# Technifying Ventures*†

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**Abstract**

The adoption of advanced technologies has important implications for employment and growth. Advanced technologies are often embraced by innovative startups, which are commonly funded by venture capital. Stylized facts are compiled, using US Census data, regarding the adoption of advanced technologies by startups and the source of funding that a startup draws upon. The relationship between technology adoption and the source of funding, on the one hand, and short- and longer-run employment and output, on the other, is studied. A model of startups is then constructed featuring decisions about technology adoption and whether venture capital funding is used. The model is matched up with Census facts about startups, employment, technology adoption, and the funding source. The implications of business taxation and subsidies for startups are examined.

*Keywords*: Advanced technology, banks, capital gains taxation, corporate income taxation, employment, reallocation effect, startups, subsidies, synergy, venture capital, technology adoption, US Census data

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*This is a report on research in progress. As such, it is *preliminary, incomplete, and subject to change.*

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# 1 Opening

Technological progress drives economic growth. Innovative startups are often the accelerants for, or developers of, new technologies. These startups play an important role in creating employment, generating growth, and encouraging the diffusion of advanced technologies. What determines the adoption of advanced technology by a startup? How does this adoption associate with a firm's employment and revenue in the short, medium, and long run? How are startups funded? Is the source of funding related to the success of a startup and technology adoption?

The startup process is studied here. In particular, the relationship between firm outcomes (employment and revenue), the adoption of advanced technologies, and the source of funding is examined. Venture capital plays an important role in the answer to the above questions. First, venture capital directs resources to places in the economy where they will have the highest return, in particular promising startups. Second, unlike banks, venture capitalists mentor startups, providing valuable advice as well as funding.

The analysis has two parts. The first part uses US Census Bureau data to document some facts concerning the relationship between venture capital, firm outcomes, and the adoption of advanced technologies (high-tech) by startups firms. Comprehensive and timely measurement of advanced technology use by US firms has been a challenge because of a lack of representative data for US firms that also contains information on advanced technology presence in firms. To fill this gap, the Census Bureau has introduced new questions on advanced technology adoption and use in several of its recent surveys. In particular, the technology modules included in the Annual Business Survey (ABS) for the years 2018-2023 have collected data at the firm level on the use of advanced technologies such as Artificial Intelligence, Robotics, and many more. Using this data, the outcomes of high-tech startups in terms of employment and revenue are compared here with those of other startups that don't adopt advanced technologies or are financed by other sources such as banks.

In the second part, a model is built to study the relationship between the adoption of advanced technologies by startups and the source of financing that startups use—either banks or venture capitalists. A venture is born by an idea that an entrepreneur has. Some ideas have more potential than others. Additionally, some ideas are better suited for the use of advanced technologies, which affect their expected payoff. Moreover, some entrepreneurs are more able than others to implement advanced technologies. Entrepreneurs take their fledgling startups to either a bank or a venture capitalist for funding. Banks finance startups by issuing a loan at a fixed interest rate. Venture capitalists provide funding in return for a share of the project's proceeds. Unlike banks, venture capitalists offer advice to startups. This requires effort by venture capitalists and the more ambitious the project is, the more time a venture capitalist will have to devote to nurturing the startup. Venture capitalists are compensated for this effort in terms of their share of any payoff. This share is subject to negotiation between the entrepreneur and venture capitalist. Successful startups are either sold through an initial public offering or through a merger and acquisition. The entrepreneur employs the source of funding that yields the highest expected payoff. Entrepreneurs make their technology adoption decision knowing the expected payoff for each type of technology, contingent on projects' potentials with each type of technology. They also make this technology choice factoring in how it will affect their funding source, which in turn impinges on the expected success of a venture.

The developed model is then calibrated to match a set of stylized facts regarding technology adoption by startups and the financing of startups by venture capital and other sources. The calibrated model is used to examine the impact of two public policies. First, the effect of capital gains and corporate income taxation is studied. These taxes influence the value that startups can be sold at and hence the incentives to adopt high-tech as well as the form of finance that is used. Second, sway of subsidies on startup formation is examined. Here two types of startup subsidies are entertained: subsidies for all startups and subsidies only for the ones that use advanced technologies.

## 2 The Advanced Technologies Studied

Five different advanced technologies are examined; namely, additive manufacturing, artificial intelligence (AI), distributed ledgers, radio-frequency identification (RFID), and robotics. These technologies were chosen as data on them were more uniformly and consistently collected in various ABS instruments during the 2018-2023 period, allowing for a large sample for the analysis.

RFID is a technology where a tag is attached to an item allowing the item to be identified and tracked. Upon activation by an electromagnetic field, the tag can transmit information to a radio receiver. The first patent for a RFID device was issued to Mario Cardullo in 1973. The business plan was to use the technology for things such as credit cards, toll systems, etc. Today the technology has a myriad of applications: access control, biometric information retrieved from passports and driver licenses, contactless credit card payments, inventory control, tracking livestock, pets, and shipments.

In 1949 William Grey Walter built battery-powered robots with three wheels that could maneuver around objects in a room. The robots resembled tortoises. They could return to their charging station when low on battery power. The robots were capable of phototaxis and move toward a source of light. Their key ingredients are incorporated into robots today: sensor technologies, feedback loops, and some form of decision making. Ten years later General Motors introduced the first robotic arm in a factory that could lift and stack hot metal parts.

A robot that incorporated AI, nicknamed Shakey, was built in 1972 by engineers at the Stanford Research Institute. Shakey was capable of breaking down commands into the steps necessary to complete a requested action. For example, it could navigate to a specified room, find a desired block, and then push it to another requested location. Today the uses of robotics are widespread. While robotics is extensively used in manufacturing, its uses have spread to other industries and the home with robotic vacuum cleaners and lawn mowers. Robots deliver food in hospitals and aid in surgery. Robotics has law enforcement and military applications (e.g., intelligence gathering, weapon systems). It is also used in space exploration.

While Alan Turing contemplated the concept of artificial intelligence in 1950. The first proof of concept was a computer program developed in 1956 by Allen Newell, Cliff Shaw, and Herbert A. Simon. The program named The Logic Theorist proved 38 of the theorems in Alfred North Whitehead's and Bertrand Russell's *Principia Mathematica*. According to Simon, the computer program was not well received at the historic 1956 Dartmouth Summer Research Project on Artificial Intelligence. An important advancement in AI was the creation of the ability for a computer to communicate with people in a natural language. In 1967 Joseph Weizenbaum created a program Eliza that could do just this using pattern matching. The early development of AI was limited by computing power. This power increased over time and in 1997 IBM's Deep Blue computer beat in a chess game the reigning world champion, Gary Kasparov. Also, techniques from economics, engineering, mathematics, and operations research were incorporated into AI programs. AI has many applications currently. It is used by advertisers to make recommendations to potential customers, by car companies to make vehicles more autonomous, by credit card agencies to detect fraud, in health care to spot diseases, in facial recognition programs for law enforcement, in various types of robots, and in voice assistants such as Alexa.

Johannes F. Gottwald patented the Liquid Metal Recorder in 1971. His machine was a modified ink jet printer. It warmed metals till they became liquid and then transferred them onto a platform while shaping them into a desired form. This was the birth of additive manufacturing (or 3D printing). Additive manufacturing has many benefits. For certain projects it allows rapid prototyping: one can go directly from design to production. In addition it can produce objects with complex shapes, allowing more freedom in design. It cuts down on waste because it eliminates cutting and milling materials. It reduces components that need to be assembled. Last, manufacturing can be done on demand.

A distributed ledger is a decentralized system for recording and verifying transactions. The ledger is comprised of data that is synchronized and shared across many locations and institutions via peer-to-peer networks using cryptography. Each node saves an identical copy of the ledger. Updates need to be verified by a majority of the nodes before a new ledger is posted uniformly across the network. The benefits are security and transparency. It also eliminates the need for a central authority to monitor transactions. The idea was introduced by Satoshi Nakamoto in 2008.

## 3 Literature Review

Using the technology module in the Census's 2018 ABS, Zolas et al. (2020) provide the very first comprehensive estimates of firms' adoption of AI and Robotics in recent years. McElheran et al. (2023), again utilizing the 2018 ABS, examine specifically the connection between early firm characteristics and AI adoption, as well as how startup growth is associated with AI use. Acemoglu et al. (2022) document, based on a different technology module included in the 2019 ABS, the prevalence of five technologies (AI, Robotics, Cloud Computing, Specialized Equipment and Specialized Software) among US firms and the effects of these technologies on firms' workforce, as self-reported by firms. They study the connection between firm characteristics and technology adoption, and the relationship between firm outcomes and technology use. Bonney et al. (2024) utilize data from Business Trends and Outlook Survey to offer the latest estimates of current and expected near-term AI use rate by firms. Dinlersoz, Dogan, and Zolas (2024) use administrative micro data underlying Census Bureau's Business Formation Statistics to study business applications that aim to develop or use AI technologies. They compare the characteristics of AI-related business applications with those of other business applications, and study the performance of businesses originating from these applications in a number of outcomes, including employment, revenue, and failure rate.

A few stylized facts have emerged from this empirical literature so far. First, the adoption rates of advanced technologies, such as AI and Robotics, are relatively small–for instance, AI is currently used only by around 6% of firms in producing goods or services (as of Fall 2024), up from about 3.2% for the period 2016-2018. Second, the adoption of advanced technologies is concentrated in large firms, and in young firms, controlling for size. Third, advanced technology users tend to exhibit better overall performance. In addition to being larger in terms of employment and revenue, they have higher labor productivity and a lower labor share. Furthermore, early characteristics of firms, including owner motivations and funding type in initial stages, are related to AI adoption. In particular, AI adoption is positively associated with early growth in startups. At the same time, users of advanced technologies overwhelmingly report overall upskilling of their workforce, together with an increase in STEM skills. However, there is little evidence that technology use necessarily results in instances of employment decline. For most technologies, a large fraction of firms report experiencing no change in employment. For those firms that experience a change in employment due to technology, an increase in employment seems to be more common than a decrease–though the relative incidences of the two outcomes depend on the technology. Despite the positive association between firm performance and technology presence, the analysis has so far not established a more causal link between firm outcomes and advanced technology use. (This paper aims to conduct additional empirical analysis towards more formally establishing a causal relationship.)

Macroeconomic models of venture capital financing are rare. Akcigit et al. (2022), Ando (2023), and Ates (2018) are three examples. Akcigit et al. (2022) show empirically, using US Census data, that startups financed by venture capital outperform those that aren't. They build a quantitative model explaining this fact that emphasizes the synergism between an entrepreneur and a venture capitalist. This synergy is absent between a banker and an entrepreneur in their model. Ando (2024) adds angel investors into the mix. He documents that firms financed by venture capitalist do better than those financed by angel investors, who in turn do better than those using banks. A firm dynamics model of this process is then calibrated and matched with the data. A Schumpeterian growth model incorporating venture capital is advanced by Ates (2022). Firms choose how far to launch their productivities relative to incumbents, a feature also in Akcigit et al. (2022). The model is not matched with data on firm startups. None of these papers examine the role that venture capital plays in incentivizing the adoption of advanced technologies. In addition, the existing papers do not explore the potentially different contribution of venture capital to high-tech versus other startups.

# 4 Data

The empirical analysis employs three data sets. The Annual Business Survey (ABS), conducted from 2018 to 2023 and designed jointly by the US Census Bureau and the National Center for Science and Engineering Statistics, provides information on advanced technology use among nonfarm employer businesses in the United States. Each survey collects data for the reference year that is prior to the survey year (e.g. the 2018 ABS collects data for the year 2017). The ABS survey is detailed in Appendix A. 850,000 businesses were sampled in the 2018 and 2023 ABS covering the Economic Census years of 2017 and 2022, and around 300,000 businesses each year from 2019 to 2021. Examples of the business technologies in the survey include artificial intelligence, robotics, cloud computing, RFID, specialized software, and specialized equipment. ABS provides information on whether a firm has used any given advanced technology during the survey reference period. In addition, the 2023 ABS contains information on the timing of the adoption of advanced technologies, measured by 5-year intervals.[1]

The presence of VC financing as a source of initial capital at the founding of a business is captured in certain years of the ABS. This information on VC financing is supplemented with data from Pitchbook and data on initial public offerings (IPOs). As a result, VC funding is identified for all firms in the ABS for the period 2017-2023.[2] The information on VC financing deals, such as deal size and equity stakes acquired by investors, is taken from Pitchbook–again, see Appendix A. This dataset is merged with the Census Bureau's Business Register/Standard Statistical Establishment List by name and address matching, and then linked to the ABS using firm identifiers in Census Bureau data sets.

Finally, characteristics of employer businesses, including employment, revenue, firm age, industry, and location are drawn from the Longitudinal Business Database (LBD). The LBD is a longitudinally linked dataset of essentially all nonfarm employer businesses in the United States from 1976 to 2022 (the latest year available at the time of the analysis). The combined data set allows for the evolution of firm size and other outcomes to be examined separately for firms that use or do not use advanced technologies, and for firms that raise or do not raise VC financing.

# 5 Empirical Results

Turn now to some stylized facts on advanced technology adoption and venture capital (VC) funding in a startup and ensuing firm outcomes. Table 1 provides some descriptive statistics, based on the main sample of firms from the Annual Business Survey (ABS) 2018-2023. The advanced technology adoption rate in the sample is 11.63%–sample weights are used so as to make the statistics representative of the US firm population. Despite the relatively small fraction of high-tech firms (i.e., firms that have adopted advanced technologies), they account for 38.76% of total employment and 44.11% of total revenue in the economy–consistent with prior findings from the ABS [see, e.g., Acemoglu et al. (2022)]. The fraction of VC-backed firms is even smaller, 0.64%. Their employment and revenue share in the economy is 13.26% and 14.98%, however, demonstrating that VC-backed firms are significantly larger than non-VC-backed firms and that they constitute an economically significant segment of firms [see also Dinlersoz et al. (2022)].

Table 2 documents the firm-level relationship between VC presence and technology adoption. It also shows the relationship between VC presence and technology adoption, on the one hand, and employment and revenue, on the other. An OLS regression is employed on a pooled cross-section for the measurement years 2017-2022. Fixed effects are included for year-industry (4-digit NAICS)

[1] See the ABS website (https://www.census.gov/programs-surveys/abs.html) for additional information.

[2] The SDC New Issues and the IPO dataset combined and released by Jay Ritter are used.

Table 1: Descriptive Statistics

| | **Fraction (%)** | **Employment Share (%)** | **Revenue Share (%)** |
|---|---|---|---|
| Tech Firms | 11.63% | 38.76% | 44.11% |
| VC-backed Firms | 0.64% | 13.26% | 14.98% |

Note: The sample consists of firms in the ABS from 2017 to 2022. Tech firms are defined as firms that have adopted advanced technologies. VC-backed firms are firms that have raised VC financing, according to the ABS (2018) or Pitchbook. Employment share is the share of employment contributed by tech firms or VC-backed firms out of total employment in the economy. Employment and revenue are obtained from the LBD (2017-2022). For firms whose revenue is missing in the LBD, sales in the ABS are used instead. All statistics are computed using the LBD weights.

Table 2: Baseline Regression

| | **(1)** | **(2)** | **(3)** |
|---|---|---|---|
| | **adoption** | ln(**emp**) | ln(**rev**) |
| VC-financed | 0.170*** | | |
| | (0.00677) | | |
| VC-tech | | 1.660*** | 1.479*** |
| | | (0.0358) | (0.0553) |
| VC-nontech | | 1.051*** | 1.048*** |
| | | (0.0298) | (0.0404) |
| NonVC-tech | | 0.483*** | 0.626*** |
| | | (0.00562) | (0.00706) |
| + Fixed effects | | | |
| R-squared | 0.0734 | 0.2403 | 0.2151 |
| N (rounded) | 1,050,000 | 1,050,000 | 1,050,000 |
| Adoption rate (all firms) | 11.63% | | |
| Adoption rate (VC firms) | 38.61% | | |

Notes: The fixed effects are: Year×NAICS (4 digits), state, and firm age. The dependent variable in column (1) is firms that have adopted advanced technologies. The dependent variables in columns (2) and (3) are firm-level ln(employment) and ln(revenue), respectively. The independent variables are binary variables that are equal to one if firms belong to the category. The regressions use LBD weights. Standard errors are shown in parentheses and are clustered by firmid. * p<0.05 ** p<0.01 *** p<0.001. N is rounded to protect confidentiality.

interactions, state, and firm age.[3] Column (1) indicates that VC-backed firms have a significantly higher technology adoption rate than non-VC-backed firms. The technology adoption rate of VC-backed firms is 38.61%, 17 percentage points higher than the overall technology adoption rate in the sample (11.63%).

Columns (2) and (3) in Table 2 compare firm-level employment and revenue, respectively, for each of the four mutually-exclusive groups of firms. Each group is defined by a pair of indicators that identify a distinct combination of VC presence and technology adoption status. VC-tech firms (i.e., firms that have VC backing and that have adopted advanced technologies) have significantly higher employment and revenue than the omitted group of firms that have no VC backing and no advanced technology. The size gap is approximately 425% [$= 100 \times (\exp(1.660) - 1)$] and 338% [$= 100 \times (\exp(1.479) - 1)$], respectively, for employment and revenue, based on the estimated coefficients. VC-backed, non-tech firms also exhibit significantly higher employment and revenue than the omitted group, but less so than VC-tech firms. Non-VC-backed, tech firms rank third in outcomes relative to the omitted group.

Regression analysis in Table 3 controls for a large number of business owner characteristics and firm business strategies/motivations related to innovation and growth. The regression examines, for robustness, the extent to which the relationships found in Table 2 are explained away by these intrinsic

[3]The year × 4-digit NAICS fixed effects absorb industry-level price differences across years, which are relevant in the regression with revenue. For multi-unit firms, the industry and state refer to the industry and state with the largest employment share.

characteristics of the businesses that may be correlated with both VC backing and firm outcomes. The large set of control variables are drawn from the 2018 and 2021 ABS and defined as follows: "Adv degree" is whether the business owner has a master's or higher degree; "Prior business" is whether the business owner has prior business ownership; "Age 35–54" is whether the business owner's age is between 35 and 54; "Age 55+" is whether the business owner's age is above 55; "Lifestyle" is whether the proprietor owns a business for a lifestyle reason (i.e., flexibility or work-life balance); "Process innov" is whether the business had process innovation (i.e., introduction or improvement in (i) methods of manufacturing, (ii) logistics, delivery or distribution methods, or (iii) supporting activities for processes); "Product innov" is whether the business had product innovation (the introduction or improvement of goods or services); "Patents" is whether the business owns patents or has pending patents; "IP important" is whether intellectual property is important for the business; and "Growth" is whether the business has a strategy to grow.[4], [5]

VC-backed firms have a higher technology adoption rate than non-VC-backed firms even after controlling for detailed business characteristics, as Table 3 shows. The coefficient on the VC-financed dummy declines from 0.106 in column (1) to 0.061 in column (3), where all control variables are employed. Firms' innovation activities are controlled for in column (3). In particular, process innovation is strongly associated with technology adoption. Columns (4)-(9) explore the relationships between the VC financing/technology adoption status and firm outcomes. The difference in firm size between VC-tech firms and the omitted group (i.e., non-VC, non-tech firms) attenuates as more business characteristics are controlled for, but remains significant (statistically and economically) in the regression with all control variables. A similar pattern is observed for the comparison between VC-backed, non-tech firms, and the omitted group, and for the comparison between non-VC, tech firms and the omitted group.

