Mechanics of Spatial Growth*

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Abstract

We develop a dynamic spatial growth model to explore the role of trade and internal migration in the process of spatial development and aggregate growth. We consider an economy in which growth is shaped by the best global and local ideas that contribute to the local stock of knowledge. Global ideas diffuse to locations that are more exposed to international trade. Local ideas diffuse across space when workers move to another location. We embed the diffusion of ideas through trade and migration into a dynamic spatial framework with trade, forward-looking migration decisions, and capital accumulation. We characterize the equilibrium properties of the model, prove uniqueness of the balanced growth path, and show how to take the model to the data to conduct counterfactual analysis. As an application, we study China's spatial and aggregate growth during the 1990s and 2000s. We find that international trade and internal migration are important mechanisms for idea diffusion that contributed to China's spatial and aggregate growth, with heterogeneous effects across space. Using patent data we provide further evidence of idea diffusion through trade and migration.

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1 Introduction

Understanding economic growth requires understanding how countries accumulate factors of production and increase the productivity of such factors. In recent decades, the world has witnessed the successful growth experiences of developing countries such as Vietnam, Laos, the Philippines, and China, among others, where high economic growth has occurred hand-in-hand with increased trade openness, more internal migration, and high productivity growth. Aggregate economic growth is shaped in part by the process of development across space within a country, namely, the dynamics of the distribution of economic activity across space, the extent to which locations have differential exposure to trade, the internal mobility of labor, the local evolution of productivity, and other local characteristics. In this paper we develop a tractable dynamic spatial growth model to study quantitatively the process of spatial development across locations in a country and how this process shapes aggregate growth.

We consider a world economy with multiple countries and multiple locations within a country. Growth in each location is shaped by the endogenous evolution of total factor productivity, which is the outcome of the diffusion of global and local ideas. In each location, a continuum of differentiated goods is produced, and there are many potential producers for each good. Producers have heterogeneous productivities (ideas) to produce goods and those who are actively producing contribute to the local pool of ideas that determines the local stock of knowledge, namely, the local fundamental productivity.

Ideas diffuse across locations because of trade in goods and the migration of workers. To make the process of diffusion tractable, we model the diffusion of ideas across space as a stochastic process. In particular, we consider that ideas arrive stochastically to each producer. The productivity of each idea is a combination of a random original component, a random insight drawn from the ideas of sellers to that location, and a random insight drawn from workers in that location. Global ideas are embedded in imported intermediate goods and diffuse more to locations relatively more exposed to international trade. Workers learn about local ideas and carry insights with them across space when they migrate and interact with local producers in the destination location. As a result, the local pool of insights from workers contains local ideas from non-migrants and ideas from migrants. Building on the results in Buera and Oberfield (2020), we show that productivities at each location follows a Fréchet distribution and that the evolution of the stock of knowledge at each location can be characterized by a system of difference equations. The evolution of fundamental productivity in each location depends on how connected the particular location is to the other locations through trade and migration as well as the quality of insights from those locations. Moreover, the idea diffusion process in our framework is endogenous since migration is shaped by the decisions of forward-looking workers, and trade is determined by the production decisions of firms.

Since the productivities at each location follow a Fréchet distribution, we apply the results in Eaton and Kortum (2002) to model the trade and production structure in our framework. Producers in each location source intermediate goods from the lowest-cost suppliers across countries and combine them with labor and capital to produce goods. The labor supply across locations is shaped by the forward-looking migration decisions of workers as in Caliendo et al. (2019). At each moment in time, workers supply labor, purchase local goods, and sort into different locations. Workers carry ideas from their previous location with them as they migrate, and they provide insights to producers in the current location. Capital accumulation at each location is determined forward-looking landlords who make investment decisions in local capital as in Kleinman et al. (2023).

With all the margins previously described, our paper's methodological contribution is to provide a tractable dynamic spatial growth model to study the role of spatial development on aggregate growth in general equilibrium. We show how to invert the model to uniquely characterize the initial stock of knowledge at each location. Given the initial stock of knowledge and data on initial allocations (production, trade, migration), we show how to compute the model without having to assume that the economy is on a balanced growth path at the initial period. We also show the existence and uniqueness of the balanced growth path equilibrium.

As an application of our framework, we study spatial growth in China. During the 1990s and far into the 2000s, China experienced fast economic growth, sustained capital accumulation, relocation of factors and production across space, and increased trade openness. This growth experience is sometimes called the China shock in the literature and has been primarily used to study the effects of import competition on labor markets and other outcomes in the United States and other countries. Less work has been devoted to understanding its internal spatial dynamics. We study the mechanics of spatial growth in China that led to fast aggregate growth during this period. We aim to study the role of idea diffusion and capital accumulation in China's aggregate growth, as well as the impact of international trade and internal migration on spatial growth during the 1990s and 2000s.

To do so, we take the model to the year 1990. We divide China into 30 provinces, and we group other countries in a rest of the world. We construct gross migration flows across provinces in China using census data. We condition gross flows by Hukou type to take into account that the Hukou system affects incentives of migration and return migration and therefore contributes to uneven spatial growth, an aspect of China's development experience that we capture in our framework. We estimate elasticities that govern the rate of idea flows from trade and migration as well as the rate of innovation. To estimate these elasticities, we apply the generalized method of moments (GMM). We first obtain cross-sectional measures of fundamental productivity using a model inversion and generate a set of time series moments that we use in our estimation. With our data and these estimates in hand, we proceed to our quantitative assessment.

We first study how initial conditions and our mechanisms shaped spatial growth in China in the 1990s and 2000s. We take the model to the data without assuming that the economy is on a balanced growth path. With the initial allocation in 1990—namely, trade openness, spatial labor mobility, factor endowments, and stock of knowledge across locations—we apply the dynamic-hat algebra method (Caliendo et al. (2019)) to study the role of 1990 fundamentals in spatial development in China. We ask the following question: How would China have developed with the initial distribution of fundamentals in 1990 and with no changes in trade and migration costs thereafter? We find a role of initial conditions in the subsequent aggregate growth in China. We find that ideas from sellers contributed more to aggregate growth than ideas from migrants. The intuition comes from the fact that all provinces benefit from trade openness and access to better global ideas from the rest of the world. The contribution of ideas from people is more uneven. In the short run, the local stock of knowledge grows faster in locations that receive migrants from high-productivity places relative to locations that receive migrants from less-productive places. Over time, migrants learn about local ideas and contribute further to the local stock of knowledge. In the case of China, due in part to the Hukou system, return migration from high-productivity locations also shapes part of China's spatial development. We also find an important role of capital accumulation; aggregate growth would have declined by around half in the absence of capital accumulation.

We also study the role of capital accumulation and idea diffusion in the speed of convergence of the economy to the detrended steady state. Specifically, we compute the half-life of real GDP convergence and find that both capital accumulation and idea diffusion result in a longer transition for the economy as a whole. While the average half-life is shorter with capital accumulation, the transition takes longer in some provinces due to their large initial labor and capital gaps from the detrended steady state. Idea diffusion leads to the economy reaching a different detrended steady state with a higher stock of knowledge, which also results in a longer transition.

Turning to the spatial growth effects, we find that aggregate growth is shaped by large heterogeneity in growth rates across space. During the 1990s, provinces located in coastal areas such as Shanghai, Guangdong, and Hainan benefited from access to better insights from the rest of the world and experienced higher growth rates. Over time, spatial growth moderated and tended to equalize as the economy moved closer to the balanced growth path. The engines of aggregate growth also changed over time; notably, Guangdong became the main contributor to aggregate growth in China while other provinces located in the central and eastern parts of China became less relevant engines for aggregate growth. We also discuss how the initial distribution of fundamentals across space shaped subsequent spatial growth in China. We find that provinces with a higher initial stock of knowledge and with more international trade openness experienced higher growth.

While initial conditions seem to be important for understanding the process of spatial development and aggregate growth in China in the 1990s and 2000s, China also experienced reforms and policies that resulted in further changes to international trade costs and internal migration costs during this period, most notably its accession to the World Trade Organization and the relaxation of some Hukou restrictions. We estimate changes in bilateral trade and migration frictions after 1990 and ask how they contributed to spatial and aggregate growth in China. We find that the change in trade costs and migration costs contributed to extra aggregate growth by about one percentage point annually and that the growth effects were very heterogeneous across space.

We also provide reduced-form evidence of idea diffusion through trade and migration. Measuring the local stock of knowledge in the data is a difficult task. To construct a proxy for it, we obtain province-level patent data and patent data for the rest of the world. We use it along with our trade and migration data to provide empirical evidence of the role played by trade and migration to diffuse ideas and to contribute to the local stock of knowledge. We find evidence of the mechanism for spatial growth through idea diffusion, namely provinces more open to trade and with more migrants from locations with larger knowledge stocks experience relative larger growth in their knowledge stock. In addition, guided by the structural relationship between the local knowledge stock and idea diffusion through trade and migration from our model, we run an instrumental variable regression and find evidence consistent with our reduced-form results.

Our research is related to different strands of existing work. While our paper contributes to a large body of work in quantitative spatial economics (see Redding and Rossi-Hansberg (2017) for a review), it mainly engages with recent work on dynamic spatial models. The general equilibrium trade structure and forward-looking migration decisions build on Caliendo et al. (2019), where locations trade goods as in Eaton and Kortum (2002). We model workers' mobility decisions subject to frictions as a dynamic discrete-choice problem as in Artuc et al. (2010). We introduce capital accumulation and spatial growth into a dynamic framework with labor market dynamics and trade. As described previously, capital accumulation in our framework features forward-looking atomistic landlords making investment decisions in local capital to maximize intertemporal utility, which follows the structure in Kleinman et al. (2023). The distinction between landlords and workers also relates to the formulations in Angeletos (2007) and Moll (2014), and as discussed later on, adds tractability in the context of a dynamic spatial model with forward-looking mobile workers. Capital accumulation in our dynamic spatial framework also connects to dynamic models of capital accumulation and international trade (e.g., Eaton et al. (2016), Alvarez (2017), Ravikumar et al. (2019), Anderson et al. (2019)), with the important difference that labor is assumed to be immobile across countries in that strand of the literature.

The distinctive feature of our dynamic spatial framework is the presence of spatial growth. The process of innovation and diffusion that gives rise to the theory of total factor productivity in our model is a discrete-time version of Buera and Oberfield (2020) enhanced to consider spatial growth through trade and migration. The model in Buera and Oberfield (2020) also relates to Kortum (1997) when there is no idea diffusion from insights, and to Jones (1995) and Atkeson and Burstein (2019) in a model with intertemporal knowledge spillovers that are not modeled explicitly as a function of insights. Our paper also relates to Cai and Xiang (2022), who study global growth and technology diffusion through multinational production. In our context, ideas diffuse not only globally but also locally. Our paper also complements recent spatial frameworks with innovation, local diffusion of technology, and spatial growth, most notably in Desmet and Rossi-Hansberg (2014) and Desmet et al. (2018), and frameworks with frictional idea diffusion across space (e.g., Berkes et al. (2022)).[1] Our framework shares some aspects with these papers such as the spatial heterogeneity in fundamentals and the geographic aspect of local idea diffusion. However, in our framework, technology diffuses spatially through trade and migration, both of which are endogenous, instead of being dictated by geographical distance or technological frictions. Our framework also departs from these papers by introducing forward-looking migration and capital accumulation decisions.

Also related to our paper, Eaton and Kortum (1999) develops a model of idea diffusion across countries where the distribution of productivities in each country follows a Fréchet distribution and the evolution of the stock of knowledge is characterized by a system of differential equations. In their model, ideas diffuse across countries exogenously, and countries are assumed to be under autarky otherwise. Building on Eaton and Kortum (1999), Cai et al. (2022) develops a trade and growth model with dynamics through innovation and technology diffusion across countries and sectors. In their model, ideas diffuse with exogenous and heterogeneous speeds across all sectors and countries. In contrast, in our model the speed of diffusion across locations is endogenous and mediated by trade and migration. Our framework also departs from these papers as it incorporates spatial growth with forward-looking

[1] See also Cruz and Rossi-Hansberg (2022), which studies the spatial effects of climate change.

migration and capital accumulation decisions.[2]

The process of idea diffusion from migrants in our framework is motivated in part by a growing literature with empirical evidence on knowledge flows resulting from interactions among people (e.g., Atkin et al. (2022), Buzard et al. (2020)), and with empirical evidence on the impact of immigrants on ideas, innovation, and growth in the United States and in other countries (e.g., Kerr (2008), Hunt and Gauthier-Loiselle (2010), Lewis (2011), Akcigit et al. (2017), Bernstein et al. (2018), Sequeira et al. (2019), Arkolakis et al. (2020), Burchardi et al. (2020), Prato (2021)). There is also recent evidence on how internal migrants impact productivity and other related outcomes in their destination in countries that have experienced large internal migration episodes (e.g., Facchini et al. (2019), Imbert et al. (2022), Pellegrina and Sotelo (2021)).

Our paper also relates to a strand of the literature that studies the role of trade and migration in shaping spatial inequality in China in the 2000s through the lens of static frameworks (e.g., Tombe and Zhu (2019) and Fan (2019)). We depart from this line of research by studying growth in China in the 1990s through the lens of a dynamic spatial growth model, which allows us to study not only the cross sectional but also the time series implication of spatial development in China. Finally, our paper also relates to other strands of the literature that have pointed to different determinants of the rise of China. Caliendo and Parro (2022) provides a review of the recent literature on the origins of the China shock. With our dynamic spatial framework we study how spatial development through our mechanisms contributed to China's growth.[3]

The rest of the paper is structured as follows. In Section 2 we develop our dynamic spatial growth model. We start by describing the process of idea diffusion for a single economy; we then introduce locations and present the dynamic spatial growth framework. We also characterize the equilibrium properties of the model. In Section 3, we describe how to take the model to the data. The section discusses data measurement and the data sources used to take the model to the Chinese economy at the province level. Additionally, the section discusses our estimation strategy of the relevant elasticities and describes the method we use for performing counterfactual analysis. Section 4 presents our quantitative results, and Section 5 provides reduced-form evidence on the contribution of idea diffusion through trade and migration to local knowledge. Section 6 concludes. We relegate all proofs,

[2] Idea diffusion through trade in our paper is also related to other recent frameworks modeling innovation and diffusion of technologies as stochastic processes to study the connection between trade and the diffusion of ideas (e.g., Lucas (2009), Perla et al. (2021), Sampson (2016)).

[3] More generally, the effects of China's trade expansion on U.S. labor markets as well as other outcomes in different countries has been the focus of an extensive body of literature (e.g., Autor et al. (2013), Acemoglu et al. (2016), Pierce and Schott (2016), Caliendo et al. (2019)).

theoretical derivations, and detailed data descriptions to the appendix.

2 Dynamic Spatial Growth Model

In this section, we develop the dynamic spatial growth model. We begin with a description of technology diffusion in a single economy given a general source distribution of insights in Subsection 2.1. We then introduce locations in the framework and describe the demand side of the model—that is, the production and trade structure—in Subsection 2.2. After that, we specify the supply of factors in our framework. In Subsection 2.3 we describe the capital accumulation decisions made by local landlords, and in Subsection 2.4 we specify the dynamic labor supply decisions made by migrants. In Subsection 2.5 we endogenize the idea diffusion process, relate it to migration and trade, and derive the evolution of the stock of knowledge across space. We also define the balanced growth path equilibrium of the economy and establish the existence and uniqueness of the balanced growth path equilibrium.

2.1 Innovation and Idea Diffusion

To simplify the exposition, consider a single economy in which there is a continuum of intermediate varieties produced in the unit interval. For each variety, there is a large set of potential producers who have different technologies to produce the good. Each potential producer is characterized by the productivity of her idea, which we denote by q, to produce an intermediate variety. Between time t and time $t+1$, producers interact with other agents in the economy and are exposed to new ideas to produce a variety. The productivity of a new idea might or might not be higher than that of the ideas the producer already has so she only adopts a new idea if the new ideas' productivity is greater than q. Both the number of new ideas and the productivity of them are stochastic, which generates randomness in the usage of the new ideas.

In particular, the number of new ideas to which a producer is exposed is stochastic and follows a Poisson distribution. Each new idea corresponds to a new productivity to produce the variety and is given by zq'^{ρ}. This new idea has two random components: z is the original component, drawn from an exogenous distribution $H(z)$; and q' is an insight drawn from a source distribution $G_t(q')$ whose evolution we describe subsequently. Producers generate new ideas originated from their internal source of ideas, drawn from their own distribution of original ideas $H(z)$. Diffusion is a component that is external to the producer and that allows her to be exposed to the ideas of other producers. These ideas diffuse at a rate that is captured by the parameter ρ. In this context, the original component of the producer's ideas can also be interpreted as randomness in the adaptation of insights from others to alternative uses.

To gain tractability, in Assumption 1 we specify the distribution of original ideas, the process for the arrival of ideas, and the parametric restrictions required to characterize the evolution of the knowledge frontier over time. We then impose these assumptions, and in Proposition 1 we characterize the frontier of knowledge in the economy and the evolution of the stock of knowledge over time.[4]

Assumption 1

a) The distribution of original ideas is Pareto; $H(z) = 1 - (z/\bar{z})^{-\theta}$ *, where* $\bar{z}$ *is the lower bound of the support and* $\theta > 1$ *is the shape parameter of the distribution.*

b) The strength of idea diffusion, $\rho \in [0, 1)$*, is strictly less than 1.*

c) The number of new ideas that arrive between t *and* $t + 1$ *follows a Poisson distribution with mean* $\Lambda_t = \alpha_t \bar{z}^{-\theta}$*.*

d) The source distribution has sufficiently thin tail; i.e. $\lim_{\bar{z} \to 0} \bar{z}^{-\theta} \left[1 - G_t\left(\left(\frac{q}{\bar{z}}\right)^{\frac{1}{\rho}}\right)\right] = 0.$

In what follows we impose Assumption 1 to solve for the distribution of productivity in the economy. The next proposition presents the result.

Proposition 1. *Under Assumption 1, between* t *and* $t + 1$*, the probability that the best new idea has a productivity no greater than* q*,* $F_t^{best\ new}(q)$*, is given by*

$$F_t^{best\ new}(q) = \exp\left(-\alpha_t q^{-\theta} \int_0^{\infty} x^{\rho\theta} dG_t(x)\right).$$

Proof. See Appendix A.

Proposition 1 shows that the probability distribution of the best new idea is Fréchet with shape parameter θ and a location parameter determined by $\alpha_t \int_0^{\infty} x^{\rho\theta} dG_t(x)$. Note that in order to obtain this result there is no need to specify the external source distribution. This is an important result that we will use when we impose more structure over the source distribution. In addition, we can use the result of Proposition 1 to characterize the frontier of knowledge and its evolution over time. In particular, we denote by $F_t(q)$ the fraction of varieties whose best producer has productivity no greater than q. In a probabilistic sense, $F_t(q)$ is also the probability that the best productivity for a specific variety is no greater than q at time t. We call this object the *frontier of knowledge*. As the new ideas that arrive

[4]We also refer the reader to Buera and Oberfield (2020) for a continuous-time-version derivation of the equilibrium evolution of technology in the economy.

might have better productivity than the current best ideas, the evolution of $F_t(q)$ between t and $t+1$ follows

$$F_{t+1}(q) = F_0(q) \cdot \prod_{\tau=0}^{t} F_\tau^{best\ new}(q).$$

Proposition 2. *Assume that the initial frontier of knowledge at time* 0 *follows a Fréchet distribution given by* $F_0(q) = \exp(-A_0 q^{-\theta})$. *It follows that* $F_t(\cdot)$ *is Fréchet at any* t *given by*

$$F_t(q) = \exp\left[-\left(A_0 + \sum_{\tau=0}^{t-1} \alpha_\tau \int_0^\infty x^{\rho\theta} dG_\tau(x)\right) q^{-\theta}\right] = \exp\left(-A_t q^{-\theta}\right),$$

where the law of motion for the knowledge stock is given by

$$A_{t+1} = A_t + \alpha_t \int_0^\infty x^{\rho\theta} dG_t(x).$$

Proof. See Appendix A.

Proposition 2 establishes two results that we use in subsequent sections. First, at each moment in time the frontier of knowledge follows a Fréchet distribution, which we use to specify the production and trade structure in our framework, as described in the next section. Second, we can see that both the arrival rate of new ideas α_t and the source distribution $G_t(\cdot)$ matter for the evolution of A_t. Later in the paper, after we describe the economic environment in our framework, we return to discuss how ideas diffuse over space and relate the source distribution $G_t(\cdot)$ to ideas from sellers and from migrants. Finally, note that it can also be shown that $F_t(\cdot)$ converges to a Fréchet asymptotically when $t \to \infty$, even without assuming that the initial frontier of knowledge follows a Fréchet distribution. Hence, the assumption about the initial frontier of knowledge is not strictly needed to obtain the result in Proposition 2.

