FRICTIONAL CAPITAL REALLOCATION*

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February 8, 2023

Abstract

This paper studies economies where firms acquire capital in primary markets, then, after idiosyncratic productivity shocks, retrade it in secondary markets incorporating bilateral trade with search, bargaining and liquidity frictions. We distinguish carefully between full or partial sales (one firm gets all or some of the other's capital), and document several long- and short-run empirical patterns between these variables and the cost of liquidity measured by inflation. Quantitatively, the model can match these patterns plus the facts deemed important in business cycle theory. We also investigate the impact of search frictions, monetary and fiscal policy and persistence in firm-specific shocks.

JEL classification nos: E22, E44

Key words: Capital, Investment, Reallocation, Liquidity

*For input we thank Phil Coyle, Jonathan Chiu, Andrea Eisfeld, Chao Gu, Pablo Ottonello, Peter Ruppert, Venky Venkateswaran, the referees and editor. We also thank participants in presentations at the 2022 SED Meeting, 2021 SAMMF online workshop, 2021 Mad Money Conference, 2021 Search and Matching European Meeting, Atlanta Fed, University of Surrey, Wenzhou-Kean University, University of Wisconsin, EIEF, Queen Mary, University of London, HKUST, and UCL. Cui acknowledges generous support (under research grant number ES/R007667/1) from the Economic and Social Research Council (ESRC); Wright acknowledges support from the Ray Zemon Chair in Liquid Assets and the Kenneth Burdett Professorship in Search Theory and Applications at the Wisconsin School of Business. The usual disclaimers apply. Email addresses: w.cui@ucl.ac.uk; randall.wright@wisc.edu; zhuyuzlf57@gmail.com.

1 Introduction

This paper study economies where firms first acquire capital in centralized primary markets, as in standard growth theory, then, after idiosyncratic productivity shocks, retrade it in decentralized secondary markets. In the interest of realism and generality, we propose a framework with secondary markets that can incorporate bilateral trade with search, matching, bargaining and liquidity frictions. A novel feature is that we distinguish explicitly between full sales, where the buyer gets all the seller's capital, and partial sales, where the buyer gets only some. Under CRS (constant returns to scale) it is socially efficient for firms with higher productivity to get all the capital in bilateral trade, but that may not happen in equilibrium, depending on financial considerations.

We first document a few facts about full and partial sales. Over the business cycle, the ratio of full sales to total capital expenditure (new investment plus reallocation) is procyclical, while the ratio of partial sales to total capital expenditure is countercyclical. In the longer run, the ratio of full sales to total capital expenditure has increased and the ratio of partial sales to total capital expenditure has decreased. Given that 42% of full sales are facilitated by cash or cash-equivalent payments (Thomson Reuters M&A Database 1971-2018), we examine the relationship between reallocation and the cost of liquidity, measured by inflation, as discussed below. In the longer run, full sales decrease while partial sales increase with inflation, while at cyclic frequencies the pattern is reversed.

In our theory, high inflation raises the cost and lowers the amount of liquidity, decreasing total reallocation and full sales, and increasing partial sales, consistent with the long-run facts. Then to get full sales increasing and partial sales decreasing with inflation at business cycle frequencies, we consider credit shocks. Easier credit increases total reallocation and full sales, decreases partial sales, and reduces the demand for money leading to a short-term jump in inflation. So with credit shocks at business cycle frequencies, total reallocation is procyclical and moves with inflation, while partial sales are countercyclical. The idea is not that credit shocks are necessarily more transitory, but that they affect the price level, implying a change in short run but not trend inflation.

While a main goal is to show the calibrated model is consistent with all these facts in the tradition of the RBC (real business cycle) literature, it can also be used to study monetary and fiscal policy. We solve for the optimal capital tax/subsidy and inflation rate, which depend on details like bargaining power not usually considered in related studies. In particular, inflation may have a nonmonotone impact on investment, output and other macro variables, and different from many models, the Friedman rule may not be optimal. We also study how search frictions and persistence in idiosyncratic shocks matter.

To motivate our interest in capital reallocation, in general, efficiency requires not only getting the right amount of investment over time, but getting capital into the hands of those best able to use it at any point in time. With idiosyncratic shocks, capital should flow from lower- to higher-productivity firms (Maksimovic and Phillips 2001; Andrade et al. 2001; Schoar 2002). Also, importantly, the ease with which capital can be retraded on secondary markets affects investment in primary markets and vice versa (as is true for lots of assets, as emphasized by, e.g., Harrison and Kreps 1978 or Lagos and Zhang 2020). However, the channel is subtle: a well-functioning secondary market encourages primary investment since, if firms have more capital than they need, it is relatively easy to sell in the secondary market; it also discourages primary investment since, if firms want more capital than they have, it is relatively easy to buy in the secondary market; and we show how the net effect depends on various factors.

Also, reallocation is sizable, with purchases of used capital reported to be between 25% and 33% of total investment (Eisfeldt and Rampini 2006; Cao and Shi 2016; Dong et al. 2016; Cui 2017; Eisfeldt and Shi 2018), which is probably an underestimate since the data ignore small firms and those that are not publicly traded, neglect mergers, and include purchases but not rentals. Studies also document several stylized facts: reallocation is procyclical while capital mismatch is countercyclical (Eisfeldt and Rampini 2006; Cao and Shi 2016); productivity dispersion is countercyclical (Kehrig 2015); the price of used capital is procyclical (Lanteri 2016); and the ratio of spending on used capital to total investment is procyclical (Cui 2017). Our goal is to match all of these facts.

As for *frictional* reallocation, many argue secondary capital markets are far from the perfectly competitive ideal (Gavazza 2010, 2011*a,b*; Kurman 2014; Ottonello 2015; Kurmann and Rabinovitz 2018; Horner 2018; Li and Whited 2015; Bierdel et al. 2021). Imperfections include financial constraints, difficulties in finding counterparties, holdup problems, and asymmetric information. Our secondary market has bilateral trade and bargaining, as in search theory.[1] It also has assets facilitating payments, as in monetary economics. While explicit modeling of this is missing from most work on capital reallocation, some studies (e.g., Buera et al. 2011; Moll 2014) argue that liquidity frictions are important, even if self financing mitigates the problem, which is just what we model.

[1] In models of capital Ottonello (2015) finds search helps fit the facts and generates more interesting propagation. Horner (2018) shows vacancy rates for commerical real estate resemble unemployment data, suggesting search is as relevant for that kind of capital as it is for labor, plus he finds disperse rents on similar structures. In aircraft markets, Pulvino (1998), Gilligan (2004) and Gavazza (2011*a,b*) show used sales are thrice new sales. Gavazza (2011*a*) shows prices vary inversely with search, and market thickness affects trading frequency, average utilization, utilization dispersion, average price and price dispersion. Also emphasized is *specificity* – capital is often customized. This all suggests search is important.

To say more about our approach to liquidity, we use the label "money" but do not mean currency per se: it can include any asset that is widely accepted as a payment instrument, or can be converted into something that is widely accepted with little cost or delay. In reality there is a spectrum of assets with varying degrees of acceptability and rate of return, implying a tradeoff between these attributes, and research on the foundations of monetary theory tries to analyze this explicitly. Kiyotaki and Wright (1989), e.g., formalizes the tradeoff rigorously, but in a way that is far too stylized for this paper, which is a study in macroeconomics.

The essence of macro is aggregation: standard models have just two uses of output, consumption or investment, and two uses of time, labor or leisure (with exceptions, like home production models with three uses for output and three uses for time). Similarly, our benchmark model has just two assets: money and capital. In reality, while cash may be the most liquid asset, there are substitutes. Hence we incorporate banking in the theory, following Berentsen et al. (2007), and define money as currency plus checkable deposits in the quantitative work. In principle other assets also provide liquidity, but as inflation lowers the return on the most liquid asset, cash, in general equilibrium it can also lower the return on other assets and thereby lower aggregate liquidity indirectly.

As Wallace (1980) says: "[inflation] is not a tax on all saving. It is a tax on saving in the form of money. But it is important to emphasize that the equilibrium rate-of-return distribution on the equilibrium portfolio does depend on [inflation]... the higher the [inflation rate] the less favorable the terms of trade – in general, a distribution – at which present income can be converted into future income." Hence we think inflation is a good way to capture the cost of liquidity, even in multiple-means-of-payment models. However, we defer a general specification to Section 6, where we add liquid real assets. While that works well in theory, models with multiple assets having different liquidity and return are harder to handle analytically and harder to take to data. So the benchmark model in Sections 3-5 has only two assets, money and capital.

The rest of the paper is organized as follows. Section 2 presents the micro and the macro evidence. Sections 3 and 4 develop the theory and derive analytic results. Section 5 provides the main quantitative results. Section 6 considers extensions. Section 7 concludes. Data details and the complicated proofs are in the Appendix.[2]

[2]When a buyer gets all a seller's capital, it resembles M&A (merger and acquisition) activity, but we are not really trying to contribute to the M&A literature in finance (see Andrade et al. 2001 and Betton et al. 2008 for surveys). That involves issues of executive compensation, management strategy, tax implications etc., and is ultimately connected to Coase's (1937) question, what is a firm? We are more in the tradition of macro, e.g., Jovanovic and Rousseau (2002), Jermann and Quadrini (2012), Del Negro et al. (2017) and perhaps especially Khan and Thomas (2013, 2017), as discussed in Section 6. Other work on reallocation

2 Evidence

2.1 Macro Data

We use US data from 1971 to 2018. Details are in Appendix A, but capital reallocation is from COMPUSTAT, where we have information on full and partial sales, measured respectively by acquisitions and sales of PPE (property, plant and equipment), plus total capital expenditures. Firm data are summed to get annual aggregate series. Reallocation is defined as full plus partial sales. We focus on the reallocation-to-expenditure ratio, called the R share, and the partial-sales-to-reallocation ratio, called the P share, capturing the importance of reallocation in investment, and the importance of partial sales in reallocation, respectively. In the early part of the sample the R share varies a lot, but it stabilizes in approximately 1984, fluctuating around 32%. Similarly, early in the sample the P share is quite high, but it stabilizes after 1984, fluctuating around 24%.[3]

As discussed, we entertain the possibility that liquidity plays a role and use inflation to measure its cost, although we also tried nominal T-bill and AAA corporate bond rates, and the results are similar. Fig. 1 (all figures are at the end of the paper) shows the R and P shares vs inflation, with different panels using the raw data, the trend and the cyclical component, after band-pass filtering, following Christiano and Fitzgerald (2003).

In the longer run, when inflation is high firms spend less on used capital relative to total investment, while within reallocation there are more partial sales. Given full sales are three times partial sales, when inflation rises the fall in reallocation is mainly driven by full sales. Of course, other secular changes may affect reallocation, and the fall in inflation may or may not have resulted from monetary policy, but in any case lower inflation is associated with more full and fewer partial sales in the longer run, while at business cycle frequencies the relationships are reversed. A plausible explanation involves credit conditions: easier credit leads to more full and fewer partial sales, plus it reduces money demand, which raises the price level, and that shows up as inflation in the short run.

We pursue this using aggregate firm debt as a proxy for credit conditions (again details are in Appendix A). Fig. 2 shows the cyclical components of debt, investment, the R and P shares, and output. Debt and the R share are procyclical, and the P share counter-

includes Ramey and Shapiro (1998,2001), Hsieh and Klenow (2009), Asker et al. (2014), Midrigan and Xu (2014), Cooper and Schott (2016), Ai et al. (2016), David et al. (2016), Lanteri and Gavazza (2019), David and Venkateswaran (2019), and Wright et al. (2018, 2020). Related papers on money and capital, if not reallocation, include Shi (1999a,b), Shi and Wang (2006), Aruoba and Wright (2003), Aruoba et al. (2011), and of course classics like Sidrauski (1967) and Tobin (1965).

[3] Since the R and P shares are more or less stationary after 1984, for calibration below we start in 1984, but here, for establishing different types of evidence we go back to 1971. This does not affect the message.

Table 1: Business Cycle Statisticss

	Debt	R share	P share	Investment	Inflation	Output
Debt	1	0.52	-0.45	0.57	0.18	0.59
R share	-	1	-0.76	0.60	0.24	0.63
P share	-	-	1	-0.49	-0.15	-0.53
Investment	-	-	-	1	0.39	0.96
Inflation	-	-	-	-	1	0.37
Output	-	-	-	-	-	1
Relative SD.	1.19	5.74	8.42	2.74	1.93	1

cyclical. So when credit conditions ease, debt goes up, part of which funds reallocation, explaining why full sales rise, partial sales fall and total reallocation rises, and notice reallocation must be more volatile than investment to get a procyclical R share. Table 1 shows investment and reallocation positively comove, and inflation is procyclical. All this is consistent with the intuitive discussion in the Introduction.

2.2 Micro Data

Next, disaggregated data in COMPUSTAT are used to present two pieces of micro evidence. First, we show money holdings have a positive effect on full purchases and a negative effect on partial purchases. (In terms of labels, we usually use full and partial *sales*, but when the focus is on the firm getting capital it seems better to use full and partial *purchases*.) Second, we examine how inflation impacts firms' money holdings.

To begin, we regress full purchases on firms' liquidity at the end of the last period, measured by holdings of cash plus cash equivalent, including assets readily convertible into cash like certificates of deposit, banker's acceptances, T bills, and commercial paper. The LHS is a binary variable that equals 1 if a firm engages in a full purchase this year and 0 otherwise, capturing the extensive margin. We use a linear probability model (a logistic model gives similar results). The second approach is to examine full purchase expenditure. For this we take logs and focus on firms engaging in a full purchase in a given year, capturing the intensive margin. For our purposes, it does not matter whether money holdings cause full purchases or anticipations of full purchases cause money holdings – both say that cash facilitates reallocation.

We control for factors that may affect purchases and money holdings, like earnings before interest and taxes (EBIT), total assets and the leverage ratio measured by short-term liabilities over shareholder equity (SEQ). Independent variables are lagged one period to reduce simultaneity problems. We include year or year-industry fixed effects (FE) defined by the first two digits of SIC codes. Total assets are in logs and normalized by the nominal price level. Other variables except the leverage ratio are also in logs and normalized by total assets of the firm, but results are similar if they are normalized by the nominal price level.

In Table 2, the first three columns give results on the probability of a full purchase. The first column includes only firm FE, the second includes firm and year FE and the third includes firm and year-industry FE. In all cases money holdings have a significant positive effect, with a 1% increase in cash raising the probability of a full purchase by 0.00019 in levels. As the average probability of a full sale is around 0.21, this means a 1% increase in cash increases the full sale probability by about 0.1%. EBIT has significant positive effects on full purchases. Total assets have a positive while leverage ratios have a negative effect.

Table 2: Full Sales and Money Holdings

	Prob	Prob	Prob	Spending	Spending	Spending
Money Holding	0.018***	0.019***	0.018***	0.189***	0.200***	0.202***
	(0.001)	(0.001)	(0.001)	(0.012)	(0.012)	(0.013)
EBIT	0.023***	0.027***	0.028***	0.268***	0.274***	0.266***
	(0.002)	(0.002)	(0.002)	(0.019)	(0.019)	(0.020)
Asset	0.085***	0.064***	0.067***	-0.317***	-0.387***	-0.380***
	(0.003)	(0.003)	(0.003)	(0.018)	(0.022)	(0.024)
Leverage	-0.001**	-0.001**	-0.001*	-0.004	-0.005	-0.005*
	(0.000)	(0.000)	(0.000)	(0.003)	(0.003)	(0.003)
Firm FE	Y	Y	Y	Y	Y	Y
Year FE		Y			Y	
Year-Industry FE			Y			Y
Adj. R^2	0.029	0.039	0.052	0.049	0.075	0.111
# observations	116228	116228	116228	33804	33804	33804

Note: $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Robust standard errors are in brackets and are clustered at firm level. All independent variables are lagged by one period. Acquisition spending, money holdings and EBIT are normalized by firms' total assets.

