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Product Recommendations and Price Parity Clauses

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Abstract

A seller can offer an experience good directly to consumers and indirectly through an intermediary. When selling indirectly, the intermediary provides recommendations based on the consumer's match value and the prices at which the product is sold. The intermediary faces the trade-off between extracting rents from consumers who strongly care about the match value versus providing less informative recommendations but also serving consumers who do not. We analyze the allocative and welfare effects of prohibiting price parity clauses and/or regulating the intermediary's recommender system. Prohibiting price parity clauses is always welfare decreasing in our model.

Keywords: intermediation, digital platforms, price parity, recommender system, MFN clause, e-commerce

JEL-classification: L12, L15, D21, D42, M37

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1 Introduction

Price-parity clauses (PPCs) are contractual obligations that stipulate sellers not to offer lower prices or better conditions on alternative sales channels.[1] In the US, PPCs are often called most-favored-customer clauses or "MFNs". Prominent examples of digital platforms imposing PPCs on sellers have been Booking.com for hotels and Apple for ebooks. PPCs have been investigated and prohibited by many competition authorities (in some instances only wide PPCs, in others also narrow PPCs).[2] In some countries, legislatures have intervened and made all PPCs illegal in a specific sector. In the European Union, the Digital Markets Act (DMA) prohibits designated gatekeeper platforms from using any PPCs in their dealings with sellers.

Digital platforms manage their ecosystems in multiple ways. This includes not only contractual obligations on pricing and listing (such as PPCs). With the frequent introduction of new products on digital marketplaces and app stores, an important role of these platforms is to steer consumer demand as they enjoy an informational advantage over sellers and consumers. In particular, platforms make purchase recommendations and consumers may follow these recommendations. In this paper, we analyze the interplay between demand steering and the use of PPCs.

If the platform is prevented from using PPCs and a seller sets a lower price in the direct channel to avoid paying commissions to the platform, this deprives the platform of revenues. As a response, the platform may steer consumers away from the seller. This reasoning is not purely theoretical. Hunold et al. (2020) find that hotels were ranked lower on hotel booking portals such as Booking.com if they offered a lower price outside the platform.[3]

Several potential anti-competitive effects of price parity are well established in economic

[1] The literature distinguishes between narrow and wide PPCs. According to the former, sellers agree not to offer a lower price on their own direct channel, but are free to set lower prices on any indirect sales channel. According to the latter, sellers agree not to offer a lower price through any other sales channel (this includes direct and indirect channels). In the main model, we consider a monopoly platform and thus focus on the effect of PPCs regarding the direct channel.

[2] For more details, see, for instance, Peitz (2022).

[3] For a discussion of the platform's incentives when designing its recommender system, see for instance Belleflamme and Peitz (2021, chapter 6).

theory and a few possible efficiency defenses have been provided. This paper adds to the latter and shows that price parity clauses (i) affect demand steering, (ii) if non-neutral, lead to more transactions on the platform, and (iii) this benefits consumers and society.

We consider an experience good setting in which some consumers (the "picky" ones) either encounter a good or a bad match, and the platform can decide whether to give a purchase recommendation to consumers in the indirect sales channel. All other consumers are "flexible" and thus do not face any uncertainty about their valuation. The seller offers the product in the direct sales channel and, through the platform, in the indirect channel, which is assumed to be more convenient for consumers and thus more efficient. We analyze this model for a particular parameter constellation such that there are gains from trade for flexible but not for picky consumers in the direct channel and gains from trade for picky consumers with a good match are higher than those for flexible consumers.

The intermediary charges a fee to the seller, which consists in a profit share. Then, the seller sets a price in the direct and in the direct channel. These prices are fed into the recommendation algorithm of the intermediary. The intermediary always recommends the product to picky consumers with a good match and may also recommend the product with positive probability to a picky consumer with a bad match; this probability is treated as observable to consumers. Based on prices, consumers decide where to shop and possibly after updating their information about the match quality whether to buy.

The novel economic mechanism underlying our result is based on the platform adjusting its purchase recommendation to the prices set by the seller in the direct and the indirect channel. The platform may want to induce the outcome such that all trade takes place in the indirect channel with purchase recommendations to picky consumers such that the conditional expected value of a picky consumer following the recommendation is equal to the willingness to pay of the flexible consumers, which is also the price set by the seller – we call this the *biased recommendation* outcome. Respecting the seller's outside option to sell to flexible consumers in the direct channel, the intermediary would like to extract all the surplus reduced by the seller's profit with the outside option. Suppose that the intermediary sets the fee (i.e. the share of profit it extracts) accordingly. A seller profitably deviates from this plan of action: It sets a high price in the indirect channel inducing the platform to recommend the product to picky consumers with a good match only and sells to flexible consumers at a lower price directly. By imposing a PPC the intermediary can rule such a behavior out and better extract surplus from the seller.

The intermediary either implements the biased recommendation outcome or *inefficient bypass*, in which sales in the indirect channel are made to picky consumers with a good match only and flexible consumers buy in the direct channel. Imposing upfront a PPC allows the platform to implement the outcome that would prevail under vertical integration (and that would also be implemented if the intermediary had full commitment power over its recommendation policy vis-a-vis sellers). Thus, PPCs serve as a substitute for the platform's ex ante commitment to the recommendation strategy. Prohibiting the use of PPCs reduces the platform's profit when the platform serves flexible and some picky consumers. It is then relatively more attractive for the platform to induce the inefficient bypass outcome. As a result, the prohibition of PPCs is welfare-reducing.

In addition or as an alternative to the prohibition of PPCs, the regulator may prohibit recommendations that would reduce total surplus. Such a regulation on purchase recommendations may increase welfare and implement the first-best outcome. However, it increases the incentives of the intermediary and the seller to make decisions that lead to trade outside the platform that is less efficient. Thus, while potentially welfare-increasing, such regulation may backfire.

Given the regulation on recommendations, prohibiting PPCs is either welfare-neutral or strictly welfare-decreasing. Thus, the two types of regulation never complement each other in our framework.

We extend our analysis to a setting in which the intermediary charges an ad valorem fee on revenues in the indirect channel. Under laissez-faire, the intermediary no longer implements the outcome under vertical integration, but implements inefficient bypass more often than in our base model. Our results regarding the different regulatory policies are qualitatively the same. However, for any given regulatory regime (which includes the laissez-faire), such a change in the pricing instrument available to the intermediary is always weakly welfare-decreasing.

We also extend our analysis to competing intermediaries in which one of them offers a higher convenience benefit and is better able to make informed recommendations. Our qualitative findings regarding the different regulatory policies are confirmed. However, introducing a competing intermediary changes the nature of the PPC (in the case, in which sellers do not have a direct sales channel they can only sell through one of the platforms) and our analysis provides a novel efficiency defense of wide PPCs.

Literature review. Our paper contributes to answering the question of how platform design responds to regulation (for more on platform design, see e.g. Teh, 2022, Choi and Jeon, 2023, Hagiu and Wright, 2024) and analyzes how the prohibition of PPCs and the regulation of recommender systems affect market outcomes. In our analysis, the platform is an intermediary that facilitates trade between a seller and consumers; our model does not feature any network effects.

Our formal setting is based on Peitz and Sobolev (forthcoming) who look at biased recommendation by a monopoly firm (that may operate as an intermediary) with full commitment power over its recommendation policy vis-a-vis buyers and, in case of an intermediary, also vis-a-vis sellers. In this context, PPCs are superfluous, whereas in our analysis the ability to use PPCs turns out to be non-neutral. Thereby, we connect to two strands of literature: one on product recommendations and the other on price-parity clauses. To the best of our knowledge, our paper is the first to look at PPCs in which the intermediary not only facilitates trade, but also plays the role of a recommender.

We consider an information design problem in which the intermediary provides recommendations to consumers and consumers correctly interpret the recommendations that are made given the information design. We consider a game played between the intermediary and the seller that determines the intermediary's information design.

Previous work has identified several reasons why an intermediary may make biased purchase recommendations. In Lee (2021), the intermediary is a mechanism designer who, for given product prices, persuades consumers to buy the recommended product. Consumers may be exposed to biased recommendations also in the presence of price effects (as in Armstrong and Zhou, 2011; Hagiu and Jullien, 2011; de Cornière and Taylor, 2019; Peitz and

Sobolev, forthcoming).[4]

Several works have pointed out anti-competitive effects of PPCs (e.g., Edelman and Wright, 2015, Boik and Corts, 2016, Johnson, 2017). A few related works have identified potentially welfare-increasing effects of PPCs.

PPCs have been defended on the grounds that they prevent consumers from free-riding on the costly efforts of intermediaries because otherwise, consumers would use the platform as a showroom and then purchase directly from the seller. Wang and Wright (2020) find that if narrow PPCs are needed for the viability of intermediaries and competition between intermediaries is sufficiently intense, narrow PPCs are in the interest of consumers and lead to higher welfare. For their result to hold, there must be competition between intermediaries. Otherwise, PPCs are welfare-reducing.[5] Shen and Wright (2019) analyze a model in which an intermediary's recommendation is essential for any product sales but sellers may free-ride on the intermediary's recommendation service and sell directly. Even though the intermediary's viability is not at risk, PPCs increase welfare as they prevent consumers from using the socially costly direct sales channel (Shen and Wright, 2019, Section 5.3).

In platform monopoly, Liu, Niu, and White (2021) show that prices may be lower thanks to PPCs when some consumers always use the direct sales channel, whereas the others choose between the direct distribution channel and the indirect channel (where the latter provides a convenience benefit as in our model). If the intermediary imposes PPCs it does so to attract low-valuation consumers, which benefits high-valuation consumers.[6] In our model, consumers do not benefit from PPCs because the intermediary makes biased product recommendations.

[4] A particular concern is the practice of self-preferencing by which a partially integrated intermediary steers consumers towards its own products (as in de Cornière and Taylor, 2019). In Hagiu, Teh, and Wright (2022) such a bias occurs off the equilibrium path and the ability to steer consumers affects equilibrium prices.

[5] Wang and Wright (2023) consider investments to reduce consumer search costs and find that wide PPCs lead to socially excessive platform investments while narrow or no PPCs lead to socially insufficient platform investments.

[6] Mariotto and Verdier (2020) assume that consumers are heterogeneous regarding the quality difference between sales channels and show that PPCs may lead to a lower price paid by consumers. Johansen and Vergé (2017) analyze a model with competing sellers who can offer their products on two competing platforms and sell directly. They show that PPCs may lead to lower platform fees and retail prices with endogenous seller participation.

Hagiu and Wright (2024) consider PPCs and other strategies by an intermediary to avoid consumers buying in the less efficient direct channel. Our work complements this article by giving the intermediary the additional role of recommender.

The paper proceeds as follows. In Section 2, we present the model. In section 3, we characterize the laissez-faire in which the intermediary can impose a PPC on the seller and make personalized purchase recommendations. In Section 4, we characterize the equilibrium outcome under three different regulatory interventions. In Section 5, we compare the market outcomes for the four regulatory options (which include the laissez-faire) and obtain welfare comparisons. Section 6 concludes. All proofs are relegated to Appendix A.

2 The model

We consider a seller that introduces an experience good and can sell through two sales channels: a direct channel D and an indirect channel I controlled by an intermediary. The seller competes against an outside option that is normalized to zero.[7]

A fraction $\alpha \in (0,1)$ of consumers care about product characteristics that they cannot observe before purchase (the picky consumers), whereas the remaining consumers are indifferent and have the willingness to pay v_m in the direct channel. Picky consumers are risk-neutral and have valuation v_h with $v_h > v_m$ for a good match with probability 1/2 and v_l with $v_l < v_m$ with remaining probability 1/2 for a bad match. We assume that $(v_l + v_h)/2 < v_m$. Consumers know whether they are picky or flexible; however, if they are picky, they do not know whether their valuation is v_h or v_l before purchase.

[7]One can interpret the outside option as a base product in competitive supply. The new product is an improved product compared to the base product. The base product provides some base utility v_0 to all consumers (with unit demand) irrespective of the sales channel and costs c_0 per unit to be put on the market. Since the base product is in competitive supply and generates the same benefit irrespective of the sales channel, it will always be sold at a price of c_0. By not buying the new product, consumers choose the base product as the outside option which gives them $v_0 - c_0$. Thus, when everybody buys the base product, consumer surplus is $v_0 - c_0$, as is the total surplus. For convenience, in what follows we renormalize consumer valuations, consumer surplus, and total surplus, reporting them only in excess of this level $v_0 - c_0$. We also report prices and unit cost of the new product as price and unit cost increments on the base product, c_0.

The indirect channel offers two advantages to consumers. First, all consumers obtain convenience benefits $b > 0$ from buying the new product in the indirect channel.[8] This is motivated by the seller's difficulties in providing the same level of customer services in the direct channel compared to the intermediary (e.g. because of the lack of scale compared to an intermediary being active in many product categories). Second, the intermediary has collected consumer data that enables it to infer the match value and provide informative recommendations about the new product. Thus, we disentangle two benefits of the indirect channel and endogenize the latter.

The per-unit production cost is $c \geq 0$. The key assumption is that $\frac{v_h+v_l}{2} + b < c < v_m$.[9] The first inequality says that selling the product to picky consumers in the indirect channel in the absence of any recommendation system leads to negative gains from trade, whereas selling to flexible consumers in the direct channel leads to positive gains from trade. Since $\frac{v_h+v_l}{2} + b < v_m$, the convenience benefit b obtained in the indirect channel can not be too large.

The intermediary makes three choices: it decides on whether to impose a PPC on the seller, sets a rate $\lambda \in [0, 1]$ as the fraction of the seller's profits it extracts (profit sharing), and, in response to the seller's retail prices makes a purchase recommend with probability $\beta \in [0, 1]$ to picky consumers with a bad match. It will always recommend the product to picky consumers with a good match. We note that it does not matter whether the intermediary makes recommendations to flexible consumers assuming that they are always aware of the option to choose the indirect channel.

Inefficiencies arise when some consumers buy in the direct channel instead of the indirect channel (this is a situation of *inefficient bypass*) or if some picky consumers with a bad match buy (this is a situation of *biased recommendations*). The latter can only occur in the indirect channel and requires that $\beta \in (0, 1)$ in which case the intermediary steers consumers with a bad match to the new products.

[8]Such an assumption is also made in other work on price parity clauses (e.g., Edelman and Wright, 2015; Mariotto and Verdier, 2020; Wang and Wright, 2020; Hagiu and Wright, 2024).

[9]For $c \in (v_l + b, \frac{v_h+v_l}{2} + b]$, our results would not change qualitatively, but would make expressions more involved. See Peitz and Sobolev (forthcoming) for the analysis in which the intermediary and seller are vertically integrated.

The *timing of the game* is as follows:

1. the intermediary sets λ;

2. the seller sets its prices p^D for the direct channel and p^I for the indirect channel if it considers selling there;

3. the intermediary observes prices and chooses its recommendation policy β,

4. consumers observe prices p^D and p^I and the recommendation policy β and then decide which sales channel to choose;

5. picky consumers in the indirect channel receive personalized recommendations and all consumers make their purchasing decisions.

We characterize subgame perfect Nash equilibria of this game in which the intermediary best responds to the seller's prices with its recommendation level β.[10] We note that consumers engage in Bayesian updating. However, this belief updating is pinned down by the recommendation policy β: If the implemented recommendation policy is β, a picky consumer who receives a recommendation updates the belief that the match is good from 1/2 to $1/(1+\beta)$. A picky consumer who does not receive a recommendation updates the belief that the match is good to 0 and, thus, is convinced that the match is bad. Consumers are not strategic players in the game and their decisions in stages 4 and 5 determine market demand in the two channels $q^I(p^I, p^D, \beta)$ and $q^D(p^I, p^D, \beta)$, respectively.

According to our timing, the intermediary responds with its recommendations to the seller's prices and, thus, lacks commitment vis-a-vis the seller.[11] In the case of algorithmic

[10] Our model is the same as Peitz and Sobolev (forthcoming) with one important difference: in our earlier paper the intermediary has full commitment power over its information policy and thus chooses a function $\beta(p^I, p^D)$.

[11] Condorelli and Szentes (2023) analyze matching algorithms in markets with heterogeneous sellers and buyers. In their Section 4, they consider price-dependent matching: sellers set prices and afterward, the platform chooses its matching algorithm, which, as they write, "may ... appear more realistic [than the platform's commitment to price-dependent matching] in light of the flexibility that platforms have in updating their algorithms." (on page 9 in the version from November 1, 2023)

recommendations this means that the algorithm is programmed to maximize profit in response to the sellers' prices. This makes the intermediary's recommendation behavior easily predictable for the seller and does not require them to trust some preannounced general recommendation policy. However, the intermediary is able to commit to β vis-à-vis consumers. Assuming observability of β on the part of the consumers means that in a repeated version of the game, consumers would be able to infer recommendations in excess of the equilibrium value of β and punish the intermediary in the future by not trusting future recommendations. We elaborate on this issue at the end of Section 3.

Since our analysis will establish biased recommendations under certain conditions, we have to clarify what we mean by biased vs. unbiased recommendations. A recommendation is unbiased from the consumers' perspective if and only if the intermediary recommends the product to consumers with valuation above price. A recommendation is unbiased from a total welfare perspective if and only if the intermediary recommends the product to consumers with a valuation above unit cost c. In this paper, we focus on recommendations that are biased from the consumers' and a total welfare perspective: the product is recommended to some consumers with valuations below unit cost c.

Before considering a mix of flexible and picky consumers, let us point out that the outcomes in the two boundary cases do not differ from the ones in the commitment case. If all consumers are flexible ($\alpha = 0$), then $p^D = v_m$ and $p^I = v_m + b$ and the seller makes a profit that is equal to $\max\{v_m - c, (1 - \lambda)(v_m + b - c)\}$. The intermediary chooses λ, such that the seller is indifferent between serving flexible consumers in the indirect and the direct channels. Thus, the optimal sharing contract is characterized by $\lambda^* = \frac{b}{v_m+b-c}$. The seller and the intermediary earn $v_m - c$ and b, respectively. Total surplus is maximal and equals to $v_m + b - c$.

If all consumers are picky ($\alpha = 1$), the seller cannot sell to consumers in the direct channel and, thus, it will accept any $\lambda \leq 1$. In every subgame with $\lambda \in [0, 1]$, the seller sets $p^I = v_h + b$ and the intermediary makes fully informative recommendations, which maximizes the intermediary's and the seller's profit. In the equilibrium of the full game, welfare is maximal and equal to $\frac{1}{2}(v_h + b - c)$. It is fully extracted by the intermediary (since $\lambda = 1$).

In the *first best*, all picky consumers with a good match and all flexible consumers buy in the indirect channel (i.e. $q^I = (1-\alpha) + (\alpha/2)$) and welfare (as total surplus) is

$$W^{FB}(\alpha) \equiv \frac{\alpha}{2}(v_h + b - c) + (1-\alpha)(v_m + b - c),$$

which is linear in α. It is increasing in α if and only if $(v_h + b - c)/2 > (v_m + b - c)$.

We introduce two other outcomes that will be relevant in the analysis. In the *inefficient bypass outcome*, only picky consumers with a good match are served in the indirect channel, whereas flexible consumers buy in the direct channel. The associated welfare is

$$W^{IE}(\alpha) \equiv \frac{\alpha}{2}(v_h + b - c) + (1-\alpha)(v_m - c).$$

In the *biased recommendation outcome*, all transactions are made in the indirect channel and a fraction

$$\beta^* \equiv \frac{v_h - v_m}{v_m - v_l}. \tag{1}$$

of picky consumers with a bad match buy the product. For this outcome welfare is

$$W^{BR}(\alpha) \equiv \frac{\alpha(1+\beta^*)}{2}(v_m + b - c) + (1-\alpha)(v_m + b - c).$$

3 Recommendations under price-parity clauses (PPC)

An important policy concern in recent years has been the use of PPCs imposed by an intermediary that restricts the pricing behavior of sellers. In our context, we consider a PPC that restricts the seller in its pricing in the direct channel relative to the indirect channel – that is, the seller is not allowed to set a lower price when selling directly, $p^D \geq p^I$. This implies that a PPC does not allow the seller to sell to the flexible consumers in the direct channel and set a high price to the picky consumers in the indirect channel, which makes the inefficient bypass outcome in which all flexible consumers buy directly and some picky consumers buy indirectly unattainable. In this section, the intermediary is unrestricted in its use of a PPC; however, it is free not to impose a PPC. In addition, it is not restricted in its choice of recommendation policy. We refer to this market environment as the laissez-faire.

