-1$ —and hence valued $p_{t-1}k_t$ at the time $t-1$ of their hiring— at a gross rental rate r_t, and the distribution of nominal profits π_t^f.[19] The representative firm, therefore, solves

$$\begin{aligned} &\max_{0 \leq k_t, h_t} \mathbb{E} \sum_{t=1}^{+\infty} \delta_f^{t-1} \frac{\pi_t^f}{p_t} \\ &\frac{\pi_t^f}{p_t} + \frac{r_t}{p_t / p_{t-1}} k_t + \frac{w_t}{p_t} h_t \leq e^{z_t} f(\kappa + k_t, h_t) \end{aligned} \tag{1}$$

—given $r_t, w_t > 0$ and stochastic processes $p_t > 0$ and z_t (with a given $p_0 > 0$ and $\mathbb{E}\, e^{z_t} = 1$),[20] for all $t = 1, 2, \ldots$— whose solution is characterized next.[21]

Proposition 3.1. *Firms' optimizing k_t, h_t are necessarily characterized, with complete markets, by*

$$\begin{aligned} \mathbb{E}\, \frac{r_t}{p_t / p_{t-1}} &= f_k(\kappa + k_t, h_t) \\ \mathbb{E}\, \frac{w_t}{p_t} &= f_h(\kappa + k_t, h_t) \end{aligned} \tag{2}$$

with

$$\frac{\pi_t^f}{p_t} = e^{z_t} f(\kappa + k_t, h_t) - \frac{r_t}{p_t / p_{t-1}} k_t - \frac{w_t}{p_t} h_t \tag{3}$$

given $r_t, w_t > 0$ and stochastic processes $p_t > 0$ and z_t (for a given $p_0 > 0$ and $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \ldots$.

It is worth noting that, without having to advance factors payments, the representative firm's problem is effectively a sequence of independent one-shot problems. As a

[18] Because of a stochastic z_t driving productivity shocks and (at equilibrium necessarily) stochastic output prices p_t.

[19] For the sake of simplicity, and without loss of generality, I assume that capital depreciates entirely in one period.

[20] For the purposes of this paper, nothing more than a value 1 for the expectation of its exponential is needed to be assumed about the process z_t, which allows for a large class of them. Indeed, the only constraint that this —and the convexity of the exponential function— imposes is that the expectation of z_t itself is negative, for any t. All expectations are to be computed with respect to the processes z_t and (equilibrium) p_t —as corresponds in a rational expectation equilibrium— and hence at the moment the optimization takes place, i.e. before time unfolds.

[21] Proofs of propositions can be found in the appendix, unless straightforward.

consequence, the firm's discounting δ_f will not play any role in the complete markets allocations.[22]

3.2 Households

The representative household maximizes the sum of all its utilities discounted by some positive $\delta_h < 1$, at all periods $t \geq 1$, from consuming c_t units of output, supplying h_t units of labor, and saving k_{t+1} units of period t output to be used by the firm as capital at $t+1$, under the budget constraint —written in real terms in (4) below— of affording with each period t labor income $w_t h_t$, capital income $r_t p_{t-1} k_t$, and distributed profits π_t^f, the cost of consumption $p_t c_t$ and saving $p_t k_{t+1}$. The representative household, therefore, solves

$$\begin{aligned} &\max_{0 \leq h_t, k_{t+1}} \mathbb{E} \sum_{t=1}^{+\infty} \delta_h^{t-1} \left[u(c_t) - v(h_t) \right] \\ &c_t + k_{t+1} \leq \frac{w_t}{p_t} h_t + \frac{r_t}{p_t / p_{t-1}} k_t + \frac{\pi_t^f}{p_t} \end{aligned} \tag{4}$$

—given $r_t, w_t > 0$ and stochastic processes $p_t > 0$ and z_t (determining through (1), at equilibrium, that of π_t^f, and for a given $p_0 > 0$ and $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \ldots$ as well as an initial endowment $k_1 > 0$— whose solution is characterized next.

Proposition 3.2. *Households' optimizing h_t, k_{t+1} are necessarily characterized, with complete markets, by*

$$\begin{aligned} \frac{\mathbb{E}\, u'(c_{t-1})}{\delta_h \mathbb{E}\, u'(c_t)} &= \mathbb{E}\, \frac{r_t}{p_t / p_{t-1}} + \frac{\operatorname{Cov}\left[u'(c_t), \frac{r_t}{p_t / p_{t-1}} \right]}{\mathbb{E}\, u'(c_t)} \\ \frac{v'(h_t)}{\mathbb{E}\, u'(c_t)} &= \mathbb{E}\, \frac{w_t}{p_t} + \frac{\operatorname{Cov}\left[u'(c_t), \frac{w_t}{p_t} \right]}{\mathbb{E}\, u'(c_t)} \end{aligned} \tag{5}$$

with

$$c_t = \frac{\pi_t^f}{p_t} + \frac{w_t}{p_t} h_t + \frac{r_t}{p_t / p_{t-1}} k_t - k_{t+1} \tag{6}$$

given $r_t, w_t > 0$ and stochastic processes $p_t > 0$ and z_t (determining through (1), at equilibrium, that of π_t^f, with a given $p_0 > 0$ and $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \ldots$ as well as an initial endowment $k_1 > 0$.

[22] Nor it does in the problem solved by the planner, who does away with the firm as a profit maximizer, and only makes use in the feasibility constraint of the production technology characterizing it.

3.3 Market equilibria with complete markets

From firms' and households' optimizing in (2-3) and (5-6) respectively, as well as market clearing, it follows straightforwardly the necessary characterization of complete markets equilibria next.

Proposition 3.3. *A market equilibrium, with complete markets, is necessarily characterized by factors k_t, h_t, factor prices $r_t, w_t > 0$, and a stochastic output prices process $p_t > 0$ satisfying*

$$\begin{aligned} \frac{\mathbb{E}\, u'(c_{t-1})}{\delta_h \mathbb{E}\, u'(c_t)} &= f_k(\kappa + k_t, h_t) + \frac{\operatorname{Cov}\left[u'(c_t), \frac{r_t}{p_t / p_{t-1}}\right]}{\mathbb{E}\, u'(c_t)} \\ \frac{v'(h_t)}{\mathbb{E}\, u'(c_t)} &= f_h(\kappa + k_t, h_t) + \frac{\operatorname{Cov}\left[u'(c_t), \frac{w_t}{p_t}\right]}{\mathbb{E}\, u'(c_t)} \end{aligned} \tag{7}$$

with

$$c_t = e^{z_t} f(\kappa + k_t, h_t) - k_{t+1} \tag{8}$$

given the stochastic process z_t (with $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \ldots$, an initial price $p_0 > 0$, and an initial endowment $k_1 > 0$.

A complete markets equilibrium steady state[23] allocation is, therefore, necessarily characterized by the conditions next.

Proposition 3.4. *A market equilibrium steady state, with complete markets, is necessarily characterized by factors k, h satisfying*

$$\begin{aligned} \frac{1}{\delta_h} &= f_k(\kappa + k, h) \\ \frac{v'(h)}{u'(c)} &= f_h(\kappa + k, h) \end{aligned} \tag{9}$$

with

$$c = f(\kappa + k, h) - k \tag{10}$$

It is worth noting that, with complete markets, not only the level of prices p_t, but also the inflation factor $\frac{p_t}{p_{t-1}}$ is left indeterminate, both at a steady state (their values) and off

[23] That is to say, for z_t being a Dirac delta distribution at 0 and k_t, h_t constant for all t.

a steady state (their expected values). The only condition on the price level and inflation imposed by equilibria is that their values (at the steady state) or their expected values (off a steady state) remain proportional to the wage and return to capital respectively, the coefficient of proportionality being respectively the reciprocals of labor and capital productivities, as required by firms' optimizing in (2).