The last three columns of Table 2 report results on full purchase spending, with a 1% increase in money leading to a 0.2% rise in spending, or a 1dollar increase in money leading to an 8cent rise, which is sizeable. Again EBIT has a positive effect on full purchase spending. We also ran dynamic panel regressions to account for the possibility full purchases are persistent; the results are similar, with coefficients on lagged purchases that are small and insignificant. This all indicates liquidity, measured by cash or cash equivalent, encourages full purchases.[4]

Now consider partial purchases. While in COMPUSTAT we cannot identify the buyer in each purchase, and hence their cash holdings, we can aggregate data to the industry level defined by the first two digits of SIC, or to the state level and investigate how cash held in an industry or in a state affects partial sales. The former aggregation would be informative about firm-level purchases if they buy mostly from firms in the same industry, the latter would if they buy mostly from firms in the same state.

Table 3: Money Holdings on P Share

	Industry Level		State Level	
	P Share	P Share	P Share	P Share
Cash Holding	-0.302*	-0.082	-0.429***	-0.349***
	(0.155)	(0.101)	(0.071)	(0.086)
EBIT	-0.186*	-0.216***	0.066	-0.099
	(0.100)	(0.080)	(0.064)	(0.074)
Log Asset	-0.381***	-0.401***	0.068	-0.046
	(0.061)	(0.086)	(0.069)	(0.073)
Leverage	0.109	0.020	-0.071	-0.159
	(0.135)	(0.070)	(0.081)	(0.096)
Year FE		Y		Y
Adj. R^2	0.174	0.302	0.169	0.278
# observations	2682	2682	2237	2237

Note: $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Robust standard errors are in brackets and are clustered at industry or state level.

Table 3 shows the results from regressing P share on cash holding at industry and state levels. P share, money held and EBIT are in logs and the latter two are normalized by total assets. Assets are in logs and normalized by the nominal price level. All the independent variables are values at the end of the last period. The first two columns report results at the industry level while the last two at the state level. We include industry FE in the industry-level regressions and state FE in the state-level regressions. The coefficients on cash holding are negative in all specifications and significant except one regression. In particular, the coefficients from the state-level regressions are significant at 1% level,

[4]While we not trying to contribute directly to the M&A literature, the findings are consistent with empirical work in that area. Harford (2005) suggests capital liquidity drives both M/B (market to book) ratios and merger waves, finding that waves are preceded by high capital liquidity, and that including capital liquidity eliminates the power of M/B to predict waves. Harford (1999) shows firms with more cash reserves are more prone to acquire others. Andrade et al. (2008) report that in 1996-2000: 26% of M&A bids are all cash; 37% are all stock; 37% are a mix of securities; and the probability of the deal going through is higher if payment is in cash. The point is not that this is surprising, but that it is consistent with our approach.

which may suggest that partial purchases normally occur locally. These results show that as cash holding increases, firms shift from partial purchases to full purchases.

Table 4: Money Holdings and Liquidity Cost

Money Holding				
Inflation	-0.848***	-	-	-
	(0.210)	-	-	-
Inflation (Cycle)	-	-3.383***	-	-
	-	(0.192)	-	-
State-Level Inflation	-	-	-1.882***	-1.550***
	-	-	(0.525)	(0.535)
EBIT	0.101***	0.120***	0.107***	0.106***
	(0.005)	(0.005)	(0.007)	(0.007)
Asset	-0.067***	-0.087***	-0.087***	-0.085***
	(0.009)	(0.009)	(0.013)	(0.013)
Leverage	-0.001**	-0.001*	-0.001*	-0.001**
	(0.000)	(0.001)	(0.001)	(0.001)
Capital Exp.	-0.097***	-0.097***	-0.088***	-0.080***
	(0.006)	(0.006)	(0.008)	(0.008)
Firm FE	Y	Y	Y	Y
Year FE			Y	
Year-Industry FE				Y
Adj. R^2	0.014	0.015	0.029	0.056
# observations	1426064	142303	92078	92078

Note: * p$<$0.10, ** p$<$0.05, *** p $<$0.01. Robust standard errors are in brackets and are clustered at firm level. Money holdings, EBIT, capital expenditures are normalized by total assets.

Next consider how money holdings depend on liquidity costs measured by inflation. While for most of the paper we use the PPI, since the theory is about producers, here we use the CPI because we are going to include state-level inflation that is not available for the PPI. The results are in Table 4, with all regressions controlling for firm FE. The first column indicates that a 1% increase in inflation reduces money holdings by about 0.848%. In our sample, inflation decreases and money holdings increase over time, so negative coefficients on inflation may result from the trends. To address this, we again use a band-pass filter to remove the trend component of inflation, and the second column in Table 4 reports results using the cyclical component. The coefficient on inflation remains negative and highly significant, and the magnitude is much larger.

To address the concern that cyclical co-movement may drive the results, we exploit cross-sectional variation in inflation rates, as COMPUSTAT has addresses of firms. Assuming firms care about local inflation, we regress money holdings on state-level inflation from Hazell et al. (2022), including year or year-industry FE. The results in the last two columns show the coefficients on state-level inflation are again negative and highly significant, and slightly larger than the first column. Hence, like the macro data, the micro evidence is consistent with our approach, and motivates developing models with full and partial sales, taking into account credit and monetary considerations.

3 Model

We build on the alternating-market structure in Lagos and Wright (2005): each period in discrete time, a continuum of infinite-lived agents interact in a frictional decentralized market, or DM, and then a frictionless centralized market, or CM.[5] In the CM, households consume a numeraire good c, supply labor hours h, settle debt d and adjust their portfolios of capital k and money m, while firms produce using h and k. In the DM, (the owners of) firms meet bilaterally and potentially retrade k after observing idiosyncratic productivity shocks. Agents discount between the CM and the next DM using $\beta \in (0,1)$, but not between the DM and CM, without loss of generality. Given time endowment 1, utility over consumption and leisure is quasi-linear: $U(c, 1-h) = u(c) + \xi(1-h)$, where $\xi > 0$ is a parameter and $u'(c) > 0 > u''(c)$.

Quasi-linearity enhances tractability, as described in Lemma 1 and 2 below, but Wong (2016) shows these results also hold for any $U(c, 1-h)$ with $U_{11}U_{22} = U_{12}^2$. Alternatively, Rocheteau et al. (2008) show the results hold for any $U(c, 1-h)$ if labor is indivisible, $h \in \{0, 1\}$, and agents trade employment lotteries as in Rogerson (1988). This is relevant for comparing our results on business cycles to those from the canonical RBC model, which we take to be Hansen (1985), as it jettisons many of the bells and whistles in Kydland and Prescott (1982) without sacrificing results, then improves on performance by using indivisible labor. A special case of our setup, with no idiosyncratic shocks, is exactly Hansen's indivisible-labor model, right down to functional forms in Section 5.[6]

[5]This is ideal for our purposes because in a stylized way the CM and DM correspond to primary and secondary trade. Also, it features an asynchronization of expenditures and receipts – reallocation occurs in the DM while profit accrues in the CM – crucial to any analysis of money or credit. Plus it proves tractable in many other applications, and flexible in that it allows various specifications for search, price determination etc. (see Lagos et al. 2017 and Rocheteau and Nosal 2017 for surveys).

[6]One implication is that, as in Hansen (1985), in the aggregate $1 - H$ can be interpreted as unemployment, or at least nonemployment, not just the leisure of all agents: it is the measure of agents with $h = 0$. Also note that Cooley and Hansen (1989) provide a monetary version of Hansen (1985), but it is not comparable, since there households use cash to buy goods while here firms use it to buy capital. So, if we shut down idiosyncratic firm shocks, money is not valued and the model reduces to Hansen, not Cooley-Hansen.

We assume each household owns its own firm; while they could hold shares in other firms, given Lemma 1 below, they do not need to. Each has a CRS production function $f(k,h) = (A\varepsilon k)^{1-\eta} h^{\eta}$, where ε is idiosyncratic and A aggregate productivity. Aggregate uncertainty from A is shut down until Section 5, when we study business cycles. The firm-specific ε has a time-invariant distribution $F(\varepsilon)$ that can be persistent: using subscript $+$ for next period, ε_+ is drawn from a conditional CDF $Q(\cdot|\varepsilon)$. As usual, investment in k at t is productive at $t+1$, and it depreciates at rate δ.

In the CM, a firm with (k,ε) chooses labor demand $\tilde{h}$ (distinct from its owner's labor supply) to maximize profit,

$$\Pi(k,\varepsilon) = \max_{\tilde{h}}\{(A\varepsilon k)^{1-\eta}\tilde{h}^{\eta} - w\tilde{h}\}.$$

Of course Π also depends on w, but that remains implicit in the notation. The solution is $\tilde{h}(k,\varepsilon) = \left(\frac{\eta}{w}\right)^{\frac{1}{1-\eta}} A\varepsilon k$, and $\Pi(k,\varepsilon) = B(w)\varepsilon k$ where

$$B(w) \equiv \left(\frac{\eta}{w}\right)^{\frac{\eta}{1-\eta}}(1-\eta)A. \tag{1}$$

Hence Π is linear in εk, which in what follows implies efficient DM reallocation entails full sales: a high ε firm should get all the capital of a low ε firm.

Let ϕ be the price of money m in terms of numeraire c (i.e., the inverse nominal price level), so real balances are $z = \phi m$ and gross inflation is $1+\pi = \phi/\phi_+$. Let the CM and DM value functions be W and V. Then

$$W(\Omega,\varepsilon) = \max_{c,h,\hat{k},\hat{z}}\left\{u(c) + \xi(1-h) + \beta\mathbb{E}_{\hat{\varepsilon}|\varepsilon}V_+(\hat{k},\hat{z},\hat{\varepsilon})\right\} \text{ st } c = \Omega + (1-\tau_h)wh - \hat{z}\phi/\phi_+ - \hat{k}, \tag{2}$$

where Ω is wealth, k and z are capital and real balances at the start of the CM while $\hat{k}$ and $\hat{z}$ are capital and real balances at the end, τ_h is a labor income tax, and $\mathbb{E}_{\hat{\varepsilon}|\varepsilon}$ denotes the expectation wrt $\hat{\varepsilon}$ conditional on ε. Note the cost of real balances next period in terms of current c is inflation. Wealth is $\Omega = (1-\tau_k)B(w)\varepsilon k + (1-\delta)k + z - d - T$, where τ_k is a capital income tax, T a lump-sum tax, and d debt from the previous DM. Using the budget equation to eliminate h, (2) becomes

$$\begin{aligned} W(\Omega,\varepsilon) &= \xi + \frac{\xi\Omega}{(1-\tau_h)w} + \max_c\left\{u(c) - \frac{\xi c}{(1-\tau_h)w}\right\} \\ &\quad + \max_{\hat{k},\hat{z}}\left\{-\frac{\xi\left(\hat{z}\phi/\phi_+ + \hat{k}\right)}{(1-\tau_h)w} + \beta\mathbb{E}_{\hat{\varepsilon}|\varepsilon}V_+\left(\hat{k},\hat{z},\hat{\varepsilon}\right)\right\}. \end{aligned} \tag{3}$$

From (3) the following results are immediate:

Lemma 1 *$W(\Omega,\varepsilon)$ is linear in Ω with slope $\xi/[(1-\tau_h)w]$.*

Lemma 2 *An interior solution for $(\hat{k},\hat{z})$ solves:*

$$\frac{\xi}{(1-\tau_h)w} = \beta\mathbb{E}_{\hat{\varepsilon}|\varepsilon}\frac{\partial V_+(\hat{k},\hat{z},\hat{\varepsilon})}{\partial\hat{k}} \quad (4)$$

$$\frac{\xi}{(1-\tau_h)w}\frac{\phi}{\phi_+} = \beta\mathbb{E}_{\hat{\varepsilon}|\varepsilon}\frac{\partial V_+(\hat{k},\hat{z},\hat{\varepsilon})}{\partial\hat{z}}. \quad (5)$$

This means $(\hat{k},\hat{z})$ is the same for all agents with the same ε, although agents with different ε choose different $(\hat{k},\hat{z})$, unless ε is i.i.d. in which case $(\hat{k},\hat{z})$ is the same for all agents.

To complete the CM problem, let $\hat{z}(\varepsilon)$ and $\hat{k}(\varepsilon)$ solve (4)-(5), and note from (3) that c solves $u'(c)=\xi/[(1-\tau_h)w]$. Then the budget gives labor supply,

$$h(\Omega,\varepsilon) = \frac{c+\hat{k}(\varepsilon)+\hat{z}(\varepsilon)\phi/\phi_+ - \Omega}{(1-\tau_h)w}. \quad (6)$$

If Γ is the distribution of (k,z,ε) at the start of a period, we have the law of motion

$$\Gamma_+(k,z,\varepsilon) = \int_{\hat{k}(x)\le k,\hat{z}(x)\le z} Q(\varepsilon|x)\,dF(x). \quad (7)$$

Without aggregate shocks, agents move around in the distribution but the cross section is constant. Even with aggregate shocks, tractability is preserved since $(\hat{k},\hat{z})$ depends on ε, but not past DM trades.

Now consider the DM, where with probability α each firm (owner) is randomly matched to a potential trading partner.[7] In a meeting, the state variables of the pair are $\mathrm{s}=(k,z,\varepsilon)$ and $\tilde{\mathrm{s}}=(\tilde{k},\tilde{z},\tilde{\varepsilon})$. When $\varepsilon>\tilde{\varepsilon}$, the ε firm is a *buyer* and the $\tilde{\varepsilon}$ firm is a *seller*, since the former should get some quantity $q(\mathrm{s},\tilde{\mathrm{s}})$ of capital from the latter. Let $p(\mathrm{s},\tilde{\mathrm{s}})$ be the cash payment by the buyer, and $d(\mathrm{s},\tilde{\mathrm{s}})$ the value of any debt, a promise of payment in the next CM, as discussed more below. Then

$$V(k,z,\varepsilon) = W(\Omega,\varepsilon) + \alpha\int_{\varepsilon>\tilde{\varepsilon}} S^b(\mathrm{s},\tilde{\mathrm{s}})d\Gamma(\tilde{\mathrm{s}}) + \alpha\int_{\varepsilon<\tilde{\varepsilon}} S^s(\tilde{\mathrm{s}},\mathrm{s})d\Gamma(\tilde{\mathrm{s}}), \quad (8)$$

[7]Similar to stories motivating frictional labor markets, $\alpha<1$ can mean it is had to find a potential trading partner – a pure search problem – or to find the right type – a matching problem, due to capital specificity.

where $S^b(\cdot)$ and $S^s(\cdot)$ are buyer and seller surpluses, which by Lemma 1 are

$$S^b(\mathrm{s}, \tilde{\mathrm{s}}) = \frac{\xi \{[(1-\tau_k)\varepsilon B(w) + 1 - \delta] q(\mathrm{s}, \tilde{\mathrm{s}}) - p(\mathrm{s}, \tilde{\mathrm{s}}) - d(\mathrm{s}, \tilde{\mathrm{s}})\}}{w(1-\tau_h)}; \tag{9}$$

$$S^s(\tilde{\mathrm{s}}, \mathrm{s}) = \frac{\xi \{p(\tilde{\mathrm{s}}, \mathrm{s}) + d(\tilde{\mathrm{s}}, \mathrm{s}) - [(1-\tau_k)\tilde{\varepsilon} B(w) + 1 - \delta] q(\tilde{\mathrm{s}}, \mathrm{s})\}}{w(1-\tau_h)}. \tag{10}$$

We distinguish between two types of reallocation, a full sale $q(\mathrm{s}, \tilde{\mathrm{s}}) = \tilde{k}$, and a partial sale $q(\mathrm{s}, \tilde{\mathrm{s}}) \in (0, \tilde{k})$. While full sales are socially efficient, they may not happen due to liquidity constraints: cash payments are constrained by $p \leq z$ while credit payments are constrained by $d \leq D$ with debt limit

$$D = \chi_0 + \chi_\Pi \Pi + \chi_q (1-\delta) q + \chi_k (1-\delta) k. \tag{11}$$

In (11) the first term represents unsecured debt, where χ_0 can be a parameter or endogenized as in Kehoe and Levine (1993); the second term is debt secured by profit, as in Holmstrom and Tirole (1998); the third and fourth are debt secured by new and existing capital, like mortgages and home equity loans, as in Kiyotaki and Moore (1997).