Together with λ, the intermediary decides whether to impose a PPC in the first stage. By imposing price parity, the intermediary can make sure that by not selling to flexible consumers in the indirect channel the seller will make profit $(1-\alpha)(v_m - c)$. This is the same profit that the seller can achieve in the absence of the indirect channel. Thus, to implement the biased recommendations outcome, the intermediary will impose a PPC and set λ such that the seller is indifferent between selling to flexible consumers and picky consumers with a good match in the indirect channel compared to selling to flexible consumers in the direct channel only. As we show in the proof of Proposition 1, the appropriate λ is

$$\lambda_0(\alpha) \equiv 1 - \frac{(1-\alpha)(v_m - c)}{\left[\frac{\alpha}{2}(1+\beta^*) + (1-\alpha)\right](v_m + b - c)}.$$

This allows the intermediary to extract all the realized industry profits above $(1-\alpha)(v_m - c)$.

If the intermediary refrains from imposing price parity it can extract the full surplus from picky consumers with a good match in the indirect channel and the seller obtains $(1-\alpha)(v_m - c)$ by selling to flexible consumers in the direct channel.

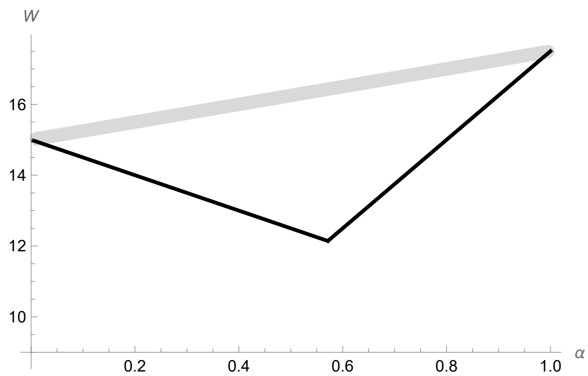
In its choice of λ and whether to impose a PPC, the intermediary faces a tradeoff between implementing the biased recommendation outcome and the inefficient bypass outcome. Since the seller's net profit is not affected by this choice, the intermediary implements the outcome that maximizes industry profits. Since consumer surplus is zero in both instances, the intermediary implements the biased recommendation outcome if $W^{BR}(\alpha) > W^{IE}(\alpha)$ and the biased recommendation outcome if the reverse inequality holds. The critical α that satifies $W^{BR}(\alpha) = W^{IE}(\alpha)$ is given by

$$\alpha_0 \equiv \frac{b}{b + \frac{1}{2}(v_h + b - c - (1+\beta^*)(v_m + b - c))}. \tag{2}$$

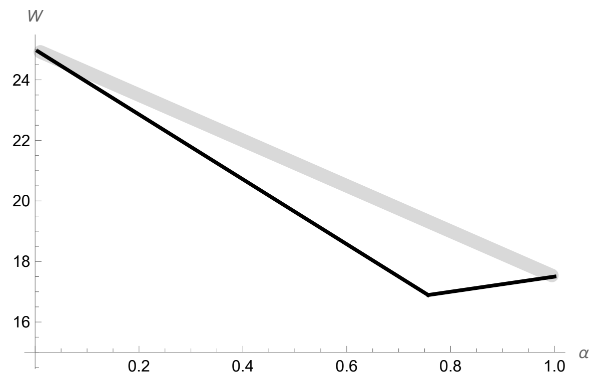
Proposition 1. *The equilibrium when the intermediary can impose a PPC is characterized as follows:*

- *for $\alpha < \alpha_0$, the intermediary imposes a PPC, sets $\lambda^* = \lambda_0(\alpha)$ and $\beta = \beta^*$ along the equilibrium path in the third stage. The seller sets prices $(p^I, p^D) = (v_m + b, v_m + b)$. All flexible consumers and the fraction $\frac{1+\beta^*}{2} = \frac{1}{2}\frac{v_h - v_l}{v_m - v_l}$ of picky consumers buy in the indirect channel;*

- *for $\alpha \geq \alpha_0$, the intermediary does not impose a PPC, sets $\lambda^* = 1$, and $\beta = 0$ along the equilibrium path in the third stage. The seller sets $(p^I, p^D) = (v_h + b, v_m)$. Half of the*



(a) $v_m = 80$



(b) $v_m = 90$

Figure 1: Total welfare in α; $v_h = 100, v_l = 20, b = 10, c = 75$. First-best outcome (grey), private solution (black).

picky consumers buy in the indirect channel and all the flexible consumers buy in the direct channel.

Figure 1 plots welfare in the laissez-faire equilibrium compared to welfare in the first best. On the left panel, we consider a situation in which welfare in the first best is increasing in α, whereas, on the right panel, it is decreasing. Welfare is $W_0(\alpha) = W^{BR}(\alpha)$ for $\alpha < \alpha_0$ and $W_0(\alpha) = W^{IB}(\alpha)$ for $\alpha \geq \alpha_0$. The laissez-faire outcome always leads to welfare strictly less than the first best for any $\alpha \in (0, 1)$. This difference is largest at α_0 which is at the kink of the welfare function under laissez-faire.

We note that the intermediary implements the vertically integrated solution.[12] Moreover, if the intermediary could commit to its recommendation policy and condition recommendations on retail prices before the seller sets its prices, the intermediary would also implement the vertically integrated solution and make the same profit as in Proposition 1 (see Peitz and Sobolev, forthcoming).

According to our timing, the intermediary can commit to β vis-à-vis consumers after observing p^D and p^I. As we have shown, the intermediary sets $\beta = 0$ in the inefficient bypass outcome and $\beta = \beta^*$ in the biased recommendation outcome. More generally, for any given p^I, to sell to picky consumers who choose the indirect channel, the intermediary must set

[12]This follows directly from Proposition 1 and Lemma 1 in Appendix B – in Appendix B we derive the vertically integrated solution.

β such that $(v_h + \beta v_l)/(1+\beta) + b \geq p^I$, because otherwise, picky consumers who receive a recommendation would, in expectation, be worse off than not buying.

A possible micro foundation of our commitment assumption vis-à-vis consumers is as follows. Suppose that time is discrete and infinite, there is discounting, and a unit mass of consumers arrives in each period, remaining active in that period only. Furthermore, suppose that the intermediary has installed a rating system that allows consumers to rate their purchase experience with the intermediary – this rating information is provided in addition to recommendations. For simplicity, suppose that individual ratings are based on the realized net surplus $v+b-p^I$ and the average rating reflects $(v_h+\beta v_l)/(1+\beta)+b-p^I$ in the past period.[13] A low average rating thus tells consumers in period t that consumers obtained on average a negative surplus. For ratings below a cut-off, period-t consumers do not trust the intermediary and do not buy in the indirect channel.[14] Recovering trust may be impossible or too costly for the intermediary (which is similar in spirit to Klein and Leffler, 1981). In order not to lose trust, recommendations have to satisfy that $(v_h + \beta v_l)/(1+\beta) + b - p^I$ is non-negative. In the inefficient bypass outcome, this means that in period t, the intermediary can not set $\beta > 0$ and in the biased recommendation outcome $\beta > \beta^*$ since future consumers would punish such a deviation to a larger β from period $t+1$ onward.

In essence, the mechanism to obtain commitment in the intermediary's recommendation behavior is based on a rating system that allows consumers to punish a deviating intermediary: consumers rate a seller's product but understand that the intermediary's recommendation behavior is key for the consumer experience. The experience of previous consumers determines whether they trust the intermediary's recommendation. If the intermediary hosts several sellers offering independent products, the intermediary may gain commitment more easily than with a single seller because consumers can pool ratings across sellers (and punish the intermediary if it was failing past consumers of some other product) because rating

[13]Consumers provide their rating based on value for money. For empirical evidence of such rating behavior, see e.g. Luca and Reshef (2021). In general, average ratings may also be affected through selection (as lower prices attract consumers with a lower valuation). In our model, the intermediary controls selection through the choice of β.

[14]This also applies to a product that has been around for more than one period and for which no recent ratings are available.

information is publicly available.

4 Regulatory interventions

In this section, we consider three regulatory interventions: (1) the prohibition of price parity clauses, (2) the prohibition of total surplus decreasing purchase recommendations, and (3) the combined prohibition of price parity clauses and total surplus decreasing purchase recommendations. We characterize market outcomes under these three regulatory regimes and evaluate their welfare effects. For each intervention, critical values of α and λ are denoted by the corresponding subscript (i.e. $\lambda_i(\alpha)$ and α_i for intervention $i \in \{1, 2, 3\}$).

4.1 The prohibition of price parity clauses

We recall consumer choices after observing prices (p^I, p^D) and possibly receiving personalized recommendations. The flexible consumers decide to buy in the indirect channel if and only if $v_m + b - p^I \geq \max\{v_m - p^D, 0\}$. Picky consumers buy in the indirect channel, after receiving a personalized recommendation, if and only if $\frac{v_h+\beta v_l}{1+\beta} + b - p^I \geq 0$. Picky consumers who did not receive a recommendation do not buy.

As under laissez-faire, by setting λ the intermediary effectively makes a choice between implementing the biased recommendation outcome and the inefficient bypass outcome. Nothing changes regarding latter.

We recall that in the biased recommendation outcome, a fraction β of picky consumers with a bad match buy. We note that whenever there are biased recommendations, the level of recommendations is the same as with price parity. However, when PPCs are prohibited, the intermediary has to leave some extra rents to the seller, as the pricing restriction under price parity is no longer effective. The seller can always offer a lower price in the direct channel to attract flexible consumers and thus make profit $(1-\lambda)\frac{\alpha}{2}(v_h + b - c) + (1-\alpha)(v_m - c)$. The critical λ that prevents the seller from doing so is

$$\lambda_1(\alpha) \equiv 1 - \frac{(1-\alpha)(v_m - c)}{\left[\frac{\alpha}{2}\frac{v_h - v_l}{v_m - v_l} + (1-\alpha)\right](v_m + b - c) - \frac{\alpha}{2}(v_h + b - c)}.$$

The critical α such that the intermediary's profit in the biased recommendation outcome is equal to its profit in the inefficient bypass outcome is denoted by α_1. Since the intermediary receives a lower profit than under laissez-faire, this critical α is lower, $\alpha_1 < \alpha_0$.

Proposition 2. *The equilibrium when the intermediary can not impose price parity is characterized as follows:*

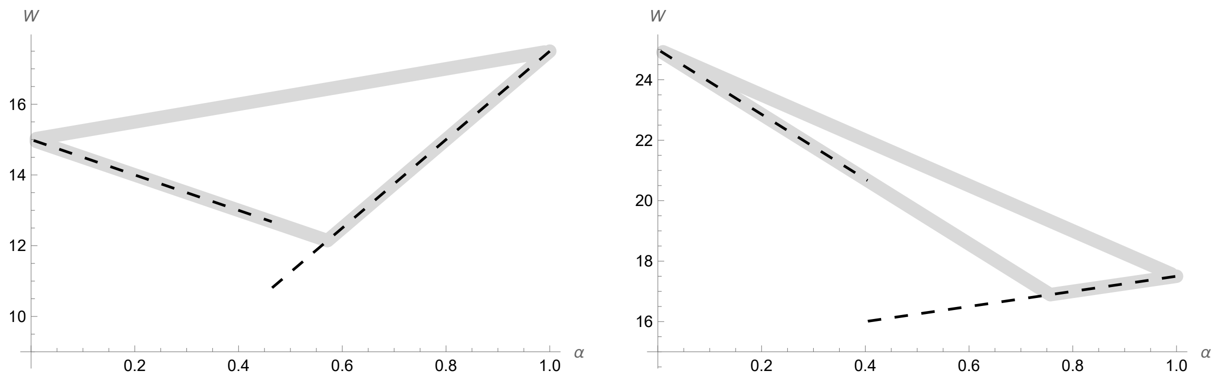
- *for $\alpha < \alpha_1$, the intermediary sets $\lambda^* = \lambda_1(\alpha)$ and $\beta = \beta^*$ along the equilibrium path. The seller sets $(p^I, p^D) = (v_m + b, v_m)$. All flexible consumers and the fraction $\frac{1+\beta^*}{2}$ of picky consumers buy in the indirect channel;*
- *for $\alpha \geq \alpha_1$, the intermediary sets $\lambda^* = 1$, and $\beta = 0$ along the equilibrium path in the third stage. The seller sets $(p^I, p^D) = (v_h + b, v_m)$. Half of the picky consumers buy in the indirect channel and all the flexible consumers buy in the direct channel.*

The proposition shows that for $\alpha \notin (\alpha_1, \alpha_0)$, the prohibition of PPCs does not change the market outcome compared to the laissez-faire. In this parameter range, the only effect of prohibiting price parity clauses is a redistribution of profits from the intermediary to the seller for $\alpha < \alpha_1$. By contrast, for (α_1, α_0), the outcome with price parity can not be obtained without it and flexible consumers choose the inefficient direct channel as a consequence of the regulatory intervention.

We plot welfare in Figure 2. Welfare is $W_1(\alpha) = W^{BR}(\alpha)$ for $\alpha < \alpha_1$ and $W_0(\alpha) = W^{IB}(\alpha)$ for $\alpha > \alpha_1$. For $\alpha \in (\alpha_1, \alpha_0)$, welfare is less without the price parity clause. The prohibition of PPCs weakens the intermediary in its relationship with the seller. This turns out to be bad for welfare.

4.2 The prohibition of biased recommendations

Instead of prohibiting price parity clauses, the intermediary may be mandated to provide unbiased recommendations. In this case, the intermediary has to set $\beta = 0$, but is free to impose a price parity clause on the seller. The intermediary will do so if it induces the outcome that picky consumers with a good match and all flexible consumers buy in the indirect channel, as this limits the profits a deviating seller can make. Given λ the



(a) $v_m = 80$ (b) $v_m = 90$

Figure 2: Total welfare in α; $v_h = 100, v_l = 20, b = 10, c = 75$. First-best outcome, private solution with PPC (grey solid), private solution without PPC (dashed).

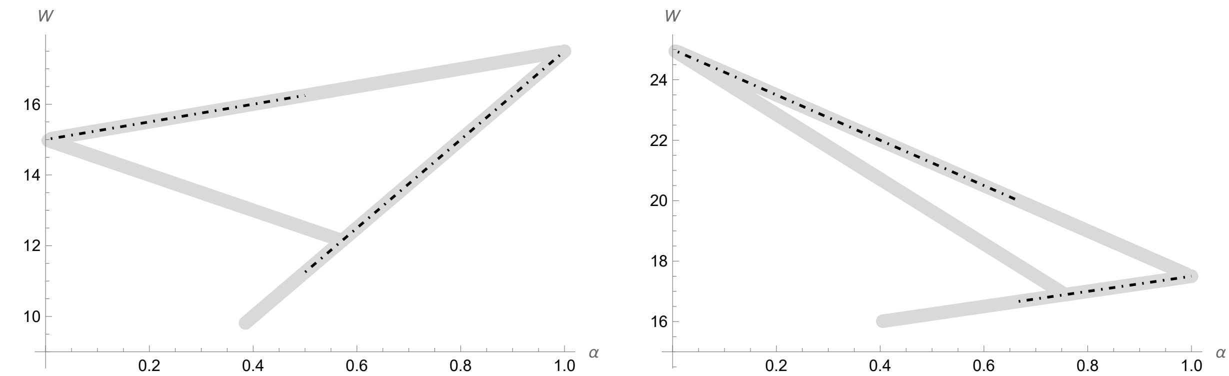
seller's profits selling at $v_m + b$ in the indirect channel and, thus, implementing the first-best outcome is $(1-\lambda)(1-\alpha+\alpha/2)(v_m+b-c)$. A deviating seller sets the price v_m in the direct channel and does not sell in the indirect channel. It will attract all flexible consumers and, thus, make profits $(1-\alpha)(v_m - c)$. The intermediary thus maximizes its profits by setting $\lambda = \lambda_2(\alpha)$, where, for given α, the profit share λ_2 is the solution to $(1-\lambda_2)(1-\alpha+\alpha/2)(v_m+b-c) = (1-\alpha)(v_m-c)$ or, equivalently,

$$\lambda_2(\alpha) \equiv \frac{\frac{\alpha}{2}(v_m - c) + (1-\frac{\alpha}{2})b}{\frac{\alpha}{2}(v_m - c) + (1-\frac{\alpha}{2})b + (1-\alpha)(v_m - c)}.$$

The intermediary has the choice between implementing the full information outcome taking the profit share λ_2 in the indirect channel or not requiring price parity, setting $\lambda = 1$ and thereby inducing the inefficient bypass outcome. The former gives profits $\lambda_2(1-\alpha+\frac{\alpha}{2})(v_m + b - c)$, while the latter gives $\frac{\alpha}{2}(v_m + b - c)$. The critical α such that both expressions are equal is denoted by $\alpha_2 \equiv \frac{b}{b+(v_h - v_m)/2}$.

Proposition 3. *When the regulator prohibits biased recommendations, the equilibrium is characterized as follows:*

- *if $\alpha < \alpha_2$, then the intermediary requires price parity, sets $\lambda^* = \lambda_2(\alpha)$, and the first-best outcome is implemented;*
- *if $\alpha \geq \alpha_2$, then the intermediary does not require price parity, sets $\lambda^* = 1$; equilibrium prices are given by $(p^I, p^D) = (v_h + b, v_m)$ and the inefficient bypass outcome is*



(a) $v_m = 80$

(b) $v_m = 90$

Figure 3: Total welfare in α; $v_h = 100, v_l = 20, b = 10, c = 75$. First-best outcome, private solution with and without a PPC (grey solid), fully informative recommendations with PPCs (dot-dashed).

implemented.

The intermediary enforces price parity off the equilibrium path for $\alpha < \alpha_2$ and thereby deprives the seller from making revenues in the direct channel. Figure 3 shows welfare as a function of α. For $\alpha < \alpha_2$, the first best is implemented, whereas for larger values of α the inefficient bypass outcome prevails. Thus, welfare is $W_2(\alpha) = W^{FB}(\alpha)$ for $\alpha < \alpha_2$ and $W_2(\alpha) = W^{IB}(\alpha)$ for $\alpha > \alpha_2$.

4.3 The prohibition of PPCs and biased recommendations

In this section, we characterize the market outcome when the regulator prohibits both, price parity clauses and total surplus decreasing recommendations. As in the previous subsection, the intermediary may want to implement the first-best outcome and take a share λ from profits in the indirect channel. Since the seller will set the price equal to $v_m + b$ when selling to all flexible consumers (and picky consumers with a good match) in the indirect channel, the remaining profit of the seller is $(1-\lambda)(1-\alpha+\alpha/2)(v_m+b-c)$. The deviating seller fully extracts the surplus from picky consumers with a good match by setting $p^I = v_h + b - c$. It sets the price $p^I = v_m$ in the direct channel and sells to flexible consumers there. Thus, deviation profits are $(1-\lambda)\frac{\alpha}{2}(v_h+b-c) + (1-\alpha)(v_m-c)$. The intermediary will then set λ as large as possible, while satisfying the seller's no deviation constraint – this critical λ is denoted by λ_3 and can be written as

$$\lambda_3(\alpha) \equiv \frac{(1-\alpha)b - (v_h - v_m)}{(1-\alpha)b - (v_h - v_m) + (1-\alpha)(v_m - c)},$$

which can be shown to be positive. Since the deviation profit is larger than in the previous subsection, it must be the case that $\lambda_3(\alpha) < \lambda_2(\alpha)$ for all α.

