Money Creation for Distributed Ledgers: Stablecoins, Tokenized Deposits, or Central Bank Digital Currencies?

Jonathan Chiu
Bank of Canada

Cyril Monnet*
University of Bern
Study Center Gerzensee

November 30, 2023

Abstract

Should a central bank digital currency (CBDC) or tokenized deposits be issued to facilitate decentralized finance (DeFi) within the crypto space? Would their introduction be a curse or a blessing for stablecoins and illicit transactions? This paper presents a general equilibrium monetary model where money creation is susceptible to moral hazard problems. The impacts of issuing a CBDC or tokenized deposits hinge upon their utilization as a means of payment or as collateral assets in the DeFi sector. In instances where surveillance is low and the interest rate remains moderate, these tokenized currencies tend to be used as a means of payment, thereby crowding out stablecoins. Conversely, when the associated interest rate and surveillance level are high, tokenized currencies tend to serve as collateral, leading to the crowding-in of stablecoins. In terms of social welfare, CBDCs generally outperform tokenized deposits. In some cases, it becomes necessary to prohibit the creation of tokenized deposits in order to implement the optimal CBDC design. Furthermore, it is deemed optimal to apply the lowest level of surveillance to a CBDC to prevent its utilization as collateral by stablecoin issuers, thus economizing on the usage of scarce collateral assets. We also extend the benchmark model to discuss various policy questions.

JEL Codes: E50, E58.

Keywords: Central Bank Digital Currencies, Crypto Banks, Tokenized Deposits, Stablecoins.

*We thank many friends and colleagues for many useful conversations. The views expressed here are those of the authors and do not necessarily reflect the position of the Bank of Canada.

1 Introduction

Recent years have witnessed a surge in blockchain-based digital innovation within payments and finance, which includes the introduction of cryptocurrencies, decentralized finance, and stablecoins. Regulators and policymakers have expressed concerns regarding potential implications for investor protection, financial stability, and criminal activities. Some policymakers believe that introducing a central bank digital currency (CBDC) can address these issues by crowding out crypto assets. Recently, the consideration of tokenizing deposits has also emerged as an alternative option.[1] However, available guidance from economic research on this matter remains limited. This paper develops a general equilibrium monetary model to inform this discussion: Should a CBDC or tokenized deposits be issued to serve the crypto space? Would their issuance be a curse or a blessing for crypto assets, stablecoins and illicit transactions? How do the answers depend on the design of these tokenized monies?

Since the introduction of Bitcoin in 2008, thousands of cryptocurrencies have been developed. In 2015, Ethereum was launched to support smart contracts running on a blockchain. Since then, blockchain-based crypto assets have been used to finance start-up projects (e.g., initial coin offerings or ICOs), facilitate institutional governance (e.g., decentralized autonomous organization or DAO), offer financial services (e.g., decentralized finance or DeFi), and manage asset ownership (e.g., non-fungible tokens or NFTs).

The demand for a stable means of payment has grown in tandem with the expansion of the crypto space. Stablecoins emerged as a solution because traditional payment methods are not readily accessible in the crypto space due to regulatory and technical constraints. Additionally, popular cryptocurrencies like Bitcoin and Ether exhibit high price volatility, making them less appealing as a store of value or unit of account.[2] The market capitalization of top stablecoins is about 125 billion US dollars in September

[1]Swiss Bank Association published a white paper on deposit tokens in March 2023. In July 2023, Bank of England Governor Andrew Bailey gave a speech to encourage private banks to consider tokenizing bank deposits. In September 2023, JP Morgan reportedly could launch blockchain-based deposit token relatively quickly after regulator approval (source: https://fortune.com/crypto/2023/09/08/jpm-jpmorgan-chase-deposit-token-blockchain/).

[2]Stablecoins are basically cryptocurrencies that peg their value to a stable asset, such as the US dollar. Stablecoins can have different designs. First, the issuance can be centralized or decentralized. Centralized stablecoins such as Tether (USDT) and USD Coin (USDC) are issued by third parties that offer off-chain custody of the reserve assets. Decentralized stablecoins such as Dai are minted through smart contracts that also hold the reserve assets on-chain, limiting the need for a trusted third party. Second, the choice and management of reserve assets differ. Stablecoins can be backed by

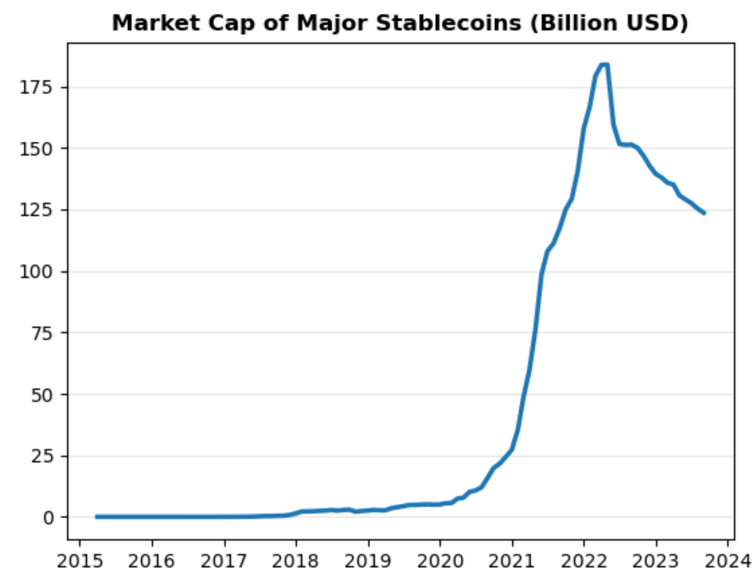

Figure 1: Market Capitalization of Major Stablecoins (Source: CoinMarketCap)

2023 (Figure 1). Figure 2 shows the shares of different stablecoins in September, 2023.

The growth of stablecoins and the issues surrounding their management have prompted regulators and policymakers to express concerns about the adverse impacts of stablecoins and crypto activities on illicit finance and financial stability.[3] Concurrently, some central bankers argue that the issuance of CBDCs can offer a solution to these problems.[4] The fundamental idea is that a high-quality outside money (i.e., CBDC) can crowd out a low-quality inside money (i.e., stablecoins).[5] In this paper, we

fiat currencies, financial assets (e.g., corporate bonds), commodities (e.g., gold) or cryptocurrencies. Stablecoins backed by cryptocurrencies (e.g., Dai) are often over-collateralized due to the price volatility of the reserve asset. Algorithmic stablecoins also exist, which are not backed explicitly by reserve assets but follow an algorithm to adjust the coin supply to maintain the peg (e.g., TerraUSD).

[3]For instance, an FSB report highlighted potential risks to financial stability if stablecoins are widely adopted (FSB, 2020). The Biden Crypto Executive Order also stressed the importance of mitigating illicit finance and national security risks associated with the misuse of digital assets.

[4]According to a BIS report, CBDC exploration is underway in over 100 countries. As of January 2023, four retail CBDCs are operational in the Bahamas, the Eastern Caribbean, Nigeria, and Jamaica. Additionally, pilots involving both wholesale and retail CBDCs are being conducted in 34 jurisdictions.

[5]Jerome Powell, the Chair of the Federal Reserve, argued that the introduction of a CBDC would eliminate the use case for cryptocurrencies. He stated, “You wouldn’t need stablecoins; you wouldn’t need cryptocurrencies if you had a

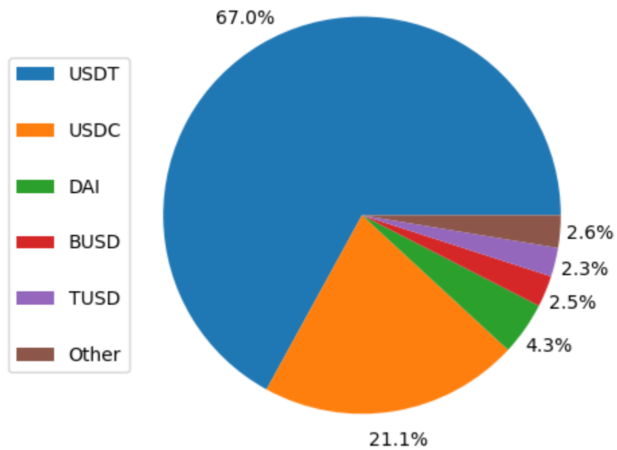

Figure 2: Market Shares of Major Stablecoins (Source: CoinMarketCap)

develop a formal model to study the issuance of two types of tokenized money: CBDC and tokenized deposits. We employ the model to examine the interactions between these tokenized monies and the crypto sector and derive conditions under which their issuance would be a blessing/curse for crypto assets, stablecoins and illicit finance. We also analyze the optimal policy regarding the issuance and design of these tokenized monies.

We model a cashless economy comprising two sectors: a traditional sector (referred to as TradFi) and a crypto sector (referred to as DeFi). In the TradFi sector, households employ bank deposits as a means of payment, which are issued by traditional banks. An important feature is that banks are susceptible to moral hazard issues, leading to an incentive constraint. Consequently, banks must maintain collateral (i.e., government securities) as a reserve to support their deposit issuance. In the DeFi sector, deposits are not feasible. Households utilize stablecoins issued by crypto banks as a means of payment. The DeFi

digital U.S. currency. I think that's one of the stronger arguments in its favor." (URL: shorturl.at/aWZ15). Reserve Bank of India Deputy Governor T. Rabi Sankar also believed that CBDC would "kill whatever little case there could be" for cryptocurrencies (URL: shorturl.at/kGO18). Academics such as Gary Gorton argue that the government can "introduce a central bank digital currency and tax private stablecoins out of existence." (Gorton, 2023) sector differs from the TradFi sector in three aspects. First, some transactions may be less desirable from the perspective of a social planner (e.g., involving criminal activities). Second, crypto banks may be less regulated compared to traditional banks, making it easier for them to abscond with their reserves. Third, crypto banks are allowed to hold crypto assets (e.g., Bitcoin, Ether) as collateral, whereas traditional banks cannot. Consequently, the regulation of crypto banks and the characteristics of crypto assets determine the tightness of the incentive constraints for crypto banks.

We use the model to examine the impact of introducing tokenized money, which can take the form of a CBDC issued by a central bank or tokenized deposits issued by private banks. We focus on two key design features: (i) the level of surveillance when tokenized money is used for transactions in the crypto space and (ii) the interest rate paid on tokenized money. We demonstrate that the issuance of tokenized money can either be a curse or a blessing for stablecoins. Specifically, the effect depends on whether tokenized money is utilized as a means of payment or as a collateral asset in the DeFi sector—an equilibrium outcome that, in turn, hinges on the design of the tokenized money. We find that tokenized money is used as a means of payment when surveillance is low, and the interest rate is moderate. In such cases, tokenized money tends to be a curse for stablecoins ("crowding-out" effects). when the associated interest rate and surveillance are high, tokenized money can serve as collateral for stablecoin issuance, and its introduction can be a blessing for stablecoins ("crowding-in" effects). Additionally, we observe that increasing surveillance can sometimes crowd-in stablecoins—an outcome that may surprise policymakers.

Next, we delve into the optimal policy regarding the issuance of tokenized money. We demonstrate that when a CBDC is in use, it is optimal to set the lowest degree of surveillance to conserve scarce collateral assets. As a result, the CBDC should be designed to prevent its use as collateral by stablecoin issuers. Concerning the optimal choice between CBDC and tokenized deposits, we find that CBDCs generally outperform tokenized deposits in terms of social welfare. One of the reasons is that private bankers may face incentive problems, whereas the central bank is considered trustworthy. Additionally, private bankers may either over-issue or under-issue tokenized money balances since they do not fully internalize the impacts of surveillance and illicit activities on the society. Finally, we provide a condition under which the use of tokenized deposits needs to be prohibited to implement the optimal CBDC design.

Our paper contributes to several lines of research in the literature. First of all, several papers such as

Ahnert et al. (2022), Andolfatto (2020), Chiu and Davoodalhosseini (2021), Chiu et al. (2023a), Keister and Sanches (2023), and Williamson (2022a) study the effects of CBDC issuance on traditional bank intermediation in normal times. In addition, Fernandez-Villaverde et al. (2020), Keister and Monnet (2022), Monnet et al. (2020), Schilling et al. (2020), and Williamson (2022b) examine the effects on bank stability in crisis times. None of these papers model the crypto sector and study its response to a CBDC issuance.[6] Furthermore, most papers in the existing literature focus on the interest rate as the main design feature of a CBDC. Our paper examines two other important design features, namely, tokenization and the degree of surveillance, generating implications useful for practical policy discussion.

Our work is also related to the emerging literature on stablecoin and decentralized finance. Theoretical papers by D'Avernas, Bourany, and Vandeweyer (2021), Li and Mayer (2021), Kozhan and Viswanath-Natraj (2021)) study decentralized stablecoins such as Dai issued by the MakerDAO. Other DeFi platforms are also actively studied. For example, Aoyagi and Itoy (2021), Capponi and Jia (2021), Lehar and Parlour (2021) study decentralized exchanges in the form of automated market makers (e.g., Uniswap), while Chiu, et al (2022) and Lehar and Parlour (2022) focus on lending platforms. Bertsch (2023) analyses the consequences of stablecoins on the fragility of the DeFi ecosystem. Cong and Mayer (2022) study the competition among national fiat currencies, cryptocurrencies, and central bank digital currencies. Chiu et al. (2023b) use a network model to investigate the relationships among different crypto tokens and DeFi activities. None of these papers evaluate the general-equilibrium impact of a CBDC and tokenized deposits on crypto activities, which is the main contribution of our analysis.