Often χ_Π, χ_k and χ_q are called *pledgeability* parameters. One story for $\chi_j < 1$ is that you can renege on promised payments, but if you do a fraction χ_j of your asset j gets seized while you abscond with the rest. Li et al. (2012) provide an alternative microfoundation, where holding more assets than you use as collateral signals quality. In any case, even at $\chi_q = 1$ credit secured using q as collateral and nothing else cannot support any DM trade, since it does not cover sellers' outside option, so buyers need other lines of credit or cash. One interpretation is that they rent capital, since in the CM it does not matter if the buyer returns $(1-\delta) q$ and pays the seller a little, or keeps it and pays a lot. Then saying that at $\chi_q = 1$ credit secured by only q cannot support DM trade is like saying you cannot rent anything if the most you promise is to return it.

To determine DM terms of trade, $q(\mathrm{s}, \tilde{\mathrm{s}})$, $p(\mathrm{s}, \tilde{\mathrm{s}})$ and $d(\mathrm{s}, \tilde{\mathrm{s}})$, several options are available, including generalized Nash bargaining (Lagos and Wright 2005), some strategic bargaining solutions (Zhu 2020) and competitive price taking or price posting (Rocheteau and Wright 2005). We use Kalai's (1977) bargaining solution, which has become popular in microfounded models of liquidity since Aruoba et al. (2007), who argue that it has several advantages (note that Kalai and Nash are the same if liquidity constraints are slack but not if they bind). The outcome can be found by solving

$$\max_{d,p,q} S^b(\mathrm{s}, \tilde{\mathrm{s}}) \text{ st } (1-\theta) S^b(\mathrm{s}, \tilde{\mathrm{s}}) = \theta S^s(\tilde{\mathrm{s}}, \mathrm{s}), \tag{12}$$

plus the constraints $q \leq \tilde{k}$, $p \leq z$ and $d \leq D$, where θ is buyers' bargaining power. Appendix B characterizes the solution as follows:

Proposition 1 *Consider a DM meeting* $(\mathbf{s}, \tilde{\mathbf{s}})$ *with* $\varepsilon > \tilde{\varepsilon}$*, and define a threshold for* ε *by* $\bar{\varepsilon} = \Psi_0 - \Psi_1 \tilde{\varepsilon}$*, where*

$$\Psi_0 \equiv \frac{[z + \chi_0 + \chi_k (1-\delta) k]/\tilde{k} - (1-\delta)\left(1-\chi_q\right)}{(1-\tau_k)\left[1-\theta-\chi_\Pi\left(1+k/\tilde{k}\right)\right] B(w)} \text{ and } \Psi_1 \equiv \frac{\theta}{1-\theta-\chi_\Pi\left(1+k/\tilde{k}\right)}. \quad (13)$$

Case (i) $\chi_\Pi < \bar{\chi}_\Pi \equiv (1-\theta)/(1+k/\tilde{k})$*: If* $\varepsilon > \bar{\varepsilon}$ *there is a partial sale,* $q = Q < \tilde{k}$*, where*

$$Q \equiv \frac{z + \chi_0 + [(1-\delta)\chi_k + (1-\tau_k) B(w) \chi_\Pi \varepsilon] k}{(1-\tau_k) B(w) [(1-\theta-\chi_\Pi)\varepsilon + \theta\tilde{\varepsilon}] + (1-\delta)\left(1-\chi_q\right)}, \quad (14)$$

and the payment constraints bind, $p = z$ *and* $d = D$*. If* $\varepsilon < \bar{\varepsilon}$ *there is a full sale* $q = \tilde{k}$ *and the the mix between* p *and* d *is irrelevant as long as*

$$p + d = \{(1-\tau_k) B(w) [(1-\theta)\varepsilon + \theta\tilde{\varepsilon}] + 1 - \delta\} \tilde{k}.$$

Case (ii) $\chi_\Pi > \bar{\chi}_\Pi$*: The results are the same except the regions of* $(\varepsilon, \tilde{\varepsilon})$ *space are reversed – i.e.,* $\varepsilon < \bar{\varepsilon}$ *implies a partial sale and* $\varepsilon > \bar{\varepsilon}$ *a full sale.*

Fig. 3 shows the case $\chi_\Pi < \bar{\chi}_\Pi$, which implies partial sales occur above the line $\varepsilon = \Psi_0 - \Psi_1 \tilde{\varepsilon}$ and full sales below it, and higher productivity firms are more likely to be constrained because the unconstrained price increases with ε. When constrained, firms use all their liquid wealth, the numerator of (14), to buy q. This occurs at low χ_Π because, while higher ε firms pay more, they also get more credit as they have higher Π, and the net effect depends its pledgeability. It is also worth reiterating that CRS makes partial sales inefficient. In Fig. 3, given $\tilde{\varepsilon}$ trade is more likely to be constrained for higher ε, so reallocation with the highest social value is most prone to inefficiency due to illiquidity.

Next consider CM clearing in m and c (by Walras' law, we can ignore h). Let the aggregate supply M grow at rate μ, with changes engineered in the CM: add seigniorage to revenue from τ_h and τ_k, subtract government spending G, and set T to balance the budget each period. Then money and goods market clearing are given by

$$\phi_+ M_+ = \int \hat{z}(\varepsilon) \, dF(\varepsilon) \text{ and } c + K_+ + G = Y + (1-\delta) K, \quad (15)$$

where $Y = B(w) \bar{K}/(1-\eta)$ is total output, $K_+ = \int \hat{k}(\varepsilon) \, dF(\varepsilon)$ is gross investment, and $\bar{K}$

is effective capital weighted by productivity after DM trade,

$$\bar{K} = \alpha \iint\limits_{\varepsilon > \tilde{\varepsilon}} \varepsilon \left[k + q\left(\mathbf{s}, \tilde{\mathbf{s}}\right)\right] d\Gamma(\tilde{\mathbf{s}}) d\Gamma\left(\mathbf{s}\right) + \alpha \iint\limits_{\varepsilon < \tilde{\varepsilon}} \varepsilon \left[k - q\left(\tilde{\mathbf{s}}, \mathbf{s}\right)\right] d\Gamma(\tilde{\mathbf{s}}) d\Gamma\left(\mathbf{s}\right) + (1-\alpha) \int \varepsilon k d\Gamma\left(\mathbf{s}\right). \tag{16}$$

Definition 1 *Given initial conditions (z, k) and paths for (μ, G, τ_h, τ_k), equilibrium is a list of nonnegative paths for $(c, \hat{z}, \hat{k}, q, p, d, \phi, w, \Gamma)$, where $\hat{z} = \hat{z}(\varepsilon)$ and $\hat{k} = \hat{k}(\varepsilon)$ for each agent while $q = q(\varepsilon, \tilde{\varepsilon})$, $p = p(\varepsilon, \tilde{\varepsilon})$ and $d = d(\varepsilon, \tilde{\varepsilon})$ for each pair, satisfying at all dates: (i) in the CM $(c, \hat{z}, \hat{k})$ solves (2); (ii) in the DM (q, p, d) are given by Prop. 1; (iii) markets clear as in (15); (iv) the distribution Γ evolves according to (7); and (v) the transversality conditions $\beta^t u'(c_t) \hat{k}_t \to 0$ and $\beta^t u'(c_t) \hat{z}_t \to 0$.*

For steady state, let (μ, G, τ_h, τ_k) be constant and note if the growth rate of M is $\mu \neq 0$ then ϕ generally changes over time, but $\phi M = z$ does not if $\phi/\phi_+ = 1 + \mu$.

Definition 2 *Steady state is a time-invariant list $(c, z, k, q, p, d, w, \Gamma)$ satisfying everything in Definition 1 except for the initial conditions.*

Monetary policy here is given by μ. We can instead target inflation π or the illiquid nominal rate ι, although in steady state these are equivalent since $\pi = \mu$ and $\iota = (1+\pi)/\beta - 1$. To be precise, we define illiquid rates as follows: $1 + r$ is the amount of c agents require in the next CM to give up 1 unit in this CM; and $1 + \iota$ is similar except m replaces c. We impose $\iota > 0$, but consider the limit $\iota \to 0$, called the Friedman rule or zero lower bound.

It is useful to derive the marginal value of capital in the DM. Using (8) we get

$$\frac{\partial V}{\partial k} = \frac{\xi}{(1-\tau_h)w} \left\{ 1 - \delta + (1-\tau_k) B(w) \left[\varepsilon + \alpha (1-\theta) \int\limits_{\mathcal{S}_s(\mathbf{s})} (\tilde{\varepsilon} - \varepsilon) d\Gamma(\tilde{\mathbf{s}}) + \alpha\theta (1-\delta) \chi_k \int\limits_{\mathcal{S}_b(\mathbf{s})} \frac{\varepsilon - \tilde{\varepsilon}}{\Delta(\varepsilon, \tilde{\varepsilon})} d\Gamma(\tilde{\mathbf{s}}) + \alpha\theta (1-\tau_k) B(w) \chi_\Pi \int\limits_{\mathcal{S}_b(\mathbf{s})} \frac{\varepsilon(\varepsilon - \tilde{\varepsilon})}{\Delta(\varepsilon, \tilde{\varepsilon})} d\Gamma(\tilde{\mathbf{s}}) \right] \right\}, \tag{17}$$

where $\Delta(\varepsilon, \tilde{\varepsilon})$ denotes the denominator in (14), while

$$\mathcal{S}_s(\mathbf{s}) = \{\tilde{\mathbf{s}} : \tilde{\varepsilon} > \varepsilon, \tilde{\varepsilon} < \overline{\varepsilon}(\tilde{\mathbf{s}}, \mathbf{s})\} \text{ and } \mathcal{S}_b(\mathbf{s}) = \{\tilde{\mathbf{s}} : \tilde{\varepsilon} < \varepsilon, \varepsilon > \overline{\varepsilon}(\mathbf{s}, \tilde{\mathbf{s}})\} \tag{18}$$

are sets of meetings where sellers are constrained by k and buyers by z.[8] Similarly,[9]

$$\frac{\partial V}{\partial z}=\frac{\xi}{(1-\tau_h)w}\left[1+(1-\tau_k)B(w)\,\alpha\theta\int\limits_{\mathcal{S}_b(\mathbf{s})}\frac{\varepsilon-\tilde{\varepsilon}}{\Delta(\varepsilon,\tilde{\varepsilon})}d\Gamma(\tilde{\mathbf{s}})\right]. \tag{19}$$

Combining (17)-(19) with the FOCs in Lemma 2, we get the Euler equations

$$\frac{1}{w}=\frac{\beta(1-\tau_k)B(w_+)}{w_+}\mathbb{E}_{\varepsilon_+|\varepsilon}\left[\varepsilon_+ +\alpha(1-\theta)I_s+\alpha\theta(1-\delta)\chi_k I_{b1}\right. \tag{20}$$
$$\left.+\alpha\theta(1-\tau_k)B(w)\chi_\Pi I_{b2}\right]+\frac{\beta(1-\delta)}{w_+}$$
$$\frac{Z}{w}=\frac{\beta Z_+}{w_+(1+\mu)}\mathbb{E}_{\varepsilon_+|\varepsilon}\left[1+(1-\tau_k)B(w_+)\,\alpha\theta I_{b1}\right], \tag{21}$$

where Z is aggregate real balances, and to save space I_s, I_{b1} and I_{b2} denote the three integrals on the RHS of (17).

Before pursuing results, let us discuss how to add banking as in Berentsen et al. (2007): after the CM closes and before the DM opens, information is revealed affecting agents' desired $\hat{z}$. This information is simply whether each agent will have a meeting in the DM – those that will not have excess cash; those that will could use more. This liquidity mismatch creates a role for banks like Diamond and Dybvig (1982), except they deal in money not goods. What makes them essential is that agents cannot easily trade liquidity among themselves using promised CM repayment, for the same reason they cannot trade DM capital using promises: lack of commitment and no concern for reputation.

Assuming bankers have reputational concerns, so their promises are credible, plus a comparative advantage in collecting debt, they have a role intermediating the exchange of liquidity (see Gu et al. 2022 for a survey of bank models along these lines). While they are not needed for the theory to be interesting, banks aid in the calibration because they prop

[8] In words, (17) says that a marginal unit of k has several potential benefits: (i) You can get the CM resale value of $1-\delta$ per unit. (ii) You can get its contribution to CM production, the first term in square brackets, which is ε because $(1-\tau_k)B(w)$ outside the brackets converts εk into income. (iii) You can get its value from a DM sale, the second term in brackets, since you sell all of k when you meet someone with $\tilde{\mathbf{s}}\in\mathcal{S}_s(\mathbf{s})$ and enjoy a share $1-\theta$ of the surplus. (iv) You can get its DM collateral value, captured by the third term, since you hit your liquidity constraint when you buy from someone with $\tilde{\mathbf{s}}\in\mathcal{S}_b(\mathbf{s})$ and enjoy a share θ of that surplus. (v) You can get the collateral value from more CM profit, captured by the fourth term, when you buy from someone with $\tilde{\mathbf{s}}\in\mathcal{S}_b(\mathbf{s})$ and enjoy a share θ of that surplus. Of course you do not get all of these benefits; you get each one with some probability.

[9] In words, (19) says a marginal unit of z has these potential benefits: (i) You can get its CM purchasing power. (ii) You can get its DM purchasing power, since you hit your liquidity constraint as a buyer when you meet someone with $\tilde{\mathbf{s}}\in\mathcal{S}_b(\mathbf{s})$ and enjoy a share θ of the surplus.

up money demand. The insight from Berentsen et al. (2007) is that the ability to retrade it makes investing in liquidity less costly, since if you find yourself with more than you need you can put it in the bank at interest financed by loans to those who want more. This is another instance of a point made in the Introduction, that the ease with which assets can be retraded on secondary markets affects demand in primary markets. Conveniently, the only impact this has on the equilibrium conditions is that 1 replaces α in (21).[10]

Let us also discuss how to add endogenous DM entry, as in many standard search models (e.g., Diamond 1982; Pissarides 2000), or monetary models featuring a cost of participating in certain markets (e.g., Chatterjee and Corbae 1992; Chiu 2014). As in Khan and Thomas (2007, 2013), the entry cost γ is random across agents, and for simplicity here entry happens before seeing the ε shocks, so only those realizing γ below a common threshold γ^* enter. With a CRS meeting technology, entry affects the measure of agents in the DM but not individual arrival rates.

Like endogenous banks, endogenous γ^* is unimportant for some purposes, but can affect calibration dynamics. In particular, an earlier version of the paper without entry had a counterfactual positive correlation between inflation and credit conditions, but with entry we get a negative correlation as in the data. Endogenous entry is also natural since the number of firms trading in secondary capital markets, and not just total trade, varies over time. Calculating the fraction of firms trading in secondary capital markets in COMPUSTAT, we find a strong positive correlation with output over the cycle.