4 Case of no on-credit factors supply and no financial intermediaries

I consider now the case in which firms cannot obtain capital and labor on credit from households, nor are there any financial intermediaries yet to borrow from, and therefore firms can only pay for factors using proceeds from previous sales of output.

4.1 Firms

The representative firm faces now the financial constraint that payments for factors and distribution of profits cannot exceed the proceeds from *previous* period output sales and, therefore, solve[24]

$$\begin{gathered} \max_{0\leq k_t,h_t} \mathbb{E} \sum_{t=1}^{+\infty} \delta_f^{t-1} \frac{\pi_t^f}{p_t} \\ \frac{\pi_t^f}{p_t} + \frac{r_t}{p_t/p_{t-1}} k_t + \frac{w_t}{p_t} h_t \leq \frac{1}{p_t/p_{t-1}} e^{z_{t-1}} f(\kappa + k_{t-1}, h_{t-1}) \end{gathered} \tag{11}$$

—given $r_t, w_t > 0$ and stochastic processes $p_t > 0$ and z_t (for a given $p_0 > 0$ and $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \ldots$— whose solution is characterized next.

Proposition 4.1. *Firms' optimizing k_t, h_t are necessarily characterized, without on-credit*

[24] Due to the lag in sales proceeds in the constraint's RHS, maximizing nominal profits is not equivalent to maximizing real profits anymore. Firms are, therefore, assumed to maximize *real* profits, in order to align their objective with that of households (the firms' owners) who care about the real value of profits in their budget constraint.

factors supply nor financial intermediaries, by

$$\begin{aligned} \mathbb{E}\,\frac{r_t}{p_t/p_{t-1}} &= \delta_f \mathbb{E}\left[\frac{1}{p_{t+1}/p_t} e^{z_t}\right] f_k(\kappa + k_t, h_t) \\ \mathbb{E}\,\frac{w_t}{p_t} &= \delta_f \mathbb{E}\left[\frac{1}{p_{t+1}/p_t} e^{z_t}\right] f_h(\kappa + k_t, h_t) \end{aligned} \tag{12}$$

with

$$\frac{\pi_t^f}{p_t} = \frac{1}{p_t/p_{t-1}} e^{z_{t-1}} f(\kappa + k_{t-1}, h_{t-1}) - \frac{r_t}{p_t/p_{t-1}} k_t - \frac{w_t}{p_t} h_t \tag{13}$$

given $r_t, w_t > 0$ and stochastic processes $p_t > 0$ and z_t (for a given $p_0 > 0$ and $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \ldots$.

4.2 Households

Households' problem does not change from that of the previous section, and the characterization of their optimal behavior is reproduced here just for the sake of completeness.

Proposition 4.2. *Households' optimizing h_t, k_{t+1} are necessarily characterized, without on-credit factors supply nor financial intermediaries, by*

$$\begin{aligned} \frac{\mathbb{E}\, u'(c_{t-1})}{\delta_h \mathbb{E}\, u'(c_t)} &= \mathbb{E}\,\frac{r_t}{p_t/p_{t-1}} + \frac{\mathrm{Cov}\left[u'(c_t), \frac{r_t}{p_t/p_{t-1}}\right]}{\mathbb{E}\, u'(c_t)} \\ \frac{v'(h_t)}{\mathbb{E}\, u'(c_t)} &= \mathbb{E}\,\frac{w_t}{p_t} + \frac{\mathrm{Cov}\left[u'(c_t), \frac{w_t}{p_t}\right]}{\mathbb{E}\, u'(c_t)} \end{aligned} \tag{14}$$

with

$$c_t = \frac{\pi_t^f}{p_t} + \frac{w_t}{p_t} h_t + \frac{r_t}{p_t/p_{t-1}} k_t - k_{t+1} \tag{15}$$

given $r_t, w_t > 0$ and the stochastic processes $p_t > 0$ and z_t (determining through (11), at equilibrium, that of π_t^f, for a given $p_0 > 0$ and $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \ldots$ as well as an initial endowment $k_1 > 0$.

4.3 Market equilibria with no on-credit factors supply and no financial intermediaries

From firms' and households' optimizing in (12-13) and (14-15) respectively, as well as market clearing, it follows straightforwardly the necessary characterization of market equilibria with no on-credit factors supply and no financial intermediaries next.

Proposition 4.3. *A market equilibrium, without on-credit factors supply nor financial intermediaries, is necessarily characterized by factors k_t, h_t, factor prices $r_t, w_t > 0$, and a stochastic output prices process $p_t > 0$ satisfying*

$$\begin{aligned} \frac{\mathbb{E}\, u'(c_{t-1})}{\delta_h \mathbb{E}\, u'(c_t)} &= \delta_f \mathbb{E}\left[\frac{1}{p_{t+1}/p_t} e^{z_t}\right] f_k(\kappa + k_t, h_t) + \frac{\mathrm{Cov}\left[u'(c_t), \frac{r_t}{p_t/p_{t-1}}\right]}{\mathbb{E}\, u'(c_t)} \\ \frac{v'(h_t)}{\mathbb{E}\, u'(c_t)} &= \delta_f \mathbb{E}\left[\frac{1}{p_{t+1}/p_t} e^{z_t}\right] f_h(\kappa + k_t, h_t) + \frac{\mathrm{Cov}\left[u'(c_t), \frac{w_t}{p_t}\right]}{\mathbb{E}\, u'(c_t)} \end{aligned} \tag{16}$$

with

$$\begin{aligned} c_t &= e^{z_t} f(\kappa + k_t, h_t) - k_{t+1} \\ \frac{p_t}{p_{t-1}} &= \frac{e^{z_{t-1}} f(\kappa + k_{t-1}, h_{t-1})}{e^{z_t} f(\kappa + k_t, h_t)} \end{aligned} \tag{17}$$

given the stochastic process z_t (with $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \ldots$, an initial price $p_0 > 0$, and an initial endowment $k_1 > 0$.

Thus, having to advance factors' payments —without the help of financial intermediaries— drives the expected ratio of the discounted productivity shock $\delta_f e^{z_t}$ to the inflation factor $\frac{p_{t+1}}{p_t}$ as a wedge between the expected factors productivities and the households' corresponding marginal rates of substitution (net of the covariances of the marginal utility with the factor prices), which is a source of inefficiency.

The proposition next follows straightforwardly from (17) in the equilibrium characterization above, and captures what should be observed in the counterfactual case of a total lack of access to credit for firms.

Proposition 4.4. *At a market equilibrium without on-credit factors supply nor financial intermediaries, nominal output is constant, i.e. prices are perfectly negatively correlated to output.*

A market equilibrium steady state[25] allocation without on-credit factors supply nor financial intermediaries is, therefore, characterized by the conditions next.

Proposition 4.5. *A market equilibrium steady state, without on-credit factors supply nor financial intermediaries, is necessarily characterized by factors k, h satisfying*

$$\begin{aligned} \frac{1}{\delta_h} &= \delta_f f_k(\kappa + k, h) \\ \frac{v'(h)}{u'(c)} &= \delta_f f_h(\kappa + k, h) \end{aligned} \tag{18}$$

with

$$\begin{aligned} c &= f(\kappa + k, h) - k \\ \frac{p_t}{p_{t-1}} &= 1 \end{aligned} \tag{19}$$

It is worth noting from the second condition in (19) that —as opposed to both the steady state price level p_t and its inflation factor $\frac{p_t}{p_{t-1}}$ not being pinned down with complete markets— prices need to be stable at a steady state when firms cannot borrow from financial intermediaries,[26] which leaves financialization as a necessary condition in this model for any drift in the level of prices —be it *sustained* inflation or any systematic departure from price stability— by effectively removing the forces pinning inflation to zero.

5 Case of no on-credit factors supply but financial intermediation

I introduce now the possibility found in actual economies for firms to borrow from financial intermediaries, in order to address the market incompleteness preventing them to hire factors on credit from households.