2.2 Production, Factor Demand, and Trade

We now consider a world with N different locations indexed by i and n. At each location i there are heterogeneous and perfectly competitive producers of varieties of intermediate goods.[5] The technology to produce these intermediate goods requires labor and capital,

[5] As explained later on, at the beginning of the period producers get insights from sellers and migrants, with randomness in the productivity of those insights for alternative uses in the destination location. At the end of the period, technology to produce a variety can be imitated, and therefore, producers decide to charge a price equal to the marginal cost. Alternatively, we could have assumed producers engage in Bertrand competition so that the lowest-cost supplier of a variety either charge the optimal markup or set a limit

which are the primary factors of production, and material inputs. The efficiency of an intermediate good producer is given by $q_{i,t}$, where we now index efficiencies by location. The output for a producer of an intermediate variety with efficiency $q_{i,t}$ in location i is given by

$$y_{i,t} = q_{i,t} \left(L_{i,t}^{\xi} K_{i,t}^{1-\xi} \right)^{\gamma} M_{i,t}^{1-\gamma},$$

where $L_{i,t}$, $K_{i,t}$, and $M_{i,t}$ are the demand for labor, capital, and material inputs, respectively. The parameters γ and $1-\gamma$ are the shares of value added and material inputs in output, and ξ and $1-\xi$ are the shares of labor and capital in value added, respectively. It follows from the cost minimization problem of the producers that the unit price of an input bundle is given by

$$x_{i,t} = B \left(w_{i,t}^{\xi} r_{i,t}^{1-\xi} \right)^{\gamma} P_{i,t}^{1-\gamma},$$

where $w_{i,t}$, $r_{i,t}$, and $P_{i,t}$ denote the price of labor, rental rate of capital, and the price of materials, respectively, and where B is a constant.[6]

We now use the results from the previous section in which we derived the law of motion of the stock of knowledge in an economy. Firms purchase intermediate goods from the lowest-cost supplier in the world. The frontier of knowledge in each location at each t is described by a Fréchet distribution with shape parameter θ and location-specific scale parameter $A_{i,t}$; namely, $F_{i,t}(q) = \exp(-A_{i,t} q^{-\theta})$.

Shipping goods across locations, from n to i, is subject to iceberg trade costs, $\kappa_{in,t}$, and therefore, the cost of purchasing an intermediate variety with efficiency q from n in location i is given by $\kappa_{in,t} x_{n,t}/q$. Hence, we can now follow the Eaton and Kortum (2002) formulation and derive the fraction of goods purchased by location i from location n (see Appendix B.1 for the derivation), which is given by

$$\lambda_{in,t} = \frac{A_{n,t} \left(\kappa_{in,t} x_{n,t} \right)^{-\theta}}{\sum_{h=1}^{N} A_{h,t} \left(\kappa_{ih} x_{h,t} \right)^{-\theta}}. \tag{1}$$

Similarly, we can solve for the price index in location i, which is given by

$$P_{i,t} = T \left(\sum_{n=1}^{N} A_{n,t} \left(\kappa_{in,t} x_{n,t} \right)^{-\theta} \right)^{-1/\theta}, \tag{2}$$

price to just undercut the second-lowest cost supplier of the variety. As shown in Bernard et al. (2003) and Buera and Oberfield (2020), with Bertrand competition aggregate costs are a fraction $\theta/(1+\theta)$ of aggregate revenues in all locations, and under the assumption that profits from local producers are spent domestically, equilibrium conditions are isomorphic to those under perfect competition except for a constant in the price index.

[6] In particular, $B = \left[\xi^{\xi} (1-\xi)^{1-\xi} \right]^{-\gamma} \gamma^{-\gamma} (1-\gamma)^{\gamma-1}$.

where T is a constant.[7] Given this environment, total expenditure in location i, which we denote by $X_{i,t}$, is given by

$$X_{i,t} = (1-\gamma)\sum_{n=1}^{N} \lambda_{ni,t} X_{n,t} + I_{i,t},$$

which reflects that the total expenditure on goods is firms' expenditure on intermediate goods plus households' expenditure where a household's income is given by $I_{i,t} = w_{i,t}L_{i,t} + r_{i,t}K_{i,t}$. The term $\sum_n \lambda_{ni,t}X_{n,t}$ is the total demand for goods produced in i from all locations. The trade balance condition is given by

$$\sum_{n=1}^{N} \lambda_{in,t} X_{i,t} = \sum_{n=1}^{N} \lambda_{ni,t} X_{n,t},$$

where the left-hand side is the total imports by location i, and the right-hand side is the total exports from i (with domestic purchases entering both sides of the equation). Finally, using the expenditure equation, trade balance, and the relative demand for capital and labor, it follows that the labor market clearing condition can be expressed as

$$w_{i,t}L_{i,t} = \sum_{n=1}^{N} \lambda_{ni,t} w_{n,t} L_{n,t}. \tag{3}$$

2.3 Capital Accumulation Across Locations

We now turn to the supply side of the model. We start by describing capital accumulation decisions across space. At each location, we assume that there are atomistic landowners who consume local goods with logarithm preferences over consumption goods and whose source of income is from renting capital structures.[8] Landowners are forward-looking and seek to maximize the present discounted value of their utility by deciding how much to consume and invest at each moment in time. Landowners are geographically immobile, have access to an investment technology in local capital, and make their investment in units of consumption goods. We follow Kleinman et al. (2023) and interpret capital as buildings and structures that are geographically immobile once installed, and we specify the problem of a landowner

[7] Intermediate varieties are aggregated with a constant elasticity of substitution η, and T is a gamma function evaluated in the argument $T = \Gamma\left(1 + (1-\eta)/\theta\right)^{1/(1-\eta)}$.

[8] Our assumption regarding the logarithm preferences of landlords is consistent with the preferences we specify for workers in the next subsection.

in location i as

$$\max_{\{C_{i,t},K_{i,t+1}\}_{t=0}^{\infty}} U = \sum_{t=0}^{\infty} \beta^t \log(C_{i,t}),$$
$$\text{s.t. } r_{i,t}K_{i,t} = P_{i,t}\left[C_{i,t} + K_{i,t+1} - (1-\delta)K_{i,t}\right] \text{ for all } t,$$

where δ is the depreciation rate and $K_{i,0}$ is taken as given. The solution to this dynamic programming problem can be characterized by the policy functions on consumption and investment,

$$\begin{aligned} C_{i,t} &= (1-\beta)\left[r_{i,t}/P_{i,t} + (1-\delta)\right]K_{i,t}, \\ K_{i,t+1} &= \beta\left[r_{i,t}/P_{i,t} + (1-\delta)\right]K_{i,t}, \end{aligned} \tag{4}$$

which give rise to the law of motion of capital accumulation across locations. In Appendix B.3 we provide the detailed derivation of these policy functions. Note that since capital structures are accumulated locally and used for local production, the evolution of capital structures in part shapes the evolution of economic activity across space. Similar to Kleinman et al. (2023), the immobility of landlords allows us to introduce forward-looking capital accumulation decisions in dynamic spatial economies with workers' mobility in a tractable way, and it prevents the number of state variables from increasing exponentially over time.[9]

We now turn to describe the dynamic labor supply decisions made by workers and migrants across locations in the model.

2.4 Dynamic Labor Supply Decisions

There is a continuum of heterogeneous forward-looking workers in the economy. Each worker observes the economic conditions and optimally decides where to locate in each period subject to mobility frictions and idiosyncratic taste shocks. We model this migration decision as a dynamic discrete-choice problem. In particular, workers maximize the present discounted value of their utility by deciding at each moment in time where to live. They supply one unit of labor inelastically at where they live, and they consume given their labor income ($w_{i,t}$) and the local price of goods ($P_{i,t}$). We denote by $U_{i,t}(c_{i,t})= log(c_{i,t})$ the current utility of a worker living in location i, where $c_{i,t} = w_{i,t}/P_{i,t}$. We assume that the decision of where to live the next period is affected by idiosyncratic amenity shocks that vary across locations denoted

[9] As a result, this framework can accommodate alternative capital accumulation formulations such as assuming decreasing return to investment, as in Lucas and Prescott (1971) and Hercowitz and Sampson (1991).

by $\epsilon_{n,t}$ and by mobility frictions of going from location i to location n, denoted by $m_{in,t}$. The presence of migration costs and idiosyncratic shocks generates a gradual adjustment of labor supply in response to changes in the economic environment.

As a result, the value of a worker in region i at time t is given by

$$v_{i,t} = log(w_{i,t}/P_{i,t}) + \max_{\{n\}_{n=1}^{N}} \{\beta E_t[v_{n,t+1}] - m_{in,t} + \nu\epsilon_{n,t}\}, \tag{5}$$

where β is the discount factor, which is assumed to be the same as the discount factor of landowners.

We assume that the idiosyncratic shocks $\epsilon_{n,t}$ are $i.i.d.$ realizations from a Gumbel (Type I Extreme Value) distribution with dispersion parameter ν. We denote by $E_t[v_{n,t+1}]$ the expectation at time t over the future realizations of the idiosyncratic shocks that shape the continuation value of each location. Using the properties of the Gumbel distribution, we can integrate both sides of equation (5) over $\epsilon_{n,t}$. We then obtain the value of location i for a representative worker in that location at time t, denoted by $V_{i,t} = E_t[v_{i,t}]$. The value of location i is given by

$$V_{i,t} = log(w_{i,t}/P_{i,t}) + \nu log\left(\sum_{n=1}^{N} exp\left(\beta V_{n,t+1} - m_{in,t}\right)^{1/\nu}\right). \tag{6}$$

We denote by $\mu_{in,t}$ the fraction of workers that moves from location i to location n, which using the properties of the Gumbel distribution can be derived in closed form as

$$\mu_{in,t} = \frac{exp\left(\beta V_{n,t+1} - m_{in,t}\right)^{1/\nu}}{\sum_{h=1}^{N} exp\left(\beta V_{h,t+1} - m_{ih,t}\right)^{1/\nu}}. \tag{7}$$

This equilibrium condition determines the gross migration flows of workers across space (see Appendix B.2 for the derivation). It shows that individuals are forward-looking and decide where to supply labor tomorrow by evaluating the relative net future value of each location. The elasticity of the migration flow $(1/\nu)$ shapes how changes to migration costs affect migration flows. This expression for gross migration flows determines the evolution of the labor supply at each location i over time. In particular, the supply of workers at location i at time $t+1$ is given by the workers who decide to migrate to location i from all locations n (including stayers in i) at time t. Therefore, the stock of workers at each location evolves according to

$$L_{i,t+1} = \sum_{n=1}^{N} \mu_{ni,t} L_{n,t}. \tag{8}$$

Having described the demand and supply sides of the model, in the next subsection we return to the idea diffusion process to specify the evolution of the local stock of knowledge across space as a result of trade and migration.

2.5 Idea Diffusion with Trade and Migration

We now specify the innovation and diffusion process described in Section 2.1 to allow for migrants and sellers to contribute to the local pool of ideas. Recall that producers in location n obtain new insights from two sources. First, they obtain insights from sellers; namely, ideas from producers in other locations are embedded in imported intermediate varieties. Second, we assume that migrants carry insights with them when they arrive in a new location. The quality of those insights does not directly affect their wages or their migration decisions, but it has a general equilibrium effect on migrant's wages through the changes in the stock of knowledge in a location. The interpretation is that a migrant becomes exposed to the local ideas in their previous location, and then as they move across locations, they randomly meet a local producer. When they meet, the migrant shares ideas from her previous location and provides insights that can contribute to the local stock of knowledge. As a result, the productivity of a new idea that arrives can be generalized to

$$q = z q_\ell^{\rho_\ell} q_m^{\rho_m},$$

where q_ℓ is the insight drawn from a source distribution that is shaped by migration and q_m the insight drawn from a source distribution that is shaped by sellers. Note that under this functional form, we assume a substitution pattern between insights from goods and people, where having both the migrants and foreign goods makes the new insight more productive than having only one of them.

The parameters $\rho_\ell, \rho_m \in [0, 1)$ capture the learning intensity from both types of insights (migration and trade) with $\rho_\ell + \rho_m < 1$. After imposing Assumption 1 and following the same steps as in Section 2.1, extending the notation by indexing the location by n, and given the results from Propositions 1 and 2, we obtain that the frontier of knowledge at each location is

$$F_{n,t}^{best\ new}(q) = exp\left(-A_{n,t} q^{-\theta}\right),$$

and the stock of knowledge evolves over time as

$$A_{n,t+1} - A_{n,t} = \alpha_t \int_0^\infty \int_0^\infty \left(q_\ell^{\rho_\ell} q_m^{\rho_m}\right)^\theta dG_{n,t}^l(q_\ell) dG_{n,t}^m(q_m).$$

We assume that since q_ℓ is drawn from people and q_m is drawn from goods, they represent two different sources of (independent) ideas. Formally, when a worker from i at the end of period t decides to move to n, she carries with her an insight q_ℓ, which is a random draw from the frontier distribution in i, whose cumulative distribution function is $F_{i,t}(q_\ell)$. At the end of time t, in location n, producers randomly meet a worker currently living in n, and the insight from this individual is the insight component of the new idea. Hence,

$$G^l_{n,t}(q_\ell) = \sum_{i=1}^{N} s_{in,t} F_{i,t}(q_\ell),$$

where $s_{in,t} = \frac{\mu_{in,t} L_{i,t}}{\sum_{h=1}^{N} \mu_{hn,t} L_{h,t}}$ is the share of workers in location n that arrived from i at the end of period t (see the derivation in Appendix A.2).

In the case of the source distribution of goods, we assume that there is learning from sellers as in Buera and Oberfield (2020); namely, that diffusion opportunities are randomly drawn from the set of best practices across all goods sold to location n. In this way the source distribution $G^m_{n,t}(q_m)$ is given by the fraction of goods for which the lowest-cost provider of the good to location n is a producer with productivity less than or equal to q_m. Under these mechanisms for idea diffusion, we derive the law of motion of the stock of knowledge across locations with idea flows from people and goods (see Appendix A.3). We obtain that the difference equation that determines the evolution of the stock of knowledge at each location is given by

$$A_{n,t+1} - A_{n,t} = \alpha_t \Gamma_{\rho_\ell,\rho_m} \underbrace{\left[\sum_{i=1}^{N} s_{in,t} \left(A_{i,t}\right)^{\rho_\ell}\right]}_{people} \underbrace{\left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}\right]}_{goods}, \tag{9}$$

where $\Gamma_{\rho_\ell,\rho_m}$ is a constant given by $\Gamma(1-\rho_\ell) \times \Gamma(1-\rho_m)$ and where $\Gamma(x)$ is gamma function evaluated at x.

Equilibrium condition (9) shows that the local stock of knowledge evolves over time according to the arrival rate of new ideas α_t, according to how the location is connected and exposed to ideas from migrants, $s_{in,t}$, and according to how open the location is to trade, $\lambda_{ni,t}$. The term $A_{i,t}/\lambda_{ni,t}$ on the right-hand side reflects the fact that location n gets insights only from the active producers (lowest-cost suppliers) in i. We also emphasize that the diffusion of ideas from migrants and sellers is endogenous since both migration and trade patterns are equilibrium objects in our framework. Additionally, it is worth noting that ideas diffuse not only from migrants and foreign sellers but also from local active producers and from non-migrants, hence the stock of knowledge also grows even in locations that are closed to trade or migration, namely where $s_{ii,t} = 1$ or $\lambda_{ii,t} = 1$. The relative strength of idea diffusion, governed by the diffusion parameters ρ_ℓ and ρ_m, shapes the importance of learning from people or goods.

The fact that there are diminishing returns to technological improvement from insights, given that the strength of idea diffusion is less than one, makes it harder to obtain insights that are good enough over time. Hence, if α_t is time-invariant, then as the knowledge frontier evolves over time, the growth rate of the stock of knowledge falls with a limiting value of zero. As a result, as the knowledge frontier evolves, ideas need to arrive faster over time in order to sustain a constant growth rate. This feature is shared by semi-endogenous growth models in Buera and Oberfield (2020), Jones (1995), Kortum (1997), and Atkeson and Burstein (2019). Given this, we make the following assumption about the arrival rate.

Assumption 2 *α_t has constant growth rate g_α, that is*

$$\alpha_t = \alpha_0(1 + g_\alpha)^t.$$

We now define formally the equilibrium of the dynamic spatial growth model.

Definition 1. Equilibrium of the Spatial Growth Model. *Given an initial distribution of the local stock of knowledge $\{A_{i,0}\}_{i=1}^{N}$ and factor endowments $\{L_{i,0}, K_{i,0}\}_{i=1}^{N}$, the evolution of fundamentals $\{\alpha_0, \kappa_{in,t}, m_{in,t}\}_{i=1,n=1,t=0}^{N,N,\infty}$, and parameters and elasticities (ρ_ℓ, ρ_m, θ, ν, γ, ξ, β), the sequential competitive equilibrium of the dynamic spatial growth model is characterized by a sequence of values, factor prices, goods prices, labor allocations, capital stocks, and stock of knowledge, $\{V_{i,t}, w_{i,t}, r_{i,t}, P_{i,t}, L_{i,t}, K_{i,t}, A_{i,t}\}_{i=1,t=0}^{N,\infty}$, that satisfies the equilibrium conditions determined by the bilateral trade shares (1), the equilibrium location prices (2), the labor market clearing condition (3), the capital accumulation condition (4), the location value function (6), the worker gross flow condition (7), the law of motion of labor (8), and the evolution of the stock of knowledge (9).*

In the long run, as the economy evolves over time, it approaches a balanced growth path equilibrium in which all equilibrium variables grow at a constant long-run rate. We now characterize the balanced growth path of the model. We first formally define the balanced growth path. We then express all equilibrium variables in the model relative to their balanced growth rate (what we refer to as the detrended variables) and then show that the equilibrium conditions of the detrended model give rise to a unique solution. Namely, we show that there exists a unique balanced growth path of the dynamic spatial growth model.

***Definition 2. Balanced Growth Path**. Along the balanced growth path all equilibrium variables grow at a constant rate. In particular, denote by g_y the growth rate of a generic variable y at the balanced growth path. At the balanced growth path the stock of knowledge grows at a rate $1 + g_A = (1 + g_\alpha)^{\frac{1}{(1-\rho_l-\rho_m)}}$, capital grows at a rate $1 + g_k = (1 + g_A)^{\frac{1}{\theta\xi\gamma}}$, and values grow at a rate $1 + g_v = (1 + g_A)^{\frac{1}{\theta\xi\gamma(1-\beta)}}$.*

Appendix C solves for the equilibrium long-run growth rates of all variables along the balanced growth path. The appendix also shows how to detrend all the equilibrium variables and equilibrium conditions, namely, how to express them relative to their balanced long-run growth. In particular,

***Definition 3. Detrended Economy.** Denote with a "~" the variable relative to its long-run growth. In the detrended economy $\tilde{y}_t \equiv y_t / (1 + g_y)^t$ for all variables y_t, where g_y is the growth rate of variable y_t at the balanced growth path.*

The next proposition establishes the existence and uniqueness of the equilibrium at the balanced growth path. At the balanced growth path all the detrended variables are not growing, and as a result, the equilibrium variables of the detrended model reach a steady state. Hence, at the balanced growth path, $\tilde{y}_{t+1} = \tilde{y}_t = \bar{y}$, and it remains constant for all t. We use an upper bar to express the detrended equilibrium variables at the balanced growth path.

***Proposition 3. Existence and Uniqueness.** Given the parameters and elasticities $(\rho_\ell, \rho_m, \theta, \nu, \gamma, \xi, \beta)$, and the fundamentals $\left\{\alpha_0, \bar{\kappa}_{in}, \bar{m}_{in}\right\}_{i=1,n=1}^{N,N}$, there exists a unique (up to scale) solution given by $\left\{\bar{w}_i, \bar{r}_i, \bar{L}_i, \bar{K}_i, \bar{V}_i, \bar{A}_i\right\}_{i=1}^{N}$ that satisfies the equilibrium conditions of the detrended model at the balanced growth path.*

Proof. See Appendix D.

Proposition 3 establishes that the model has a unique balanced growth path. The proof extends the results of Kleinman et al. (2023) , and shows that the spectral radius of the matrix of power elasticities and parameters $(\rho_\ell, \rho_m, \theta, \nu, \gamma, \xi, \beta)$ of the non-linear system at the balanced growth path is less than or equal to one, which establishes the uniqueness of the balanced growth path equilibrium in our spatial growth model up to a normalization.

In the proof, we solve for six eigenvalues that characterize the system of equilibrium conditions. The eigenvalues are $(1, 1, \frac{-b\pm\sqrt{b^2-4ac}}{2a}, 0, \rho_\ell + \rho_m)$ where $a = \beta + \nu + \theta\gamma\nu\xi$, $b = -\nu(1 + \beta - \gamma\xi(1 + \theta(1 - \beta)))$, and $c = \beta(\nu - 1 - \gamma\xi\nu(1 + \theta))$. We show that $\frac{-b\pm\sqrt{b^2-4ac}}{2a} < 1$, which establishes uniqueness and existence given that all the other eigenvalues are less than or equal to one. The Appendix D also presents the proof of existence and uniqueness for alternative versions of the model. For example, we present results for a version of the model with no idea flows. In this case, with no growth, we show that there are four eigenvalues, which are given by $(1, 1, \frac{-b\pm\sqrt{b^2-4ac}}{2a})$. We then consider the case of an economy with idea flows only from sellers. We show that the equilibrium eigenvalues are $(1, 1, \frac{-b\pm\sqrt{b^2-4ac}}{2a}, \rho_m)$, where the additional eigenvalue compared to the model with no idea flows is exactly given by ρ_m, the strength of idea flows from trade. Similarly, in a model with only idea flows from migration, one obtains that the new eigenvalue is given by ρ_ℓ, the strength of idea flows from migration.