4 Analytic Results

While banking, entry and persistent ε shocks are useful in quantitative work, we now study the theory without those features. For this, assume χ_Π is not too big, to get uniqueness (at least without entry, one reason to not have it here). First, make a change of variables by defining

$$L \equiv \frac{(Z+\chi_0)/K - (1-\delta)\left(1-\chi_q-\chi_k\right)}{(1-\tau_k)B(w)}, \tag{22}$$

a normalized notion of liquidity determining when the constraint binds. Then write

$$\mathcal{S}_b(L) \equiv \left\{(\varepsilon,\tilde{\varepsilon}) : \varepsilon > \tilde{\varepsilon},\, \varepsilon > \frac{L-\theta\tilde{\varepsilon}}{1-\theta-2\chi_\Pi}\right\} \text{ and } \mathcal{S}_s(L) \equiv \left\{(\varepsilon,\tilde{\varepsilon}) : \varepsilon > \tilde{\varepsilon},\, \tilde{\varepsilon} < \frac{L-\theta\varepsilon}{1-\theta-2\chi_\Pi}\right\}$$

[10] For details see, e.g., He et al. (2015), but the intuition is clear: 1 replaces α in (21) since by depositing you effectively lend $\hat{z}$ to someone who will have a DM meeting, so you get the same marginal benefit. Setting $\alpha = 1$ without banks also props up money demand, but is different, as it has other implications.

for the sets of meetings where partial and full sales occur now as functions of L. Also, write the effective capital stock, defined in (16), as $\bar{K} = J(L, w) K$ with

$$J(L,w) \equiv \mathbb{E}\varepsilon + \alpha I_s(L) + \alpha \left[(1-\tau_k) B(w) L + \left(1-\chi_q\right)(1-\delta)\right] I_{b1}(L) + \alpha \chi_\Pi (1-\tau_k) B(w) I_{b2}(L)$$

where I_s, I_{b1} and I_{b2} are the integrals from (17) but now as functions of L. Then

$$\frac{r+\delta}{B(w)(1-\tau_k)} = \mathbb{E}\varepsilon + \alpha(1-\theta) I_s(L) + (1-\delta)\chi_k \iota + \chi_\Pi B(w)(1-\tau_k)\alpha\theta I_{b2}(L) \quad (23)$$

$$\iota = \alpha\theta \iint_{\mathcal{S}_b(L)} \frac{(\varepsilon - \tilde{\varepsilon})\, dF(\tilde{\varepsilon}) dF(\varepsilon)}{(1-\theta-\chi_\Pi)\varepsilon + \theta\tilde{\varepsilon} + \frac{(1-\delta)\left(1-\chi_q\right)}{(1-\tau_k)B(w)}} \quad (24)$$

are the steady state Euler equations, where r and ι are the illiquid real and nominal rates defined above. Also, goods market clearing becomes

$$u'^{-1}\left[\frac{\xi}{(1-\tau_h) w}\right] + G = \left[\frac{B(w) J(L,w)}{1-\eta} - \delta\right] K. \quad (25)$$

Now (23)-(25) are three equations in (K, Z, w). For what it's worth, (23) and (24) are the classical IS and LM curves: demand for Investment equals supply of Savings and demand for Liquidity equals supply of Money. While not the textbook IS-LM model, these can be used similarly by shifting curves (see fn. 11). Of course, w is endogenous, but in principle (25) can be solved for it and used to write (23)-(24) to get two equations in (K, Z). We cannot solve for w explicitly, however, except in special cases, like $\theta = 1$, which is too restrictive in theory, or $f(k, h)$ linear in h, which is too restrictive for quantitative work.

In any case, while in principle it may be nice to work in (K, Z) space – after all, the paper is about capital and liquidity – in practice it is better to regard (23)-(24) as two equations in (L, B). As shown in Fig. 5, intersections of (23)-(24) in (L, B) space are monetary steady states, then K, Z and w follow from (10), (22) and (25). As an important component of the theoretical analysis, Appendix B establishes existence and uniqueness of monetary steady state under certain conditions. These conditions are needed since, as is known from related work, monetary equilibrium does not exist if θ is too small, ι is too big, or credit is too easy.

Proposition 2 *Assume θ is not too small, while χ_0, χ_Π and χ_k are not too big. Then there exists a unique monetary steady state iff $\iota < \bar{\iota}$, where $\bar{\iota} > 0$.*

Table 5: Comparative Statics

	ι	χ_0	χ_k	χ_q	χ_Π	τ_k	τ_h	G	A	ι^*	$\iota^\dagger$
K	$\pm$	0	$+$	$\pm$	$\pm$	$-$	$-$	$+$	$+$	$\pm$	$+$
Z	$\pm$	$-$	$\pm$	$\pm$	$\pm$	$\pm$	$-$	$+$	$+$	$\pm$	$\pm$
w	$\pm$	0	$+$	$+$	$+$	$-$	0	0	$+$	$-$	$+$
c	$\pm$	0	$+$	$+$	$+$	$-$	$-$	0	$+$	$-$	$+$
H	$\pm$	0	$\pm$	$\pm$	$\pm$	$\pm$	$-$	$+$	$\pm$	$\pm$	$\pm$
Y	$\pm$	0	$+$	$\pm$	$\pm$	$-$	$-$	$+$	$+$	$\pm$	$+$
Φ	$-$	0	$+$	$+$	$\pm$	$-$	0	0	0	$-$	$-$

Notes: All effects assume χ_q is big and χ_π small; $*$ also assumes χ_k is small; † assumes χ_k and θ are big.

Table 5 shows the effects of parameters on standard macro variables, plus the probability of a full sale in any given meeting, denoted Φ. As mentioned, here we are restricting χ_Π to be not too big, and for Table 5 we also assume χ_q is not too small. While these are not unnatural restrictions, still many entries are ambiguous, shown by $\pm$. This is not because the theory is messy; this because it is rich enough to make some effects nonmonotone – e.g., standard macro variables can be nonmonotone in ι, as can be verified numerically. In particular, we can have $\partial K/\partial\iota > 0$ or $\partial K/\partial\iota < 0$, with the former reminiscent of the Mundell-Tobin effect reflecting the fact that K and Z are substitutes, but as payment instruments here, not arguments of utility.

Some results in Table 5 are unambiguous, including most of the effects of fiscal policy. A particularly relevant result is $\partial Z/\partial\chi_0 < 0$, which says that an increase in credit reduces money demand, pushing up the price level, and showing up as inflation in the short run even if that is pinned down by $\phi/\phi_+ = 1+\mu$ in the long run. Similarly, most of the effects on Φ are unambiguous, including $\partial\Phi/\partial\iota < 0$, $\partial\Phi/\partial\chi_k > 0$ and $\partial\Phi/\partial\chi_q > 0$, which together with $\partial Z/\partial\chi_0 < 0$ relate directly to the discussion in the Introduction about how we intend to explain the facts.[11]

In addition to existence, uniqueness and comparative statics, welfare can be studied analytically. Appendix B compares the solution to the planner problem and equilibrium with $\tau_k = \tau_h = 0$. For q to be efficient in equilibrium we need full sales in all meetings, which for an arbitrary ε distribution requires $\iota \to 0$. For K to be efficient we need sellers

[11] As mentioned, the results in Table 5 can be illustrated by shifting curves in Fig. 5, and we use this in the proofs in Appendix B. While there we study a more general case, note that restricting $\chi_k = 0$ makes graphical analysis especially easy since then, e.g., increasing ι shifts LM but not IS, leading to lower L, which implies fewer full sales, more partial sales and less total reallocation, plus higher B, which implies more profit per unit of K. If χ_k is too big, however, L still decreases but B can increase or decrease. One can similarly handle fiscal policy, which is especially easy if $\chi_q = 1$, since then τ_k affects IS but not LM.

in the DM to reap the full benefit of their investments, meaning $\theta = 0$, but there is no monetary equilibrium at $\theta = 0$. More generally, there is a two-sided holdup problem: given $\iota > 0$, high θ is needed to get money demand right; low θ is needed to get capital demand right; and we can't have both.

However, for any $\theta > 0$, efficiency obtains when $\iota \to 0$ if we set $\tau_h = 0$ and implement a corrective subsidy on capital formation financed by the lump sum tax T.

Proposition 3 *Efficiency is not possible at $\iota > 0$. When $\iota \to 0$, monetary steady state is efficient if $\tau_h = 0$ and $\tau_k = \tau_k^*$, where $\tau_k^* \leq 0$ with strict inequality unless $\theta = 0$, is given by*

$$\tau_k^* = 1 - \frac{\int_0^\infty \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha \int_{\hat{\varepsilon}<\tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon}) \, dF(\tilde{\varepsilon}) \, dF(\hat{\varepsilon})}{\int_0^\infty \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha (1-\theta) \int_{\hat{\varepsilon}<\tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon}) \, dF(\tilde{\varepsilon}) \, dF(\hat{\varepsilon})}. \tag{26}$$

This does not mean the Friedman rule is always optimal. If $\tau_h \neq 0$ or $\tau_k \neq \tau_k^*$, it is possible that welfare can be increasing in ι over some range, as can be verified numerically.

Before moving to the numerical work, consider a pure credit setup with $\phi M = 0$ (no money) and $\chi_0 > 0$, $\chi_j = 0$ for $j \neq 0$ (no collateral). This does not mean perfect credit, in general, since DM trade is constrained by $d \leq \chi_0$. This version has one nice feature: it immediately gives two equations in (K, w). The reason we consider it is that most papers on capital reallocation do not have money, and while that may be a deficiency, the nonmonetary version of our framework facilitates comparison, and this is taken up quantitatively in Section 5.4. The Appendix proves:

Proposition 4 *With pure credit steady state exists. It is unique if α is not too big. It is efficient if χ_0 is not too small, $\tau_h = 0$ and $\tau_k = \tau_k^*$ as given in (26).*

5 Quantitative Results

5.1 Calibration

Many of the parameters are standard in the RBC literature, and we follow those methods wherever possible. Also, we assume ε is i.i.d. for now, but that is relaxed in Section 6. Our sample period is 1984 to 2018, where the P and R shares are relatively stable. Parameter values are given in Table 6, and here is how they are set:

For fiscal policy, we use $G/Y = 0.20$, $\tau_k = 0.25$ and $\tau_h = 0.22$, which is a good approximation for the US (Gomme and Rupert 2007). For monetary policy, we measure π by annual PPI inflation, which averages 2.68% in the period. For the illiquid nominal

Table 6: Calibrated Parameters and Targets

Parameter	Value	Explanation	Parameter	Value	Explanation
ι	0.067	nominal AAA yield	μ_γ	-1.554	R share
β	0.962	real AAA yield	χ_q	0.884	P share
ξ	2.39	labor hours	χ_Π	0.113	cash/output
η	0.60	labor share	τ_k	0.25	capital tax rate
δ	0.10	depreciation	τ_h	0.22	labor tax rate
σ_ε	1.30	COMPUSTAT	G	0.201	gov't share
σ_γ	0.275	acquisition elasticity	θ	0.50	symmetry

rate ι, the average AAA corporate bond yield is 6.72%, and the illiquid real rate solves $1 + r = (1 + i)/(1 + \pi) = 1.0393$, implying $\beta = 0.962$.[12] Then we use $u(c) = \log(c)$ and set the coefficient on leisure to get hours worked as a fraction of discretionary time 33%, a standard target from time-use surveys (Gomme et al. 2004). For technology, labor's share is $\eta = 0.60$, in the range usually used (Christiano 1988 argues one can reasonably say it is anywhere from 0.57 to 0.75, depending on data interpretation). Then depreciation is $\delta = 0.10$ to match $K/Y = 2.20$. This completes the standard RBC parameters.

For the rest, to begin with credit, we set $\chi_0 = \chi_k = 0$ for this reason: The general framework allows several kinds of credit that are interesting in principle, but it can be difficult to identify them all in practice. Most papers use just one; we use two, χ_q and χ_Π. As regards χ_q, it is reasonable to think new capital secures purchases of new capital the way houses secure mortgage loans (Kiyotaki and Moore 1997). As regards χ_Π, the pledgeability of profit is regarded as crucial in finance (Holmstrom and Tirole 1998; see also Li 2022), and we want to integrate that into a macro model. Hence we focus on χ_q and χ_Π, but before calibrating them we must discuss other parameters that are set jointly.

Assume ε is log-normal, with normalized $\mu_\varepsilon = 0$ and $\sigma_\varepsilon = 1.30$, in line with previous studies (e.g., Imrohoroglu and Tuzel 2014). To explain, it is standard to fit an AR(1) for the log of productivity, which gives a persistence coefficient 0.70 and standard deviation 0.37 in COMPUSTAT, so the unconditional variance and standard deviation are $0.37^2/(1-0.7^2)$ and 0.52. That is for total productivity, while ε in the model is capital productivity, so $\sigma_\varepsilon = 0.52/(1 - \eta) = 1.30$. Assume the entry cost γ is also log-normal with mean μ_γ and standard deviation σ_γ, calibrated jointly with χ_q and χ_Π to match four targets: average R

[12] Similar to using the PPI rather than CPI, we use AAA corporate bonds rather than T-bills since we are studying firms. Also, corporate bonds may have more risky yields, but our agents are risk neutral wrt yield by Lemma 1, so these bonds correspond well to our definition of ι (the dollars agents require in the next CM to give up one in this CM). Also, it is generally agreed in finance that corporate bonds are less liquid than T-bills, or less convenient (Krishnamurthy and Vissing-Jorgensen 2012), which we interpret as less liquid.

and P shares of 0.32 and 0.24; average firm money holdings over output of 4.2%; and an elasticity of acquisition spending wrt inflation of -0.64.[13]

Buyers' bargaining power is set to $\theta = 1/2$, since symmetric bargaining is natural with ex ante identical agents, but we check below how the results vary with θ. The final detail involves measurement of the DM entry cost. As always, one has to decide whether it is in terms of, say, utility, labor or output. We assume entry requires labor services from a financial sector, which is not modeled explicitly, but the idea is simple enough: participating in the DM uses hours employed in this service multiplied by w, and total output is that plus CM output. The parameter capturing this – i.e., how much labor is needed to enter the DM – is then set to 1.7% to match the share of financial services in total output from BEA data, which seems a reasonable way to do the accounting.

5.2 Long-Run Results

The first exercise concerns the impact of inflation on steady state. Fig. 5 shows this for three bargaining powers, $\theta = 0.50$, 0.45 and 0.55, recalibrating other parameters in each case. The top row shows standard macro variables, Y, K, C and H; the middle row shows reallocation variables, the R and P shares, plus the probability Φ of a full sale and the average DM price; the bottom row shows productivity, welfare, money and credit. A vertical line indicates a threshold beyond which monetary equilibria vanish.[14]

First notice that although output is nonmonotone in π, in general, it is decreasing in π at calibrated parameters. Second notice there is a Phillips curve: H increases with π, and recall (from fn. 6) that $1 - H$ can be interpreted as unemployment, not leisure. Also, K increases with π over a large range, and the effect is sizable: at $\theta = 0.5$, as π goes from 0 to 10%, K rises about 6%. However, the effect of π on K is not monotone: it decreases K around the Friedman rule (not plotted). An implication is that if one were to look at the relationship between K and π, one would get different results depending on the range of π in the sample. Still, as π rises agents consume less and work more, so welfare falls.

[13] Firm money includes checkable deposits plus currency held by nonfinancial corporate and noncorporate businesses, from FRED, divided by output to make it stationary. The elasticity of acquisition spending wrt inflation is estimated using COMPUSTAT controlling for various factors, as discussed in Section 2, and here acquisitions mean full sales. Depending on the specification, estimates range from -0.29 to -0.99, and we settled on -0.64. Generally, this is meant to emulate methods for household money demand, where cash over output and its elasticity are key targets. Of course, there is past work on firm money demand, too, but we like getting the numbers from the same data used for reallocation variables.