5.1 Firms

While holding the same objective, the representative firm faces now the financial constraint that, at any period t, payments for factors $w_t h_t$ and $r_t p_{t-1} k_t$, distribution of profits

[25] See footnote 23.

[26] Nor hire factors on credit from households.

π_t^f, and *repayment of principal and interest of previous period's loan* $r_{t-1}^l l_{t-1}$, cannot exceed the proceeds from *previous* period's output sale $p_{t-1}e^{z_{t-1}}f(\kappa + k_{t-1}, h_{t-1})$ *plus any new loan* l_t, which is bounded above by some multiple θ of the value of the period t output $p_t e^{z_t} f(\kappa + k_t, h_t)$.[27] The representative firm, therefore, solves[28]

$$\begin{aligned} &\max_{0\leq k_t, h_t} \mathbb{E}\sum_{t=1}^{+\infty}\delta_f^{t-1}\frac{\pi_t^f}{p_t} \\ \frac{\pi_t^f}{p_t} + \frac{r_t}{p_t/p_{t-1}}k_t + \frac{w_t}{p_t}h_t + \frac{r_{t-1}^l}{p_t/p_{t-1}}\frac{l_{t-1}}{p_{t-1}} &\leq \frac{1}{p_t/p_{t-1}}e^{z_{t-1}}f(\kappa + k_{t-1}, h_{t-1}) + \frac{l_t}{p_t} \\ \frac{l_t}{p_t} &\leq e^{z_t}f(\kappa + k_t, h_t)\theta \end{aligned} \tag{20}$$

—given $r_t, w_t, r_{t-1}^l > 0$ and stochastic processes $p_t > 0$ and z_t (for a given $p_0 > 0$ and $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \ldots$ as well as θ— whose solution is characterized next.

Proposition 5.1. *Firms' optimizing* k_t, h_t *are necessarily characterized,*[29] *with no on-credit factors supply but financial intermediation, by*

$$\begin{aligned} \mathbb{E}\,\frac{r_t}{p_t/p_{t-1}} &= \mathbb{E}\left[\left(\theta + \delta_f\frac{1 - r_t^l\theta}{p_{t+1}/p_t}\right)e^{z_t}\right]f_k(\kappa + k_t, h_t) \\ \mathbb{E}\,\frac{w_t}{p_t} &= \mathbb{E}\left[\left(\theta + \delta_f\frac{1 - r_t^l\theta}{p_{t+1}/p_t}\right)e^{z_t}\right]f_h(\kappa + k_t, h_t) \end{aligned} \tag{21}$$

[27] Although θ just needs to be positive, the empirically relevant case is $\theta > 1$ meaning that a portion of firms' debt remains uncollateralized. Indeed, on average, only 40% of the total value of loans is secured by collateral in US data —as shown by the share of loans value secured by collateral for all commercial and industry loans at all commercial banks in the US between 1998 and 2016 in time series ESANQ from FRED II database of the Federal Reserve Bank of St. Louis, Dávila and Lukmanova (2025)— with firms facing varying collateral requirements depending on their characteristics —Chodorow-Reich et al. (2021) find, analyzing firm-level differences in collateral requirements, that larger US firms often have access to unsecured debt, while smaller firms typically encounter stricter collateral requirements.

[28] Footnote 24 applies here too.

[29] Whenever the expected real borrowing rate is, at every period $t = 1, 2, \ldots$, smaller than the implicit rate that the firm uses for discounting, i.e.

$$\mathbb{E}\,\frac{r_t^l}{p_{t+1}/p_t} < \frac{1}{\delta_f}$$

so that the borrowing constraint is binding. In spite of the maximization in (20) making it explicit, it might be worth drawing the attention to the fact that the optimization takes place ex ante —and, accordingly, the expectation is over the realization of the stochastic process z_t at all periods t— which, along with the condition above, guarantees time consistency.

with

$$\begin{aligned}\frac{\pi_t^f}{p_t} &= e^{z_t} f(\kappa + k_t, h_t)\theta + \frac{1 - r_{t-1}^l \theta}{p_t / p_{t-1}} e^{z_{t-1}} f(\kappa + k_{t-1}, h_{t-1}) - \frac{r_t}{p_t / p_{t-1}} k_t - \frac{w_t}{p_t} h_t \\ \frac{l_t}{p_t} &= e^{z_t} f(\kappa + k_t, h_t)\theta\end{aligned} \tag{22}$$

given $r_t, w_t, r_{t-1}^l > 0$ *and stochastic processes* $p_t > 0$ *and* z_t *(for a given* $p_0 > 0$ *and* $\mathbb{E}\, e^{z_t} = 1$*), for all* $t = 1, 2, \dots$ *, as well as* θ*.*

5.2 Households

Households receive now each period —besides labor income $w_t h_t$, a capital income $r_t p_{t-1} k_t$, and dividends π_t^f from the firm— dividends π_t^b and remunerated past deposits $r_{t-1}^d d_{t-1}$ from the financial intermediary, in order to pay for consumption $p_t c_t$, lending to firms as capital to be used in the next period $p_t k_{t+1}$, and new deposits d_t. The representative household, therefore, solves

$$\begin{gathered}\max_{0 \le h_t, k_{t+1}, \frac{d_t}{p_t}} \mathbb{E} \sum_{t=1}^{+\infty} \delta_h^{t-1} \big[u(c_t) - v(h_t) \big] \\ c_t + k_{t+1} + \frac{d_t}{p_t} \le \frac{r_{t-1}^d}{p_t / p_{t-1}} \frac{d_{t-1}}{p_{t-1}} + \frac{w_t}{p_t} h_t + \frac{r_t}{p_t / p_{t-1}} k_t + \frac{\pi_t^f}{p_t} + \frac{\pi_t^b}{p_t}\end{gathered} \tag{23}$$

—given $r_t, w_t, r_{t-1}^d > 0$ and the stochastic processes $p_t > 0$ and z_t (determining through (20), at equilibrium, that of π_t^f, for a given $p_0 > 0$ and $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \dots$ as well as an initial endowment $k_1 > 0$, whose solution is characterized next.

Proposition 5.2. *Households' optimizing* h_t, k_{t+1} *are necessarily characterized, with no on-credit factors supply but financial intermediation, by*

$$\begin{aligned}\frac{\mathbb{E}\, u'(c_{t-1})}{\delta_h \mathbb{E}\, u'(c_t)} &= \mathbb{E}\, \frac{r_t}{p_t / p_{t-1}} + \frac{\mathrm{Cov}\left[u'(c_t), \frac{r_t}{p_t / p_{t-1}} \right]}{\mathbb{E}\, u'(c_t)} \\ \frac{v'(h_t)}{\mathbb{E}\, u'(c_t)} &= \mathbb{E}\, \frac{w_t}{p_t} + \frac{\mathrm{Cov}\left[u'(c_t), \frac{w_t}{p_t} \right]}{\mathbb{E}\, u'(c_t)}\end{aligned} \tag{24}$$

and

$$\begin{aligned} c_t &= \frac{\pi_t^f}{p_t} + \frac{\pi_t^b}{p_t} + \frac{w_t}{p_t} h_t + \frac{r_t}{p_t / p_{t-1}} k_t - k_{t+1} + \frac{r_{t-1}^d}{p_t / p_{t-1}} \frac{d_{t-1}}{p_{t-1}} - \frac{d_t}{p_t} \\ r_t &= r_{t-1}^d \end{aligned} \tag{25}$$

given $r_t, w_t > 0$ and the stochastic processes $p_t > 0$ and z_t (determining through (20), at equilibrium, that of π_t^f, with a given $p_0 > 0$ and $\mathbb{E}\, e^{z_t} = 1$), and π_t^b,[30] for all $t = 1, 2, \ldots$ as well as an initial endowment $k_1 > 0$.

5.3 Market equilibria with no on-credit factors supply but financial intermediation

From firms', and households' optimizing in (21–22) and (24–25) respectively, it follows straightforwardly the necessary characterization of market equilibria with no on-credit factors supply but with financial intermediaries next.