This paper is organized as follows. Section 2 presents the model environment. Sections 3 and 4 study respectively the problem of a crypto bank and the effects of introducing tokenized deposits. Section 5 considers CBDC and its optimal design. Section 6 analyzes whether CBDC is better than tokenized deposits with respect to a welfare function. Section 7 examines various extensions of the basic model and Section 8 concludes.

[6]See Auer et al. (2022) for a review of some policy issues and the academic literature.

2 Environment

Time is discrete and continues forever: $t = 0, 1, 2, ...$ Each period consists of two sub-periods with two alternating markets a la Lagos and Wright (2005). In the first sub-period, a frictional market opens and we call it AM. In the second sub-period, a Walrasian centralized market opens and we call it PM. The AM market consists of two segmented sectors, denoted by TradFi and DeFi. There is an equal measure one of infinitely lived buyers and sellers that transact consumption goods in these sectors.

A numeraire good is traded in the PM. In each PM, a large number of new bankers enter the economy and exit in the following PM. They are either traditional banks specializing in TradFi or crypto banks specializing in DeFi. The role of the banks is to provide buyers with a means to pay in the two sectors, with a technology that we specify below. We consider a digital and cashless economy where there is no supply of physical cash, but only digital money.

Government

Each period in the PM, the government issues a fixed supply B of illiquid one-period bonds trading at price q (in terms of the numeraire good) which is endogenously determined. Each unit of bonds pays one unit of numeraire good in the following PM. The redemption is financed by lump-sum taxation in the PM.

TradFi Sector

A measure one of buyers and sellers trade goods x_T in the TradFi sector. A buyer's per period utility is given by

$$U(y) - h + u(x_T),$$

where the utility from the numeraire good is $U(y)$, and the utility from the TradFi consumption good is $u(x_T)$. We assume $u(0) = 0$, $u'(0) = \infty$, $U'(y) > 0$, $u'(x_T) > 0$, $U''(y) \leq 0$, and $u''(x_T) < 0$. The discount factor between the PM and the next AM is β. Sellers' per period utility is given by

$$U(y) - h - x_T.$$

They produce TradFi goods subject to a linear cost function. AM transactions are facilitated by deposits. Traditional banks issue deposits d. However, they cannot commit to repay their outstanding claims, and they need to back them by holding government bonds b. Banks can abscond with a fraction $1-\rho$ of reserve assets, so we call ρ the pledgeability parameter of traditional banks.

DeFi Sector

A measure one of buyers and sellers trade consumption goods x_D in the DeFi sector. In the AM market DeFi sector, the utility and production functions are the same as those in TradFi. However, we allow the social welfare to assign different weights on consumption in different sectors to capture the fact that some illicit activities are conducted in the DeFi sector. Hence the regulator may want to discourage DeFi transactions.

Trades have to be facilitated by some forms of digital money. We will consider three types of digital money: stablecoins issued by crypto banks in the DeFi sector, tokenized deposits issued by traditional banks and (tokenized) CBDC issued by the central bank.

Crypto banks can issue **stablecoins** s backed by crypto assets e which pay a real return $R^e < 1/\beta$ in the PM each period, or by other assets in available in the DeFi sector, such as tokenized money. As traditional banks, crypto banks cannot commit and they can abscond with a fraction $1-\kappa$ of their assets. So their pledgeability parameter is κ. The case of $\kappa < \rho$ captures the idea that, unlike traditional banks, crypto banks are unregulated. Note that regulation precludes traditional banks from holding crypto assets, as in reality.

The central bank can issue **CBDC**, M. The central bank can fully commit, but backs the issuance of CBDC with government bonds b_C. The CBDC has two design features: First, the CBDC can pay a real rate R^m. Second, the central bank can control the degree of surveillance. Specifically, a fraction $1-\mu \in [0,1]$ of CBDC transactions will be monitored so that the CBDC will be voided when used in the DeFi sector. Lower μ implies higher surveillance.

Finally, traditional banks can issue **tokenized deposits**, denoted by D. These are "normal" deposits – therefore backed by bonds – with the feature that they can be recorded and transferred on a blockchain, facilitating DeFi transactions. Tokenized deposits are subject to the same regulatory surveillance as the CBDC.

We refer to "tokenized money" as comprising both tokenized deposits and CBDC (which is a tokenized form of fiat money). Both the CBDC and the tokenized deposits can be used by DeFi households as a means of payments, and as a reserve asset held by crypto banks.

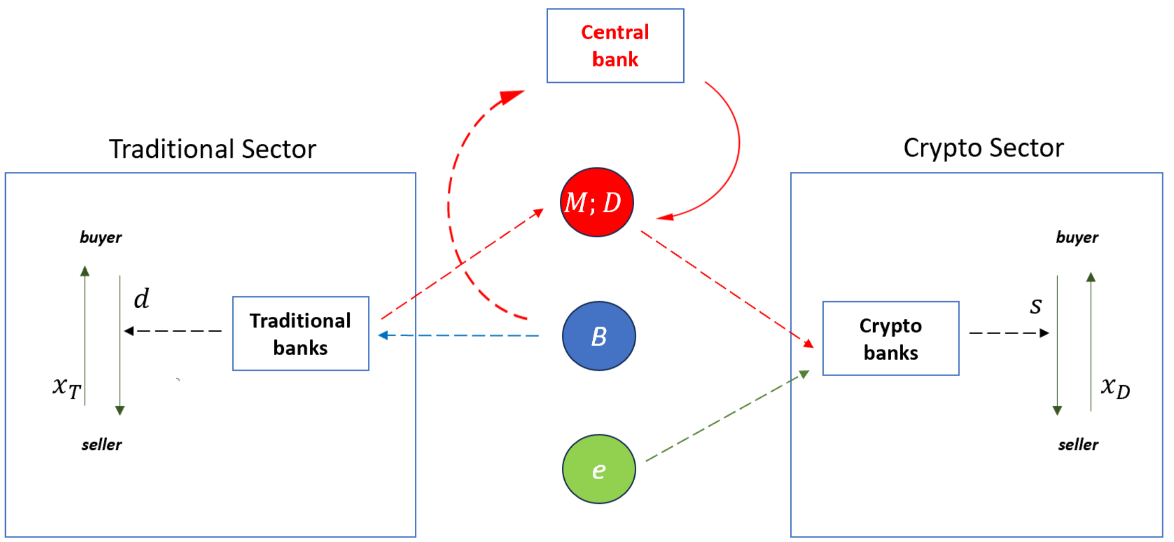

Figure 3: Model Setup

3 Crypto banks' problem

It is convenient to first analyze the optimal problem of a crypto bank, taking as given the supply of tokenized money, including its remuneration R^m and the degree of government surveillance μ. In latter sections, we endogenize the supply of tokenized money, by considering the choice of traditional banks to offer tokenized deposits and the supply of CBDC by the central bank.

In the DeFi sector, a buyer can use either stablecoins s or tokenized money $\tilde{m}$ as a means of payment to buy consumption goods. Since tokenized money is subject to compliance, the seller knows that the payment will be confiscated w.p. $1 - \mu$. Therefore the seller applies the discount μ to a payment in tokenized money.

Given μ, R^m and R^e, a crypto bank maximizes its users' payoff by choosing the users' investment into the bank a, the quantity of tokenized money $\tilde{m}$ and stablecoins s directly held by its users, as well as the reserves of tokenized money m and crypto assets e that the crypto bank will hold to back its issuance of stablecoins,[7]

$$\max_{a,s,e,m,\tilde{m}} \left[-a - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\right]$$

$$\text{subject to :} \quad \underbrace{a - m - e + \beta \left[R^e e + R^m m - s\right]}_{\text{net worth}} \;\geq\; 0, \qquad (PC)$$

$$\kappa \left(R^e e + R^m m\right) \;\geq\; s, \qquad (IC)$$

where $\kappa \in (0, 1)$ denotes the pledgeability parameter of assets for the crypto bank. It is obvious that (PC) binds, so that the problem becomes

$$\max_{a,s,e,m,\tilde{m}} \left[-m - e + \beta \left[R^e e + R^m m - s\right] - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\right]$$

$$\text{subject to:} \qquad \kappa \left(R^e e + R^m m\right) \;\geq\; s. \qquad (IC)$$

Let $c = m, e$ denote the asset held by the crypto as collateral. If $\beta R^c < 1$ the crypto bank only holds asset $c = m, e$ if it relaxes IC, and it is indifferent when $\beta R^c = 1$. Also, the crypto bank holds the cheapest asset to satisfy its IC, that is, it will hold m whenever $R^m > R^e$, e whenever $R^e > R^m$, and it will be indifferent otherwise. With this understanding we can rewrite the problem of the crypto bank as

$$\max_{a,s,e,m,\tilde{m}} \left[-c + \beta \left[R^c c - s\right] - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\right] + \beta\lambda \left[\kappa R^c c - s\right]$$

where λ is the Lagrange multiplier on the crypto bank's IC. The first order conditions are

$$\begin{aligned} s: \quad u'(s + \mu R^m \tilde{m}) &\leq 1 + \lambda, \\ c: \quad (1 + \lambda\kappa)\beta R^c &\leq 1, \\ \tilde{m}: \quad \mu\beta R^m u'(s + \mu R^m \tilde{m}) &\leq 1. \end{aligned}$$

From these first order conditions,. we can derive three regions for the parameter space.

i.) Region $\mathcal{A}_e$ (with only stablecoins which are secured with crypto assets): $e > 0$ and $m = \tilde{m} = 0$, where

$$u'(z) = \frac{1 - (1 - \kappa)\beta R^e}{\kappa\beta R^e},$$

$$R^m < \min \left\{ \frac{\kappa R^e}{\mu \left[1 - (1 - \kappa)\beta R^e\right]}, R^e \right\} \equiv R_1^m.$$

[7]This problem is equivalent to maximizing the bank's payoff subject to the user's participation constraint.

Under these parameters, the level of surveillance μ is so high and the return on tokenized money R^m is so low that sellers (and the crypto bank) do not value tokenized money much. At the same time, the return on crypto assets is large enough for the crypto bank to issue stablecoins backed only by crypto assets.

ii.) Region $\mathcal{A}_m$ (with only stablecoin which are secured with tokenized money): : $m > 0$ and $e = \tilde{m} = 0$ where

$$u'(x_D) = \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m},$$
$$R^m > \max\left\{\frac{\mu-\kappa}{\beta\mu(1-\kappa)}, R^e\right\} \equiv R_2^m.$$

Under these parameters, the level of surveillance is still too high that sellers do not value tokenized money much. At the same time, the return on tokenized money is now sufficiently high for the crypto bank to issue stablecoins backed by tokenized money. Therefore, buyers only trade with stablecoins. Notice that $R_2^m \geq R_1^m$.

iii.) Region $\mathcal{A}_{\tilde{m}}$ (with only tokenized money and no stablecoins) : $\tilde{m} > 0$ and $e = m = 0$ where

$$\mu R^m u'(\mu R^m \tilde{m}) = \frac{1}{\beta},$$
$$\frac{\kappa R^e}{\mu[1-(1-\kappa)R^e\beta]} < R^m < \frac{\mu-\kappa}{\beta\mu(1-\kappa)}.$$

Under these parameters, the level of surveillance is low enough that sellers value tokenized money. However the return on tokenized money or crypto asset are too low for the crypto bank to issue stablecoins. Therefore, buyers trade directly with tokenized money and the role of the crypto bank is just to purchase tokenized money on behalf of its users.

In the Appendix we derive all the possible equilibria and the conditions for their existence, including the knife-edge cases.[8] The following proposition is a direct consequence of the equilibrium characterization.

[8]There are also knife-edge cases, where the crypto bank backs its stablecoins with both crypto assets and tokenized money (when $R^e = R^m$), or when buyers are indifferent between using both stablecoins and tokenized money (when $\mu\beta R^m\left[1+\frac{1-\beta R^c}{\kappa\beta R^c}\right] = 1$ and $R^c = R^e$ if crypto assets are backing the stablecoins or $R^c = R^m$ if tokenized money are backing the stablecoins). We consider these knife-edge cases in all the proofs but do not consider them in the text.

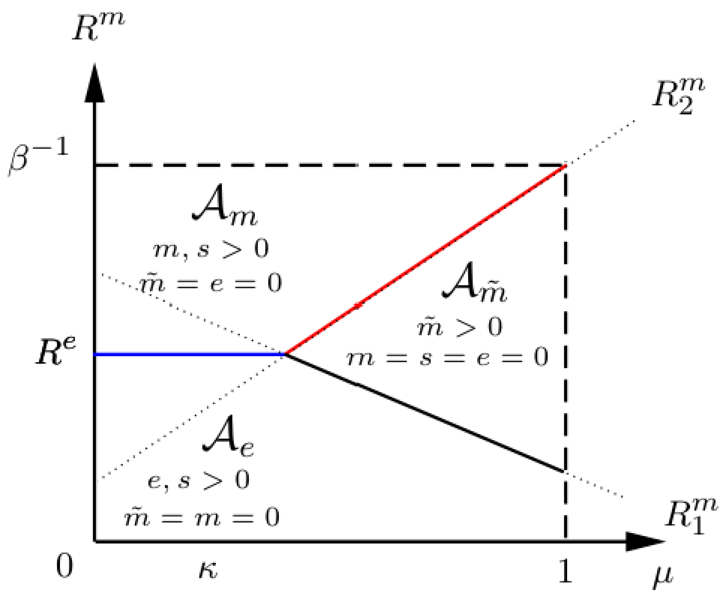

Figure 4: Distribution of Equilibria in the (R^m, μ) space

Proposition 1. *Introducing tokenized money has the following effects:*

a.) it crowds out stablecoin for $R^m \in (R_1^m, R_2^m)$

b.) it crowds in stablecoin for $R^m \geq R_2^m$,

c.) it promotes consumption of the crypto bank users for $R^m > R_1^m$.