[14] The threshold is the $\bar{\iota}$ from Prop. 2. We do not take it too seriously, however, as monetary equilibria would survive beyond $\bar{\iota}$ if, say, cash were also demanded by households, or by firms for reasons other than reallocating k. Also notice some variables jump at $\bar{\iota}$; this is not numerial error, but what theory predicts.

Welfare is maximized over the range shown at $\pi = 0$, but over a larger range the Friedman rule is optimum for these parameters, if not in general. The bottom row shows the cost of inflation defined in a standard way: the amount of consumption agents would give up to go from π to 0, as a fraction of some benchmark, which here is consumption at calibrated parameters and the average π in the sample. For each θ the curves start at the origin, by construction. At $\theta = 0.5$, going from 10% inflation to 0 is worth around 1.4% of consumption, but again $\pi = 0$ is not the best we can do, and going to the Friedman rule from 10% is worth around 4.7% of consumption.[15]

The middle row of Fig. 5 concerns reallocation. As in the long-run data, higher π decreases the R share, increases the P share, and decreases the probability of full sales. Also, higher π raises the DM price $(p + d)/q$, consistent with the facts, and note that this is the average price, since the q a firm gets and the p it pays depend on the meeting – and in case it is not obvious, we note that the law of one price does not hold, consistent with the stylized facts on secondary capital markets (recall fn. 1). The bottom row show higher π lowers average productivity by hindering reallocation.

5.3 Medium-Run Results

Next, inspired by some referee remarks, we ask if the model matches medium-run observations. The idea is that one might worry the empirical findings in Section 2, on the long-run relationship between π and reallocation, are basically driven by two observations: first there was high π with a low R and high P share; then there was low π with a high R and low P share.

Fig. 6 plots the empirical R and P shares starting in 1971 along with the predictions given by the calibrated model when we input empirical inflation rates, focusing on the medium run. Medium run means that we average π in the data over 5-year subperiods and look at steady state equilibrium in that same subperiod. We do this in two ways, using fixed-window and rolling averages, always performing the same calculations on the model and data. Holding other parameters fixed, this shows how well inflation explains the medium-run patterns in reallocation.

[15] For comparison, in models of household money demand without capital, like Lagos and Wright (2005), going from 10% to 0 is worth 4.6% of consumption and going to the Friedman rule is worth 6.8%. In similar models with capital, like Aruoba et al. (2011), the numbers are somewhat lower, while in reduced-form monetary models with or with capital, like Cooley and Hansen (1989) or Lucas (2000), they are much lower. We initially expected bigger numbers here since: (i) macro public finance tells us taxing capital is generally a bad idea; and (ii) micro public finance tells us big distortions come from taxing things with close substitutes, and CM k is a reasonably close substitute for DM k. These effects are present, but attenuated by inflation stimulating investment, which tends to be too low due to holdup problems in DM bargaining.

The model tracks the data well: changes in inflation account for much of the pattern in the R share and P share, not only over two broad episodes, one with high and one with low π, but in the medium run as well. Of course the fit is not perfect, as many factors not in the experiment could affect things, but overall the fit is remarkably good.[16]

5.4 Short-Run Results

Next we ask how well the model accounts for business cycles. Motivated by the earlier discussion of economic intuition and empirical findings, we allow shocks to aggregate productivity A and to credit conditions captured by χ_q. Namely,

$$\begin{aligned} \ln A_t &= \rho_A \ln A_{t-1} + \varsigma_{A,t} \\ \ln \chi_{q,t} - \ln \chi_q &= \rho_\chi \left(\ln \chi_{q,t-1} - \ln \chi_q \right) + \varsigma_{\chi,t}, \end{aligned}$$

where $\varsigma_{A,t} \sim N(0, \sigma_A^2)$ and $\varsigma_{\chi,t} \sim N(0, \sigma_\chi^2)$ are i.i.d. and orthogonal. We set $\rho_A = \rho_\chi = 0.83$, as is standard in yearly models, corresponding to 0.95 in quarterly models. When using only A shocks, σ_A is set to 4.58% to match the volatility of output; when using both shocks, σ_A is set to 3.69% and σ_χ to 3.00% to match the volatility of output and the R share; then ask how well we capture the volatility and correlation with output for other variables.

The first three columns of Table 7 show standard deviations from the data, the model with A shocks, and the model with both shocks; other columns show correlations with Y; and model as well as data statistics are computed after taking logs and filtering out lower frequencies. For Y, C, I and H, the model does well accounting for volatility and correlation, by the standards of the literature, and indeed is similar to the textbook RBC model with only A shocks or with both shocks. This shows our new features do not impair performance of RBC theory in capturing the basic business cycle facts.

How about reallocation dynamics? On that, the model with only A shocks is really way off. For the R share, the standard deviation is too small, and the correlation with Y is -0.86 instead of $+0.63$. For the P share, the standard deviation is far too small, only 1.35 compared to 8.42 in the data, and the correlation with Y again takes the wrong sign, this time $+0.96$ instead of -0.53. The conclusion is that with only aggregate productivity shocks, the model does not capture reallocation dynamics at all.

[16] Potentially relevant factors not in the experiment include technical progress, tax changes and financial innovation. Also, we do not plot firm money holdings since that is hard to fit. One reason is that firms value liquidity for purposes other than capital reallocation. Others include the factors mentioned above, especially financial innovation and regulatory changes over the period, which make the empirical notion of money a moving target (see, e.g., Lucas and Niolini 2015). Hence we focus on dynamics in the R and P shares.

Table 7: Business Cycle Statistics

	Standard deviation			Correlation with output		
	Data	A only	A & χ_q	Data	A only	A & χ_q
Output	1.92	1.92	1.92	1.00	1.00	1.00
Consumption	0.63	0.61	0.90	0.91	0.99	0.93
Investment	2.74	3.19	2.67	0.96	0.99	0.93
Employment	0.74	0.55	0.62	0.90	0.96	0.96
TFP	0.62	0.71	0.60	0.79	0.99	0.95
R share	5.74	1.22	5.74	0.63	-0.86	0.47
P share	8.42	1.35	11.62	-0.53	0.96	-0.50
Inflation	1.93	0.07	0.46	0.37	-0.99	0.39

Note: Standard deviation of other variables are relative to output.

What about the model with both χ_q and A shocks? It obviously does much better, and by the standards of the literature it accounts for the data quite well. Of course it matches the volatility of the R share, since σ_χ is calibrated to that, but we did not target the correlation between the R share and Y, which now has a similar magnitude and the correct sign. For the P share, the standard deviation and correlation with Y look reasonably good, and certainly better than with only A shocks.

In terms of subsidiary results, first, with only A shocks inflation is counterfactually countercyclical and not volatile enough, but with both shocks its correlation with Y has the correct sign and its volatility is not too far off. Also, with both shocks inflation is correlated positively with the R share and negatively with the P share (not shown in the Table but correlations with Y are 0.78 and -0.92), so they have the right signs if higher magnitudes than the data (0.24 and -0.14). We do not push this, however, as the theory abstracts from features that may influence inflation dynamics, e.g., a more detailed model of monetary policy. We prefer to emphasize that the average DM price is procyclical (not shown but the correlation with Y is 0.61), and productivity dispersion measured by the coefficient of variation is countercyclical (not shown but the correlation with Y is -0.64). Hence, the model matches all the stylized capital reallocation facts stressed in the literature and summarized in the Introduction .

It is also interesting to see how the two shocks affect different variables – e.g., the results on the DM price are driven mainly by A shocks, while the results on productivity dispersion are driven by χ_q shocks. Moreover, having two shocks has an advantage stressed by Christiano and Eichenbaum (1992): it breaks the tight relationship between A and H in one-shock RBC models that is not in the data. Although the labor market is not our main focus, we can report a correlation between A and H of 0.61, which is a big

Table 8: Business Cycle Statistics without Money

	SD			Corr with output		
	Data	A only	A and χ_q	Data	A only	A and χ_q
Output	1.92	1.92	1.92	1.00	1.00	1.00
Consumption	0.63	0.54	0.77	0.91	0.97	0.88
Investment	2.74	3.39	3.09	0.96	0.99	0.94
Employment	0.74	0.49	0.54	0.90	0.96	0.96
TFP	0.62	0.78	0.71	0.79	0.99	0.96
R share	5.74	2.68	5.74	0.63	-0.98	0.01
P share	8.42	1.40	4.73	-0.53	0.96	-0.17
Inflation	1.93	-	-	0.37	-	-

Note: SD is standard deviation relative to output, except for output itself.

step in the right direction compared to one-shock models, and the standard deviation of H is not bad. So the model not only captures reallocation dynamics without sacrificing performance on the business cycle facts, it sometimes improves on the latter (although not always, e.g., consumption is a bit too volatile).

For comparison, consider the nonmonetary model from Prop. 4, to see if our results are due mainly to credit shocks, or if money also matters.[17] The results are in Table 8, with parameters re-calibrated, and in particular credit shocks re-set to still match volatility of the R share. On standard macro variables and at least on the volatility of reallocation variables, the nonmonetary model does well, but it does not deliver a positive correlation between the R share and Y, or the same degree of negative correlation between the P share and Y. Having money evidently helps get these right. One intuition is that without money we need a large χ_q to match first moments, but then the credit shocks generating the right volatility in the R share do not induce enough correlation between reallocation variables and Y. Also, a monetary model is needed for all the results on inflation and reallocation in the short, medium and long run. So, yes, money matters.

6 Extensions and Ideas for Future Work

We now pursue Wallace's insights about inflation and multiple assets, then study the impact of search and taxation, then add heterogeneity from persistent firm-specific shocks. As each of these can become quite involved, details are left to future work, but we want to show the framework has many potentially interesting applications.

[17] The nonmonetary model resembles Kahn and Thomas (2013). They do not examine capital reallocation, but they argue that credit shocks capture key features of more standard macro variables in the data.

6.1 Multiple Liquid Assets

Consider the baseline model, except now, in addition to money, there is a long-lived real asset a in fixed supply, with CM price ψ and dividend ρ (the proverbial Lucas tree, but we also considered bonds). In the DM both z and a can be used for payments. For generality we allow three types of DM meetings: with probability α_1 only z is accepted; with probability α_2 only a is accepted, and with probability α_3 both are accepted. We also allow general χ_z and χ_a, but fix $\chi_q = 1$ and $\chi_k = \chi_\pi = 0$, because we are more interested in asset liquidity than credit for this exercise.

Now wealth Ω includes $(\psi + \rho)\, a$, and the CM problem becomes

$$\begin{aligned} W(\Omega, \varepsilon) &= \max_{c,h,\hat{a},\hat{k},\hat{z}} \left\{ u(c) - \xi h + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} V_+(\hat{a}, \hat{k}, \hat{z}, \hat{\varepsilon}) \right\} \\ \text{st } c &= \Omega + (1-\tau_h) w h - \hat{z}\phi/\phi_+ - \hat{k} - \psi \hat{a}. \end{aligned}$$

It is routine to derive the first order conditions and Euler equations. Normalizing the supply of the real asset to 1, letting $Z_a = (\rho + \psi)\, \chi_a$ and emulating earlier methods, we get

$$\begin{aligned} \frac{r+\delta}{B(w)(1-\tau_k)} &= \mathbb{E}\varepsilon + (1-\theta)\left[\alpha_1 I_s(L_1) + \alpha_2 I_s(L_2) + \alpha_3 I_s(L_1+L_2)\right], \\ \iota &= \alpha_1 \chi_z \lambda(L_1) + \alpha_3 \chi_z \lambda(L_1+L_2), \\ r Z_a &= (1+r)\chi_a \rho + \beta Z_a \chi_a \left[\alpha_2 \lambda(L_2) + \alpha_3 \lambda(L_1+L_2)\right] \end{aligned}$$

in steady state, where

$$\lambda(L) \equiv \iint_{\mathcal{S}_b(L)} \frac{\alpha\theta(\varepsilon - \tilde{\varepsilon})\, dF(\tilde{\varepsilon}) dF(\varepsilon)}{(1-\theta)\varepsilon + \theta\tilde{\varepsilon}},\ L_1 \equiv \frac{\chi_z Z}{(1-\tau_k) B(w) K} \text{ and } L_2 \equiv \frac{\chi_a Z_a}{(1-\tau_k) B(w) K}.$$

Notice L_1 and L_2 represent the liquidity embodied in money and in real assets.

If $\theta = 1$ the system reduces to two equations in (L_1, L_2),

$$\begin{aligned} \iota &= \chi_z\left[\alpha_1 \lambda(L_1) + \alpha_3 \lambda(L_1+L_2)\right], \\ r &= \Upsilon + \chi_a\left[\alpha_2 \lambda(L_2) + \alpha_3 \lambda(L_1+L_2)\right], \end{aligned}$$

where $\Upsilon \equiv (1+r)\chi_a \rho/(1-\tau_k) B(w) K L_2$. Suppose ρ is small. Then so are $\partial\Upsilon/\partial L_1$ and $\partial\Upsilon/\partial L_2$. Given that, it is routine to derive

$$\frac{\partial L_1}{\partial \iota} < 0,\ \frac{\partial L_2}{\partial \iota} > 0 \text{ and } \frac{\partial(L_1+L_2)}{\partial \iota} < 0.$$

Intuitively, as ι rises the liquidity embodied in cash falls, so agents try to substitute into real assets (another Mundell-Tobin effect). This increases the price and hence the liquidity embodied in real assets, but total liquidity, $L_1 + L_2$, falls. By continuity, if $\theta < 1$ is not too small and $\rho > 0$ not too big, the results still hold. Wallace (1980) did not derive these effects – his framework at the time used Walrasian markets where liquidity has no role – but they are consistent in spirit with his words quoted above. The key point is that inflation can reduce overall liquidity even if it directly taxes only cash. It would be interesting to quantify this version, but much more complicated, so it is left to future research.

6.2 Effects of Search Frictions

We now turn to a quantitative question motivated by claims (recall fn. 1) for the relevance of search in secondary capital markets: How much does the DM arrival rate α matter? One issue is how the business cycle results in Table 7 depend on α. For standard macro variables, α does not matter much, but that may not be too surprising given both $\alpha = 0$ (the textbook RBC model) and our calibrated α do well on that. Of course, it matters for reallocation dynamics – indeed, at low α the DM shuts down – but we are more interested in less obvious results.

Fig. 7 shows steady states as varies α from 0 to 1 in three specifications: perfect credit; constrained credit without money; and constrained credit with money. These can generate the same outcome – e.g., having easy credit, captured here by big χ_k, and running the Friedman rule in a monetary equilibrium, are equivalent to perfect credit. To keep outcomes distinct, in the second case we set $\chi_k = 0.11$, and in the third $\iota = 0.01$, with other parameters at calibrated values. First note the DM is closed (no one enters) unless α is above a threshold around $1/2$. Second, high α makes the economy better off, naturally, but the magnitudes are interesting. As α goes from $1/2$ to 1, output rises between 10% and 15% with perfect credit, and almost as much with money. To put this in perspective, with perfect credit the impact on welfare of raising α to 1 is bigger than the impact of lowering π from 10% to 0 in the monetary model.

Since search fictions apparently matter a lot, it would be useful to investigate alternative specifications, such as directed search and price posting, rather than random search and bargaining, since this is known to affect quantitative results in models of household liquidity (e.g., Bethune et al. 2020). That is not a trivial extension, however, since standard models of directed search and posting have agents exogenously partitioned into buyers and sellers, while here that would be a choice. Hence it is relegated to future work.