As in the previous section, the intermediary has the choice between implementing the full information outcome taking the share λ_3 of revenues in the indirect channel or setting $\lambda = 1$ and thereby inducing the inefficient bypass outcome. The former gives profits $\lambda_3(\alpha)(1 - \alpha + \frac{\alpha}{2})(v_m + b - c)$, while the latter gives $\frac{\alpha}{2}(v_m + b - c)$. The critical α such that both expressions are equal is denoted by α_3, which is necessarily smaller than α_2 since $\lambda_3(\alpha) < \lambda_2(\alpha)$.

Proposition 4. *When the regulator prohibits price parity clauses and biased recommendations, the equilibrium is characterized as follows:*

- *if $\alpha < \alpha_3$, then the intermediary sets $\lambda^* = \lambda_3(\alpha)$ and the first-best outcome is implemented;*
- *if $\alpha \geq \alpha_3$, then the intermediary sets $\lambda^* = 1$; equilibrium prices are given by $(p^I, p^D) = (v_h + b, v_m)$ and the inefficient bypass outcome is implemented.*

Figure 4 shows welfare as a function of α. Welfare is $W_3(\alpha) = W^{FB}(\alpha)$ for $\alpha < \alpha_3$ and $W_3(\alpha) = W^{IB}(\alpha)$ for $\alpha > \alpha_3$.

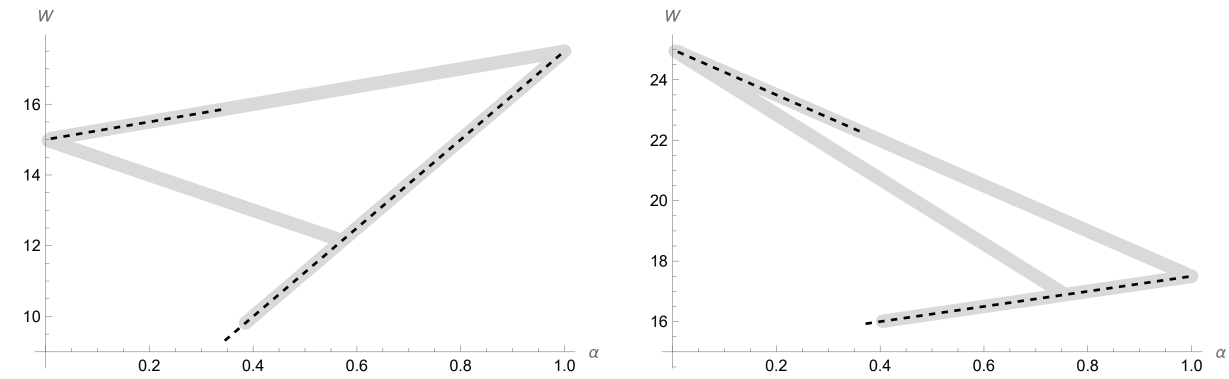
5 Comparison between different regulatory options

The comparison of the different regulatory interventions including laissez-faire (intervention 0) follows from Propositions 1-4 and the following proposition regarding the critical values of α:

Proposition 5. *The critical values of α satisfy the following inequalities:*

$$0 < \alpha_3 < \min\{\alpha_1, \alpha_2\} \leq \max\{\alpha_1, \alpha_2\} < \alpha_0 < 1.$$

There exists a unique $\hat{v}_m \in (c, v_h)$ such that $\alpha_2 > \alpha_1$ if $v_m > \hat{v}_m$, and $\alpha_2 \leq \alpha_1$ otherwise.



(a) $v_m = 80$ (b) $v_m = 90$

Figure 4: Total welfare in α; $v_h = 100, v_l = 20, b = 10, c = 75$. First-best outcome, private solution with and without a PPC, fully informative recommendations with a PPC (grey solid), fully informative recommendations without a PPC (dashed).

It is a priori unclear whether $\alpha_1 < \alpha_2$ or the reverse inequality holds. Under the first regulation, the intermediary's profit is $\lambda_1(\alpha)(\alpha(1+\beta^*)/2+(1-\alpha))(v_m+b-c)$ when all sales occur in the indirect channel. Under the second regulation, it is $\lambda_2(\alpha)(\alpha/2+(1-\alpha))(v_m+b-c)$. Under the second regulation, the intermediary is prohibited from biasing recommendations and, therefore, can not make purchase recommendations to some of the picky consumers with a bad match (i.e. $\beta = 0$). This leads to a smaller number of transactions on the platform. However, since the intermediary is allowed to use a PPC, it can extract a larger share of profits from the seller than under the first regulation: indeed, one can show that $\lambda_2(\alpha) > \lambda_1(\alpha)$.

Note that the equilibrium outcome for any regulatory intervention is either the first best (FB), the biased recommendations outcome (BR), or the inefficient bypass outcome (IB). Respective industry profits are

$$\begin{aligned}
\Pi^{FB}(\alpha) &= \left(\frac{\alpha}{2} + (1-\alpha)\right)(v_m + b - c), \\
\Pi^{BR}(\alpha) &= \left(\frac{\alpha}{2}(1+\beta^*) + (1-\alpha)\right)(v_m + b - c), \\
\Pi^{IB}(\alpha) &= \frac{\alpha}{2}(v_h + b - c) + (1-\alpha)(v_m - c).
\end{aligned}$$

We observe that $\Pi^{FB}(\alpha) \leq \Pi^{BR}(\alpha)$. We have that $\Pi^{FB}(\alpha) < \Pi^{IB}(\alpha)$ if and only if $\alpha > \alpha_2$. Furthermore, $\Pi^{BR}(\alpha) > \Pi^{IB}(\alpha)$ if and only if $\alpha < \alpha_0$. These observations imply that

Table 1: Profit and welfare ranking when $\alpha_2 < \alpha_1$

Parameter range	Profit ranking	Welfare ranking
$\alpha \in [0, \alpha_3)$	$\Pi_0 = \Pi_1 > \Pi_2 = \Pi_3 = \Pi^{FB}$	$W_0 = W_1 < W_2 = W_3 = W^{FB}$
$\alpha \in (\alpha_3, \alpha_1)$	$\Pi_0 = \Pi_1 > \Pi_2 = \Pi^{FB} > \Pi_3$	$W_3 < W_0 = W_1 < W_2 = W^{FB}$
$\alpha \in (\alpha_1, \alpha_2)$	$\Pi_0 > \Pi_2 = \Pi^{FB} > \Pi_1 = \Pi_3$	$W_1 = W_3 < W_0 < W_2 = W^{FB}$
$\alpha \in (\alpha_2, \alpha_0)$	$\Pi_0 > \Pi_1 = \Pi_2 = \Pi_3 > \Pi^{FB}$	$W_1 = W_2 = W_3 < W_0 < W^{FB}$
$\alpha \in (\alpha_0, 1]$	$\Pi_0 = \Pi_1 = \Pi_2 = \Pi_3 > \Pi^{FB}$	$W_0 = W_1 = W_2 = W_3 < W^{FB}$

$\Pi^{BR}(\alpha) > \Pi^{FB}(\alpha) > \Pi^{IB}(\alpha)$ for $\alpha \in [0, \alpha_2)$, $\Pi^{BR}(\alpha) > \Pi^{IB}(\alpha) > \Pi^{FB}(\alpha)$ for $\alpha \in (\alpha_2, \alpha_0)$, and $\Pi^{IB}(\alpha) > \Pi^{BR}(\alpha) > \Pi^{FB}(\alpha)$ for $\alpha \in (\alpha_0, 1]$. Together with Propositions 1 to 5, these inequalities fully characterize the ordering of industry profits for the four possible regulatory interventions. If parameters satisfy that $\alpha_2 > \alpha_1$, the profit rankings of the four regulatory interventions $i \in \{0, 1, 2, 3\}$ are reported in the second column of Table 1. If the reverse inequality $\alpha_2 > \alpha_1$ holds, they are reported in Table 2.

Notably, when the intermediary is prohibited from using a PPC and from biasing recommendations, for $\alpha \in (\alpha_3, \alpha_2)$, the intermediary implements the inefficient bypass outcome even though the first best would lead to higher industry profits – that is, $\Pi^{FB} > \Pi_2$ holds (see Tables 1 and 2). Here, regulation ties the hands of the intermediary so much that the intermediary does not have an incentive to "invite" the seller to serve flexible consumers in the indirect channel. This can also be the case when only PPCs are prohibited: $\Pi^{FB} > \Pi_2$ holds if $\alpha_1 < \alpha_2$ and $\alpha \in (\alpha_1, \alpha_2)$ (see Table 1).

We now turn to the welfare ranking of the different regulatory regimes. Suppose that $\alpha_2 > \alpha_1$, which holds for the parameter constellation reported in Figure 3. This gives rise to the welfare ranking reported in Table 1.

Suppose instead that $\alpha_2 < \alpha_1$. The corresponding welfare ranking is reported in Table

Table 2: Profit and welfare ranking when $\alpha_1 < \alpha_2$

Parameter range	Profit ranking	Welfare ranking
$\alpha \in [0, \alpha_3)$	$\Pi_0 = \Pi_1 > \Pi_2 = \Pi_3 = \Pi^{FB}$	$W_0 = W_1 < W_2 = W_3 = W^{FB}$
$\alpha \in (\alpha_3, \alpha_2)$	$\Pi_0 = \Pi_1 > \Pi_2 = \Pi^{FB} > \Pi_3$	$W_3 < W_0 = W_1 < W_2 = W^{FB}$
$\alpha \in (\alpha_2, \alpha_1)$	$\Pi_0 = \Pi_1 > \Pi_2 = \Pi_3 > \Pi^{FB}$	$W_2 = W_3 < W_0 = W_1 < W^{FB}$
$\alpha \in (\alpha_1, \alpha_0)$	$\Pi_0 > \Pi_1 = \Pi_2 = \Pi_3 > \Pi^{FB}$	$W_1 = W_2 = W_3 < W_0 < W^{FB}$
$\alpha \in (\alpha_0, 1]$	$\Pi_0 = \Pi_1 = \Pi_2 = \Pi_3 > \Pi^{FB}$	$W_0 = W_1 = W_2 = W_3 < W^{FB}$

2. The key change compared to the reverse constellation $\alpha_2 > \alpha_1$ is that the welfare ranking between the first and the second regulation is no longer unambiguous. Prohibiting biased recommendations instead of prohibiting PPCs increases welfare for $\alpha < \alpha_2$ but decreases welfare for $\alpha \in (\alpha_2, \alpha_1)$ because in this parameter range the intermediary subject to the prohibition of biased recommendations induces inefficient bypass, whereas the intermediary subject to the prohibition of PPCs continues to induce an outcome with biased recommendation such that all picky consumers with a purchase recommendation and all flexible consumers buy in the superior indirect channel.

The following remarks directly follow from our previous results and apply independent of whether α_1 or α_2 is larger than the other. The following two remarks show that prohibiting price parity clauses is never welfare-increasing in our setting independent of whether or not the regulator regulates the intermediary's recommender system.

Remark 1. *Laissez-faire leads to weakly higher welfare than prohibiting the use of PPCs – that is,* $W_0(\alpha) \geq W_1(\alpha)$ *for all* α*.*

Remark 2. *Not prohibiting PPCs leads to weakly higher welfare than doing so given that the regulator prohibits total surplus decreasing recommendations – that is,* $W_2(\alpha) \geq W_3(\alpha)$ *for* *all α.*

Take the prohibition of price parity clauses as given (e.g. in the case of gatekeeper platforms in the EU because of the regulation in the DMA).[15] The question for both intermediary and seller is whether the seller should sell to flexible consumers directly or indirectly. If flexible consumers buy in the direct channel, either all surplus is extracted from them or all flexible consumers buy in the direct channel as well. What are the seller's incentives to set prices, such that flexible consumers buy in the indirect channel? Under regulation with $\beta = 0$, the seller obtains a fraction $1 - \lambda$ of the joint profit $\left(\frac{\alpha}{2} + 1 - \alpha\right)(v_m + b - c)$, while, under laissez-faire, it obtains a fraction of $(\alpha/2(1 + \beta^*) + (1 - \alpha))(v_m + b - c)$. Since the former is less than the latter, the intermediary has to give a larger fraction of joint profits to the seller under regulation. Since the joint profit from selling to flexible consumers in the indirect channel is strictly smaller under regulation and the intermediary has to confine itself with a strictly smaller share λ, the critical α below which flexible consumers buy in the indirect channel must be strictly lower under regulation of the recommendation policy than without; that is, $\alpha_3 < \alpha_1$ (see the proof of Proposition 5).

Remark 3. *Given the prohibition of price parity clauses, if the regulator mandates fully informative recommendations ($\beta = 0$), this leads to higher welfare for $\alpha < \alpha_3$, but lower welfare for $\alpha \in (\alpha_3, \alpha_1)$. Welfare is unchanged for $\alpha \geq \alpha_1$.*

This shows that prohibiting total surplus-decreasing recommendations can increase welfare (and implement the first best), but runs the risk of backfiring. This holds in case the regulator prohibits the use of price parity clauses. Qualitatively, the same result holds if the intermediary is free to use PPCs in its contracting with the seller.

Remark 4. *Given that the intermediary is not restricted in its use of price parity clauses, if the regulator mandates fully informative recommendations ($\beta = 0$), this leads to higher welfare for $\alpha < \alpha_2$, but lower welfare for $\alpha \in (\alpha_2, \alpha_0)$. Welfare is unchanged for $\alpha \geq \alpha_0$.*

Since, in our model, there are only two ex-ante types and the seller is a monopolist, consumer surplus results are straightforward. Consumers can only gain in net surplus if the

[15] The DMA does not give any discretion to the European Commission in this respect. It must enforce the prohibition on PPCs, which applies to gatekeeper platforms concerning their core platform services.

intermediary is prohibited from biasing its purchase recommendation. In this case, for $\alpha < \alpha_1$ (or $\alpha < \alpha_3$ when also PPCs are prohibited), picky consumers gain since their gross gain is $v_h + b$ and they pay the price $v_m + b$ if they obtain a purchase recommendation, which happens if and only if the match is good. Thus, consumer surplus increases by $(\alpha/2)(v_h - v_m)$.

The regulator may care about the welfare of the users of the intermediation service, which is total welfare minus the intermediary's profit (i.e., user welfare is $W - \Pi$), instead of or in addition to total welfare and/or consumer welfare. We observe that, for $\alpha \geq \alpha_1$, user welfare is unaffected by the prohibition of PPCs given that the intermediary's recommendation policy is unregulated because the seller always makes profit $(1 - \alpha)(v_m - c)$. For $\alpha < \alpha_1$, the seller benefits from the regulation (and consumers have zero surplus with and without the regulation) and, thus, the regulation increases the user welfare. Qualitatively the same result holds for user welfare when prohibiting biased recommendations given that PPCs are allowed: for $\alpha \geq \alpha_2$, the regulation is neutral because the seller always makes profit $(1 - \alpha)(v_m - c)$ and consumers a net surplus of zero; for $\alpha < \alpha_2$, the regulation increases user welfare because the seller's profit is unaffected by the regulation and consumers benefit from the regulation (as their net surplus increases from zero to $(\alpha/2)(v_h - v_m)$). If, given the prohibition of PPCs, the regulator also forbids biased recommendations (i.e. intervention 1 is superseded by intervention 3), this additional regulatory intervention may backfire also concerning user welfare $W - \Pi$. Backfiring prevails for $\alpha \in [\alpha_3, \alpha_1)$ because seller profit drops and consumer welfare is unchanged. For $\alpha < \alpha_3$, the additional regulation increases user welfare because total welfare goes up and the intermediary's profit goes down (here, the seller and consumers are better off).

Another regulatory intervention that we did not consider so far is fee regulation. Since the intermediary sets $\lambda = 1$ under inefficient bypass, any cap $\bar{\lambda} < 1$ reduces the intermediary's profits when it implements the inefficient bypass outcome. Consider such a cap that is not binding under any of the regulatory regimes considered above when the inefficient bypass outcome was not implemented. Such a regulatory intervention leads to a critical $\alpha_0' > \alpha_0$ implying that the biased recommendation outcome is implemented on a larger set of parameters. This is welfare-reducing.[16]

[16]By contrast, Gomes and Mantovani (forthcoming) consider optimal fee regulation under price parity with

If fee regulation is combined with prohibiting PPCs, the critical value of α satisfies $\alpha_1' > \alpha_1$. This implies that for $\bar{\lambda}$ sufficiently close to one adding fee regulation on top of prohibiting PPCs is welfare-increasing. However, the combined regulation always performs worse than the laissez-faire.

If fee regulation is combined with prohibiting biased recommendations, the critical value of α satisfies $\alpha_2' > \alpha_2$. Hence, for $\bar{\lambda}$ sufficiently close to one, adding fee regulation on top of prohibiting biased recommendations is welfare-increasing. Furthermore, for b sufficiently small, one can show that the combined regulation (appropriate fee regulation and the prohibition of biased recommendations) increases welfare compared to the laissez-faire ($\alpha_2' \geq \alpha_0$), which implies that fee regulation and the prohibition of biased recommendations are complements from the viewpoint of a welfare-maximizing regulator.

6 Extensions

6.1 Commission fee on revenues

In many real-world settings the intermediary charges an ad valorem fee on the seller's revenues in the indirect channel, as this information is readily available to the intermediary. By contrast, an ad valorem fee on the seller's profits is informationally more demanding because the intermediary must be able to monitor the seller's costs. The intermediary may then decide not to implement the vertically integrated outcome even under laissez-faire:[17] In Proposition 6 in Appendix C, we show that there exists $\alpha_0^{RS} \leq \alpha_0$ such that the intermediary induces the biased recommendation outcome for $\alpha < \alpha_0^{RS}$ and the inefficient bypass outcome for $\alpha \geq \alpha_0^{RS}$.

For sufficiently large marginal costs c and a sufficiently large convenience benefit b, we have that $\alpha_0^{RS} < \alpha_0$ and, thus, the intermediary implements the inefficient bypass outcome for

sellers as oligopolists and show that fee regulation increases welfare.

[17]This is in contrast to what happens in a setting in which the intermediary can fully commit to its recommendation policy: As shown in Peitz and Sobolev (forthcoming), the intermediary implements the vertically integrated outcome for a variety of price instruments including the two considered in the present paper.

a strictly larger set of values for α than under vertical integration. This implies that industry profits (and total surplus) are lower than what the intermediary could achieve. The reason is that by setting a high price ($p^I = v_h + b$) the seller induces the intermediary to recommend the product only to those picky consumers who have a good match ($\beta = 0$). However, this is not in the interest of the intermediary. Since, for high c, the seller is attracted by this possibility, the intermediary must reduce the commission it charges in order to induce the seller to set the price $p^I = v_m + b$. This implies that the intermediary earns less when charging an ad valorem fee on revenues instead of one on profits and, hence, $\alpha_0^{RS} < \alpha_0$ (Proposition 6 in Appendix C).

Outcomes under the different regulations are qualitatively the same as in the setting in which the intermediary charges an ad valorem fee on the seller's profit in the indirect channel, with the important difference that thresholds under revenue sharing, α_i^{RS}, are now lower (depending on the regulation, they are either always strictly lower or strictly lower under some parameter constraints on marginal costs and convenience benefit) – that is $\alpha_i^{RS} \leq \alpha_i$ for regime $i \in \{1, 2, 3\}$. This is shown in Propositions 7–9 and implies that the inability of the intermediary to monitor costs and, thus, to tax seller profits instead of seller revenues in the indirect channel comes at a social cost.

The ordering of the critical thresholds is preserved under the alternative price instrument; that is,

$$0 < \alpha_3^{RS} < \min\{\alpha_1^{RS}, \alpha_2^{RS}\} \leq \max\{\alpha_1^{RS}, \alpha_2^{RS}\} \leq \alpha_0^{RS} < 1$$

(as established in Proposition 10). Thus the same regulatory trade-offs apply as in Section 5.

6.2 Competing intermediaries

An intermediary may not just compete with a direct sales channel, but alternatively or in addition with other intermediaries. Our setting extends to a model with a less efficient competing intermediary.

Suppose that we replace the direct channel by a competing, less efficient indirect channel operated by an intermediary who neither provides the convenience benefit b nor is able to make informed recommendations. This intermediary decides about its fee μ and whether to impose a PPC *after* the more efficient intermediary does.[18] If an intermediary imposes PPC this means in the present context that the seller is not allowed to offer a lower price on the competing indirect sales channel. Otherwise, the model is as before.