Proof. Denote the equilibrium DeFi consumption by $x_D = s + \mu R^m \tilde{m}$. Before introducing tokenized money,

$$x_D = u'^{-1}\left[\frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}\right], \text{ and } \quad s = x_D \tag{1}$$

When $R^m \in (1, R_1^m)$, tokenized money is not used and hence s and x_D stay unchanged and they are independent of R^m. When $R^m \in (R_1^m, R_2^m)$,

$$x_D = u'^{-1}\left[\frac{1}{\beta\mu R^m}\right], \text{ and } s = 0. \tag{2}$$

Relative to the case without tokenized money, x_D is higher (by definition of $\mathcal{A}_{\tilde{m}}$), and s is lower. Also, x_D is increasing in R^m while $s = 0$ is independent of R^m. When $R^m \geq R_2^m$,

$$x_D = u'^{-1}\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}\right], \text{ and } s = x_D, \tag{3}$$

Relative to the case without tokenized money, x_D is higher (by definition of $\mathcal{A}_m$), and s is higher too. Also, both are increasing in R^m. □

The following comparative statics from raising the interest rate on tokenized money (R^m) are a direct consequence of the last proof: for $R^m \in (1, R_1^m)$, consumption of crypto bank users and stablecoin issuance are independent of R^m (hence $\partial x_D/\partial R^m = \partial s/\partial R^m = 0$). However, for $R^m \in (R_1^m, R_2^m)$, consumption is increasing in R^m, while the crypto bank does not issue stablecoin which therefore does not react to R^m. Finally, for $R^m \in (R_2^m, \beta^{-1})$, both consumption of the crypto bank users and the issuance of stablecoin are increasing in R^m: the crypto bank uses tokenized money as collateral and the higher interest rate R^m relaxes the IC of the crypto bank allowing it to issue more stablecoins, thus increasing the consumption of crypto bank users.

Proposition 2. *Increasing the degree of surveillance can crowd in stablecoins.*

The intuition for Proposition 2 is straightforward : Suppose the crypto bank is not issuing stablecoin (in region $\mathcal{A}_m$). Increasing the degree of surveillance (i.e., lowering μ) directly lowers the payoff of buyers from using tokenized money to trade with sellers, because sellers will discount the sales as the government may confiscate their profit. This encourages buyers to use stablecoin instead, which is not subject to surveillance.

Proof. Inside the regions $\mathcal{A}_e$, $\mathcal{A}_m$ and $\mathcal{A}_{\tilde{m}}$, an increasing μ has no effect on s. Moving from $\mathcal{A}_e$, $\mathcal{A}_m$ into $\mathcal{A}_{\tilde{m}}$, however, lowers s. □

4 Tokenized Deposits

In this section we consider the problem of a traditional bank issuing deposits to the traditional sector. At the same time, it also tokenizes its deposits for buyers to purchase goods in the DeFi sector. The traditional bank has to secure all of its deposits by holding government bonds. Recall that the government issues B units of one-period bonds. The price of newly issued bonds is q. Each bond yields one unit of goods in the next period, financed by lump-sum taxation.

A traditional bank takes q and the equilibrium interest rate on tokenized deposits R^m as given. It issues d units of deposits to traditional depositors but it can also issue tokenized deposits D for the crypto sector. All deposits have to be secured by government bonds, with pledgeability ρ. Therefore the traditional bank's problem is

$$\max_{a,d,D,b} [-a + \beta u(x_T)]$$

subject to $d = x_T$ and to its net worth being positive

$$a + D - qb + \beta [b - R^m D - d] \quad \geq \quad 0, \qquad (PC)$$

and the collateral constraint

$$\rho b \quad \geq \quad R^m D + d. \qquad (IC)$$

The PC necessarily binds and when the IC also binds, the first order conditions are

$$\begin{aligned} D: \qquad & R^m u'(\rho b - R^m D) && \geq \frac{1}{\beta}, \\ b: \quad & \rho\beta u'(\rho b - R^m D) + \beta(1-\rho) && = q. \end{aligned}$$

and $b \leq B$, with equality in equilibrium. The intuition for the first order condition with respect to b is that it costs q to acquire one additional unit of bonds, a fraction ρ of it can be used as collateral to boost consumption which has value $u'(x_T)$ and the remaining fraction $1-\rho$ is held to maturity, bringing a unit (discounted) payoff of β.

The solution of the above problem can potentially lead to two outcomes. First, the traditional bank chooses optimally not to issue tokenized deposits i.e., $D = 0$. Then the consumption of traditional depositors is $x_T = \rho B$ and the price of government bonds q is given by

$$q = \rho\beta u'(\rho B) + \beta(1-\rho).$$

This is an equilibrium if

$$R^m u'(\rho B) \geq \frac{1}{\beta}.$$

Second, the traditional bank issues tokenized deposits $D > 0$ to the crypto bank. Then the consumption of traditional depositors is reduced to $x_T = \rho B - R^m D$, and the supply of tokenized deposits solves

$$\frac{1}{R^m} = \beta u'(\rho B - R^m D)$$

while (given ρ and B) the price of the government bonds is increased to

$$q = \rho\beta u'(\rho B - R^m D) + \beta(1-\rho) = \rho\frac{1}{R^m} + \beta(1-\rho).$$

This is an equilibrium if it is cheap to issue tokenized deposits (R^m is low) and/or there is an abundance of government bonds

$$R^m u'(\rho B) < \frac{1}{\beta}.$$

We first define an equilibrium with tokenized deposits.

Definition 1. *An equilibrium with tokenized deposits is a list* $(x_T, d, D, b, x_D, e, m, \tilde{m}, b, q, R^m)$ *such that given prices* (q, R^m, R^e) *the traditional bank optimally chooses* (x_T, d, D, b)*, and the crypto bank optimally chooses* $(x_D, e, m, \tilde{m})$*, and markets clear, so that* $B = (d + R^m D)/\rho$ *and* $D = m + \tilde{m}$.

Proposition 3. *An equilibrium with positive tokenized deposits* $(D > 0)$ *exists iff*

$$u'(\rho B)\beta R^e \min\left\{1, \frac{\kappa}{\mu\left[1-(1-\kappa)\beta R^e\right]}\right\} < 1.$$

Proof. Recall that the crypto bank demands tokenized money whenever $R_1^m \leq R^m$, while the traditional bank supplies tokenized deposits whenever $u'(\rho B)\beta R^m < 1$. The result follows from combining both conditions and the definition of R_1^m. □

Proposition 3 shows that an equilibrium with tokenized deposits exists whenever the supply of government bonds and their pledgeability is large, while the return on crypto assets and their pledgeability is small. Also, a low degree of surveillance (a high μ) makes the existence of that equilibrium more likely because it increases the exchange value of tokenized deposits.

Figure 5 shows the demand and supply of tokenized deposits in the case of log utility. In this case, there exists a unique equilibrium. Deposits are tokenized iff

$$\rho B > \beta R^e \min\left\{1, \frac{\kappa}{\mu\left[1-(1-\kappa)\beta R^e\right]}\right\}.$$

5 Central Bank Digital Currency

In this section we study the case where tokenized money takes the form of a central bank digital currency, while the traditional bank is not allowed to issue tokenized deposits and it is not allowed to hold CBDC as reserves. There are two main differences compared to tokenized deposits: First, central bank will not abscond and thus it is not subject to a pledgeability constraint. But, as in reality, the central bank still

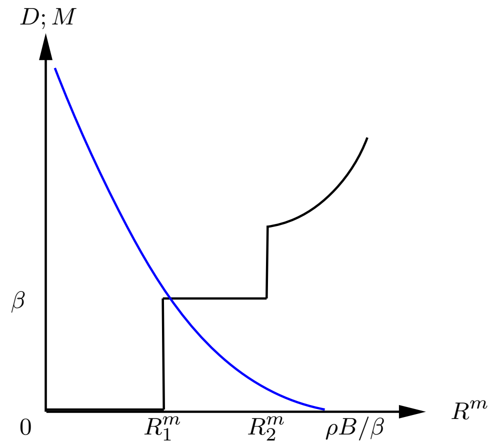

Figure 5: Tokenized Deposit Supply and Demand (with log utility)

needs to back the issuance of CBDC with government bonds, and when it holds government bonds, it competes with the traditional banks. Second, the central bank sets the interest rate on CBDC, implying that R^m is no longer an equilibrium object.

Suppose the central bank issues M units of CBDC bearing an interest rate R^m, backed by bonds. The problem of a traditional bank is the same as above except that $D = 0$ since it cannot issue tokenized deposits. When both the PC and IC are binding, the traditional bank problem gives their bond demand b as the solution to

$$\beta\rho u'(\rho b) + \beta(1-\rho) = q. \tag{4}$$

When the central bank issues M units of CBDC, its balance sheet constraint requires

$$R^m M = b_C,$$

and the market clearing condition for bonds is $b + b_C = B$. From the solution from the crypto bank problem we know that the demand for CBDC is

$$R^m M = \begin{cases} 0, & R^m \in (0, R_1^m) \\ \frac{1}{\mu} u'^{-1}\left[\frac{1}{\beta\mu R^m}\right], & R^m \in (R_1^m, R_2^m) \\ \frac{1}{\kappa} u'^{-1}\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}\right], & R^m \in (R_2^m, \beta^{-1}) \end{cases} \tag{5}$$

Definition 2. *An equilibrium with CBDC is a list* $(x_T, d, b, x_D, e, m, \tilde{m}, b_C, M, q, R^m)$ *such that given prices* (q, R^m, R^e)*, the traditional bank optimally chooses* (x_T, d, b)*, the crypto bank optimally chooses*

$(x_D, e, m, \tilde{m})$, *the central bank sets* R^m *and issues* M *backed by bonds* b_C *and markets clear, so that* $B = d/\rho + R^m M$ *and* $M = m + \tilde{m}$.

Proposition 4. *An equilibrium with positive CBDC,* $M > 0$, *exists iff* $R^m \geq R_1^m$.

Notice that for the existence it only requires that the crypto bank demands CBDC, which necessitates that the central bank sets $R^m \geq R_1^m$. Then the central bank will supply $R^m M$ as given by (5) and the price of the government bond q is given by (4) with $b = B - R^m M$.

5.1 Optimal degree of surveillance μ

With the central bank issuing a CBDC the question of privacy and state surveillance is looming large. On the one hand, a central bank does not want to encourage illicit trade and tax evasion by providing an anonymous means to pay and to store wealth. On the other hand, broad adoption of CBDC requires that it preserves some level of privacy for its users. In this section we analyze the optimal degree of surveillance μ when the central bank issues CBDC that can be used to conduct illicit activities. In particular we abstract from the cost of relinquishing privacy, but still we show the following result,

Proposition 5. *When the central bank issues CBDC, no surveillance is optimal, i.e.* $\mu = 1$.

Proof. Suppose $\mu < 1$. Welfare can be (weakly) improved by increasing μ as long as $x_T < x_T^*$ defined as $u'(x_T^*) = 1$. Consider first the three interior equilibria:

regions	bounds	$R^m M$	x_D	x_T
$\mathcal{A}_e$	$R^m \leq \min\left\{\frac{\kappa R^e}{\mu[1-\beta(1-\kappa)R^e]}, R^e\right\} = R_1^m$	0	$u'(x_D) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}$	ρB
$\mathcal{A}_{\tilde{m}}$	$R_1^m \leq R^m \leq R_2^m$	$\frac{1}{\mu}u'^{-1}\left[u'(x_D)\right]$	$u'(x_D) = \frac{1}{\beta\mu R^m}$	$\rho(B - R^m M)$
$\mathcal{A}_m$	$R^m \geq \max\left\{\frac{\mu-\kappa}{\beta\mu(1-\kappa)}, R^e\right\} = R_2^m$	$\frac{1}{\kappa}u'^{-1}\left[u'(x_D)\right]$	$u'(x_D) = \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}$	$\rho(B - R^m M)$

In the first and the third cases, $\tilde{m} = 0$ and hence increasing μ has no effects.

In the second case, setting $\mu' > \mu$ and $R^{m\prime} = \frac{R^m\mu}{\mu'} < R^m$ will keep x_D unchanged while x_T is increased, since $R^m M$ will decline. This is welfare improving whenever $x_T < x_T^*$.

For the four corner solutions:

	R^m	x_D	x_T
$e > 0, m > 0$	$R^m = R^e$	$u'(x_D) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}$	$\rho(B - mR^m)$
$m > 0, \tilde{m} > 0$	$\beta\mu R^m = \frac{\mu-\kappa}{1-\kappa}$	$u'(x_D) = \frac{1-\kappa}{\mu-\kappa}$	$\rho(B - (m + \tilde{m})R^m)$
$e > 0, \tilde{m} > 0$	$\beta\mu R^m = \frac{\beta\kappa R^e}{[1-(1-\kappa)\beta R^e]}$	$u'(x_D) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}$	$\rho(B - \tilde{m}R^m)$
$e > 0, \tilde{m} > 0, m > 0$	$\beta\mu R^m = \frac{\beta\kappa R^e}{[1-(1-\kappa)\beta R^e]} = \frac{\kappa R^m}{[1-(1-\kappa)\beta R^m]}$	$u'(x_D) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}$	$\rho(B - (m + \tilde{m})R^m)$

When $e > 0, m > \tilde{m} = 0$: marginally increasing μ will maintain the indifference condition, having no effects on the equilibrium allocation.

When $e = 0, m > 0, \tilde{m} > 0$: marginally increasing μ to μ' and lowering R^m to $R^{m\prime}$ will enter the $\mathcal{A}_{\tilde{m}}$ region. This will keep x_D unchanged at $u'(x_D) = \frac{1-\kappa}{\mu-\kappa}$ while x_T will go up as R^m declines.