6.3 Fiscal Policy

There are many experiments one could run on taxes, but Fig. 8 plots steady state against τ_k, ranging from the calibrated $\tau_k = 0.25$ down to large negative values. Clearly τ_k has big effects on standard macro variables, but that is also true in other models (e.g., McGrattan et al. 1997; McGrattan 2012). Yet it does not matter much for the P and R shares. In fact, the absolute amount of reallocation moves a lot, but the R share is approximately constant since investment also moves a lot. We do see an impact on money demand, since reallocating capital is less lucrative when taxation is heavier. One reason for the big effect of τ_k on some variables is that the revenue change here is made up by adjusting the lump sum T, which may not be very realistic.

Still, we can report that steady state welfare is maximized, given the calibration, including τ_h and ι, at $\tau_k = -0.57$.[18] Alternatively, the optimum when $\tau_h = 0$ and $\iota \to 0$, from Prop. 3, is $\tau_k^* = -0.86$. Further investigating fiscal policy seems interesting. Aruoba and Chugh (2010) solve a Ramsey optimal policy problem in a model of household liquidity, and find search-and-bargaining frictions matter a lot, so it seems worth pursuing optimal taxation in models of firm liquidity, but again it is left to future work.

6.4 Persistent Shocks

The firm-specific shocks used above are i.i.d. Suppose ε can be decomposed into a persistent component ε_1 and a transient component ε_2,

$$\log \varepsilon = \log \varepsilon_1 + \log \varepsilon_2.$$

Assume $\varepsilon_1 \in \{1 - x, 1 + x\}$, with $x \in [0, 1)$, so ε_1 is a two-state Markov process, with $\log \varepsilon_2$ i.i.d. normal. Firms' $(\hat{k}, \hat{z})$ choices in the CM now depend on their persistent component ε_2. Hence, we lose the degeneracy of the (k, z) distribution at the start of the DM, but maintain history independence – i.e., Lemma 2 holds for each ε_1.[19]

The first observation is that adding persistence does not have much impact on the business cycle properties of standard aggregate variables reported in Table 7. This is reminiscent of Rios-Rull (1996), who asks how results in RBC models change when one

[18] In terms of accounting, this means $\$1$ in profit gets 57 cents in subsidy; equivalently, $\$1$ of investment at t gets $\tau_k B(1-\delta)^{t+i} J(L, w)$ in subsidy for every $i > 0$, which in total is worth 82 cents at t.

[19] Because the formulation with this kind of heterogeneity is computationally harder, for this exercise we shut down entry by fixing the number of agents in the DM. This is not a big deal, however, because our only intent is to compare versions of this model with different productivity gaps, not this model vs the benchmark specification.

incorporates realistic life cycles rather than a representative agent. His finding is that the responses of aggregate variables to aggregate shocks do not change much, which is nice since models with a representative agent are much easier to solve. Yet a life-cycle economy generates more than aggregate statistics, it generates cross sections due to age heterogeneity. Hence, there are many more business cycle predictions one can take to the data, e.g., the volatility of H for different age groups (Gomme et al. 2004). Similarly, our model with persistent firm shocks generates more business cycle predictions one can take to data, but a serious analysis of that is beyond the scope of the paper.

Less ambitiously, for now, we concentrate on the impact of x on steady state. We keep other parameters the same and use a switching probability $1-\omega$ to get a stationary distribution where half of the firms have $\varepsilon_1 = 1-x$ and half have $\varepsilon_1 = 1+x$. Setting $\omega = 0.75$ and varying x but keeping average productivity the same, we show the results in Figure 9, where the horizontal axis is the gap $2x$, and we report variables for both high- and low-productivity firms.[20]

If x is larger, high ε_1 firms invest in more k in the CM, and low ε_1 firms less, reflecting differences in expected future productivity, but cash holdings can go up or down depending on details including bargaining power θ. Perhaps surprisingly, for these parameters high ε_1 firms hold less cash. This is because they know their productivity is likely to be higher, and find it optimal to shift investment from the DM to the CM for two reasons: they may not be able to get enough k in the DM; and for them the liquidity value of k is big due to χ_Π. Similarly, low ε_1 firms substitute out of capital and into cash for two reasons: after the CM closes it is easier to trade cash (given our frictionless banking specification) than capital; and for them the liquidity value of k is small.

Liquidity in terms of cash plus credit can increase or decrease with the gap. Even if high ε_1 firms have lower liquidity, they need not be more constrained: high ε_1 buyers are less constrained than low ε_1 buyers when trading with low ε_1 sellers, since the former can leverage their size advantage. Also, notice the R share falls slightly with the gap because the DM is used to partially insure the i.i.d. component of idiosyncratic shocks. As the gap increases, the i.i.d. component contributes less volatility and investment shifts to the primary market, so the R share drops. The P share also drops, as some firms get bigger and hold less cash.

The last row of Figure 9 shows the composition of full sales in terms of trading parties.

[20] Notice that when ε_1 is a 2-state process there is a two-point distribution of (k, m) after the CM, but a much more interesting distribution after the DM. Obviously it may be even more interesting to use an N-state process, although that would be computationally more intense. In principle, the framework could be used to try to match the empirical distribution of firm size by K or Z, or even by H.

The first panel is the fraction of full sales where both the buyer and the seller are big, i.e., high ε_1 firms; the second is fraction where the buyer is big and the seller small; and so on. As ε_1 increases, it is easier for big firms to fully purchase small firms, and the reverse is harder. A full sale is most likely to occur when a big firm meets a small firm. Interestingly, there are also a fair number of full sales where small firms buy small firms. Small firms hold a lot of cash, which allows them to fully purchase other small firms if their productivity turns out to be high.[21]

7 Conclusion

This paper developed a model consistent with empirical relationships related to different types of capital reallocation and inflation. Theory predicts higher inflation lowers liquidity, which decreases (increases) full (partial) sales. This captures long- and medium-run patterns in the data. Then we added credit shocks. Better credit conditions reduce the demand for money, increasing short- but not long-run inflation, as well as increasing (decreasing) full (partial) sales. This captures business-cycle patterns in the data. Importantly, the model can also account for business cycle patterns in standard macro variables.

For some observations a nonmonetary version of the model does a decent job; for several other observations money matters. The framework also provides qualitative and quantitative insights into how bargaining, search and fiscal policy affect reallocation and standard macro variables. Additionally, it allows us to study how persistence in idiosyncratic shocks affects capital and liquidity positions. The model yields analytic results on existence, uniqueness and comparative statics, and is amenable to calibration. This suggests there may be other applications for the framework, some of which were sketched above. There is much left for additional research.

[21] We thank the Editor Harald Uhlig for motivating this by emphasizing the fact that it is big firms that typically "swallow" small firms. That made us think hard about heterogeneity, induced here by persistent shocks. Other types of heterogeneity may be relevant, too, but this is enough to match the fact. Without going into much detail we also can mention some applications suggested by the referees. One is to notice that reallocation is currently high despite an uptick in expected inflation. A lot has been going on recently, including a pandemic, and we do not claim to fit every nuance of the current episode, only broad business cycle patterns. Still, trying to explain the current situation would be interesting. Another idea is to ask if decisions between partial and full sales are driven mainly by liquidity costs, agency considerations, or implications for taxation. We abstract from complicated tax and agency factors to see how far the model can go without them. It goes quite far. Still, future work could pursue this issue. One can also introduce vintages of capital, where partial sales may arise not only due to liquidity, but because a buyer wants some but not all types of a seller's capital. And one could consider modeling firms selling off capital to raise cash, which does not happen here, but could be interesting.

Appendix A: More on Data

Financial data are from the Flow of Funds Accounts (Z1 Report of the FRB). We use the Coded Table released in 2018; new editions may use different coding. Corporate plus noncorporate nominal debt is the sum of Debt Securities (Table F.102, item 30) and Loans (Table F.102, item 34), with the GDP implicit price deflator (Table 1.1.9 in NIPA) putting these in 2012 dollars. Aggregate output is measured by business value-added from Table 1.3.5 in NIPA. Aggregate consumption and investment are from Table 1.1.5 in NIPA, excluding inventory, government and net exports, in 2012 dollars (after we use Table 1.1.9 in NIPA).

The AAA corporate bond yield, PPI and CPI inflation are from FRED. For total factor productivity (TFP), we use the annual table Fernald (2014) for the growth rate, then normalize the starting year observation to 1. From this we construct TFP for each year until 2018. For labor supply, we use BEA's hours worked by full-time and part-time employees.

For capital reallocation, COMPUSTAT (North America) has information on ownership changes of productive assets starting in 1971. Capital reallocation is measured by sales of property, plant and equipment (SPPE, data item 107 with combined data code entries excluded), plus full purchases (AQC, data item 129 with combined data code entries excluded) from 1971 to 2018. We also use capital spending (CAPX, data item 128). Since capital spending in COMPUSTAT excludes full sales, capital expenditures for each firm is the sum of AQC and CAPX. For the micro data in Section 2, industries are excluded with standard industry classification (SIC) codes below 1000 (agriculture, forestry and fishing), above 9000 (public and non-classified), and between 6000 and 6500 (financial).

Appendix B: Proofs

Prop 1: Either the constraints on p and d both bind or both are slack. Suppose they bind, and consider solving (12) ignoring the constraint on q. The Kalai condition $(1-\theta)\, S^b(\mathrm{s}, \tilde{\mathrm{s}}) = \theta S^s(\tilde{\mathrm{s}}, \mathrm{s})$ yields $q = Q$. If $Q < \tilde{k}$ then the true, constrained, solution is $q = Q$ and the constraints on p and d at equality. If $Q > \tilde{k}$ then the true solution is $q = \tilde{k}$ and the Kalai condition gives total payment. Finally, the threshold comes from rearranging $Q < \tilde{k}$. ■

Prop 2: Without loss of generality, set $\chi_\Pi = 0$. Consider IS. If $L \leq \underline{L} \equiv \varepsilon_L$, the integral in (23) is 0, and $B = \overline{B} \equiv (r+\delta) / [(1-\tau_k)\mathbb{E}\varepsilon + (1-\delta)\,\chi_k \iota]$. IS is decreasing and $B \to \widetilde{B}$ as $L \to \infty$, where

$$\widetilde{B} \equiv \frac{r+\delta}{(1-\tau_k)\left[\mathbb{E}\varepsilon + \alpha(1-\theta) \iint\limits_{\varepsilon<\tilde{\varepsilon}} (\tilde{\varepsilon}-\varepsilon)\, dF(\tilde{\varepsilon}) dF(\varepsilon)\right] + (1-\delta)\,\chi_k \iota}.$$

Intuitively, if L is larger the liquidity constraint is looser and opportunities for resale are better, so firms invest in more k even if the benefit from production B is low.

Now consider LM. If $L \le \underline{L}$, buyers are always constrained and $B = \underline{B}$ where

$$\iota = \iint\limits_{\varepsilon > \tilde{\varepsilon}} \frac{\alpha B \theta (1-\tau_k)(\varepsilon - \tilde{\varepsilon})}{\Delta(\varepsilon, \tilde{\varepsilon})} dF(\tilde{\varepsilon}) dF(\varepsilon).$$

Notice that $\underline{B}$ increases with ι and $\underline{B} = 0$ at $\iota = 0$. As L increases, buyers become less constrained. To make them willing to hold money it must be that the benefit B from reallocation is higher. Notice $B \to \infty$ as $L \to \tilde{L}$, where $\tilde{L}$ solves

$$\iota = \iint\limits_{\mathcal{S}_1(\tilde{L})} \frac{\alpha \theta (\varepsilon - \tilde{\varepsilon})}{(1-\theta-\chi_\Pi)\varepsilon + \theta \tilde{\varepsilon}} dF(\tilde{\varepsilon}) dF(\varepsilon).$$

If monetary steady state exists, it uniquely pins down B and L, and they uniquely determine w and Z/K. It remains to show K is unique. By the definition of $J(L,w)$ and (23), $J(L,w)B(w) \ge (r+\delta)/(1-\tau_k) > r$. So there is a unique $K > 0$ solving (25), implying steady state is unique. Existence is standard, so details are omitted. ■

Prop 3: We solve the planner problem given the DM frictions. First note that in the CM labor should be allocated to firms according to

$$h^*(k,\varepsilon) = \left[\frac{\eta u'(c)}{\xi}\right]^{\frac{1}{1-\eta}} A\varepsilon k. \tag{27}$$

Aggregating across firms gives total hours, and $h \le 1$ is assumed slack. Also, when two firms meet in the DM the higher ε firm should get all the capital. Given these observations, consider a planner choosing a path for k to maximize utility of the representative agent, subject to an initial k_0 and resource feasibility after government takes G_t units of x. Assuming ε is i.i.d., for simplicity, $\hat{k}$ is the same for all agents in the CM.

Then the problem can be written

$$\begin{aligned} W^*(k_0) &= \max_{k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \xi(1-h_t)\right] \\ \text{st } c_t &= y_t + (1-\delta)k_t - G_t - k_{t+1} \\ y_t &= (1-\alpha)\int_0^\infty (A\hat{\varepsilon}k_t)^{1-\eta} h^*(k_t,\hat{\varepsilon})^\eta dF(\hat{\varepsilon}) \\ &\quad + \alpha \int_{\hat{\varepsilon} > \tilde{\varepsilon}} (A\hat{\varepsilon}2k_t)^{1-\eta} h^*(2k_t,\hat{\varepsilon})^\eta dF(\hat{\varepsilon}) dF(\tilde{\varepsilon}) \end{aligned} \tag{28}$$

where output y_t includes production by the $1-\alpha$ measure of firms that did not have a DM meeting, the α measure that had a meeting and increased k, plus the α measure that had a meeting and decreased k. Routine methods and (27) yield the Euler equation

$$r_t+\delta=(1-\eta)\,A\left[\frac{\eta u'(c_{t+1})}{\xi}\right]^{\frac{\eta}{1-\eta}}\left[\int_0^{\infty}\hat{\varepsilon}dF(\hat{\varepsilon})+\alpha\int_{\hat{\varepsilon}<\tilde{\varepsilon}}(\tilde{\varepsilon}-\hat{\varepsilon})\,dF(\tilde{\varepsilon})\,dF(\hat{\varepsilon})\right]. \tag{29}$$

where r_t satisfies $1+r_t=u'(c_t)/\beta u'(c_{t+1})$. Recall that in equilibrium with $\iota\to 0$, transactions are efficient in the DM and the Euler equation for k_t is

$$r_t+\delta=(1-\tau_k)\,B(w_{t+1})\left[\int_0^{\infty}\hat{\varepsilon}dF(\hat{\varepsilon})+\alpha(1-\theta)\int_{\hat{\varepsilon}<\tilde{\varepsilon}}(\tilde{\varepsilon}-\hat{\varepsilon})\,dF(\tilde{\varepsilon})\,dF(\hat{\varepsilon})\right],$$

where $1+r_t=u'(c_t)/\beta u'(c_{t+1})$, $B(w)=(\eta/w)^{\frac{\eta}{1-\eta}}(1-\eta)\,A$ and $u'(c)=\xi/[(1-\tau_h)\,w]$. Comparing this with (29), one can see that $\theta>0$ implies agents do not fully internalizes the benefits of investment, so there is under accumulation of capital under under at $\tau_k=\tau_h=0$. But if $\tau_h=0$ and τ_k is given by (26), the first best is achieved. ■

Prop 4: The equilibrium is the same as the monetary equilibrium with $\iota=0$. Then, (23) defines a unique k for any $w\in(\underline{w},\bar{w})$, where

$$B(\underline{w})=\frac{r+\delta}{\mathbb{E}\varepsilon(1-\tau_k)},\ B(\bar{w})=\frac{r+\delta}{[\mathbb{E}\varepsilon+\alpha(1-\theta)\,I_s(\infty)]\,(1-\tau_k)}.$$

Suppose ε is bounded away from 0 and ∞. If $w=\underline{w}$, any sufficiently large K solves (23). If $w=\bar{w}$, any sufficiently small K solves (23). Also, (25) implies w is increasing in k. Moreover $w=0$ if $k=0$ and k is finite if $w=\bar{w}$. By continuity, there is a steady state. If α is not too big, the curve defined by (23) is decreasing in w, implying uniqueness. ■

Comparative statics: In what follows we set $\chi_{\Pi}=0$. Then (23) defines a downward sloping IS curve and an upward sloping LM curve.