As in the main analysis, there are three possible outcomes: (i) biased recommendations, (ii) first best, and (iii) inefficient bypass. Whenever outcome (i) or (ii) prevails, the less efficient intermediary makes zero profit (and sets $\mu = 0$). For these outcomes, nothing else changes compared to our base model. When outcome (iii) prevails, both intermediaries set their fees equal to 1. The seller sets the retail price with the less efficient intermediary equal to v_m and, thus, the less efficient intermediary makes profit $(1-\alpha)(v_m - c)$, while the seller makes zero profit. The more efficient intermediary is in the position to pick its preferred outcome and the cutoffs are the same as in our base model. We formally establish this result in Proposition 11 in Appendix D.

It is also straightforward to introduce a competing intermediary in addition to the direct channel. There are now intermediaries A and B operating indirect channels and a direct sales channel. While B is less efficient than A (as in the model without the direct channel), it may offer a convenience benefit that is different from the direct channel.

Suppose that the direct channel offers less convenience than the indirect channel B; this difference is denoted by d. For the direct channel to have any impact we assume that $v_m - d - c > 0$. In this case, the inferior intermediary cannot extract the full surplus that the seller makes from flexible consumers under inefficient bypass selling through the less

[18] If intermediaries decide simultaneously, for a range of parameters, a pure-strategy equilibrium does not exist and we would need to characterize mixed-strategy equilibria. The reason for the nonexistence of pure-strategy equilibrium under laissez-faire is the following best-response behavior: Suppose that in equilibrium flexible consumers buy from the less efficient intermediary. Then, the efficient intermediary sets $\lambda = 1$, imposes no PPC, and obtains all profits from the picky consumers with a good match. The less efficient intermediary responds by setting $\mu = 1$, implying that the seller earns zero profits. If $(\alpha/2(1+\beta^*) + 1 - \alpha)(v_m + b - c) > (\alpha/2)(v_h + b - c)$ (which is satisfied for α slightly above α_0), then the more efficient intermediary would be willing to slightly undercut the fee and induce the biased recommendations outcome. In response, the less efficient intermediary would set $\mu < 1$ to induce the seller to sell to flexible consumers through the less efficient indirect channel. In return, the more efficient intermediary sets a lower λ to induce the biased recommendation outcome. The undercutting continues until it becomes profitable for the more efficient intermediary to set $\lambda = 1$ and induce inefficient bypass.

efficient intermediary (as the seller can guarantee itself a profit of $(1-\alpha)(v_m - d - c)$). Under laissez-faire, the more efficient intermediary prohibits lower prices on all other channels when implementing the biased recommendations outcome and is always able to achieve the profit of the vertically integrated firm minus $(1-\alpha)(v_m - c)$, which is left for the seller and the less efficient intermediary. The critical cut-offs under laissez-faire and the regulatory regimes 1–3 remain unaffected.

In the extension with two indirect channels and no direct channel, we considered regulations that may prohibit wide PPCs. However, in the present extension, we can distinguish between the prohibition of wide and narrow price parity clauses. Thus, there are two additional possibilities for regulatory interventions: the prohibition of wide PPCs with no restrictions on narrow PPCs (regime $1'$)[19] and the combined prohibition of wide PPCs and total-surplus-decreasing recommendations, with no restrictions on narrow PPCs (regime $3'$). Denote the respective critical α as $\alpha_{1'}$ and $\alpha_{3'}$. However, for $d > 0$, the prohibition of wide PPCs according to regime $1'$ is ineffective and $\alpha_{1'} = \alpha_0$. The reason is that under outcomes (i) and (ii), the less efficient intermediary sets $\mu = 0$, which makes the indirect channel B more attractive than the direct channel for the seller. Since narrow PPCs do not entail any price restrictions on indirect channel B, this intervention is ineffective and the laissez-faire outcome prevails under regime $1'$. One can also show that $\alpha_{3'} = \alpha_2$. Thus, the prohibition of wide PPCs (in regimes $1'$ and $3'$) does not have any bite.

This is no longer the case if the direct channel gives a higher benefit than the indirect channel operated by intermediary B; that is, $d < 0$ (assuming that $|d| < b$). We note that in regimes 0–3, the presence of intermediary B does not matter and, therefore, introducing such an intermediary does not add to the analysis of the base model. However, in regimes $1'$ and $3'$ it becomes relevant and we have that $\alpha_{1'} \in (\alpha_1, \alpha_0)$ and $\alpha_{3'} \in (\alpha_3, \alpha_2)$.[20] In the case of outcome (i) or (ii), if the more efficient intermediary imposes a narrow PPC, the most profitable deviation of the seller is to set prices such that it diverts the flexible consumers

[19] In practice, some competition authorities have been drawing the line between wide and narrow PPCs, prohibiting the former, but allowing the latter – see, e.g., Peitz (2022).

[20] For $d < 0$, the definitions of α_i, $i \in \{0, 1, 2, 3\}$ from Sections 3 and 4 have to be adjusted to account for the fact that the seller can sell to the flexible consumers at the price of $v_m - d$ (instead of v_m in the main model).

to the less efficient intermediary and serves the picky consumers with a good match through the more efficient intermediary – such prices are feasible under a narrow PPC. This deviation is less profitable for the seller than the most profitable deviation under no PPC (as in the latter case, it could divert flexible consumers to the direct channel which is superior than the indirect channel operated by intermediary B). Moreover, this deviation is more profitable than what it could achieve with a deviation under a wide PPC.

7 Conclusion

A seller can sell directly and indirectly through an intermediary to consumers. The intermediary offers a convenience benefit to consumers and, thanks to the information it has on consumers, may make purchase recommendations that are conditioned on the match value that will be realized by consumers after purchase. In return, the intermediary asks for a share of the seller's profit.

Picky consumers rely on the purchase recommendation for gains from trade to arise. In the first best, all consumers with a net surplus equal to or larger than unit costs would buy the product in the direct channel. The intermediary asks upfront for a share of the seller's profit made in the direct channel and then makes recommendations to consumers after observing the prices set by the seller.

Absent regulatory interventions, the intermediary either chooses to bias recommendations such that some of the consumers with a bad match buy (biased recommendation outcome) or does not bias recommendations and asks for the full profit in which case the seller will cater to some consumers in the direct channel (inefficient bypass outcome). The profit-maximizing strategy of the intermediary in the former case contains a PPC imposed by the intermediary.

Regulatory interventions that prohibit the use of PPCs reduce the ability of the intermediary to punish unwanted retail prices. As a result, the intermediary is more inclined to implement the inefficient bypass outcome and regulation is welfare-reducing. Prohibiting biased recommendations by requiring the intermediary to make a purchase recommendation only if the consumer valuation is above unit costs has the potential to improve welfare. However, also this policy can backfire and lead to lower welfare than under laissez-faire.

Our paper explains the use of sales channels as a function of market fundamentals and the regulatory setting. Our framework can be extended to allow for the possibility that some consumers can use the indirect channel to update their prior on the match value and then may make the transaction in the direct channel (showrooming). In response, the intermediary may modify its monetization model and collect a fee that does not depend on the profits in the indirect channel.

Appendix

A Relegated proofs

Proof of Proposition 1. We note that the seller can always serve only flexible consumers in the direct channel and guarantee a profit of $(1-\alpha)(v_m - c)$. Thus, in any equilibrium, the seller cannot earn any less. Consequently, the intermediary cannot earn strictly more than the profit of a vertically integrated firm (that has control over recommendation policy and prices) net of the minimal guaranteed profits to the seller, $(1-\alpha)(v_m-c)$. For any $\alpha \in (0,1)$, we show that the intermediary can reach the profit of the vertically integrated firm net of $(1-\alpha)(v_m - c)$. This implies that the proposed intermediary's strategies are optimal.

In our proof, we consider the case of $\alpha \geq \alpha_0$ and $\alpha < \alpha_0$, where α_0 is determined in equation (2), separately.

First, suppose that $\alpha \geq \alpha_0$. By Lemma 1 in Appendix B, the vertically-integrated solution is characterized by the inefficient bypass outcome, in which flexible consumers are served in the direct channel at a price $p^D = v_m$ and picky consumers receive fully informative recommendations in the indirect channel ($\beta = 0$) and buy at a price $p^I = v_h + b$ if they have a good match.

We show that for $\alpha \geq \alpha_0$ the intermediary can decentralize the outcome with inefficient bypass. Suppose that the intermediary sets $\lambda = 1$ and imposes no PPC. The seller cannot make positive profits by selling through the indirect channel and therefore, generates profits only by serving flexible consumers in the direct channel at $p^D = v_m$. The resulting profit of the seller is $(1-\alpha)(v_m-c)$. The seller is indifferent between any price $p^I > v_m + b$. We break the indifference in the favour of the intermediary and assume that the seller sets $p^I = v_h + b$.[21] In the third stage, the intermediary fully reveals the match value to picky consumers, i.e., $\beta = 0$. As a result, flexible consumers buy directly at $p^D = v_m$, and picky consumers with a good match buy indirectly at $p^I = v_h + b$, which constitutes the outcome with inefficient bypass. Since the seller makes the minimal guaranteed profit $(1-\alpha)(v_m - c)$, the proposed strategy is optimal for the intermediary. Note that in this case the intermediary must not

[21] Note that for any λ slightly smaller than 1, the seller strictly prefers to set $p^I = v_h + b$.

impose a PPC: under PPCs we have that $p^D \geq p^I$, and the outcome with inefficient bypass would be unattainable.

Second, suppose that $\alpha < \alpha_0$. By Lemma 1, the vertically-integrated solution is characterized by the outcome with biased recommendations, in which all flexible consumers and the fraction $(1+\beta^*)/2$ of all picky consumers buy in the indirect channel at $p^I = v_m + b$.

We show that for $\alpha < \alpha_0$ the intermediary can decentralize the outcome with biased recommendations. Suppose that the intermediary sets

$$\lambda = 1 - \frac{(1-\alpha)(v_m - c)}{\left[\frac{\alpha}{2}\frac{v_h - v_l}{v_m - v_l} + (1-\alpha)\right](v_m + b - c)},$$

and imposes PPCs. Consider the possible strategies of the seller. If the seller sets $p^D = v_m$ in order to serve flexible consumers in the direct channel, then the PPCs require to set $p^I \leq v_m$ in the indirect channel. Clearly, at any such p^I, flexible consumers would buy through the indirect channel as this would increase their surplus at least by b. If the seller decides to serve consumers only through the indirect channel, then, since λ is already fixed, then both the seller and the intermediary have incentives to maximize the total industry profit. By Lemma 1, for $\alpha < \alpha_0$, the total industry profit is maximized if the outcome with biased recommendations is implemented. Thus, the seller sets $p^D = p^I \geq v_m + b$ and the intermediary sets $\beta = \beta^*$ in the third stage. All flexible consumers as well as a fraction of $\frac{1+\beta^*}{2}$ picky consumers buy in the indirect channel. The seller's equilibrium profit is $(1-\alpha)(v_m - c)$ and therefore, the proposed strategy is optimal for the intermediary.

We also note that the intermediary cannot obtain the maximal profits without imposing a price parity clause. If the intermediary does not impose a PPC, then for any $\lambda > 0$, the seller can set $p^D = v_m$ and $p^I = v_h + b$ that would guarantee profits that strictly greater than $(1-\alpha)(v_m - c)$. Therefore, the intermediary must impose a PPC at the optimum.

□

Proof of Proposition 2. By Proposition 1, the intermediary does not have incentives to impose a price-parity clause for $\alpha \geq \alpha_0$. Thus, for this range of α, the intermediary sets $\lambda = 1$ and $\beta = 0$ along the equilibrium path in the third stage of the game, inducing the inefficient bypass outcome.

In the following, we focus on $\alpha < \alpha_0$. If the intermediary sets $\lambda = 1$, then the seller expects no profits earned in the indirect channel and, therefore, sets $p^D = v_m$ to serve flexible consumers in the direct channel. The seller is indifferent between any prices $p^I > v_m + b$ and we assume that it sets a price that maximizes the intermediary's profit from the picky consumers – that is, $p^I = v_h + b$. The resulting profit of the intermediary is

$$\frac{\alpha}{2}(v_h + b - c).$$

Suppose that the intermediary sets $\lambda \in (0, 1)$. Then, the equilibrium seller's profit in this subgame cannot be smaller than the profit from serving flexible consumers in the direct channel at $p^D = v_m$ and serving all picky consumers with a good match in the indirect channel at $p^I = v_h + b$. This profit equals to

$$(1-\alpha)(v_m - c) + (1-\lambda)\frac{\alpha}{2}(v_h + b - c).$$

If the seller decides to serve all flexible consumers only through the indirect channel, then the intermediary, which has already fixed λ, eventually picks β that maximizes the total industry profit. Since $\alpha < \alpha_0$, Lemma 1 implies that $\beta = \beta^* = \frac{v_h - v_m}{v_m - v_l}$. The resulting seller's profit from the outcome with biased recommendations is given by

$$(1-\lambda)\left(\frac{\alpha(1+\beta^*)}{2} + (1-\alpha)\right)(v_m + b - c).$$

Thus, the seller decides to serve consumers only through the indirect channel if and only if

$$(1-\lambda)\left[\left(\frac{\alpha(1+\beta^*)}{2} + (1-\alpha)\right)(v_m + b - c) - \frac{\alpha}{2}(v_h + b - c)\right] \geq (1-\alpha)(v_m - c). \tag{3}$$

Note that since $\alpha < \alpha_0$, the expression in square brackets is always positive and greater than $(1-\alpha)(v_m - c)$. Thus, we can define $\lambda_1 = \lambda_1(\alpha)$ on $(0, \alpha_0]$, which makes (3) binding:

$$\lambda_1(\alpha) = 1 - \frac{(1-\alpha)(v_m - c)}{\left[\frac{\alpha}{2}\frac{v_h - v_l}{v_m - v_l} + (1-\alpha)\right](v_m + b - c) - \frac{\alpha}{2}(v_h + b - c)}. \tag{4}$$

It is easy to see that $\lambda_1(\cdot)$ strictly decreases from $b/(v_m + b - c)$ to 0 when α goes from 0 to α_0.

The maximal profit of the intermediary that induces the outcome with biased recommendations is greater than the profit from inducing the outcome with inefficient bypass if and only if

$$\lambda_1(\alpha)\left(\frac{\alpha(1+\beta^*)}{2}+(1-\alpha)\right)(v_m+b-c)-\frac{\alpha}{2}(v_h+b-c)\geq 0. \tag{5}$$

Note that $\lambda_1(\alpha)$ as well as the total industry profit from the biased recommendation outcome strictly decrease in α on $[0,\alpha_0]$. This implies that the first term, and consequently, the left-hand side of this inequality strictly decreases in α. Moreover, the left-hand side is positive at $\alpha=0$ and negative at $\alpha=\alpha_0$. Thus, there exists a unique $\alpha_1\in(0,\alpha_0)$, such that for all $\alpha\leq\alpha_1$ the equilibrium features biased recommendations with $\lambda^*=\lambda_1$. If instead $\alpha>\alpha_1$, then the equilibrium features inefficient bypass.

□

Proof of Proposition 3. By Lemma 2, the vertically integrated monopoly, operating under the regulation that prohibits biased recommendations, induces the first-best outcome for $\alpha<\alpha_2$ and the inefficient bypass outcome for $\alpha\geq\alpha_2$, where α_2 is given in equation (11). We show that the intermediary decentralizes the vertically integrated solution under biased recommendation regulation for any $\alpha\in(0,1)$ The minimal profit of the seller that it can always guarantee by serving the flexible consumers through the direct channel is $(1-\alpha)(v_m-c)$. First, suppose that $\alpha<\alpha_2$. Consider the intermediary imposing a PPC and setting

$$\lambda_2(\alpha)\equiv 1-\frac{(1-\alpha)(v_m-c)}{\left(\frac{\alpha}{2}+1-\alpha\right)(v_m+b-c)}. \tag{6}$$

Since $\alpha<\alpha_2$, we have that

$$\left(\frac{\alpha}{2}+1-\alpha\right)(v_m+b-c)>\frac{\alpha}{2}(v_h+b-c)+(1-\alpha)(v_m-c),$$

implying that the seller's incentives compatibility constraints are satisfied: $(1-\lambda_2(\alpha))(\alpha/2+1-\alpha)(v_m+b-c)>(1-\lambda_2(\alpha))\alpha/2(v_h+b-c)$ and $(1-\lambda_2(\alpha))(\alpha/2+1-\alpha)=(1-\alpha)(v_m-c)$. Thus, the seller weakly prefers serving flexible consumers and picky consumers with a good match via the indirect channel, resulting in the first-best outcome. Moreover, the seller earns the minimal guaranteed profits of $(1-\alpha)(v_m-c)$. Therefore, by Lemma 2, we have that for $\alpha<\alpha_2$, the intermediary induces the first-best outcome.

Next, suppose that $\alpha\geq\alpha_2$. By Lemma 2, the vertically-integrated firm implements the outcome with inefficient bypass. The intermediary imposing no PPCs and setting $\lambda^*=1$

also induce the outcome with inefficient bypass, in which it earns the profits of the vertically-integrated firm reduced by $(1-\alpha)(v_m - c)$. Thus, for $\alpha < \alpha_2$, the inefficient bypass outcome is implemented.

□

Proof of Proposition 4. By Proposition 3, for $\alpha \geq \alpha_2$, the intermediary would not require a price parity even if it was allowed to use it. This implies that if $\alpha \geq \alpha_2$, we have that the intermediary sets $\lambda^* = 1$ and the inefficient bypass outcome is implemented.

Next, we consider the case of $\alpha < \alpha_2$. Suppose that the intermediary sets $\lambda \in [0,1]$. Then, the seller prefers to serve flexible consumers and picky consumers with a good match in the indirect channel rather than using the indirect channel for picky consumers only if

$$(1-\lambda)\left(\frac{\alpha}{2}+1-\alpha\right)(v_m+b-c) \geq (1-\alpha)(v_m-c)+(1-\lambda)\frac{\alpha}{2}(v_h+b-c).$$

Note that for any $\alpha < \alpha_2$, there exists $\lambda_3 = \lambda_3(\alpha) \in (0,1)$, which makes the seller indifferent between the first-best outcome and serving only picky consumers in the indirect channel. Solving for λ_3, we obtain:

$$\lambda_3(\alpha) \equiv 1 - \frac{(1-\alpha)(v_m-c)}{\left(\frac{\alpha}{2}+1-\alpha\right)(v_m+b-c) - \frac{\alpha}{2}(v_h+b-c)}. \tag{7}$$

Rearranging terms, we have that

$$\lambda_3(\alpha) = 1 - \frac{(1-\alpha)(v_m-c)}{(1-\alpha)(v_m-c)+(\alpha_2-\alpha)\left(b+\frac{v_h-v_m}{2}\right)},$$

where α_2 was defined in equation (11). This can be directly observed from the expression for λ_3 that $\lambda_3(\cdot)$ strictly decreases from $b/(v_m+b-c)$ to 0 when α goes from 0 to α_2.

The highest possible profits that the intermediary can make when the first-best outcome is implemented is given by $\lambda_3(\alpha)(1-\alpha+\alpha/2)(v_m+b-c)$. Alternatively, the intermediary can set $\lambda^* = 1$, induce the inefficient bypass outcome, making profits of $\frac{\alpha}{2}(v_h+b-c)$. The former strategy is strictly more profitable if and only if

$$\lambda_3(\alpha)\left(1-\frac{\alpha}{2}\right)(v_m+b-c) - \frac{\alpha}{2}(v_h+b-c) > 0.$$

Since $\lambda_3(\cdot)$ strictly decreases in α, we have that the left-hand side of this inequality also strictly decreases in α on $(0,\alpha_2)$. This implies that there exists $\alpha_3 \in (0,\alpha_2)$ such that for all

$\alpha \leq \alpha_3$, the intermediary sets $\lambda^* = \lambda_3$ and the first-best outcome is implemented. If instead $\alpha > \alpha_3$, then the equilibrium features inefficient bypass. $\square$

Proof of Proposition 5. It was shown that $\alpha_1 < \alpha_0$ in the proof of Proposition 1. We also showed that $\alpha_3 < \alpha_2$ in the proof of Proposition 3.