Proposition 5.3. *A market equilibrium, with no on-credit factors supply but financial intermediaries, is necessarily characterized by factors k_t, h_t, factor and credit prices $r_t, w_t, r_t^l > 0$, and an output prices process $p_t > 0$ satisfying*

$$\begin{aligned} \frac{\mathbb{E}\, u'(c_{t-1})}{\delta_h \mathbb{E}\, u'(c_t)} &= \mathbb{E}\left[\left(\theta + \delta_f \frac{1 - r_t^l \theta}{p_{t+1} / p_t}\right) e^{z_t}\right] f_k(\kappa + k_t, h_t) + \frac{\mathrm{Cov}\left[u'(c_t), \frac{r_t}{p_t / p_{t-1}}\right]}{\mathbb{E}\, u'(c_t)} \\ \frac{v'(h_t)}{\mathbb{E}\, u'(c_t)} &= \mathbb{E}\left[\left(\theta + \delta_f \frac{1 - r_t^l \theta}{p_{t+1} / p_t}\right) e^{z_t}\right] f_h(\kappa + k_t, h_t) + \frac{\mathrm{Cov}\left[u'(c_t), \frac{w_t}{p_t}\right]}{\mathbb{E}\, u'(c_t)} \end{aligned} \tag{26}$$

with

$$c_t = e^{z_t} f(\kappa + k_t, h_t) - k_{t+1} \tag{27}$$

given the stochastic process z_t (with $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, \ldots$ an initial price $p_0 > 0$, and an initial endowment $k_1 > 0$, as well as θ.

A few important remarks follow from the characterization above.

Firstly, Proposition 5.3 holds regardless the way in which financial intermediaries are modeled. This is the consequence of it being driven exclusively by relations between households' optimal marginal rates of substitution and firms' optimal marginal

[30] Determined by the (unmodeled, since unnecessary, see the remarks next) financial intermediaries optimizing.

productivities that need to hold (to exhaust gains from trade) at equilibrium in *any* model, independently of how the model might be completed to spell out financial intermediation explicitly. As a consequence, for the purposes of identifying the implications of households and firms optimizing at equilibrium that are being considered here, the financial intermediaries can be made abstraction of.[31] It should be noted, nonetheless, that any modeling of the financial intermediaries needs to capture their ability to create money in the process of lending. As a consequence of this ability, profits π_t^b in (23) are not limited to be the reimbursement and returns of past lending $r_{t-1}^l l_{t-1}$ (along with new deposits d_t) net of the new loans granted l_t (along with the reimbursement and remuneration of past deposits $r_{t-1}^d d_{t-1}$) since that would reduce banks to intermediating existing loanable funds, contradicting their creation of money when extending loans (see Jakab and Kumhof (2019)).

Secondly, the differences in wedges between marginal rates of substitution and marginal productivities driving the results next are, moreover, independent of the fundamentals of the economy —i.e. households' preferences represented by u and v, as well as firms' technology f and equity κ. Only the conditions under which firms can borrow —namely, the gross interest rate r_t^l and the factor θ by which current and future output can be leveraged as collateral— as well as the factor δ_f by which they discount future profits, drive the results. Indeed, the two equations in (26) —that follow from the households' and firms' FOCs having to advance factors payments, with financial intermediaries— modify the equilibrium wedge between firms' factors productivities and households' corresponding marginal rates of substitution, to take into account the lending rate r_t^l and leverage bound θ.

Thirdly, financial intermediation decreases the distortion with respect to efficient allocations, compared to the distortion in the absence financial intermediaries and of on-credit factors supply.[32] Indeed, the wedges between the household's marginal rates of substitution and firm's productivities that the absence of on-credit factors supply drives —with and without financial intermediation, which are respectively $\mathbb{E}\left[\left(\theta + \delta_f \frac{1-r_t^l\theta}{p_{t+1}/p_t}\right)e^{z_t}\right]$ and $\delta_f \mathbb{E}\left[\frac{1}{p_{t+1}/p_t}e^{z_t}\right]$— are ranked, relative to the complete markets case

[31] See the remarks in the concluding section.

[32] Whenever $\theta > 1$, which is the empirically relevant case, see footnote 27.

—that is to say, the case of a unit "wedge" factor (i.e. no wedge)— as follows

$$\delta_f \mathbb{E}\left[\frac{1}{p_{t+1}/p_t} e^{z_t}\right] < \mathbb{E}\left[\left(\theta + \delta_f \frac{1 - r_t^l \theta}{p_{t+1}/p_t}\right) e^{z_t}\right] \leq 1 \tag{28}$$

implying that financial intermediation reduces the inefficiency,[33] but does not necessarily eliminate it, as stated in the proposition next.

Proposition 5.4. *Financial intermediation improves the efficiency of the allocation of resources —if productivity shocks and inflation do not have a too strong negative covariance— without necessarily delivering full efficiency.*

The conditions under which full efficiency is attained will be treated in Section 7.

Fourth, from Proposition 5.3 follows that a market equilibrium steady state[34] allocation without on-credit factors supply but financial intermediation is, therefore, characterized by the conditions in the Proposition 5.5 next.

Proposition 5.5. *A market equilibrium steady state, with no on-credit factors supply but financial intermediation, is necessarily characterized by factors k, h, credit prices $r_t^l > 0$, and an output prices process $p_t > 0$, satisfying*

$$\begin{aligned} \frac{1}{\delta_h} &= \left[\theta + \delta_f \mathbb{E}\, \frac{1 - r_t^l \theta}{p_{t+1}/p_t}\right] f_k(\kappa + k, h) \\ \frac{v'(h)}{u'(c)} &= \left[\theta + \delta_f \mathbb{E}\, \frac{1 - r_t^l \theta}{p_{t+1}/p_t}\right] f_h(\kappa + k, h) \end{aligned} \tag{29}$$

with

$$c = f(\kappa + k, h) - k \tag{30}$$

given θ.

At a steady state, the banks' ability to create money, with the households' and firms' constraints binding, implies the following steady state relation between money creation and price inflation, where m_t is the amount of money created in period t.[35]

[33] In the case that positive productivity shocks are not too disinflationary (see the proof of Proposition 5.4 in Appendix).

[34] See footnote 23.

[35] The proof follows straightforwardly from the aggregation, at any period, of the household's and firm's binding constraints, along with the feasibility condition.

Proposition 5.6. *At a market equilibrium steady state k, h, with no on-credit factors supply but financial intermediation, price inflation and bank money creation satisfy*

$$\frac{m_t / p_t}{f(\kappa + k, h)} = 1 - \frac{1}{p_t / p_{t-1}} \tag{31}$$

implying that the higher the ratio of per period created money over produced output, the higher the period inflation of prices.

Besides the quantitiative theoy of money flavour of Proposition 5.6, it can be noted in Proposition 5.5 that the steady state equilibrium *allocation* is, with financial intermediation, not pinned down yet since it depends on the constant ratio

$$\mathbb{E}\, \frac{1 - r_t^l \theta}{p_{t+1} / p_t} \tag{32}$$

in terms of the gross lending rate r_t^l and the expected reciprocal of the inflation factor $\mathbb{E}\frac{1}{p_{t+1}/p_t}$, given a degree of leverage θ. As a consequence, different combinations of inflation targets $\frac{p_{t+1}}{p_t}$ and rate policies r_t^l pin down different steady states factors h, k and, hence, consumption c, which allows to wonder what combinations decentralize the first-best steady state. This question will be addressed in Section 7.