Comparative statics wrt ι: If ι increases, the IS curve shifts down while the LM curve shifts up, resulting in a decrease in L and Φ. Other variables may go up or down. If $\chi_k=0$, the IS curve does not change, B goes up, hence c and w go down. By continuity, this holds if χ_k are small. If θ is big and χ_k is not too small, we can use (24) to eliminate ι in (23). This defines B as an increasing function in L, which we referred to as IS' curve. The unique intersection of IS' and LM corresponds to the equilibrium. An increase in ι does not change the IS' curve but shifts the LM curve up. Because the IS' curve is upward sloping, both B and L increase. Therefore, w increases and Φ decreases. Now notice that

$$J(L,w) \equiv \int \varepsilon dF(\varepsilon) + \alpha \iint\limits_{\varepsilon > \tilde{\varepsilon}} (\varepsilon - \tilde{\varepsilon}) \min\left\{1, \frac{L + \chi_\Pi \varepsilon + \frac{(1-\chi_q)(1-\delta)}{B(1-\tau_k)}}{(1-\theta-\chi_\Pi)\varepsilon + \theta\tilde{\varepsilon} + \frac{(1-\chi_q)(1-\delta)}{B(1-\tau_k)}}\right\} dF(\varepsilon)dF(\tilde{\varepsilon}).$$

Therefore, both B and J decreases. Then (25) implies K and Y increase.

Comparative statics wrt χ_0: As χ_0 does not affect (23)-(25), w, K, Y and L stay the same. Therefore, $(Z+\chi_0)/K$ is constant. If χ_0 increases, Z decreases.

Comparative statics wrt χ_k: Higher χ_k shifts the IS curve down and does not affect the LM curve. Hence B and L decrease, so w increases. If $\chi_q = 1$ then L stays constant and B decreases. Thus w and K increase. Additionally, Y increases because both c and K increase. By continuity, the same is true if χ_q is not too small.

Comparative statics wrt χ_q: If χ_q increases LM shifts down and IS stays the same. Hence L increases and B decreases, w and c go up and Φ increases. If θ is close to 1, the change in B is close to 0. As a result, w and c are almost unchanged. $B(w)J(w,L)/(1-\eta)$ increases because L increases, so K and Y decrease. If θ is not close to 1, the effects on K and Y are ambiguous. Φ increases because L increases.

Comparative statics wrt χ_Π: If χ_Π increases both LM and IS shift down. Hence B decreases, w and c go up and L may go up or down.

Comparative statics wrt τ_k: This shifts up both LM and IS, so $B(w)$ increases, L increases if χ_q close to 1, and w decreases. So Φ increases, and since χ_q is close to 1, $B(w)J(w,L)/(1-\eta)$ increases, so K, c and Y decrease.

Comparative statics wrt τ_h: This does not change B or L, so w and Φ stay the same, while c decreases. Then K decreases, which implies Y decreases. Also, Z decreases because L is unchanged and K decreases. Lastly, $H = (\eta/w)^{\frac{1}{1-\eta}} AJ(L,w)K$ decreases.

Comparative statics wrt A: This does not change B or L, so Φ stays unchanged. Because $B = (\eta/w)^{\frac{\eta}{1-\eta}} A$, w increases, which implies c increases. Then both K and Y increase by the goods market clearing condition. Also, Z increases because L is unchanged. The effect on H is unknown.

Comparative statics wrt G: The argument for G is similar to A.

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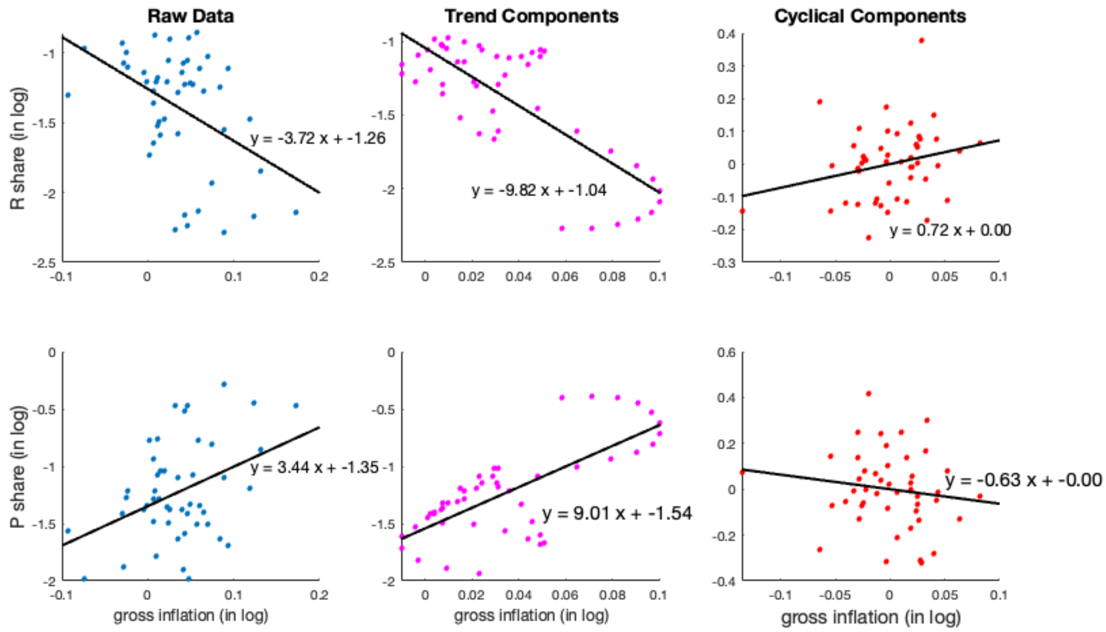
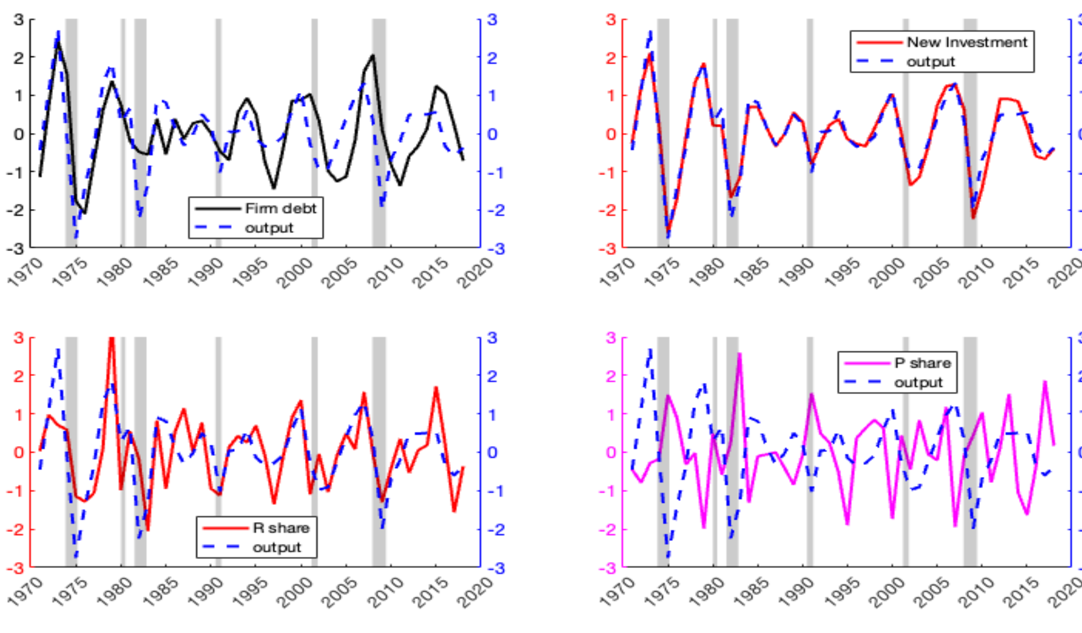
Figure 1: Reallocation and the Cost of Liquidity



Note: The R^2 values are (0.19, 0.60, 0.06) for graphs in the first row and (0.16, 0.53, 0.02) in the second row. The t-statistics for the slopes are (-3.32, -8.24, 1.83) in the first row and (2.95, 7.13, -1.41) in second row. In a one-sided test, we reject inflation has no effect on P share in business cycles at 10% level.

Figure 2: Debt, Investment and Reallocation



Note: shaded areas denote NBER recession dates.

Figure 3: DM Trade: Full Sale or Partial Sale

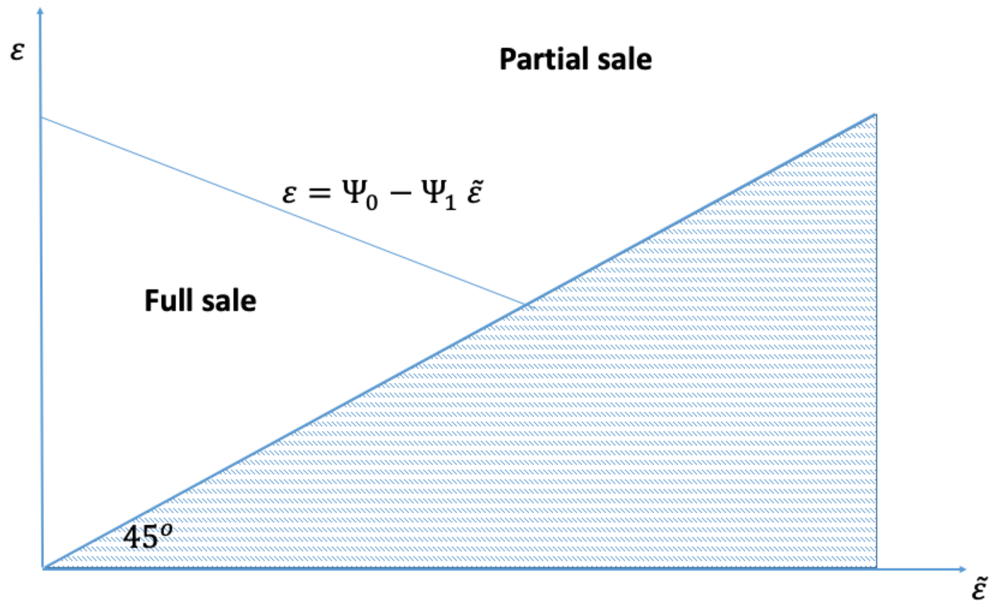


Figure 4: Steady-state Equilibrium

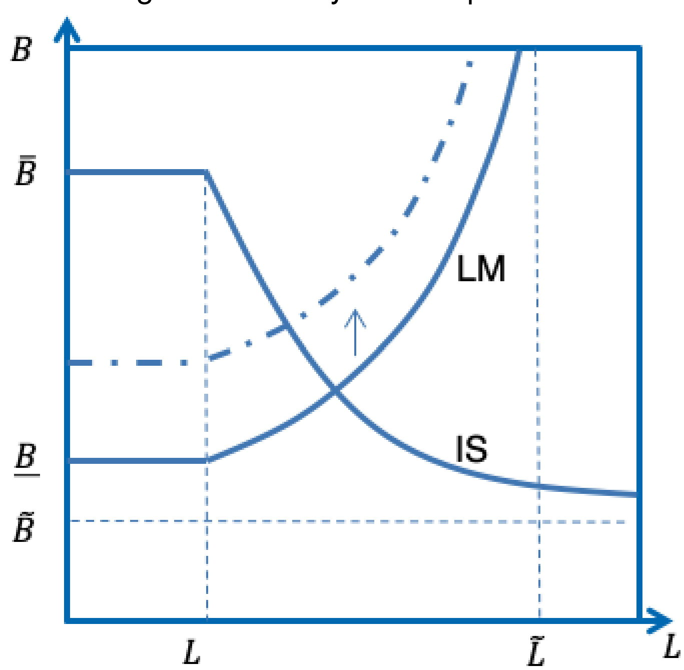
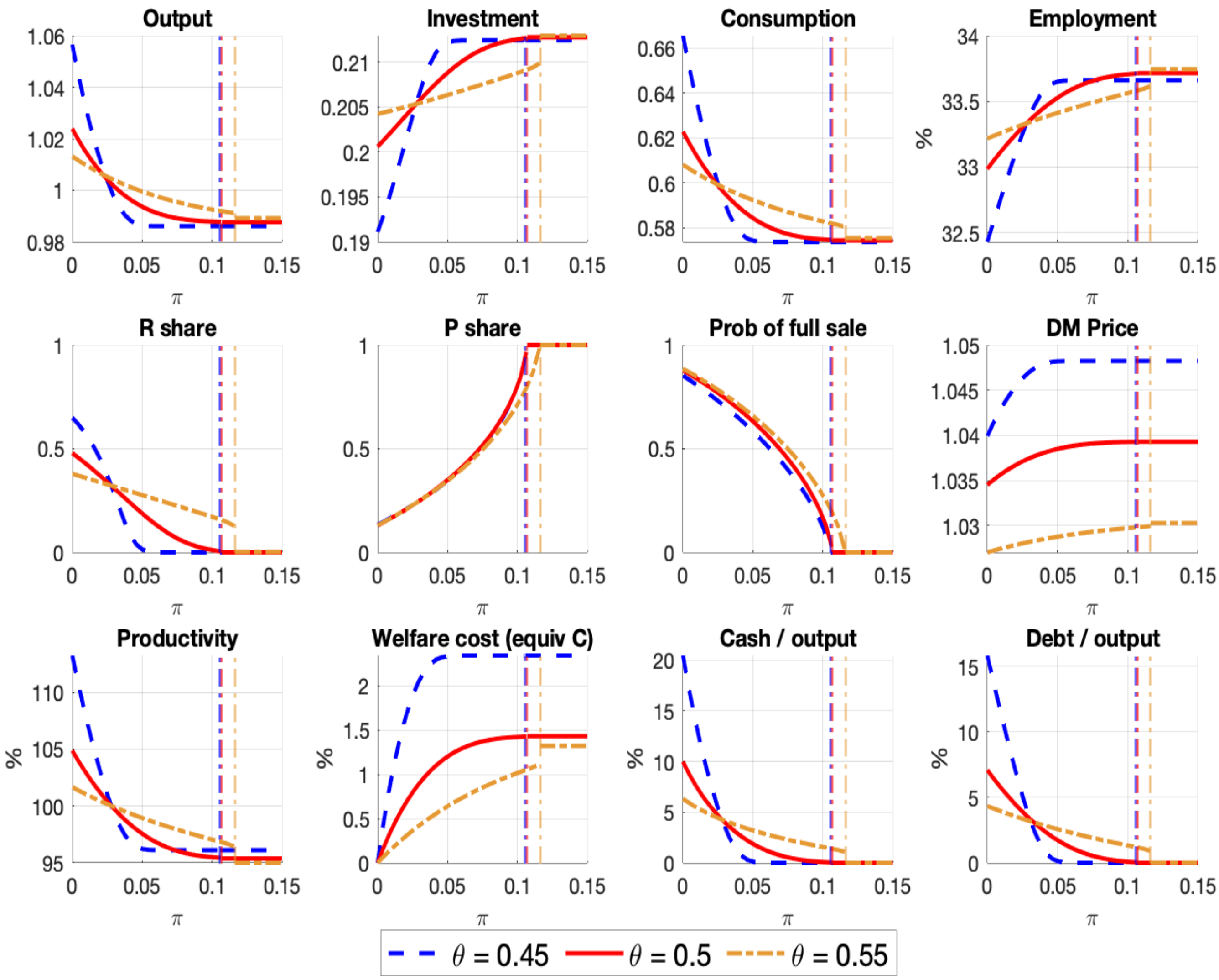
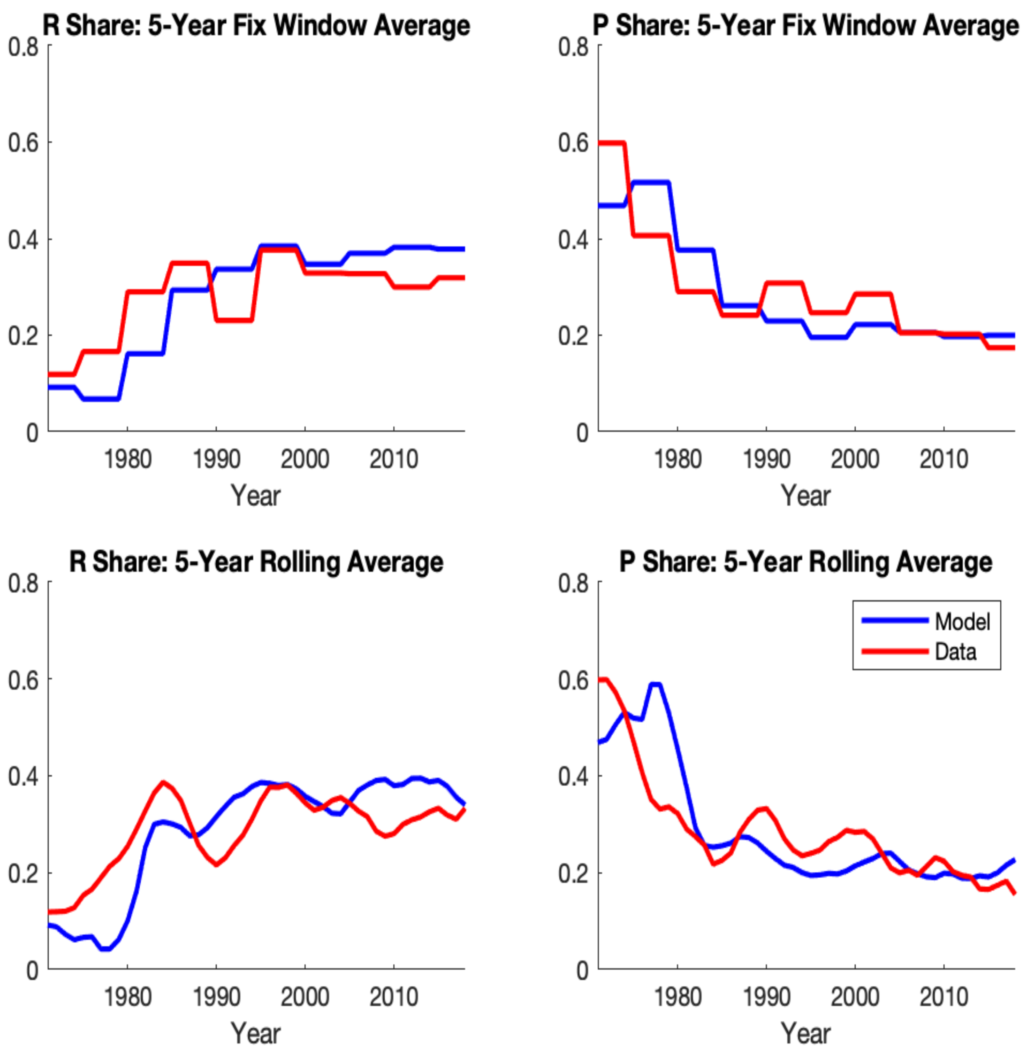