We now turn to quantitatively study the importance of our mechanisms for aggregate and spatial growth. To do so, we apply our framework to study spatial growth in China, an economy that features heterogeneous locations in terms of stock of knowledge, initial supply of labor and supply of capital, exposure to international trade, and mobility flows.

3 Quantitative Analysis

During the 1990s and far into the 2000s, China experienced fast economic growth, considerable capital accumulation, shifts in the distribution of economic activity and factors of production across space, increased productivity, and trade openness. Caliendo and Parro (2022) reviews recent literature that describes the macroeconomic performance of China during the 1990s and 2000s and the different factors that contributed to China's growth.

We now turn to study spatial growth in China in the 1990s and 2000s through the lens of the dynamic spatial growth model developed in the previous section. We take the model to year 1990 in a world composed of 30 Chinese provinces and a constructed rest of the world. In doing so, we use migration, production, and value added data. We also use trade data between provinces and the rest of the world. Importantly in the case of China, where there are well-defined mobility frictions across provinces, we condition gross migration flows across provinces by Hukou status. To understand how the Hukou system works, think about a province-level "passport" that identifies an individual based on their province of origin and restricts non-locals' access to certain amenities.

Accordingly, in the quantitative analysis we extend our framework to take into account these considerations. In particular, we allow for workers with different Hukou statuses to value locations differently, as Hukou restrictions give them access to different amounts of amenities, and we also allow workers to face different mobility restrictions. In equilibrium, this implies different mobility rates across provinces for individuals with different Hukou statuses that we discipline in the data.

Hence, the equilibrium conditions of the dynamic labor supply decisions of workers are now given by

$$V_{i,t}^{H} = log(\psi_i^H w_{i,t}/P_{i,t}) + \nu log\left(\sum_{n=1}^{N} exp\left(\beta V_{n,t+1}^{H} - m_{in,t}^{H}\right)^{1/\nu}\right), \tag{10}$$

$$\mu_{in,t}^{H} = \frac{exp\left(\beta V_{n,t+1}^{H} - m_{in,t}^{H}\right)^{1/\nu}}{\sum_{g=1}^{N} exp\left(\beta V_{g,t+1}^{H} - m_{ig,t}^{H}\right)^{1/\nu}}, \tag{11}$$

$$L_{i,t+1} = \sum_{H}\sum_{n=1}^{N} \mu_{ni,t}^{H} L_{n,t}^{H}, \tag{12}$$

where the H index denotes the Hukou status and ψ_i^H is the amenity parameter of location i for an individual with Hukou status H. Once in the same location, workers with different Hukou statuses consume the same basket of goods and earn the same real wages although their levels of utility are different because they have access to different amenities. In this way, we aim to capture a characteristic of this economy in transition: that is, that migrants to a given province registered in a different province have access to different amounts of amenities, face different mobility costs, and as a result, make different migration decisions compared with migrants registered in the destination province. We later provide some descriptive evidence of the importance of two-way migration across provinces in China in part due to the Hukou restrictions.

We now proceed to describe the data sources we use in our quantitative analysis. In Appendix G we further describe data sources and data construction.

3.1 Data

To bring the model to the data, we need data across provinces in China and for the rest of the world on bilateral trade shares $\lambda_{in,t}$, total expenditure $X_{i,t}$, value added $w_{i,t}L_{i,t} + r_{i,t}K_{i,t}$, the distribution of employment $L_{i,t}$, and migration flows across provinces conditional on Hukou type $\mu_{in,t}^{H}$. We also need the share of value added in gross output γ, the share of labor in value added ξ, and the initial capital stocks $K_{i,0}$. In addition, we need estimates of the trade elasticity θ, the migration elasticity $1/\nu$, the discount factor β, and the depreciation rate δ. We later describe how we discipline the elasticities that govern innovation and idea diffusion $(\alpha_0, \rho_l, \rho_m)$.

We consider a model in which each period represents five years. Hence, we use a discount factor β of 0.86, equivalent to an annual discount factor of 0.97, which implies a yearly interest rate of roughly 4 percent. The trade elasticity $\theta = 4.55$ is obtained from Caliendo and Parro (2015). We set a migration elasticity of $1/\nu = 0.15$, which is the value estimated by Cruz (2021) for a five-year period in a sample of developing countries. We set a depreciation rate $(1 - \delta) = 0.95^5$, which corresponds to an annual depreciation rate of 5 percent. We compute the values of $\gamma = 0.38$ and $\xi = 0.54$, which correspond to the parameter values for the year 1990 from the world's aggregates in the Eora multi-region input-output table. Finally, we set a value of $g_\alpha = 0.014$ that matches the long-term productivity growth in the U.S. economy that during the great moderation period in the 1990s was arguably on a balanced growth path.

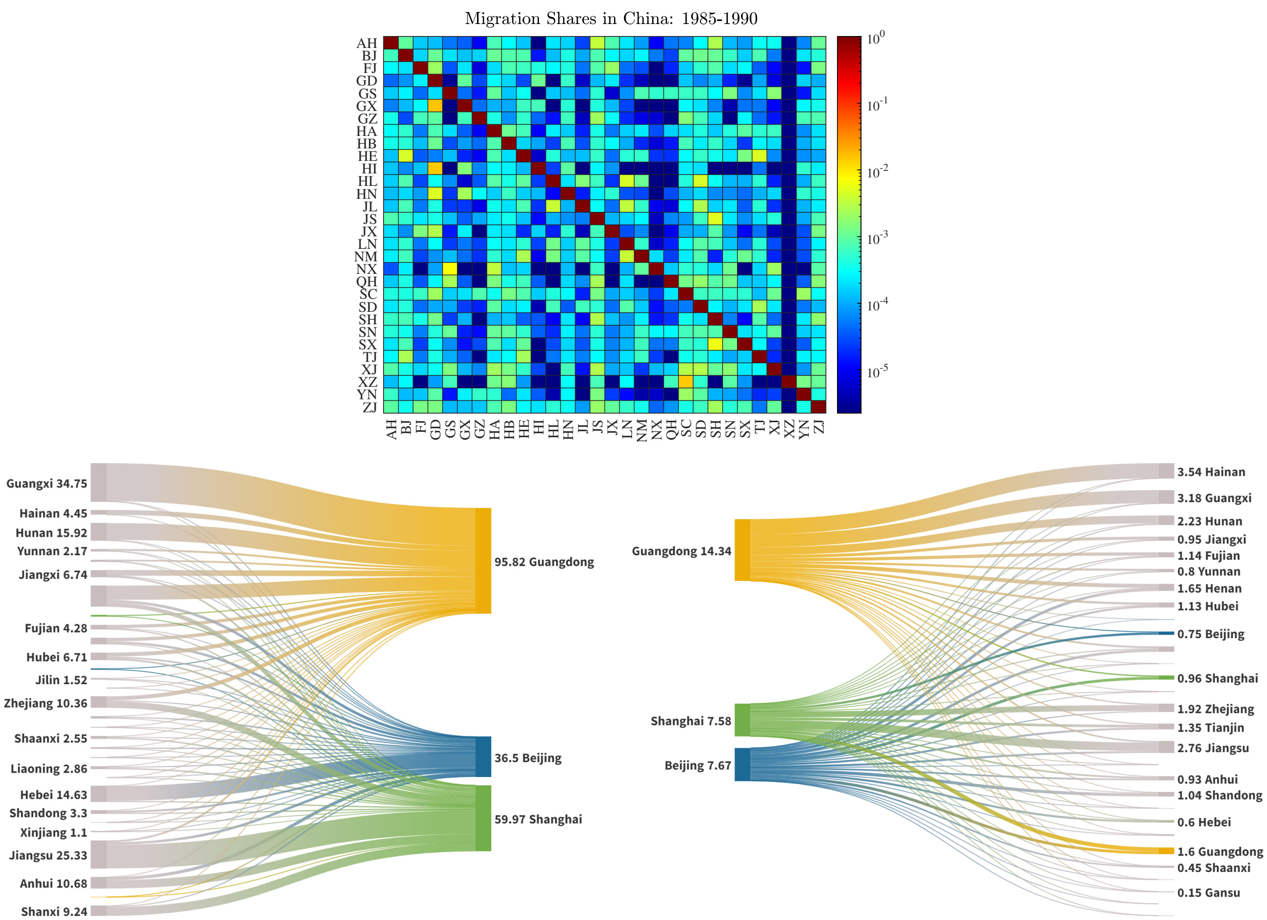
Gross Migration Flows. We obtain five-year mobility rates across provinces in China from the 1 percent sample of the 1990 census from IPUMS. The census data contains both the location (province) in 1990 and the location (province) five years ago. We take the working-age (15-64 years old) population as our sample. Furthermore, we keep respondents who are actively employed in 1990. To check the representativeness of our sample, we compute the employment share of each province out of the nationwide employment, and we compare it to the data counterpart provided in the 1991 China Statistics Yearbook.

To condition the gross flows on Hukou type, we proceed as follows. We use information from the 1990 census on the status and nature of registration. In particular, if the individual has the status "residing and registered here", we use the current location in 1990 as the registration location. If the individual has the category "residing here over 1 year, but registered elsewhere", "living here less than 1 year and absent from registration place over 1 year", or "living here with registration unsettled", we use the person's location in 1985 as the registration location; otherwise, the individual was living abroad, and we drop such observations (less than 0.02 percent of the observations). Finally, for those who registered non-locally yet resided in the same province in 1985 and 1990, we assign their registration location with a probability given by the immigration share from that location.

As an illustration of the mobility patterns across provinces in China, Figure 1 presents the five-year mobility flows across provinces. In the upper panel, we present a heat map with the migration shares across all provinces in China. We can see the heterogeneity of mobility patterns across provinces. As expected, the larger flows are stayers (the diagonal in the heat map); however, we can also see the importance of two-way migration across provinces.

To see more clearly these patterns, observe the bottom panel of the figure, which presents the mobility flows from other provinces to Beijing, Shanghai, and Guangdong (left-hand panel) and from these three provinces to the rest of the provinces in China (right-hand panel). Origin provinces are on the left axis and destination provinces are on the right axis, and a thicker line in the figure indicates a larger flow. As we describe in the next section, these three provinces have higher initial measured productivity, and as expected, in the left-hand panel we see how they receive migrants from all provinces in China. In the right-hand panel, we also observe how migrants move from these high-productivity places to the rest of China, which is an indication of the importance of return migration in China due in part to the Hukou restrictions as well as how return migrants diffuse knowledge from high-productivity places.

Figure 1: Mobility across provinces in China (1985-1990)



Note: The figure presents the mobility flows across provinces in China. The upper panel presents a heat map with the migration shares across all provinces in China, where the y-axis shows the origin provinces and the x-axis presents the destination provinces. The lower panels display the mobility flows (in 10,000 people) for the selected provinces, where the left axis presents the origin provinces and the right axis shows the destination provinces.

Trade and Production Data. We obtain export and import data between Chinese provinces and the rest of the world from the China Compendium of Statistics and the China's Statistics Yearbook that we discuss extensively in Appendix G. We also obtain GDP and employment data across provinces from the same source. The GDP for the rest of the world is obtained from the Penn World Table 10.0 (PWT). The PWT reports real GDP at constant 2017 national prices; hence, we convert real GDP for the rest of the world to 1990 prices using the world GDP deflator from the World Bank's World Development Indicators. To estimate the series of capital stock across provinces, we follow Shan (2008) and apply the perpetual inventory method, using fixed capital formation from the China Compendium of Statistics as the measure of investment and estimates of capital stocks at a base year from Young (2003). For the rest of the world, we obtain the capital stock at constant 2017 national prices from the PWT, which we convert to 1990 prices using GDP deflators from the same source. Using our constructed series of capital stock and equation (4), we obtain the initial real rental rates across locations.

Finally, we point out that in the quantitative analysis we abstract from trade across provinces and sectoral heterogeneity given the lack of data along these dimensions in the Chinese statistics for the 1990s.[10] As a result, our quantitative analysis will center on the role of local idea diffusion through internal migration and global idea diffusion through international trade.

3.2 Initial Stock of Knowledge

To estimate the initial stock of knowledge across locations, we start with the definition of real GDP. In our model, real GDP in location n at $t = 0$ is given by

$$Real\ GDP_{n,0} = \frac{w_{n,0}L_{n,0} + r_{n,0}K_{n,0}}{P_{n,0}} = \left(A_{n,0}/(\lambda_{nn,0}\Upsilon)\right)^{\frac{1}{\gamma\theta}} \left(K_{n,0}\right)^{(1-\xi)} \left(L_{n,0}\right)^{\xi}, \tag{13}$$

where $\Upsilon = (BT)^{\theta} (1-\xi)^{(1-\xi)\gamma\theta} (\xi)^{\xi\gamma\theta}$.[11] Real GDP in our model is determined by factor accumulation (capital, labor) and by measured productivity. In particular, measured productivity is captured by the term $\left(A_{n,0}/(\lambda_{nn,0}\Upsilon)\right)^{\frac{1}{\gamma\theta}}$. It has two main components: fundamental productivity $A_{n,o}$, and trade openness captured by the inverse of the domestic expenditure share $\lambda_{nn,0}$. The intuition is that in a closed economy— namely, when $\lambda_{nn,0} = 1$—measured productivity is the same as fundamental productivity $A_{n,o}$ (scaled by a constant power coefficient), which is the average efficiency of the set of goods produced and consumed in n. In an open economy, firms purchase a fraction of goods from abroad and produce only that set of goods of which they are the lowest-cost supplier in the world. Hence, a smaller domestic expenditure share $\lambda_{nn,0}$ results in firms in n producing a smaller set of goods with higher marginal efficiency.

[10] In Appendix G, we provide a detailed discussion of available data sources that could be used to impute inter-province trade flows under certain assumptions.

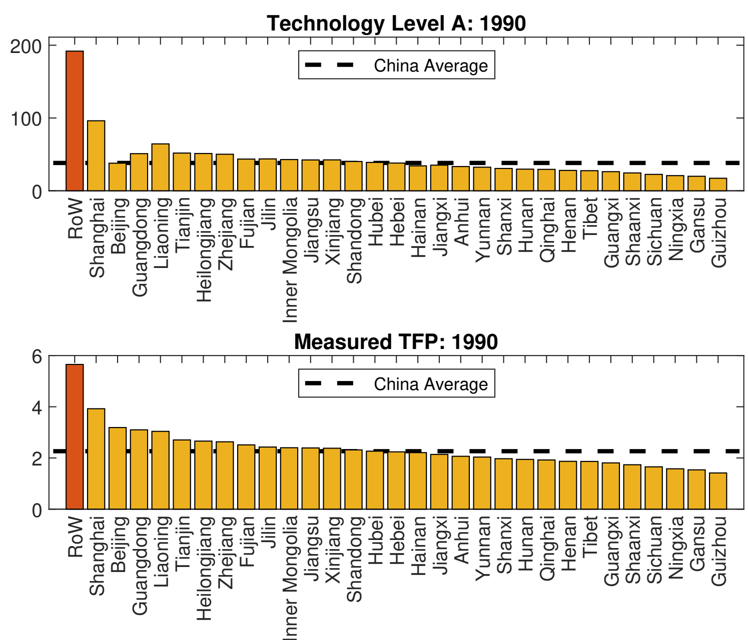
[11] See Appendix H.1 for the details of this derivation.

Inverting equation (13), and solving for fundamental productivity $A_{n,0}$, we obtain

$$A_{n,0} = \Upsilon \left(\frac{Real\ GDP_{n,0}}{(K_{n,0})^{1-\xi} (L_{n,0})^{\xi}} \right)^{\gamma\theta} \lambda_{nn,0}. \tag{14}$$

Using the data described in the previous subsection, we compute the initial stock of knowledge across provinces in China as well as for the rest of the world.[12] Figure 2 presents the initial stock of knowledge (year 1990) across locations. In the upper panel, we see that the 1990 stock of knowledge for provinces in China is smaller than that for the rest of the world. Across provinces in China, the initial stock of knowledge is very heterogeneous, with Shanghai, Liaoning, and Guangdong being the top three provinces in terms of the initial stocks of knowledge, and Gansu, Guizhou, and Ningxia the bottom three provinces. The bottom panel presents the 1990 measured productivity across locations, which corrects for the impact of trade as previously explained. Again we observe that the rest of the world has higher measured productivity in 1990 than the provinces in China. We can see that Shanghai, Beijing, and Guangdong are the top three provinces with the highest measured productivity, whereas Gansu, Guizhou, and Ningxia are the bottom three provinces.

Figure 2: Initial stock of knowledge and measured productivity across locations (1990)



Note: The figures present the initial stock of knowledge (upper panel), computed as described in this section, and measured TFP (bottom panel), computed as $(A_{n,0}/(\lambda_{nn,0}/\Upsilon))^{1/\gamma\theta}$.

[12]We set a value of $\eta = 2$ in the gamma function in equation (14).

3.3 Estimation of Idea Diffusion from Trade and Migration

In our dynamic spatial growth model, three parameters discipline productivity growth and idea diffusion across locations: the strength of idea diffusion through sellers ρ_m, the strength of idea diffusion through workers ρ_l, and the arrival rate of insights α_0. To discipline these parameters in the our dynamic spatial model, we proceed as follows.

We first measure fundamental productivity, $A_{n,t}$, by geography for different periods of time. We do this using equation (14), as described in Section 3.2. The model inversion described in Section 3.2 delivers model-consistent cross-sections of fundamental productivities across locations at different points in time that we use to discipline the idea diffusion parameters. It is important to emphasize that our estimated fundamental productivities for the various periods of time are cross-sectional measures; thus, we do not impose any structure on how each of these measures might be related over time. Therefore, we create moment conditions related to the evolution of fundamental productivity over time and compute the same moments using the model-implied fundamental productivity from equation (9). We then use the GMM to estimate the parameters of interest following Hansen and Singleton (1982) and Newey (1985).

We use fundamental productivity estimates for the period 1990-2000 and five moment conditions: the first moment is the average change in fundamental productivity levels across locations; the second moment is the average growth rate in fundamental productivities; the third moment is the variance in the time changes in fundamental productivity levels; the fourth moment is the covariance between the initial fundamental productivities and the change in fundamental productivity levels; and the fifth moment is the covariance between the initial fundamental productivities and the growth rate in fundamental productivities.

To provide further intuition on how these five moments help identify the innovation and diffusion parameters, we note that the first two moments help us identify α_0 since the initial arrival rate of ideas scales up productivity everywhere. The third moment helps us separate α_0 from the diffusion parameters ρ_m and ρ_l since they provide information about how heterogeneity in trade openness and mobility flows result in cross-province variations in the stock of knowledge over time. The last two moments provide information to disentangle ρ_m from ρ_l. The intuition is that provinces with a higher initial stock of knowledge tend to be more open to trade and therefore benefit more from the global diffusion of ideas from the rest of the world. Hence, ideas from sellers tend to generate a positive covariance between the initial stock of knowledge and subsequent changes in productivity. On the other hand, ideas from people are not necessarily associated with a positive covariance; this depends on whether locations receive migrants from places with relatively good insights.

Finally, we must confront the fact that the observed evolution of TFP is partly influenced by determinants outside our model. To address this, we assume the arrival rate of ideas is subject to *i.i.d.* location-specific shocks resulting in unobserved residuals in the TFP evolution, which are not predictable by migrants and are therefore not part of their migration decisions. Consequently, our empirical strategy allows for an unobserved residual that captures the effects of factors influencing TFP besides idea diffusion. Appendix H.2 presents the empirical moment conditions and the model-implied moments. In addition, in Appendix H.3 we show that the unobserved residuals from our estimation are uncorrelated over time, consistent with our *i.i.d.* assumption of the TFP shocks. Also, in Appendix H.4 we compute the contribution of idea diffusion to the observed changes in TFP. Using this procedure, we obtain our preferred estimates of $\rho_l = 0.18$, $\rho_m = 0.61$ and $\alpha_0 = 0.14$.

3.4 Computing Counterfactuals

To compute the dynamic spatial growth model, we apply dynamic-hat algebra techniques developed in Caliendo et al. (2019) and show that by expressing the equilibrium conditions in relative time differences, we are able to compute the model without needing to estimate the levels of exogenous fundamentals or assuming that the economy is in the balanced growth path in the initial period. The intuition is that solving the model in relative time differences requires conditioning the model on observable allocations, which contain all the information about the fundamentals, and matching the cross-section of the actual economy in the initial year that does not need to be in a balanced growth path. The next proposition establishes the result.

Proposition 4. Dynamic-Hat Algebra. *Define the variable $\hat{y}_{t+1}$ as the relative time difference of the detrended endogenous variable denoted by $\tilde{y}$; namely, $\hat{y}_{t+1} = \tilde{y}_{t+1}/\tilde{y}_t$. Given an initial observed allocation $\left\{\left\{\lambda_{in,0}\right\}_{i=1,n=1}^{N,N}, \left\{\mu_{in,0}\right\}_{i=1,n=1}^{N,N}, \left\{w_{i,0}L_{i,0}\right\}_{i=1}^{N}, \left\{K_{i,0}\right\}_{i=1}^{N}, \left\{L_{i,0}\right\}_{i=1}^{N}\right\}$, the parameters and elasticities $(\rho_\ell, \rho_m, \theta, \nu, \gamma, \xi, \beta)$, the initial rate and growth rate in the arrival of ideas (α_0, g_α) and a convergent sequence of future changes in fundamentals under perfect foresight $\left\{\hat{\kappa}_{in,t}, \hat{m}_{in,t}\right\}_{i=1,n=1,t=1}^{N,N,\infty}$, the solution for the sequence of changes in the model's endogenous variables in the detrended model $\left\{\hat{y}_{t+1}\right\}_{t=1}^{\infty}$ does not require information on the level of fundamentals (trade and migration costs).*

Proof. See Appendix E.