Comparing the analytical expression for α_0 and α_2 given in equations (2) and (11), respectively, and using the fact that $v_h - v_m > v_h + b - c - (1+\beta^*)(v_m + b - c)$, we have that $\alpha_2 \in (\alpha_3, \alpha_0)$.

To show the ranking of the different critical values of α, it remains to show that $\alpha_3 < \alpha_1$. In the proof of Proposition 2, we showed that the intermediary induces the outcome with biased recommendations if and only if $g(\alpha) \geq 0$, where

$$g(\alpha) \equiv \lambda_1(\alpha)\left(\frac{\alpha(1+\beta^*)}{2} + (1-\alpha)\right)(v_m + b - c) - \frac{\alpha}{2}(v_h + b - c). \tag{8}$$

It can be directly seen from comparing expressions for $\lambda_1(\cdot)$ and $\lambda_3(\cdot)$, given in equations (4) and (7), respectively, that $\lambda_1(\alpha) > \lambda_3(\alpha)$ for all $\alpha < \alpha_2$. As $\alpha_3 < \alpha_2$, we have that $\lambda_1(\alpha_3) > \lambda_3(\alpha_3)$. Using this inequality, we have that

$$g(\alpha_3) > \lambda_3(\alpha_3)\left(\frac{\alpha_3}{2} + (1-\alpha_3)\right)(v_m + b - c) - \frac{\alpha_3}{2}(v_h + b - c) = 0.$$

Since $g(\alpha_1) = 0$ and $g(\alpha)$ is strictly decreasing on $(0, \alpha_0)$, we have that $\alpha_3 < \alpha_1$.

Finally, we compare α_1 and α_2. By Proposition 2, the intermediary induces the outcome with biased recommendations if and only if $g(\alpha) \geq 0$, where $g(\alpha)$ was defined in equation (8).

Plugging $\alpha = \alpha_2$ into equation (8), and using equation (4) for $\lambda_1(\cdot)$, we have that

$$\begin{aligned}
g(\alpha_2) = &\left(\frac{\alpha_2}{2} + (1-\alpha_2)\right)(v_m + b - c) - (1-\alpha_2)(v_m - c) - \frac{\alpha_2}{2}(v_h + b - c) \\
&+ \frac{\alpha_2}{2}\beta^*(v_m + b - c) - (1 - \lambda_1(\alpha_2))\frac{\alpha_2}{2}(v_h + b - c) \\
=&\frac{\alpha_2}{2}\left(\beta^*(v_m + b - c) - (1-\lambda_1(\alpha_2))(v_h + b - c)\right),
\end{aligned}$$

where we used $\alpha_2 = b/(b + (v_h - v_m)/2)$ to obtain the final expression. In the proof of Proposition 2, we showed that $g(\alpha)$ is strictly decreasing in α on $(0, \alpha_0)$. Therefore, as $g(\alpha_1) = 0$, we have that $\alpha_2 > (\leq)\alpha_1$ if

$$(1 - \lambda_1(\alpha_2))(v_h + b - c) - \beta^*(v_m + b - c) > (\leq) 0.$$

Define the function

$$h(v_m) \equiv (v_m - v_l)(v_h + b - c) - \frac{(v_h - v_m)(v_m + b - c)}{1 - \lambda_1(\alpha_2)}$$

and note that $\alpha_2 > \alpha_1$ if $h(v_m) > 0$, and $\alpha_2 \leq \alpha_1$ otherwise. Using analytical expressions for α_2 and $\lambda_1(\cdot)$, given in equations (4) and (11), respectively, we have that

$$\begin{aligned} h(v_m) =&(v_m - v_l)(v_h + b - c) \\ &- \frac{(v_h - v_m)(v_m + b - c)\left(\left(\frac{\alpha_2}{2}(1 + \beta^*) + (1 - \alpha_2)\right)(v_m + b - c) - \frac{\alpha_2}{2}(v_h + b - c)\right)}{(1 - \alpha_2)(v_m - c)} \\ =&(v_m - v_l)(v_h + b - c) - \frac{(v_h - v_m)(v_m + b - c)\left((1 - \alpha_2)(v_m - c) + \frac{\alpha_2}{2}\beta^*(v_m + b - c)\right)}{(1 - \alpha_2)(v_m - c)} \\ =&(v_m - v_l)(v_h + b - c) - \frac{(v_h - v_m)(v_m + b - c)\left(v_m - c + b\frac{v_m + b - c}{v_m - v_l}\right)}{v_m - c} \\ =&(v_m - v_l)(v_h + b - c) - (v_h - v_m)(v_m + b - c) - b\frac{v_h - v_m}{v_m - v_l}\frac{(v_m + b - c)^2}{v_m - c}. \end{aligned} \tag{9}$$

Note that $h(v_m)$ tends to $-\infty$ as v_m goes to c and $h(v_m)$ tends to $(v_h - v_l)(v_h + b - c) > 0$ as v_m goes to v_h. This implies that the parameter ranges such that $\alpha_1 < \alpha_2$ and the one such that $\alpha_1 > \alpha_2$ are both non-empty.

Next, we show that $h(v_m)$ is strictly increasing in v_m on (c, v_h). The derivative of h with respect to v_m is given by

$$\begin{aligned} h'(v_m) &= 2(v_m + b - c) \\ &\quad - b\left(\frac{(v_h - v_m)(v_m + b - c)}{(v_m - v_l)(v_m - c)} - \frac{b(v_h - v_m)(v_m + b - c)}{(v_m - c)^2(v_m - v_l)} - \frac{(v_h - v_l)(v_m + b - c)^2}{(v_m - v_l)^2(v_m - c)}\right) \\ &= (v_m + b - c)\left[2 - \frac{b(v_h - v_m)(v_m - c - b)}{(v_m - c)^2(v_m - v_l)} + \frac{b(v_h - v_l)(v_m + b - c)^2}{(v_m - v_l)^2(v_m - c)}\right] \\ &> (v_m + b - c)\left[2 - \frac{b(v_h - v_m)(v_m - c - b)}{(v_m - c)^2(v_m - v_l)}\right]. \end{aligned}$$

If $v_m \in (c, c + b]$, then the expression in square brackets is strictly positive, implying that $h'(v_m) > 0$. If instead $v_m \in (c + b, v_h)$, we have that

$$\begin{aligned} h'(v_m) &> (v_m + b - c)\left[2 - \frac{v_h - v_m}{v_m - v_l}\frac{v_m - c - b}{v_m - c}\frac{b}{v_m - c}\right] \\ &> (v_m + b - c)\left[2 - \frac{b}{v_m - c}\right] \\ &> 2b > 0, \end{aligned}$$

where we used the fact that $(v_m + b - c)(2 - b/(v_m - c))$ is increasing in v_m on $(c + b, v_h)$.

To sum up, we have established that $h(v_m)$ is strictly increasing on (c, v_h) and changes its sign from negative to positive on this interval. This implies a unique solution $\hat{v}_m \in (c, v_h)$ to the equation

$$(1 - \lambda_1(\alpha_2))(v_h + b - c) - \beta^*(\hat{v}_m + b - c) = 0. \tag{10}$$

Therefore, we obtain that $\alpha_2 > \alpha_1$ for $v_m > \hat{v}_m$, and $\alpha_2 \leq \alpha_1$ otherwise. □

B The vertically integrated solution

We look at the vertically integrated solution, which is the outcome of the problem in which a vertically integrated firm acts as recommender and price setter. Essentially the same problem was analyzed in Section 3 of Peitz and Sobolev (forthcoming), which focused on the cost of bypassing the indirect channel. We do not consider the costs of bypass and adapt their analysis to the convenience benefits $b > 0$.

In the following, we focus on the case in which $c \in (v_l + b, v_m)$. The vertically integrated monopoly faces a trade-off between implementing price discrimination between picky and flexible consumers across two sales channels and serving all consumers at a uniform price in the superior indirect channel.

First, suppose that the monopoly decides to serve consumers only through the indirect channel and sets price $p^I < v_m + b$ and β that makes the picky consumer indifferent between buying indirectly and taking the outside option – that is, $p^I = (v_h + \beta v_l)/(1 + \beta) + b$. Since $v_l + b < c$, the profits from the picky consumers strictly increase in p^I. This implies that the monopoly, serving consumers only through the indirect channel, obtains its maximal profits by setting prices $p^I = v_m + b$, $p^D \geq v_m$ and recommendation policy β^* that solves $v_m = (v_h + \beta^* v_l)/(1 + \beta^*)$ or, equivalently $\beta = (v_h - v_m)/(v_m - v_l)$. We refer to this outcome as the outcome with *biased recommendation*. The corresponding profit is

$$\left[\frac{\alpha}{2}(1 + \beta) + (1 - \alpha)\right](v_m + b - c) = \left[\frac{\alpha}{2}\frac{v_h - v_l}{v_m - v_l} + (1 - \alpha)\right](v_m + b - c).$$

Second, if the monopoly decides to serve consumers through two channels, then it sets $\beta = 0$, serves picky consumers with a good match at $p^I = v_h + b$ in the indirect channel and serves flexible consumers in the direct channel at $p^D = v_m$. We refer to this outcome as the outcome with *inefficient bypass*. The corresponding profit is

$$\frac{\alpha}{2}(v_h + b - c) + (1 - \alpha)(v_m - c).$$

Comparing the two cases above, we find that the maximal profit is given by

$$(1 - \alpha)(v_m - c) + \max\left\{\frac{\alpha}{2}\frac{v_h - v_l}{v_m - v_l}(v_m + b - c) + (1 - \alpha)b, \frac{\alpha}{2}(v_h + b - c)\right\}.$$

In the proof of Lemma 1, there is a uniquely defined $\alpha_0 \in (0, 1)$, such that the two cases give the same profit, which is given by $\frac{\alpha_0}{2}\frac{v_h - v_l}{v_m - v_l}(v_m + b - c) + (1 - \alpha_0)b = \frac{\alpha_0}{2}(v_h + b - c)$. For $\alpha < \alpha_0$, the firm maximizes its profit by inducing the outcome with biased recommendations, while, for $\alpha \geq \alpha_0$, it does so by inducing inefficient bypass.

Lemma 1. *Suppose that $c \in (v_l + b, v_m)$. Then, the vertically integrated solution is characterized as follows:*

- *for $\alpha < \alpha_0$, where $\alpha_0 \in (0, 1)$ the firm sets $\beta^* = \frac{v_h - v_m}{v_m - v_l}$ and $(p^I, p^D) = (v_m + b, v_m)$. All consumers go to the indirect channel. All flexible consumers and all picky consumers with a recommendation buy. Welfare losses compared to the first best are given by $\frac{\alpha}{2}\frac{v_h - v_m}{v_m - v_l}(c - b - v_l)$.*
- *for $\alpha \geq \alpha_0$, the firm sets $\beta = 0$ and $(p^I, p^D) = (v_h + b, v_m)$. Picky consumers go to the indirect channel and buy if they receive the recommendation to buy, whereas all flexible consumers buy in the direct channel. Welfare losses compared to the first best are given by $(1 - \alpha)b$.*

Proof. Comparing the maximal profit in the two possible solutions, the critical α_0 is implicitly determined by

$$\frac{\alpha_0}{2}\frac{v_h - v_l}{v_m - v_l}(v_m + b - c) + (1 - \alpha_0)b = \frac{\alpha_0}{2}(v_h + b - c),$$

which can be rewritten as

$$\frac{\alpha_0}{2}(v_h + b - c - (1 + \beta^*)(v_m + b - c)) = (1 - \alpha_0)b$$

and, after further manipulation gives the explicit expression for α_0 reported in the main text:

$$\alpha_0 = \frac{b}{b + \frac{1}{2}(v_h + b - c - (1+\beta^*)(v_m + b - c))}.$$

To show that $\alpha_0 \in (0,1)$, it is sufficient show that the expression $v_h + b - c - (1+\beta^*)(v_m + b - c)$ is positive. Dividing by $v_m + b - c$, we have that

$$\frac{v_h + b - c}{v_m + b - c} - \frac{v_h - v_l}{v_m - v_l} > 0,$$

where we use the fact that function $\frac{v_h - x}{v_m - x}$ is increasing in x and $v_l < c - b$ to obtain the final inequality. □

Next, we consider the case in which the regulator prohibits surplus decreasing product recommendation, implying that $\beta = 0$. Define

$$\alpha_2 \equiv \frac{b}{b + \frac{1}{2}(v_h - v_m)}. \tag{11}$$

The vertically integrated solution is characterized in the following lemma.

Lemma 2. *When the regulator prohibits biased recommendations, the vertically integrated solution is characterized as follows:*

- *for $\alpha < \alpha_2$, where $\alpha_2 < \alpha_0$, the firm sets $(p^I, p^D) = (v_m + b, v_m)$ and the first-best outcome is implemented.*

- *for $\alpha \geq \alpha_2$, the firm sets $(p^I, p^D) = (v_h + b, v_m)$ and the inefficient bypass outcome is implemented.*

Proof. The monopoly either serves consumers only through the indirect channel or engages in price discrimination between picky and flexible consumers across two sales channels. In the former case, the monopoly's induces the first-best outcome by serving picky consumers with a good match and flexible consumers through the indirect channel. The resulting profits are $(\alpha/2 + 1 - \alpha)(v_m + b - c)$. In the latter case, the firm earns $\alpha/2(v_h + b - c) + (1-\alpha)(v_m - c)$. Thus, the monopoly prefers to induce the first-best outcome if

$$\left(\frac{\alpha}{2} + 1 - \alpha\right)(v_m + b - c) > \frac{\alpha}{2}(v_h + b - c) + (1-\alpha)(v_m - c),$$

which is equivalent to $\alpha < \alpha_2$. If instead $\alpha \geq \alpha_2$, the monopoly induces the outcome with inefficient bypass. It directly follows from equations (5) and (11) that $\alpha_2 < \alpha_0$. □

C Revenue sharing

In this appendix, we consider the model in which in the first stage, the intermediary sets the fee $\tau \in [0,1]$ as the fraction of the seller's revenues in the indirect channel that the seller has to pay. This formalizes the discussion in Section 6.1.

Lemma 3. *Consider a subgame starting in the second stage with or without the regulatory interventions from Sections 4.1-4.3. Then, for any $\tau \in [0,1]$, the seller responds by either serving some consumers through the indirect channel at a price $p^I \in \{v_m + b, v_h + b\}$ or by not selling through the indirect channel at all.*

Proof. Note that for any $\tau > 1 - c/(v_h + b)$, serving any consumer through the indirect channel results in negative profits for the seller, implying that the seller does not sell through the indirect channel.

First, consider the case in which the regulator allows total-surplus-decreasing purchase recommendations and either allows or prohibits PPCs. Suppose that the intermediary sets $\tau \leq 1 - c/(v_h + b)$ and either imposes (if allowed) or does not impose a PPC (if not allowed or not in its interest). Then, for any price $p^I \in (v_m + b, v_h + b]$ set by the seller in the indirect channel, the intermediary's best response is to set a recommendation policy β such that the picky consumers are indifferent between buying and taking the outside option – that is, $\frac{v_h+\beta v_l}{1+\beta} + b = p^I$. The seller's profit from the picky consumers is given by

$$\frac{\alpha}{2}(1+\beta)[(1-\tau)p^I - c] = \frac{\alpha}{2}(1-\tau)(v_h - v_l)\frac{p^I - c/(1-\tau)}{p^I - (v_l + b)},$$

which is increasing in p^I, since $c/(1-\tau) > v_l + b$. Thus, the seller strictly prefers to set $p^I = v_h + b$ to any price in $(v_m + b, v_h + b)$.[22] If the seller sets a price $p^I < v_m + b$, then the intermediary responds by setting recommendation policy β satisfying $\frac{v_h+\beta v_l}{1+\beta} + b = p^I$. The flexible consumers buy through the indirect channel if $p^D > p^I - b$. Similar to the previous argument, the seller can slightly increase its price and make strictly higher profits from the picky consumers and weakly higher profits from the flexible consumers. Thus, if the seller serves some consumers through the indirect channel it sets a price $p^I \in \{v_m + b, v_h + b\}$.

[22] If $\tau = 1 - c/(v_h + b)$, then the seller's profit from the picky consumers is maximal for any $p^I \geq v_h + b$ and equals to zero. We break this indifference in favor of the intermediary and assume that the seller sets $p^I = v_h + b$.

Second, consider the case in which the regulator mandates unbiased recommendations ($\beta = 0$) and either allows or prohibits PPCs. Suppose that the intermediary sets $\tau \leq 1 - c/(v_h + b)$ and either imposes (if allowed) or does not impose PPCs (it not allowed or not in its interest). Since $\beta = 0$, the picky consumers are informed about their match values. Therefore, the seller sets either $p^I = v_h + b$ and sells only to the picky consumers with a good match in the indirect channel; or it sets $p^I = v_m + b$ and also serves flexible consumers in the indirect channel, or does not sell through the indirect channel at all (if τ is sufficiently large). □

We start with the analysis of the laissez-faire. Define $\psi(\alpha) \equiv \alpha/2(1 + \beta^*) + 1 - \alpha$,

$$\gamma_1(\alpha) \equiv \frac{(\psi(\alpha) - \alpha/2)c}{\psi(\alpha)(v_m + b) - \alpha/2(v_h + b)} \quad \text{and} \quad \gamma_2(\alpha) \equiv \frac{\psi(\alpha)c + (1 - \alpha)(v_m - c)}{\psi(\alpha)(v_m + b)}.$$

We also define

$$\tau_0(\alpha) \equiv 1 - \max\{\gamma_1(\alpha), \gamma_2(\alpha)\}. \tag{12}$$

In the following proposition, we show that there exists a threshold α_0^{RS} under a revenue-sharing contract with $\alpha_0^{RS} \leq \alpha_0$ such that the intermediary induces the biased recommendation outcome for $\alpha < \alpha_0^{RS}$ and the inefficient bypass outcome for $\alpha \geq \alpha_0^{RS}$.

Proposition 6. *Under laissez-faire, the equilibrium when the intermediary sets an ad valorem fee on revenues in the indirect channel, τ, is characterized as follows: There exists a threshold $\alpha_0^{RS} \leq \alpha_0$ such that*

- *for $\alpha < \alpha_0^{RS}$, the intermediary imposes a PPC, sets $\tau^* = \tau_0(\alpha)$ and $\beta = \beta^*$ along the equilibrium path in the third stage. The seller sets prices $(p^I, p^D) = (v_m + b, v_m + b)$. All flexible consumers and the fraction $\frac{1+\beta^*}{2}$ of picky consumers buy in the indirect channel.*

- *For $\alpha \geq \alpha_0^{RS}$, the intermediary does not impose a PPC, sets $\tau^* = 1 - c/(v_h + b)$, and $\beta = 0$ along the equilibrium path in the third stage. The seller sets $(p^I, p^D) = (v_h + b, v_m)$. Half of the picky consumers buy in the indirect channel and all the flexible consumers buy in the direct channel.*

Moreover, there exist a threshold $\bar{c} \in ((v_h + v_l)/2, v_m)$ and a function $\bar{b}(c) \in (0, c - (v_h + v_l)/2)$, such that $\alpha_0^{RS} < \alpha_0$ if and only if $c > \bar{c}$ and $b > \bar{b}(c)$. Otherwise, $\alpha_0^{RS} = \alpha_0$.

Proof. Suppose that $\alpha \geq \alpha_0$. By Lemma 1, the vertically integrated firm induces the outcome with inefficient bypass. Suppose that the intermediary sets an ad valorem fee on revenues in the indirect channel, $\tau = 1 - c/(v_h + b)$, and does not impose a PPC. By Lemma 3, the seller either sets $p^I \in \{v_m + b, v_h + b\}$ and serves some picky consumers through the indirect channel or does not sell through the indirect channel at all and serves only flexible consumers in the direct channel earning $(1-\alpha)(v_m - c)$. If $p^I = v_m + b$, then the profit from the picky consumers is negative. If $p^I = v_h + b$, then the profit from the picky consumers equals zero. Given $\tau = 1 - c/(v_h + b)$, the seller sets $p^I = v_h + b$, and the intermediary responds with $\beta = 0$ along the equilibrium path. Therefore, for any $\alpha \geq \alpha_0$, the intermediary can decentralize the outcome with inefficient bypass by setting $\tau = 1 - c/(v_h + b)$ and not imposing a PPC.