Figure 5: The Long-Run Effects of Inflation



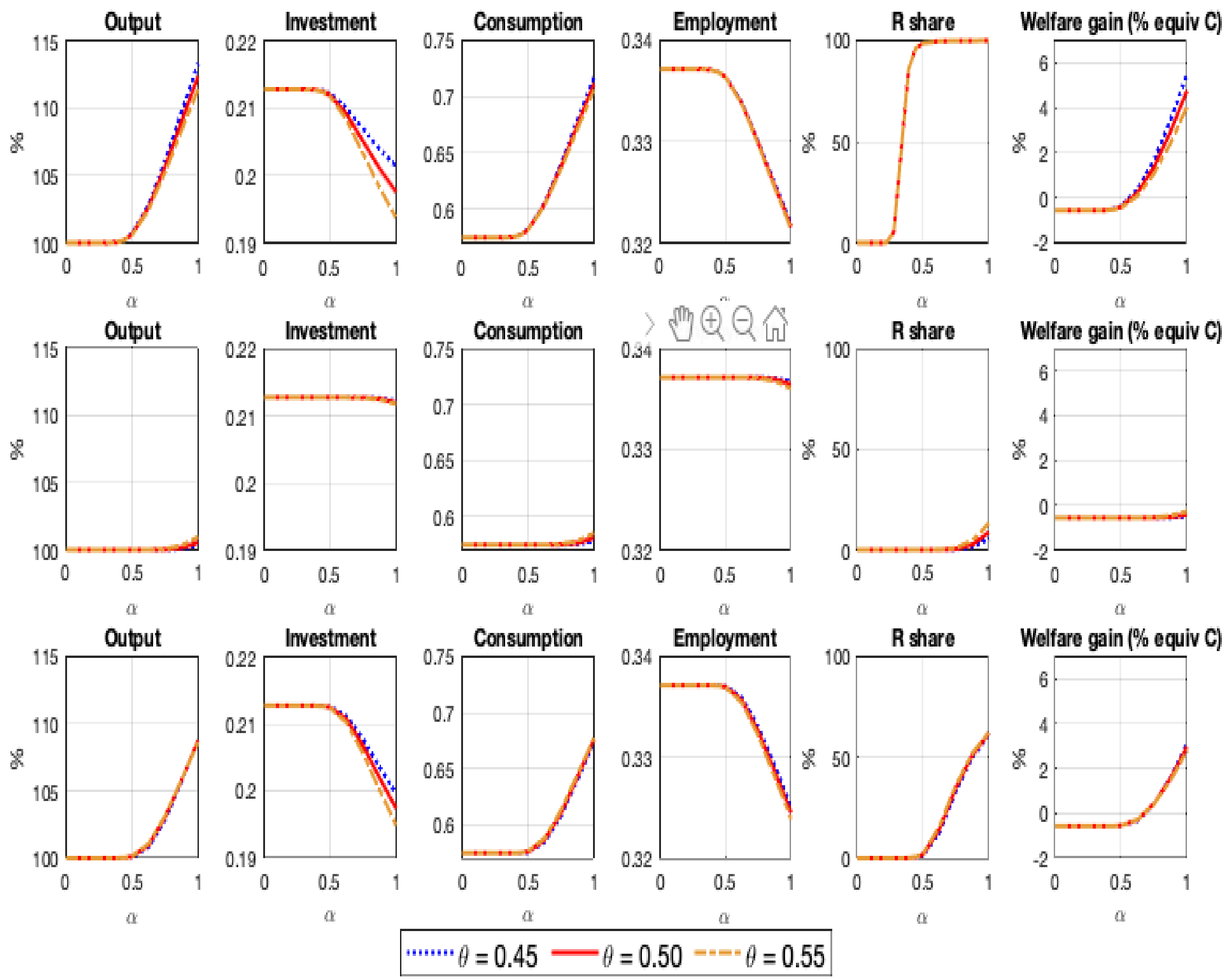
Note: The vertical lines divide non-monetary (left) and monetary (right) regions. For output, investment, consumption, productivity, and welfare, the corresponding levels in the economy with no reallocation are used as the normalization.

Figure 6: The Medium-Run Effects of Inflation: Data vs Model



Note: The figure shows R share and P share from model (blue) and data(red). Graphs in the first row are obtained by taking average in fixed 5 year windows. Therefore, the time series are constant within each 5-year window. Graphs in the second row are calculated by taking 5-year rolling average.

Figure 7: Long-run Effects of Search Frictions



Note: The first row has perfect credit. The second row has imperfect credit and no money and the last row has both.

Figure 8: Long Run Effect of Capital Taxation

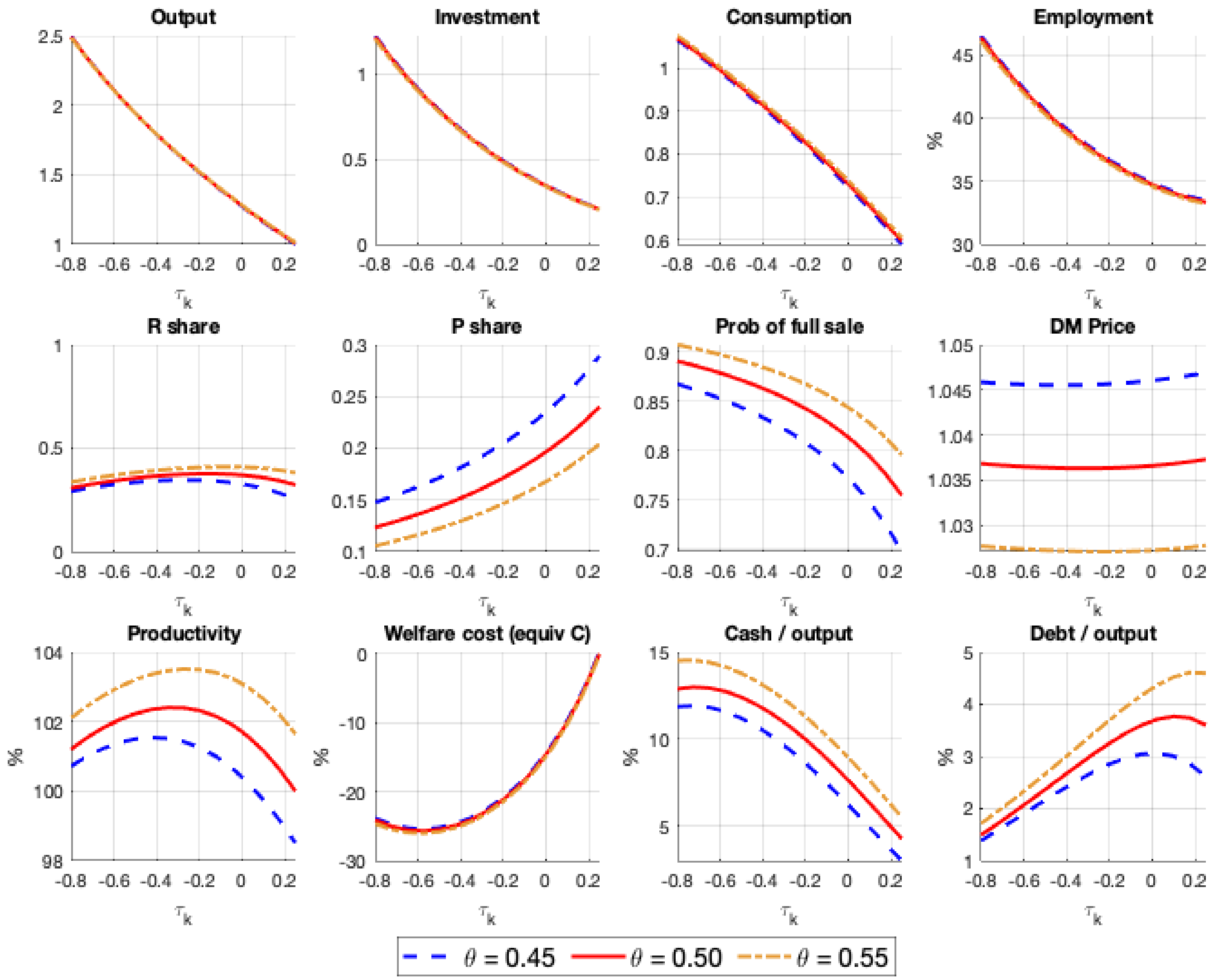


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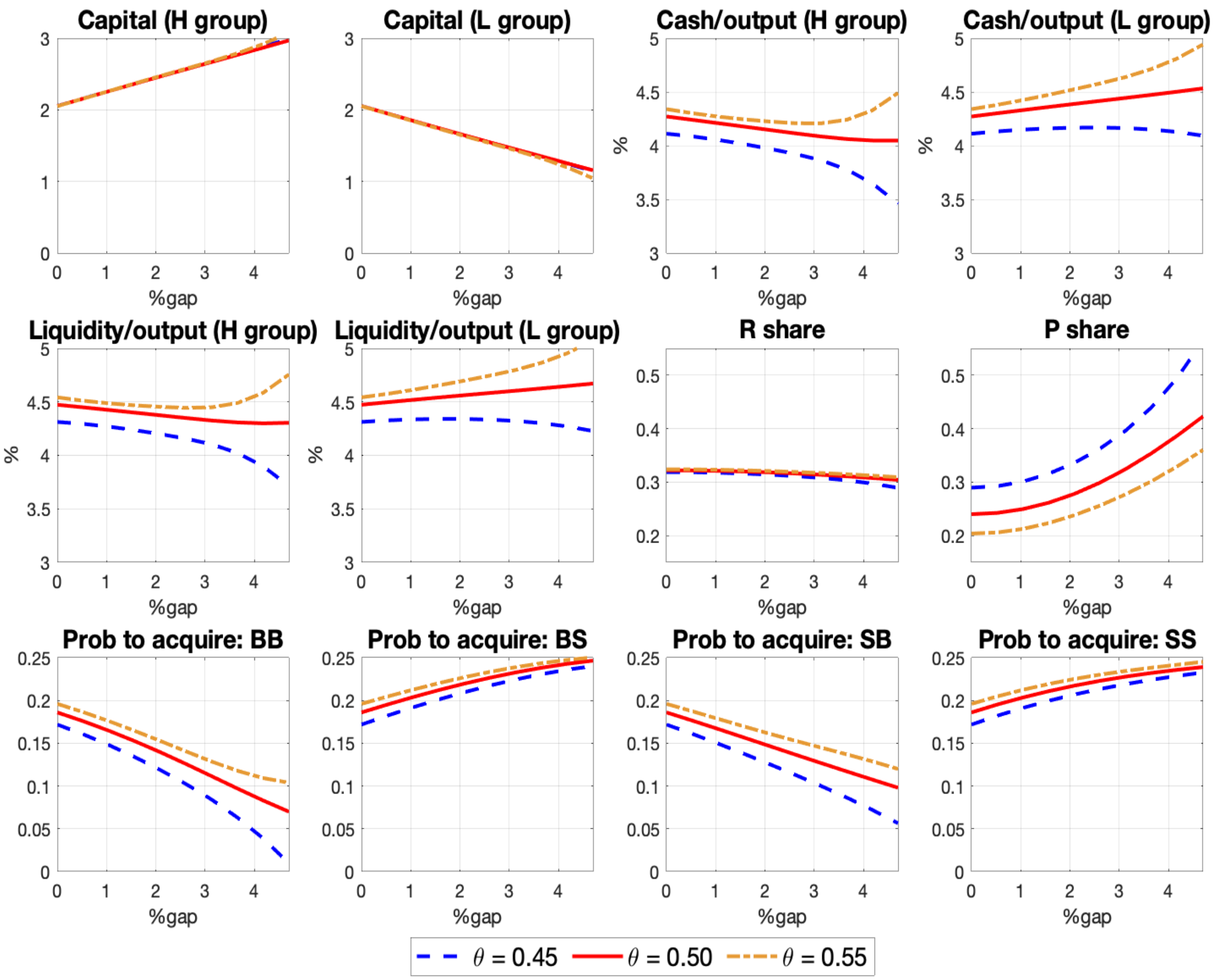
Figure 9: Effect of Differences in Persistent Component of Productivity



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