In the detrended balanced growth path, $\hat{A}_n = 1$, and therefore $\hat{y} = 1$ for all variables $\tilde{y}$. We use this property of the detrended model to develop an algorithm to compute counterfactuals in the dynamic spatial growth model, which is described in Appendix F. In addition, as the proposition establishes, solving the model in relative time differences requires conditioning the model on the initial observable allocations $\lambda_{in,0}$, $w_{i,0}L_{i,0}$, $L_{i,0}$, $\mu_{in,0}$, and $K_{i,0}$, and parameters and elasticities θ, ν, β, δ, ρ_ℓ, ρ_m, and α_0. The previous sections have described our process for collecting these initial allocations and disciplining the parameters and elasticities in our framework.

4 Mechanics of Spatial Growth in China

In this section we describe our quantitative analysis of the mechanics of spatial growth in China. In Subsection 4.1 we study the role of the initial distribution of fundamentals in shaping spatial and aggregate development in China in the 1990s and 2000s. In particular, Subsection 4.1.1 describes how initial conditions shaped the distribution of economic activity across space in China during the 1990s and 2000s, Subsection 4.1.2 studies the role of idea diffusion and capital accumulation on aggregate growth for China, and Subsection 4.1.3 describes the speed of convergence of the economy. In Subsection 4.1.4 we discuss the process of spatial growth in China shape aggregate growth, and Subsection 4.1.5 studies the role of the initial distribution of fundamentals in shaping subsequent spatial growth in China. Finally, in Subsection 4.2 we explore the effects of changes in fundamentals (international trade costs and internal migration restrictions) after 1990 on spatial and aggregate growth.

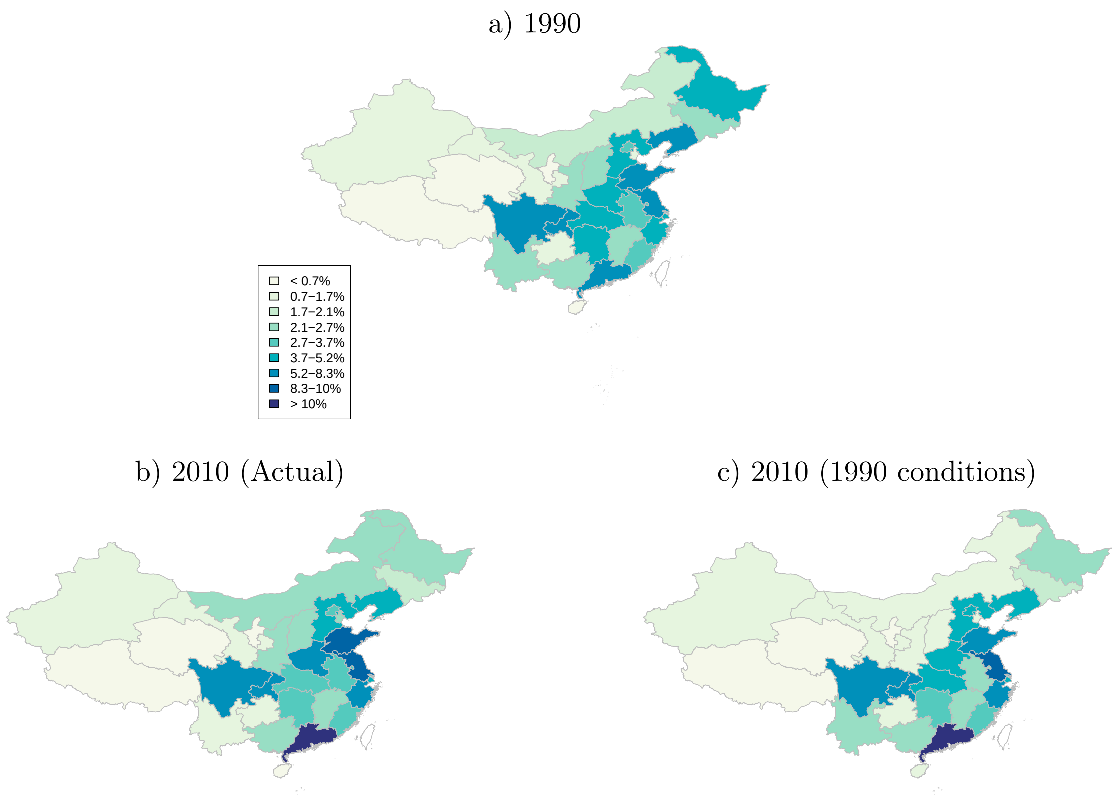
4.1 Initial Conditions, Idea Diffusion, and Capital Accumulation

We first use our framework to study how the initial distribution of fundamentals shaped spatial growth in China in the 1990s and 2000s. To do so, as we described before, we take the model to the data in the year 1990. The model does not assume that the economy is in a balanced growth path in the initial year since the model is taken to the data in the initial period conditioning on observed allocations. We then compute the economy with 1990 fundamentals and the endogenous evolution of productivity; namely, we answer the counterfactual question: How would the provinces in China and the rest of the world have looked if fundamentals (trade and migration costs) had stayed at their 1990 levels and the changes in the stock of knowledge had operated solely through the mechanisms in the spatial growth model?

4.1.1 Regional Distribution of Economic Activity

We start by describing the initial distribution of economic activity in China and its evolution in subsequent decades, and we explore the role of initial conditions in shaping the distribution of economic activity in China across the 1990s and 2000s. Figure 3 presents the actual GDP shares across provinces in the year 1990 and the GDP shares twenty years later in 2010. The left column of the bottom panels shows the actual GDP shares across provinces in China in 2010, and the right panel presents the predicted GDP shares in 2010 under the initial distribution of fundamentals.

Figure 3: Regional distribution of economic activity (GDP shares)



Note: The figures show the distribution of economic activity across provinces in China, measured as GDP shares, in the data and with 1990 fundamentals over the period 1990-2010.

Starting with the initial GDP shares across provinces in China, we can see that economic activity tends to concentrate in the center and coastal areas of China, with Guangdong, Shandong, and Jiangsu being the largest provinces in terms of GDP in 1990. In 2010, in the bottom left panel, we can see a persistent concentration of economic activity in the same areas of China, but unlike in 1990, in 2010, economic activity tends to move from the central part of China to the coastal areas. For instance, Guangdong, Jiangsu, Zhejiang, Fujian, and Shandong are all provinces that increased their GDP shares, and these provinces are located in the coastal areas of China. In the bottom right panel, we see a similar pattern predicted by the model, pointing to the role of initial conditions in shaping the redistribution of economic activity across space during the subsequent two decades. In Appendix I.1 we present the evolution of the GDP shares across provinces in China every five years, which displays the same pattern described in this subsection.

4.1.2 Aggregate Growth in China

We next study the implication of initial conditions and spatial development described in the previous subsection for aggregate growth in China during the 1990s and 2000s. Table 1 presents the annual real GDP growth in China for different time frames over the period 1990-2020 if fundamentals (trade and migration costs) had stayed constant at the 1990 levels. We also quantify the contribution of idea diffusion through goods and people, and the contribution of capital accumulation, to aggregate growth in China.

We find that initial conditions play a role in subsequent growth in China during the 1990s and 2000s. For instance, with the initial distribution of fundamentals in 1990 and with no changes in migration costs and trade costs thereafter, China's real GDP would have increased at an annual rate of about ten percent in the 1990s and about nine percent in the 2000s. As discussed in the review by Caliendo and Parro (2022), many reforms in China that involve changes to trade and industrial policy took place before the 1990s. In our model, these reforms are captured by the initial conditions. As previously described, our methodology does not assume that the economy is on the balanced growth path in the initial period. The framework is taken to the actual data in 1990, and therefore, the actual initial allocations contain information about fundamentals and policies in the Chinese economy and the rest of the world up to that year. We find a role of these initial conditions in aggregate growth in China in the decades after 1990.

In the next two rows of the table, we evaluate the contribution of idea flows from people and from goods to aggregate growth in China. In the second row, we compute the model assuming $\rho_l = 0$; namely, that productivity evolves endogenously only due to idea flows from goods. We find that without idea diffusion through people, aggregate growth in China would have been smaller but still significant. In the third row of the table, we quantify aggregate growth in China with idea diffusion from people only; namely, in a model with $\rho_m = 0$. We find that without idea flows from goods, aggregate growth in China would have been even smaller. Intuitively, two factors explain the larger importance of idea flows from goods for aggregate growth in China. First, as described in Section 3.2, the initial stock of knowledge across provinces in China is lower than it is in the rest of the world; hence, international trade makes an important contribution to growth through the diffusion of good ideas from the rest of the world to all provinces in China. At the same time, the contribution of idea flows from people can have offsetting effects on growth since return migration from high-productivity places fosters growth in the stock of knowledge in the destination province but receiving migrants from low-productivity locations slows down the process of knowledge accumulation. As we described in Section 3.1, both migration patterns are salient in China. Second, the estimated elasticity that governs the diffusion of ideas through people is smaller than the one that disciplines the diffusion of ideas through goods. In addition, and as discussed before, the extent to which locations have differential exposure to trade and are connected to migrants from different locations results in spatial heterogeneity in the relative contributions of ideas from goods and ideas from people to growth across provinces in China, which we document in Appendix I.4.

In the second to last row of the table, we quantify the importance of capital accumulation for aggregate growth in China. We find that in a model without capital accumulation, initial conditions in 1990 would have resulted in around a half of the aggregate growth in subsequent decades. In the last row of the table, we compute the aggregate growth with capital accumulation and no idea diffusion. Comparing the last two rows of the table, we can see that capital accumulation played a more important role than idea diffusion as an engine for aggregate growth in the early 1990s, but idea diffusion became more important over time. In the absence of changes in fundamentals, capital accumulation had a relatively stable contribution to aggregate growth. Different from capital accumulation, the contribution of idea diffusion to aggregate growth increased over time. As knowledge diffused and locations increased their stock of knowledge, people contributed with better insights to their locations and to other locations when moving, and provinces more opened to international trade also benefited more from better global insights as the stock of knowledge in the rest of the world also increased.

Table 1: Annual GDP growth rate

	90-95	90-00	90-05	90-10	90-15	90-20
With fundamentals in 1990	10.69%	10.15%	9.68%	9.29%	8.94%	8.64%
W/o ideas from people ($\rho_l = 0$)	8.22%	7.42%	6.81%	6.32%	5.92%	5.59%
W/o ideas from goods ($\rho_m = 0$)	6.12%	5.13%	4.40%	3.84%	3.41%	3.05%
W/o capital accumulation	5.02%	4.89%	4.77%	4.66%	4.56%	4.47%
W/o idea diffusion ($\rho_l = 0$, $\rho_m = 0$)	6.06%	5.05%	4.32%	3.76%	3.32%	2.96%

Note: GDP growth with 1990 fundamentals is computed by solving the dynamic spatial growth model with constant fundamentals. The growth rate without idea flows from people is obtained by computing the model with $\rho_l = 0$, and the growth rate without idea flows from goods is obtained by computing the model with $\rho_m = 0$. The second to last row presents the aggregate GDP growth in the absence of capital accumulation. The last row presents the aggregate growth with capital accumulation and no idea diffusion, obtained by computing the model with $\rho_l = 0$ and $\rho_m = 0$.

4.1.3 Speed of Convergence

We then study the role of idea diffusion and capital accumulation on the speed of convergence of the economy to the detrended steady state. In particular, we compute the half-life of real GDP convergence across provinces in China under different versions of the model. Appendix I.2 reports the results for each province in China, and in what follows we discuss the main findings.

We find that capital accumulation increases the average pace at which the economy converges to the detrended steady state. However, we find spatial differences in the speed of convergence. In fact, the transition of the economy as whole takes longer with capital accumulation since some provinces, such as Shanxi and Guizhou experience a much longer half-life of convergence with capital accumulation. The reason is that these provinces have a much larger initial labor force than in the steady-state, leading to higher marginal productivity of capital and faster capital accumulation at the beginning of the transition. As the labor shifts out during the transition, capital eventually declines towards the detrended steady state. The overshooting of capital in these provinces contributes to a more extended transition period of the Chinese economy with capital accumulation. These results align with the findings of Kleinman et al. (2023), who show that a positive correlation between the capital and labor gaps from the steady-state across different locations results in slower convergence.

We find that with idea diffusion, the convergence of the economy to the detrended steady state takes longer and it is more homogeneous across provinces. The intuition comes from the fact that with technology growth, the detrended steady state is farther away from the initial condition as the stock of knowledge accumulates over time due to the insights from global and local ideas. As a result, all the sources of dynamics—labor mobility, capital accumulation, and growth in the stock of knowledge—need to converge to their respective detrended steady state for the GDP convergence of the economy as whole to be completed, which makes the transition longer. Finally, idea diffusion from goods makes the transition of the economy longer than idea diffusion from people. Chinese provinces benefit from global insights with idea diffusion from goods, resulting in a higher stock of knowledge in the detrended steady-state and a longer transition. Conversely, the growth in knowledge stock through idea diffusion from people balances the varying quality of insights gained by locations from migrants, resulting in a lower stock of knowledge in the detrended steady state and a shorter transition.

4.1.4 Spatial Growth

We turn to describe spatial growth across provinces in China. In particular, in this subsection we study how the aggregate growth in China described in Section 4.1.2 was shaped by spatial growth. To do so, Figure 4 displays the real GDP growth across provinces in China during different time frames over the period 1990-2020 as predicted by our model under the initial conditions in 1990. In each panel, the upper figure presents the annual real GDP growth and the lower panel displays the contribution of each province to the aggregate real GDP growth in China during that period.

Several results emerge from the figure. Looking first at the upper figures in each panel, in the 1990s we find large heterogeneity in spatial growth. We can see that Hainan, Shanghai, Beijing, and Guangdong are the provinces with the highest growth rates in the 1990s. Among them, the last three are the ones with the highest initial measured productivity, as described in Section 3.2. At the same time, these are provinces located in the coastal areas with better access to foreign goods. Hence, we observe some divergence after 1990 in the form of higher growth in provinces with better initial technology, which is in part shaped by the idea diffusion through goods purchased from the rest of the world. Over the 2000s, we can see that growth rates tend to moderate and converge across provinces, as shown in the remaining upper figures, as the Chinese economy starts approaching the balanced growth path. Also, ideas from people diffuse across space, and insights from migrants coming from provinces with a relatively low stock of knowledge slow down growth in destinations with a relatively higher stock of knowledge, fostering some convergence in the stock of knowledge across space.

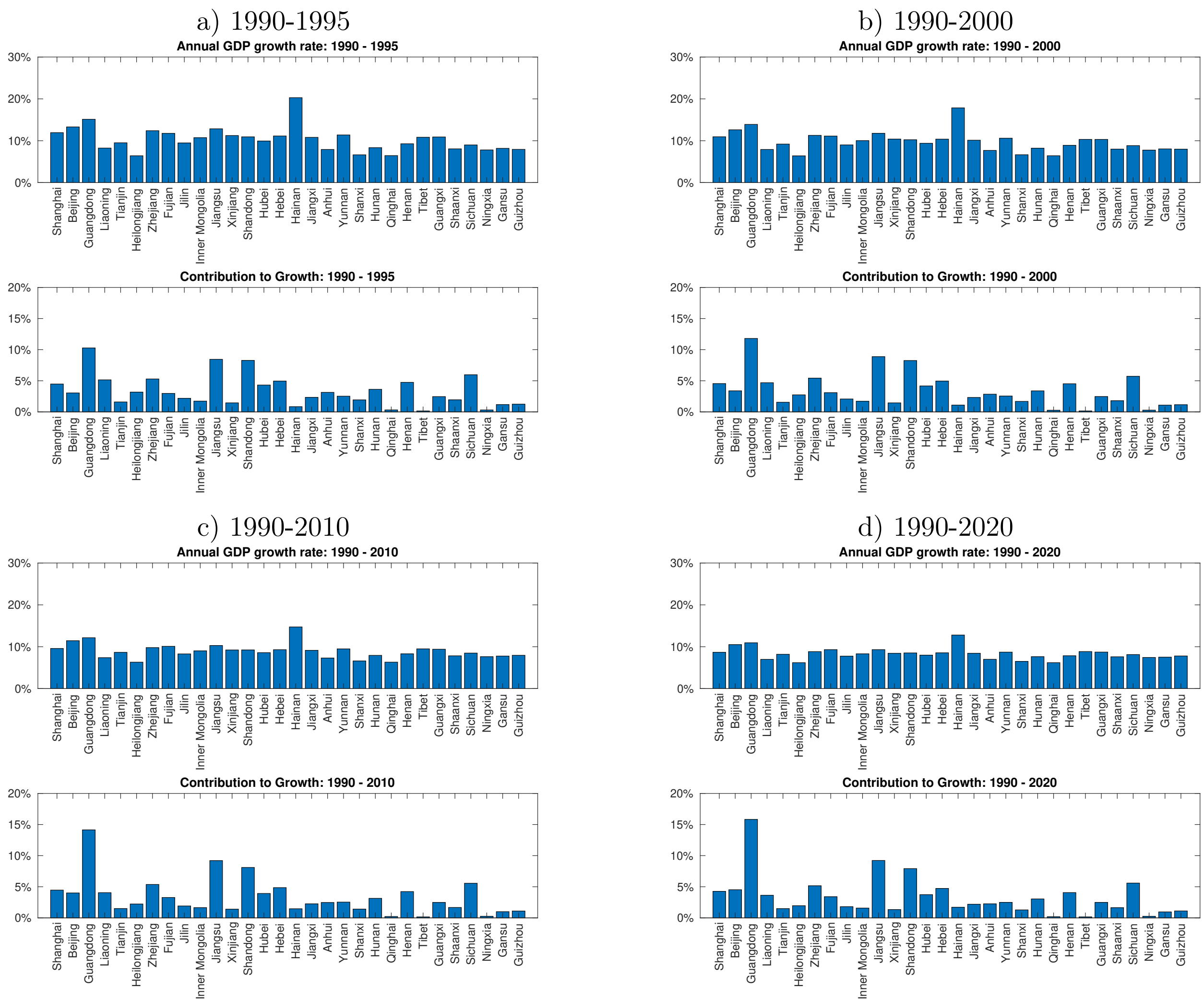
In the lower figures of each panel, we present the contribution of each province to aggregate growth in China in each period of time. From these figures, we can see how Guangdong became a much more important engine of aggregate growth in China over time, as it benefits relatively more from ideas from the rest of the world, especially given its advantaged location on the coast near Hong Kong. The figures also show that other provinces like Beijing became more important contributors to aggregate growth in China. At the same time, other large provinces like Shandong, Henan, and Hubei decreased their importance for aggregate growth over time.[13]

Overall, this subsection illustrates how aggregate growth was shaped by heterogeneous spatial growth across provinces in China over time. The mechanics of spatial growth across provinces are in turn shaped by initial conditions, the dynamics of productivity resulting

[13] Appendix I.3 presents spatial growth effects for additional time frames, which deliver the same conclusions discussed in this section.

from idea diffusion through goods and people, changes in trade openness and migration, and the dynamics of labor markets and capital accumulation across space.

Figure 4: Spatial growth (annual, percent)



Note: The figures show the annual real GDP growth across provinces and the contribution of each province to the aggregate growth in China in different time frames over the period 1990-2020. Spatial growth in each figure is computed in the model under the initial 1990 conditions.

4.1.5 Initial Distribution of Fundamentals and Spatial Development

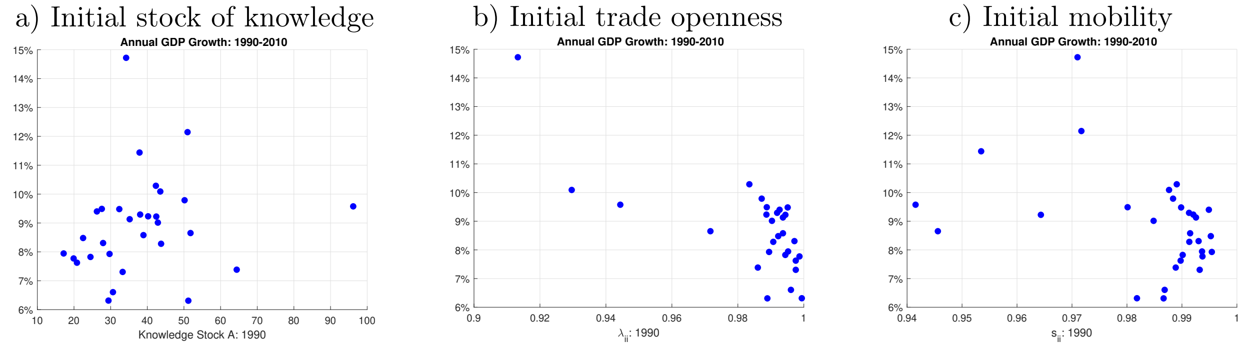
In this subsection we study how the spatial growth across provinces during the 1990s and 2000s that we described in the previous section correlates with the initial distribution of fundamentals; namely, the initial stock of knowledge, the initial level of trade openness, and initial mobility frictions.