The difference in firm-size distributions between VC-backed and non-VC-backed firms, and between tech and non-tech firms, is examined separately in Table 4. Firm size is measured in terms of employment and revenue. The Kolmogorov-Smirnov test is employed to test whether one distribution stochastically dominates the other based on the difference between two cumulative distribution functions. The table shows that the VC-backed firm-size distribution stochastically dominates that of non-VC-backed firms (0 vs 1), because statistically speaking the cumulative distribution function of the latter lies above the former, while the opposite does not hold (1 vs 0). Likewise, the tech firm-size distribution stochastically dominates that of non-tech firms, but again no dominance in the opposite direction is detected.

Finally, the regression analysis in Table 5 exploits information on the timing of technology adoption. The ABS 2023 asked firms about the timing of AI adoption. The response options were five-year intervals (prior to 1990, 1991–1995, ..., 2016–2020, 2021–present). This data is linked to the Longitudinal Business Database (LBD) using firm identifiers. The dummy variable, Adoption (during), indicates that the year for the dependent variable, either employment or revenue, was within the five-year interval during which AI adoption occurred, and the dummy variable, Adoption (post), indicates that the year was after the adoption interval. These two dummy variables take on a value of zero for the years before the adoption interval and in all years for firms that have never adopted AI. Similarly, the dummy variable, VC (during), indicates that the firm raised its first VC financing in the year recorded for either employment or revenue, and the dummy variable, VC (post), means that the firm had raised VC financing before the year. Similar to the adoption timing case, the omitted category here is all years before VC financing and all years for firms that had no VC funding. The linear regression

[4] Similar control variables are also used in McElheran et al. (2023) in their analysis of AI adoption. Here, the definition of technology includes a broad set of advanced technologies beyond AI. Moreover, firm outcomes are examined as a function of the interaction between VC backing and technology adoption.

[5] A business owner is defined as the person who owns the largest percentage in the business. If several owners have the same percentage in the business, "Owner 1" in the ABS form is chosen as the owner.

Table 3: Regression with detailed control variables using ABS (2018)

| | (1) adoption | (2) adoption | (3) adoption | (4) ln(emp) | (5) ln(emp) | (6) ln(emp) | (7) ln(rev) | (8) ln(rev) | (9) ln(rev) |
|---|---|---|---|---|---|---|---|---|---|
| VC-financed | 0.106*** | 0.102*** | 0.0606*** | | | | | | |
| | (0.00837) | (0.00836) | (0.00819) | | | | | | |
| VC-tech | | | | 0.999*** | 0.919*** | 0.638*** | 0.821*** | 0.730*** | 0.362*** |
| | | | | (0.0485) | (0.0486) | (0.048) | (0.0761) | (0.0768) | (0.0774) |
| VC-nontech | | | | 0.661*** | 0.604*** | 0.459*** | 0.686*** | 0.618*** | 0.428*** |
| | | | | (0.0287) | (0.0286) | (0.0284) | (0.0384) | (0.0386) | (0.0386) |
| nonVC-tech | | | | 0.397*** | 0.384*** | 0.276*** | 0.484*** | 0.469*** | 0.327*** |
| | | | | (0.00681) | (0.00677) | (0.00678) | (0.00895) | (0.00889) | (0.00894) |
| Adv degree | | 0.0232*** | 0.0153*** | | 0.131*** | 0.109*** | | 0.174*** | 0.146*** |
| | | (0.00183) | (0.0018) | | (0.00629) | (0.00619) | | (0.00878) | (0.00864) |
| Prior business | | 0.0194*** | 0.0131*** | | 0.181*** | 0.162*** | | 0.184*** | 0.159*** |
| | | (0.00121) | (0.00119) | | (0.00423) | (0.00419) | | (0.0059) | (0.00583) |
| Age 35–54 | | 0.0013 | 0.00908*** | | 0.011 | 0.0316*** | | 0.0218 | 0.0496*** |
| | | (0.00258) | (0.00256) | | (0.00907) | (0.00902) | | (0.0128) | (0.0128) |
| Age 55+ | | -0.00402 | 0.00857** | | -0.172*** | -0.134*** | | -0.291*** | -0.240*** |
| | | (0.00269) | (0.00266) | | (0.00948) | (0.00943) | | (0.0134) | (0.0133) |
| Lifestyle | | -0.000642 | -0.00774*** | | -0.156*** | -0.207*** | | -0.163*** | -0.232*** |
| | | (0.00113) | (0.00113) | | (0.00417) | (0.00414) | | (0.00569) | (0.00565) |
| Process innov | | | 0.0905*** | | | 0.155*** | | | 0.215*** |
| | | | (0.00176) | | | (0.00521) | | | (0.00714) |
| Product innov | | | 0.0176*** | | | -0.00345 | | | 0.000273 |
| | | | (0.00108) | | | (0.00401) | | | (0.00552) |
| Patents | | | 0.0372*** | | | 0.261*** | | | 0.349*** |
| | | | (0.0052) | | | (0.0152) | | | (0.0211) |
| IP important | | | 0.0872*** | | | 0.273*** | | | 0.328*** |
| | | | (0.00192) | | | (0.00574) | | | (0.00777) |
| Growth | | | 0.0222*** | | | 0.301*** | | | 0.411*** |
| | | | (0.0011) | | | (0.00415) | | | (0.00574) |
| + Fixed fx | | | | | | | | | |
| R-squared | 0.0363 | 0.0377 | 0.0719 | 0.2086 | 0.2203 | 0.2472 | 0.1745 | 0.1860 | 0.2132 |
| N (rounded) | 346,000 | 346,000 | 346,000 | 346,000 | 346,000 | 346,000 | 346,000 | 346,000 | 346,000 |
| Adopt rate | 10.03% | | | | | | | | |
| VC-adopt rate | 22.98% | | | | | | | | |

*Notes*: Included fixed effects are: year×NAICS (4 digits), state, and firm age. All independent variables are binary variables. The regressions are weighted by the LBD weights. Standard errors are in parentheses and are clustered by firmid. * $p<0.05$ ** $p<0.01$ *** $p<0.001$. N is rounded to protect confidentiality.

Table 4: Kolmogorov-Smirnov test

| Variable | Context | D(0 vs 1) | D(1 vs 0) | p(0 vs 1) | p(1 vs 0) |
|---|---|---|---|---|---|
| ln(emp) | VC | 0.3331 | 0.0000 | 0.0000 | 1.0000 |
| ln(emp) | Tech | 0.2217 | 0.0000 | 0.0000 | 1.0000 |
| ln(rev) | VC | 0.3159 | -0.0087 | 0.0000 | 0.1141 |
| ln(rev) | Tech | 0.2316 | 0.0000 | 0.0000 | 1.0000 |

*Notes*: Depending on the context, 0 and 1 refer to the empirical cumulative distribution functions for either ~VC and VC funded, respectively, or ~high-tech and high tech. D(0 vs 1) is the test statistic that measures the distance between the cumulative distribution functions 0 and 1. D(1 vs 0) is the test statistic that measures the distance between the cumulative distributions functions 1 and 0. p(0 vs 1) and p(1 vs 0) give the significance level (probability) of the tests. The sample consists of firms in the ABS from 2017 to 2022.

Table 5: Regression with Timing of AI Adoption (Two-Way Fixed-Effect Model)

| | (1) | (2) |
|---|---|---|
| | ln(**emp**) | ln(**rev**) |
| Adoption (during) | 0.136*** | 0.174*** |
| | (0.0116) | (0.0146) |
| Adoption (post) | 0.168*** | 0.260*** |
| | (0.0291) | (0.0379) |
| VC (during) | 0.266*** | 0.118 |
| | (0.0453) | (0.103) |
| VC (post) | 0.844*** | 1.103*** |
| | (0.0516) | (0.102) |
| + Fixed effects | | |
| R-squared | 0.871 | 0.894 |
| N (rounded) | 6,590,000 | 4,800,000 |

*Notes*: The included fixed effects are: year×NAICS (4 digits), and firm age. The sample consists of firms in ABS (2023) linked to LBD (1978-2021). Revenue is obtained from the LBD (1997-2021). The regressions are weighted by sampling weights [tabulation weights in ABS (2023) in the first column and by tabulation weights × probability weights associated with LBD revenue in the second column]. Standard errors are shown in parentheses and are clustered by firmid. * p<0.05 ** p<0.01 *** p<0.001. N is rounded to protect confidentiality.

absorbs fixed effects of the firm, year-industry interactions, and firm age. Therefore, the coefficient on Adoption (during) examines how firm size changes during the years of AI adoption relative to years before AI adoption within the firm, controlling for year-industry effects and firm age effects. The table shows that firm size, measured as ln(employment) and ln(revenue), increases during AI adoption and even further after AI adoption. The size of VC-backed firms also increases after the VC funding year. The magnitudes of the estimated effects are larger for VC financing compared to AI adoption.

The current findings are in line with a growing literature that documents that the adoption of automation technologies is associated with an increase in employment at the firm level–this literature is surveyed in Restrepo (2023). This literature is further extended here by documenting the increase in employment and revenue after technology adoption in a representative sample of US firms. The findings are also consistent with the findings of Dinlersoz et al. (2022) that firms perform better after VC funding. Note that the analysis in Table 5 does not have a causal interpretation. The assessment of the causal relationship between technology adoption and firm growth, as well as the one between VC funding and firm outcomes, requires further analysis. A goal for future versions of this research is to explore the causal links.

# 6 The Model

The analysis develops a model with three phases of a firm's life cycle that focuses on the startup process. In the first phase of life, new entrepreneurs have an idea but no funding. Adolescent startups are funded by either a bank or a venture capitalist in the second phase. In the third phase, adult firms hire capital and labor on frictionless spot markets. The setup for the model is described first. Then, the analysis proceeds backwards in time starting with production by adult firms. Moving reversely, it then examines adolescent startups. Two types of funding contracts are considered: the partnership agreement between entrepreneurs and venture capitalists and the terms of a loan from bankers to entrepreneurs. After this, the source of funding for an adolescent startup is determined. Subsequently, production by a newborn startup in the first phase of life is formulated. Then, the choice of technology by a newborn entrepreneur is analyzed. Finally, an equilibrium for the model is specified.

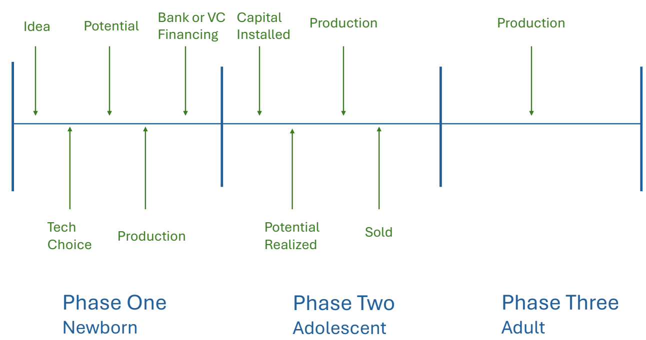


Figure 1: Timing of Events.

## 6.1 Setting the Stage

In the first phase a unit mass of entrepreneurs are blessed with ideas. Newly born startups employ one of two production technologies: an advanced production technology (dubbed high tech) and a conventional one (dubbed non-high tech). An entrepreneur's production technology has two attributes: its current level of productivity and its potential for the future. It's possible for a startup's current productivity, and hence output, to be low while its future potential looks promising. The joint distributions over initial productivities and potentials differ across the advanced and conventional production technologies. Some ideas are better suited for using the advanced technology than others. An entrepreneur decides in the first phase which production technology to use based on its expected profits, which depends on odds of receiving bank or VC financing, and the technology's implementation cost. They then approach financiers, either banks or venture capitalists, for funding of their startups. The funding is used to acquire the capital needed for production in the second phase. Adolescent startups grow into adult firms in the third phase of the life cycle. Adolescent startups are sold off at the end of the second phase in anticipation of this. The amount received depends on the success of the startup. Labor is hired on a spot market in each period using current revenue. There is one unit of labor available in the economy. Behind the scene, banks and venture capitalists borrow funds from a representative consumer/worker at a fixed interest rate.

Projects are funded by venture capital based on their potential. The success of a startup is realized in the second phase. While potential is positively correlated with future output, it is an imperfect signal of that output. At the time of funding in the first phase, the entrepreneur and the venture capitalist decide on a sharing rule for phase-two profits. The sharing rule is determined by Nash bargaining. In the case of bank financing, the entrepreneur borrows funds from the bank in the first phase at a fixed interest rate. The loan is payed back in the second phase. Funding by a venture capitalist increases the likelihood that a startup will be successful due to a synergistic effect. That is, unlike a bank, a venture capitalist plays an instrumental role in starting up a venture. This requires effort by the venture capitalist. The contract between an entrepreneur and a venture capitalist rewards the latter for their value added to the project as well as for the effort they expend. The discussion below works backwards in time from phase-three production to the choice of technology in the first phase. Figure 1 summarizes the timing of events in a startup's life cycle.

## 6.2 Production by Adult Firms

By the third phase adult firms have been sold off by the founders at the end of the second phase either through a merger and acquisition or through an initial public offering. There are four generic types of adult firms operating in this period. Some adult firms became high-tech in the startup phase and were financed by either a bank or venture capitalists. Others chose non-high-tech production and utilized one of the two sources of financing. In the the third phase adult firms hire capital, $k^3_{\tau f}$, and labor, $l^3_{\tau f}$, on spot markets and produce output, $o^3_{\tau f}$, according to

$$o^3_{\tau f} = (z^3_{\tau f})^{\zeta}(k^3_{\tau f})^{\kappa}(l^3_{\tau f})^{\lambda}, \text{ with } \zeta + \kappa + \lambda = 1,$$

where the subscript $\tau = h, n$ denotes the high-tech, $h$, and the non-high-tech, $n$, production and the subscript $f = b, v$ represents the (initial) source of finance, either a bank, $b$, or a venture capitalist, $v$. Total factor productivity is given by $z^3_{\tau f}$. Total factor productivity in the third phase, $z^3_{\tau f}$, is a function of the level of total factor productivity realized in the second phase, $z^2_{\tau f}$. Specifically,

$$z^3_{\tau f} = \chi_{\tau f} z^2_{\tau f},$$

where the growth factor, $\chi_{\tau f}$, depends both on the type of technology and the source of finance. Labor is hired in the third phase at the wage rate $w$ and capital can be raised at cost $r$.

The maximization problem for an adult firm is entirely standard and given by

$$\tilde{\pi}^3_{\tau f} = \max_{l^3_{\tau f}, k^3_{\tau f}} \{(z^3_{\tau f})^{\zeta}(k^3_{\tau f})^{\kappa}(l^3_{\tau f})^{\lambda} - wl^3_{\tau f} - rk^3_{\tau f}\}, \text{ for } \tau = h, n \text{ and } f = b, v, \tag{1}$$

where $\tilde{\pi}^3_{\tau f}$ is the per period profits for an adult firm. This above problem yields the familiar first-order conditions

$$\lambda(z^3_{\tau f})^{\zeta}(k^3_{\tau f})^{\kappa}(l^3_{\tau f})^{\lambda-1} = w \tag{2}$$

and

$$\kappa(z^3_{\tau f})^{\zeta}(k^3_{\tau f})^{\kappa-1}(l^3_{\tau f})^{\lambda} = r. \tag{3}$$

The discounted stream of profits from an adult firm at the beginning of phase 3 is

$$\pi^3_{\tau f} = \frac{1}{1-\delta^3}\tilde{\pi}^3_{\tau f} = \frac{1}{1-\delta^3} z^3_{\tau f}(1-\kappa-\lambda)[(\frac{\kappa}{r})^{\kappa}(\frac{\lambda}{w})^{\lambda}]^{1/\zeta},$$

where $\delta^3$ is the survival-adjusted discount factor.

## 6.3 Production by Adolescent Startups

Output in the second phase is governed by

$$o^2_{\tau f} = (z^2_{\tau f})^{\zeta}(k^2_{\varsigma f})^{\kappa}(l^2_{\tau f})^{\lambda}, \text{ for } \tau = h, n \text{ and } f = b, v.$$

As can be seen, there are four generic types of adolescent firms. Total factor productivity, $z^2_{\tau f}$, is drawn in the second phase according to

$$\ln z^2_{\tau f} = \ln p_{\tau} + \ln \varepsilon_{\tau f},$$

and depends on two factors $p_{\tau}$ and $\varepsilon_{\tau f}$. The first factor is the project's potential, $p_{\tau}$, which is known in the first phase. The second factor is a random shock, $\varepsilon_{\tau f}$, which is drawn in the second phase. The distribution function for this shock depends both on the source of finance, either a bank, $b$, or a venture capitalist, $v$. In particular,

$$\ln \varepsilon_{\tau f} \sim N(\gamma_{\tau f}, \sigma^2_{\epsilon_{\tau}}), \text{ for } \tau = h, n \text{ and} f = b, v,$$

where $\gamma_{\tau v} > \gamma_{\tau b} = 0$. Therefore, a high-tech project funded by a venture capitalist draws the phase-two shock from a distribution function with a higher mean than a similar bank-funded venture. This captures the synergy effect from venture capital. The input of capital is decided in the first phase before $\varepsilon_{\tau f}$ is known. Labor is hired in the second phase after the shock $\varepsilon_{\tau f}$ is realized.

## 6.4 Venture Capital Financing

The partnership agreement between a newborn entrepreneur and a venture capitalist takes place in the first phase before the phase-two technology shock is known. At this time the technology employed for the project, either $h$ or $n$, and their associated potentials, $p_h$ and $p_n$, are known. The two parties agree on three things that are contingent on the technology employed: the initial investment in capital, $k^2_{\tau v}$, the amount of labor to be hired in the second phase contingent on the shock, $l^2_{\tau v}(z^2_{\tau v})$, and the venture capitalist's share of realized revenue, $s_\tau$. The venture capitalist must expend effort, $e_\tau$, overseeing the project according to

$$e_\tau = \alpha_\tau + \xi p_\tau, \text{ for } \tau = h, n,$$

which is increasing in a venture's potential, $p_\tau$.

The Nash bargaining problem appears as

$$\max_{k^2_{\tau v}, l^2_{\tau v}(z^2_{\tau v}), s_\tau} \Big\{ E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2 \pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^\eta$$
$$\times E\Big[s_\tau[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l_\tau(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2 \pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{1-\eta}\Big\}, \text{ for } \tau = h, n, \quad (4)$$

where $\mathfrak{b}_\tau$ is the threat point of the entrepreneur or what they could receive from bank funding. The term on the first line is the entrepreneur's expected share of profits (after their threat point). This includes expected payoff from selling the startup just before phase-3 production starts, which is discounted at rate $\delta^2$. This term is weighted by the entrepreneur's bargaining power, $\eta$. The term on the second line is the venture capitalist's expected share of profits net of their exertion on effort. Expected profits are conditioned both on an enterprise's potential, $p_\tau$, and its source of finance, $v$.