Next, suppose that $\alpha < \alpha_0$. By Lemma 1, the vertically integrated solution is characterized by the outcome with biased recommendations. Suppose that the intermediary sets a revenue sharing contract $\tau \leq 1 - c/(v_h + b)$ and imposes a PPC.[23] By Lemma 3, the seller either serves some consumers through the indirect channel at a price $p^I \in \{v_m + b, v_h + b\}$ or does not sell through the indirect channel at all. Thus, the seller sets prices $p^I = p^D = v_m + b$ and the outcome with biased recommendations is induced if

$$\left(\frac{\alpha}{2}(1+\beta^*) + 1 - \alpha\right)[(1-\tau)(v_m + b) - c] \geq \max\left\{\frac{\alpha}{2}[(1-\tau)(v_h + b) - c], (1-\alpha)(v_m - c)\right\}.$$

Rearranging the incentive compatibility constraint of the seller, we have that the intermediary inducing the biased recommendation outcome sets the highest τ that satisfies the following constraints:[24]

$$1 - \tau \geq \frac{(\psi(\alpha) - \alpha/2)c}{\psi(\alpha)(v_m + b) - \alpha/2(v_h + b)} = \gamma_1(\alpha),$$
$$1 - \tau \geq \frac{\psi(\alpha)c + (1-\alpha)(v_m - c)}{\psi(\alpha)(v_m + b)} = \gamma_2(\alpha).$$

We determine the sign of $\delta(\alpha) \equiv \gamma_2(\alpha) - \gamma_1(\alpha)$. We show that $\delta(\alpha)$ is strictly decreasing in

[23]The intermediary generates maximal profits with the biased recommendation outcome given that it imposes a PPC.

[24]For any $\alpha < \alpha_0$, we have that $\psi(\alpha)(v_m + b - c) > \alpha/2(v_h + b - c) + (1-\alpha)(v_m - c)$. This implies that $\gamma_1(\alpha)$ and $\gamma_2(\alpha)$ belong to $(0, 1)$.

α on $(0, \alpha_0]$. The derivative of $\gamma_1(\alpha)$ with respect to α is given by

$$\begin{aligned}\gamma_1'(\alpha) =& \frac{(\psi'(\alpha) - 1/2)((\psi(\alpha) - \alpha/2)(v_m + b) - \alpha/2(v_h - v_m))c}{(\psi(\alpha)(v_m + b) - \alpha/2(v_h + b))^2} \\ &- \frac{(\psi(\alpha) - \alpha/2)((\psi'(\alpha) - 1/2)(v_m + b) - 1/2(v_h - v_m))c}{(\psi(\alpha)(v_m + b) - \alpha/2(v_h + b))^2} \\ &= \frac{(\psi(\alpha) - \alpha\psi'(\alpha))c(v_h - v_m)/2}{(\psi(\alpha)(v_m + b) - \alpha/2(v_h + b))^2} \\ &= \frac{c(v_h - v_m)/2}{(\psi(\alpha)(v_m + b) - \alpha/2(v_h + b))^2} > 0,\end{aligned}$$

which implies that $\gamma_1(\alpha)$ is strictly increasing in α on $(0, \alpha_0]$. Moreover, $\gamma_2(\alpha)$ is strictly decreasing in α on $(0, \alpha_0]$, since

$$\gamma_2'(\alpha) = \frac{\partial}{\partial \alpha}\left(\frac{c}{v_m + b} + \frac{1-\alpha}{\psi(\alpha)}\frac{v_m - c}{v_m + b}\right) = -\frac{(1+\beta^*)/2}{\psi^2(\alpha)}\frac{v_m - c}{v_m + b} < 0.$$

This implies that $\delta(\alpha)$ is also strictly decreasing in α on $(0, \alpha_0]$. Since $\delta(0) = (v_m - c)/(v_m + b) > 0$ and $\delta(\alpha)$ is decreasing in α on $(0, \alpha_0]$, we have that either $\gamma_2(\alpha) \geq \gamma_1(\alpha)$ for any $\alpha \in (0, \alpha_0)$ or there exists $\hat{\alpha} \in (0, \alpha_0)$ such that $\gamma_2(\alpha) \geq \gamma_1(\alpha)$ on $(0, \hat{\alpha}]$ and $\gamma_2(\alpha) < \gamma_1(\alpha)$ on $[\hat{\alpha}, \alpha_0)$. Note that $\gamma_2(\alpha) \geq \gamma_1(\alpha)$ for any $\alpha \in (0, \alpha_0)$ if and only if $\delta(\alpha_0) \geq 0$.

Next, we characterize the necessary and sufficient conditions under which $\delta(\alpha_0) \geq 0$. First, we show that for every v_h, v_m, v_l and c there is a function $\bar{b}(c) \in (0, v_m - (v_h + v_l)/2)$ such that $\delta(\alpha_0) \geq 0$ if and only if $b \leq \bar{b}(c)$. Second, we characterize the conditions under which $(v_h + v_l)/2 + \bar{b}(c) \geq c$.

Rearranging terms, we have that $\delta(\alpha_0)$ can be rewritten as

$$\begin{aligned}\delta(\alpha_0) &= 1 - \gamma_1(\alpha_0) - (1 - \gamma_2(\alpha_0)) \\ &= \frac{\psi(\alpha_0)(v_m + b - c) - \alpha_0/2(v_h + b - c)}{\psi(\alpha_0)(v_m + b) - \alpha_0/2(v_h + b)} - \frac{\psi(\alpha_0)(v_m + b - c) - (1 - \alpha_0)(v_m - c)}{\psi(\alpha_0)(v_m + b)}.\end{aligned}$$

In the proof of Lemma 1, we showed that α_0 solves $\psi(\alpha_0)(v_m + b - c) = \alpha_0/2(v_h + b - c) + (1 - \alpha_0)(v_m - c)$. Applying this identity, the expression for $\delta(\alpha_0)$ can be further rearranged as follows:

$$\delta(\alpha_0) = \frac{(1 - \alpha_0)(v_m - c)}{(1 - \alpha_0)(v_m - c) + (\psi(\alpha_0) - \alpha_0/2)c} - \frac{\alpha_0/2(v_h + b - c)}{\psi(\alpha_0)(v_m + b)}.$$

Plugging in

$$\alpha_0 = \frac{b}{b + \frac{1}{2}(v_h + b - c - (1 + \beta^*)(v_m + b - c))} = \frac{b}{b + \frac{\beta^*}{2}(c - (v_l + b))},$$

we have that

$$\psi(\alpha_0) = \frac{b(1+\beta^*)/2 + \beta^*/2(c-(v_l+b))}{b+\beta^*(c-(v_l+b))} = \frac{\frac{1}{2}(b+\beta^*(c-v_l))}{b+\beta^*(c-(v_l+b))}.$$

This implies that

$$\frac{\alpha_0/2}{\psi(\alpha_0)} = \frac{b/2}{b+\beta^*(c-(v_l+b))} \frac{b+\beta^*/2(c-(v_l+b))}{(b+\beta^*(c-v_l))/2} = \frac{b}{b+\beta^*(c-v_l)}$$

and

$$\begin{aligned} \frac{\psi(\alpha_0)-\alpha_0/2}{1-\alpha_0} &= \frac{\beta^*\alpha_0/2}{1-\alpha_0} + 1 = \frac{b\beta^*/2}{\beta^*/2(c-(v_l+b))} + 1 \\ &= 1 + \frac{b}{c-(v_l+b)}. \end{aligned}$$

Plugging both expression into the formula for $\delta(\alpha_0)$, we obtain:

$$\begin{aligned} \delta(\alpha_0) &= \frac{v_m - c}{v_m - c + \frac{\psi(\alpha_0)-\alpha_0/2}{1-\alpha_0}c} - \frac{\alpha_0/2}{\psi(\alpha_0)} \frac{v_h + b - c}{v_m + b} \\ &= \frac{v_m - c}{v_m + \frac{cb}{c-(v_l+b)}} - \frac{b}{b+\beta^*(c-v_l)} \frac{v_h+b-c}{v_m+b} \\ &= \left(\frac{(v_m-c)(c-(v_l+b))}{c-(v_l+b)\frac{v_m}{v_m+b}} - \frac{b(v_h+b-c)}{b+\beta^*(c-v_l)} \right) \Big/ (v_m+b). \end{aligned} \tag{13}$$

Since $(v_h - c)/(c - v_l)$ is decreasing in c, we have that $(v_h - c)/(c - v_l) > \beta^*$, implying that $v_h + b - c > b + \beta^*(c - v_l)$. It follows that

$$\delta(\alpha_0) < \left(v_m - c - \frac{b(v_h+b-c)}{b+\beta^*(c-v_l)} \right) \Big/ (v_m+b) < \frac{v_m - c - b}{v_m + b}.$$

Thus, $\delta(\alpha_0)$ evaluated at $b = v_m - (v_h + v_l)/2$ is strictly smaller than $((v_h+v_l)/2-c)/(2v_m - (v_h+v_l)/2)$, which is strictly negative. Moreover, note that $\delta(\alpha_0)$ evaluated at $b = 0$ equals $(v_m - c)/v_m > 0$. By the intermediate value theorem, there exists a solution $\bar{b}(c) \in (0, v_m - (v_h + v_l)/2)$ such that $\delta(\alpha_0) = 0$ at $b = \bar{b}(c)$. Next, we show that the solution is unique.

The partial derivative of $\delta(\alpha_0)$ with respect to b at $b = \bar{b}(c)$ is

$$\frac{\partial \delta(\alpha_0)}{\partial b}\Big|_{b=\bar{b}(c)} = \frac{1}{v_m + \bar{b}(c)} \frac{\partial}{\partial b} \left(\frac{(v_m-c)(c-(v_l+b))}{c-(v_l+b)\frac{v_m}{v_m+b}} - \frac{b(v_h+b-c)}{b+\beta^*(c-v_l)} \right) \Big|_{b=\bar{b}(c)}.$$

The first term of this derivative is negative, which follows from

$$\frac{\partial}{\partial b}\left(\frac{c-(v_l+b)}{c-(v_l+b)\frac{v_m}{v_m+b}}\right)=\frac{-c+(v_l+b)\frac{v_m}{v_m+b}-(c-(v_l+b))\left[-\frac{v_m}{v_m+b}+\frac{v_m(v_l+b)}{(v_m+b)^2}\right]}{\left(c-(v_l+b)\frac{v_m}{v_m+b}\right)^2}$$
$$=-\left(\frac{cb}{v_m+b}+(c-(v_l+b))\frac{v_m(v_l+b)}{(v_m+b)^2}\right)\Big/\left(c-(v_l+b)\frac{v_m}{v_m+b}\right)^2$$
$$<0.$$

Since $b/(b+\beta^*(c-v_l))$ is increasing in b we have that the second term of the derivative is strictly positive. Therefore, the derivative of $\delta(\alpha_0)$ with respect to b at $b=\bar{b}(c)$ is negative. This implies that for every v_h, v_m, v_l, and c, there exists a unique $\bar{b}(c)\in(0,v_m-(v_h+v_l)/2)$ that solves $\delta(\alpha_0)=0$. It also follows that $\delta(\alpha_0)\geq 0$ if $b\leq\bar{b}(c)$ and $\delta(\alpha_0)<0$, otherwise.

Next, we establish conditions under which $\bar{b}(c)\geq c-(v_h+v_l)/2$, implying that $\delta(\alpha_0)\geq 0$ for all $b\in(0,c-(v_h+v_l)/2)$. In particular, we show that there exists a unique $\bar{c}$ such that $\bar{b}(c)\geq c-(v_h+v_l)/2$ if and only if $c\leq\bar{c}$. Define $\hat{b}\equiv c-(v_h+v_l)/2$ and

$$z(c)\equiv\delta(\alpha_0)\Big|_{b=\hat{b}}=\left(\frac{v_m-c}{c-(v_l+b)\frac{v_m}{v_m+\hat{b}}}-\frac{\hat{b}}{\hat{b}+\beta^*(c-v_l)}\right)\frac{(v_h-v_l)/2}{v_m+\hat{b}}$$
$$=\frac{\chi(c)(v_h-v_l)/2}{\hat{b}((v_m+\hat{b})c-v_m(v_l+\hat{b}))(\hat{b}+\beta^*(c-v_l))},$$

where

$$\chi(c)\equiv(v_m+\hat{b})(v_m-c)(\hat{b}+\beta^*(c-v_l))-\hat{b}((v_m+\hat{b})c-v_m(v_l+\hat{b}))$$
$$=(v_m+\hat{b})(v_m-c)(\hat{b}+\beta^*(c-v_l))-\hat{b}(\hat{b}\bar{c}+v_m(v_h-v_l)/2).$$

Note that $\hat{b}=0$ at $c=(v_h+v_l)/2$, which implies that $z((v_h+v_l)/2)>0$. Moreover, we have that $z(v_m)<0$, implying that by the intermediate value theorem, there exists $\bar{c}$ such that $z(\bar{c})=0$. The derivative of the function $z(c)$ with respect to c at $c=\bar{c}$ is given by

$$z'(\bar{c})=\frac{\chi'(\bar{c})(v_h-v_l)/2}{\hat{b}((v_m+\hat{b})\bar{c}-v_m(v_l+\hat{b}))(\hat{b}+\beta^*(\bar{c}-v_l))},$$

where

$$\chi'(c)=-(\hat{b}+c)(2\hat{b}+\beta^*(c-v_l))+(1+\beta^*)(v_m+\hat{b})(v_m-c)-(\hat{b}c+v_m(v_h-v_l)/2).$$

The first and the third terms of $\chi'(c)$ are decreasing in c, the derivative of the second term is $-(1+\beta^*)(\hat{b}+c) < 0$, implying that $\chi'(c)$ is strictly decreasing in c. Thus,

$$\begin{aligned}\chi'(\bar{c}) &< \chi'\left(\frac{v_h+v_l}{2}\right)\\ &= -\beta^*\frac{(v_h+v_l)(v_h-v_l)}{4} + (1+\beta^*)v_m\left(v_m - \frac{v_h+v_l}{2} - \frac{v_m-v_l}{2}\right)\\ &= -\beta^*\frac{(v_h+v_l)(v_h-v_l)}{4} - (1+\beta^*)\frac{v_m(v_h-v_m)}{2} < 0.\end{aligned}$$

It follows that $z(\bar{c}) < 0$, which implies that there exists a unique $\bar{c} \in ((v_h+v_l)/2, v_m)$ such that $\delta(\alpha_0)$ evaluated at $b = c-(v_h+v_l)/2$ is positive if $c > \bar{c}$ and is negative, otherwise. We conclude that for any $c \leq \bar{c}$, we have that $\delta(\alpha_0) \geq 0$ for every $b \in (0, c-(v_h+v_l)/2)$.

If instead $c > \bar{c}$, then there exists $\bar{b}(c) \in (0, c-(v_h+v_l)/2)$ such that $\delta(\alpha_0) \geq 0$ if and only if $b \leq \bar{b}(c)$. If $c > \bar{c}$ and $b > \bar{b}(c)$, we have that $\delta(\alpha_0) < 0$.

We are now in the position to characterize the equilibrium for $\alpha < \alpha_0$. First, suppose that either $c \leq \bar{c}$ or $c > \bar{c}$ and $b \leq \bar{b}(c)$. It follows from the previous argument that $\gamma_2(\alpha) \geq \gamma_1(\alpha)$ for any $\alpha < \alpha_0$. Thus, the intermediary setting $\tau = 1-\gamma_2(\alpha)$ and imposing a PPC can induce the outcome with biased recommendations. In this outcome the seller earns $(1-\alpha)(v_m-c)$. Thus, the intermediary can decentralize the outcome with biased recommendations for any $\alpha < \alpha_0$. Consequently, $\alpha_0^{RS} = \alpha_0$.

Second, suppose that $c > \bar{c}$ and $b \in (\bar{b}(c), c-(v_h+v_l)/2)$. In this case, we have that $\delta(\alpha_0) < 0$, implying that there exists $\hat{\alpha} \in (0, \alpha_0)$ such that $\gamma_2(\alpha) \geq \gamma_1(\alpha)$ for $\alpha \leq \hat{\alpha}$ and $\gamma_1(\alpha) > \gamma_2(\alpha)$, otherwise. If $\alpha \leq \hat{\alpha}$, the intermediary setting $\tau = 1-\gamma_2(\alpha)$ and imposing a PPC can decentralize the outcome with biased recommendations. Next, consider $\alpha \in (\hat{\alpha}, \alpha_0)$. The highest profit that the intermediary can reach inducing the outcome with biased recommendations is $(1-\gamma_1(\alpha))\psi(\alpha)(v_m+b)$. The intermediary can also induce the outcome with inefficient bypass by setting $\tau = 1-c/(v_h+b)$ and imposing no PPC. In this case, the profit of the intermediary is $\alpha/2(1-c/(v_h+b))(v_h+b) = \alpha/2(v_h+b-c)$. Implementing the biased recommendation outcome instead of the inefficient bypass outcome leads to the difference in the intermediary's profits of

$$\Delta_0(\alpha) \equiv (1-\gamma_1(\alpha))\psi(\alpha)(v_m+b) - \alpha/2(v_h+b-c).$$

Since the intermediary can decentralize the vertically-integrated solution at $\alpha = \hat{\alpha} < \alpha_0$, we have that $\Delta_0(\hat{\alpha}) > 0$. Moreover,

$$\begin{aligned}\Delta_0(\alpha_0) &= (1-\gamma_1(\alpha))\psi(\alpha)(v_m+b) - \alpha_0/2(v_h+b-c)\\ &< (1-\gamma_2(\alpha))\psi(\alpha)(v_m+b) - \alpha_0/2(v_h+b-c)\\ &= \left(\frac{\alpha_0}{2}(1+\beta^*)+1-\alpha_0\right)(v_m+b-c) - (1-\alpha_0)(v_m-c) - \alpha_0/2(v_h+b-c)\\ &= 0.\end{aligned}$$

Since $1-\gamma_1(\alpha)$ is strictly decreasing on $(0,\alpha_0)$, we have that $\Delta_0(\alpha)$ is strictly decreasing in α and changes its sign on $(\hat{\alpha},\alpha_0)$. Thus, there exists a unique threshold $\alpha_0^{RS} \in (\hat{\alpha},\alpha_0)$ such that for $\alpha < \alpha_0^{RS}$, the intermediary sets $\tau_0(\alpha) = 1-\gamma_1(\alpha)$, imposes a PPC, and induces the outcome with biased recommendations. For $\alpha \geq \alpha_0^{RS}$, the intermediary sets $\tau = 1-c/(v_h+b)$, imposes no PPC, and induces the outcome with inefficient bypass. □

Next, we explore the regulation that prohibits PPCs but does not prohibit biased recommendations. We define

$$\tau_1(\alpha) \equiv 1 - \frac{(\psi(\alpha)-\alpha/2)c + (1-\alpha)(v_m-c)}{\psi(\alpha)(v_m+b) - \frac{\alpha}{2}(v_h+b)}. \tag{14}$$

We also define α_1^{RS} as the solution to the following equation:

$$\psi(\alpha)\tau_1(\alpha)(v_m+b) - \alpha/2(v_h+b-c) = 0.$$

In the following proposition, we show that α_1^{RS} is uniquely-defined and is strictly smaller than α_0^{RS}.