The upper panel of Figure 5 shows a scatter plot between the initial stock of knowledge and the real GDP growth across provinces over the period 1990-2010. We can see a positive correlation, meaning that provinces with a relatively higher initial fundamental productivity tend to grow more in the subsequent decades. It is important to emphasize that our dynamic spatial growth model allows for this correlation to have any sign. On the one hand, provinces with a higher initial stock of knowledge might also be more open to trade and as a result, they can benefit more from ideas diffused from the rest of the world relative to provinces with a lower initial stock of knowledge. On the other hand, provinces with relatively higher fundamental productivity might be attractive to migrants from other provinces who bring relatively few good-quality insights, which might slow down growth in those provinces.

Panel (b) presents a scatter plot between real GDP growth and the initial level of trade openness measured by domestic expenditure share λ_{ii}, where a smaller value of λ_{ii} means higher trade openness. The figure shows a clear negative correlation, meaning that a higher initial level of trade openness leads to higher growth in subsequent decades. This correlation is intuitive since provinces more open to trade benefit more from idea diffusion through goods purchased from the rest of the world. Finally, Panel (c) presents a scatter plot between growth and initial mobility measured by the fraction of stayers in a province denoted by s_{ii}, where a smaller value of s_{ii} means lower mobility frictions and higher mobility. The negative correlation is less clear than in the case of trade openness. As explained before, provinces that receive migrants experience faster growth in knowledge only if the insights from the provinces from which they migrate are of sufficiently good quality. All these scatter plots deliver the same message for the period 1990-2020.

Figure 5: Real GDP growth versus initial conditions

a) Initial stock of knowledge b) Initial trade openness c) Initial mobility



Note: The figures show scatter plots of annual GDP growth across provinces in China over the period 1990-2010 against initial conditions in 1990: initial knowledge stock in Panel (a), initial level of domestic expenditure share, λ_{ii}, in Panel (b) (two outlier provinces were trimmed), and initial share of stayers, s_{ii}, in Panel (c).

4.2 Changes in Fundamentals

Our quantitative analysis in the previous sections shows that initial conditions seem to be important for understanding and quantifying the process of spatial development and aggregate economic growth in China. It also shows the importance of the general equilibrium interactions of the mechanisms in our framework such as idea diffusion through trade and migration, labor market dynamics, and capital accumulation.

Over the 1990s and 2000s, China also undertook reforms related to changes in trade costs and migration frictions that might have also impacted spatial growth. In particular, when China joined the World Trade Organization, provinces more exposed to trade might have developed more relative to the less exposed provinces. Likewise, Hukou reforms might have fostered idea flows by increasing mobility across provinces. In this section, we explore quantitatively through the lens of our framework the impact of these reforms on spatial growth in China.

We capture the changes in trade costs between China and the rest of the world using the time variation in bilateral trade shares relative to domestic expenditure shares across provinces. In other words, from our model we back up changes in trade costs as $\frac{\hat{\lambda}_{in,t}\hat{\lambda}_{ni,t}}{\hat{\lambda}_{ii,t}\hat{\lambda}_{nn,t}} = (\hat{\kappa}_{in,t}\hat{\kappa}_{ni,t})^{-\theta}$.[14] We obtain changes in trade frictions between provinces in China and the rest of the world over the period 1990-2010, and then ask how spatial growth in China would have looked if the only change in fundamentals over the period 1990-2010 had been international trade costs. To do so, we compare this counterfactual economy with the evolution of the economy with the 1990 initial conditions described in the previous sections.

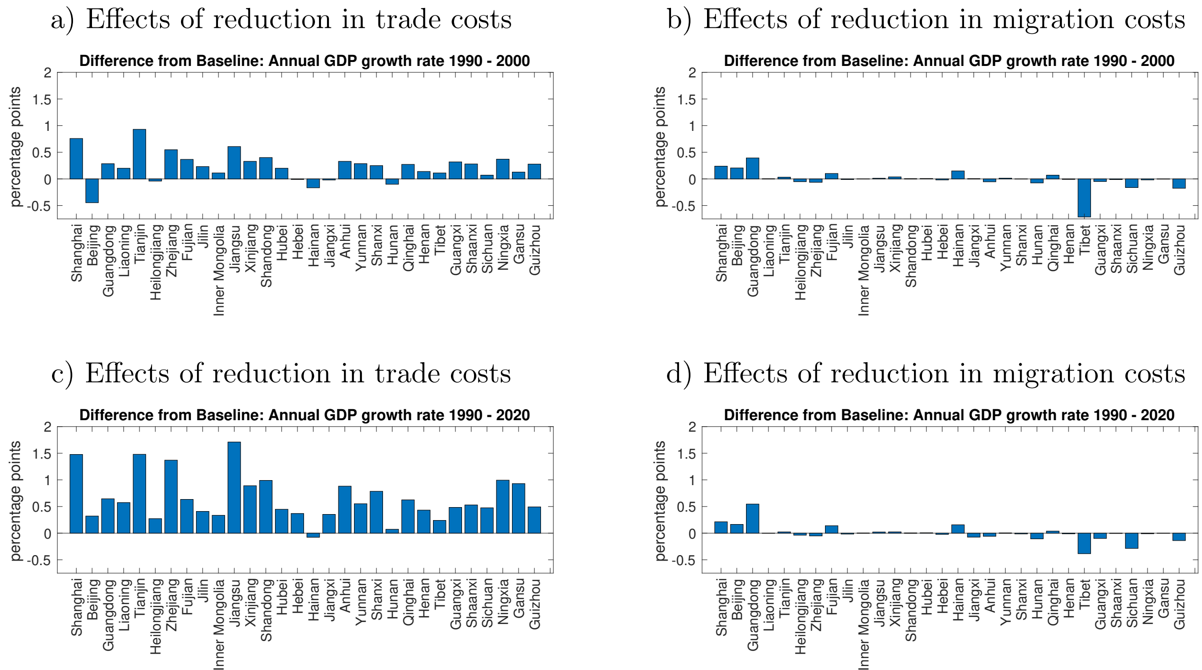
We also explore Hukou reforms in a simple way. We capture the changes in migration frictions across provinces in China using the cross-variation in five-year mobility rates from 1985-1990 to 1995-2000 as $\frac{\hat{\mu}_{in,t}\hat{\mu}_{ni,t}}{\hat{\mu}_{ii,t}\hat{\mu}_{nn,t}} = (\hat{m}_{in,t}\hat{m}_{ni,t})^{-\frac{1}{v}}$, and apply this change in mobility frictions to all Hukou types.[15] We then ask the counterfactual question of how spatial growth in China would have looked if the only change in fundamentals after 1990 were the change in mobility frictions to Hukou types. To do so, we compare this counterfactual economy with the evolution of the economy with 1990 initial conditions described in the previous sections.

The spatial growth effects of changes in trade costs and mobility frictions are presented in Figure 6. The left-hand panels display the effects of changes in trade costs, and the right-hand panels present the effects of Hukou restrictions. We present the results for the periods 1990-2000 and 1990-2020, and in Appendix I.5 we present results for additional time frames.

[14] This statistic is known as the Head-Ries index (Head and Ries (2001)) and is widely used in the trade and spatial literature to measure bilateral trade frictions.

[15] Note that since Hukou type is assigned to either the origin or the destination province in the data, changes in mobility frictions are isomorphic to changes in amenities by Hukou type.

Figure 6: Effects of trade and migration costs on spatial growth (percentage points)



Note: The figures show the percentage point change in real GDP growth across provinces due to the trade and migration restrictions in different time frames. The left-hand panels present the effects of changes in trade costs and the right-hand panels show the effects of migration restrictions. All effects are computed relative to the baseline economy with 1990 trade and migration costs.

We find that the change in trade costs contributed to growth effects that were very heterogeneous across space. Panel (a) shows the spatial growth effects of changes in trade costs over the 1990s. We can see that changes in trade costs increase growth in almost all provinces in China relative to the baseline economy with initial trade and migration costs. The coastal provinces of Shanghai, Tianjin, and Jiangsu experience the largest growth effects from changes in trade costs as they benefit relatively more from trade openness compared with provinces located farther away. Some provinces that initially are relatively more open to trade, such as Beijing, Hainan, and Hunan, see growth slightly decline in the 1990s, as they face increased competition from other provinces, especially in the coastal areas that benefit from trade with the rest of the world. Moving down to the bottom panel, we can see that changes in trade costs foster growth across all provinces over time as ideas from the rest of the world continue to diffuse to provinces in China.

In the right-hand panels, we present the spatial growth effects of changes to migration restrictions. We can see that changes in mobility restrictions lead to smaller growth effects than changes in trade costs. However, the growth effects of changes in mobility restrictions are more heterogeneous than the growth effects of trade costs, with increases in growth in some provinces and decreases in growth in others relative to the baseline economy with 1990 conditions. Consistent with the intuition provided in previous sections, changes in mobility frictions benefit more open provinces that have scaled-up production and provinces that benefit from the ideas from migrants coming from places with a higher stock of knowledge, while they slow down growth in provinces left behind by international trade and internal migration. In Appendix I.5 we present the same figures with the spatial growth effects from changes in trade costs and Hukou restrictions together across provinces in China.

Table 2 presents the aggregate growth in China due to both 1990 fundamentals and the changes in international trade costs and migration restrictions over the 1990s and 2000s. To facilitate the analysis, the first row of the table reproduces the growth rate with 1990 fundamentals displayed in Table 1. The next three rows present the growth effects with the 1990 fundamentals and the changes in trade costs and migration restrictions. We can see from the table that both changes in trade costs and changes in mobility restrictions added about one percentage point of extra annual aggregate growth in China by the 2000s, mostly coming from the changes in international trade costs. The changes in migration restrictions did not have a significant impact on aggregate growth, although they had heterogeneous effects across locations, as described previously.

Table 2: Annual GDP growth rate: Changes in fundamentals

	90-95	90-00	90-05	90-10	90-15	90-20
With fundamentals in 1990	10.69%	10.15%	9.68%	9.29%	8.94%	8.64%
Fund. in 1990 & change in trade cost	10.77%	10.42%	10.19%	9.95%	9.68%	9.41%
Fund. 1990 & change in mig. restrictions	10.69%	10.19%	9.75%	9.37%	9.03%	8.72%
Fund. in 1990 & change in fundamentals	10.77%	10.46%	10.26%	10.03%	9.77%	9.50%

Note: The first row of the table reproduces the growth rate with 1990 fundamentals displayed in Table 1. The second row presents the annual growth rate with 1990 fundamentals and changes in international trade costs. The third row presents the annual growth rate with 1990 fundamentals and changes in migration restrictions. The fourth row presents the annual growth rate with 1990 fundamentals and changes in international trade costs and migration restrictions.

5 Empirical Evidence of Idea Diffusion

In the previous section we highlighted the importance of the spatial mechanisms in our framework for shaping spatial development and aggregate growth, and in particular, the role of idea diffusion through trade and migration. We did so through the lens of the spatial dynamic growth model developed in Section 2. In this section, we complement the structural analysis by providing reduced-form evidence of idea diffusion from trade and migration. To do so, we obtain province-level patent data as a proxy for the local stock of knowledge and use it along with our trade and migration data to provide empirical evidence of the role played by trade and migration in the diffusion of ideas.

We obtain province-level patent data from the China Statistics Yearbooks. To proxy the measure of knowledge stock, $A_{n,t}$, we obtain the cumulative approved patents at the province level for each year over the period 1985-2010. We then compute the change in the stock of knowledge every five years from 1985 to 2010. For the approved patents of the rest of the world, we obtain data from Google Patent from 1985-2010 following Liu and Ma (2021).[16] In Appendix J we provide more details on these patent data.

Using our patent data, as well as the migration and trade data described in Section 3.1 and in Appendix G, we run the following empirical specification:

$$\log\left(A_{n,t+1} - A_{n,t}\right) = \tau + \beta_m \log \lambda_{nn,t} + \beta_l \log\left(migration_{n,t}\right) + \tau_n + \tau_t + \epsilon_{n,t}.$$

The term $\lambda_{nn,t}$ is the domestic expenditure share and captures the (inverse) level of trade openness of province n. We expect the coefficient β_m to be negative, indicating that provinces more exposed to international trade benefit more from the global diffusion of ideas and experience higher growth in the stock of knowledge as a result. We define $\log\left(migration_{n,t}\right) = \log\left[\sum_{i=1}^{N} s_{in,t} A_{i,t}\right]$, which equals the weighted average of the stock of knowledge diffusing to location n at time t through both migrants and locals. This term in our model captures the idea diffusion through people, and as a result, we expect the coefficient β_l to be positive. Finally, the term τ_n controls for province fixed effects, τ_t is a year fixed effect, and $\epsilon_{n,t}$ is an error term following an i.i.d. standard normal distribution. Since individual-level population census data for the year 1995 do not exist, migration flows are unavailable for the period 1990-1995; hence, we run the regression using data for five-year intervals from 1995 to 2010.

Table 5 reports the results. In Column (1) we first show that faster growth in the stock of knowledge is associated with a higher degree of trade openness, and Column (2) shows that the growth in knowledge stock positively correlates with idea diffusion through people, controlling for year and province fixed effects. In Column (3), we show that trade openness and idea diffusion through people together contribute to the growth in the stock of knowledge in the way that our theory suggests. In each of these specifications, the coefficients of interest, β_m and β_l, have the expected signs and are statistically significant.[17]

[16]We are grateful to Song Ma for sharing the Google Patent data.

[17]For completeness, in Appendix J.1 we present scatter plots with correlations between the change in the local stock of knowledge constructed with our patent data, and our measures of trade openness and idea diffusion through people. These correlations also show a positive relationship between trade openness and

Table 3: Estimates of knowledge diffusion through trade and people

	$\log(A_{n,t+1} - A_{n,t})$		
	(1)	(2)	(3)
$\log \lambda_{nn,t}$	-6.067***		-5.715**
	(2.214)		(2.288)
$\log(migration_{n,t})$		0.353*	0.276*
		(0.201)	(0.154)
Constant	8.697***	5.448**	5.943***
	(0.112)	(2.003)	(1.543)
Observations	90	90	90
R-squared	0.542	0.473	0.562
Year FE	✓	✓	✓
Province FE	✓	✓	✓

Note: Robust standard errors in parentheses.*** p<0.01, ** p<0.05, * p<0.1.

In the previous regression we documented the contribution of idea diffusion through people to the local stock of knowledge. One might wonder whether this contribution is entirely driven by local ideas, as a significant share of locals stay in the same location during each five-year window. To address this question, we distinguish between idea diffusion through the locals (stayers) and that through non-locals (immigrants) with the following specification,

$$\log(A_{n,t+1} - A_{n,t}) = \tau + \beta_m \log \lambda_{nn,t} + \beta_i \log(immigration_{n,t}) + \beta_s \log A_{n,t} + \tau_n + \tau_t + \epsilon_{n,t},$$

where $\log(immigration_{n,t}) = \log\left[\sum_{i \neq n} s_{in,t} A_{i,t}\right]$ captures the weighted sum of the knowledge brought by migrants to n from locations other than n. In this specification we control for $\log A_{n,t}$, which is the measure of local knowledge. Therefore, the coefficient β_i captures the idea diffusion through non-locals (immigrants), and the coefficient β_s captures the knowledge diffusion through locals (stayers).

Table 5 reports the results. In Column (1), we start by showing the positive and significant coefficient of $\log A_{n,t}$, β_s, which reveals that idea diffusion through local knowledge contributes to growth in the local knowledge. Column (2) suggests that in addition to the local knowledge, higher knowledge growth tends to be seen in locations with more exposure to international trade. In Column (3), the positive and significant coefficient of the term $\log(immigration_{n,t})$, β_i, shows that after controlling for the local knowledge stock, the knowledge brought by non-local immigrants also contributes to the growth in the stock of

idea diffusion through people, and growth in the local stock of knowledge.

knowledge. Column (4) shows that international trade openness, ideas brought by migrants, and local knowledge stock all contribute to the growth in local knowledge stock, in line with the spatial mechanisms in our model. Alternatively, we could replace the dependent variable by the growth rate in the knowledge stock. In Column (5), the dependent variable is $\log\left(\frac{A_{n,t+1}-A_{n,t}}{A_{n,t}}\right)$, and the purpose of this alternative specification is to normalize the change in knowledge stock in each location by the local knowledge stock. Again, the results suggest that international trade and ideas diffused by non-locals contribute significantly to the growth in the stock of knowledge.

We have provided reduced-form evidence of the contribution of idea diffusion through both international trade and migration to the stock of local knowledge. In Appendix J we provide further evidence of idea diffusion by implementing an instrumental variable strategy to estimate the effect of idea diffusion on the local stock of knowledge. We also derive empirical specifications using the model's structure. Consistent with the reduced-form evidence presented in this section, we find a statistically significant contribution of idea diffusion through trade and migration to the local stock of knowledge.

Table 4: Estimates of knowledge diffusion through trade and migration

	$\log(A_{n,t+1}-A_{n,t})$				$\log\left(\frac{A_{n,t+1}-A_{n,t}}{A_{n,t}}\right)$
	(1)	(2)	(3)	(4)	(5)
$\log \lambda_{nn,t}$		-5.176**		-5.148**	-4.518*
		(2.409)		(2.397)	(2.590)
$\log(immigration_{n,t})$			0.133*	0.129*	0.170**
			(0.076)	(0.070)	(0.073)
$\log A_{n,t}$	0.655***	0.552***	0.703***	0.599***	
	(0.211)	(0.186)	(0.218)	(0.190)	
Constant	3.121*	3.802**	2.012	2.723	-0.817*
	(1.775)	(1.555)	(2.049)	(1.732)	(0.406)
Observations	90	90	90	90	90
R-squared	0.537	0.608	0.550	0.621	0.827
Year FE	✓	✓	✓	✓	✓
Province FE	✓	✓	✓	✓	✓

Note: Robust standard errors in parentheses.*** p<0.01, ** p<0.05, * p<0.1.

6 Concluding Remarks

In this paper we have developed a dynamic spatial growth model to study, understand, and quantify the role of spatial growth on aggregate economic activity. In our model, internal migration and trade provide the mechanics for spatial growth. Producers and migrants share ideas with other producers, and the flow of ideas across space and time serves as the main mechanism that generates spatial growth. We characterized the equilibrium properties of our model and showed that it has a unique balanced growth path. We also showed how to take the model to the data in order to perform quantitative exercises without assuming that the economy is initially in a balanced growth path.

As an application, we studied the importance of trade and migration as engines of growth for the Chinese economy after 1990. Initial conditions and our spatial mechanisms that operate through international trade and internal migration played an important role in the spatial development and aggregate growth during the 1990s and 2000s in China. The changes in fundamentals due to trade openness and migration restrictions also contributed to aggregate growth and heterogeneous spatial development in China.

Our study has developed a dynamic spatial growth framework that can be used to explore a wide range of questions related to spatial and aggregate growth. This opens up interesting avenues for research, such as the study of optimal policy. Our model assumes that firms and workers do not consider the impact of their insights on the stock of knowledge across locations, leading to externalities. In such a context, there is a role for trade policy to correct these externalities. Furthermore, our framework can be used to investigate the consequences of the increasing isolation observed in many countries on their dynamic spatial growth and inequality. It can also be applied to understand the growth patterns of cities and towns, and gain a better understanding of issues such as optimal infrastructure investment across different regions, and the aggregate growth implications of place-based policies. Also, in our study we have abstracted from a sectoral analysis, where insights from producers and people in the same industry might be better than those from other industries. Our framework is tractable enough to accommodate this extension.

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Appendix: Mechanics of Spatial Growth

The appendix includes detailed theoretical derivations and proofs, additional quantitative results, and detailed data descriptions described in the paper.

A Proofs and Derivations of Idea Diffusion

In this appendix, we derive the idea diffusion process with a generic source distribution of insights. We then endogenize the source distribution as a result of idea diffusion from migrants and sellers.

A.1 Law of Motion of the Stock of Knowledge

We start by providing a proof of Proposition 1 and then derive the law of motion with idea flows after specifying the external source of ideas. In this section we use uppercase letters for random variables, and lowercase letters for their realized values.

Proposition 1. *Under Assumption 1, between t and $t+1$, the probability that the best new idea has productivity no greater than q, $F_t^{best\ new}(q)$, is given by*

$$F_t^{best\ new}(q) = \exp\left(-\alpha_t q^{-\theta} \int_0^\infty x^{\rho\theta} dG_t(x)\right)$$

in the limiting case when $\bar{z} \to 0$.

Proof. For any new idea that arrives between time t and $t+1$, the probability at time t that its productivity is no greater than q is given by

$$\begin{aligned}
&F_t^{new}(q) \\
&= \Pr[ZQ'^\rho \le q] \\
&= \int_0^\infty \Pr\left[Z \le \frac{q}{Q'^\rho}\middle| q'\right] dG_t(q') \\
&= \int_0^{(q/\bar{z})^{1/\rho}} \Pr\left[Z \le \frac{q}{Q'^\rho}\middle| q'\right] dG_t(q') + \int_{(q/\bar{z})^{1/\rho}}^\infty \Pr\left[Z \le \frac{q}{Q'^\rho}\middle| q'\right] dG_t(q') \\
&= \int_0^{(q/\bar{z})^{1/\rho}} \Pr\left[Z \le \frac{q}{Q'^\rho}\middle| q'\right] dG_t(q'). \\
&= \int_0^{(q/\bar{z})^{1/\rho}} H\left(\frac{q}{q'^\rho}\right) dG_t(q'),
\end{aligned}$$

where the fourth equality follows from the fact that $\Pr\left[Z \leq \frac{q}{Q'^{\rho}} \middle| Q' > (q/\bar{z})^{1/\rho}\right] = \Pr\left[Z \leq \bar{z}\right] = 0$.