It is worth distinguishing the seeming indeterminacy in Proposition 5.5 from the classical nominal indeterminacy of flexible-price monetary models absent a nominal anchor. In the latter, indeterminacy concerns the price level and is payoff-irrelevant: any two equilibria differing only by a rescaling of nominal prices are identical in every real respect. Here, by contrast, only a knife-edge affine relationship between the inflation target and the lending rate, given by (41) below, restores the planner's steady state — the vast majority of (target, rate) pairs do not. Moreover, the same model, absent access to financial intermediation, delivers a fully determinate steady-state inflation rate of exactly zero (Proposition 4.5). The seeming indeterminacy is therefore not an artifact of an unspecified monetary policy rule, but a direct consequence of the credit market structure: it is financialization itself, not the absence of a nominal anchor, that leads to it.

As a matter of fact, the apparent indeterminacy in Proposition 5.5 is not a failure of the model to close, in the sense of leaving the equilibrium under-identified given the policy environment. Actually, for any given, exogenously fixed lending rate r_t^l, condi-

tion (41) below pins down a unique steady-state inflation factor p_{t+1}/p_t — the equilibrium is fully determinate conditional on the policy rate. What Proposition 5.5 characterizes, along with condition (41), is not an indeterminacy of the equilibrium given policy, but the set of policy choices available to the monetary authority: the one-dimensional freedom in (target, rate) pairs reflects the authority's genuine latitude in selecting which inflation target to aim in order to implement the first-best steady state, each supported by its corresponding lending rate, rather than a mathematical underdetermination left by an unspecified policy rule. In this sense the result is closer in spirit to the multiplicity of Ramsey-optimal policy instruments delivering the same allocation than to the classical nominal indeterminacy that arises from omitting any nominal anchor altogether.

Finally, and most importantly, since the market steady state k, h in (29) depends on the expected reciprocal of inflation $\mathbb{E}\frac{1}{p_{t+1}/p_t}$, the question of what is the inflation that should be targeted, if any, in order to implement the first-best steady state can be addressed. In order to provide an answer, I characterize first in the next section the planner's steady state levels k, h and c of capital, labor supply, and consumption.

6 Planner's allocations

The planner's allocations provide the welfare benchmark against which the decentralized equilibria are evaluated. They are therefore characterized here to address in this next section whether monetary policy can decentralize this benchmark.

The planner maximizes the representative household's objective under the feasibility constraint, i.e. it solves

$$\begin{aligned} \max_{0\leq h_t, k_{t+1}} \ & \mathbb{E}\sum_{t=1}^{+\infty} \delta_h^{t-1}\big[u(c_t) - v(h_t)\big] \\ & c_t + k_{t+1} \leq e^{z_t} f(\kappa + k_t, h_t) \end{aligned} \tag{33}$$

—given the stochastic process z_t (with $\mathbb{E}\, e^{z_t} = 1$) and an initial $k_1 > 0$— whose solution is characterized next.

Proposition 6.1. *A planner's allocation h_t, k_{t+1} is necessarily characterized by*

$$\begin{aligned} \frac{\mathbb{E}\, u'(c_{t-1})}{\delta_h \mathbb{E}\left[u'(c_t)e^{z_t}\right]} &= f_k(\kappa + k_t, h_t) \\ \frac{v'(h_t)}{\mathbb{E}\left[u'(c_t)e^{z_t}\right]} &= f_h(\kappa + k_t, h_t) \end{aligned} \tag{34}$$

with

$$c_t = e^{z_t} f(\kappa + k_t, h_t) - k_{t+1} \tag{35}$$

given the stochastic process z_t (with $\mathbb{E}\, e^{z_t} = 1$), for all $t = 1, 2, ...$ and an initial $k_1 > 0$.

A planner steady state[36] allocation is, therefore, necessarily characterized by the conditions in the proposition next, which shows it to coincide with the complete markets equilibrium steady state allocation.

Proposition 6.2. *A planner's steady state k, h is necessarily characterized by*

$$\begin{aligned} \frac{1}{\delta_h} &= f_k(\kappa + k, h) \\ \frac{v'(h)}{u'(c)} &= f_h(\kappa + k, h) \end{aligned} \tag{36}$$

with

$$c = f(\kappa + k, h) - k \tag{37}$$

7 Optimal inflation targets and implementing lending rates

It follows from Proposition 5.5 and Proposition 6.2, that the market decentralization of the planner steady state requires to have the wedge between the household marginal rates of substitution and factor productivities in (29) removed, i.e. to choose an inflation target *and* a lending rate satisfying the condition next

$$\theta + \delta_f\, \mathbb{E}\, \frac{1 - r_t^l \theta}{p_{t+1}/p_t} = 1 \tag{38}$$

[36] See footnote 23.

so that the optimal steady state (reciprocal of the) inflation factor is, in expectation,

$$\mathbb{E}\,\frac{1}{p_{t+1}/p_t} = \frac{1}{\delta_f}\cdot\frac{\theta-1}{r_t^l\theta-1} \tag{39}$$

or, approximately,[37]

$$\frac{p_{t+1}}{p_t} = \delta_f\cdot\frac{r_t^l\theta-1}{\theta-1} \tag{40}$$

which *depends on the lending rate*.

Following from the result above, it is worth noting that

1. there is actually a range of (expected reciprocals of) gross inflation rates —and corresponding lending rates— implementing the planner's steady state, and this range is unbounded

2. a steady state (expected reciprocal of) inflation target is optimal if, and only if, the (gross) lending rate is set at

$$r_t^l = \frac{1}{\theta}\cdot 1 + (1-\frac{1}{\theta})\cdot\frac{1}{\delta_f}\frac{p_{t+1}}{p_t} \tag{41}$$

In (41) the gross lending rate rendering an inflation target optimal appears to be a convex combination[38] of 1 and the inflation target $\frac{p_{t+1}}{p_t}$,[39] *compounded with* the return implied by firms' discounting $\frac{1}{\delta_f}$ —the weight on 1 increasing with the degree of collateralisation of firms' borrowing $\frac{1}{\theta}$, thus (in case of inflation) the gross lending rate should be lower the less firms can leverage the period output, and conversely higher (and eventually over inflation) the more they can leverage it.

Equation (41) makes clear that, for any leverage degree θ of firms' period output as borrowing collateral, and any discount δ_f by firms, the implementation of the planner's steady state can be attained with a variety of inflation targets, depending on the lending rate that the monetary authority may choose to implement through its policy rate.

[37] For an equilibrium price process p_t such that (most of) the support of $\frac{p_{t+1}}{p_t}$ is within a small enough neighborhood around 1, which is to be expected except in rare episodes of hyperinflation or even rarer rapid deflations.

[38] If $\theta > 1$, which is the empirically relevant case (see footnote 27).

[39] To be precise, the reciprocal of the expected reciprocal of the inflation target $\frac{1}{\mathbb{E}\frac{p_{t+1}}{p_t}}$.

Figure 1 below shows the dependence of the optimal inflation factor $\frac{p_{t+1}}{p_t}$ on its implementing gross nominal lending rate r_t^l, for a firm discounting $\delta_f = .98$,[40] and a sequence of leverage factors $\theta = 1.2, 1.5, 2, 2.5, 3, 3.5, 4$, in the interval $[1.2, 4.0]$,[41] as implied in (40) or (41). The central vertical dashed blue line corresponds to an optimal inflation of 2% —the US Federal Reserve and ECB targets, among other major central banks. The two other dashed vertical blue lines correspond to targets of 1.5% and 2.5%.

It follows from the relation found —according to the ordinates at the crossings, in Figure 1, of the dashed blue 2% inflation target and the affine lines generated by (41)— that for a 2% inflation target to be optimal, the implementing lending rate runs from 0.68%,[42] to 3.06%,[43] for a firms' leverage range of $1.2 - 4.0$ times the value of their period output. The upper limit for the lending rate implementing an optimal 2% inflation is 4.08%,[44] —the ordinate at the crossing of the dashed blue 2% inflation target and the dashed red limit with slope $\delta_f = .98$. Slightly higher policy rates are needed, for each degree of leverage, for a 2.5% inflation target to be optimal while, similarly, they need to be slightly lower at each leverage for a 1.5% inflation target.