When $e > 0, \tilde{m} > m = 0$: marginally increasing μ to μ' and lowering R^m to $R^{m\prime}$ will maintain the indifference condition. This will keep x_D unchanged at $u'(x_D) = \frac{[1-(1-\kappa)\beta R^e]}{\beta\kappa R^e}$ while x_T goes up as R^m declines.

When $e > 0, \tilde{m} > m = 0$: marginally increasing μ to μ' and lowering R^m to $R^{m\prime}$ will induce an equilibrium with $e = 0, m > 0, \tilde{m} > 0$. By setting $e' = e$ and $\tilde{m}' = \tilde{m} + m$, we can support the same x_D while x_T goes up as R^m declines.

□

The intuition for Proposition 5 is that government bonds are not used efficiently whenever $\mu < 1$. Recall that bonds are necessary to back the deposits of traditional banks as well as CBDC. When bonds are in scarce supply, CBDC takes resources away from traditional banks and distorts their allocation. Setting $\mu < 1$ implies that DeFi sellers value CBDC at a discount, increasing the distortion in the traditional sector. Setting $\mu = 1$ can eliminate this inefficiency. If it is important that sellers value CBDC "less", the central bank can achieve the same target value for CBDC (μR^m) by increasing μ and reducing R^m. This improves social welfare.

Note that, when $\mu = 1$, CBDC is not used as a collateral. Hence, we have the following result.

Corollary 1. *It is optimal to design the CBDC so that it is not used as a collateral in the crypto space.*

6 Is a CBDC better than tokenized deposits?

The previous two sections study separately the equilibrium with tokenized deposits and the equilibrium with CBDC. This section considers the case where the two instruments co-exist and derives conditions under which a CBDC can achieve a higher level of welfare.

Definition 3. *An equilibrium with CBDC and tokenized deposits is a list* $(x_T, d, D, b, x_D, e, m, \tilde{m}, b, b_C, M, q, R^m)$ *such that given prices* (q, R^m, R^e) *the traditional bank optimally chooses* (x_T, d, D, b)*, the crypto bank optimally chooses* $(x_D, e, m, \tilde{m})$*, the central bank sets* R^m *and issues* M *backed by bonds* b_C*,and markets clear, so that* $B = (d + R^m D)/\rho + R^m M$ *and* $D + M = m + \tilde{m}$*.*

6.1 CBDC dominates tokenized deposits

Given any equilibrium where $D > 0$, the following proposition shows that replacing D by M is welfare improving.

Proposition 6. *Replacing tokenized deposits by a CBDC can (weakly) increase welfare.*

The proof consists of several steps. First, we analyze the case where the pledgeability parameter $\rho < 1$. This means that the traditional bank is not as good as the central bank to provide tokenized money (since the central bank can pledge its assets fully when it issues CBDC). In this case it is intuitive that the equilibrium with CBDC will achieve a (weakly) higher welfare than the one with tokenized deposits. Then we analyze the case with $\rho = 1$. We show that given the interest rate R^m in an equilibrium where $D > 0$, there exists an equilibrium with CBDC only with a slightly higher or lower interest rate such that the welfare level is higher. In other words, replacing D by M with an appropriate interest rate adjustment is welfare improving. The reasons is that the central bank internalizes the cost of surveillance ($\mu < 1$) – which reduces the value of CBDC – or the cost of conducting illicit activities ($\omega < 1$), while private agents do not. Therefore, with tokenized deposits, there may be too much DeFi trades (x_D too large) when ω is small, or too little of it when μ is small.

Proof. When banks with $\rho < 1$ tokenize deposits, collateral is not used efficiently in the creation of tokenized deposits. Replacing tokenized deposits by CBDC that are traded at the same rate can support better allocation. We use superscript "0" to denote variables associated with the original equilibrium where and tokenized deposits are allowed, and use “1” to denote the new equilibrium where tokenized deposits are replaced by a CBDC. In particular, $M^1 = D^0 + M^0, D^1 = 0$. The welfare levels associated with the two equilibria are

$$W^1 = u(x_T^1) - x_T^1 + \omega \left[u(x_D^1) - x_D^1\right]$$
$$W^0 = u(x_T^0) - x_T^0 + \omega \left[u(x_D^0) - x_D^0\right]$$

where $x_D^1 = x_D^0$, $x_T^0 = \rho(B - R^m M^0) - R^m D^0$, and $x_T^1 = \rho(B - R^m M^0 - R^m D^0)$. Obviously $x_T^1 > x_T^0$, implying that $W^1 \geq W^0$.

We now turn to the case where $\rho = 1$ and show that CBDC can still dominate a tokenized deposit by offering a higher or a lower R^m. Consider $R^{m,0} \in (R_1^m, R_2^m)$ when tokenized money $D^0 + M^0$ are used as a means of payments (region $\mathcal{A}_{\tilde{m}}$). The central bank can offer a CBDC with rate $R^{m,1}$ to maximize welfare:

$$\max_{R^{m,1}} W^1 = u(x_T^1) - x_T^1 + w[u(x_D^1) - x_D^1]$$
$$\text{subject to:} \qquad u'(x_D^1) = \frac{1}{\mu\beta R^{m,1}}, \text{ and } \quad x_T^1 = \rho(B - x_D^1/\mu)$$

The market clearing condition implies that $\partial x_T^1/\partial R^{m,1} = -\rho\left(\partial x_D^1/\partial R^{m,1}\right)/\mu$. Note that we can set the interest rate at the original level, $R^{m,1} = R^{m,0}$, and replace the tokenized deposits by CBDC (i.e., $M^1 = D^0 + M^0, D^1 = 0$) to support exactly the same allocations (i.e., $x_T^1 = x_T^0, x_D^1 = x_D^0$). Note also that the original equilibrium satisfies $u'(x_T^0) = \mu u'(x_D^0)$. Starting from the original equilibrium rate, the marginal effect of changing $R^{m,1}$ is

$$\frac{dW^1}{dR^{m,1}} = [u'(x_T^1) - 1]\frac{\partial x_T^1}{\partial R^{m,1}} + \omega[u'(x_D^1) - 1]\frac{\partial x_D^1}{\partial R^{m,1}}.$$

Using the fact that $u'(x_T^1) = \mu u'(x_D^1)$ when $R^{m,1} = R^{m,0}$, we can rewrite this expression as

$$\frac{dW^1}{dR^{m,1}}|_{R^{m,1}=R^{m,0}} = \left\{\frac{\rho}{\mu} - \omega - (\rho - \omega)\frac{1}{\mu\beta R^{m,1}}\right\}\frac{\partial x_D}{\partial R^{m,1}}. \tag{6}$$

Since $\rho = 1$ and $\partial x_D/\partial R^{m,1} > 0$, it is optimal to decrease R^m if $\beta R^{m,0} < \frac{1-\omega}{1-\mu\omega}$ and to increase it otherwise. In general, offering a CBDC can improve welfare by varying R^m. The result is intuitive: the central bank prefers to reduce R^m to make tokenized money less valuable when $\mu = 1$. Decreasing μ below 1 would already make the tokenized money less attractive so that (if μ is low enough) it could be optimal to raise the value of tokenized money by increasing R^m, that is, whenever $\beta R^m \geq \frac{1-\omega}{1-\mu\omega}$.

We now consider the case with $R^{m,0} \geq R_2^m$, where tokenized money are used as collateral (region $\mathcal{A}_m$), with $\rho = 1$, the central bank solves

$$\max_{R^{m,1}} W^1 = u(x_T^1) - x_T^1 + \omega[u(x_D^1) - x_D^1]$$

$$\text{subject to:} \qquad u'(x_D^1) = \frac{1 - (1-\kappa)\beta R^{m,1}}{\kappa \beta R^{m,1}},$$

and feasibility requires $x_T^1 = B - x_D^1/\kappa$. The feasibility constraint implies that $\partial x_T^1/\partial R^{m,1} = -\left(\partial x_D^1/\partial R^{m,1}\right)/\kappa$. Note that, in this region, the original equilibrium with tokenized deposits satisfies $u'(x_T^0) = (1-\kappa) + \kappa u'(x_D^0)$. Hence the marginal effect of changing $R^{m,1}$ is

$$\begin{aligned}
\frac{dW^1}{dR^m}|_{R^{m,1}=R^{m,0}} &= [u'(x_T^1) - 1]\frac{\partial x_T^1}{\partial R^{m,1}} + \omega[u'(x_D^1) - 1]\frac{\partial x_D^1}{\partial R^{m,1}} \\
&= (\omega - 1)\left[u'(x_D^0) - 1\right]\frac{\partial x_D^1}{\partial R^{m,1}}
\end{aligned}$$

Therefore, $dW/dR^m \leq 0$ whenever $\omega < 1$: the central bank wants to reduce the interest rate to lower the value of tokenized money when it is used as collateral, inducing crypto banks to issue less stablecoins. If $\omega = 1$, the central bank finds the allocation with tokenized deposits optimal and issuing CBDC with the same interest rate achieves the same welfare. □

6.2 Prohibiting tokenized deposits

Given the result above, it is natural to expect that prohibiting tokenized deposits is welfare improving. The following proposition provides a condition under which it is necessary to prohibit tokenized deposits in order to achieve the optimal CBDC design. The reason is that, when the optimal CBDC rate is so low that traditional banks find the cheap funding cost very attractive, they will issue more tokenized deposits than desirable, driving the interest rate up to mop up the excess supply of (tokenized) deposits and distorting the allocation.

Proposition 7. *Suppose $\omega < \rho$. The optimal CBDC design requires that tokenized deposits be prohibited.*

Proof. We have shown that the optimal CBDC design has $\mu = 1$. The optimal CBDC design then solves

$$\max_{R^m} W = u(x_T) - x_T + \omega[u(x_D) - x_D]$$

$$\text{subject to:} \qquad u'(x_D) = \frac{1}{\beta R^m}, \text{ and} \quad x_T = \rho(B - x_D)$$

Hence the FOC satisfies

$$[u'(x_T) - 1]\rho = \omega[u'(x_D) - 1].$$

Note that the traditional bank has an incentive to issue an infinite amount of tokenized deposits when

$$u'(x_T) < \frac{1}{\beta R^m} = u'(x_D),$$

which is satisfied because, according to the above FOC,

$$\begin{aligned} & u'(x_T) - u'(x_D) \\ = \quad & [u'(x_D) - 1]\frac{\omega - \rho}{\rho} < 0. \end{aligned}$$

□

Since market forces will always constrain the central bank if they are allowed to play, optimality requires that tokenized deposits be prohibited so that the central bank can set the interest rate at the optimal level.

7 Extensions

In this section, we examine a few extensions of the model to generalize our results.

7.1 Private credit creation by banks

Our benchmark model implies that there are no real benefits of issuing tokenized deposits. In some sense, everything the traditional bank does, the central bank can do better. Here, we analyze an extension where traditional banks extend loans to firms and can use these loans to back deposits. It is important to consider lending to the real sector because the CBDC policy affects the interest rate, impacting the lending decisions of traditional banks.

We assume traditional banks can purchase bonds from the one firm representing the aggregate productive sector. Each bond has price q and gives a real return of 1. The representative firm's production function is $F(k)$ and the firms' profit is $F(k) - (1 + r)k$ where k is the capital invested by the firm.

In equilibrium, when the firm issues L bonds, it will be able to invest $k = qL$ and its profit will be $F(qL) - L$.

Hence the supply of corporate bonds L_s is given by the solution to the firm's problem

$$\max_{L_s} F(qL_s) - L_s$$

and the FOC gives $qF'(qL_s) = 1$. Therefore if $F(.)$ is very concave such that $-qL_s \frac{F''(qL_s)}{F'(qL_s)} > 1$, then $\partial L_s/\partial q < 0$. Otherwise, $\partial L_s/\partial q > 0$. We assume the latter so that increasing q (decreasing the interest rate) increases the supply of corporate bonds. For example, if $F(k) = Ak^{\alpha}$, the FOC gives $A\alpha q\,(qL)^{\alpha-1} = 1$ so $L^{1-\alpha} = A\alpha q^{\alpha}$ and L is increasing with q whenever $\alpha < 1$.

The problem of the traditional bank then is

$$\max_{a,d,b,L} [-a + \beta u(x_T)]$$

subject to $x_T = d$ and

$$\begin{aligned} a + D - q(b+L) + \beta\,(b + L - R^m D - d) &\geq 0 \\ \rho(b+L) &\geq d \end{aligned}$$

Notice that the problem is now a function of $\tilde{b} = b + L$. So when both constraints bind, an equilibrium with $D > 0$ is characterized by $x_T = \rho(B + L_s(q)) - R^m D$ and

$$\begin{aligned} u'(x_T) &= \frac{1}{\beta R^m} \\ \rho\beta u'(x_T) + \beta(1-\rho) &= q \end{aligned}$$

So $\partial x_T/\partial R^m > 0$ and so $\partial q/\partial R^m < 0$, and D is given by the demand for tokenized deposits, as in the previous section.