Using Assumption 1 a), on the functional form of $H(\cdot)$, we obtain

$$F_t^{new}(q) = \int_0^{(q/\bar{z})^{1/\rho}} \left[1 - \left(\frac{q/\bar{z}}{q'^{\rho}}\right)^{-\theta}\right] dG_t(q').$$

Note that in order to derive this expression, we do not need to specify the source distribution of the insights. Assumption 1 c) implies that between t and $t+1$, the probability that the best new idea has productivity no greater than q is given by

$$\begin{aligned}
&F_t^{best\ new}(q)\\
&= \Pr\left[\text{all new ideas are no greater than } q\right]\\
&= \sum_{s=0}^{\infty} \Pr[\#\text{ new ideas} = \text{s}] \cdot \Pr\left[\text{all new ideas are no greater than } q \mid \#\text{ new ideas} = \text{s}\right]\\
&= \sum_{s=0}^{\infty} \frac{(\alpha_t \bar{z}^{-\theta})^s e^{-(\alpha_t \bar{z}^{-\theta})}}{s!} \cdot F_t^{new}(q)^s\\
&= \underbrace{\sum_{s=0}^{\infty} \frac{[\alpha_t \bar{z}^{-\theta} F_t^{new}(q)]^s \cdot e^{-(\alpha_t \bar{z}^{-\theta}) F_t^{new}(q)}}{s!}}_{=1} \cdot e^{-(\alpha_t \bar{z}^{-\theta})(1 - F_t^{new}(q))},
\end{aligned}$$

and therefore we obtain that

$$F_t^{best\ new}(q) = e^{-(\alpha_t \bar{z}^{-\theta})(1 - F_t^{new}(q))}.$$

In order to characterize the probability distribution of the best new ideas, we hold α_t constant and investigate the limiting case where $\bar{z} \to 0$. We then have that

$$\begin{aligned}
\lim_{\bar{z}\to 0} \alpha_t \bar{z}^{-\theta}(1 - F_t^{new}(q)) &= \lim_{\bar{z}\to 0} \alpha_t \bar{z}^{-\theta} \left(1 - \int_0^{(q/\bar{z})^{1/\rho}} \left[1 - \left(\frac{q/\bar{z}}{q'^{\rho}}\right)^{-\theta}\right] dG_t(q')\right)\\
&= \lim_{\bar{z}\to 0} \alpha_t \bar{z}^{-\theta} \left(1 - G_t\left((\frac{q}{\bar{z}})^{\frac{1}{\rho}}\right) + \int_0^{(q/\bar{z})^{1/\rho}} \left[\left(\frac{q/\bar{z}}{q'^{\rho}}\right)^{-\theta}\right] dG_t(q')\right)\\
&= \alpha_t \lim_{\bar{z}\to 0} \bar{z}^{-\theta} \left[1 - G_t\left((\frac{q}{\bar{z}})^{\frac{1}{\rho}}\right)\right]\\
&+ \alpha_t \lim_{\bar{z}\to 0} \bar{z}^{-\theta} \int_0^{(q/\bar{z})^{1/\rho}} \left[\left(\frac{q/\bar{z}}{q'^{\rho}}\right)^{-\theta}\right] dG_t(q')\\
&= \alpha_t \lim_{\bar{z}\to 0} \bar{z}^{-\theta} \left[1 - G_t\left((\frac{q}{\bar{z}})^{\frac{1}{\rho}}\right)\right] + \alpha_t \int_0^{\infty} \left(\frac{q}{q'^{\rho}}\right)^{-\theta} dG_t(q'),
\end{aligned}$$

where the first term on the right-hand side is zero by Assumption 1 d). In the limiting case when $\bar{z} \to 0$, the expression is equal to the second term only, which is $-\alpha_t q^{-\theta} \int_0^\infty x^{\rho\theta} dG_t(x)$.

Henceforth, we assume $\bar{z} \to 0$ and focus on the limiting case. The best new idea then follows

$$F_t^{best\ new}(q) = \exp\left(-\alpha_t q^{-\theta} \int_0^\infty x^{\rho\theta} dG_t(x)\right).$$

The productivity of the economy depends on the frontier of knowledge, $F_t(q)$. The frontier of knowledge denotes the fraction of varieties whose best producer has productivity no greater than q. In a probabilistic sense, $F_t(q)$ is also the probability that the best productivity for a specific variety is no greater than q at time t.

Proposition 2. *Assume that the initial frontier of knowledge at time* 0 *follows a Fréchet distribution given by* $F_0(q) = \exp(-A_0 q^{-\theta})$.

Imposing this assumption, then it follows that $F_t(\cdot)$ *is Fréchet at any* t *given by*

$$\begin{aligned} F_t(q) &= \exp\left[-\left(A_0 + \sum_{\tau=0}^{t-1} \alpha_\tau \int_0^\infty x^{\rho\theta} dG_\tau(x)\right) q^{-\theta}\right] \\ &= \exp\left(-A_t q^{-\theta}\right), \end{aligned}$$

where the law of motion for the knowledge stock is given by

$$A_{t+1} = A_t + \alpha_t \int_0^\infty x^{\rho\theta} dG_t(x).$$

Proof. The frontier $F_t(q)$ changes from t to $t+1$ because some new ideas might have better productivity than the current best. At $t+1$, we then have

$$\begin{aligned} F_{t+1}(q) &= \Pr\left[\text{the best productivity is no greater than } q \text{ at } t+1\right] \\ &= \Pr\left[\text{the best productivity is no greater than } q \text{ at } t\right] \cdot \\ &\quad \Pr[\text{no new ideas greater than } q \text{ between } t \text{ and } t+1] \\ &= F_t(q) \cdot F_t^{best\ new}(q) \\ &= F_0(q) \cdot \prod_{\tau=0}^{t} F_\tau^{best\ new}(q), \end{aligned}$$

where the last line follows from iteration back to $t = 0$.

Assume that the initial distribution at time 0 follows a Fréchet distribution; namely,

$$F_0(q) = \exp(-A_0 q^{-\theta}).$$

Then it follows that $F_t(\cdot)$ is Fréchet at any t:

$$\begin{aligned} F_t(q) = \exp &\left[- \left(A_0 + \sum_{\tau=0}^{t-1} \alpha_\tau \int_0^\infty x^{\rho\theta} dG_\tau(x) \right) q^{-\theta} \right] \\ = \exp &\left(-A_t q^{-\theta} \right). \end{aligned}$$

It also follows that the law of motion of the knowledge stock is

$$A_{t+1} = A_t + \alpha_t \int_0^\infty x^{\rho\theta} dG_t(x).$$

As we can see from this equation, both the arrival rate of new ideas α_t and the learning pool $G_t(\cdot)$ matter for the evolution of A_t.

A.2 Migration and the Source Distribution of Insights

Assume that at time t in location n, when a new idea arrives, the insight from a randomly drawn person currently living in n is the insight component of the new idea. Then

$$\begin{aligned} G_{n,t}(q') &= \Pr[\text{the insight component is no greater than } q'] \\ &= \sum_{i=1}^{N} \Pr[\text{the person with the insight lives in } i \text{ at } t] \cdot \\ &\qquad \Pr[\text{the insight is no greater than } q' | \text{the person with the insight lives in } i \text{ at } t] \\ &= \sum_{i=1}^{N} s_{in,t} F_{i,t}(q'), \end{aligned}$$

where $s_{in,t}$ is the share of households from location i living in location n. In particular, we denote by $\mu_{in,t}$ the fraction of households that relocate from from i to n. We then have $s_{in,t} = \frac{\mu_{in,t} L_{i,t}}{\sum_{h=1}^{N} \mu_{hn,t} L_{h,t}}$ and

$$\int_0^\infty x^{\rho_\ell \theta} dG_t(x) = \Gamma(1-\rho_\ell) \sum_{i=1}^N s_{in,t}(A_{i,t})^{\rho_\ell}.$$

Finally, the law of motion of the stock of knowledge with ideas from people is given by

$$A_{n,t+1} - A_{n,t} = \alpha_{n,t}\Gamma(1-\rho_\ell) \sum_{i=1}^N s_{in,t}(A_{i,t})^{\rho_\ell}.$$

A.3 Derivation of the Law of Motion of Knowledge with Ideas from Migrants and Sellers

Now we derive the law of motion of the knowledge stock with idea flows from both trade and migration.

We impose the following version of Assumption 1 to incorporate both sources of idea flows:

Assumption 1'
a) The same as Assumption 1 a)
b) The strength of idea diffusion, $\rho_m + \rho_l \in [0,1)$, is strictly less than 1.
c) The same as Assumption 1 c)
d) The source distribution has a sufficiently thin tail such that for any monotonically decreasing sequence of $\bar{z}_n \to 0$, $\alpha_t \lim_{n\to\infty} \bar{z}_n^{-\theta} \left[1 - \int\int_{B(\bar{z}_n)} dG_t^l(q_\ell) dG_t^m(q_m)\right] = 0$, where $B(\bar{z}) := \{(x_1,x_2) : \bar{z} x_1^{\rho_l} x_2^{\rho_m} < q\} \subset \mathbb{R}^2$. In addition, the integral $\int\int \left(\frac{q}{q_\ell^{\rho_\ell} q_m^{\rho_m}}\right)^{-\theta} dG_t^l(q_\ell) dG_t^m(q_m)$ exists.

Proposition 1'. *Under Assumption 1', between t and $t+1$, the probability that the best new idea has productivity no greater than q, $F_t^{best\ new}(q)$, is given by*

$$F_t^{best\ new}(q) = \exp\left(-\alpha_t q^{-\theta} \int_0^\infty \int_0^\infty \left(q_\ell^{\rho_\ell} q_m^{\rho_m}\right)^\theta dG_t^l(q_\ell) dG_t^m(q_m)\right)$$

in the limiting case where $\bar{z} \to 0$.

Proof: For any new idea that arrives between time t and $t+1$, the probability at t that its productivity is no greater than q is given by

$$
\begin{aligned}
&F_t^{new}(q) \\
&= \Pr[ZQ_\ell^{\rho_\ell}Q_m^{\rho_m} \le q] \\
&= \int\int_{\mathbb{R}_+^2} \Pr\left[Z \le \frac{q}{Q_\ell^{\rho_\ell}Q_m^{\rho_m}}\middle| q_\ell, q_m\right] dG_t^l(q_\ell)dG_t^m(q_m) \\
&= \int\int_{B(\bar{z})} \Pr\left[Z \le \frac{q}{Q_\ell^{\rho_\ell}Q_m^{\rho_m}}\middle| q_\ell, q_m\right] dG_t^l(q_\ell)dG_t^m(q_m) \\
&+ \int\int_{\mathbb{R}_+^2\setminus B(\bar{z})} \Pr\left[Z \le \frac{q}{Q_\ell^{\rho_\ell}Q_m^{\rho_m}}\middle| q_\ell, q_m\right] dG_t^l(q_\ell)dG_t^m(q_m) \\
&= \int\int_{B(\bar{z})} \Pr\left[Z \le \frac{q}{Q_\ell^{\rho_\ell}Q_m^{\rho_m}}\middle| q_\ell, q_m\right] dG_t^l(q_\ell)dG_t^m(q_m),
\end{aligned}
$$

where $B(\bar{z})$ is defined in Assumption 1' d). Using Assumption 1' a), we obtain

$$F_t^{new}(q) = \int\int_{B(\bar{z})} \left[1 - \left(\frac{q/\bar{z}}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta}\right] dG_t^l(q_\ell)dG_t^m(q_m).$$

The probability that the best new idea has productivity no greater than q is the same as before: $F_t^{best\ new}(q) = e^{-(\alpha_t\bar{z}^{-\theta})(1-F_t^{new}(q))}$. Consider a monotonically decreasing sequence of $\bar{z}_n \to 0$. We prove by the dominated convergence theorem that $\lim_{n\to\infty} \alpha_t\bar{z}_n^{-\theta}(1-F_t^{new}(q)) = \alpha_t \int\int \left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta} dG_t^l(q_\ell)dG_t^m(q_m)$. The integral exists under Assumption 1' d).
Define $g_n : \mathbb{R}_+ \to \mathbb{R}$,

$$
\begin{aligned}
g_n(q) &= \bar{z}_n^{-\theta}(1 - F_t^{new}(q)) \\
&= \bar{z}_n^{-\theta}\left(1 - \int\int_{B(\bar{z}_n)} \left[1 - \left(\frac{q/\bar{z}}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta}\right] dG_t^l(q_\ell)dG_t^m(q_m)\right) \\
&= \bar{z}_n^{-\theta}\left[1 - \int\int_{B(\bar{z}_n)} dG_t^l(q_\ell)dG_t^m(q_m)\right] + \int\int_{B(\bar{z}_n)} \left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta} dG_t^l(q_\ell)dG_t^m(q_m) \\
&= \bar{z}_n^{-\theta}\left[1 - \int\int_{B(\bar{z}_n)} dG_t^l(q_\ell)dG_t^m(q_m)\right] + \int\int \left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta} \mathbf{1}_{B(\bar{z}_n)} dG_t^l(q_\ell)dG_t^m(q_m).
\end{aligned}
$$

By Assumption 1' d), we have $\lim_{n\to\infty} \bar{z}_n^{-\theta}\left[1 - \int_{B(\bar{z}_n)} dG_t^l(q_\ell)dG_t^m(q_m)\right] = 0$. Since $\forall q \ge 0$, $\forall n$,

$$\left|\left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta} \mathbf{1}_{B(\bar{z}_n)}\right| \le \left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta},$$

and

$$\lim_{n\to\infty}\left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta}\mathbf{1}_{B(\bar{z}_n)}=\left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta},$$

by the dominated convergence theorem, we have

$$\lim_{n\to\infty}\int\int\left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta}\mathbf{1}_{B(\bar{z}_n)}dG_t^l(q_\ell)dG_t^m(q_m)=\int\int\left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta}dG_t^l(q_\ell)dG_t^m(q_m),$$

so

$$\begin{aligned}\lim_{n\to\infty}g_n(q)&=\lim_{n\to\infty}\alpha_t\bar{z}_n^{-\theta}(1-F_t^{new}(q))\\&=\lim_{n\to\infty}\bar{z}_n^{-\theta}\left[1-\int\int_{B(\bar{z}_n)}dG_t^l(q_\ell)dG_t^m(q_m)\right]+\lim_{n\to\infty}\int\int\left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta}\mathbf{1}_{B(\bar{z}_n)}dG_t^l(q_\ell)dG_t^m(q_m)\\&=\int\int\left(\frac{q}{q_\ell^{\rho_\ell}q_m^{\rho_m}}\right)^{-\theta}dG_t^l(q_\ell)dG_t^m(q_m).\end{aligned}$$

Henceforth, we assume $\bar{z}\to 0$ and focus on the limiting case. The best new idea then follows

$$F_t^{best\ new}(q)=\exp\left(-\alpha_t q^{-\theta}\int\int\left(q_\ell^{\rho_\ell}q_m^{\rho_m}\right)^{\theta}dG_t^l(q_\ell)dG_t^m(q_m)\right)$$

or, using the Riemann integral,

$$F_t^{best\ new}(q)=\exp\left(-\alpha_t q^{-\theta}\int_0^\infty\int_0^\infty\left(q_\ell^{\rho_\ell}q_m^{\rho_m}\right)^{\theta}dG_t^l(q_\ell)dG_t^m(q_m)\right).$$

As in the previous section, in this section it follows that the frontier distribution $F_{n,t}(\cdot)$ follows a Fréchet distribution with location parameter $A_{n,t}$ and shape parameter θ, and the law of motion of $A_{n,t}$ is

$$A_{n,t+1}=A_{n,t}+\alpha_t\int_0^\infty\int_0^\infty\left(q_\ell^{\rho_\ell}q_m^{\rho_m}\right)^{\theta}dG_{n,t}^l(q_\ell)dG_{n,t}^m(q_m).$$

Then the law of motion becomes

$$A_{n,t+1}=A_{n,t}+\alpha_t\int_0^\infty q_\ell^{\theta\rho_\ell}dG_{n,t}^l(q_\ell)\int_0^\infty q_m^{\theta\rho_m}dG_{n,t}^m(q_m).$$

The first integral,

$$\int_0^\infty q_\ell^{\theta\rho_\ell}dG_{n,t}^l(q_\ell)=\Gamma(1-\rho_\ell)\sum_{i=1}^N s_{in,t}(A_{i,t})^{\rho_\ell},$$

is the same as in the previous section. The derivation of this term follows the previous section of this appendix. For the second integral, we assume learning from sellers as in Buera and Oberfield (2020). Namely, the insights from goods are randomly drawn from the set of goods sold locally. To simplify the notation, we omit intermediate goods in the derivation that follows. In this case,

$$\begin{aligned}
G^m_{n,t}(x) &= \sum_i \mathbb{P}\left[q_i \leq x,\ \text{i is the lowest-cost supplier to n at t}\right] \\
&= \sum_i \mathbb{P}\left[q_i \leq x, q_j \leq \frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}} q_i\ \forall j\right] \\
&= \sum_i \int_0^x f_{i,t}(q)\left(\prod_{j\neq i} F_{j,t}\left(\frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}} q\right)\right) dq,
\end{aligned}$$

where $F_{i,t}(\cdot)$ and $f_{i,t}(\cdot)$ are the cumulative distribution function (CDF) and probability density function (PDF) of a Fréchet distribution with location parameter $A_{i,t}$ and shape parameter θ, respectively:

$$\begin{aligned}
F_{i,t}(q) &= \exp\left(-A_{i,t} q^{-\theta}\right), \\
f_{i,t}(q) &= A_{i,t}\theta q^{-\theta-1}\exp\left(-A_{i,t} q^{-\theta}\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
G^m_{n,t}(x) &= \sum_i \int_0^x f_{i,t}(q)\left(\prod_{j\neq i} F_{j,t}\left(\frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}} q\right)\right) dq \\
&= \sum_i \int_0^x A_{i,t}\theta q^{-\theta-1}\exp\left(-A_{i,t}q^{-\theta}\right)\exp\left(-\sum_{j\neq i} A_{j,t}\left(\frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}}\right)^{-\theta} q^{-\theta}\right) dq \\
&= \sum_i \int_0^x A_{i,t}\theta q^{-\theta-1}\exp\left(-\sum_{j} A_{j,t}\left(\frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}}\right)^{-\theta} q^{-\theta}\right) dq \\
&= \sum_i \frac{A_{i,t}\left(w_{i,t}\kappa_{ni}\right)^{-\theta}}{\sum_j A_{j,t}\left(w_{j,t}\kappa_{nj}\right)^{-\theta}}\exp\left(-\sum_{j} A_{j,t}\left(\frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}}\right)^{-\theta} x^{-\theta}\right) \\
&= \sum_i \pi_{ni,t}\exp\left(-\sum_{j} A_{j,t}\left(\frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}}\right)^{-\theta} x^{-\theta}\right).
\end{aligned}$$

It follows that the second integral, which represents the learning from goods, is given by

$$
\begin{aligned}
\int_0^\infty q_m^{\theta\rho_m} dG_{n,t}^m(q_m) &= \int_0^\infty q_m^{\theta\rho_m} d\sum_i \lambda_{ni,t} \exp\left(-\sum_j A_{j,t}\left(\frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}}\right)^{-\theta} q_m^{-\theta}\right) \\
&= \sum_i \lambda_{ni,t} \int_0^\infty q_m^{\theta\rho_m} d\exp\left(-\sum_j A_{j,t}\left(\frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}}\right)^{-\theta} q_m^{-\theta}\right)
\end{aligned}
$$

Using change of variables, define $x = \sum_j A_{j,t}\left(\frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}}\right)^{-\theta} q_m^{-\theta}$, and we have

$$
\begin{aligned}
\int_0^\infty q_m^{\theta\rho_m} dG_{n,t}^m(q_m) &= \sum_i \lambda_{ni,t} \int_0^\infty \sum_j A_{j,t}^{\rho_m}\left(\frac{w_{j,t}\kappa_{nj,t}}{w_{i,t}\kappa_{ni,t}}\right)^{-\theta\rho_m} x^{-\rho_m} d\exp(-x) \\
&= \Gamma(1-\rho_m)\sum_i \lambda_{ni,t}\left(\frac{A_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}.
\end{aligned}
$$

Therefore, the law of motion of $A_{n,t}$ is given by

$$
A_{n,t+1} - A_{n,t} = \alpha_t \Gamma(1-\rho_\ell)\Gamma(1-\rho_m)\left[\sum_{i=1}^N s_{in,t}(A_{i,t})^{\rho_\ell}\right]\left[\sum_{i=1}^N \lambda_{ni,t}\left(\frac{A_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}\right].
$$

B Additional Derivations

In this section, we provide detailed derivations of the trade shares, migration shares, and the solution to landowner consumption and investment decisions.