As the next subsection shows, realistic variations for the value of δ_f do not, by continuity in (41), change much the implementing lending rate values obtained, nor would different reasonable inflation targets, above or below 2%, with the only caveat that higher inflation targets can only be optimal for higher lending rates for any given maximum leverage of firms output, and symmetrically for lower inflation targets.

[40] This choice is consistent with calibrations used to match US firm dynamics in Clementi and Palazzo (2016) and European firm-level date in Arellano et al. (2012). Importantly, it is consistent too with the evidence used in Luttmer (2011) on exit rates of large, mature firms which are the relevant ones here since we are focusing on steady states, while representing a conservative choice (smaller, younger firms have higher exit rates) that only strengthens the results, since they are thus shown to hold even with firms in the lower end of a reasonable discounting range. At any rate, by continuity in (41), the specific value is without consequence for the essence of the results.

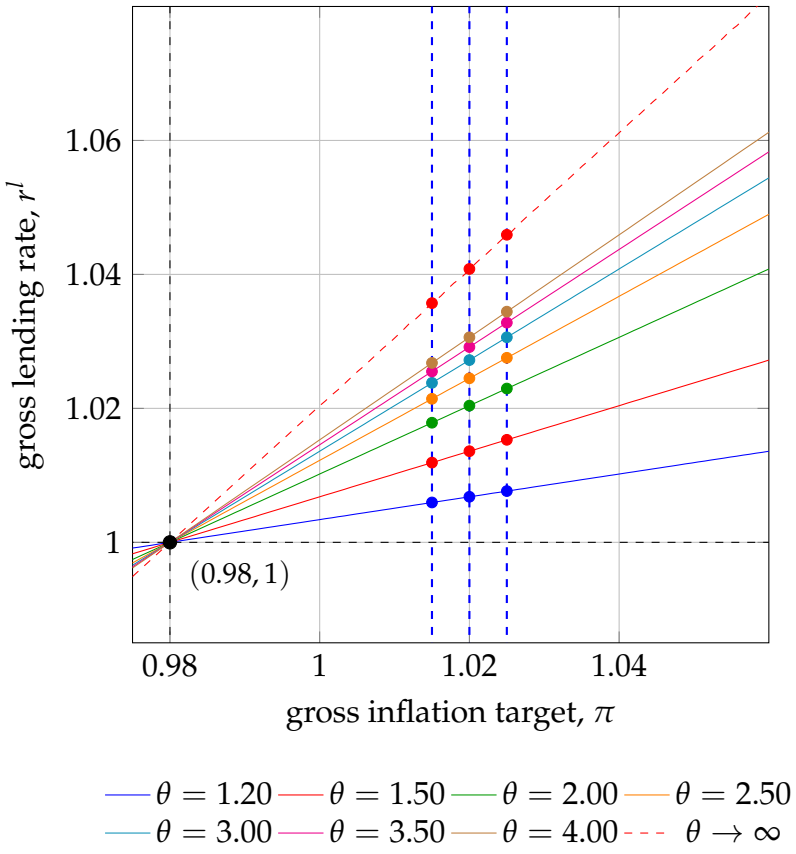
[41] The leverage parameter is calibrated as $\theta = \lambda/s$, where s is the share of the debt of non-financial corporations based on specific-asset liquidation value, and hence secured, and λ is the average loan-to-value ratio on that debt. Lian and Ma (2021) find $s = 0.2$ for U.S. nonfinancial firms. Since privately-held firms —which rely more on secured bank lending and account for roughly 60% of total outstanding debt of U.S. nonfinancial firms— push the value-weighted s above the Lian-Ma large-firm estimate, we take $s \in [0.2, 0.5]$ as the relevant range —indeed, Berger and Udell (1998) report that about 40% of small business loans and close to 60% of loan dollars are guaranteed and/or secured. Corporate-finance calibrations in Falato et al. (2022) put loan-to-value ratios on genuinely asset-based corporate lending at $\lambda \in [0.6, 0.8]$. This implies $\theta = \lambda/s \in [1.2, 4.0]$.

[42] Lending rate of the ordinate at 1.02 for $\theta = 1.2$ in Figure 1, up to two decimals.

[43] Lending rate of the ordinate at 1.02 for $\theta = 4.0$ in Figure 1, up to two decimals.

[44] Lending rate of the ordinate at 1.02 for $\theta = \infty$ in Figure 1, up to two decimals.

Figure 1: **Optimal inflation and lending rates, contingent on leverage**
abscissae: optimal inflation as p_{t+1}/p_t ; ordinates: implementing gross lending rate r_t^l (for $\delta_f = .98$ and, from bottom to top, $\theta = 1.2, 1.5, 2, 2.5, 3, 3.5, 4, ...$ with the limit as $\theta \to +\infty$ in dashes).



7.1 Quantitative relevance and robustness

The calibration above is intended to illustrate the theoretical relationship between optimal inflation targets and implementing lending rates rather than to estimate an inflation target for a particular economy. The purpose of this subsection is therefore not structural estimation, but to assess whether the mechanism operates over empirically plausible parameter values.

The characterization in equation (41) shows that the implementing lending rate depends exclusively on three objects: the inflation target, firms' effective discount factor δ_f, and the leverage parameter θ. The remaining features of preferences and technology affect the first-best steady state allocation itself but not the inflation–lending rate relationship required to decentralize it. Consequently, the empirical relevance of the mechanism reduces to asking whether reasonable values of δ_f and θ imply lending rates within the range observed in modern economies.

The benchmark calibration adopts $\delta_f = 0.98$, consistent with the effective discounting commonly used in firm-dynamics models to capture survival probabilities, exit risk and limited contract enforceability —e.g. Luttmer (2011); Clementi and Palazzo (2016); Arellano et al. (2012). Likewise, the leverage interval $\theta \in [1.2, 4.0]$ is chosen to encompass plausible borrowing capacities implied by evidence on corporate leverage and collateralization rates —see Lian and Ma (2021), Chodorow-Reich et al. (2021), Drechsel (2023).

Within this parameter region, for a 2% inflation target to implement the first-best steady state allocation lending rates need to run from 0.68% to 3.06% —extended to $0.4\% - 4.0\%$ for a range of discounting factors $\delta_f \in [.97, .99]$, see Figure 2— with an asymptotic upper bound of 4.08% as leverage becomes arbitrarily large. These values fall comfortably within the range of nominal lending rates observed in advanced economies during periods of relatively stable inflation, suggesting that the mechanism is quantitatively plausible rather than requiring implausible financial conditions. The qualitative prediction —that each inflation target is associated with a unique first-best steady state implementing lending rate for a given leverage ratio— remains unchanged throughout the parameter space considered, indicating that the central result is robust to moderate variations in firms' intertemporal discounting. Figure 2 illustrates that the continuity in (41) guarantees no "knife-edge" situations, and hence robustness.

More importantly, equation (41) implies that the relationship is remarkably stable.

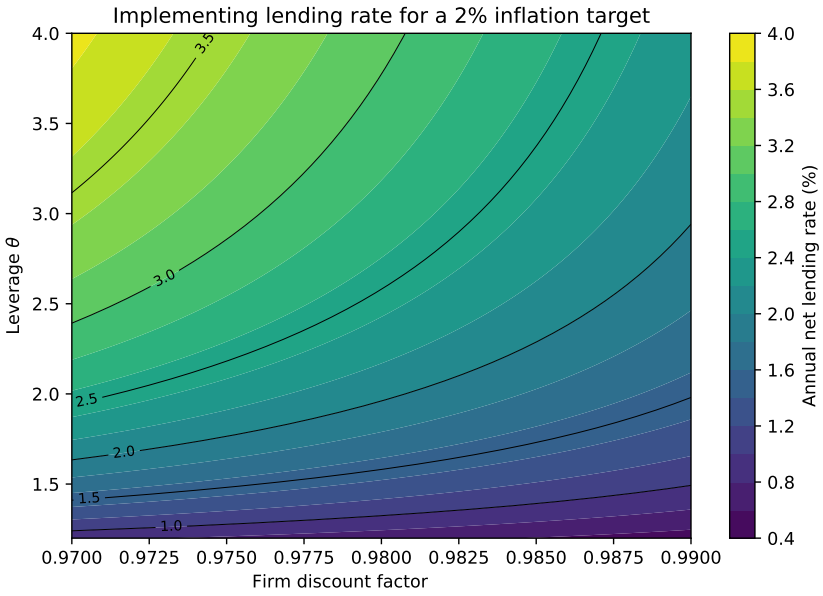


Figure 2: Annual net lending rates implementing a 2% inflation target for alternative values of firms' effective discount factor δ_f and leverage bound θ. The rates are calculated from equation (40). Contour lines report lending rates in percentage points.