Welfare is

$$W^T = u(x_T) - x_T + F(qL) - L + \omega\,[u(x_D) - x_D]$$

If $\rho < 1$ we have the same result as before (CBDC does better than tokenized deposits because it uses collateral more efficiently). Let us set $\rho = 1$. A change in R^m affects welfare in the following way

$$\frac{\partial W^T}{\partial R^m} = [u'(x_T) - 1]\,\frac{\partial x_T}{\partial R^m} + \underbrace{[qF'(qL) - 1]}_{=0}\,\frac{\partial L}{\partial R^m} + LF'(qL)\frac{\partial q}{\partial R^m} + \omega\,[u'(x_D) - 1]\,\frac{\partial x_D}{\partial R^m}$$

In particular, increasing R^m now reduces firms' production. Also, from market clearing, $x_T = B + L_s(q) - x_D/\nu$ (where $\nu = \mu$ in region $\mathcal{A}_{\tilde{m}}$ and $\nu = \kappa$ in region $\mathcal{A}_m$). Therefore, it requires:

$$\frac{\partial x_T}{\partial R^m} = L_s'(q)\frac{\partial q}{\partial R^m} - \frac{1}{\nu}\frac{\partial x_D}{\partial R^m}.$$

and $\partial x_T/\partial R^m$ is affected by the change in q (since traditional banks are the only one that purchase corporate bonds). It is straightforward to show that $\mu = 1$ using the same steps as in the previous section. The reason is that the previous proof only requires that R^m be reduced, which here plays to increase q and so L.

Then, following again the same steps as before, it is easy to show that, starting from an equilibrium with tokenized deposits, the central bank can achieve a better allocation by issuing a CBDC at a lower rate R^m_{cbdc} than the prevailing equilibrium interest rate, as long as $F(\cdot)$ is not too concave (so that L is increasing with q. However, a necessary condition is that the supply of government bonds B is large enough to allow the central bank to issue the sufficient amount of CBDC. Otherwise it is not clear that CBDC will dominate tokenized deposits. We summarize in the following result,

Corollary 2. *Suppose B is large enough or that the central bank can back CBDC with real assets. Then in a lending economy ($L > 0$) replacing tokenized deposits by a CBDC can (weakly) increase welfare.*

Finally, we illustrate why legalizing tokenized deposits can be welfare-improving when B is small. Consider first an economy without tokenized deposits. Suppose $B \to 0$ and $R^e \to 0$. Then it is optimal for the central bank to devote all the government bonds B to issue CBDC for the DeFi sector if

$$[u'(\rho L) - 1]\rho < \omega[u'(B) - 1],$$

and the price of corporate bonds is $q_L < 1$. The former condition ensures that allocating government bonds to CBDC creation dominates using bonds to create deposits, and the latter condition ensures that this does not drive the corporate bond price so high that there is over production. Now if banks are allowed to issue some tokenized deposits, they will have an incentive to do so by purchasing more corporate bonds and allocating more consumption to the DeFi sector as long as

$$[u'(\rho L) - 1] < [u'(B) - 1].$$

A sufficient condition is $\rho > \omega$. One can then show that, at the margin, permitting some tokenized deposits can improve social welfare by improving (i) consumption allocation between the two sectors and (ii) production efficiency.

7.2 Endogenous crypto asset returns R^e

Suppose there is a fixed supply E of the crypto assets. These assets pay a dividend δ each period. These assets are initially held by risk neutral agents that discount the future at rate β. These agents are active in the PM and crypto AM, and they can trade their asset there to obtain utility $v(x)$. Assume $v'(0)$ is finite, so that these agents may want to sell all their assets. Also they can work to produce the PM good with a linear technology. The problem of these agents in the PM is to sell $e^s \leq E$ to maximize

$$\max_{e^s,x} p_t^e e^s + \beta v(x) + \beta(\delta + p_{t+1}^e)\left[E - e^s - x\right]$$

subject to

$$x \leq \left(p_{t+1}^e + \delta\right)\left(E - e^s\right).$$

The assumption here is that sellers acquire the crypto asset in the AM by producing x, and sell these assets back in the crypto PM. So the value of one unit of the crypto asset for sellers is $(p_{t+1}^e + \delta)$. So bringing $E - e^s$ in the crypto PM, crypto consumers can get at most $\left(p_{t+1}^e + \delta\right)\left(E - e^s\right)$ of the crypto good. The FOC with respect to x gives

$$\beta v'(x) - \beta(\delta + p_{t+1}^e) - \lambda_x = 0$$

Hence if these agents constraint binds in the AM, $x = \left(p_{t+1}^e + \delta\right)\left(E - e^s\right)$, the FOC gives

$$p_t^e = \beta v'\left(\left(\delta + p_{t+1}^e\right)\left(E - e^s\right)\right) > \beta(\delta + p_{t+1}^e)$$

while if the constraint does not bind, then

$$p_t^e = \beta(\delta + p_{t+1}^e).$$

So the "natural" price of these assets after they have paid the dividend δ is

$$p_t^e = \beta\left(\delta + p_{t+1}^e\right)$$

and in steady state,

$$p^e = \frac{\beta\delta}{1-\beta}.$$

If the constraint binds, $p_t^e > \beta(\delta + p_{t+1}^e)$.

To be consistent with the model notation, define the return at time t from the asset as

$$R^e = \frac{(\delta + p_{t+1}^e)}{p_t^e}$$

and in steady state the equilibrium return gives the price,

$$\beta R^e = \frac{\beta(\delta + p^e)}{p^e} \leq 1$$

where $\beta R^e < 1$ if the selling constraint binds. Notice that

$$p^e = \frac{\delta}{R^e - 1}$$

Let's analyze an equilibrium in region $\mathcal{A}_e$ we have $e > 0$ and $m = \tilde{m} = 0$ – the equilibrium in the other two regions is straightforward because the crypto bank does not demand any e, so that $e^s = 0$ in equilibrium – where

$$\begin{aligned} u'(\kappa R^e e) &= \frac{1 - (1-\kappa)\beta R^e}{\kappa \beta R^e}, \\ R^m &< \min\left\{\frac{\kappa R^e}{\mu\left[1 - (1-\kappa)\beta R^e\right]}, R^e\right\} \equiv R_1^m \end{aligned}$$

Case 1. $\beta R^e = 1$ and $e < E$. Then the equilibrium in the crypto sector is given by

$$u'\left(\kappa \frac{e}{\beta}\right) = 1$$

and market clearing gives $e = e^s$ such that

$$p^e = \frac{\beta\delta}{1-\beta} = \beta v'\left((\delta + p^e)(E - e)\right).$$

So this is a knife edge case. This is an equilibrium if

$$R^m < \min\left\{\frac{1}{\beta\mu}, \frac{1}{\beta}\right\} \equiv R_1^m.$$

Case 2. $\beta R^e < 1$ and $e < E$. Then the equilibrium in the crypto sector is given by

$$u'(\kappa R^e e) = \frac{1 - (1-\kappa)\beta R^e}{\kappa \beta R^e} > 1,$$

and using market clearing

$$p^e = \frac{\delta}{R^e - 1} = \beta v'\left((\delta + p^e)(E - e)\right).$$

These two equations give the equilibrium (R^e, e). This is an equilibrium iff

$$R^m < \min\left\{\frac{\kappa R^e}{\mu\left[1 - (1-\kappa)\beta R^e\right]}, R^e\right\} \equiv R_1^m$$

Case 3. $\beta R^e < 1$ and $e = E$. Then the equilibrium rate of return in the crypto sector R^e is given by

$$u'(\kappa R^e E) \quad = \quad \frac{1 - (1-\kappa)\beta R^e}{\kappa\beta R^e} > 1.$$

and using market clearing

$$p^e = \frac{\delta}{R^e - 1} > \beta v'(0),$$

so the price is so high that the holders of crypto assets forgo the gains from trading with crypto assets, by selling them all to the crypto bank. This is an equilibrium iff

$$R^m < \min\left\{\frac{\kappa R^e}{\mu\left[1 - (1-\kappa)\beta R^e\right]}, R^e\right\} \equiv R_1^m.$$

Bottom line:

Since $v(.)$ is concave, crowding out crypto assets from the crypto bank by offering a higher rate of return CBDC will reduce the price of crypto assets p^e. We can see this from the equilibrium equation $p^e = v'\left((\delta + p^e)(E - e)\right)$ and using the implicit function theorem

$$dp^e = (E - e)v''(x)\, dp^e - (\delta + p^e)v''(x)\, de$$

Hence

$$\frac{dp^e}{de} = \frac{-(\delta + p^e)v''(x)}{1 - (E - e)v''(x)} > 0$$

and the more the crypto bank demands crypto assets, the higher their price p^e.

7.3 Supervision of crypto banks ($\kappa \uparrow$)

In the benchmark model, we assume that κ is a parameter. In reality, κ can likely be affected by government regulations. For example, supervision may raise the value of κ. This consideration is relevant as there is currently a debate on whether authorities ought to spend resources regulating crypto shadow banks. It is believed that regulation can make these banks more trustworthy so that the crypto sector can develop in a safe and sound way; abstaining from regulating them, it is thought that crypto banks cannot stand the test of time. While some economists argue that regulators should not act and just "let it burn" (e.g., Cecchetti and Schoenholtz, 2022; Summer 2022), others point out that some regulations could be socially desirable (Waller, 2022). To shed some light on that debate, in this section we analyze the optimal level of pledgeability κ: indeed, in our model, a strictly regulated crypto bank will have a higher pledgeability parameter, because the regulation should make it more difficult for the crypto bank to abscond with their assets.

We measure optimality by the following welfare function

$$W = u(x_T) - x_T + \omega \left[u(x_D) - x_D\right],$$

where x_T is the consumption of depositors in the traditional banking sector and x_D is the consumption of crypto space users, while ω is a welfare weight on the surplus generated in this sector. ω captures the fact that the regulator considers the crypto space to be mostly useful to conduct illicit transactions (but not only which could explain why $\omega > 0$). In addition, there are foreign transactions that do not contribute to the welfare of a country.

The optimal choice of κ maximizes W subject to the constraint that (x_D, x_T) are equilibrium allocations of the economy with tokenized deposits. To facilitate the presentation of the analysis, we summarize the equilibrium allocations with tokenized deposits (x_D, x_T) in the following table.

	x_D	x_T
$\mathcal{A}_e$ or $R^m < R_1^m$	$x_D = \kappa R^e e \quad u'(x_D) = \frac{1-(1-\kappa)\beta R^e}{\kappa \beta R^e}$	$x_T = \rho B \quad u'(x_T) \geq \frac{1}{\beta R^m}$
$\mathcal{A}_{\tilde{m}}$ or $R_2^m \geq R^m \geq R_1^m$	$x_D = \mu R^m D \quad u'(x_D) = \frac{1}{\beta \mu R^m}$	$x_T = \rho B - R^m D \quad u'(x_T) = \frac{1}{\beta R^m}$
$\mathcal{A}_m$ or $R^m \geq R_2^m$	$x_D = \kappa R^m D \quad u'(x_D) = \frac{1-(1-\kappa)\beta R^m}{\kappa \beta R^m}$	$x_T = \rho B - R^m D \quad u'(x_T) = \frac{1}{\beta R^m}$

In region $\mathcal{A}_e$, there is no demand for tokenized deposits. As a consequence there is a dichotomy between the traditional and crypto sectors. In particular, the trade surplus in the traditional sector is independent of κ, while the surplus in the crypto sector is increasing in κ. Therefore the optimal policy in this region is to increase κ to 1, thus maximizing the pledgeability of the assets held by the crypto banks. Also, increasing κ will increase R_1^m so that this region applies for more parameters: it is more likely that the crypto will not rely on tokenized money thus maintaining the (potentially) desirable separation between the crypto and the traditional sectors. But starting from an equilibrium in region $\mathcal{A}_e$ and increasing κ, the equilibrium remains in region $\mathcal{A}_e$.

In region $\mathcal{A}_{\tilde{m}}$, the crypto bank only acquires tokenized deposits from the traditional bank and passes it to its users who use them directly in the crypto space. Therefore κ does not affect the allocation in this region, because the crypto bank is a mere pass-through vehicle. However κ affects the possibility of falling in this region: indeed increasing κ above some threshold value will imply that $R_1^m = R_2^m = R^e$ so that this region vanishes. Then the crypto bank will turn from simple pass-through to an intermediary. To the contrary, decreasing κ, e.g., by relaxing regulations, makes falling in $\mathcal{A}_{\tilde{m}}$ more likely, where crypto banks have no meaningful role. This is how limited regulation induces the crypto bank to "burn" in our model.

In region $\mathcal{A}_m$, the crypto bank acquires tokenized deposits and use them as collateral to secure its issuance of stablecoins. Intuitively, keeping x_D constant, it is clear that increasing κ implies a reduction in D, so that the consumption in the traditional sector (and surplus) increases. The reason is that a higher degree of pledgeability of the collateral in the crypto sector frees up resources in the traditional sector. However, x_D does not need to remain constant when κ changes. In particular, if the substitution effect is stronger than the income effect, then increasing κ may increase the demand for tokenized money so much that $R^m D$ increases. The proposition below says that if the coefficient of relative risk aversion ξ is high enough then the substitution effect is sufficiently muted that increasing κ reduces the demand for tokenized deposits (cum interest, so $R^m D$) while still increasing the surplus for crypto users.

More precisely, in region $\mathcal{A}_m$ the equilibrium is given by the following two equations in the two unknowns (R^m, D),

$$\begin{aligned} \kappa\beta R^m u'(\kappa R^m D) + (1-\kappa)\beta R^m &= 1 \\ \beta R^m u'(\rho B - R^m D) &= 1 \end{aligned}$$

In the Appendix we show that if the (constant) coefficient of relative risk aversion ξ is greater than some threshold $\bar{\xi}$ defined in the proof, then a rise in κ increases the surplus in both the traditional and the crypto sectors.

Proposition 8. *There is $\bar{\xi}$ such that if the coefficient of relative risk aversion is $\xi > \bar{\xi}$, then $\kappa = 1$ is* *optimal.*

With log-utility ($\xi = 1$), it will be optimal to set $\kappa = 1$. According to the above proposition, the optimal regulation depends on (i) the severity of crypto banks' incentive problem, (ii) whether and how traditional bank liabilities are demanded in the crypto space, and (iii) the response of such demand to the regulation.