**Lemma 1.** *(Nash Bargaining) The upshot of the Nash bargaining problem (4) is the following set of efficiency conditions (for $\tau = h, n$):*

$$\kappa E[(z^2_{\tau v})^\zeta (k^2_{\tau v})^{\kappa-1} l^2_{\tau v}(z^2_{\tau v})^\lambda | p_\tau] = r, \quad (5)$$

$$\lambda (z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^{\lambda-1} = w, \quad (6)$$

and the sharing rule

$$s_\tau = 1 - \eta + \frac{\eta(\alpha + \xi p_\tau) - (1-\eta)\mathfrak{b}_\tau}{E[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2 \pi^3_{\tau v} | p_\tau]}. \quad (7)$$

*Proof.* See Appendix B. □

The first condition sets the expected marginal product of capital equal to the rental rate. Labor is hired to the point where the realized marginal product of labor equals the wage rate, as the second condition specifies. Thus, capital and labor are hired in an efficient manner. The last condition gives the venture capitalist's share of profits. It states that the venture capitalist is entitled to the fraction $1-\eta$ of profits, plus the fraction $\eta$ of the venture capitalist's effort and less the fraction $1-\eta$ of the entrepreneur's value elsewhere, both expressed as shares of profits. Denote the entrepreneur's expected profits from the Nash bargaining problem by

$$E[\pi^2_{\tau v} | p_\tau] = E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2 \pi^3_{\tau v}] | p_\tau\Big], \quad (8)$$

which includes the discounted expected profits from selling off a successful startup at the end of phase 2.

## 6.5 Bank Financing

Entrepreneurs can also approach banks for financing. Once again capital is installed in place before the technology shock is realized in the second phase. Labor is hired after the shock is known. Banking is a competitive industry. The banker and entrepreneur sign a loan contract. The contract specifies that for a loan of size $k_{\tau b}^2$ the entrepreneur will have to pay back the amount $\hat{r}_\tau(k_{\tau b}^2; z_{\tau b}^2, p_\tau)$. The loan payment, $\hat{r}_\tau(k_{\tau b}^2; z_{\tau b}^2, p_\tau)$, has two mutually exclusive parts: a fixed interest part, $\tilde{r}_\tau(k_{\tau b}^2; p_\tau)$, when the venture can cover its loan payment, and a default payment, $i_\tau(z_{\tau b}^2, k_{\tau b}^2)$, when it can't. To understand the default payment, note that in some states the entrepreneur will not be able currently to repay all of the fixed loan payment, $\tilde{r}_\tau(k_{\tau b}^2; p_\tau)$, because

$$(z_{\tau b}^2)^\zeta (k_{\tau b}^2)^\kappa l_{\tau b}^2 (z_{\tau b}^2)^\lambda - w l_{\tau v}^2 (z_{\tau v}^2) + \delta^2 \pi_{\tau b}^3 < \tilde{r}_\tau(k_{\tau b}^2; p_\tau).$$

If this is the case, the bank seizes the startup, takes the current profits, $(z_{\tau b}^2)^\zeta (k_{\tau b}^2)^\kappa l_{\tau b}^2 (z_{\tau b}^2)^\lambda - w l_{\tau b}^2 (z_{\tau b}^2)$, and subsequently sells the enterprise for $\delta^2 \pi_{\tau b}^3$. Thus, upon a default the bank receives

$$i_\tau(z_{\tau b}^2, k_{\tau b}^2) = (z_{\tau b}^2)^\zeta (k_{\tau b}^2)^\kappa l_{\tau b}^2 (z_{\tau b}^2)^\lambda - w l_{\tau b}^2 (z_{\tau b}^2) + \delta^2 \pi_{\tau b}^3. \tag{9}$$

Define $z_\tau^{2*}$ to be the value of $z_{\tau b}^2$ at which the entrepreneur can just make his fixed interest payment. Thus, $z_\tau^{2*}$ solves

$$i_\tau(z_\tau^{2*}, k_{\tau b}) = (z_\tau^{2*})^\zeta k_{\tau b}^\kappa l_{\tau b} (z_\tau^{\prime *})^\lambda - w l_{\tau b}(z_\tau^{2*}) + \delta^2 \pi_{\tau b}^3 = \tilde{r}_\tau(k_{\tau b}^2; p_\tau). \tag{10}$$

This threshold is a function of $k_{\tau b}^2$, which in turn is a function of the enterprise's potential, $p_\tau$. It also is a function of the fixed interest payment $\tilde{r}_\tau(k_{\tau b}^2; p_\tau)$. Therefore the loan payment takes the following form:

$$\hat{r}_\tau(k_{\tau b}^2; z_{\tau b}^2, p_\tau) = \begin{cases} i_\tau(z_{\tau b}^2, k_{\tau b}^2), & \text{if } z_{\tau b}^2 < z_\tau^{2*}; \\ \tilde{r}_\tau(k_{\tau b}^2; p_\tau), & \text{if } z_{\tau b}^2 \geq z_\tau^{2*}. \end{cases}$$

The bank's zero-profit condition reads

$$r k_{\tau b}^2 = \Pr(z_{\tau b}^2 \geq z_\tau^{2*} | p_\tau) \tilde{r}_\tau(k_{\tau b}^2; p_\tau) + [1 - \Pr(z_{\tau b}^2 \geq z_\tau^{2*} | p_\tau)] E[i_\tau(z_{\tau b}^2, k_{\tau b}^2) | z_{\tau b}^2 < z_\tau^{2*}, p_\tau],$$

where $r$ is the interest rate paid to savers. The first term on the righthand side is the fixed interest payment, $\tilde{r}_\tau(k_{\tau b}^2; p_\tau)$, that is received with probability $\Pr(z_{\tau b}^2 \geq z_\tau^{\prime *} | p_\tau)$. The second term is the expected value of the default payment, $E[i_\tau(z_{\tau b}^2, k_{\tau b}^2) | z_{\tau b}^2 < z_\tau^{2*}, p_\tau]$, an event that occurs with the odds $1 - \Pr(z_{\tau b}^2 \geq z_\tau^{2*} | p_\tau)$. This zero-profit condition implies that the fixed interest component, $\tilde{r}_\tau(k_{\tau b}^2; p_\tau)$, can be expressed as

$$\tilde{r}_\tau(k_{\tau b}^2; p_\tau) = \frac{r k_{\tau b}^2 - [1 - \Pr(z_{\tau b}^2 \geq z_\tau^{2*} | p_\tau)] E[i_\tau(z_{\tau b}^2, k_{\tau b}^2) | z_{\tau b}^2 < z_\tau^{2*}, p_\tau]}{\Pr(z_{\tau b}^2 \geq z_\tau^{2*} | p_\tau)}. \tag{11}$$

The entrepreneur's choice of capital and labor (for $\tau = h, n$) is given by

$$E[\pi_{\tau b}^2 | p_\tau] = \max_{k_{\tau b}^2, l_{\tau b}^2(z_{\tau b}^2)} \Big\{ \Pr(z_{\tau b}^2 \geq z_\tau^{2*} | p_\tau) E[(z_{\tau b}^2)^\zeta (k_{\tau b}^2)^\kappa l_{\tau b}^2 (z_{\tau b}^2)^\lambda - \tilde{r}_\tau(k_{\tau b}^2; p_\tau) - w l_{\tau b}^2 (z_{\tau b}^2) + \delta^2 \pi_{\tau b}^3 | z_{\tau b}^2 \geq z_\tau^{2*}, p_\tau] \Big\}. \tag{12}$$

The entrepreneur's expected profits, $E[\pi_{\tau b}^2 | p_\tau]$, incorporates the discounted expectation of selling off the startup just before phase-3 production starts. Note that $\pi_{\tau b}^2$ is the entrepreneur's threat point in the Nash Bargaining problem (4) so that $E[\mathfrak{b}_\tau] = E[\pi_{\tau b}^2 | p_\tau]$.

**Lemma 2.** *(Bank Financing) The solution to the bank financing problem (12) is the following set of efficiency conditions for capital and labor:*

$$\kappa E[(z_{\tau b}^2)^\zeta (k_{\tau b}^2)^{\kappa - 1} l_{\tau b}^2 (z_{\tau b}^2)^\lambda | p_\tau] = r \tag{13}$$

*and*

$$\lambda (z_{\tau b}^2)^\zeta (k_{\tau b}^2)^\kappa l_{\tau b}^2 (z_{\tau b}^2)^{\lambda - 1} = w. \tag{14}$$

*Proof.* Once again see Appendix B, where equation (27) gives the entrepreneur's expected profits, $E[\pi^2_{\tau b}|p_\tau]$. $\square$

The lemma implies that capital accumulation and the hiring of labor are done efficiently. This transpires because: (i) banking is a competitive industry, (ii) both bankers and entrepreneurs are risk neutral, and (iii) upon a default bankers can seize the full value of a startup. The entrepreneur's expected profits are same as if they could finance the capital themselves, at an opportunity of cost of $r$ per unit of lost savings–this can be seen from the form of the objective function (27) in Appendix B. The benefit from using venture capital is the synergy effect. The form of the contract resembles the costly state verification models of Townsend (1979) and Williamson (1986), but here there is no private information problem.

### 6.6 Determination of Financing

The entrepreneur selects the type of financing that yields the highest expected payoff. The payoff depends on the type of technology that they use and its potential. The financing decision depends on the type of technology used and is given by (for $\tau = h, n$)

$$\begin{array}{rl} \texttt{VC}, & \text{if } E[\pi^2_{\tau v}|p_\tau] \geq E[\pi^2_{\tau b}|p_\tau]; \\ \texttt{Bank}, & \text{if } E[\pi^2_{\tau v}|p_\tau] < E[\pi^2_{\tau b}|p_\tau]. \end{array} \tag{15}$$

This can be rewritten in terms of a threshold rule. Specifically,

$$\begin{array}{rl} \texttt{VC}, & \text{if } p_\tau \geq p^*_\tau; \\ \texttt{Bank}, & \text{if } p_\tau < p^*_\tau, \end{array}$$

where the threshold $p^*_\tau$ solves the indifference condition

$$E[\pi^2_{\tau v}|p^*_\tau] = E[\pi^2_{\tau b}|p^*_\tau]. \tag{16}$$

The fraction of type-$\tau$ projects funded by venture capitalists is given by $\Pr[p_\tau \geq p^*_\tau]$, while the fraction receiving loans from banks is $1 - \Pr[p_\tau \geq p^*_\tau]$. The determination of financing is portrayed by Figure 2.

### 6.7 Production by Newborn Startups

Output in the first phase is produced before the newly born entrepreneur approaches a bank or venture capitalists for funding but after the entrepreneur has made a technology choice. Phase-one output is governed by the production function (for $\tau = h, n$)

$$o^1_\tau = (z^1_\tau)^\zeta m^\kappa (l^1_\tau)^\lambda,$$

where $m$ is a fixed amount of intangible entrepreneurial capital in the startup. So, there are two generic types of newborn startups. The technology shocks $z^1_\tau$ is not perfectly correlated with the potential $p_\tau$. Specifically, the two follow the bi-variate Normal distribution

$$\ln z^1_\tau, \ln p_\tau \sim N(\mu_{z^1_\tau}, \mu_{p_\tau}, \sigma^2_{z^1_\tau}, \sigma^2_{p_\tau}, \sigma_{z^1_\tau, p_\tau}).$$

This feature implies that the initial employment and output of a very young firm will only imperfectly predict whether a startup will get VC funding. An infant startup using technology $\tau = h, n$ chooses labor to maximize its profits in line with

$$\pi^1_\tau = \max_{l^1_\tau}\{(z^1_\tau)^\zeta m^\kappa (l^1_\tau)^\lambda - w l^1_\tau\}, \tag{17}$$

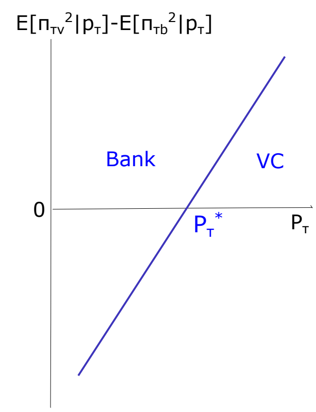


Figure 2: The determination of financing. At low levels of potential, $p_\tau < p^*_\tau$, the entrepreneur prefers using a bank. This transpires because of the fixed cost, $\alpha_\tau$, associated with a venture capitalist's effort. As the firm's potential rises the benefit of VC financing in terms of a *likely* higher level of productivity, $z^2_{\tau v} > z^2_{\tau b}$, overcomes the presence of the fixed cost.

which yields the standard looking first-order condition

$$\lambda(z^1_\tau)^\zeta m^\kappa (l^1_\tau)^{\lambda-1} = w. \tag{18}$$

The entrepreneur lives off of all of the return from his entrepreneurial capital and the profits generated by the newborn startup.

**Choice of Technology**

At the beginning of phase one a newly born entrepreneur is endowed with an idea. They can implement the idea using either the high-tech or non-high-tech technology. They make this choice before knowing the potential of the respective technologies. Therefore, they do not know the type of finance that they will procure. Implementing the advanced technology involves a fixed cost, $\phi$, which is distributed according to a Gumbel distribution:

$$\phi \sim G(\mathfrak{g}, \mathfrak{l} = 1),$$

where $\mathfrak{g}$ and $\mathfrak{l}$ are the shape and location parameters. This captures the feature that some ideas are more amenable to high tech than are others. The entrepreneur picks the technology that yields the highest discounted expected profits, after factoring in its implementation cost. The presence of venture capital in the economy does affect this choice. The unconditional expected payoffs from using the advanced and conventional technologies, $E[\pi^1|h]$ and $E[\pi^1|n]$, can be written as

$$E[\pi^1|h] = \pi^1_h + \Pr[p_h \geq p^*_h]\delta^1 E[\pi^2_{hv}|p_h \geq p^*_h] + \Big[1 - \Pr[p_h \geq p^*_h]\Big]\delta^1 E[\pi^2_{hb}|p_h < p^*_h]$$

and

$$E[\pi^1|n] = \pi^1_n + \Pr[p_n \geq p^*_n]\delta^1 E[\pi^2_{nv}|p_n \geq p^*_n] + (1 - \Pr[p_n \geq p^*_n])\delta^1 E[\pi^2_{nb}|p_n < p^*_n].$$

To understand these formulas take the first one. The probability of obtaining VC funding using the high-tech technology is $\Pr[p_h \geq p^*_h]$ while the expected discounted phase-2 payoff with VC funding is

$\delta^1 E[\pi^2_{hv}|p_h \geq p^*_h]$, where $\delta^1$ is the discount factor applied to phase two profits. With the complementary probability $1-\Pr[p_h \geq p^*_h]$ the entrepreneur obtains a bank loan from which phase-2 discounted profits are expected to be $\delta^1 E[\pi^2_{nb}|p_n < p^*_n]$.

The choice of technology is then summarized by

$$\begin{array}{ll} \texttt{High}-\texttt{Tech}, & \text{if } E[\pi^1|h] - \phi \geq E[\pi^1|n]; \\ \texttt{Non}-\texttt{High}-\texttt{Tech}, & \text{if } E[\pi^1|h] - \phi < E[\pi^1|n]. \end{array} \tag{19}$$

The fraction of high-tech startups reads

$$\Pr\left[E[\pi^1|h] - \phi \geq E[\pi^1|n]\right].$$

## 6.8 Equilibrium

In the background, think about a representative consumer/worker living in a stationary equilibrium–for more detail see Appendix C. This consumer/worker supplies labor to firms and savings to banks and venture capitalists. The person earns income from their labor and savings and reaps the profits from firms and venture capital operation. Banks and venture capitalists can borrow funds at the fixed interest rate, $\iota$, which represents the consumer/worker's rate of time preference. If the depreciation rate of physical capital is $\mathfrak{d}$, then the rental rate for capital, $r$, is

$$r = \iota + \mathfrak{d}. \tag{20}$$

Each period a unit mass of newborn startups flow into the economy. Newborn startups either survive to become adolescent startups with probability $\mathfrak{s}_n$ or suffer an infant death. Likewise, adolescent startups become adult ones with survival odds $\mathfrak{s}_a$. Similarly, adult firms survive a period with probability $\mathfrak{s}_e$. So, there will be 1 unit of newborn startups, $\mathfrak{s}_n$ units of adolescent startups, and $\mathfrak{s}_n\mathfrak{s}_a/(1-\mathfrak{s}_e)$ units of adult firms. The survival adjusted discount factors, $\delta^1$ and $\delta^2$, are accordingly given by

$$\delta^1 = \frac{\mathfrak{s}_n}{1+\iota} \text{ and } \delta^2 = \frac{\mathfrak{s}_a}{1+\iota}. \tag{21}$$

For an equilibrium to obtain the labor market must clear. There are 10 types of generic firms: newborn startups using either high-tech or non-high-tech production technology, adolescent high-tech and non-high-tech adopting startups that are financed by either banks or venture capitalists, and adult high-tech and non-high firms that are initially financed by one of the two sources. Thus, labor-market-clearing condition reads

$$\begin{aligned}
&\Pr\left[E[\pi^1|h] - \phi \geq E[\pi^1|n]\right]E[l^1_h(z^1_h)] + \Pr\left[E[\pi^1|h] - \phi < E[\pi^1|n]\right]E[l^1_n(z^1_n)] \\
&+ \mathfrak{s}_n\Big\{ \Pr\left[E[\pi|h] - \phi \geq E[\pi|n]\right]\{\Pr[p_h \geq p^*_h]E[l^2_{hv}(z^2_{\tau v})|p_h \geq p^*_h] + \Pr[p_h < p^*_h]E[l^2_{hb}(z^2_{hb})|p_h < p^*_h]\}\Big\} \\
&+ \mathfrak{s}_n\Big\{ \Pr\left[E[\pi|h] - \phi < E[\pi|n]\right]\{\Pr[p_n \geq p^*_n]E[l^2_{nv}(z^2_{nv})|p_n \geq p^*_n] + \Pr[p_n < p^*_n]E[l^2_{nb}(z^2_{nb})|p_n < p^*_n]\}\Big\} \\
&+ \mathfrak{s}_n\mathfrak{s}_a\frac{1}{1-\mathfrak{s}_e}\Big\{ \Pr\left[E[\pi|h] - \phi \geq E[\pi|n]\right]\{\Pr[p_h \geq p^*_h]\frac{\mathfrak{s}_a}{1-\mathfrak{s}_e}E[l^3_{hv}(z^3_{hv})|p_h \geq p^*_h] + \Pr[p_h < p^*_h]E[l^2_{hb}(z^3_{hb})|p_h < p^*_h]\}\Big\} \\
&+ \mathfrak{s}_n\mathfrak{s}_a\frac{1}{1-\mathfrak{s}_e}\Big\{ \Pr\left[E[\pi|h] - \phi < E[\pi|n]\right]\{\Pr[p_n \geq p^*_n]\frac{\mathfrak{s}_a}{1-\mathfrak{s}_e}E[l^3_{nv}(z^3_{nv})|p_n \geq p^*_n] + \Pr[p_n < p^*_n]E[l^3_{nb}(z^3_{nb})|p_n < p^*_n]\}\Big\} \\
&= 1.
\end{aligned} \tag{22}$$

The first line gives labor demand from newborn startups using high-tech and non-high-tech tech. The second line give labor demand from adolescent VC- and bank-funded firms using high-tech tech. The third line replicates the second line for the conventional technology. The fourth and fifth lines repeat the second and third lines for adult firms. The last line is labor supply.