Proposition 7. *Under the regulation that prohibits the use of PPCs, the equilibrium when the intermediary sets an ad valorem fee on revenues in the indirect channel is characterized as follows: There is a critical threshold α_1^{RS} with $\alpha_1^{RS} < \alpha_0^{RS}$ such that*

- *for $\alpha < \alpha_1^{RS} < \alpha_0^{RS}$, the intermediary sets $\tau^* = \tau_1(\alpha)$ and $\beta = \beta^*$ along the equilibrium path in the third stage. The seller sets prices $(p^I, p^D) = (v_m+b, v_m+b)$. All flexible consumers and the fraction $\frac{1+\beta^*}{2}$ of picky consumers buy in the indirect channel.*

- *For $\alpha \geq \alpha_1^{RS}$, the intermediary sets $\tau^* = 1-c/(v_h+b)$ and $\beta = 0$ along the equilibrium path in the third stage. The seller sets $(p^I, p^D) = (v_h+b, v_m)$. Half of the picky* *consumers buy in the indirect channel and all the flexible consumers buy in the direct channel.*

Moreover, $\alpha_1^{RS} < \alpha_1$.

Proof. Suppose that $\alpha \geq \alpha_0^{RS}$, where α_0^{RS} is defined in the proof of Proposition 6. If imposing a PPC were allowed for the intermediary, by Proposition 6, the intermediary would not make use of this option for $\alpha \geq \alpha_0^{RS}$. Therefore, for $\alpha > \alpha_0^{RS}$, the intermediary induces the inefficient bypass outcome and the equilibrium coincides with the one in Proposition 6.

In the following, we consider $\alpha < \alpha_0^{RS}$. Suppose that the intermediary sets a fee $\tau \leq 1 - c/(v_h + b)$. By Lemma 3, if the seller serves some consumers through the indirect channel, it sets a price $p^I \in \{v_m + b, v_h + b\}$. If the seller sets $p^I = v_h + b$, then the intermediary responds with recommendation policy $\beta = 0$, resulting in the outcome with inefficient bypass. If the intermediary induces the outcome with inefficient bypass, then its maximal profit equals $\alpha/2(v_h + b - c)$. This profit level can be achieved by setting the fee $\tau = 1 - c/(v_h + b)$. The seller earns $(1-\alpha)(v_m - c)$ and is indifferent between any prices greater than $v_m + b$. Regarding tie-breaking, we assumed that when indifferent, the seller sets a price that maximizes the intermediary's profit from the picky consumers — that is, $p^I = v_h + b$ is set along the equilibrium path when $\tau = 1 - c/(v_h + b)$.

If the seller sets $p^I = v_m + b$, then the flexible consumers are served through the indirect channel. The seller's incentive compatibility constraint is given by:

$$\psi(\alpha)[(1-\tau)(v_m + b) - c] \geq \frac{\alpha}{2}[(1-\tau)(v_h + b) - c] + (1-\alpha)(v_m - c).$$

The highest fee that the intermediary can set to induce the outcome with biased recommendations is given by

$$\tau_1(\alpha) = 1 - \frac{(\psi(\alpha) - \alpha/2)c + (1-\alpha)(v_m - c)}{\psi(\alpha)(v_m + b) - \frac{\alpha}{2}(v_h + b)} < \min\{1 - \gamma_1(\alpha), 1 - \gamma_2(\alpha)\} = \tau_0(\alpha),$$

where $\tau_1(\alpha)$ is weakly greater than zero (this holds since $\alpha < \alpha_0^{RS} \leq \alpha_0$). The resulting profit of the intermediary is $\psi(\alpha)\tau_1(\alpha)(v_m + b)$.

The intermediary's profit difference with biased recommendations compared to inefficient bypass is given by:

$$\Delta_1(\alpha) \equiv \psi(\alpha)\tau_1(\alpha)(v_m + b) - \alpha/2(v_h + b - c).$$

Note that $\Delta_1(0) = b > 0$ and

$$\begin{aligned}\Delta_1(\alpha_0^{RS}) &= \tau_1(\alpha_0^{RS})\psi(\alpha_0^{RS})(v_m + b) - \alpha_0^{RS}/2(v_h + b - c)\\ &< \tau_0(\alpha^{RS})\psi(\alpha_0^{RS})(v_m + b) - \alpha_0^{RS}/2(v_h + b - c).\end{aligned}$$

If $\alpha_0^{RS} = \alpha_0$, then $\tau_0(\alpha_0^{RS}) = 1 - \gamma_2(\alpha_0)$, implying that

$$\begin{aligned}\Delta_1(\alpha_0^{RS}) &< (1 - \gamma_2(\alpha_0))\psi(\alpha_0)(v_m + b) - \alpha_0/2(v_h + b - c)\\ &= \psi(\alpha_0)(v_m + b - c) - (1 - \alpha_0)(v_m - c) - \alpha_0/2(v_h + b - c)\\ &= 0.\end{aligned}$$

If instead $\alpha_0^{RS} < \alpha_0$, then $\tau_0(\alpha_0^{RS}) = 1 - \gamma_1(\alpha_0)$, implying that

$$\Delta_1(\alpha_0^{RS}) < \tau_0(\alpha^{RS})\psi(\alpha_0^{RS})(v_m + b) - \alpha_0^{RS}/2(v_h + b - c) = 0.$$

Taking the derivative of $\tau_1(\alpha)$ with respect to α, we obtain

$$\begin{aligned}\tau_1'(\alpha) &= \frac{((1/2 - \psi'(\alpha))c + (v_m - c)) + (\psi'(\alpha)(v_m + b) - 1/2(v_h + b))\,(1 - \tau_1(\alpha))}{\psi(\alpha)(v_m + b) - \frac{\alpha}{2}(v_h + b)}\\ &< \frac{1 - \tau_1(\alpha)}{\psi(\alpha)(v_m + b) - \frac{\alpha}{2}(v_h + b)}[(v_m - c) + \psi'(\alpha)(v_m + b - c) - 1/2(v_h + b - c)]\\ &= -\frac{1 - \tau_1(\alpha)}{\psi(\alpha)(v_m + b) - \frac{\alpha}{2}(v_h + b)}\left[b + \frac{1}{2}(v_h + b - c - (1 + \beta^*)(v_m + b - c))\right]\\ &= -\frac{1 - \tau_1(\alpha)}{\psi(\alpha)(v_m + b) - \frac{\alpha}{2}(v_h + b)}\frac{b}{\alpha_0} < 0,\end{aligned}$$

where we used that $\psi'(\alpha) = (1 + \beta^*)/2 - 1 < 0$ to obtain the second and the third lines, and equation (2) to obtain the final expression. This implies that $\tau_1(\alpha)$ is decreasing in α. Thus, $\Delta_1(\alpha)$ is also strictly decreasing in α and changes its sign on $(0, \alpha_0^{RS})$. We conclude that there exists a unique $\alpha_1^{RS} < \alpha_0^{RS}$ such that for $\alpha < \alpha_1^{RS}$, the intermediary sets $\tau^* = \tau_1(\alpha)$ and induces the outcome with biased recommendations. For $\alpha \geq \alpha_1^{RS}$, the intermediary sets $\tau^* = 1 - c/(v_h + b)$ and induces the outcome with inefficient bypass.

It remains to show that $\alpha_1^{RS} < \alpha_1$. Recall that α_1 solves equation (5), where $\lambda_1(\alpha)$ is given in equation (4). Rearranging equations (14) and (4), we have that

$$\begin{aligned}\tau_1(\alpha)\left(\psi(\alpha)(v_m + b) - \frac{\alpha}{2}(v_h + b)\right) &= \psi(\alpha)(v_m + b - c) - \frac{\alpha}{2}(v_h + b - c) - (1 - \alpha)(v_m - c)\\ &= \lambda_1(\alpha)\left(\psi(\alpha)(v_m + b - c) - \frac{\alpha}{2}(v_h + b - c)\right).\end{aligned}$$

Solving for $\alpha_1/2(v_h+b-c)$ from equation (5) and plugging it into equation (14), we have

$$\begin{aligned}\Delta_1(\alpha_1) &= \psi(\alpha_1)\tau_1(\alpha_1)(v_m+b) - \frac{\alpha_1}{2}(v_h+b-c)\\ &= \psi(\alpha_1)\tau_1(\alpha_1)(v_m+b) - \psi(\alpha)\lambda_1(\alpha_1)(v_m+b-c),\end{aligned}$$

Plugging in the expression for $\lambda_1(\alpha)$, we obtain that

$$\begin{aligned}\Delta_1(\alpha_1) &= \psi(\alpha_1)\tau_1(\alpha_1)\left(v_m+b-\frac{\psi(\alpha_1)(v_m+b)-\alpha_1/2(v_h+b)}{\psi(\alpha_1)(v_m+b-c)-\alpha_1/2(v_h+b-c)}(v_m+b-c)\right)\\ &= -\frac{\psi(\alpha_1)\tau_1(\alpha_1)\alpha/2(v_h-v_m)c}{\psi(\alpha_1)(v_m+b-c)-\alpha_1/2(v_h+b-c)}\\ &< 0.\end{aligned}$$

Since $\Delta_1(\alpha)$ is strictly decreasing in α on $(0,\alpha_0)$ and $\Delta_1(\alpha_1^{RS})=0$, we have that $\alpha_1^{RS}<\alpha_1$.

□

Next, we consider the regulation that prohibits total-surplus-decreasing purchase recommendations.

Define $\psi(\alpha)\equiv\alpha/2(1+\beta^*)+1-\alpha$,

$$\tilde{\gamma}_1(\alpha)\equiv\frac{(1-\alpha)c}{(1-\alpha)(v_m+b)-\alpha/2(v_h-v_m)} \quad \text{and} \quad \tilde{\gamma}_2(\alpha)\equiv\frac{(1-\alpha)v_m+\alpha/2c}{(\alpha/2+1-\alpha)(v_m+b)}=\tilde{\gamma}_2(\alpha).$$

We also define

$$\tau_2(\alpha)\equiv 1-\max\left\{\tilde{\gamma}_1(\alpha),\tilde{\gamma}_2(\alpha)\right\}. \tag{15}$$

We show that α_2^{RS} is uniquely-defined and strictly smaller than α_0^{RS} and α_2 in the following proposition.

Proposition 8. *The equilibrium when the intermediary sets a commission fee on revenues and the regulator prohibits biased recommendations is characterized as follows: There exists a threshold α_2^{RS} with $\alpha_2^{RS}\leq\max\{\alpha_0^{RS},\alpha_2\}$, such that*

- *for $\alpha<\alpha_2^{RS}$, the intermediary imposes a PPC, sets $\tau^*=\tau_2(\alpha)$, and the first-best outcome is implemented.*
- *For $\alpha\geq\alpha_2^{RS}$, the intermediary does not impose a PPC and sets $\tau^*=1-c/(v_h+b)$; equilibrium prices are given by $(p^I,p^D)=(v_h+b,v_m)$ and the inefficient bypass outcome is implemented.*

Moreover, there exist a threshold $\bar{c}_2 \in ((v_h + v_l)/2, v_m)$ *and a function* $\bar{b}_2(c) \in (0, c - (v_h + v_l)/2)$*, such that* $\alpha_2^{RS} < \alpha_2$ *if and only if* $c > \bar{c}_2$ *and* $b > \bar{b}_2(c)$*. Otherwise,* $\alpha_2^{RS} = \alpha_2$*.*

Proof. Suppose that $\alpha \geq \alpha_2$. By Lemma 2, the vertically-integrated firm restricted to providing no biased recommendations induces the outcome with inefficient bypass. Thus, for $\alpha \geq \alpha_2$, it is optimal for the intermediary to implement the outcome with inefficient bypass by setting $\tau = 1 - c/(v_h + b)$ and imposing no PPCs.

Next, suppose that $\alpha < \alpha_2$. By Lemma 3, if the seller serves some consumers in the indirect channel, then it either sets $p^I = v_m + b$ resulting in the first-best outcome or $p^I = v_h + b$, which leads to the outcome with inefficient bypass. If a PPC is imposed, then the seller prefers to set $p^I = v_m + b$ and serve flexible consumers and the picky consumers with a good match through the indirect channel if

$$\left(\frac{\alpha}{2} + 1 - \alpha\right)((1-\tau)(v_m + b) - c) \geq \max\left\{\frac{\alpha}{2}((1-\tau)(v_h + b) - c), (1-\alpha)(v_m - c)\right\}.$$

If a PPC is not imposed, then the IC-constraint of the seller inducing the first-best outcome is given by

$$\left(\frac{\alpha}{2} + 1 - \alpha\right)((1-\tau)(v_m + b) - c) \geq \frac{\alpha}{2}((1-\tau)(v_h + b) - c) + (1-\alpha)(v_m - c).$$

Comparing the seller's incentives compatibility constraints with and without a PPC, we obtain that for any τ at which the seller prefers to sell only through the indirect channel under a PPC, it will also prefer this if a PPC is not imposed. Therefore, the intermediary inducing the first-best outcome earns weakly greater profits by imposing a PPC.

Rearranging the incentive compatibility constraint of the seller, we have that the intermediary inducing the first-best outcome sets the highest τ that satisfies the following constraints:[25]

$$1-\tau \geq \frac{(1-\alpha)c}{(1-\alpha)(v_m+b)-\alpha/2(v_h-v_m)} = \tilde{\gamma}_1(\alpha),$$
$$1-\tau \geq \frac{(1-\alpha)v_m+\alpha/2c}{(\alpha/2+1-\alpha)(v_m+b)} = \tilde{\gamma}_2(\alpha).$$

We explore the sign of $\tilde{\gamma}_2(\alpha)-\tilde{\gamma}_1(\alpha)$. The derivative of $\tilde{\gamma}_1(\alpha)$ with respect to α is given by

$$\tilde{\gamma}_1'(\alpha) = \frac{c(v_h-v_m)/2}{((1-\alpha)(v_m+b)-\alpha/2(v_h-v_m))^2} > 0,$$

and the derivative of $\tilde{\gamma}_2$ with respect to α is given by

$$\tilde{\gamma}_2'(\alpha) = -\frac{1/2}{(1-\alpha/2)^2}\frac{v_m-c}{v_m+b} < 0.$$

It follows that $\tilde{\delta}(\alpha) \equiv \tilde{\gamma}_2(\alpha)-\tilde{\gamma}_1(\alpha)$ is strictly decreasing in α on $(0,\alpha_2]$. Moreover, note that $\tilde{\delta}(0) = (v_m-c)/(v_m+b) > 0$, implying that either *i)* $\tilde{\gamma}_2(\alpha) \geq \tilde{\gamma}_1(\alpha)$ for all $\alpha < \alpha_2$ or *ii)* there exists a unique $\hat{\alpha} < \alpha_2$ such that $\tilde{\gamma}_2(\alpha) \geq \tilde{\gamma}_1(\alpha)$ for any $\alpha \in (0,\hat{\alpha}]$ and $\tilde{\gamma}_1(\alpha) > \tilde{\gamma}_2(\alpha)$ for any $\alpha \in (\hat{\alpha},\alpha_2)$. Note that $\tilde{\delta}(\alpha) \geq 0$ for any $\alpha \in (0,\alpha_2)$ if and only if $\tilde{\delta}(\alpha_2) \geq 0$.

Next, we characterize the necessary and sufficient conditions under which $\tilde{\delta}(\alpha_2) \geq 0$. Rearranging terms and applying the identity

$$(1-\alpha_2/2)(v_m+b-c) = \alpha_2/2(v_h+b-c)+(1-\alpha_2)(v_m-c),$$

we obtain:

$$\begin{aligned}
\tilde{\delta}(\alpha_2) &= 1-\tilde{\gamma}_1(\alpha_2)-(1-\tilde{\gamma}_2(\alpha_2)) \\
&= \frac{(1-\alpha_2/2)(v_m+b-c)-\alpha_2/2(v_h+b-c)}{(1-\alpha_2/2)(v_m+b)-\alpha_2/2(v_h+b)} - \frac{(1-\alpha_2/2)(v_m+b-c)-(1-\alpha_2)(v_m-c)}{(1-\alpha_2/2)(v_m+b)} \\
&= \frac{v_m-c}{v_m} - \frac{\alpha_2/2}{1-\alpha_2/2}\frac{v_h+b-c}{v_m+b} \\
&= \frac{v_m-c}{v_m} - \frac{b}{v_m+b}\frac{v_h+b-c}{v_h+b-v_m}.
\end{aligned}$$

[25] In the proof of Lemma 3, we showed that $\alpha < \alpha_2$ if and only if $(\alpha/2+1-\alpha)(v_m+b-c) > \alpha/2(v_h+b-c)+(1-\alpha)(v_m-c)$. It follows that $(1-\alpha)(v_m+b)-\frac{\alpha}{2}(v_h-v_m) > (1-\alpha)v_m > (1-\alpha)c$, which guarantees that $\tilde{\gamma}_1(\alpha) \in (0,1)$ for any $\alpha < \alpha_2$. Moreover, it follows that

$$\left(\frac{\alpha}{2}+1-\alpha\right)(v_m+b) > \frac{\alpha}{2}(v_h+b)+(1-\alpha)v_m > \frac{\alpha}{2}c+(1-\alpha)v_m,$$

implying that $\tilde{\gamma}_2(\alpha) \in (0,1)$ for any $\alpha < \alpha_2$.

The derivative of $\tilde{\delta}(\alpha_2)$ with respect to b is given by

$$\frac{\partial\tilde{\delta}(\alpha_2)}{\partial b} = -\frac{(v_h+b-c+b)(v_m+b)(v_h+b-v_m)-b(v_h+b-c)(v_m+b+v_h+b-v_m)}{((v_m+b)(v_h+b-v_m))^2}$$
$$= \frac{b(v_m+b)(v_m-c)-v_m(v_h+b-c)(v_h+b-v_m)}{((v_m+b)(v_h+b-v_m))^2}.$$

The derivative of the numerator with respect to b is given by

$$(v_m+2b)(v_m-c)-v_m(2v_h+2b-v_m-c) = -2bc-2v_m(v_h-v_m) < 0,$$

implying that $\tilde{\delta}(\alpha_0)$ is strictly decreasing in b. Note that $\tilde{\delta}(\alpha_2)$ is positive at $b=0$. Next, we explore the sign of $\tilde{\delta}(\alpha_2)$ at $b=c-(v_h+v_l)/2$. Define $\tilde{z}(c)$ as $\tilde{\delta}(\alpha_2)$ evaluated at $b=c-(v_h+v_l)/2$; that is,

$$\tilde{z}(c) = \frac{v_m-c}{v_m} - \frac{c-(v_h+v_l)/2}{v_m+c-(v_h+v_l)/2}\frac{(v_h-v_l)/2}{(v_h-v_l)/2-(v_m-c)}$$

Note that $\tilde{z}((v_h+v_l)/2)$ is positive and $z(v_m)$ is negative. Moreover, the derivative of $\tilde{z}(c)$ with respect to c is given by

$$\tilde{z}'(c) = \left(\frac{\partial\tilde{\delta}(\alpha_2)}{\partial b}+\frac{\partial\tilde{\delta}(\alpha_2)}{\partial c}\right)\Big|_{b=c-(v_h+v_l)/2} < 0,$$

since $\partial\tilde{\delta}(\alpha_2)/\partial b < 0$ (it was shown above) and

$$\frac{\partial\tilde{\delta}(\alpha_2)}{\partial c} = -\frac{b^2+(v_m+b)(v_h-v_m)}{(v_m+b)(v_h+b-v_m)} < 0.$$

Thus, there exists a unique $\bar{c}_2 \in (v_h+v_l)/2, v_m$ such that $\tilde{\delta}(\alpha_2)$ evaluated at $b=c-(v_h+v_l)/2$ is non-negative if $c \le \bar{c}_2$ and is negative, otherwise. By the intermediate value theorem and the fact that $\tilde{\delta}(\alpha_2)$ is strictly decreasing in b, we have that $\tilde{\delta}(\alpha_2) \ge 0$ for any $c \le \bar{c}$ and $b \in (0, c-(v_h+v_l)/2)$. If instead, $c > \bar{c}_2$, then there exists a function $\bar{b}_2(c) \in (0, c-(v_h+v_l)/2)$ such that $\tilde{\delta}(\alpha_2)$ if and only if $b \le \bar{b}_2(c)$. If $c > \bar{c}_2$ and $b > \bar{b}_2(c)$, we have that $\delta(\alpha_2) < 0$.