7.4 Outright ban of crypto banks ($\kappa = 0$)

Instead of driving up κ through supervision, regulators may also drive κ to zero by banning crypto banks outright. When $\omega > 0$, setting $\kappa = 0$ to ban crypto banks cannot improve welfare. In region $\mathcal{A}_m$, it does not matter. In the other two regions, welfare drops as X_D becomes zero. When $\omega < 0$, however, it is optimal to drive X_D to zero by setting $\kappa = 0$ and to stop the issuance of CBDC and tokenized deposits.

7.5 Licit and illicit crypto transactions

Suppose not all crypto transactions are illicit. In particular, there are two types of crypto consumers: licit and illicit, where licit consumers only purchase licit goods, while illicit consumers purchase illicit goods. The measure of licit consumers is λ while the measure of illicit consumers is $1 - \lambda$. The sellers are the same and they produce licit or illicit goods using the same production technology (so the price is $p = 1$). With surveillance, a CBDC transaction is monitored so that an illicit payment is confiscated with probability $1 - \mu$ while a licit transaction is not affected.

The crypto bank offers a contract to licit consumers and another to illicit consumers. The contract to the illicit consumers solves

$$\max_{a,s,e,m,\tilde{m}} \left[-c + \beta \left[R^c c - s\right] - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\right] + \beta\lambda \left[\kappa R^c c - s\right]$$

while the contract to the licit consumers solves the same problem with $\mu = 1$.

Assume that licit consumers find it optimal to use CBDC as a means to pay. Since $\mu = 1$ for licit transactions, licit consumers choose

$$R^m M^\ell = x_D^\ell$$

with

$$u'(x_D^\ell) \quad = \quad \frac{1}{\beta R^m}$$

while illicit consumers' consumption x_D^i and traditional depositors' consumption x_T are governed by

regions	bounds	$R^m M^i$	x_D^i	x_T
$\mathcal{A}_e$	$R^m \leq \min\left\{\frac{\kappa R^e}{\mu[1-\beta(1-\kappa)R^e]}, R^e\right\} = R_1^m$	0	$u'(x_D^i) = \frac{1-(1-\kappa)\beta R^e}{\kappa \beta R^e}$	$\rho\left(B - \lambda x_D^\ell\right)$
$\mathcal{A}_{\tilde{m}}$	$R_1^m \leq R^m \leq R_2^m$	$\frac{1}{\mu} u'^{-1}\left[u'(x_D^i)\right]$	$u'(x_D^i) = \frac{1}{\beta \mu R^m}$	$\rho(B - \lambda x_D^\ell - (1-\lambda) R^m M^i)$
$\mathcal{A}_m$	$R^m \geq \max\left\{\frac{\mu-\kappa}{\beta\mu(1-\kappa)}, R^e\right\} = R_2^m$	$\frac{1}{\kappa} u'^{-1}\left[u'(x_D^i)\right]$	$u'(x_D^i) = \frac{1-(1-\kappa)\beta R^m}{\kappa \beta R^m}$	$\rho(B - \lambda x_D^\ell - (1-\lambda) R^m M^i)$

We measure optimality using the following objective function:

$$W(\mu) = u(x_T) - x_T + \lambda\left[u(x_D^\ell) - x_D^\ell\right] + \omega(1-\lambda)\left[u(x_D^i) - x_D^i\right]$$

where x_D^ℓ is the consumption of licit consumers and x_D^i is the one of illicit consumers. Let's consider $\omega > 0$.

In Region $\mathcal{A}_e$ and $\mathcal{A}_m$ there is no effect of changing μ. In Region $\mathcal{A}_{\tilde{m}}$: Fixing R^m, we have (since x_D^ℓ does not depend on μ):

$$\begin{aligned}
W'(\mu) &= \left[u'(x_T) - 1\right]\frac{\partial x_T}{\partial \mu} + \lambda\left[u'(x_D^\ell) - 1\right]\frac{\partial x_D^\ell}{\partial \mu} + \omega(1-\lambda)\left[u'(x_D^i) - 1\right]\frac{\partial x_D^i}{\partial \mu} \\
&= \left[u'(x_T) - 1\right]\frac{\partial x_T}{\partial \mu} + \omega(1-\lambda)\left[u'(x_D^i) - 1\right]\frac{\partial x_D^i}{\partial \mu} \\
&= \left[u'(x_T) - 1\right]\left(-\rho(1-\lambda)R^m\frac{\partial M^i}{\partial \mu}\right) + \omega(1-\lambda)\left(\frac{1}{\beta\mu R^m} - 1\right)\frac{\partial x_D^i}{\partial \mu}
\end{aligned}$$

Using the implicit function theorem, we get $\partial x_D^i / \partial \mu$

$$\begin{aligned}
u''(x_D^i)dx_D^i &= -\frac{1}{\beta\mu^2 R^m}d\mu \\
\frac{dx_D^i}{d\mu} &= -\frac{u'(x_D^i)}{\mu u''(x_D^i)} \\
\frac{dx_D^i}{d\mu} &= -\frac{u'(x_D^i)}{\mu u''(x_D^i)x_D^i}x_D^i = \frac{x_D^i}{\sigma\mu}
\end{aligned}$$

where σ is the coefficient of relative risk aversion, and $\partial M^i/\partial\mu$ from $\mu R^m M^i = x_D^i$

$$\begin{aligned}
R^m M^i d\mu + \mu R^m dM^i &= dx_D^i \\
\mu R^m \frac{dM^i}{d\mu} &= \frac{dx_D^i}{d\mu} - R^m M^i \\
\mu R^m \frac{dM^i}{d\mu} &= \frac{x_D^i}{\sigma\mu} - \frac{x_D^i}{\mu} \\
\mu R^m \frac{dM^i}{d\mu} &= \frac{(1-\sigma)}{\sigma\mu}x_D^i
\end{aligned}$$

Therefore,

$$\begin{aligned} W'(\mu) &= [u'(x_T)-1]\left(-\rho(1-\lambda)R^m\frac{\partial M^i}{\partial\mu}\right)+\omega(1-\lambda)\left(\frac{1}{\beta\mu R^m}-1\right)\frac{\partial x_D^i}{\partial\mu} \\ &= [u'(x_T)-1]\left(-\frac{\rho(1-\lambda)}{\mu}\frac{(1-\sigma)}{\sigma\mu}x_D^i\right)+\omega(1-\lambda)\left(\frac{1}{\beta\mu R^m}-1\right)\frac{x_D^i}{\sigma\mu} \\ &= \left\{\omega\frac{1-\beta\mu R^m}{\beta R^m}-[u'(x_T)-1]\,\rho(1-\sigma)\right\}(1-\lambda)\frac{x_D^i}{\sigma\mu^2} \end{aligned}$$

Hence if $\sigma > 1$, $W'(\mu) > 0$ and $\mu = 1$ is the solution.

7.6 Stablecoins backed by government bonds ($\kappa_b < \kappa_e$)

In this section we assume that the crypto bank can back the issuance of its stable coin with crypto assets, tokenized assets, or government bonds.

Given μ, R^b, R^m and R^e, a crypto bank maximizes its users' payoff by choosing the users' investment into the bank a, the quantity of tokenized money $\tilde{m}$ and stablecoins s directly held by its users, as well as the reserves of tokenized money m, bonds b, and crypto assets e that the crypto bank will hold to back its issuance of stablecoins,[9]

$$\max_{a,s,e,m,\tilde{m}} \left[-a-\tilde{m}+\beta u(s+\mu R^m\tilde{m})\right]$$

$$\begin{aligned} \text{subject to :} \qquad \underbrace{a-m-e-b+\beta\left[R^e e+R^b b+R^m m-s\right]}_{\text{net worth}} &\geq 0, \qquad (PC) \\ \kappa_e R^e e+\kappa_b R^b b+\kappa_m R^m m &\geq s. \qquad (IC) \end{aligned}$$

where $\kappa_c \in (0,1)$ denotes the pledgeability parameter of asset $c = b, e, m$ for the crypto bank. It is obvious that (PC) binds, so that the problem becomes

$$\begin{aligned} \max_{a,s,e,m,\tilde{m}} &\left[-b-e-m+\beta\left[R^b b+R^e e+R^m m-s\right]-\tilde{m}+\beta u(s+\mu R^m\tilde{m})\right] \\ &\qquad +\beta\lambda\left[\kappa_e R^e e+\kappa_b R^b b+\kappa_m R^m m-s\right] \end{aligned}$$

where $\beta\lambda$ is the multiplier on the IC of the crypto bank. We can rewrite this problem as

$$\max_{a,s,e,m,\tilde{m}} \left[-b-e-m+\sum_{c=b,e,m}(1+\lambda\kappa_c)\beta R^c c-\beta(1+\lambda)s-\tilde{m}+\beta u(s+\mu R^m\tilde{m})\right]$$

[9]This problem is equivalent to maximizing the bank's payoff subject to the user's participation constraint.

Let $c = m, e, b$ denote the asset held by the crypto as collateral. If $\beta R^c < 1$ the crypto bank only holds asset $c = m, e, b$ if it relaxes IC, and it is indifferent when $\beta R^c = 1$. Also, the crypto bank holds the cheapest asset to satisfy its IC, that is, it will hold m whenever $(1 + \lambda\kappa_m)R^m > \max\left\{(1 + \lambda\kappa_e)R^e, (1 + \lambda\kappa_b)R^b\right\}$, e whenever $(1 + \lambda\kappa_e)R^e > \max\left\{(1 + \lambda\kappa_b)R^b, (1 + \lambda\kappa_m)R^m\right\}$, and b otherwise. It will be indifferent between two assets c_1, c_2 whenever $(1+\lambda\kappa_{c_1})R^{c_1} = (1+\lambda\kappa_{c_2})R^{c_2}$. With this understanding we can rewrite the problem of the crypto bank as

$$\max_{a,s,e,m,\tilde{m}} \left[-c + \beta\left[(1 + \lambda\kappa_c)R^c c - (1 + \lambda)s\right] - \tilde{m} + \beta u(s + \mu R^m \tilde{m})\right]$$

The first order conditions are

$$\begin{aligned} s: \quad u'(s + \mu R^m \tilde{m}) &\leq 1 + \lambda, \\ c: \quad (1 + \lambda\kappa_c)\beta R^c &\leq 1, \\ \tilde{m}: \quad \mu\beta R^m u'(s + \mu R^m \tilde{m}) &\leq 1. \end{aligned}$$

1. First suppose there is c such that $\beta R^c = 1$. Then $\lambda = 0$, $u'(s + \mu R^m \tilde{m}) = 1$ and $\tilde{m} = 0$ if $\mu < 1$ since $\mu < 1$ implies $\mu\beta R^m u'(s + \mu R^m \tilde{m}) < 1$.

2. Next suppose $\beta R^c < 1$ for all c.

   (a) If $s = 0$ then $\tilde{m} > 0$ and it solves $\mu\beta R^m u'(\mu R^m \tilde{m}) = 1$. This is the case iff for all c,

$$u'(\mu R^m \tilde{m}) = \frac{1}{\mu\beta R^m} = 1 + \lambda$$

$$\left(1 - \kappa_c + \frac{1}{\mu\beta R^m}\kappa_c\right)\beta R^c \leq 1.$$

   Hence, stable coin is not issued whenever

$$\beta R^m \leq \frac{\mu - \kappa_m}{\mu\left(1 - \kappa_m\right)}$$

$$\frac{\kappa_e \beta R^e}{\mu\left[1 - \left(1 - \kappa_e\right)\beta R^e\right]} \leq \beta R^m$$

   and

$$\frac{\kappa_b \beta R^b}{\mu\left[1 - \left(1 - \kappa_b\right)\beta R^b\right]} \leq \beta R^m$$

(b) If $s > 0$ then

$$u'(s + \mu R^m \tilde{m}) = 1 + \lambda$$

and for at least one c,

$$\begin{aligned}(1 + \lambda\kappa_c)\beta R^c &= 1. \\ \lambda &= \frac{1 - \beta R^c}{\beta R^c \kappa_c}.\end{aligned}$$

Hence for the one c,

$$u'\left(\kappa_c R^c c + \mu R^m \tilde{m}\right) = \frac{1 - \beta R^c(1 - \kappa_c)}{\beta R^c \kappa_c} > 1,$$

with $\tilde{m}$ is given by

$$\tilde{m} = \begin{cases} 0 & \text{if } \mu\beta R^m \frac{1-\beta R^c(1-\kappa_c)}{\beta R^c \kappa_c} < 1 \\ \geq 0 & \text{if } \mu\beta R^m \frac{1-\beta R^c(1-\kappa_c)}{\beta R^c \kappa_c} = 1 \end{cases}$$

i. If the **stablecoin is backed by crypto assets**, then

$$u'\left(\kappa_e R^e e + \mu R^m \tilde{m}\right) = \frac{1 - \beta R^e(1 - \kappa_e)}{\beta R^e \kappa_e},$$

and

$$\begin{aligned}(1 + \lambda\kappa_e)\beta R^e &= 1 > \max\left\{(1 + \lambda\kappa_m)\beta R^m, (1 + \lambda\kappa_b)\beta R^b\right\} \\ & \quad 1 > \max\left\{\left(1 + \frac{1 - \beta R^e}{\beta R^e \kappa_e}\kappa_m\right)\beta R^m, \left(1 + \frac{1 - \beta R^e}{\beta R^e \kappa_e}\kappa_b\right)\beta R^b\right\}\end{aligned}$$

Hence,

$$\begin{aligned}\beta R^m &< \frac{\beta R^e \kappa_e}{1 + \beta R^e(\kappa_e - \kappa_m)} \iff \frac{\beta R^m}{\kappa_e - \beta R^m(\kappa_e - \kappa_m)} < \beta R^e \\ \beta R^b &< \frac{\beta R^e \kappa_e}{1 + \beta R^e(\kappa_e - \kappa_b)} \iff \frac{\beta R^b}{\kappa_e - \beta R^b(\kappa_e - \kappa_b)} < \beta R^e\end{aligned}$$

ii. If the **stablecoin is backed by bonds**, then

$$u'\left(\kappa_b R^b b + \mu R^m \tilde{m}\right) = \frac{1 - \beta R^b(1 - \kappa_b)}{\beta R^b \kappa_b},$$

and, symmetrically to the condition above,

$$\begin{aligned}\beta R^m &< \frac{\beta R^b \kappa_b}{1 + \beta R^b(\kappa_b - \kappa_m)}, \\ \beta R^e &< \frac{\beta R^b \kappa_b}{1 + \beta R^b(\kappa_b - \kappa_e)}.\end{aligned}$$

iii. If the **stablecoin is backed by tokenized money**, then

$$u'\left(\kappa_m R^m m + \mu R^m \tilde{m}\right) = \frac{1 - \beta R^m (1-\kappa_m)}{\beta R^m \kappa_m},$$

and, symmetrically to the condition above,

$$\begin{aligned}\beta R^e &< \frac{\beta R^m \kappa_m}{1+\beta R^m(\kappa_m - \kappa_e)},\\ \beta R^b &< \frac{\beta R^m \kappa_m}{1+\beta R^m(\kappa_m - \kappa_b)}.\end{aligned}$$

Notice that if $\kappa_m = \kappa_b$ and tokenized money has a liquidity premium relative to bonds ($\beta R^m \leq \beta R^b$) then this can't be the case.