**Definition.** An equilibrium consists of a solution for: (i) the fractions of newborn startups using the advanced and conventional technologies, $\Pr\Big[E[\pi^1|h] - \phi \geq E[\pi^1|n]\Big]$ and $\Pr\Big[E[\pi^1|h] - \phi < E[\pi^1|n]\Big]$; (ii) the labor hired by newborn startups using the high tech and conventional technologies, $l_h^1(z_h^1)$ and $l_n^1(z_n^1)$; (iii) the fractions of high-tech adopting adolescent startups funded by venture capital and banks, $\Pr[p_h \geq p_h^*]$ and $\Pr[p_h < p_h^*]$; (iv) the capital and labor hired by such adolescent startups,$k_{hv}^2(p_h \geq p_h^*)$, $k_{hb}^2(p_h < p_h^*)$, $l_{hv}^2(z_{hv}^2)$, and $l_{hb}^2(z_{hb}^2)$; (v) the fractions of conventional technology adopting adolescent startups funded by venture capital and banks, $\Pr[p_n \geq p_n^*]$ and $\Pr[p_n < p_n^*]$; (vi) the capital and labor hired by such adolescent startups, $k_{nv}^2(p_n \geq p_n^*), k_{nb}^2(p_n < p_n^*)$, $l_{nv}^2(z_{nv}^2)$, and $l_{nb}^2(z_{nb}^2)$; (vii) the threshold rules for venture capital funding of advanced and conventional technology projects, $p_h^*$ and $p_n^*$; (viii) the amounts capital, $k_{hv}^3(z_{hv}^3), k_{hb}^3(z_{hb}^3), k_{nv}^3(z_{nv}^3)$, and $k_{nb}^3(z_{nb}^3)$, and labor hired by adult firms, $l_{hv}^3(z_{hv}^3), l_{hb}^3(z_{hb}^3), l_{nv}^3(z_{nv}^3)$, and $l_{nb}^3(z_{nb}^3)$; and (ix) the wage rate, $w$. These allocations are determined such that:

1. The fractions of newborn startups using the advanced and conventional technologies, $\Pr\Big[E[\pi^1|h] - \phi \geq E[\pi^1|n]\Big]$ and $\Pr\Big[E[\pi^1|h] - \phi < E[\pi^1|n]\Big]$, are determined by the technology choice decision (19).

2. Newborn high-tech and conventional startups hire labor, $l_h^1(z_h^1)$ and $l_n^1(z_n^1)$, to maximize the entrepreneur's profits in accordance with (18).

3. The fractions of advanced technology adolescent startups funded by venture capitalists and banks, $\Pr[p_h \geq p_h^*]$ and $\Pr[p_h < p_h^*]$, are governed by the financing decision (15).

4. Adolescent high-tech and conventional startups that are funded by venture capitalists hire labor, $l_{hv}^2(z_{\tau v}^2)$ and $l_{nv}^2(z_{nv}^2)$, in keeping with the solution to the Nash Bargaining problems as specified by (5).

5. The fractions of conventional technology adolescent startups funded by venture capitalists and banks, $\Pr[p_n \geq p_n^*]$ and $\Pr[p_n < p_m^*]$, are ruled by the financing decision (15).

6. Adolescent startups that are funded by banks hire labor, $l_{hb}^2(z_{hb}^2)$ and $l_{nb}^2(z_{nb}^2)$, to maximize the entrepreneur's profits as stated by (14).

7. The threshold rules for VC funding for advanced and conventional technologies, $p_h^*$ and $p_n^*$, are governed by (16).

8. Capital is hired by VC-funded adolescent startups, $k_{hv}^2(p_h \geq p_h^*)$ and $k_{nv}^2(p_n \geq p_n^*)$, in line with the outcome of the Nash Bargaining problem or with equation (6).

9. Capital is hired by bank-funded adolescent startups that adopt high tech, $k_{hb}^2(p_h < p_h^*)$ and $k_{nb}^2(p_n < p_n^*)$, as dictated by (13), to maximize the entrepreneur's profits.

10. Adult firms hire capital, $k_{hv}^3(z_{hv}^3), k_{hn}^3(z_{hn}^3), k_{nb}^3(z_{nb}^3)$, and $k_{nb}^3(z_{nb}^3)$, and labor, $l_{hv}^3(z_{hv}^3)$, $l_{hn}^3(z_{hn}^3)$, $l_{nb}^3(z_{nb}^3)$, and $l_{nb}^3(z_{nb}^3)$, to maximize profits as specified by (2) and (3).

11. The labor market clears in accordance with (22). This determines wages, $w$.

12. The rental rate on capital, $r$, and the survival adjusted discount factors, $\delta^1$ and $\delta^2$, are pinned down by (20) and (21).

# 7 Quantitative Analysis

The big picture for the quantitative analysis is this. Startups are small in terms of employment. The modal startup remains small over its lifetime, in line with the findings in Hurst and Pugsley (2011). By contrast, high-tech, VC-financed firms grow to become very large. They furnish a lot of workers to employment even though they represent a tiny fraction of firms. VC-backed firms that use conventional technologies also become large, but not as big as the high-tech ones. They contribute non-negligibly to employment, yet are still a fraction of firms. Last, most firms start off as bank financed—for quantitative analysis all non-VC-financed firms are dubbed as bank-financed firms. They are numerous and account for the majority of employment. Bank-financed, high-tech firms are bigger than the ones using conventional technologies, but are still much smaller than VC-financed firms. The modal firm in the economy is bank-financed and uses conventional technologies. These are very small in size, but given their large numbers contribute substantially to employment. VC-funded firms have a higher proclivity to use high-tech relative to bank-financed ones. The question is whether or not the developed model can match such facts.

To answer this question, the model is calibrated to replicate a rich set of data targets detailed shortly. The vast majority of data targets are grouped into three categories of firms: newborn startups, adolescent startups, and adult firms. A period in the model is taken to be a year. A startup is classified as newborn if it hasn't reached its fourth birthday. Adolescent startups are those that have had their third birthday but not their eleventh. Adults firms are 11+ years old.

To make the computational analysis more interesting, labor supply is endogenized. Again, think about a representative consumer/worker living in a stationary equilibrium–Appendix C presents more detail. Endow this person with a momentary utility function of the following form:

$$u = \ln(c - \upsilon \frac{l^{1+\theta}}{1+\theta}),$$

where $c$ is consumption and $l$ is labor effort. This utility function has the simple solution for labor supply

$$l = w^{1/\theta}, \tag{23}$$

so it is easy to append an endogenous labor supply onto the framework. So, now the labor-market-clearing condition (22) will have $l$ instead of 1 on the righthand side.

Since the three phases are of different lengths some adjustments have to be made to theory in order to match the data. Recall that each period a unit mass of new startups is born. If the annual survival rate for newborns is $\mathfrak{s}_n$, then $\mathfrak{s}_n^3$ of these will survive into the adolescent phase. By summing across the 3 periods in the newborn phase, the mass of newborn startups is $(1-\mathfrak{s}_n^3)/(1-\mathfrak{s}_n)$. Suppose that the annual survival rate for an adolescent startup is $\mathfrak{s}_a$. Then, the mass of startups surviving into adulthood is $\mathfrak{s}_n^3\mathfrak{s}_a^8$. Taking into account that an adolescent startups potentially have eight periods of life implies that there will be $\mathfrak{s}_n^3(1-\mathfrak{s}_a^8)/(1-\mathfrak{s}_a)$ of them. Last, there will be $\mathfrak{s}_n^3\mathfrak{s}_a^8/(1-\mathfrak{s}_e)$ adult firms, where $\mathfrak{s}_e$ is the annual survival rate. When taking the theory to data, the survival-adjusted discounted profits for each phase need to be computed and the labor-market-clearing condition has to be modified to take into account the mass of firms in each phase–see Appendix D for the details.

The annual real interest rate is given a standard value of 4% implying that $\iota = 0.04$. From the US Census Bureau's Business Dynamics Statistics (BDS), the annual survival rates for newborns, adolescents, and adults are $\mathfrak{s}_n = 0.859$, $\mathfrak{s}_a = 0.920$,and $\mathfrak{s}_e = 0.949$. Hence, the annual survival-adjusted discount factors within the three phases are $0.859/(1.04) = 0.826$, $0.920/(1.04) = 0.885$ and $0.949/(1.04) = 0.913$. The annual depreciation rate of capital is taken to have a standard value of 8%, so that the rental rate on capital is $r = 0.04 + 0.08 = 0.12$. Appendices E to G derive the key formulas used for computing the model and matching it with the data.

## 7.1 Adolescent Startups, age 4 to 10 yrs

Now, start with data targets for adolescent startups. The facts for them are:

1. Average employment for adolescent high-tech adopting startups, computed separately for (a) bank and (b) VC financing.
2. The standard deviation of log employment for adolescent high-tech adopting startups, computed separately for (a) bank and (b) VC financing.
3. Average employment for adolescent non-high-tech adopting startups, computed separately for (a) bank and (b) VC financing.
4. The standard deviation of log employment for adolescent non-high-tech adopting startups, computed separately for (a) bank and (b) VC financing.
5. The fractions of VC funded adolescent firms that are (a) high-tech- and (b) non-high-tech adopting.
6. The share of high-tech adolescent startups in all firms.
7. The equity share of venture capitalists in adolescent startups.

The above facts are presented in Table 6. As can be seen from the table, for a given method of financing, adolescent startups using advanced technologies are much larger in terms of employment than those using conventional technologies. VC-backed startups are bigger than bank-financed ones, regardless of the technology used. The standard deviation of employment is large for all types of adolescent startups. For a given level of financing, the variance of employment is larger for high-tech startups. Holding the type of technology fixed, the variance of employment for VC-funded startups is bigger than for bank-financed ones. Not surprisingly, high-tech adolescent startups do not account for a large percentage of firms ($< 4\%$). VC-funded startups comprise a very small fraction of firms. The annual exit rate for adolescent startups is $100 \times (1 - \mathfrak{s}_a) = 8.0\%$. Last, to compute the average equity share earned by venture capitalist the sample of firms that had a first VC deal before 2010 was analyzed. The average equity share for a firm's last deal before age 10 was 55%. Some intuition about how these data targets help to identify some of the model's parameters is provided now.

*High-tech startups.* Start with high-tech adolescent startups. Data targets 1(a), 1(b), 2(a), 2(b), and 5(a) are useful for pinning down the mean and variance of high-tech startup's potentials, $\mu_{p_h}$ and $\sigma^2_{p_h}$, the variance of the second-period random productivity shock for high-tech firms, $\sigma^2_{\epsilon v}$, and for computing the VC threshold for high-tech startups, $p^*_h$. Intuitively, data target 5(a) is instrumental for determining the threshold for VC funding, $p^*_h$. Given this threshold, targets 1(a), 2(a), and 2(b) provide information on employment that can be used to back out the properties of the productivity distributions that determine the distributions for employment.

*Non-high-tech startups.* Likewise, for non-high-tech adolescent startups, data targets 3(a), 3(b), 4(a), 4(b), and 5(b) provide information for calibrating $\mu_{p_n}, \sigma^2_{p_n}, \sigma^2_{\epsilon b}$ and $p^*_n$.

*Costs and benefits of VC.* The information provided by data targets 5(a) and 5(b) help to pin down the venture capitalist's overseeing costs parameters, $\alpha_h$,$\alpha_n$,$\xi$, and the benefit from using VC, $\gamma_{hv}$ and $\gamma_{nv}$.[6] Heuristically speaking, a venture capitalist's share of an adolescent venture reflects the benefits and costs of using VC.

*High-tech adoption cost.* Last, data target 6 ties down the shape parameter, $\mathfrak{s}$, for the Gumbel distribution, which governs the fixed cost for a high-tech startup.

*Adolescent survival rate.* This is calculated using data from the BDS.

*VC Equity Share.* This number is useful for calibrating the venture capitalist's bargaining power, $1 - \eta$.

[6] The indifference condition (16) between using bank and VC funding for each technology implies that two of these variables can be solved for in terms of the others.

Table 6: Calibration Targets, Adolescent Startups, 4 to 10yrs

| **Target** | | **US Data** | **Model** |
|---|---|---|---|
| *Employment Shares (% of all employment)* | | | |
| T1 | High-Tech, VC financed | 0.28 (57.13)* | 0.28 |
| T2 | High-Tech, Bank financed | 2.398 (16.39) | 0.29 |
| T3 | Non-High-Tech, VC financed | 0.29 (39.36) | 2.398 |
| T4 | Non-High-Tech, Bank financed | 10.62 (9.446) | 10.62 |
| *Standard Deviation of* ln *Employment* | | | |
| T5 | High-Tech, VC financed | 1.528 | 1.302 |
| T6 | High-Tech, Bank financed | 1.302 | 1.302 |
| T7 | Non-High-Tech, VC financed | 1.47 | 1.15 |
| T8 | Non-High-Tech, Bank financed | 1.15 | 1.15 |
| *Share of Startups (% of all firms)* | | | |
| T9 | High-Tech, VC financed | 0.1147 | 0.1109 |
| T10 | High-Tech, Bank financed | 3.398 | 3.285 |
| T11 | Non-High-Tech, VC financed | 0.1712 | 0.1736 |
| T12 | Non-High-Tech, Bank financed | 26.12 | 26.48 |
| *Equity Share (%)* | | | |
| T13 | All VC financed firms | 55 | 66 |

* Numbers in parenthesis refer to absolute average employment size.

## 7.2 Newborn Startups, age $\leq 3$ yrs

A set of stylized facts is also collected for newborn startups–see Table 7. The facts concerning employment are now just broken down by technology. Newborn startups are much smaller than adolescent ones. They haven't had time to grow. Still the ones using advanced technologies are bigger than those using conventional ones. The standard deviation of employment is also large for newborn startups. It is larger for the high-tech newborn startups relative to the non-high-tech ones. Additionally, the correlation of a newborn's employment with that obtaining in its adult phase is also computed, where these are broken down both by the source of funding and the type of technology adopted. For VC-financed firms the correlation between their initial employment and the employment 11 plus years later (conditional on survival) is quite high. Unsurprisingly, newborn startups have a high exit rate of $100 \times (1 - \mathfrak{s}_n) = 14.1\%$. These facts are useful for pinning down the mean and variance, $\mu_{z_\tau^1}$ and $\sigma^2_{z_\tau^1}$ for $\tau = h, n$, of a newborn startup's productivity as well as the covariance, $\sigma_{z_\tau^1 p_\tau}$, between a newborn's productivity and its potential.

## 7.3 Adult Firms, age 11+ yrs

Last, a limited set of facts is provided for adults firms–Table 8. High-tech adult firms are much larger than non-high-tech ones, holding fixed the method of finance. High-tech, VC-financed firms average almost 3,000 employees. VC-backed firms are much larger than bank-financed ones, regardless of the technology used. By comparison average firm size in the United States is only about 23 employees. VC-backed firms (using either advanced or conventional technologies) are only a small fraction of all firms ($< 0.2\%$). Yet, they still account for roughly 12.56% of employment. High-tech, VC-backed firms makeup only 0.07% of firms in the economy, but they still comprise 8.6% of employment. Bank-financed adult firms using conventional technologies comprise by far the largest slice of firms in the economy and these firms tend to be small; actually, smaller than the average firm size in the economy. Bank-financed, high-tech firms are bigger than the ones using conventional technologies, but they are still much smaller than VC-funded firms regardless of the technology used by the latter. VC-funded firms are 4 times more likely to adopt high-tech than bank-financed ones.

Table 7: Calibration Targets, Newborn Startups, $\leq$ 3yrs

| **Target** | | **US Data** | **Model** |
|---|---|---|---|
| *Employment Shares (% of all employment)* | | | |
| T14 | High-Tech | 0.73 (8.167 )* | 0.73 |
| T15 | Non-High-Tech | 4.23 (6.026 ) | 4.23 |
| *Standard Deviation of* ln *Employment* | | | |
| T16 | High-Tech | 1.141 | 1.141 |
| T17 | Non-High-Tech | 1.05 | 1.05 |
| *Correlation of Newborn with Adult* ln *Employment* | | | |
| T18 | High-Tech, VC financed | 0.6349 | - |
| T19 | High-Tech, Bank financed | 0.6239 | - |
| *Share of Startups (% of all firms)* | | | |
| T20 | High-Tech | 2.078 | 2.287 |
| T21 | Non-High-Tech | 16.31 | 17.95 |

* Numbers in parenthesis refer to absolute average employment size.

Table 8: Calibration Targets, Adult Firms, 11+yrs

| **Target** | | **US Data** | **Model** |
|---|---|---|---|
| *Employment Shares (% of all employment)* | | | |
| T22 | High-Tech, VC financed | 8.63 (2,944)* | 8.63 |
| T23 | High-Tech, Bank financed | 26.72 (104) | 26.72 |
| T24 | Non-High-Tech, VC financed | 3.96 (736.8) | 3.96 |
| T25 | Non-High-Tech, Bank financed | 42.13 (21.43) | 42.13 |
| *Share of Adult Firms (% of all firms)* | | | |
| T26 | High-Tech, VC financed | 0.06807 | 0.183 |
| T27 | High-Tech, Bank financed | 5.967 | 5.434 |
| T28 | Non-High-Tech, VC financed | 0.1247 | 0.287 |
| T29 | Non-High-Tech, Bank financed | 45.66 | 43.8 |

* Numbers in parenthesis refer to absolute average employment size.

Table 9: Odds Ratios for Employment

| **Phase** | **Odds Ratios (%)** | | | |
|---|---|---|---|---|
| *Adolescent Startups* | High-Tech, VC | High-Tech, Bank | Non-High-Tech, VC | Non-High-Tech, Bank |
| | 246.00 | 70.58 | 169.49 | 40.68 |
| *Newborn Startups* | High-Tech | | Non-High-Tech | |
| | 35.17 | | 24.94 | |
| *Adult Firms* | High-Tech, VC | High-Tech, Bank | Non-High-Tech, VC | Non-High-Tech, Bank |
| | 12,677.04 | 447.83 | 3,172.71 | 92.28 |

The lefthand side panel of Figure 3 shows the model generated firm-size distributions for each of the four types of adult firms. As can be clearly seen, there is a greater mass of VC-funded firms in the right tail of the firm-size distributions for employment relative to bank-funded firms, regardless of the technologies that firms use. This is due to both selection and synergy effects. For a given type of finance, the firm-size distributions for high-tech firms dominate those using conventional technologies.