Moderate changes in either firms' discount factor or leverage alter the implementing lending rate only gradually because of the mapping's continuity. The qualitative implication—that every inflation target is associated with a first-best steady state implementing lending rate—therefore does not depend on knife-edge calibrations. Higher leverage increases the lending rate compatible with any given inflation target, whereas lower effective discounting shifts the entire schedule almost proportionally without changing its shape.

The quantitative exercise should therefore be interpreted as establishing plausibility rather than identifying the "true" optimal inflation target. The model does not predict that central banks ought to target 2% inflation. Instead, it predicts that, once firms' financing conditions are taken into account, different inflation targets can decentralize the first-best steady state provided that lending conditions adjust consistently with equation (40). The empirical relevance of the mechanism lies precisely in replacing a unique optimal inflation target with a family of optimal inflation–lending-rate combinations whose location depends on observable characteristics of firms' financing opportunities.

Using the Federal Reserve's EEANQ weighted-average effective lending rate on all commercial and industrial loans, equation (40), evaluated at a 2% inflation target and

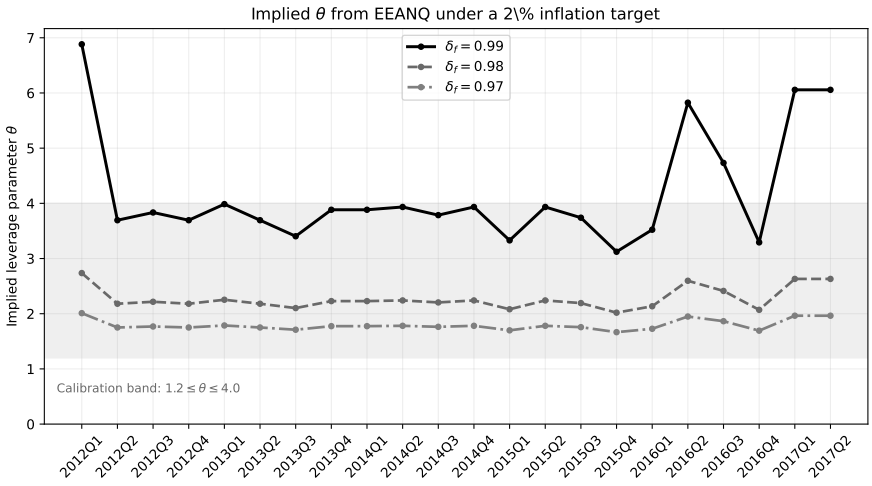


Figure 3: Leverages implied by lending rates —measured by EEANQ series, from the Fed adoption of the 2% target in 2012Q1 through the final observation in 2017Q2)— for the target to be optimal, for alternative values of firms' effective discount factor δ_f, calculated from equation (40). Leverages outside the band of calibrated values call into question the optimality of the target in the quarter.

$\delta_f = 0.98$, implies a leverage parameter between 2.02 and 2.74 over 2012Q1–2017Q2 —from the Fed adoption of the 2% target in 2012Q1 through the final observation in 2017Q2—[45] with a median of 2.22, as shown in Figure 3. These values lie entirely within the calibration range considered in the quantitative exercise, which means that the optimality of the target was not contradicted by the likely leverage during the period. For robustness, the implied leverages are presented for $\delta_f = .97$ and $\delta_f = .99$. Only under the latter assumption of extremely low discounting is the optimality of the target compromised in some quarters. This historical consistency under the U.S. inflation-targeting regime is not a formal validation of the model, but it provides evidence that the affine relationship is compatible with the observed lending rates and plausible leverage values during a period in which the Federal Reserve explicitly targeted 2% inflation.

8 Concluding remarks

It is worth stressing first that the results above follow from core conditions that need to hold at any equilibrium, namely the way households' marginal rates of substitution

[45] This period of relative calm, in between the shocks of the 2007-08 great financial crisis and the COVID-19 pandemic in 2020, is at any rate more likely to help check the validity of a condition that, after all, is supposed to hold in a steady state.

and firms' marginal productivities need to exhaust gains from trade at their respective optimal choices, with or without access to funds intermediated by banks or other financial intermediaries. They are therefore robust to any way in which the model may be completed to make explicit a financial intermediation that accounts for banks' ability to create money. For the same reason, the results are bound to be robust to the introduction of more realistic aspects in the model —like heterogeneity across households and firms— that cannot escape to the logic imposed by the first-order conditions at the heart of the results.

It is also worth noting that the results do not depend on fundamentals of the economy like households' preferences and endowments or firms' technologies, but rather on the conditions under which firms can borrow and how they discount the future, reinforcing their robustness to any way in which the model is completed to explicitly model financial intermediaries choices, since it is not the details of how they behave but their mere presence and borrowing conditions they allow —expressed as r_t^l and θ— that matters for the results to hold. It is interesting too that, while financial intermediation improves the efficiency the allocation, there is the posibility that this is not the case should productivity shocks and inflation co-variate too much.

Finally, that different inflation targets might be optimal when accompanied with the adequate policy rate actually makes room to accommodate, in a broader model of an open economy to be developed, the need to manage exchange rates and trade too. Indeed, if the choice of the lending rate was constrained by exchange rate considerations —say, in the case of an open small economy— the optimal inflation rate would be determined by the leverage of firms, the higher the latter the smaller the optimal inflation, and vice-versa.

9 Appendix

9.1 proof of Proposition 3.1

The firm's problem is, equivalently,

$$\max_{0\leq k_t,h_t} \mathbb{E}\sum_{t=1}^{+\infty}\delta_f^{t-1}\left[e^{z_t}f(\kappa+k_t,h_t)-\frac{r_t}{p_t/p_{t-1}}k_t-\frac{w_t}{p_t}h_t\right] \tag{42}$$

which has as FOCs

$$\begin{pmatrix} \delta_f^{t-1}\Big[f_k(\kappa+k_t,h_t)\mathbb{E}\,e^{z_t}-\mathbb{E}\,\frac{r_t}{p_t/p_{t-1}}\Big] \\ \delta_f^{t-1}\Big[f_h(\kappa+k_t,h_t)\mathbb{E}\,e^{z_t}-\mathbb{E}\,\frac{w_t}{p_t}\Big] \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{43}$$

for all t, whence the claim follows, since $\mathbb{E}\,e^{z_t}=1$.

9.1.1 Proof of Proposition 3.2

The household's problem is, equivalently,

$$\max_{0\le h_t,k_{t+1}} \mathbb{E}\sum_{t=1}^{+\infty}\delta_h^{t-1}\Big[u\big(\frac{\pi_t^f}{p_t}+\frac{w_t}{p_t}h_t+\frac{r_t}{p_t/p_{t-1}}k_t-k_{t+1}\big)-v(h_t)\Big] \tag{44}$$

which has as FOCs

$$\begin{pmatrix} \delta_h^{t-1}\Big[\mathbb{E}[u'(c_t)\frac{w_t}{p_t}]-v'(h_t)\Big] \\ \delta_h^{t-1}\Big[\delta_h\mathbb{E}[u'(c_{t+1})\frac{r_{t+1}}{p_{t+1}/p_t}]-\mathbb{E}u'(c_t)\Big] \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{45}$$

with

$$c_t=\frac{\pi_t^f}{p_t}+\frac{w_t}{p_t}h_t+\frac{r_t}{p_t/p_{t-1}}k_t-k_{t+1}$$

for all t, whence the claim follows.