We are in the position to characterize the equilibrium for $\alpha < \alpha_2$. First, suppose that either $c \le \bar{c}_2$ or $c > \bar{c}$ and $b \le \bar{b}_2(c)$. We established that in this parameter range, $\tilde{\gamma}_2(\alpha) \ge \tilde{\gamma}_1(\alpha)$ for any $\alpha < \alpha_2$. The intermediary setting a commission fee $\tau = 1-\tilde{\gamma}_2(\alpha)$ and imposing a PPC can induce the first-best outcome, in which the seller earns $(1-\alpha)(v_m-c)$. Therefore, the intermediary can decentralize the solution of the vertically-integrated firm for any $\alpha < \alpha_2$ (see Lemma 2). Consequently, $\alpha_2^{RS} = \alpha_2$.

Second, suppose that $c > \bar{c}_2$ and $b \in (\bar{b}_2, c - (v_h + v_l)/2)$. In this parameter range, we have that $\tilde{\delta}(\alpha_2) < 0$, implying that there exists $\hat{\alpha} < \alpha_2$ such that $\tilde{\gamma}_2(\alpha) \geq \tilde{\gamma}_1(\alpha)$ for any $\alpha \leq \hat{\alpha}$ and $\tilde{\gamma}_1(\alpha) > \tilde{\gamma}_2(\alpha)$ for any $\alpha \in (\hat{\alpha}, \alpha_2)$. If $\alpha \leq \hat{\alpha}$, the intermediary can decentralize the first-best outcome by setting $\tau = 1 - \tilde{\gamma}_2(\alpha)$ and imposing a PPC. If $\alpha \in (\hat{\alpha}, \alpha_2)$, the intermediary can earn $\alpha/2(v_h + b - c)$ by inducing the outcome with inefficient bypass or $(1 - \tilde{\gamma}_1(\alpha))(1 - \alpha/2)(v_m + b)$ by inducing the first-best outcome. Consider the intermediary's profit difference in the first best compared to inefficient bypass:

$$\Delta_2(\alpha) \equiv (1 - \tilde{\gamma}_1(\alpha))(1 - \alpha/2)(v_m + b) - \alpha/2(v_h + b - c).$$

Since the intermediary can decentralize the vertically-integrated solution at $\alpha = \hat{\alpha} < \alpha_2$, we have that $\Delta_2(\hat{\alpha}) > 0$. Moreover,

$$\begin{aligned}
\Delta_2(\alpha_2) &= (1 - \tilde{\gamma}_1(\alpha))(1 - \alpha/2)(v_m + b) - \alpha_2/2(v_h + b - c) \\
&< (1 - \tilde{\gamma}_2(\alpha))(1 - \alpha/2)(v_m + b) - \alpha_0/2(v_h + b - c) \\
&= (1 - \alpha_2/2)(v_m + b - c) - (1 - \alpha_2)(v_m - c) - \alpha_2/2(v_h + b - c) \\
&= 0.
\end{aligned}$$

Since $\tilde{\gamma}_1(\alpha)$ is strictly increasing in α, we have that $\Delta_2(\alpha)$ is strictly decreasing in α, implying that there exists a unique $\alpha_2^{RS} < \alpha_2$ such that for any $\alpha < \alpha_2^{RS}$ the intermediary sets $\tau = 1 - \gamma_1(\alpha)$, imposes a PPC and induces the first-best outcome. For $\alpha \geq$, the intermediary sets $\tau = 1 - c/(v_h + b)$, imposes no PPCs and induces the outcome with inefficient bypass.

It remains to show that $\alpha_2^{RS} \leq \alpha_0^{RS}$. In Proposition 6, we showed that for $\alpha \geq \alpha_0^{RS}$, the intermediary facing no restrictions on biased recommendations sets $\tau = 1 - c/(v_h + b)$, imposes no PPCs and induces the outcome with inefficient bypass (which satisfies the prohibition of total surplus decreasing recommendations). Therefore, $\alpha_2^{RS} \leq \alpha_0^{RS}$.

□

Finally, we explore the regulation that prohibits PPCs and total-surplus-decreasing product recommendations. We define

$$\tau_3(\alpha) \equiv 1 - \frac{(1 - \alpha)v_m}{\left(1 - \frac{\alpha}{2}\right)(v_m + b) - \frac{\alpha}{2}(v_h + b)}. \tag{16}$$

We also define α_3^{RS} as the solution to the following equation:

$$\left(1-\frac{\alpha}{2}\right)\tau_3(\alpha)(v_m+b)-\alpha/2(v_h+b-c)=0.$$

We show that α_3^{RS} is uniquely-defined and is strictly smaller than α_3 in the following proposition.

Proposition 9. *Under the regulation that prohibits the use of PPCs and biased recommendations, the equilibrium when the intermediary sets an ad valorem fee on revenues in the indirect channel is characterized as follows: There exists a threshold α_3^{RS} with $\alpha_3^{RS} < \max\{\alpha_2^{RS}, \alpha_3\}$, such that*

- *for $\alpha < \alpha_3^{RS}$, the intermediary sets $\tau^* = \tau_3(\alpha)$ and the first-best outcome is implemented.*
- *For $\alpha \geq \alpha_3^{RS}$, the intermediary sets $\tau^* = 1 - c/(v_h + b)$, equilibrium prices are given by $(p^I, p^D) = (v_h + b, v_m)$, and the inefficient bypass outcome is implemented.*

Proof. Suppose that $\alpha \geq \alpha_2^{RS}$. By Proposition 8, the intermediary that is allowed to use price parity, finds it optimal to induce the inefficiency bypass outcome that does not require either biased recommendations or using price parity clauses. Thus, for $\alpha \geq \alpha_2^{RS}$, we have that the equilibrium coincides with the one described in Proposition 8.

In the following, we consider $\alpha < \alpha_2^{RS}$. Suppose that the intermediary sets a commission fee on revenues $\tau \leq 1 - c/(v_h + b)$. By Lemma 3, the seller sets a price $p^I \in \{v_m + b, v_h + b\}$ in the indirect channel. The seller sets $p^I = v_m + b$ and induces the first-best outcome if

$$\left(1-\frac{\alpha}{2}\right)[(1-\tau)(v_m+b)-c] \geq \frac{\alpha}{2}[(1-\tau)(v_h+b)-c]+(1-\alpha)(v_m-c).$$

Then, the highest fee that the intermediary can set is given by

$$\tau_3(\alpha)=1-\frac{(1-\alpha)v_m}{\left(1-\frac{\alpha}{2}\right)(v_m+b)-\frac{\alpha}{2}(v_h+b)} < \min\{1-\tilde{\gamma}_1(\alpha), 1-\tilde{\gamma}_2(\alpha)\}=\tau_2(\alpha),$$

where $\tau_3(\alpha)$ is also weakly greater than zero, since $\alpha < \alpha_2^{RS} \leq \alpha_2$. The resulting profit of the intermediary is $(1-\alpha/2)\tau_3(\alpha)(v_m+b)$.

The intermediary's profit difference in the first best compared to inefficient bypass outcomes is given by:

$$\Delta_3(\alpha) \equiv \left(1 - \frac{\alpha}{2}\right) \tau_3(\alpha)(v_m + b) - \frac{\alpha}{2}(v_h + b - c).$$

Note that $\Delta_3(0) = b > 0$. Next, we show that $\Delta_3(\alpha_2^{RS}) < 0$. Since $\tau_3(\alpha) < \tau_2(\alpha)$, we have that

$$\begin{aligned}\Delta_3(\alpha_2^{RS}) &= \left(1 - \frac{\alpha_2^{RS}}{2}\right) \tau_3(\alpha_2^{RS})(v_m + b) - \alpha_2^{RS}/2(v_h + b - c)\\ &< \left(1 - \frac{\alpha_2^{RS}}{2}\right) \tau_2(\alpha_2^{RS})(v_m + b) - \alpha_2^{RS}/2(v_h + b - c).\end{aligned}$$

In the proof of Proposition 8, we showed that if $\alpha_2^{RS} = \alpha_2$, defined in equation (11), then $\tau_2(\alpha_2^{RS}) = 1 - \tilde{\gamma}_2(\alpha_2)$, implying that

$$\begin{aligned}\Delta_3(\alpha_2^{RS}) &< \left(1 - \frac{\alpha_2}{2}\right) (1 - \tilde{\gamma}_2(\alpha_2))(v_m + b) - \alpha_2/2(v_h + b - c)\\ &= \left(1 - \frac{\alpha_2}{2}\right) (v_m + b - c) - (1 - \alpha_2)(v_m - c) - \alpha_2/2(v_h + b - c)\\ &= 0.\end{aligned}$$

If instead $\alpha_2^{RS} < \alpha_2$, then $\tau_2(\alpha_2^{RS}) = 1 - \tilde{\gamma}_1(\alpha_2)$, implying that

$$\Delta_3(\alpha_2^{RS}) < \left(1 - \frac{\alpha_2^{RS}}{2}\right) (1 - \tilde{\gamma}_1(\alpha_2))(\alpha_2^{RS})(v_m + b) - \alpha_2^{RS}/2(v_h + b - c) = 0.$$

The derivative of $\tau_3(\alpha)$ with respect to α is given by

$$\begin{aligned}\tau_3'(\alpha) &= \frac{v_m \left(\left(1 - \frac{\alpha}{2}\right)(v_m + b) - \frac{\alpha}{2}(v_h + b) - \frac{1-\alpha}{2}(v_m + b + v_h + b)\right)}{\left(\left(1 - \frac{\alpha}{2}\right)(v_m + b) - \frac{\alpha}{2}(v_h + b)\right)^2}\\ &= -\frac{(v_h - v_m)/2}{\left(\left(1 - \frac{\alpha}{2}\right)(v_m + b) - \frac{\alpha}{2}(v_h + b)\right)^2}\\ &< 0.\end{aligned}$$

It follows that $\Delta_3(\alpha)$ is strictly decreasing in α on $(0, \alpha_2^{RS})$ and changes its sign. Thus, there exists a unique $\alpha_3^{RS} < \alpha_2^{RS}$ such that for $\alpha < \alpha_3^{RS}$, the intermediary induces the first-best outcome and for $\alpha \geq \alpha_3^{RS}$, the intermediary implements the outcome with inefficient bypass.

Next, we show that $\alpha_3^{RS} < \alpha_3$. Rearranging equations (16), we have that

$$\begin{aligned}
&\tau_3(\alpha)\left(\left(1-\frac{\alpha}{2}\right)(v_m+b)-\frac{\alpha}{2}(v_h+b)\right)\\
&\quad=\left(1-\frac{\alpha}{2}\right)(v_m+b-c)-\frac{\alpha}{2}(v_h+b-c)-(1-\alpha)(v_m-c)\\
&\quad=\lambda_3(\alpha)\left(\left(1-\frac{\alpha}{2}\right)(v_m+b-c)-\frac{\alpha}{2}(v_h+b-c)\right),
\end{aligned}$$

where we used equation (7) to obtain the third line. In the proof of Proposition 4, we established that α_3 solves equation

$$\lambda_3(\alpha)\left(1-\frac{\alpha}{2}\right)(v_m+b-c)-\frac{\alpha}{2}(v_h+b-c)=0,$$

where $\lambda_3(\alpha)$ is given in equation (7). Therefore,

$$\begin{aligned}
\Delta_3(\alpha_3)&=\left(1-\frac{\alpha_3}{2}\right)\tau_3(\alpha_3)(v_m+b)-\frac{\alpha_3}{2}(v_h+b-c)\\
&=\left(1-\frac{\alpha_3}{2}\right)\tau_3(\alpha_3)(v_m+b)-\lambda_3(\alpha_3)\left(1-\frac{\alpha_3}{2}\right)(v_m+b-c).
\end{aligned}$$

Plugging in the expression for $\lambda_3(\alpha)$ from above, we obtain that

$$\begin{aligned}
&\Delta_3(\alpha_3)=\left(1-\frac{\alpha_3}{2}\right)\tau_3(\alpha_3)\left(v_m+b-\frac{(1-\alpha_3/2)(v_m+b)-\alpha_3/2(v_h+b)}{(1-\alpha_3/2)(v_m+b-c)-\alpha_3/2(v_h+b-c)}(v_m+b-c)\right)\\
&\quad=-\frac{(1-\alpha_3/2)\tau_3(\alpha_3)\alpha_3/2(v_h-v_m)c}{(1-\alpha_3/2)(v_m+b-c)-\alpha_3/2(v_h+b-c)}\\
&\quad<0.
\end{aligned}$$

Since $\Delta_3(\alpha)$ is strictly decreasing in α on $(0,\alpha_2)$ and $\Delta(\alpha_3^{RS})=0$, we have that $\alpha_3^{RS} < \alpha_3$.

□

The following proposition summarizes the ordering of critical values of α in the model, where the intermediary sets an ad valorem fee on revenues in the indirect channel.

Proposition 10. *The critical values of α satisfy the following inequalities:*

$$0<\alpha_3^{RS}<\min\{\alpha_1^{RS},\alpha_2^{RS}\}\le\max\{\alpha_1^{RS},\alpha_2^{RS}\}\le\alpha_0^{RS}<1.$$

Proof. In Propositions 7 and 8, we showed that $\alpha_1^{RS} < \alpha_0^{RS}$ and $\alpha_2^{RS} \le \alpha_0^{RS}$. In Proposition 2, we showed that $\alpha_3^{RS} < \alpha_2^{RS}$. It remains to show that $\alpha_3^{RS} \le \alpha_1^{RS}$.

Since $\psi(\alpha) = \alpha(1+\beta^*)/2+1-\alpha > 1-\alpha/2$ and $\tau_1(\alpha)$ (equation (14)) is strictly increasing in $\psi(\alpha)$, we have that $\tau_1(\alpha) > \tau_3(\alpha)$ on $(0, \alpha_3)$. Therefore,

$$\left(\frac{\alpha_3^{RS}(1+\beta^*)}{2} + 1 - \alpha_3^{RS}\right)\tau_1(\alpha_3^{RS})(v_m + b) - \alpha_3^{RS}/2(v_h + b - c)$$
$$> \left(\frac{\alpha_3^{RS}}{2} + 1 - \alpha_3^{RS}\right)\tau_3(\alpha_3^{RS})(v_m + b) - \alpha_3^{RS}/2(v_h + b - c) = 0,$$

implying that at $\alpha = \alpha_3^{RS}$, the intermediary that is prohibited from using PPCs but can use biased recommendation induces the outcome with inflated recommendations. Thus, by Proposition 7, we have that $\alpha_3^{RS} < \alpha_1^{RS}$. □

D Competing intermediaries

In this appendix, we consider two competing intermediaries, A and B, in the absence of a direct channel. Intermediary A provides the convenience benefits $b > 0$ and possibly informative product recommendations, whereas intermediary B is less efficient and does not provide either of those services.

The intermediaries A and B are assumed to move sequentially by setting profit-sharing rates λ and μ, respectively, and deciding whether to impose wide PPCs. Then, the seller observes the fees and sets prices p^m in the indirect channel of m if it considers selling through intermediary m, $m \in \{A, B\}$. From then on, the game evolves as in the base model.

The following proposition characterizes the equilibrium for the regimes 0–3.

Proposition 11. *In the model with two competing intermediaries and no direct channel, the equilibrium outcomes under laissez-faire (regime 0) and the regulatory interventions (regimes 1-3) considered in Sections 3 and 4.1-4.3, respectively, are characterized as follows:*

- *for $\alpha < \alpha_i$, where $\alpha_0, \ldots, \alpha_3$ are the critical values of α from Propositions 1-4, the more efficient intermediary imposes a PPC (if allowed), sets $\lambda^* = \lambda_i(\alpha)$ and $\beta = \beta^*$ (if not ruled out by regulation and $\beta^* = 0$, otherwise) along the equilibrium path in the third stage. In response, the less efficient intermediary sets $\mu = 0$. The seller sets prices $(p^A, p^B) = (v_m + b, v_m + b)$. All flexible consumers and the fraction $(1+\beta^*)/2$ of picky consumers buy through the more efficient intermediary.*

- *For $\alpha \geq \alpha_i$, the more efficient intermediary does not impose a PPC, sets $\lambda^* = 1$ and $\beta = 0$ along the equilibrium path. In response, the less efficient intermediary sets $\mu = 1$. The seller sets prices $(p^A, p^B) = (v_h + b, v_m)$ and the inefficient bypass outcome is implemented.*

Proof. By contradiction, assume that the flexible consumers do not buy in equilibrium. Then, the seller serves picky consumers with a good match at price $p^A = v_h + b$ through the more efficient intermediary I_1. We note that the highest fee that the more efficient intermediary can set is strictly below 1, as otherwise, the inefficient intermediary would earn positive profits by accommodating sales to the flexible consumers resulting in a contradiction. For any $\lambda < 1$, the more efficient intermediary can deviate by setting $\lambda = 1$ and imposing no PPC, extracting all surplus from sales to the picky consumers with a good match. Since this is a profitable deviation, the flexible consumers must buy in any equilibrium. Note that this argument holds under laisezz-faire and regulatory interventions 1-3.

Suppose that the flexible consumers buy through the more efficient intermediary in equilibrium. Competition between intermediaries drives down the fee of the less efficient intermediary to $\mu = 0$, resulting in zero profits for B. The highest fee that the efficient intermediary can set in such an equilibrium is $\lambda_i(\alpha)$ (inducing either the biased recommendation or the first-best outcomes). If instead it set $\lambda > \lambda_i(\alpha)$, then the IC-constraints of the seller as reported in the proofs of Propositions 1-4 would be violated. This implies that the less efficient intermediary would profitably attract the seller to serve the flexible consumers through the indirect channel of B. Thus, the most profitable deviation of intermediary A is to set $\lambda = 1$ and impose no PPCs. This yields profits of $\alpha/2(v_h + b - c)$ for intermediary A. Therefore, if there is no regulation against biased recommendations, the biased recommendation outcome constitutes an equilibrium if and only if

$$\lambda_i(\alpha)\left(\frac{\alpha(1+\beta^*)}{2} + 1 - \alpha\right)(v_m + b - c) \geq \frac{\alpha}{2}(v_h + b - c),$$

which is equivalent to $\alpha < \alpha_i$ for $i \in \{0, 1\}$. If instead the total-surplus-decreasing recommendations are forbidden, the first-best outcome constitutes an equilibrium if and only if

$$\lambda_i(\alpha)\left(1 - \frac{\alpha}{2}\right)(v_m + b - c) \geq \frac{\alpha}{2}(v_h + b - c),$$

which is equivalent to $\alpha < \alpha_i$ for $i \in \{2, 3\}$. Thus, for $\alpha < \alpha_i$, $i \in \{0, 1, 2, 3\}$, the equilibrium is characterized by the biased recommendations outcome (the first-best outcome in case of the regulation against biased recommendations), in which the two intermediaries set $\lambda = \lambda_i(\alpha)$ and $\mu = 0$, respectively.

Next, suppose that the flexible consumers buy through the less efficient intermediary in equilibrium. Following the argument above, it must be that $\lambda > \lambda_i(\alpha)$; otherwise, the seller would serve the flexible consumers through the more efficient intermediary. For any $\lambda \in (\lambda_i(\alpha), 1)$, the efficient seller has a profitable deviation by setting $\lambda = 1$ and imposing no PPCs. Thus, in equilibrium, the more efficient seller sets $\lambda = 1$, imposes no PPC, and earns profits from accommodating sales to picky consumers with a good match. In response, the less efficient seller sets $\mu = 1$. The seller induces the inefficient bypass outcome and earns zero profits.

Consider the more efficient intermediary's deviation to $\lambda = \lambda_i(\alpha)$, which induces the seller to serve the flexible consumers through the indirect channel of A. Then, the less efficient intermediary responds by setting $\mu = 0$. Thus, because of the IC constraints of the more efficient intermediary such a deviation is unprofitable if and only if $\alpha \geq \alpha_i$ for regime $i \in \{0, 1, 2, 3\}$. We conclude that for $\alpha \geq \alpha_i$, the equilibrium regime $i \in \{0, 1, 2, 3\}$ is characterized by the inefficient bypass outcome, in which intermediaries set $\lambda = 1$ and $\mu = 1$.

□

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