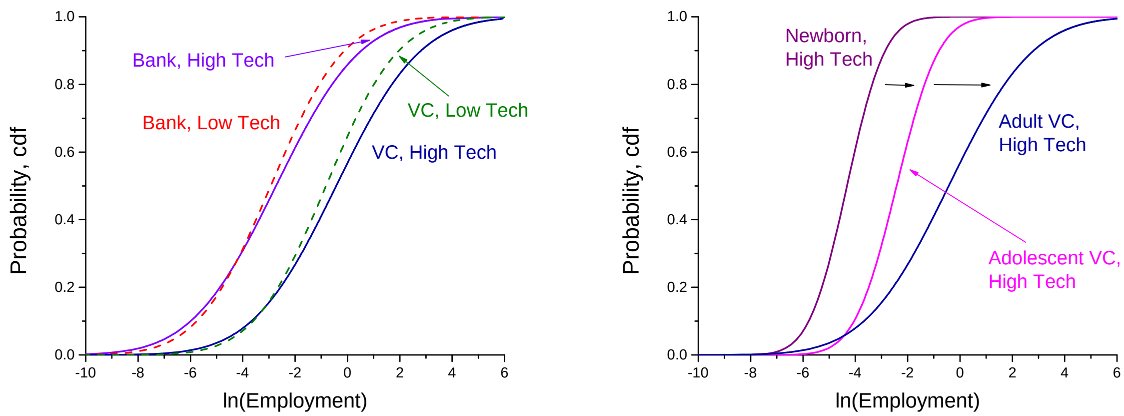


Figure 3: Firm-size distributions in the model measured by $ln$ employment. The plots are in cumulative distribution function form. Left panel, the firm-size distributions are shown for the four types of adult firms. Right panel, the growth in the firm-size distributions from a newborn, high-tech startup to an adult, VC-financed, high-tech firm.

The fact that VC-funded adult firms are punching well above their weight is evident from Table 9, which displays the odds ratios for the various types of firms. The odds ratios give each type of firm's share of total employment relative to their share in the total number of firms. The odds ratios for VC-funded adult firms are huge, especially for firms using advanced technologies. Additionally, bank-financed, high-tech adult firms also contribute proportionally more to employment than their numbers warrant. Bank-financed firms using conventional technologies underperform in terms of employment; i.e, their odds ratio is less than 100%. Last, both high- and low-tech newborn firms are underachievers in terms of generating employment just because their employment is very low when starting out.

The shares of employment for the various types of adult firms are useful for calibrating the growth factors, $\chi_{\tau f}$ for $\tau = h, n$ and $f = b, v$. From Tables 6 and 8 it can be calculated that VC-funded, high-tech firms grow a phenomenal 52 times between the adolescent and adult phases. This is followed by VC-funded, non-high-tech firms that increase by a factor of 19 times. Bank-financed, high-tech firms have a growth factor of 6 times. Bank-financed, non-high-tech firms are relative laggards, only roughly doubling in size. The evolution, or rightward shifts, of the firm-size distributions for VC-financed,

Table 10: Calibrated Parameter Values

| **Parameter Value** | **Description** | **Identification** |
|---|---|---|
| *Production function–all firms* | | |
| $\lambda = 0.6$ | Labor share | Standard |
| $\kappa = 0.2$ | Capital share | Standard |
| $\zeta = 0.2$ | Profit share | Standard |
| $m = 0.1$ | Entrepreneurial capital | Normalization |
| *Production shocks, adolescent startups* | | |
| $\mu_{p_h} = -3.831, \sigma_{p_h} = 0.34, \sigma_{\epsilon_h} = 2.527$ | Potential means, variances–high-tech | T1, T2, T5, T6 |
| $\mu_{p_n} = -2.953, \sigma_{p_n} = 0.391, \sigma_{\epsilon_n} = 2.169$ | Potential means, variances–non-high tech | T3, T4, T7, T8 |
| *Production shocks, newborn startups* | | |
| $\mu_{z_h^1} = -3.831, \mu_{z_n^1} = -4.239$ | Productivity means, newborns | T14, T15 |
| $\sigma_{z_h^1} = 2.282, \sigma_{z_n^1} = 2.100$ | Potential variances, newborns | T16, T17 |
| $\sigma_{z_h^1, p_h} = 0, \sigma_{z_n^1 p_n} = 0$ | Covariance, newborn prod and potential | To do |
| *Discount factors, rental rates, survival rates–annual* | | |
| $r = 0.04 + 0.08 = 0.12$ | Capital rental rate, $r = \iota + \mathfrak{d}$ | Standard |
| $\mathfrak{s}^1 = 0.859, \mathfrak{s}^2 = 0.920, \mathfrak{s}^3 = 0.949$ | Survival rates | BDS |
| $\delta^1 = 0.826, \delta^2 = 0.885, \delta^3 = 0.913$ | Discount factors, $\mathfrak{s}^i/(1.04)$ | Surv Adj |
| *Venture capital* | | |
| $\alpha_h = 3.619$ | Oversight cost, constant, high tech | T9, T10 |
| $\alpha_n = 1.087$ | Oversight cost, constant, non-high tech | T11, T12 |
| $\xi = 0.5$ | Oversight cost, intercept | Imposed |
| $\gamma_{hv} = 0.502, \gamma_{nv} = 0.386$ | Synergy effect | T1, T3 |
| $\eta = 0.75$ | Entrepreneur's bargaining power | T13 |
| *High-tech adoption cost* | | |
| $\mathfrak{g} = 0.33$ | Gumbel distribution, shape | T20, T21 |
| *Adult firm growth* | | |
| $\chi_{hv} = 3.748, \chi_{hb} = 1.366$ | Growth factors, high-tech | T22, T24 |
| $\chi_{nv} = 2.542, \chi_{nb} = 0.739$ | Growth factors, non-high-tech | T23, T25 |

high-tech startups is shown in the righthand side panel of Figure 3. The annual exit rate for adult firms is $100 \times (1 - \mathfrak{s}_e) = 5.1\%$. Exit rates decline with age; i.e., 5.1<8.20<14.1%. The evolution, or rightward shifts, in the employment distributions starting from a newborn, high-tech startup to an adult, high-tech, VC-financed firm is shown in the righthand side panel of Figure 3. The parameter values obtained from the calibration are displayed in Table 10.

# 8 Public Policy

The impacts on startups of business taxation and subsidies are now examined. Consider two tax-cum-subsidy regimes, $A$ and $B$. The equivalent variation, $\varepsilon$, associated with a shift from $A$ to $B$ is given by

$$\varepsilon = 100\% \times \frac{\exp[c^B - \upsilon(l^B)^{1+\theta}/(1+\theta)] - \exp[c^A - \upsilon(l^A)^{1+\theta}/(1+\theta)]}{c^A},$$

where $c^j$ and $l^j$, for $j = A, B$, are the consumptions and labor supplies in the two regimes. This measures the amount, in terms of a percentage of regime A's consumption, that either a person would be willing to pay or have to compensated to move from $A$ to $B$.

## 8.1 Taxing Startups

An interesting question is how business taxation affects startups. Suppose that a startup is setup as a C Corporation, the organizational form favored by venture capitalists. When operating as an adult firm the business will pay the corporate income tax at rate $\tau_c$. Traditional public finance theory states that the corporate income tax is nondistorting. As a profit tax it does not affect the choice of capital and labor in the adult firm problem (1), because it just multiplies the term in braces by $1-\tau_c$. It influences the type of financing and technologies used by startups, however. When a startup is sold, the sellers are taxed at the capital gains tax rate, $\tau_g$. The presence of the corporate income tax reduces the sale value of a startup by a factor of $1-\tau_c$. This combined with the capital gains tax implies that the proceeds from the sale are effectively lessened by the factor $(1-\tau_c)(1-\tau_g)$.[7] In the current framework, the expected payoff that a venture capitalist earns, $E\Big[s_\tau[(z^2_{\tau v})^\varsigma(k^2_{\tau v})^\kappa l_\tau(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}]|p_\tau\Big]$, is reduced–recall problem (4). Since the venture capitalist's effort is not tax deductible, this affects how the profits from the venture are split between the entrepreneur and venture capitalist, as specified by (7).[8] Therefore, $E[\pi^2_{\tau v}|p_\tau]$ is affected, which in turn impacts the financing decision (15).

Currently, the corporate income tax is 21%. Capital gains are taxed at 20%. Suppose as thought experiment both the corporate income and capital gains tax rates are raised to 28%.[9] Tax receipts are rebated back to consumers via lump-sum transfer payments. The results of this thought experiment for adult firms are displayed in Table 11. As can be seen, the share of both VC-funded adolescent startups and adult firms in employment drops quite drastically. The average size of VC-funded firms actually increases because the threshold for funding, $p^*_\tau$, rises. The average size of non-VC-funded adolescent and adult firms also moves up. This is a reallocation effect. First, as the share of VC-funded startups and firms drops, labor moves into the non-VC-funded startups and firms. Note, too, that since the threshold for VC funding has risen, at the margin the productivity of non-VC-funded startups and firms increases. Not surprisingly, there is a loss in welfare connected to the rise in taxes. The equivalent variation (EV) is -0.6% of aggregate consumption. The excess burden (EB) from the tax change is 55.9, implying that for each dollar raised in tax revenue there is a welfare loss of 55.9 cents due to the distorting nature of business income taxation.[10] The huge drop in the shares of the capital stock, employment, and the output of VC-funded startups and firms is mirrored by falls in the aggregate capital stock, employment, and output. The high-tech adoption rate tumbles as well. Total factor productivity, as conventionally measured, decreases.

The change in aggregate employment (or in fact income and the capital stock) can be decomposed into the difference in each type of firm's average employment and the shift in the share of each type of firm in the economy. The formula for doing this is

$$l^B - l^A = \sum_{j=1}^{10}[(\frac{\#^B_j + \#^A_j}{2})(l^B_j - l^A_j) + (\frac{(l^B_j + l^A_j)}{2})(\#^B_j - \#^A_j)],$$

where $l^A_j$ and $l^B_j$ are the average employments for a type-$j$ firm in the two situations and $\#^A_j$ and $\#^B_j$ are the numbers for each type of firm. The results are displayed in Table 12. For each type of firm, average employment actually increases, so their contribution to the decline in aggregate employment

[7] Startups can also be set up as partnerships. The profits from a partnership are taxed at the personal income tax rate, $\tau_l$. Qualified business income can be deduced at the rate $d$. So, which business organization form is preferred for tax purposes depends on whether $(1-\tau_c)(1-\tau_g) \gtrless 1-\tau_l(1-d)$. Currently, $\tau_l = 0.30$ and $d = 0.20$. Accountants believe that there is slight tax advantage to partnerships, yet they acknowledge that venture capitalists are very smart implying there must be other considerations.

[8] It is easy to see that the profits to the entrepreneur from bank financing are lowered by the factor $(1-\tau_c)(1-\tau_g)$. Hence, the entrepreneur's threat point, $\mathfrak{b}_\tau$, drops by this. By eyeballing the VC share equation (7), it is immediate that if the VC effort term, $\alpha + \xi p_\tau$, is reduced by $(1-\tau_c)(1-\tau_g)$, then there would be no effect on the venture capitalist's share, $s$. This transpires because then both the numerator and denominator would scale down by the same proportion.

[9] According to the National Tax Foundation this was one of the 2024 presidential election campaign proposals.

[10] Excess burden is computed as EB $= -$(Equivalent Variation)/$\{\Delta$(Revenue-Subsidy)$\times$signum[$\Delta$(Revenue-Subsidy)]$\}$.

Table 11: Effect of a Change in Business Tax Rates, %

| | **Empl Sh** | **Empl** | **Cap Sh** | **Cap** | **Opt Sh** | **Opt** | **Firm Sh** |
|---|---|---|---|---|---|---|---|
| ***Adolescent Startups*** | | | | | | | |
| *High-Tech, VC* | -58.9 | 20.1 | -58.8 | 18.9 | -58.9 | 18.9 | -67.0 |
| *High-Tech, Bank* | 1.8 | 9.1 | 2.2 | 7.3 | 1.8 | 7.3 | -9.3 |
| *Non-High-Tech, VC* | -56.1 | 21.4 | -55.9 | 19.4 | -56.1 | 19.4 | -64.8 |
| *Non-High-Tech, Bank* | 12.8 | 7.7 | 13.2 | 5.9 | 12.8 | 5.9 | 1.9 |
| ***Adult Firms*** | | | | | | | |
| *High-Tech, VC* | -58.9 | 20.9 | -58.8 | 18.9 | -58.9 | 18.9 | -67.0 |
| *High-Tech, Bank* | 1.8 | 9.1 | 2.2 | 7.3 | 1.8 | 7.3 | -9.3 |
| *Non-High-Tech, VC* | -56.1 | 21.4 | -55.9 | 19.4 | -56.1 | 19.4 | -64.8 |
| *Non-High-Tech, Bank* | 12.8 | 7.7 | 13.2 | 5.9 | 12.8 | 5.9 | 1.9 |
| | | ***Aggregate Variables*** | | | | | |
| *Output* | -2.3 | | *Wage Rate* | | -1.7 | | EV -0.6 |
| *Employment* | -2.8 | | *Productivity* | | -1.8 | | EB 55.9 |
| *Capital Stock* | -4.7 | | *Adoption Rate* | | -11.1 | | |

is negative. This is especially true for adult firms. The rise in average employment by adult bank-financed firms works to offset significantly the decline in aggregate employment. This occurs for two reasons. First, there is a reallocation of some high productivity firms away from VC financing to bank financing. Second, the wage rate has fallen. Both of these effects work to stimulate employment by adult bank-financed firms. The reallocation effect away from VC financed to bank-financed firms is strong. As can be seen, the largest contribution to the decline in aggregate employment results from a drop in the number of VC-financed adult firms. These were highly productive enterprises hiring a lot of labor. While it is true that the number of adolescent VC-funded startups also falls, at such a young stage they don't hire much labor.

## 8.2 Subsidizing Startups

The subsidization of startups is often proposed as a public policy. Two policies are entertained here. The first policy is the subsidization of all startups, whereas the second policy provides a subsidy only for high-tech startups. The first policy is operationalized by providing a subsidy equal to 10% of the cost of capital for an adolescent startup. The second policy gives the 10% subsidy only to high-tech startups. The two policies amount to a reduction in $r$ for adolescent startups.

The first policy increases the employment and output shares of all types of adolescent startups, as Table 13 shows. It also increases the levels of employment and output for all adolescent startups. Not surprisingly, the shares of adult firms in employment and output drop. More surprising is the fact that the levels of employment and output drop as well. This transpires for two reasons. First, the policy drives up the wage rate, and second, adult firms do not receive the subsidy. There is a moderate increase in aggregate output. The aggregate capital stock increases more because the policy has a large stimulus effect on capital accumulation by adolescent firms. This effect does not carry over to adult firms. Capital accumulation by adult firms drops because of the rise in wages. Total factor productivity, as conventionally measured, falls negligibly due to the fact the output falls while the capital stock rises (weighted by capital's share of income). Welfare actually increases, as measured by the equivalent variation. This transpires because the subsidy works to offset partially the disincentive effects of corporate taxation. The excess burden of the subsidy is positive because tax revenue net of the subsidy decreases.

From Table 14 it is clear that the rise in aggregate employment is primarily driven by in the increase in average employment by adolescent startups. Observe that the share in all firms of both adolescent and adult high-tech, VC-funded firms declines. This works to decrease aggregate employment. This

Table 12: Reallocation Effect: Tax Experiment

| Decomposition of Δ Employment, % | | | |
|---|---|---|---|
| | **Δ Empl** | **Δ Firms** | **Δ Total** |
| ***Newborn Startups*** | | | |
| *High-Tech* | -1.1 | 3.0 | 1.9 |
| *Non-High-Tech* | -6.6 | -2.2 | -8.8 |
| ***Adolescent Startups*** | | | |
| *High-Tech, VC* | -1.4 | 7.5 | -6.0 |
| *High-Tech, Bank* | -7.5 | 8.4 | -0.9 |
| *Non-High-Tech, VC* | -1.5 | 7.5 | 6.0 |
| *Non-High-Tech, Bank* | -29.9 | -7.4 | -37.2 |
| ***Adult Firms*** | | | |
| *High-Tech, VC* | -43.3 | 230.5 | 187.2 |
| *High-Tech, Bank* | -83.5 | 93.4 | -9.8 |
| *Non-High-Tech, VC* | -20.7 | 102.6 | 81.9 |
| *Non-High-Tech, Bank* | -118.5 | -21.3 | -147.8 |
| ***Aggregate Variables*** | | | |
| *Employment* | -2.8 | | |

Since the change in aggregate employment is negative, for the newborn, adolescent, and adult categories, a negative number means that the entry increased employment, while a positive one implies it decreased it.

Table 13: Effect of a Subsidy for All Startups, %

| | **Empl Sh** | **Empl** | **Cap Sh** | **Cap** | **Opt Sh** | **Opt** | **Firm Sh** |
|---|---|---|---|---|---|---|---|
| ***Adolescent Startups*** | | | | | | | |
| *High-Tech, VC* | 8.2 | 10.1 | 18.1 | 22.7 | 8.1 | 10.4 | -1.2 |
| *High-Tech, Bank* | 10.1 | 9.9 | 20.2 | 22.4 | 10.1 | 10.2 | 0.7 |
| *Non-High-Tech, VC* | 10.0 | 9.7 | 20.2 | 22.3 | 10.1 | 10.0 | 0.8 |
| *Non-High-Tech, Bank* | 9.2 | 9.8 | 19.2 | 22.4 | 9.2 | 10.1 | -0.1 |
| ***Adult Firms*** | | | | | | | |
| *High-Tech, VC* | -2.6 | -0.9 | -4.4 | -0.6 | -2.6 | -0.6 | -1.2 |
| *High-Tech, Bank* | -0.9 | -1.1 | -2.6 | -0.8 | -0.9 | -0.8 | 0.7 |
| *Non-High-Tech, VC* | -0.9 | -1.3 | -2.7 | -1.0 | -0.9 | -1.0 | 0.8 |
| *Non-High-Tech, Bank* | -1.7 | -1.2 | -3.4 | -0.9 | -1.7 | -0.9 | -0.1 |
| ***Aggregate Variables*** | | | | | | | |
| *Output* | 0.8 | | *Wage Rate* | 0.3 | | EV | 0.3 |
| *Employment* | 0.5 | | *Productivity* | -0.0 | | EB | 85.5 |
| *Capital Stock* | 2.6 | | *Adoption Rate* | -0.6 | | | |

Table 14: Reallocation Effect: Subsidy, All

| Decomposition of Δ Employment, % | | | |
|---|---|---|---|
| | **Δ Empl** | **Δ Firms** | **Δ Total** |
| ***Newborn Startups*** | | | |
| *High-Tech* | -1.1 | 1.0 | -0.1 |
| *Non-High-Tech* | -6.3 | -0.7 | -7.0 |
| ***Adolescent Startups*** | | | |
| *High-Tech, VC* | 5.9 | -0.8 | 5.1 |
| *High-Tech, Bank* | 49.7 | 3.7 | 53.3 |
| *Non-High-Tech, VC* | 5.9 | 0.5 | 6.4 |
| *Non-High-Tech, Bank* | 217.7 | -2.0 | 215.7 |
| ***Adult Firms*** | | | |
| *High-Tech, VC* | -15.9 | -22.9 | -38.9 |
| *High-Tech, Bank* | -60.9 | 38.6 | -22.3 |
| *Non-High-Tech, VC* | -10.4 | 6.7 | -3.7 |
| *Non-High-Tech, Bank* | -101.0 | -7.5 | -108.6 |
| ***Aggregate Variables*** | | | |
| *Employment* | 0.5 | | |

transpires because the policy favors bank-funded high-tech firms. When using bank funding an entrepreneur reaps all of the benefit from the subsidy. By contrast, the entrepreneur must share part of the benefit with the venture capitalist when using VC funding. Also note that non-high-tech, bank-funded firms also decline. The rise in wages hurts low productivity bank-funded firms.