9.1.2 Proof of Proposition 4.1

The firm's problem is, equivalently,

$$\max_{0\le k_t,h_t} \mathbb{E}\sum_{t=1}^{+\infty}\delta_f^{t-1}\Big[\frac{1}{p_t/p_{t-1}}e^{z_{t-1}}f(\kappa+k_{t-1},h_{t-1})-\frac{r_t}{p_t/p_{t-1}}k_t-\frac{w_t}{p_t}h_t\Big] \tag{46}$$

which has as FOCs

$$\begin{pmatrix} \delta_f^{t-1}\Big[\delta_f\mathbb{E}[\frac{1}{p_{t+1}/p_t}e^{z_t}]f_k(\kappa+k_t,h_t)-\mathbb{E}\frac{r_t}{p_t/p_{t-1}}\Big] \\ \delta_f^{t-1}\Big[\delta_f\mathbb{E}[\frac{1}{p_{t+1}/p_t}e^{z_t}]f_h(\kappa+k_t,h_t)-\mathbb{E}\frac{w_t}{p_t}\Big] \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{47}$$

for all t, whence the claim follows.

9.1.3 Proof of Proposition 5.1

The firm's problem is, for expected real lending rates below firms' implied discounting return, [46] equivalently,

$$\max_{0\leq k_t,h_t} \mathbb{E}\sum_{t=1}^{+\infty}\delta_f^{t-1}\Big[\frac{1-r_{t-1}^l\theta}{p_t/p_{t-1}}e^{z_{t-1}}f(\kappa+k_{t-1},h_{t-1})+e^{z_t}f(\kappa+k_t,h_t)\theta-\frac{r_t}{p_t/p_{t-1}}k_t-\frac{w_t}{p_t}h_t\Big] \tag{48}$$

which has as FOCs

$$\begin{pmatrix}\delta_f^{t-1}\Big[\mathbb{E}\big[(\theta+\delta_f\frac{1-r_t^l\theta}{p_{t+1}/p_t})e^{z_t}\big]f_k(\kappa+k_t,h_t)-\mathbb{E}\frac{r_t}{p_t/p_{t-1}}\Big]\\ \delta_f^{t-1}\Big[\mathbb{E}\big[(\theta+\delta_f\frac{1-r_t^l\theta}{p_{t+1}/p_t})e^{z_t}\big]f_h(\kappa+k_t,h_t)-\mathbb{E}\frac{w_t}{p_t}\Big]\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix} \tag{49}$$

for all t, whence the claim follows.

9.1.4 Proof of Proposition 5.2

The household's problem is, equivalently,

$$\max_{0\leq h_t,k_{t+1},d_t} \mathbb{E}\sum_{t=1}^{+\infty}\delta_h^{t-1}\Big[u\big(\frac{\pi_t^f}{p_t}+\frac{\pi_t^b}{p_t}+\frac{w_t}{p_t}h_t+\frac{r_t}{p_t/p_{t-1}}k_t-k_{t+1}+\frac{r_{t-1}^d}{p_t/p_{t-1}}\frac{d_{t-1}}{p_{t-1}}-\frac{d_t}{p_t}\big)-v(h_t)\Big] \tag{50}$$

which has as FOCs

$$\begin{pmatrix}\delta_h^{t-1}\Big[\mathbb{E}[u'(c_t)\frac{w_t}{p_t}]-v'(h_t)\Big]\\ \delta_h^{t-1}\Big[\delta_h\mathbb{E}[u'(c_{t+1})\frac{r_{t+1}}{p_{t+1}/p_t}]-\mathbb{E}u'(c_t)\Big]\\ \delta_h^{t-1}\Big[\delta_h\mathbb{E}[u'(c_{t+1})\frac{r_t^d}{p_{t+1}/p_t}]-\mathbb{E}u'(c_t)\Big]\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \tag{51}$$

with

$$c_t=\frac{\pi_t^f}{p_t}+\frac{\pi_t^b}{p_t}+\frac{w_t}{p_t}h_t+\frac{r_t}{p_t/p_{t-1}}k_t-k_{t+1}+\frac{r_{t-1}^d}{p_t/p_{t-1}}\frac{d_{t-1}}{p_{t-1}}-\frac{d_t}{p_t}$$

for all t, whence the claim follows.

[46] That is to say, for

$$\mathbb{E}\,\frac{r_t^l}{p_{t+1}/p_t}<\frac{1}{\delta_f}$$

so that the borrowing constraint is binding.

9.1.5 Proof of Proposition 5.4

The first inequality in (28) is, after cancelling out terms and factors,

$$\mathbb{E}\left[\left(\delta_f \frac{r_t^l}{p_{t+1}/p_t} - 1\right) e^{z_t}\right] < 0 \tag{52}$$

i.e.

$$\mathbb{E}\left(\delta_f \frac{r_t^l}{p_{t+1}/p_t} - 1\right)\mathbb{E}\, e^{z_t} + \mathrm{Cov}\left[\left(\delta_f \frac{r_t^l}{p_{t+1}/p_t} - 1\right), e^{z_t}\right] < 0 \tag{53}$$

where the first factor of the first term in the LHS (the second is 1) is negative when firms' borrowing is binding, and the sum will also be negative even with a positive covariance[47] if it remains below the the opposite of the first term.

As for the second inequality, it is

$$\mathbb{E}\left[\left(\theta + \delta_f \frac{1 - r_t^l \theta}{p_{t+1}/p_t} - 1\right) e^{z_t}\right] \leq 0 \tag{54}$$

—after substituting $\mathbb{E}\, e^{z_t}$ to 1 in its RHS and rearranging— i.e.

$$\mathbb{E}\left(\theta + \delta_f \frac{1 - r_t^l \theta}{p_{t+1}/p_t} - 1\right)\mathbb{E}\, e^{z_t} + \mathrm{Cov}\left[\left(\theta + \delta_f \frac{1 - r_t^l \theta}{p_{t+1}/p_t} - 1\right), e^{z_t}\right] < 0 \tag{55}$$

the first factor of the the first term in the LHS (the second is 1) is negative since

$$\delta_f \mathbb{E}\, \frac{1}{p_{t+1}/p_t} - 1 < \left[\delta_f \mathbb{E}\, \frac{r_t^l}{p_{t+1}/p_t} - 1\right]\theta \tag{56}$$

whenever $r_t^l, \theta > 1$ (see footnote 27 on the latter inequlity), while the second term is negative in that same case whenever positive shocks are disinflationary .

9.1.6 Proof of Proposition 6.1

The planner's problem is equivalently

$$\max_{0 \leq h_t, k_{t+1}} \mathbb{E} \sum_{t=1}^{+\infty} \delta_h^{t-1} \left[u(e^{z_t} f(\kappa + k_t, h_t) - k_{t+1}) - v(h_t)\right] \tag{57}$$

[47] Positive productivity shocks tend to be disinflationary.

has as FOCs

$$\begin{pmatrix} \delta_h^{t-1}\left[\mathbb{E}\left[u'(c_t)e^{z_t}\right]f_h(\kappa+k_t,h_t)-v'(h_t)\right] \\ \delta_h^{t-1}\left[\delta_h\mathbb{E}\left[u'(c_{t+1})e^{z_{t+1}}\right]f_k(\kappa+k_{t+1},h_{t+1})-\mathbb{E}\,u'(c_t)\right] \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{58}$$

with

$$c_t = e^{z_t}f(\kappa+k_t,h_t)-k_{t+1} \tag{59}$$

for all t, whence the claim follows.

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