7.7 CBDC for the traditional sector

Many papers have already studied the competition between CBDC and bank deposits (see the literature review in Section 1). In the benchmark environment, if CBDC is circulated only in the traditional sector, then it is optimal to crowd out traditional banks as central banks are more efficient in creating money whenever $\rho < 1$. We can restore a welfare trade-off when only banks can offer loans to firms, as modelled above.

7.8 Regulating CBDC collaterallization

Our benchmark model assumes that central bank cannot confiscate CBDC balances that are used by crypto banks to back the issuance of stablecoins while it can confiscate CBDC balances that are used as a means of payment. The reason is that using CBDC as a means of payments requires processing all payment transactions directly on the CBDC ledger, providing an easy way for the central bank to enforce regulations. By contrast, using a CBDC-backed stablecoins as a means of payments merely requires holding some CBDC balances by the issuers, with all the payments processed on the issuer's ledger. The central bank can potentially prohibit people sending CBDC to the issuer's account address. But as long as the issuer can create a new address easily to hold the CBDC it is very hard for the central bank to completely prevent stablecoins from utilizing CBDC as a reserve. In a sense, stablecoins are just providing a vehicle for people to avoid surveillance and use CBDC indirectly to conduct crypto transactions, rather than use CBDC directly as a payment instrument. It should be straightforward to generalize the model so that CBDC reserves can be confiscated with probability $1-\mu_m$. We expect that the basic results hold as long as it is more difficult to confiscate CBDC reserves i.e., $\mu_m > \mu$.

7.9 Privacy loss induced by surveillance

Suppose government surveillance generates a privacy loss, e.g., $\ell(\mu)$ with $\ell'(1) = 0$ $\ell'(\mu) < 0$. This will further increase the social costs of surveillance and strengthen the results of minimizing surveillance.

8 Conclusion

This paper develops a monetary model to study money creation for the crypto space, and assess how introducing tokenized money can affect the issuance and circulation of stablecoins. Our analysis highlights that, depending on its design, a CBDC can facilitate stablecoin creation and promote illicit activities. In addition, in contrast with some policy recommendations, we show that private banks can over or under supply tokenized deposits, hindering the implementation of optimal policy. Our findings suggest that policymakers need to clarify their objectives, pay attention to multiple channels and consider several design features before deciding whether CBDC issuance is an appropriate response to crypto sector development.

References

[1] Ahnert, T., Hoffmann, P. and Monnet, C., (2022). The digital economy, privacy, and CBDC.

[2] Andolfatto, D. (2020) Assessing the Impact of Central Bank Digital Currency on Private Banks. The Economic Journal 131.634 (2021): 525-540.

[3] Aoyagi, J. and Y. Itoy (2021). Coexisting Exchange Platforms: Limit Order Books and Automated Market Makers. url: https://ssrn.com/abstract=3808755.

[4] Auer, R., Frost, J., Gambacorta, L., Monnet, C., Rice, T., and Shin, H. S. (2022). Central bank digital currencies: motives, economic implications, and the research frontier. Annual review of economics, 14, 697-721.

[5] Bertsch, C. "Stablecoins: Adoption and fragility." (2023).

[6] Capponi, A. and R. Jia (2021). Decentralized Stablecoins and Collateral Risk. url: https://arxiv.org/pdf/2103.08842.pdf.

[7] Chiu, J., and Mohammad Davoodalhosseini. (2021) Central bank digital currency and banking: Macroeconomic benefits of a cash-like design. No. 2021-63. Bank of Canada.

[8] Chiu, J., E. Ozdenoren, K. Yuan, and S. Zhang. (2022) On the Fragility of DeFi Lending. Available at SSRN 4328481.

[9] Chiu, J., Davoodalhosseini, S. M., Jiang, J., and Zhu, Y. (2023a). Bank market power and central bank digital currency: Theory and quantitative assessment. Journal of Political Economy, 131(5), 1213-1248.

[10] Chiu, J., T. Koeppl, H. Yu, and S. Zhang. (2023b) Understanding DeFi Through the Lens of a Production-Network Model. No. 2023-42. Bank of Canada, 2023.

[11] Cong, L. W., and S. Mayer. (2022) The coming battle of digital currencies. The SC Johnson College of Business Applied Economics and Policy Working Paper Series 2022-04.

[12] D'Avernas, A, Bourany T, Q. Vandeweyer (2021). Are stablecoins stable? Workin g Paper. 2021 Feb 2.

[13] Fernandez-Villaverde, J., Sanches, D., Schilling, L., and Uhlig, H. (2021). Central bank digital currency: Central banking for all? Review of Economic Dynamics, 41, 225-242.

[14] Gorton, G. B., and J. Y. Zhang. (2023) Taming wildcat stablecoins. U. Chi. L. Rev. 90: 909.

[15] Keister, T., and C. Monnet. (2022) Central bank digital currency: Stability and information. Journal of Economic Dynamics and Control 142: 104501.

[16] Keister, T. and Sanches, D., (2023) Should central banks issue digital currency? The Review of Economic Studies, 90(1), pp.404-431.

[17] Kozhan, R. and G.F. Viswanath-Natraj (2021). Decentralized Stablecoins and Collateral Risk. url: https://dx.doi.org/10.2139/ssrn.3866975.

[18] R. Lagos and R. Wright (2005) "A Unified Framework of Monetary Theory and Policy Analysis." *Journal of Political Economy* 113, 463-484.

[19] Lehar, A. and C. A. Parlour (2021). Decentralized Exchanges. url: https : //dx.doi.org/10.2139/ssrn.3905316.

[20] Li, Y. and S. Mayer (2021). Money Creation in Decentralized Finance: A Dynamic Model of Stablecoin and Crypto Shadow Banking. url: https://dx.doi.org/10.2139/ ssrn.3757083.

[21] C. Monnet, A. Petursdottir, and M. Rojas-Breu (2020). "Central Bank Account for All: Efficiency and Stability." Mimeo.

[22] Schilling, Linda, JesÃºs FernÃ¡ndez-Villaverde, and Harald Uhlig. (2020) Central bank digital currency: When price and bank stability collide. No. w28237. National Bureau of Economic Research.

[23] Williamson, S. (2022a) Central bank digital currency: Welfare and policy implications. Journal of Political Economy 130.11 (2022): 2829-2861.

[24] Williamson, S. (2022b) Central bank digital currency and flight to safety. Journal of Economic Dynamics and Control 142 (2022): 104146.

9 Appendix

9.1 Equilibrium cases

Using (PC) rewrite as

$$\max_{s,e,m,\tilde{m}} -m - e + \beta\left[R^e e + R^m m - s\right] - \tilde{m} + \beta u(s + \mu R^m \tilde{m})$$
$$+\beta\lambda\left[\kappa\left(R^e e + R^m m\right) - s\right]$$

FOC:

$$\begin{aligned}
s: &\quad u'(s+\mu R^m\tilde{m}) &&= 1+\lambda \\
e: &\quad (1+\lambda\kappa)\beta R^e &&= 1 \\
m: &\quad (1+\lambda\kappa)\beta R^m &&\leq 1 \\
\tilde{m}: &\quad u'(s+\mu R^m\tilde{m})\beta\mu R^m &&\leq 1
\end{aligned}$$

We will focus on the parameter space where (IC) is binding. **We will provide conditions under which $\lambda > 0$.** Hence the problem becomes

$$\max_{e,m,\tilde{m}} -m - \tilde{m} - e + \beta(1-\kappa)\left[R^e e + R^m m\right] + \beta u\underbrace{\left(\kappa R^e e + \kappa R^m m + \mu R^m \tilde{m}\right)}_{=z}$$

FOC

$$\begin{aligned}
e: &\quad (1-\kappa)R^e + \kappa R^e u'(z) &&\leq \frac{1}{\beta} \\
m: &\quad (1-\kappa)R^m + \kappa R^m u'(z) &&\leq \frac{1}{\beta} \\
\tilde{m}: &\quad \mu R^m u'(z) &&\leq \frac{1}{\beta}
\end{aligned}$$

When $e > 0$, the consumption is given by

$$u'(z) \quad = \quad \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}.$$

Note that z is below the first-best level unless $\kappa = 1$ or $\beta R^e = 1$. The solution is $e, m, \tilde{m} > 0$ iff

$$R^m = R^e = \frac{(\mu-\kappa)}{\beta\mu(1-\kappa)},$$

which is a knife edge case. We first consider three conditions. First, $m > 0$ and $\tilde{m} = 0$ iff

$$R^m \geq \mathbf{C}_{m\tilde{m}}(\mu, \kappa) \equiv \frac{(\mu - \kappa)}{\beta\mu(1-\kappa)},$$

with the RHS is increasing in μ and decreasing in κ. Also, $\mathbf{C}_{m\tilde{m}}(1, \kappa) = \beta^{-1}$.. Second, $e > 0$ and $m = 0$ iff

$$R^m \leq \mathbf{C}_{em}(R^e) \equiv R^e.$$

Third, $e > 0$ and $\tilde{m} = 0$ iff

$$R^m \leq \mathbf{C}_{e\tilde{m}}(\mu, \kappa, R^e) \equiv \frac{\kappa R^e}{\mu\left[1 - (1-\kappa)\beta R^e\right]},$$

with the RHS is increasing in $R^e$and κ and decreasing in μ. We plot the three conditions in the (R^m, μ) space in Figure 4. Note that the lines defined by these conditions intersect at the point $R^m = R^e$ and $\mu = \frac{\kappa}{1-R^e\beta(1-\kappa)} < \kappa$. Three equilibrium regions are identified:

(i) $\mathcal{A}_e$ with $e > 0$ ($m = \tilde{m} = 0$) where

$$u'(z) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e},$$
$$R^m < \min\left\{R^e, \frac{\kappa R^e}{\mu\left[1-(1-\kappa)\beta R^e\right]}\right\}.$$

(ii) $\mathcal{A}_m$ with $m > 0$ ($e = \tilde{m} = 0$) where

$$u'(z) = \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m},$$
$$\max([\frac{(\mu-\kappa)}{\beta\mu(1-\kappa)}, R^e]) < R^m.$$

(iii) $\mathcal{A}_{\tilde{m}}$ with $\tilde{m} > 0$ ($e = m = 0$) where

$$\mu R^m u'(\mu R^m \tilde{m}) = \frac{1}{\beta},$$
$$\frac{\kappa R^e}{\mu[1-(1-\kappa)R^e\beta]} < R^m < \frac{\mu-\kappa}{\beta\mu(1-\kappa)}.$$

The following table lists the complete set of potential equilibria.

	e	m	$\tilde{m}$
(a)	>0	0	0
(b)	0	>0	0
(c)	0	0	>0
(d)	>0	>0	0
(e)	0	>0	>0
(f)	>0	0	>0
(g)	>0	>0	>0

The conditions for their existence are derived below.

Case (a): $e > 0, m = \tilde{m} = 0$**:**

$$\begin{aligned} e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &= \frac{1}{\beta} \\ m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &< \frac{1}{\beta} \\ \tilde{m}: \quad \mu R^m u'(z) &< \frac{1}{\beta} \end{aligned}$$

$$u'(z) = \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e}$$

and

$$R^m < R^e$$

and

$$\begin{aligned} \mu R^m u'(z) = \mu R^m \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e} &< \frac{1}{\beta}, \\ R^m &< \frac{\kappa R^e}{\mu\left[1-(1-\kappa)\beta R^e\right]}. \end{aligned}$$

This equilibrium exists in area $\mathcal{A}_e$.

Case (b): $e = 0, m > 0, \tilde{m} = 0$**:**

$$\begin{aligned} e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &< \frac{1}{\beta} \\ m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &= \frac{1}{\beta} \\ \tilde{m}: \quad \mu R^m u'(z) &< \frac{1}{\beta} \end{aligned}$$

and $R^e < R^m$ and

$$u'(z) = \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}$$

$$\begin{aligned} \mu u'(z) &< (1-\kappa)+\kappa u'(z) \\ \frac{(\mu-\kappa)}{\beta\mu(1-\kappa)} &< R^m \end{aligned}$$

This equilibrium exists in area $\mathcal{A}_m$.