Turn now to the second policy. Not surprisingly, the share of high-tech adolescent startups in employment and output rises significantly. There is a large uplift in the levels of employment and output for these types of firms. This comes at the expense of non-high-tech adolescent firms that are hurt by the climb in wages. The spike in the share of high-tech firms in employment and output carries over to adult firms. This happens mechanically, because an adolescent high-tech startup must mature into adult high-tech firm and high-tech firms have higher levels of employment and output relative to non-high-tech one. The levels of employment and output for non-high-tech firms fall however, again due to the rise in wages. Aggregate output increases by a moderate amount due to the rise in high-tech adolescent output. The aggregate capital stock shoots up because of the tremendous boost in capital accumulation by high-tech adolescent firms. Total factor productivity moves up slightly–note that the weight on capital is small. Again, welfare increases, as measured by the equivalent variation, because the subsidy works to mitigate somewhat the negative effects of corporate taxation. The excess burden from the subsidy is huge, because in this situation both welfare increases significantly and revenue net of the subsidy rises slightly.

The rising share in the total number of firms of adult high-tech firms contribute significantly to the increase in aggregate employment–see Table 16. This occurs because adult high-tech firms are large on average. Non-high-tech, bank-financed adult firms provide the biggest offset to the rise in employment, because both of the decline in their average employment and in their share in the total number of firms.

# 9 Closing

Not all startups are born equal. Some entrepreneurs have exciting ideas, others more pedestrian. Some startups adopt advanced technologies (or high-tech), while others don't. Some are backed by venture capitalists, while others obtain funding from other sources. Newborn firms start out very small, having around 6 employees on average. The vast majority of these startups barely grow, reaching fewer than

Table 15: Effect of a Subsidy for High-Tech Startups, %

| | **Empl Sh** | **Empl** | **Cap Sh** | **Cap** | **Output Sh** | **Output** | **Firm Sh** |
|---|---|---|---|---|---|---|---|
| ***Adolescent Startups*** | | | | | | | |
| *High-Tech, VC* | 13.1 | 10.3 | 25.2 | 22.8 | 13.1 | 10.5 | 2.9 |
| *High-Tech, Bank* | 13.0 | 10.3 | 25.0 | 22.8 | 13.0 | 10.5 | 2.7 |
| *Non-High-Tech, VC* | -5.0 | -0.2 | -5.4 | -0.0 | -5.0 | -0.0 | -4.5 |
| *Non-High-Tech, Bank* | -1.3 | -0.7 | -1.7 | -0.5 | -1.3 | -0.5 | -0.3 |
| ***Adult Firms*** | | | | | | | |
| *High-Tech, VC* | 1.8 | -0.8 | 1.4 | -0.6 | 1.8 | -0.6 | 2.9 |
| *High-Tech, Bank* | 1.6 | -0.7 | 1.2 | -0.6 | 1.6 | -0.6 | 2.7 |
| *Non-High-Tech, VC* | -5.0 | -0.2 | -5.4 | -0.0 | -5.0 | -0.0 | -4.5 |
| *Non-High-Tech, Bank* | -1.3 | -0.7 | -1.7 | -0.5 | -1.3 | -0.5 | -0.3 |
| | | | ***Aggregate Variables*** | | | | |
| *Output* | 0.5 | | *Wage Rate* | 0.2 | | EV | 0.2 |
| *Employment* | 0.3 | | *Productivity* | 0.1 | | EB | -379.1 |
| *Capital Stock* | 0.9 | | *Adoption Rate* | 2.7 | | | |

Table 16: Reallocation Effect: Subsidy, High Tech

| Decomposition of Δ Employment, % | | | |
|---|---|---|---|
| | **Δ Empl** | **Δ Firms** | **Δ Total** |
| ***Newborn Startups*** | | | |
| *High-Tech* | -1.1 | 6.4 | 5.3 |
| *Non-High-Tech* | -6.3 | -4.7 | -11.1 |
| ***Adolescent Startups*** | | | |
| *High-Tech, VC* | 9.6 | 2.8 | 12.4 |
| *High-Tech, Bank* | 81.5 | 22.2 | 103.7 |
| *Non-High-Tech, VC* | -0.2 | -4.3 | -4.5 |
| *Non-High-Tech, Bank* | -23.9 | -11.0 | -34.9 |
| ***Adult Firms*** | | | |
| *High-Tech, VC* | -21.5 | 80.4 | 58.9 |
| *High-Tech, Bank* | -65.1 | 234.3 | 169.3 |
| *Non-High-Tech, VC* | -2.5 | -58.3 | -60.8 |
| *Non-High-Tech, Bank* | -94.9 | -43.4 | -138.3 |
| | ***Aggregate Variables*** | | |
| *Employment* | 0.3 | | |

31 employees. High-tech firms that are VC financed are much larger on average than firms that use conventional technologies and that obtain financing from other sources. The former have roughly 3,000 employees, even though they represent only 0.07% of firms. The latter represent 46% of firms, but hire 42% of workers. VC-funded firms are 4 times more likely to use advanced technologies than firms obtaining funding elsewhere.

A model is built for the startup process. It is used to study the relationship between the adoption of advanced technologies by startups and their source of funding. Two decisions are incorporated into the startup process: the choice to adopt either an advanced or conventional technology and the selection between bank or VC funding. Quantitative analysis is undertaken to see if model can match a rich set of data targets taken from US Census Bureau databases. It can. The quantitative analysis then moves on to consider two types of public policy. First, the impact of a shift in the capital gains and corporate income taxation is investigated. Second, the effect of startup subsidies is examined. Specifically, subsidies for the cost of capital are entertained, either for all startups or for just for high-tech ones.

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# A Data

The empirical analysis uses data from the Annual Business Survey (ABS) and Pitchbook. The technological variables are obtained from the ABS. For the most part, information on venture capital funding is harvested from Pitchbook with a limited amount of facts taken from the ABS.

## A.1 The Use of Technology by Firms

The ABS queries firms about their use of various technology for the period 2017-2022, captured by the ABS waves over 2018-2023. The questions asked about the use of technology differ by year of the survey. Only the technologies examined in the current analysis are listed for each question.

- **ABS (yr1 - 2018) Business Technologies**. The question asked was: *In 2017, to what extent did this business use the following technologies in producing goods or services?*
  Augmented reality, Automated guided vehicles (AGV) or AGV systems, Automated storage and retrieval systems, Machine learning, Machine vision software, Natural language processing, Radio-frequency identification (RFID) inventory system, Robotics, Voice recognition software.[11] Extent was gauged by the choice selected in the following list: No use, Testing but not using in production or service, In use for less than 5% of production or service, In use for between 5%–25% of production or service, In use for more than 25% of production or service, Don't know. In the analysis the technology is counted as used if the firm indicated in "use for less than 5%" or higher.

- **ABS (yr2 - 2019) Production Technology for Goods and Services**: The question asked was: *During the three years 2016 to 2018, to what extent did this business use the following technologies in production processes for goods or services?*
  Artificial Intelligence, Robotics.[12] Extent was gauged by the choice selected in the following list: Did not use, Tested, but did not use in production or service, Low use, Moderate use, High use, Don't know. In the analysis the technology is counted as being used if the firm indicated "low use" or higher.

- **ABS (yr3 - 2020) Use of Digital Technologies**: *During the three years 2017 to 2019, to what extent does this business use the following digital technologies for innovation activities?*
  Artificial Intelligence, Digital technologies for distributed ledgers (blockchain).[13] Extent was gauged by the choice selected in the following list: A great extent, To some extent, To a small extent, Not at all. A technology was counted as being used if the firm indicated to "some extent" or higher.

- **ABS (yr4 - 2021) Business Technologies**: *In 2020, did this business produce goods or provide services by using or applying any of the following technologies?*

[11] The following technology is excluded in the analysis because it is thought to be too commonplace: Touchscreens/ kiosks for customer interface (Examples: self-checkout, self-check-in, touchscreen ordering).

[12] Cloud-Based Computing Systems and Applications, Specialized Software, Specialized Equipment are excluded. In general, the year-2 survey sampled a different set of firms in 2018 than the year-1 survey.

[13] Excluded are: Computer infrastructure (server technologies), cloud computing, automation, Internet-connected devices, mobile communication technologies, the use of digital technologies for collaboration, communication (i.e., through social media), digital technologies for planning and management (i.e., enterprise resource planning, customer relationship management).

Augmented reality, Automated guided vehicles (AGV) or AGV systems, Machine learning, Machine vision, Natural language processing, Radio-frequency identification (RFID) system, Robotics, Voice recognition software, Additive manufacturing (3D printing), including prototyping. Extent was gauged by the choice selected in the following list: In use, In testing, but not in use, Not in use nor testing, Don't know. The technology is counted as being used if the firm indicated "in use."

- **ABS (yr5 - 2022) Use of Technologies**: *During 2021, to what extent did this business use the following technologies?*
  Advanced sensing (e.g., machine vision, voice recognition, networked sensors and sensing, millimeter-wave radar, LIDAR, RFID, biointegrated sensors, electric grid measurement), Artificial intelligence (e.g., machine learning, planning, reasoning, and decision making), Autonomous systems and robotics (e.g., industrial robotics, automated guided vehicles (surface, aerospace, maritime)), Additive manufacturing (e.g., additive manufacturing (3-D Printing), smart manufacturing), Biotechnology (e.g., genetic engineering, DNA synthesis, genetic sequencing, tissue engineering, biomanufacturing, bioinformatics), Human-machine interfaces (e.g., augmented reality, virtual reality, brain-computer interfaces, human-machine teaming), Communication and networking technologies (e.g., radio frequency and mixed signal circuits, antennas and components, spectrum management technologies, communications and network security, mesh networks /infrastructure independent communication technologies), Advanced financial technologies (e.g., digital technologies for distributed ledgers, blockchain), Advanced semiconductors and microelectronics (e.g., beyond CMOS electronics (including next generation semiconductor materials), design and electronic design automation (EDA) tools, manufacturing technologies and tooling, advanced lithography), Advanced engineering materials (e.g., materials by design, metamaterials, nanomaterials, smart materials, AM alloys, biomimetic, flexible electronics, material property characterization additive manufactured parts), Renewable energy generation and storage (e.g., wind, solar, and bio-based generation, electric and hybrid engines, batteries and grid backup/storage), Advanced gas turbine engine technologies [e.g., aero, marine, industrial (power generation), enabling component technologies], Advanced nuclear energy technologies [e.g., aero, marine, industrial (power generation), enabling component technologies][14] Extent was gauged by the choice selected in the following list: A lot, Somewhat, A little, Not at all. The technology is counted as being used if the firm indicated "a little" or higher.

- ***ABS (yr6 - 2023) Production Technology for Goods and Services***: *During the three years 2020 to 2022, did this business adopt/use the following technologies?*
  Artificial Intelligence, Cloud-Based Computing Systems and Applications, Specialized Software, Robotics, Specialized Equipment.
  For each technology, the survey asks the timing of adoption. For example, *Timing of Adoption for Artificial Intelligence Technology–Processes and Methods: Approximately what year did this business first adopt or use Artificial Intelligence in processes and methods?*
  Prior to 1990, 1991–1995, 1996–2000, 2001–2005, 2006–2010, 2011–2015, 2016–2020, 2021–Present, Don't know.

## A.2 Venture Capital Funding for Firms

For the most part, data on venture capital funding is obtained from Pitchbook. Pitchbook collects data on venture capital financing through public sources as well as its network. It is a major provider of VC data, and the annual reports by the National Venture Capital Association are based on the Pitchbook data. The dataset is merged with the Census Bureau's Business Register through name and address matching, utilizing the information on company name, state, city, zip code, and street

[14]Excluded: Advanced computing (e.g., supercomputing, edge computing, cloud computing, data storage, advanced computing architectures).

address. According to its glossary, venture capital is described as a "type of private equity investing that focuses on startups and early-stage companies with long-term, high-growth potential." For the year 2018 the information from Pitchbook is augmented by the answer to a question in the ABS.

- ***ABS (yr1 - 2018) Capital Funding***: *What was the source(s) of capital used to start or initially acquire this business*?:
  Personal/family savings of owner(s), Personal/family assets other than savings of owner(s), Personal/family home equity loan, Personal credit card(s) carrying balances, Business credit card(s) carrying balances, Government-guaranteed business loan from a bank or financial / institutions, including SBA-guaranteed loans, Business loan from a bank or financial institution, Business loan from a federal, state, or local government, Business loan/investment from family/friend(s), Investment by venture capitalist(s), Grants, Other source(s) of capital, Don't know, None needed.

# B Theory

## Nash Bargaining–Lemma 1, Proof

The first-order conditions for $k^2_{\tau v}$,$l^2_{\tau v}(z^2_{\tau v})$, and $s_\tau$ associated with the Nash bargaining problem (4) between an entrepreneur and a venture capitalist are (for $\tau = h, n$):

$$\begin{aligned}
&\eta E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta-1} (1-s_\tau)E[\kappa(z^2_{\tau v})^\zeta (k^2_{\tau v})^{\kappa-1} l^2_{\tau v}(z^2_{\tau v})^\lambda - r|p_\tau] \\
&\quad\times E\Big[s_\tau[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{1-\eta} \\
&\quad= E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta} \\
&\quad\times (1-\eta)E\Big[s_\tau[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{-\eta} \\
&\qquad\times s_\tau E[\kappa(z^2_{\tau v})^\zeta (k^2_{\tau v})^{\kappa-1} l^2_{\tau v}(z^2_{\tau v})^\lambda - r|p_\tau],
\end{aligned} \tag{24}$$

$$\begin{aligned}
&\eta E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta-1} (1-s_\tau)[\lambda(z^2_{\tau v})^\zeta (k^2_{\tau v})^{\kappa} l^2_{\tau v}(z^2_{\tau v})^{\lambda-1} - w] \\
&\quad\times E\Big[s_\tau[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{1-\eta} \\
&\quad= E\Big[[(1-s_\tau)(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta} \\
&\quad\times (1-\eta)E\Big[s_\tau[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{-\eta} \\
&\qquad\times s_\tau[\lambda(z^2_{\tau v})^\zeta (k^2_{\tau v})^{\kappa} l^2_{\tau v}(z^2_{\tau v})^{\lambda-1} - w],
\end{aligned} \tag{25}$$

and

$$\begin{aligned}
&\eta E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta-1} \\
&\qquad\times E[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau] \\
&\quad\times E\Big[s_\tau[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{1-\eta} \\
&\quad= E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau | p_\tau\Big]^{\eta} \\
&\quad\times (1-\eta)E\Big[s_\tau[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau | p_\tau\Big]^{-\eta} \\
&\qquad\times E[(z^2_{\tau v})^\zeta (k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau].
\end{aligned} \tag{26}$$

Start with the first order condition (24). It is automatically satisfied when (5) holds. Likewise, (6) will guarantee that (25) is fullfilled. To derive the solution for $s_\tau$ note that the term $E[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau]$ cancels out on both sides of (26). Then, divide both sides of equation (26) by $E\Big[(1-s_\tau)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \mathfrak{b}_\tau|p_\tau\Big]^{\eta-1}$ and $E\Big[s_\tau[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}] - \alpha - \xi p_\tau|p_\tau\Big]^{-\eta}$, and then solve for $s_\tau$ to get equation (7).

### Bank Financing–Lemma 2, Proof

In the bank financing problem (12) the entrepreneur will make the interest payment $\tilde{r}_\tau(k^2_{\tau b};p_\tau)$ with probability $\Pr(z^2_{\tau b} \geq z^{2*}_\tau|p_\tau)$. Solving out for $\tilde{r}_\tau(k^2_{\tau b};p_\tau)$ in (12) by using the bank's zero-profit condition (11) gives

$$E[\pi^2_{\tau b}|p_\tau] = \max_{k^2_{\tau b}, l^2_{\tau b}(z^2_{\tau b})} \Big\{ \Pr(z^2_{\tau b} \geq z^{2*}_\tau|p_\tau)E[(z^2_{\tau b})^\zeta(k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - wl^2_{\tau b}(z^2_{\tau b}) + \delta^2\pi^3_{\tau b}|z^2_{\tau b} \geq z^{2*}_\tau, p_\tau] \\ - rk^2_{\tau b} + [1 - \Pr(z^2_{\tau b} \geq z^{2*}_\tau|p_\tau)]E[i_\tau(z^2_{\tau b}, k^2_{\tau b})|z^2_{\tau b} < z^{2*}_\tau, p_\tau]\Big\}.$$

Next, use (9) to substitute out for $i_\tau(z^2_{\tau b}, k^2_{\tau b})$ in the above maximization problem, which yields

$$E[\pi^2_{\tau b}|p_\tau] = \max_{k^2_{\tau b}, l^2_{\tau b}(z^2_{\tau b})} \Big\{ \Pr(z^2_{\tau b} \geq z^{2*}_\tau|p_\tau)E[(z^2_{\tau b})^\zeta(k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - wl^2_{\tau b}(z^2_{\tau b}) + \delta^2\pi^3_{\tau b}|z^2_{\tau b} \geq z^{2*}_\tau, p_\tau] \\ - rk^2_{\tau b} + [1 - \Pr(z^2_{\tau b} \geq z^{2*}_\tau|p_\tau)]E[(z^2_{\tau b})^\zeta(k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - wl^2_{\tau b}(z^2_{\tau b}) + \delta^2\pi^3_{\tau b}|z^2_{\tau b} < z^{2*}_\tau, p_\tau]\Big\}.$$

Finally, this gives the maximization problem shown below which has the associated first-order conditions (13) and (14).

$$E[\pi^2_{\tau b}|p_\tau] = \max_{k^2_{\tau b}, l^2_{\tau b}(z^2_{\tau b})} \Big\{E[(z^2_{\tau b})^\zeta(k^2_{\tau b})^\kappa l^2_{\tau b}(z^2_{\tau b})^\lambda - rk^2_{\tau b} - wl^2_{\tau b}(z^2_{\tau b}) + \delta^2\pi^3_{\tau b}|p_\tau]\Big\}. \tag{27}$$

# C Supplemental Material, Households

Behind the scenes is a representative household residing in a stationary equilibrium. The household solves the following intertemporal maximization problem:

$$\max_{c_t, l_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \ln(c_t - \upsilon\frac{l^{1+\theta}}{1+\theta}),$$

subject to

$$c_t + k_{t+1} = w_t l_t + r_t k_t + \pi_t + (1 - \mathfrak{d})k_t,$$

where $c_t, l_t, k_t$, and $\pi_t$ are period-$t$ aggregate consumption, labor supply, capital, and profits. The solution to this problem gives the consumption Euler equation and the consumption/labor efficiency condition

$$\frac{1}{c_t} = \beta\frac{1}{c_{t+1}}(r_t + 1 - \mathfrak{d})$$

and

$$\upsilon l^\theta_t = w_t, \text{ cf. equation (23).}$$

## C.1 Stationary Equilibrium

It is trivial to deduce equation from the consumption Euler equation that in a stationary equilibrium,

$$r = 1/\beta - 1 + \mathfrak{d} = \iota + \mathfrak{d}, \text{ cf. equation (20)},$$

where the subjective rate of time preference is $\iota \equiv 1/\beta - 1$. The consumer's budget constraint can be written as

$$c = wl + rk + \pi - \mathfrak{d}k.$$

Now, the national income identity implies that aggregate output, $o$, is given by,[15]

$$o = wl + rk + \pi$$

so that

$$c = o - \mathfrak{d}k.$$

# D Supplemental Material, Adjusting for Period Length

When the model is matched with the US data the 3 phases are of different potential lengths: the newborn (3 years), adolescent (8 years), and adult ($\infty$ years). Some adjustments are required to the labor-market-clearing conditions, discounted profits, and employment.