B.1 Derivation of the Trade Shares

Let Ω be the variety space and intermediate variety $\omega \in \Omega$. Let $p_{in,t}(\omega)$ be the price that firms in location i pay for good ω purchased from location n at time t. Then perfect competition implies

$$
p_{in,t}(\omega) = \frac{\kappa_{in,t} x_{n,t}}{q(\omega)},
$$

where $x_{n.t}$ is the unit cost of inputs to produce in location n. Since $\{q(\omega)\}_{\omega\in\Omega}$ are i.i.d., for all $\omega \in \Omega$, they have the same distribution. Let $H_{in,t}(p)$ be the cumulative distribution of prices, i.e., $H_{in,t}(p) = \mathbb{P}\left[p_{in,t}(\omega) \leq p\right]$. Then

$$
\begin{aligned}
H_{in,t}(p) &= \mathbb{P}[p_{in,t}(\omega) \leq p] \\
&= \mathbb{P}\left[\frac{\kappa_{in,t} x_{n,t}}{q(\omega)} \leq p\right] \\
&= \mathbb{P}\left[q(\omega) \geq \frac{\kappa_{in,t} x_{n,t}}{p}\right] \\
&= 1 - \mathbb{P}\left[q(\omega) \leq \frac{\kappa_{in} x_{n,t}}{p}\right] \\
&= 1 - F_{n,t}\left(\frac{\kappa_{in,t} x_{n,t}}{p}\right) \\
&= 1 - \exp\left\{-A_{n,t}\left(\frac{\kappa_{in,t} x_{n,t}}{p}\right)^{-\theta}\right\},
\end{aligned}
\tag{B.1}
$$

where $F_{n,t}(\cdot)$ denotes the Fréchet distribution with scale parameter $A_{n,t}$ and shape parameter θ.

Let $\lambda_{in,t}$ be the fraction of goods purchased by location i from n. For location i to buy good ω from n, n must be the lowest-cost supplier among all locations. By the law of large numbers, we have

$$
\begin{aligned}
\lambda_{in,t} &= \mathbb{P}\left[p_{in,t}(\omega) \leq \min_{h \in S \setminus \{i\}} p_{ih,t}(\omega)\right] \\
&= \int_0^\infty \mathbb{P}\left[\min_{h \in S \setminus \{i\}} p_{ih,t}(\omega) \geq p\right] dH_{in,t}(p) \\
&= \int_0^\infty \mathbb{P}\left[\bigcap_{h \in S \setminus \{i\}} \{p_{ih,t}(\omega) \geq p\}\right] dH_{in,t}(p) \\
&= \int_0^\infty \prod_{h \in S \setminus \{i\}} \mathbb{P}\left[p_{ih,t}(\omega) \geq p\right] dH_{in,t}(p) \\
&= \int_0^\infty \prod_{h \in S \setminus \{i\}} \left[1 - H_{ih,t}(p)\right] dH_{in,t}(p),
\end{aligned}
$$

where the law of iterated expectation is used for the second equality and independence is used for the fourth equality.

Using the expression of price distribution derived in (B.1), we have

$$\lambda_{in,t} = \int_0^\infty \prod_{h \in S \setminus \{i\}} \exp\left\{-A_{h,t}\left(\frac{\kappa_{ih,t}x_{h,t}}{p}\right)^{-\theta}\right\} \exp\left\{-A_{n,t}\left(\frac{\kappa_{in,t}x_{n,t}}{p}\right)^{-\theta}\right\} A_{n,t}(\kappa_{in}x_{in,t})^{-\theta} dp^\theta$$

$$= A_{n,t}(\kappa_{in,t}x_{n,t})^{-\theta} \int_0^\infty \exp\left\{-\sum_{h=1}^N A_{h,t}(\kappa_{ih,t}x_{h,t})^{-\theta} p^\theta\right\} dp^\theta$$

$$= \frac{A_{n,t}(\kappa_{in,t}x_{n,t})^{-\theta}}{\sum_{h=1}^N A_{h,t}(\kappa_{ih,t}x_{h,t})^{-\theta}}.$$

B.2 Derivation of Gross Flows Equation

Let $\mu_{in,t}$ be the fraction of individuals who relocate from location i to location n at time t. By definition, we have

$$\mu_{in,t} = \mathbb{P}\left[\frac{\beta V_{n,t+1} - m_{in,t}}{\nu} + \epsilon_{n,t} \geq \max_{l \neq n}\left\{\frac{\beta V_{l,t+1} - m_{il,t}}{\nu} + \epsilon_{l,t}\right\}\right]$$

$$= \int_\infty^\infty \mathbb{P}\left[\frac{\beta V_{n,t+1} - m_{in,t}}{\nu} + x \geq \max_{l \neq n}\left\{\frac{\beta V_{l,t+1} - m_{il,t}}{\nu} + \epsilon_{l,t}\right\}\right] dM(x)$$

$$= \int_{-\infty}^\infty \mathbb{P}\left[\bigcap_{l \neq n}\left\{\frac{\beta V_{n,t+1} - m_{in,t}}{\nu} + x \geq \frac{\beta V_{l,t+1} - m_{il,t}}{\nu} + \epsilon_{l,t}\right\}\right] dM(x)$$

$$= \int_{-\infty}^\infty \prod_{l \neq n} \mathbb{P}\left[\frac{\beta V_{n,t+1} - m_{in,t}}{\nu} + x \geq \frac{\beta V_{l,t+1} - m_{il,t}}{\nu} + \epsilon_{l,t}\right] dM(x)$$

$$= \int_{-\infty}^\infty \prod_{l \neq n} \mathbb{P}\left[\epsilon_{l,t} \leq \frac{\beta(V_{n,t+1} - V_{l,t+1}) - (m_{in,t} - m_{il,t})}{\nu} + x\right] dM(x)$$

$$= \int_{-\infty}^\infty \prod_{l \neq n} M\left(\frac{\beta(V_{n,t+1} - V_{l,t+1}) - (m_{in,t} - m_{il,t})}{\nu} + x\right) dM(x),$$

where $M(\cdot)$ denotes the cumulative distribution function of a Gumbel Type I distribution.

Define $\bar{\epsilon}_{ln,t} = \frac{\beta(V_{n,t+1} - V_{l,t+1}) - (m_{in,t} - m_{il,t})}{\nu}$ with $\bar{\epsilon}_{nn,t} = 0$. Using this notation and the expression of $M(\cdot)$, we have

$$\mu_{in,t} = \int_{-\infty}^\infty \exp\{-e^{-x-\gamma}\} e^{-x-\gamma} \exp\{-e^{-x-\gamma} \sum_{l \neq n} e^{-\bar{\epsilon}_{ln,t}}\} dx$$

$$= \int_{-\infty}^\infty e^{-x-\gamma} \exp\{-e^{-x-\gamma} \sum_{l=1}^N e^{-\bar{\epsilon}_{ln,t}}\} dx.$$

Define $\Xi_{in} = \log(\sum_{l=1}^{N} e^{-\bar{\epsilon}_{ln,t}})$ and $y = x + \gamma - \Xi_{in}$. Then

$$\begin{aligned}\mu_{in,t} &= \int_{-\infty}^{\infty} e^{-y-\Xi_{in}} e^{-e^{-y}} dy \\ &= e^{\Xi_{in}}.\end{aligned}$$

Finally, plugging in the expression of Ξ_{in}, we have

$$\begin{aligned}\mu_{in,t} &= \frac{1}{\sum_{l=1}^{N} \exp\left\{\frac{\beta(V_{l,t+1}-V_{n,t+1})-m_{il,t}+m_{in,t}}{\nu}\right\}} \\ &= \frac{\exp(\beta V_{n,t+1} - m_{in,t})^{\frac{1}{\nu}}}{\sum_{l=1}^{N} \exp(\beta V_{l,t+1} - m_{il,t})^{\frac{1}{\nu}}}.\end{aligned}$$

B.3 Landlord's Problem

The landlord's problem is defined as

$$\begin{aligned}\max_{\{C_{i,t},K_{i,t+1}\}_{t=0}^{\infty}} U &= \sum_{t=0}^{\infty} \beta^t \log(C_{i,t}), \\ \text{s.t. } r_{i,t}K_{i,t} &= P_{i,t}\left[C_{i,t} + K_{i,t+1} - (1-\delta)K_{i,t}\right] \text{ all } t,\end{aligned}$$

where δ is the depreciation rate and $K_{i,0}$ is taken as given. Set up the Lagrangian equation,

$$\mathcal{L} = \left[\beta^t\{\log(C_{i,t}) + \lambda_t[r_{i,t}K_{i,t} - P_{i,t}(C_{i,t} + K_{i,t+1} - (1-\delta)K_{i,t})]\}\right], \tag{B.2}$$

where λ_t is the Lagrangian multiplier for the constraint in period t.

The first-order conditions for the problem are

$$\begin{aligned}\frac{1}{C_{i,t}} &= \lambda_t P_{i,t} \\ \lambda_t P_{i,t} &= \beta\left[\lambda_{t+1}[r_{i,t+1} + P_{i,t+1}(1-\delta)]\right].\end{aligned}$$

Define $R_{i,t} = 1 - \delta + \frac{r_{i,t}}{P_{i,t}}$. Then eliminating λ_t yields the Euler equation,

$$\frac{1}{C_{i,t}} = \beta\left[R_{i,t+1}\frac{1}{C_{i,t+1}}\right], \tag{B.3}$$

together with the budget constraint

$$R_{i,t}K_{i,t} = C_{i,t} + K_{i,t+1}. \tag{B.4}$$

To solve this problem, we use the guess-and-verify strategy. We guess that $C_{i,t} = \varsigma R_{i,t}K_{i,t}$, where ς is a constant to be determined. Plugging in (B.4), we have

$$K_{i,t+1} = (1-\varsigma)R_{i,t}K_{i,t}. \tag{B.5}$$

Combining equations (B.5) and (B.3), we have

$$\frac{1}{\varsigma R_{i,t}K_{i,t}} = \beta \left[R_{i,t+1} \frac{1}{\varsigma R_{i,t+1}(1-\varsigma)R_{i,t}K_{i,t}} \right].$$

The undetermined coefficient method implies that $\varsigma = 1-\beta$. Hence, the consumption and saving policy functions are as follows:

$$\begin{aligned} C_{i,t} &= (1-\beta)[r_{i,t}/P_{i,t} + (1-\delta)]K_{i,t}, \\ K_{i,t+1} &= \beta[r_{i,t}/P_{i,t} + (1-\delta)]K_{i,t}. \end{aligned}$$

C Detrended Model and Balanced Growth Path

In this appendix we characterize the long-run growth rates of the equilibrium variables of the model at the balanced growth path. In what follows, we denote the long-run growth rate of any variable y_t by $(1+g_y)$, and we also refer to a variable with a "~" as a detrended variable. In particular, $\tilde{y}_t = y_t/\left(1+g_y\right)^t$.

The equilibrium conditions of the detrended model are given by

$$\tilde{V}_{i,t} = \beta log\left(1+g_v\right) + log\left(\frac{\tilde{w}_{i,t}}{\tilde{P}_{i,t}}\right) + \nu log\left(\sum_{n=1}^{N} exp\left(\beta \tilde{V}_{n,t+1} - m_{in,t}\right)^{1/\nu}\right), \tag{C.1}$$

$$\tilde{P}_{i,t} = T\left(\sum_{n=1}^{N} \tilde{A}_{n,t}\left(\kappa_{in,t}\tilde{x}_{n,t}\right)^{-\theta}\right)^{-1/\theta}, \tag{C.2}$$

$$\tilde{w}_{i,t}L_{i,t} = \sum_{n=1}^{N} \tilde{A}_{i,t}\left(\frac{\kappa_{ni,t}\tilde{x}_{i,t}}{\tilde{P}_{n,t}/T}\right)^{-\theta} \tilde{w}_{n,t}L_{n,t}, \tag{C.3}$$

$$\tilde{r}_{i,t}\tilde{K}_{i,t} = \sum_{n=1}^{N} \tilde{A}_{i,t} \left(\frac{\kappa_{ni,t}\tilde{x}_{i,t}}{\tilde{P}_{n,t}/T} \right)^{-\theta} \tilde{r}_{n,t}\tilde{K}_{n,t}, \tag{C.4}$$

$$L_{i,t+1} = \sum_{n=1}^{N} \mu_{ni,t} L_{n,t}, \tag{C.5}$$

$$\tilde{K}_{i,t+1} = \frac{\beta}{(1+g_k)} \left(\tilde{r}_{i,t}/\tilde{P}_{i,t} + (1-\delta) \right) \tilde{K}_{i,t}, \tag{C.6}$$

$$\tilde{A}_{n,t+1} - \frac{\tilde{A}_{n,t}}{(1+g_A)} = \frac{\alpha_0 \Gamma_{\rho_\ell,\rho_m}}{(1+g_A)} \sum_{i=1}^{N} s_{in,t} \left(\tilde{A}_{i,t} \right)^{\rho_l} \sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{\tilde{A}_{i,t}}{\lambda_{ni,t}} \right)^{\rho_m}, \tag{C.7}$$

where we note that since there is no population growth, employment does not have a long-run growth rate; namely, $\tilde{L}_{n,t} = L_{n,t}$. Since values grow at the same rate in the long run, it follows that $\tilde{\mu}_{ni,t} = \mu_{ni,t}$, as we show below.

We start with the evolution of the stock of knowledge. At the balanced growth path, $A_{n,t}$ for all n grow at a rate $1+g_A$. From the law of motion of the stock of knowledge, we have

$$A_{n,t+1} - A_{n,t} = \alpha_t \Gamma_\rho \sum_{i=1}^{N} s_{in,t} \left(A_{i,t} \right)^{\rho_l} \sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}} \right)^{\rho_m},$$

using Assumption 2 and after detrending the variables, we obtain

$$\begin{aligned} &\tilde{A}_{n,t+1} \left(1+g_A\right)^{t+1} - \tilde{A}_{n,t} \left(1+g_A\right)^{t} \\ &= \alpha_0 \left(1+g_\alpha\right)^t \Gamma_\rho \sum_{i=1}^{N} s_{in,t} \left(\tilde{A}_{i,t} \left(1+g_A\right)^t \right)^{\rho_l} \sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{\tilde{A}_{i,t} \left(1+g_A\right)^t}{\lambda_{ni,t}} \right)^{\rho_m} \end{aligned}$$

or

$$\tilde{A}_{n,t+1} \left(1+g_A\right) - \tilde{A}_{n,t} = \left(1+g_\alpha\right)^t \left(1+g_A\right)^{t(\rho_l+\rho_m-1)} \alpha_0 \Gamma_\rho \sum_{i=1}^{N} s_{in,t} \left(\tilde{A}_{i,t} \right)^{\rho_l} \sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{\tilde{A}_{i,t}}{\lambda_{ni,t}} \right)^{\rho_m},$$

which then implies that the long-run growth rate of the stock of knowledge is related to the growth rate of the arrival of ideas in the following way:

$$1+g_A = \left(1+g_\alpha\right)^{\frac{1}{(1-\rho_l-\rho_m)}}.$$

As a result, the detrended equilibrium evolution of the local stock of knowledge evolves according to

$$\tilde{A}_{n,t+1} - \frac{\tilde{A}_{n,t}}{(1+g_A)} = \frac{\tilde{\alpha}_0 \Gamma_\rho}{(1+g_A)} \sum_{i=1}^{N} s_{in,t} \left(\tilde{A}_{i,t}\right)^{\rho_l} \sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{\tilde{A}_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}$$

or

$$\frac{\tilde{A}_{n,t+1}}{\tilde{A}_{n,t}} = \frac{1}{1+g_A} + \frac{\alpha_0 \Gamma_\rho}{(1+g_A)\,\bar{A}_{n,t}} \sum_{i=1}^{N} s_{in,t} \left(\tilde{A}_{i,t}\right)^{\rho_l} \sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{\tilde{A}_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m} .$$

We now consider the detrended value functions of the workers. Let $e^{V_{i,t}} = e^{\tilde{V}_{i,t}} (1+g_v)^t$. We then have

$$\tilde{V}_{i,t} + log\,(1+g_v)^t = log\left(\frac{\tilde{w}_{i,t}}{\tilde{P}_{i,t}}\left(1+g_{w/p}\right)^t\right) + \nu log\left(\sum_{n=1}^{N} exp\left(\beta\tilde{V}_{n,t+1} + \beta log\,(1+g_v)^{t+1} - m_{in,t}\right)^{1/\nu}\right), \tag{C.8}$$

where $g_{w/p}$ is the growth rate of $\tilde{w}_{i,t}/\tilde{P}_{i,t}$ at the balanced growth path. It follows that

$$\tilde{V}_{i,t} + log\,(1+g_v)^t = log\left(\frac{\tilde{w}_{i,t}}{\tilde{P}_{i,t}}\right) + log\left(1+g_{w/p}\right)^t + log\,(1+g_v)^{\beta(t+1)} + \nu log\left(\sum_{n=1}^{N} exp\left(\beta\tilde{V}_{n,t+1} - m_{in,t}\right)^{1/\nu}\right),$$

which immediately implies that

$$(1+g_v)^{(1-\beta)t} = \left(1+g_{w/p}\right)^t ,$$

$$1+g_v = \left(1+g_{w/p}\right)^{\frac{1}{(1-\beta)}} .$$

Hence, the detrended equilibrium values become

$$\tilde{V}_{i,t} = log\left(\tilde{w}_{i,t}/\tilde{P}_{i,t}\right) + log\,(1+g_v)^\beta + \nu log\left(\sum_{n=1}^{N} exp\left(\beta\tilde{V}_{n,t+1} - m_{in,t}\right)^{1/\nu}\right).$$

This result immediately implies that $\mu_{in,t}$ is not growing in the long run since

$$\mu_{in,t} = \frac{exp\left(\beta V_{n,t+1} - m_{in,t}\right)^{1/\nu}}{\sum_{l=1}^{N} exp\left(\beta V_{l,t+1} - m_{il,t}\right)^{1/\nu}} = \frac{exp\left(\beta\tilde{V}_{n,t+1} - m_{in,t}\right)^{1/\nu}}{\sum_{l=1}^{N} exp\left(\beta\tilde{V}_{l,t+1} - m_{il,t}\right)^{1/\nu}}.$$

It also implies that $L_{i,t}$ does not have long-run growth since

$$\begin{aligned}
L_{i,t+1} &= \sum_{n=1}^{N} \mu_{ni,t} L_{n,t}, \\
&= \sum_{n=1}^{N} \frac{exp\left(\beta V_{i,t+1} - m_{ni,t}\right)^{1/\nu}}{\sum_{l=1}^{N} exp\left(\beta V_{l,t+1} - m_{nl,t}\right)^{1/\nu}} L_{n,t} \\
&= \sum_{n=1}^{N} \frac{exp\left(\beta \tilde{V}_{i,t+1} - m_{ni,t}\right)^{1/\nu}}{\sum_{l=1}^{N} exp\left(\beta \tilde{V}_{l,t+1} - m_{nl,t}\right)^{1/\nu}} L_{n,t}.
\end{aligned}$$

Let us now consider the labor market clearing condition,

$$w_{i,t} L_{i,t} = \sum_{n=1}^{N} A_{i,t} \left(\frac{\kappa_{ni,t} x_{i,t}}{P_{n,t}/T}\right)^{-\theta} w_{n,t} L_{n,t}.$$

First note that

$$\begin{aligned}
x_{i,t} = \tilde{x}_{i,t} \left(1+g_x\right)^t &= B \left(\left(\frac{\tilde{w}_{i,t}}{\tilde{P}_{i,t}} \left(1+g_{w/p}\right)^t \right)^{\xi} \left(\frac{\tilde{r}_{i,t}}{\tilde{P}_{i,t}} \left(1+g_{r/p}\right)^t \right)^{1-\xi} \right)^{\gamma} \tilde{P}_{i,t} \left(1+g_p\right)^t \\
&= \tilde{x}_{i,t} \left(1+g_{w/p}\right)^{t\xi\gamma} \left(1+g_{r/p}\right)^{t(1-\xi)\gamma} \left(1+g_p\right)^t .
\end{aligned}$$

Using this expression, we express the labor market clearing condition in a detrended form as

$$\begin{aligned}
&\tilde{w}_{i,t} \left(1+g_w\right)^t L_{i,t} \\
&= \sum_{n=1}^{N} \tilde{A}_{i,t} \left(1+g_A\right)^t \left(\frac{\kappa_{ni,t} \tilde{x}_{i,t} \left(1+g_{w/p}\right)^{t\xi\gamma} \left(1+g_{r/p}\right)^{t(1-\xi)\gamma} \left(1+g_p\right)^t}{\tilde{P}_{n,t} \left(1+g_p\right)^t / T} \right)^{-\theta} \tilde{w}_{n,t} \left(1+g_w\right)^t L_{n,t},
\end{aligned}$$

where we use the fact that $L_{i,t}$ does not grow in the long run. It follows that

$$1 = \left(1+g_A\right)^t \left(\left(1+g_{w/p}\right)^{t\xi\gamma} \left(1+g_{r/p}\right)^{t(1-\xi)\gamma} \right)^{-\theta},$$

$$\left(1+g_{w/p}\right)^{\theta\xi\gamma} \left(1+g_{r/p}\right)^{\theta(1-\xi)\gamma} = \left(1+g_A\right). \tag{C.9}$$

We follow the same steps for the capital accumulation equation. Then the detrended labor and capital market equilibrium conditions become

$$\tilde{w}_{i,t} L_{i,t} = \sum_{n=1}^{N} \tilde{A}_{i,t} \left(\frac{\kappa_{ni,t} \tilde{x}_{i,t}}{\tilde{P}_{n,t}/T} \right)^{-\theta} \tilde{w}_{n,t} L_{n,t},$$

$$\tilde{r}_{i,t} \tilde{K}_{i,t} = \sum_{n=1}^{N} \tilde{A}_{i,t} \left(\frac{\kappa_{ni,t} \tilde{x}_{i,t}}{\tilde{P}_{n,t}/T} \right)^{-\theta} \tilde{r}_{n,t} \tilde{K}_{n,t},$$

where $\tilde{K}_{n,t}$ is the detrended value of capital that we subsequently characterize.