Case (c): $e = 0, m = 0, \tilde{m} > 0$**:**

$$\begin{aligned} e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &< \frac{1}{\beta} \\ m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &< \frac{1}{\beta} \\ \tilde{m}: \quad \mu R^m u'(z) &= \frac{1}{\beta} \end{aligned}$$

and

$$\mu R^m u'(\mu R^m \tilde{m}) = \frac{1}{\beta}$$

which requires

$$\begin{aligned} (1-\kappa)R^m + \frac{\kappa}{\mu}\frac{1}{\beta} &< \frac{1}{\beta} \\ \rightarrow R^m &< \frac{\mu-\kappa}{\beta\mu(1-\kappa)} \end{aligned}$$

$$\begin{aligned} (1-\kappa)R^e + \kappa R^e \frac{1}{\beta\mu R^m} &< \frac{1}{\beta} \\ \rightarrow \frac{\kappa R^e}{\mu[1-(1-\kappa)R^e\beta]} &< R^m \end{aligned}$$

This equilibrium exists in area $\mathcal{A}_{\tilde{m}}$.

Case (d): $e > 0, m > \tilde{m} = 0$**:**

$$\begin{aligned} e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &= \frac{1}{\beta} \\ m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &= \frac{1}{\beta} \\ \tilde{m}: \quad \mu R^m u'(z) &< \frac{1}{\beta} \end{aligned}$$

Hence $R^m = R^e$ and

$$\begin{aligned} \mu R^m u'(z) &< \frac{1}{\beta} \\ \mu R^m \frac{1-(1-\kappa)\beta R^e}{\kappa\beta R^e} &< \frac{1}{\beta} \\ \frac{\mu-\kappa}{\beta\mu(1-\kappa)} &< R^e = R^m \end{aligned}$$

This equilibrium exists along the line defined by $R^m = \mathbf{C}_{em}$.

Case (e): $e = 0, m > 0, \tilde{m} > 0$**:**

$$\begin{aligned} e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &< \frac{1}{\beta} \\ m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &= \frac{1}{\beta} \\ \tilde{m}: \quad \mu R^m u'(z) &= \frac{1}{\beta} \end{aligned}$$

then $R^e < R^m$ and from the last two

$$R^m = \frac{\mu-\kappa}{\beta\mu(1-\kappa)}$$

so that

$$u'(z) = \frac{1-(1-\kappa)\beta\frac{\mu-\kappa}{\beta\mu(1-\kappa)}}{\kappa\beta\frac{\mu-\kappa}{\beta\mu(1-\kappa)}} = \frac{1-\kappa}{\mu-\kappa}.$$

This equilibrium exists along the line defined by $R^m = \mathbf{C}_{m\tilde{m}}$.

Case (f): $e > 0, \tilde{m} > m = 0$**:**

$$\begin{aligned} e: \quad (1-\kappa)R^e + \kappa R^e u'(z) &= \frac{1}{\beta} \\ m: \quad (1-\kappa)R^m + \kappa R^m u'(z) &< \frac{1}{\beta} \\ \tilde{m}: \quad \mu R^m u'(z) &= \frac{1}{\beta} \end{aligned}$$

Hence

$$R^m < R^e$$

and

$$(1-\kappa)R^e + \kappa R^e \frac{1}{\mu R^m \beta} = \frac{1}{\beta}$$

so that

$$R^m = \frac{\kappa R^e}{\mu\left[1-(1-\kappa)\beta R^e\right]}$$

Since $R^m < R^e$,

$$\begin{aligned} \frac{\kappa R^e}{\mu\left[1-(1-\kappa)\beta R^e\right]} &< R^e \\ R^e &< \frac{\mu-\kappa}{\beta\mu(1-\kappa)} \end{aligned}$$

This equilibrium exists along the line defined by $R^m = \mathbf{C}_{e\tilde{m}}$.

Case (g): $e, m, \tilde{m} > 0$

$R^m = R^e$ and

$$\begin{aligned} (1-\kappa)R^m + \kappa R^m u'(z) &= \mu R^m u'(z) \\ u'(z) &= \frac{(1-\kappa)}{(\mu-\kappa)} \end{aligned}$$

and

$$\begin{aligned} \mu R^m u'(z) &= \frac{1}{\beta} \\ R^m &= \frac{(\mu-\kappa)}{\beta\mu(1-\kappa)} \end{aligned}$$

which is a knife edge case given by the interaction point of the three lines.

9.2 Proof $\kappa = 1$ Optimal for $\xi > \bar{\xi}$

$$\begin{aligned} \kappa\beta R^m u'(\kappa R^m D) + (1-\kappa)\beta R^m &= 1 \\ \beta R^m u'(\rho B - R^m D) &= 1 \end{aligned}$$

Define

$$x = R^m D$$

then we have

$$\begin{aligned} \kappa x u'(\kappa x) + (1-\kappa)x &= D/\beta \\ x u'(\rho B - x) &= D/\beta \end{aligned}$$

Hence, total derivatives give

$$\begin{aligned}\left[xu'(\kappa x)+\kappa x^2u''(\kappa x)-x\right]d\kappa+\left[\kappa u'(\kappa x)+\kappa^2xu''(\kappa x)+(1-\kappa)\right]dx &= dD/\beta\\ \left[u'(\rho B-x)-xu''(\rho B-x)\right]dx &= dD/\beta\end{aligned}$$

from the last equation, we have $dD/dx>0/$. We use a constant coefficient of risk aversion ξ, so that

$$\begin{aligned}x\left[u'(\kappa x)\left(1+\frac{\kappa xu''(\kappa x)}{u'(\kappa x)}\right)-1\right]d\kappa+\kappa\left[u'(\kappa x)\left(1+\frac{\kappa xu''(\kappa x)}{u'(\kappa x)}\right)+\frac{(1-\kappa)}{\kappa}\right]dx &= dD/\beta\\ u'(\rho B-x)\left[1-x\frac{u''(\rho B-x)}{u'(\rho B-x)}\right]dx &= dD/\beta\end{aligned}$$

and

$$\begin{aligned}x\left[u'(\kappa x)(1-\xi)-1\right]d\kappa+\kappa\left[u'(\kappa x)(1-\xi)+\frac{(1-\kappa)}{\kappa}\right]dx &= dD/\beta\\ u'(\rho B-x)\left[1+\frac{x}{\rho B-x}\xi\right]dx &= dD/\beta\end{aligned}$$

Using the FOC,

$$\begin{aligned}u'(\kappa R^mD) &= \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}\\ u'(\rho B-R^mD) &= \frac{1}{\beta R^m}\end{aligned}$$

we obtain

$$\begin{aligned}x\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)-1\right]d\kappa+\kappa\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)+\frac{(1-\kappa)}{\kappa}\right]dx &= dD/\beta\\ \frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right]dx &= dD/\beta\end{aligned}$$

Combining both equations we get $dx/d\kappa$:

$$x\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)-1\right]d\kappa+\kappa\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)+\frac{(1-\kappa)}{\kappa}\right]dx = \frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right]dx$$

or

$$\frac{dx}{d\kappa}=\frac{x\left[1-\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)\right]}{\kappa\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)+\frac{(1-\kappa)}{\kappa}\right]-\frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right]}$$

If $\xi \geq 1$, the nominator is positive, and the denominator is negative whenever

$$\begin{aligned}
\frac{1-(1-\kappa)\beta R^m}{\beta R^m}(1-\xi)+(1-\kappa)-\frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right] &< 0 \\
\left[1-(1-\kappa)\beta R^m\right](1-\xi)+\beta R^m(1-\kappa)-\left[1+\frac{x}{\rho B-x}\xi\right] &< 0 \\
(1-\xi)+(1-\kappa)\beta R^m\xi-\left[1+\frac{x}{\rho B-x}\xi\right] &< 0 \\
-1+(1-\kappa)\beta R^m-\frac{x}{\rho B-x} &< 0
\end{aligned}$$

which is always the case since $\beta R^m \leq 1$. Hence if $\xi \geq 1$ we have $dx/d\kappa \leq 0$. Therefore, $dD/d\kappa \leq 0$. Hence, as κ increases the consumption in the traditional sector increases.

Finally, we want to know the sign of $d(\kappa x)/d\kappa$, this is given by the sign of

$$\begin{aligned}
x+\kappa\frac{dx}{d\kappa} &= x+\kappa\frac{x\left[1-\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)\right]}{\kappa\left[\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)+\frac{(1-\kappa)}{\kappa}\right]-\frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right]} \\
&= x\left[1+\frac{1-\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)}{\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)+\frac{(1-\kappa)}{\kappa}-\frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right]}\right] \\
&= x\left[1-\frac{\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)-1}{\frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}(1-\xi)-1+\frac{1}{\kappa}-\frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right]}\right]
\end{aligned}$$

this is positive whenever

$$\frac{1}{\kappa}-\frac{1}{\beta R^m}\left[1+\frac{x}{\rho B-x}\xi\right]<0$$

so that the fraction in the expression in [.] is less than 1. Hence, we require

$$\begin{aligned}
\frac{\beta R^m}{\kappa} &< 1+\frac{R^m D}{\rho B-R^m D}\xi \\
\frac{\beta R^m}{\kappa}-1 &< \frac{R^m D}{\rho B-R^m D}\xi \\
\left[\frac{\beta R^m}{\kappa}-1\right]\left(\frac{\rho B}{R^m D}-1\right) &< \xi
\end{aligned}$$

Since $\rho B/R^m D > 1$and we are in region $\mathcal{A}_m$ we know $1 \geq \beta R^m \geq \beta \max\left\{\frac{\mu-\kappa}{\beta\mu(1-\kappa)}, R^e\right\} = \beta R_2^m$and

$$u'(\kappa R^m D) = \frac{1-(1-\kappa)\beta R^m}{\kappa\beta R^m}$$

if βR^mincrease, $R^m D$ has to increase, so $R^m D \geq \frac{1}{\kappa}u'^{-1}\left(\frac{1-(1-\kappa)\beta R_2^m}{\kappa\beta R_2^m}\right)$ and $R^m D \leq \frac{1}{\kappa}u'^{-1}(1)$. Hence a sufficient condition for $d(\kappa x)/d\kappa > 0$ is

$$\xi > \left(\frac{1}{\kappa} - 1\right)\left(\frac{\kappa\rho B}{u'^{-1}\left(\frac{1-(1-\kappa)\beta R_2^m}{\kappa\beta R_2^m}\right)} - 1\right) \equiv \bar{\xi}$$

9.2.1 With log-utility

Assume log utility, government sets κ to maximize welfare.

Note that

$$\frac{d}{d\kappa}R_1^m > 0$$
$$\frac{d}{d\kappa}R_2^m < 0$$

With log utility:

	$R^m M$	z	d
$\mathcal{A}_e$	0	$\frac{\kappa\beta R^e}{1-(1-\kappa)\beta R^e}$	ρB
$\mathcal{A}_{\tilde{m}}$	βR^m	$\beta\mu R^m$	βR^m
$\mathcal{A}_m$	$\frac{\beta R^m}{1-(1-\kappa)\beta R^m}$	$\frac{\kappa\beta R^m}{1-(1-\kappa)\beta R^m}$	βR^m

Replacing for the equilibrium values for R^m:

	$R^m M$	z	d
$\mathcal{A}_e$	0	$\frac{\kappa\beta R^e}{1-(1-\kappa)\beta R^e}$	ρB
$\mathcal{A}_{\tilde{m}}$	βR^m	$\mu\rho B/2$	$\rho B/2$
$\mathcal{A}_m$	$\frac{\beta R^m}{1-(1-\kappa)\beta R^m}$	$\frac{\kappa\beta R^m}{1-(1-\kappa)\beta R^m}$	βR^m

In the first region,

$$x_C = \frac{\kappa\beta R^e}{1-(1-\kappa)\beta R^e}$$

which is increasing in β. Hence, within this regio, it is optimal to maximize κ too.

In the second region, welfare is independent of κ.

In the third region:

Traditional consumption is

$$x_T = \beta R^m = \frac{[2+\rho B(1-\kappa)] - \sqrt{4+((1-\kappa)\rho B)^2}}{2(1-\kappa)} = \frac{2+v-\sqrt{4+v^2}}{2v}\rho B$$

where $v = \rho B(1-\kappa)$. We know that x_T is increasing in κ as

$$\begin{aligned}
\frac{d}{dv}\frac{2+v-\sqrt{4+(v)^2}}{v} &= \frac{v - v^2[4+(v)^2]^{-0.5} - 2 - v + \sqrt{4+(v)^2}}{v^2} \\
&= -\frac{1}{v^2}\left(\frac{v^2}{\sqrt{4+v^2}} + 2 - \sqrt{4+v^2}\right) \\
&= -\frac{1}{v^2\sqrt{4+v^2}}\left(v^2 + 2\sqrt{4+v^2} - 4 - v^2\right) \\
&= -\frac{2}{v^2\sqrt{4+v^2}}\left(\sqrt{4+v^2} - 2\right) < 0.
\end{aligned}$$

Hence

$$\frac{d}{d\kappa}x_T = \frac{d}{d\kappa}\beta R^m > 0$$

Since crypto consumption is

$$x_C = \frac{\kappa\beta R^m}{1-(1-\kappa)\beta R^m}$$

which is increasing in κ given R^m and increasing in R^m given κ. Hence $dx_C/d\kappa > 0$. Incresaing κ reduces D and hence releases resources to the traditional sector.

In this region, it is optimal to set $\kappa = 1$.