## D.1 Labor-Market-Clearing Condition

From Section 7, the mass of newborn startups is $\mathfrak{m}_n = (1 - \mathfrak{s}_n^3)/(1 - \mathfrak{s}_n)$, the mass of adolescent startup is $\mathfrak{m}_a = \mathfrak{s}_n^3(1 - \mathfrak{s}_a^8)/(1 - \mathfrak{s}_a)$, and the mass of adult firms is $\mathfrak{s}_n^3\mathfrak{s}_a^8/(1 - \mathfrak{s}_e)$. Given this, the labor-market-clearing condition (22) rewrites as

$$\begin{aligned}
&\mathfrak{m}_n\{\Pr\Big[E[\pi^1|h] - \phi \geq E[\pi^1|n]\Big]E[l_h^1(z_h^1)] + \Pr\Big[E[\pi^1|h] - \phi < E[\pi^1|n]\Big]E[l_n^1(z_n^1)]\} \\
&+ \mathfrak{m}_a\Big\{\Pr\Big[E[\pi|h] - \phi \geq E[\pi|n]\Big]\{\Pr[p_h \geq p_h^*]E[l_{hv}^2(z_{\tau v}^2)|p_h \geq p_h^*] + \Pr[p_h < p_h^*]E[l_{hb}^2(z_{hb}^2)|p_h < p_h^*]\}\Big\} \\
&+ \mathfrak{m}_a\Big\{\Pr\Big[E[\pi|h] - \phi < E[\pi|n]\Big]\{\Pr[p_n \geq p_n^*]E[l_{nv}^2(z_{nv}^2)|p_n \geq p_n^*] + \Pr[p_n < p_n^*]E[l_{nb}^2(z_{nb}^2)|p_n < p_n^*]\}\Big\} \\
&+ \mathfrak{m}_e\Big\{\Pr\Big[E[\pi|h] - \phi \geq E[\pi|n]\Big]\{\Pr[p_h \geq p_h^*]E[l_{hv}^3(z_{hv}^3)|p_h \geq p_h^*] + \Pr[p_h < p_h^*]E[l_{hb}^2(z_{hb}^3)|p_h < p_h^*]\}\Big\} \\
&+ \mathfrak{m}_e\Big\{\Pr\Big[E[\pi|h] - \phi < E[\pi|n]\Big]\{\Pr[p_n \geq p_n^*]E[l_{nv}^3(z_{nv}^3)|p_n \geq p_n^*] + \Pr[p_n < p_n^*]E[l_{nb}^3(z_{nb}^3)|p_n < p_n^*]\}\Big\} \\
&= 1
\end{aligned}$$

## D.2 Discounted Profits and Employments

The annual survival-adjusted discount factors for each phase are $\delta_n = \mathfrak{s}_n/(1 + \iota)$, $\delta_a = \mathfrak{s}_a/(1 + \iota)$, and $\delta_e = \mathfrak{s}_e/(1 + \iota)$. There are two relevant concepts: discounted profits (left) and employment (right) for each type of newborn startup.

1. Type-$\tau$ newborn startups

$$(1 + \delta_n + \delta_n^2)\pi_\tau^1 = \frac{1 - \delta_n^3}{1 - \delta_n}\pi_\tau^1 \text{ and } (1 + \mathfrak{s}_n + \mathfrak{s}_n^2)E[l_\tau^1(z_\tau^1)] = \frac{1 - \mathfrak{s}_n^3}{1 - \mathfrak{s}_n}E[l_\tau^1(z_\tau^1)];$$

[15] For each firm $j$ it is the case that $o_j = wl_j + rk_t + \pi_t$. So, the national income identity just sums over all firms.

2. Type-$\tau f$ adolescent startups

$$\frac{1-\delta_a^8}{1-\delta_a}\pi_{\tau f}^2 \text{ and } \frac{1-\mathfrak{s}_a^8}{1-\mathfrak{s}_a}E[l(z_{\tau f}^2)];$$

3. Type-$\tau f$ adult firms

$$\frac{1}{1-\delta_e}\tilde{\pi}_{\tau f}^3 \text{ and } \frac{1}{1-\mathfrak{s}_e}E[l_{\tau f}^3(z_{\tau f}^3)].$$

# E Supplemental Material, Firms

Formulas from the firms' problems are derived that are used to compute the model's equilibrium.

## E.1 Adult Firms

By combining (2) and (3), the solutions for capital and labor for an adult firm are

$$k_{\tau f}^3(z_{\tau f}^3) = z_{\tau f}^3[(\frac{\kappa}{r})^{1-\lambda}(\frac{\lambda}{w})^{\lambda}]^{\frac{1}{\zeta}}$$

and

$$l_{\tau f}^3(z_{\tau f}^3) = z_{\tau f}^3[(\frac{\kappa}{r})^{\kappa}(\frac{\lambda}{w})^{1-\kappa}]^{\frac{1}{\zeta}}. \tag{28}$$

Accordingly, by substituting these solutions into (1), momentary profits during the adult phase are given by

$$\tilde{\pi}_{\tau f}^3(z_{\tau f}^3) = (1-\kappa-\lambda)z_{\tau f}^3[(\frac{\kappa}{r})^{\kappa}(\frac{\lambda}{w})^{\lambda}]^{1/\zeta}.$$

## E.2 Adolescent Startups

From (6), (5), (13), and (14), it can be seen that the generic first-order conditions for an adolescent startup's capital and labor read

$$\lambda(z_{\tau f}^2)^{\zeta}(k_{\tau f}^2)^{\kappa} l_{\tau f}^2(z_{\tau f}^2)^{\lambda-1} = w \tag{29}$$

and

$$\kappa E[(z_{\tau f}^2)^{\zeta}(k_{\tau f}^2)^{\kappa-1} l_{\tau f}^2(z_{\tau f}^2)^{\lambda}|p_\tau] = r.$$

Combining the two first-order conditions yields the following expression for capital investment:

$$k_{\tau f}^2 = \left[\left(\frac{\kappa}{r}\right)^{1-\lambda}\left(\frac{\lambda}{w}\right)^{\lambda}\right]^{\frac{1}{\zeta}} E\left[(z_{\tau f}^2)^{\frac{\zeta}{1-\lambda}}|p_\tau\right]^{\frac{1-\lambda}{\zeta}}. \tag{30}$$

To use this equation requires knowledge of the conditional expectation $E\left[(z_{\tau f}^2)^{\frac{\zeta}{1-\lambda}}|p_\tau\right]$. This can be computed using the properties of the log-normal distribution.

To this end, let $\tilde{z}_{\tau f} \equiv (z_{\tau f}^2)^{\frac{\zeta}{1-\lambda}}$. Then,

$$\ln \tilde{z}_{\tau f} = \frac{\zeta}{1-\lambda}\ln z_{\tau f}^2 = \frac{\zeta}{1-\lambda}(\ln p_\tau + \ln \varepsilon_{\tau f}).$$

Now, the the sum of two independently distributed normal variables is normal, with mean and variance given by the sum of the means and the variance of the two original distributions. Thus,

$$\ln \tilde{z}_{\tau f} | p_\tau \sim \mathcal{N}(\mu_{\tau f}, \sigma^2_{\tau f}),$$

where $\mu_{\tau f} = \frac{\zeta}{1-\lambda}(\ln p_\tau + \gamma_{\tau f})$ and $\sigma^2_{\tau f} = \frac{\zeta^2}{(1-\lambda)^2}\sigma^2_{\varepsilon_\tau}$ (note at this stage the variance of $p_\tau$ is zero). Then, using the formula for the expected value of a variable that distributed according to a log-normal distribution gives

$$E\left[(z^2_{\tau f})^{\frac{\zeta}{1-\lambda}} | p_\tau\right] = E[\tilde{z}_{\tau f} | p_\tau] = \exp\left\{\mu_{\tau f} + \frac{1}{2}\sigma^2_{\tau f}\right\}.$$

Formula (30) for $k^2_{\tau f}$ now appears as

$$k^2_{\tau f}(p_\tau) = p_\tau \exp\left\{\gamma_{\tau f} + \frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\left[\left(\frac{\kappa}{r}\right)^{1-\lambda}\left(\frac{\lambda}{w}\right)^{\lambda}\right]^{\frac{1}{\zeta}}. \tag{31}$$

By substituting (31) into (29), a solution can be derived for labor, $l^2_{\tau f}(z^2_{\tau f})$, conditional both on potential, $p_\tau$, and the productivity shock, $\varepsilon_{\tau f}$. Specifically,

$$l^2_{\tau f}(z^2_{\tau f}) = p_\tau \varepsilon_{\tau f}^{\frac{\zeta}{1-\lambda}} \exp\left\{\frac{\kappa}{1-\lambda}\gamma_{\tau f} + \frac{\kappa\zeta}{(1-\lambda)^2}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\left(\frac{\kappa}{r}\right)^{\frac{\kappa}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\zeta+\lambda\kappa}{(1-\lambda)\zeta}}. \tag{32}$$

Last, the adolescent startup's expected profits, conditional on its potential, read

$$E[\hat{\pi}^2_{\tau f}|p_\tau] \equiv E\left[(z^2_{\tau f})^{\zeta}\left(k^2_{\tau f}(p_\tau)\right)^{\kappa} l^2_{\tau f}(z^2_{\tau f})^{\lambda} - r k^2_{\tau f}(p_\tau) - w l^2_{\tau f}(z^2_{\tau b})|p_\tau\right].$$

These profits may *not* be what either an entrepreneur or a financier earns. By using the policy functions (31) and (32), it follows that

$$E[\hat{\pi}^2_{\tau f}|p_\tau] = p_\tau(1-\lambda-\kappa)\exp\left\{\gamma_{\tau f} + \frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\left[(\frac{\kappa}{r})^{\kappa}(\frac{\lambda}{w})^{\lambda}\right]^{\frac{1}{\zeta}}. \tag{33}$$

**Expected Selling Value**

The selling value of a startup at the end of the adolescent phase, conditional on its potential is $E[\pi^3_{\tau b}|p_\tau]$ . Now,

$$\pi^3_{\tau f} = \frac{1}{1-\delta^3}\tilde{\pi}^3_{\tau f} = \frac{1}{1-\delta^3}(1-\lambda-\kappa)z^3_{\tau f}[(\frac{\kappa}{r})^{\kappa}(\frac{\lambda}{w})^{\lambda}]^{1/\zeta},$$

where $z^3_{\tau f} = \chi_{\tau f} z^2_{\tau f} = \chi_{\tau f} p_\tau \varepsilon_{\tau f}$. So to compute the expected selling value, $E[\pi^3_{\tau b}|p_\tau]$, requires a solution for $E[z^3_{\tau f}|p_\tau]$. By using the properties of the log-normal, $E[z^3_{\tau f}|p_\tau]$ can be rewritten as

$$E[z^3_{\tau f}|p_\tau] = \chi_{\tau f} p_\tau E[\varepsilon_{\tau f}|p_\tau] = \chi_{\tau f} p_\tau \exp\left\{\gamma_{\tau f} + \frac{1}{2}\sigma^2_{\varepsilon_\tau}\right\}.$$

Thus, the expected selling value of an adolescent startup rewrites as

$$E[\pi^3_{\tau b}|p_\tau] = \frac{1}{1-\delta^3}(1-\lambda-\kappa)\chi_{\tau f} p_\tau \exp\left\{\gamma_{\tau f} + \frac{1}{2}\sigma^2_{\varepsilon_\tau}\right\}[(\frac{\kappa}{r})^{\kappa}(\frac{\lambda}{w})^{\lambda}]^{1/\zeta}, \tag{34}$$

where $f = v$, if $p_\tau > p^*_\tau$, and $f = b$, if $p_\tau < p^*_\tau$.

**Entrepreneur's Expected Profits, Bank Financing**

It is easy to deduce that in the adolescent phase the entrepreneur's expected profits from bank financing are

$$E[\pi^2_{\tau b}|p_\tau] = E[\hat{\pi}^2_{\tau f}|p] + \delta^2 E[\pi^3_{\tau b}|p_\tau],$$

where momentary expected profits are given by $E[\hat{\pi}^2_{\tau f}|p]$ and the discounted expected selling value by $\delta^2 E[\pi^3_{\tau b}|p_\tau]$. Equations (33) and (34) allows this to be modified to

$$E[\pi^2_{\tau b}|p_\tau] = p_\tau(1-\lambda-\kappa)\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}\left[\exp\{\gamma_{\tau b}+\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\}+\frac{\delta^2}{1-\delta^3}\chi_{\tau b}\exp\{\gamma_{\tau b}+\frac{1}{2}\sigma^2_{\varepsilon_\tau}\}\right]. \quad (35)$$

**Entrepreneur's Expected Profits, Venture Capital Financing**

From the Nash Bargaining problem (4) it is clear that the entrepreneur's expected profits in the adolescent phase from venture capital financing are

$$E[\pi^2_{\tau v}|p_\tau] = E\left[\Big(1-s_\tau(p_\tau)\Big)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}]|p_\tau\right].$$

Equation (7) in turn implies that

$$\begin{aligned}&E\left[\Big(1-s_\tau(p_\tau)\Big)[(z^2_{\tau v})^\zeta(k^2_{\tau v})^\kappa l_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v} - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}]|p_\tau\right]\\ &= (1-\eta)E[\pi^2_{\tau b}|p_\tau]+\eta E\left[(z^2_{\tau v})^\zeta\left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v} - \alpha - \xi p_\tau|p_\tau\right].\end{aligned} \quad (36)$$

Following steps parallel to the derivation of (35), it can be established that

$$\begin{aligned}&E\left[(z^2_{\tau v})^\zeta\left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right] =\\ &\quad p_\tau(1-\lambda-\kappa)\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}\left[\exp\{\gamma_{\tau v}+\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\}+\frac{\delta^2}{1-\delta^3}\chi_{\tau v}\exp\{\gamma_{\tau v}+\frac{1}{2}\sigma^2_{\varepsilon_\tau}\}\right].\end{aligned} \quad (37)$$

Finally, plugging (37) into (36) and rearranging yields the expected profits from venture capital financing:

$$\begin{aligned}&E[\pi^2_{\tau v}|p_\tau] = p_\tau(1-\lambda-\kappa)\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}\times\\ &\left\{\exp\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\}[\eta(e^{\gamma_{\tau v}}-e^{\gamma_{\tau b}})+e^{\gamma_{\tau b}}]+\frac{\delta^2}{1-\delta^3}\exp\{\frac{1}{2}\sigma^2_{\varepsilon_\tau}\}\chi_{\tau b}\left[\eta(\frac{\chi_{\tau v}}{\chi_{\tau b}}e^{\gamma_{\tau v}}-e^{\gamma_{\tau b}})+e^{\gamma_{\tau b}}\right]\right\}-\eta(\alpha+\xi p_\tau).\end{aligned} \quad (38)$$

## E.3 Newborn Startups

From (18) the solution for the labor hired by a newborn startup is

$$l^1_\tau(z^1_\tau) = \left[\frac{\lambda}{w}(z^1_\tau)^\zeta m^\kappa_\tau\right]^{\frac{1}{1-\lambda}}. \quad (39)$$

Using this in (17) then gives the momentary profits for a newborn startup:

$$\pi^1_\tau(z^1_\tau) = (1-\lambda)\left[\left(\frac{\lambda}{w}\right)^\lambda(z^1_\tau)^\zeta m^\kappa_\tau\right]^{\frac{1}{1-\lambda}}. \quad (40)$$

**Entrepreneur's Expected Profits**

For a newborn startup the entrepreneur's unconditional expected profits from using technology $\tau$ are given by

$$E[\pi^1|\tau] = E[\pi^1_\tau] + \delta^1 E[\pi^2_\tau],$$

where

$$E[\pi^2_\tau] = \Pr[p_\tau \geq p^*_\tau]E[\pi^2_{\tau v}|p_\tau \geq p^*_\tau] + \Big[1 - \Pr[p_\tau \geq p^*_\tau]\Big]E[\pi^2_{\tau b}|p_\tau < p^*_\tau].$$

Here $E[\pi^1_\tau]$ is the expected momentary profits from the newborn phase and $E[\pi^2_\tau]$ is the expected profits from the adolescent phase. The terms for $E[\pi^1_\tau]$ and $E[\pi^2_\tau]$ are now computed, starting with the latter.

By using (35) and (38), the expected profits from the adolescent phase are

$$\begin{aligned}
E[\pi^2_\tau] = \Big[1 - \Pr[p_\tau \geq p^*_\tau]\Big] &\times E[p_\tau|p_\tau < p^*_\tau](1-\lambda-\kappa)\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}} \\
&\times \left\{\exp\{\gamma_{\tau b} + \frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau b}\exp\{\gamma_{\tau b} + \frac{1}{2}\sigma^2_{\varepsilon_\tau}\}\right\} \\
&+ \Pr[p_\tau \geq p^*_\tau] \times E[p_\tau|p_\tau > p^*_\tau](1-\lambda-\kappa)\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}} \times \\
\{\Big[\exp\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\}\left[\eta(e^{\gamma_{\tau v}} - e^{\gamma_{\tau b}}) + e^{\gamma_{\tau b}}\right] &+ \frac{\delta^2}{1-\delta^3}\exp\{\frac{1}{2}\sigma^2_{\varepsilon_\tau}\}\chi_{\tau b}\left[\eta(\frac{\chi_{\tau v}}{\chi_{\tau b}}e^{\gamma_{\tau v}} - e^{\gamma_{\tau b}}) + e^{\gamma_{\tau b}}\right]\Big] \\
&- \eta\left(\alpha + \xi E[p_\tau|p_\tau > p^*_\tau]\right)\},
\end{aligned}$$

Next, using the formula for the expected value of variable with a truncated log-normal distribution (see Appendix G) allows the expected values for the potentials, $E[p_\tau|p_\tau < p^*_\tau]$ and $E[p|p > p^*]$, to be put forth as

$$E[p_\tau|p_\tau < p^*_\tau] = E\left[e^{\ln p_\tau}|\ln p_\tau < \ln p^*_\tau\right] = \exp\left\{\mu_{p_\tau} + \frac{\sigma^2_{p_\tau}}{2}\right\}\frac{\Phi_{\mu^*_{p_\tau},\sigma^2_{p_\tau}}(\ln p^*_\tau)}{\Phi_{\mu_{p_\tau},\sigma^2_{p_\tau}}(\ln p^*_\tau)} \tag{41}$$

and

$$E[p_\tau|p_\tau > p^*_\tau] = E\left[e^{\ln p_\tau}|\ln p_\tau > \ln p^*_\tau\right] = \exp\left\{\mu_p + \frac{\sigma^2_p}{2}\right\}\frac{1-\Phi_{\mu^*_{p_\tau},\sigma^2_{p_\tau}}(\ln p^*_\tau)}{1-\Phi_{\mu_{p_\tau},\sigma^2_{p_\tau}}(\ln p^*_\tau)}, \tag{42}$$

where $\Phi_{\mu^*_{p_\tau},\sigma^2_{p_\tau}}$ denotes the cumulative distribution function for a normal distribution with mean $\mu^*_{p_\tau} \equiv \mu_{p_\tau} + \sigma^2_{p_\tau}$ and variance $\sigma^2_{p_\tau}$.

Last, using (40), the unconditional momentary profits from the newborn phase, $E[\pi^1_\tau]$, can be expressed as

$$E[\pi^1_\tau] = (1-\lambda)\left[\left(\frac{\lambda}{w}\right)^\lambda m^\kappa_\tau\right]^{\frac{1}{1-\lambda}} E\left[(z^1_\tau)^{\frac{\zeta}{1-\lambda}}\right].$$

The properties of the bivariate log-normal distribution allow this to read

$$E\left[(z^1_\tau)^{\frac{\zeta}{1-\lambda}}\right] = \exp\left\{\frac{\zeta}{1-\lambda}\mu_{z^1_\tau} + \left(\frac{\zeta}{1-\lambda}\right)^2\frac{\sigma^2_{z^1_\tau}}{2}\right\}.$$

# F Supplemental Material, Data Targets

Formulas for the data targets are presented here. In what follows, let $\Phi_{\mu,\sigma^2}(x)$ represent the cumulative normal distribution for a variable $x$ with mean $\mu$ and variance $\sigma^2$. The associated density function is denoted by $\phi_{\mu,\sigma^2}(x)$. When subscripts are omitted, $\Phi(x)$ and $\phi(x)$ refer to the cumulative distribution and density functions for the standard normal distribution with mean zero and variance of one. Finally, $\mu^*_{p_\tau} \equiv \mu_{p_\tau} + \sigma^2_{p_\tau}$ where $\mu_{p_\tau}$ and $\sigma^2_{p_\tau}$ are the mean and variance of the marginal distribution for the log of potential, $\ln p_\tau$.

## F.1 Average Employment

**Newborn Startups**

From equation (39), average employment in a newborn startup can be expressed as

$$E[l^1_\tau] = \left[\frac{\lambda}{w} m^\kappa_\tau\right]^{\frac{1}{1-\lambda}} E\left[(z^1_\tau)^{\frac{\zeta}{1-\lambda}}\right],$$

where by using the properties of a log-normal distribution it can be calculated that

$$E\left[(z^1_\tau)^{\frac{\zeta}{1-\lambda}}\right] = \exp\left\{\frac{\zeta}{1-\lambda}\mu_{z^1_\tau} + \left(\frac{\zeta}{1-\lambda}\right)^2 \frac{\sigma^2_{z^1_\tau}}{2}\right\}.$$

**Adolescent Startups, Bank Financing**

By using (32), average employment in an adolescent bank-backed startup is

$$E[l^2_{\tau b}] = E[p_\tau | p_\tau < p^*_\tau] E\left[\varepsilon_{\tau b}^{\frac{\zeta}{1-\lambda}}\right] \exp\left\{\frac{\kappa}{1-\lambda}\gamma_{\tau b} + \frac{\kappa\zeta}{(1-\lambda)^2}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\left(\frac{\kappa}{r}\right)^{\frac{\kappa}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\zeta+\lambda\kappa}{(1-\lambda)\zeta}}. \tag{43}$$

Exploiting the properties of the log-normal distribution gives

$$E\left[\varepsilon_{\tau f}^{\frac{\zeta}{1-\lambda}}\right] = \exp\left\{\frac{\zeta}{1-\lambda}\gamma_{\tau f} + \left(\frac{\zeta}{1-\lambda}\right)^2 \frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}. \tag{44}$$

Therefore, substituting in for $E[p_\tau | p_\tau < p^*_\tau]$, by using (41), allows (43) to be rewritten as

$$E[l^2_{\tau b}] = \exp\left\{\gamma_{\tau b} + \mu_{p_\tau} + \frac{\sigma^2_{p_\tau}}{2} + \frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\left(\frac{\kappa}{r}\right)^{\frac{\kappa}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\zeta+\lambda\kappa}{(1-\lambda)\zeta}} \frac{\Phi_{\mu^*_{p_\tau},\sigma^2_{p_\tau}}(\ln p^*_\tau)}{\Phi_{\mu_{p_\tau},\sigma^2_{p_\tau}}(\ln p^*_\tau)}. \tag{45}$$

**Adolescent Startups, VC Financing**

Equation (32) implies that average employment in an adolescent VC-backed startup is

$$E[l^2_{\tau v}] = E[p_\tau | p_\tau > p^*_\tau] E\left[\varepsilon_{\tau v}^{\frac{\zeta}{1-\lambda}}\right] \exp\left\{\frac{\kappa}{1-\lambda}\gamma_{\tau v} + \frac{\kappa\zeta}{(1-\lambda)^2}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\left(\frac{\kappa}{r}\right)^{\frac{\kappa}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\zeta+\lambda\kappa}{(1-\lambda)\zeta}}.$$

Next, using (42) and (44) results in

$$E[l^2_{\tau v}] = \exp\left\{\gamma_{\tau v} + \mu_{p_\tau} + \frac{\sigma^2_{p_\tau}}{2} + \frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\left(\frac{\kappa}{r}\right)^{\frac{\kappa}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\zeta+\lambda\kappa}{(1-\lambda)\zeta}} \frac{1-\Phi_{\mu^*_{p_\tau},\sigma^2_{p_\tau}}(\ln p^*_\tau)}{1-\Phi_{\mu_{p_\tau},\sigma^2_{p_\tau}}(\ln p^*_\tau)}.$$

### Adult Firms, Bank Financing

Using (28) average employment in an adult bank-backed firm can be expressed as

$$E[l_{\tau b}^3] = \chi_{\tau b} E[p_\tau | p_\tau < p_\tau^*] E[\varepsilon_{\tau b}][(\frac{\kappa}{r})^\kappa (\frac{\lambda}{w})^{1-\kappa}]^{\frac{1}{\zeta}}.$$

The above expression can be rewritten, using (41) and the properties of the log-normal, as

$$E[l_{\tau b}^3] = \chi_{\tau b} \exp\left\{\gamma_{\tau b} + \frac{1}{2}\sigma_{\varepsilon_\tau}^2 + \mu_{p_\tau} + \frac{1}{2}\sigma_{p_\tau}^2\right\} [(\frac{\kappa}{r})^\kappa (\frac{\lambda}{w})^{1-\kappa}]^{\frac{1}{\zeta}} \frac{\Phi_{\mu_{p_\tau}^*, \sigma_{p_\tau}^2}(\ln p_\tau^*)}{\Phi_{\mu_{p_\tau}, \sigma_{p_\tau}^2}(\ln p_\tau^*)}.$$

### Adult Firms, VC Financing

Average employment in an adult VC-backed firm can be expressed using (28) as

$$E[l_{\tau v}^3] = \chi_{\tau v} E[p_\tau | p_\tau > p_\tau^*] E[\varepsilon_{\tau v}][(\frac{\kappa}{r})^\kappa (\frac{\lambda}{w})^{1-\kappa}]^{\frac{1}{\zeta}}.$$

Using (42) together with the properties of the log-normal permits the following rewrite:

$$E[l_{\tau b}^3] = \chi_{\tau v} \exp\left\{\gamma_{\tau v} + \frac{1}{2}\sigma_{\varepsilon_\tau}^2 + \mu_{p_\tau} + \frac{1}{2}\sigma_{p_\tau}^2\right\} [(\frac{\kappa}{r})^\kappa (\frac{\lambda}{w})^{1-\kappa}]^{\frac{1}{\zeta}} \frac{1 - \Phi_{\mu_{p_\tau}^*, \sigma_{p_\tau}^2}(\ln p_\tau^*)}{1 - \Phi_{\mu_{p_\tau}, \sigma_{p_\tau}^2}(\ln p_\tau^*)}.$$

## F.2 Variances

### Newborn Startups

It is straightforward to see from equation (39) that the variance of log employment for newborn startup is

$$V\left(\ln l_\tau^1\right) = \left(\frac{\zeta}{1-\lambda}\right)^2 \sigma_{z_\tau}^2.$$

### Adolescent Startups, Bank Financing

Using (32), the variance of log employment in adolescent bank-backed startup can be written as

$$V(\ln l_{\tau b}^2) = V\left(\ln p_\tau | \ln p_\tau < \ln p_\tau^*\right) + \left(\frac{\zeta}{1-\lambda}\right)^2 \sigma_{\varepsilon_\tau}^2.$$

The first term is

$$V\left(\ln p_\tau | \ln p_\tau < \ln p_\tau^*\right) = E\left[(\ln p_\tau)^2 | \ln p_\tau < \ln p_\tau^*\right] - E[\ln p_\tau | \ln p_\tau < \ln p_\tau^*]^2.$$

Let $\tilde{p_\tau} \equiv (\ln p_\tau - \mu_{p_\tau})/\sigma_{p_\tau}$. Using the formula for the variance of a variable distributed according to a truncated normal distribution (see Appendix G), it transpires that

$$E\left[(\ln p_\tau)^2 | \ln p_\tau < \ln p_\tau^*\right] = \sigma_{p_\tau}^2 + \mu_{p_\tau}^2 - \frac{\sigma_{p_\tau}^2 \tilde{p_\tau} \phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})} - 2\mu_{p_\tau}\sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})}.$$

Then, using the formula for the mean of a truncated normal distribution (Appendix G), one can derive that

$$E[\ln p_\tau | \ln p_\tau < \ln p_\tau^*] = \mu_{p_\tau} - \sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})}.$$

Therefore, the variance of log-employment can be written as follows

$$V\left(\ln l_{\tau b}^2\right) = \sigma_{p_\tau}^2 + \mu_{p_\tau}^2 - \frac{\sigma_{p_\tau}^2 \tilde{p_\tau}\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})} - 2\mu_{p_\tau}\sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})} - \left(\mu_{p_\tau} - \sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{\Phi(\tilde{p_\tau})}\right)^2 + \left(\frac{\zeta}{1-\lambda}\right)^2 \sigma_{\varepsilon_\tau}^2.$$

**Adolescent Startups, VC Financing**

This derivation parallels the one above. The variance of log employment in adolescent VC-backed startup can be expressed, using (32), as

$$V(\ln l^2_{\tau v}) = V\left(\ln p_\tau | \ln p_\tau > \ln p^*_\tau\right) + \left(\frac{\zeta}{1-\lambda}\right)^2 \sigma^2_{\varepsilon_\tau}.$$

Rewrite the first term as

$$V\left(\ln p_\tau | \ln p_\tau > \ln p^*_\tau\right) = E\left[(\ln p_\tau)^2 | \ln p_\tau > \ln p^*_\tau\right] - E[\ln p_\tau | \ln p_\tau > \ln p^*_\tau]^2.$$

Again, let $\tilde{p_\tau} \equiv (\ln p_\tau - \mu_{p_\tau})/\sigma_{p_\tau}$. Using the formula for the variance of a truncated normal distribution (Appendix G) delivers

$$E\left[(\ln p_\tau)^2 | \ln p_\tau > \ln p^*_\tau\right] = \sigma^2_{p_\tau} + \mu^2_{p_\tau} + \frac{\sigma^2_{p_\tau}\tilde{p_\tau}\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})} + 2\mu_{p_\tau}\sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})}.$$

Then, using the formula for the mean of a truncated normal distribution (Appendix G) results in

$$E[\ln p_\tau | \ln p_\tau > \ln p^*_\tau] = \mu_{p_\tau} + \sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})}.$$

The variance of log-employment therefore is

$$V\left(\ln l^2_{\tau v}\right) = \sigma^2_{p_\tau} + \mu^2_{p_\tau} + \frac{\sigma^2_{p_\tau}\tilde{p_\tau}\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})} + 2\mu_{p_\tau}\sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})} - \left(\mu_{p_\tau} + \sigma_{p_\tau}\frac{\phi(\tilde{p_\tau})}{1-\Phi(\tilde{p_\tau})}\right)^2 + \left(\frac{\zeta}{1-\lambda}\right)^2 \sigma^2_{\varepsilon_\tau}.$$

## F.3 VC's Profit Share

The formula for a venture capitalist's share of a type-$\tau$ adolescent startup's profits is derived now. From (7), it can be seen that a venture capitalist's share of profits is

$$s(p_\tau) = \frac{\eta(\alpha_\tau + \xi p_\tau) + (1-\eta)\left[E\left[(z^2_{\tau v})^\zeta \left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right] - E[\pi^2_{\tau b}|p_\tau]\right]}{E\left[(z^2_{\tau v})^\zeta \left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right]}. \tag{46}$$

Using (35) and (37), it can be shown that

$$(1-\eta)\frac{\left[E\left[(z^2_{\tau v})^\zeta \left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right] - E[\pi^2_{\tau b}|p_\tau]\right]}{E\left[(z^2_{\tau v})^\zeta \left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right]} =$$
$$\frac{(1-\eta)}{e^{\gamma_{\tau v}}}\frac{\exp\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\}(e^{\gamma_{\tau v}} - e^{\gamma_{\tau b}}) + \frac{\delta^2}{1-\delta^3}\exp\{\frac{1}{2}\sigma^2_{\varepsilon_\tau}\}(\chi_{\tau v}e^{\gamma_{\tau v}} - \chi_{\tau b}e^{\gamma_{\tau b}})}{\exp\left\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau v}\exp\left\{\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}}. \tag{47}$$

Additionally, one can express

$$\frac{\eta(\alpha_\tau + \xi p_\tau)}{E\left[(z^2_{\tau v})^\zeta \left(k^2_{\tau v}(p_\tau)\right)^\kappa l^2_{\tau v}(z^2_{\tau v})^\lambda - rk^2_{\tau v}(p_\tau) - wl^2_{\tau v}(z^2_{\tau v}) + \delta^2\pi^3_{\tau v}|p_\tau\right]} =$$
$$\frac{\eta(\alpha_\tau p_\tau^{-1} + \xi)}{(1-\lambda-\kappa)e^{\gamma_{\tau v}}\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}\left\{\exp\left\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau v}\exp\left\{\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\right\}}. \tag{48}$$

Finally, to compute $E[s(p_\tau)]$ requires solving for $E[1/p_\tau|p_\tau > p^*_\tau]$. Using the formula for the mean of a truncated normal distribution (Appendix G) gives the average share of profits for a type-$\tau$ project or

$$E[s(p_\tau)] = \frac{(1-\eta)}{e^{\gamma_{\tau v}}} \frac{\exp\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\}(e^{\gamma_{\tau v}} - e^{\gamma_{\tau b}}) + \frac{\delta^2}{1-\delta^3}\exp\{\frac{1}{2}\sigma^2_{\varepsilon_\tau}\}(\chi_{\tau v}e^{\gamma_{\tau v}} - \chi_{\tau b}e^{\gamma_{\tau b}})}{\exp\left\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau v}\exp\left\{\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}} +$$
$$+ \frac{\eta}{(1-\lambda-\kappa)e^{\gamma_{\tau v}}\left[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda\right]^{\frac{1}{\zeta}}} \times \frac{\alpha E\left[p^{-1}_\tau|p_\tau > p^*_\tau\right] + \xi}{\left\{\exp\left\{\frac{\zeta}{1-\lambda}\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\} + \frac{\delta^2}{1-\delta^3}\chi_{\tau v}\exp\left\{\frac{\sigma^2_{\varepsilon_\tau}}{2}\right\}\right\}}. \quad (49)$$

# G Supplemental Material, Properties of the Normal and Log-Normal Distributions

Three properties of the normal and the log-normal distribution are listed here.[16] In what follows, let $\Phi(x)$ and $\phi(x)$ refer to the standard normal cumulative distribution and density functions with mean zero and variance of one for the random variable $x$.

1. Expected values for one-sided truncations of a normally distributed variable with mean $\mu$ and standard deviation $\sigma$:

$$E[x|x > a] = \mu + \sigma\frac{\phi\Big((a-\mu)/\sigma\Big)}{1-\Phi\Big((a-\mu)/\sigma\Big)}$$

and

$$E[x|x < a] = \mu - \sigma\frac{\phi\Big((a-\mu)/\sigma\Big)}{\Phi\Big((a-\mu)/\sigma\Big)}.$$

2. Variances for one-sided truncations of a normally distributed variable with mean $\mu$ and standard deviation $\sigma$:

$$V(x|x > a) = \sigma^2\{1 + [(a-\mu)/\sigma]\frac{\phi\Big((a-\mu)/\sigma\Big)}{1-\Phi\Big((a-\mu)/\sigma\Big)} - [\frac{\phi\Big((a-\mu)/\sigma\Big)}{1-\Phi\Big((a-\mu)/\sigma\Big)}]^2\}$$

and

$$V(x|x < a) = \sigma^2\{1 - [(a-\mu)/\sigma]\frac{\phi\Big((a-\mu)/\sigma\Big)}{\Phi\Big((a-\mu)/\sigma\Big)} - [\frac{\phi\Big((a-\mu)/\sigma\Big)}{\Phi\Big((a-\mu)/\sigma\Big)}]^2\}.$$

3. Expected values for one-sided truncations of a log-normally distributed variable $x$ with mean $\mu$ and standard deviation $\sigma$:

$$E[x|x > a] = e^{\mu+\sigma^2/2}\frac{\Phi\Big((\mu+\sigma^2-\ln a)/\sigma\Big)}{1-\Phi\Big((\ln a-\mu)/\sigma\Big)}$$

and

$$E\left[x|x < a\right] = e^{\mu+\sigma^2/2}\frac{\Phi\Big((\ln a-\mu-\sigma^2)/\sigma\Big)}{\Phi\Big((\ln a-\mu)/\sigma\Big)}.$$

[16]See https://en.wikipedia.org/wiki/Truncated_normal_distribution
and
https://en.wikipedia.org/wiki/Log-normal_distribution.