We now detrend the price index equilibrium condition,

$$P_{i,t} = T \left(\sum_{n=1}^{N} A_{n,t} \left(\kappa_{in,t} x_{n,t} \right)^{-\theta} \right)^{-1/\theta},$$

which once detrended can be expressed as

$$\tilde{P}_{i,t} \left(1+g_p\right)^t = T \left(\sum_{n=1}^{N} \tilde{A}_{n,t} \left(1+g_A\right)^t \left(\kappa_{in,t} \tilde{x}_{n,t} \left(1+g_{w/p}\right)^{t\xi\gamma} \left(1+g_{r/p}\right)^{t(1-\xi)\gamma} \left(1+g_p\right)^t \right)^{-\theta} \right)^{-1/\theta}.$$

Using equation (C.9), we obtain the detrended equilibrium condition for the price index:

$$\tilde{P}_{i,t} = T \left(\sum_{n=1}^{N} \tilde{A}_{n,t} \left(\kappa_{in,t} \tilde{x}_{n,t} \right)^{-\theta} \right)^{-1/\theta}.$$

Now note that since in equilibrium we have that

$$w_{i,t} L_{i,t} = \frac{\xi}{1-\xi} r_{i,t} K_{i,t},$$

then

$$\frac{\tilde{w}_{i,t}}{\tilde{P}_{i,t}} \left(1+g_{w/p}\right)^t L_{i,t} = \frac{\xi}{1-\xi} \frac{\tilde{r}_{i,t}}{\tilde{P}_{i,t}} \left(1+g_{r/p}\right)^t \tilde{K}_{i,t} \left(1+g_k\right)^t,$$

which immediately implies that

$$1+g_{w/p} = \left(1+g_{r/p}\right)\left(1+g_k\right). \tag{C.10}$$

We now detrend the law of motion of capital accumulation,

$$K_{i,t+1} = \beta \left(r_{i,t}/P_{i,t} + (1-\delta) \right) K_{i,t},$$

which can be written as

$$\tilde{K}_{i,t+1}\left(1+g_k\right)^{t+1}=\beta\left(\left(1+g_{r/p}\right)^t\frac{\tilde{r}_{i,t}}{\tilde{P}_{i,t}}+(1-\delta)\right)\bar{K}_{i,t}\left(1+g_k\right)^t$$

or

$$\tilde{K}_{i,t+1}=\frac{\beta}{(1+g_k)}\left(\left(1+g_{r/p}\right)^t\frac{\tilde{r}_{i,t}}{\tilde{P}_{i,t}}+(1-\delta)\right)\tilde{K}_{i,t}.$$

We then require that

$$g_{r/p}=0,\ \Rightarrow,\ g_r=g_p,$$

and in this way, the detrended capital accumulation equation becomes

$$\tilde{K}_{i,t+1}=\frac{\beta}{(1+g_k)}\left(\frac{\tilde{r}_{i,t}}{\tilde{P}_{i,t}}+(1-\delta)\right)\tilde{K}_{i,t}.$$

From equation (C.9) we obtain that

$$1+g_{w/p}=(1+g_A)^{\frac{1}{\theta\xi\gamma}},$$

and from equation (C.9) we also obtain that

$$1+g_k=(1+g_A)^{\frac{1}{\theta\xi\gamma}}.$$

D Existence and Uniqueness

Proposition 3. Existence and Uniqueness. *Given the parameters and the elasticities* $(\rho_\ell,\rho_m,\theta,\nu,\gamma,\xi,\beta)$*, and fundamentals* $\{\alpha_0,\bar{\kappa}_{in},\bar{m}_{in}\}_{i=1,n=1}^{N,N}$*, there exists a unique (up to scale) solution given by* $\{\bar{w}_i,\bar{r}_i,\bar{L}_i,\bar{K}_i,\bar{V}_i,\bar{A}_i\}_{i=1}^{N}$ *that satisfies the equilibrium conditions of the detrended model at the balanced growth path.*

Proof. At the balanced growth path none of the detrended variables are growing, and as a result, the equilibrium variables of the detrended model reach a steady state. Hence, at the balanced growth path, $\tilde{y}_{t+1}=\tilde{y}_t=\bar{y}$, and it remains constant for all t. We use an upper bar to express the detrended equilibrium variables at the balanced growth path.

From the first-order condition of the firm's problem, we have that $\bar{w}_i\bar{L}_i=\frac{\xi}{1-\xi}\bar{r}_i\bar{K}_i$. It follows that $\bar{r}_i=\bar{w}_i\frac{\bar{L}_i(1-\xi)}{\bar{K}_i\xi}$ in the balanced growth path of the detrended model. Hence, we have that

$$\frac{\bar{K}_i}{\bar{L}_i} = \frac{\bar{w}_i}{\bar{P}_i} \frac{(1-\xi)}{\xi} \frac{\beta}{(1+g_k) - \beta(1-\delta)}.$$

Now we use that $\bar{x}_i = B\left(\bar{w}_i^{\xi} \bar{r}_i^{1-\xi}\right)^{\gamma} \bar{P}_i^{1-\gamma}$ to obtain that

$$\begin{aligned}
\bar{x}_i &= B\left(\bar{w}_i^{\xi} \bar{r}_i^{1-\xi}\right)^{\gamma} \bar{P}_i^{1-\gamma} = B\left(\bar{w}_i^{\xi}\left(\bar{w}_i \frac{\bar{L}_i(1-\xi)}{\bar{K}_i \xi}\right)^{1-\xi}\right)^{\gamma} \bar{P}_i^{1-\gamma} \\
&= B\left(\frac{(1-\xi)}{\xi}\right)^{(1-\xi)\gamma} (\bar{w}_i)^{\gamma} \left(\frac{\bar{L}_i}{\bar{K}_i}\right)^{(1-\xi)\gamma} \bar{P}_i^{1-\gamma} \\
&= B\left(\frac{(1-\xi)}{\xi}\right)^{(1-\xi)\gamma} \left(\frac{((1+g_k) - \beta(1-\delta))\xi}{(1-\xi)\beta}\right)^{(1-\xi)\gamma} \left(\frac{\bar{P}_i}{\bar{w}_i}\right)^{-\xi\gamma} \bar{P}_i \\
&= \Psi (\bar{w}_i)^{\xi\gamma} \left(\bar{P}_i\right)^{1-\xi\gamma},
\end{aligned}$$

where $\Psi = B\left(\frac{(1-\xi)}{\xi}\right)^{(1-\xi)\gamma} \left(\frac{((1+g_k)-\beta(1-\delta))\xi}{(1-\xi)\beta}\right)^{(1-\xi)\gamma}$.

Hence, we can rewrite the labor market clearing condition as

$$\begin{aligned}
\bar{w}_i \bar{L}_i &= \sum_{n=1}^{N} \bar{A}_i \left(\frac{\bar{\kappa}_{ni} \bar{x}_i}{\bar{P}_n / T}\right)^{-\theta} \bar{w}_n \bar{L}_n \\
&= \sum_{n=1}^{N} \bar{A}_i \left(\frac{\bar{\kappa}_{ni} \Psi (\bar{w}_i)^{\xi\gamma} \left(\bar{P}_i\right)^{1-\xi\gamma}}{\bar{P}_n / T}\right)^{-\theta} \bar{w}_n \bar{L}_n
\end{aligned}$$

or

$$(\bar{w}_i)^{1+\xi\gamma\theta} \bar{L}_i \left(\bar{P}_i\right)^{\theta(1-\xi\gamma)} \left(\bar{A}_i\right)^{-1} = \sum_{n=1}^{N} (\bar{\kappa}_{ni} \Psi T)^{-\theta} \left(\bar{P}_n\right)^{\theta} \bar{w}_n \bar{L}_n.$$

Analogously, the price index can be written as

$$\begin{aligned}
\bar{P}_i^{-\theta} &= T^{-\theta} \sum_{n=1}^{N} \bar{A}_n (\bar{\kappa}_{in} \bar{x}_n)^{-\theta} \\
&= T^{-\theta} \sum_{n=1}^{N} \bar{A}_n \bar{\kappa}_{in}^{-\theta} \Psi^{-\theta} (\bar{w}_n)^{-\xi\gamma\theta} \left(\bar{P}_n\right)^{-\theta(1-\xi\gamma)}.
\end{aligned}$$

Turning to the value functions, we use the following change of variables:

$$\widetilde{m}_{in} \equiv exp\left(\bar{m}_{in}\right)^{-1/\nu},$$
$$\bar{\phi}_i = \sum_{n=1}^{N} exp\left(\beta \bar{V}_n - \bar{m}_{in}\right)^{1/\nu}.$$

Using these conditions, we express

$$\exp\left(\frac{\beta}{\nu}\bar{V}_i\right) = \left(\zeta \bar{w}_i/\bar{P}_i\right)^{\frac{\beta}{\nu}} \left(\sum_{n=1}^{N} \widetilde{m}_{in} \exp\left(\frac{\beta}{\nu}\bar{V}_n\right)\right)^{\beta},$$

and using the definition of $\bar{\phi}_i$ we have that

$$\exp\left(\frac{\beta}{\nu}\bar{V}_i\right) = \left(\zeta \bar{w}_i/\bar{P}_i\right)^{\frac{\beta}{\nu}} \bar{\phi}_i^{\beta},$$

with $\zeta = (1+g_v)^{\beta}$. Hence,

$$\bar{\phi}_i = \sum_{n=1}^{N} \widetilde{m}_{in} \left(\zeta \bar{w}_n/\bar{P}_n\right)^{\frac{\beta}{\nu}} \bar{\phi}_n^{\beta}.$$

We also re-express the gross flows equation as

$$\bar{\mu}_{ni} = \frac{exp\left(\beta \bar{V}_i - \bar{m}_{ni}\right)^{1/\nu}}{\sum_{h=1}^{N} exp\left(\beta \bar{V}_h - \bar{m}_{nh}\right)^{1/\nu}} = \frac{\widetilde{m}_{ni}\left(\zeta \bar{w}_i/\bar{P}_i\right)^{\frac{\beta}{\nu}} \bar{\phi}_i^{\beta}}{\bar{\phi}_n}.$$

Hence, we express the law of motion of labor as

$$\begin{aligned}
\bar{L}_i &= \sum_{n=1}^{N} \bar{\mu}_{ni} \bar{L}_n, \\
\bar{L}_i &= \sum_{n=1}^{N} \frac{\widetilde{m}_{ni}\left(\zeta \bar{w}_i/\bar{P}_i\right)^{\frac{\beta}{\nu}} \bar{\phi}_i^{\beta}}{\bar{\phi}_n} \bar{L}_n, \\
\bar{w}_i^{-\frac{\beta}{\nu}} \bar{P}_i^{\frac{\beta}{\nu}} \bar{\phi}_i^{-\beta} \bar{L}_i \quad \zeta^{-\frac{\beta}{\nu}} &= \sum_{n=1}^{N} \widetilde{m}_{ni} \bar{\phi}_n^{-1} \bar{L}_n.
\end{aligned}$$

Finally, the evolution of technology is given by

$$\bar{A}_n = \frac{\alpha_0 \Gamma_{\rho}}{g_A} \sum_{i=1}^{N} \bar{s}_{in} \left(\bar{A}_i\right)^{\rho_l} \sum_{i=1}^{N} \bar{\lambda}_{ni} \left(\frac{\bar{A}_i}{\bar{\lambda}_{ni}}\right)^{\rho_m}$$

and using $\bar{\lambda}_{in} = \frac{\bar{A}_n(\bar{\kappa}_{in}\bar{x}_n)^{-\theta}}{\left(\bar{P}_i/T\right)^{-\theta}}$, $\bar{\mu}_{ni} = \frac{\widetilde{m}_{ni}\left(\zeta\bar{w}_i/\bar{P}_i\right)^{\frac{\beta}{\nu}}\bar{\phi}_i^{\beta}}{\bar{\phi}_n}$, and $\bar{s}_{in} = \frac{\bar{\mu}_{in}\bar{L}_i}{\sum_{h=1}^{N}\bar{\mu}_{hn}\bar{L}_h}$, we obtain

$$\begin{aligned}
\bar{A}_n &= \frac{\alpha_0\Gamma_\rho}{g_A}\sum_{i=1}^{N}\bar{s}_{in}\left(\bar{A}_i\right)^{\rho_l}\sum_{i=1}^{N}\left(\bar{\lambda}_{ni}\right)^{1-\rho_m}\left(\bar{A}_i\right)^{\rho_m}\\
&=\frac{\alpha_0\Gamma_\rho}{g_A}\sum_{i=1}^{N}\frac{\bar{\mu}_{in}\bar{L}_i}{\bar{L}_n}\left(\bar{A}_i\right)^{\rho_l}\sum_{i=1}^{N}\left(\frac{\left(\bar{\kappa}_{ni}\bar{x}_i\right)^{-\theta}}{\left(\bar{P}_n/T\right)^{-\theta}}\right)^{1-\rho_m}\bar{A}_i\\
&=\frac{\alpha_0\Gamma_\rho}{g_A}\sum_{i=1}^{N}\frac{L_i}{L_n}\frac{\widetilde{m}_{in}\left(\zeta\bar{w}_n/\bar{P}_n\right)^{\frac{\beta}{\nu}}\bar{\phi}_n^{\beta}}{\bar{\phi}_i}\left(\bar{A}_i\right)^{\rho_l}\sum_{i=1}^{N}\left(\frac{\left(\bar{\kappa}_{ni}\Psi\left(\bar{w}_i\right)^{\xi\gamma}\left(\bar{P}_i\right)^{1-\xi\gamma}\right)^{-\theta}}{\left(\bar{P}_n/T\right)^{-\theta}}\right)^{1-\rho_m}\bar{A}_i.
\end{aligned}$$

Finally, rearranging this expression, we obtain

$$\begin{aligned}
&\bar{w}_n^{-\frac{\beta}{\nu}}\bar{P}_n^{\frac{\beta}{\nu}-(1-\rho_m)\theta}\bar{\phi}_n^{-\beta}\bar{A}_n\bar{L}_n\\
&=\frac{\alpha_0\Gamma_\rho\left(\Psi T\right)^{-(1-\rho_m)\theta}}{\zeta^{-\frac{\beta}{\nu}}g_A}\sum_{i=1}^{N}\widetilde{m}_{in}\bar{\phi}_i^{-1}\bar{A}_i^{\rho_l}\bar{L}_i\sum_{i=1}^{N}\bar{\kappa}_{ni}^{-(1-\rho_m)\theta}\bar{w}_i^{-(1-\rho_m)\theta\xi\gamma}\bar{P}_i^{-(1-\rho_m)\theta(1-\xi\gamma)}\bar{A}_i.
\end{aligned}$$

We end up with the following system of equations to solve for the equilibrium variables at the balanced growth path:

$$\bar{w}_i^{1+\xi\gamma\theta}\bar{P}_i^{\theta(1-\xi\gamma)}\bar{L}_i\bar{A}_i^{-1} = \left(\Psi T\right)^{-\theta}\sum_{n=1}^{N}\bar{\kappa}_{ni}^{-\theta}\bar{w}_n\bar{P}_n^{\theta}\bar{L}_n, \tag{D.1}$$

$$\bar{P}_i^{-\theta} = \left(\Psi T\right)^{-\theta}\sum_{n=1}^{N}\bar{\kappa}_{in}^{-\theta}\bar{w}_n^{-\theta\xi\gamma}\bar{P}_n^{-\theta(1-\xi\gamma)}\bar{A}_n, \tag{D.2}$$

$$\bar{w}_i^{-\frac{\beta}{\nu}}\bar{P}_i^{\frac{\beta}{\nu}}\bar{L}_i\bar{\phi}_i^{-\beta}\zeta^{-\frac{\beta}{\nu}} = \sum_{n=1}^{N}\widetilde{m}_{ni}\bar{L}_n\bar{\phi}_n^{-1}, \tag{D.3}$$

$$\bar{\phi}_i = \sum_{n=1}^{N}\widetilde{m}_{in}\zeta^{\frac{\beta}{\nu}}\bar{w}_n^{\frac{\beta}{\nu}}\bar{P}_n^{-\frac{\beta}{\nu}}\bar{\phi}_n^{\beta}, \tag{D.4}$$

$$\begin{aligned}
&\bar{w}_n^{-\frac{\beta}{\nu}}\bar{P}_n^{\frac{\beta}{\nu}-(1-\rho_m)\theta}\bar{\phi}_n^{-\beta}\bar{A}_n\bar{L}_n\\
&=\frac{\zeta^{\frac{\beta}{\nu}}\alpha_0\Gamma_\rho}{g_A(\Psi T)^{(1-\rho_m)\theta}}\sum_{i=1}^{N}\widetilde{m}_{in}\bar{\phi}_i^{-1}\bar{A}_i^{\rho_l}\bar{L}_i\sum_{i=1}^{N}\kappa_{ni}^{-(1-\rho_m)\theta}\bar{w}_i^{-(1-\rho_m)\theta\xi\gamma}\bar{P}_i^{-(1-\rho_m)\theta(1-\xi\gamma)}\bar{A}_i.
\end{aligned} \tag{D.5}$$

To prove the existence and uniqueness of the balanced growth path equilibrium in the detrended model, we start with the following change of variables:

$$\bar{A}_n = \bar{A}_n^l \bar{A}_n^m,$$

$$\bar{A}_n^l = \sum_{i=1}^{N} \frac{\bar{\mu}_{in} \bar{L}_i}{\bar{L}_n} \left(\bar{A}_i\right)^{\rho_l},$$

$$\bar{A}_n^m = \frac{\alpha_0 \Gamma_\rho}{g_A} \sum_{i=1}^{N} \bar{\lambda}_{ni} \left(\frac{\bar{A}_i}{\bar{\lambda}_{ni}}\right)^{\rho_m}.$$

Using this change of variables in the equilibrium system, we can write the matrices Λ and Γ representing the exponents of $\{\bar{w}_i, \bar{P}_i, \bar{L}_i, \bar{\phi}_i, \bar{A}_i^l, \bar{A}_i^m\}$ on the left-hand side and right-hand side of the system of equations, respectively. These matrices are given by

$$\Lambda = \begin{pmatrix} 1+\theta\xi\gamma & \theta\left(1-\xi\gamma\right) & 1 & 0 & -1 & -1 \\ 0 & -\theta & 0 & 0 & 0 & 0 \\ -\frac{\beta}{\nu} & \frac{\beta}{\nu} & 1 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{\beta}{\nu} & \frac{\beta}{\nu} & 1 & -\beta & 1 & 0 \\ 0 & -\left(1-\rho_m\right)\theta & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\Gamma = \begin{pmatrix} 1 & \theta & 1 & 0 & 0 & 0 \\ -\theta\xi\gamma & -\theta\left(1-\xi\gamma\right) & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ \frac{\beta}{\nu} & -\frac{\beta}{\nu} & 0 & \beta & 0 & 0 \\ 0 & 0 & 1 & -1 & \rho_l & \rho_l \\ -\left(1-\rho_m\right)\theta\xi\gamma & -\left(1-\rho_m\right)\theta\left(1-\xi\gamma\right) & 0 & 0 & 1 & 1 \end{pmatrix}.$$

We then define the matrix $\Omega = \Gamma\Lambda^{-1}$. Following Kleinman et al. (2023) and Allen et al. (2020), we show that if the spectral radius of Ω is equal to one ($\rho\left(\Omega\right) = 1$) and if Ω is invertible, then the solution must be unique up to scale. Evaluating the eigenvalues of Ω, we have

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \frac{-b+\sqrt{b^2-4ac}}{2a} \\ \frac{-b-\sqrt{b^2-4ac}}{2a} \\ 0 \\ \rho_m + \rho_l \end{bmatrix},$$

where

$$\begin{aligned} a &= \beta + \nu + \theta\gamma\nu\xi, \\ b &= -\nu\left(1 + \beta - \gamma\xi\left(1 + \theta\left(1 - \beta\right)\right)\right), \\ c &= \beta\left(\nu - 1 - \gamma\xi\nu\left(1 + \theta\right)\right). \end{aligned}$$

We proceed to show that $|\zeta_3| < 1$ and $|\zeta_4| < 1$.

First, $\left|-b + \sqrt{b^2 - 4ac}\right| < 2a$. If $-b + \sqrt{b^2 - 4ac} > 0$, then this is equivalent to show that $b^2 - 4ac < (2a + b)^2$ or that $b^2 - 4ac < (2a + b)^2 = 4a^2 + 4ab + b^2$, or equivalently that $0 < a + b + c$ which holds since $a + b + c = \gamma\nu\xi(1 - \beta)(1 + 2\theta) > 0$. Otherwise, if $-b + \sqrt{b^2 - 4ac} < 0$, we need to show that $b - \sqrt{b^2 - 4ac} < 2a$, or that $-\sqrt{b^2 - 4ac} < 2a - b$ but note that $2a - b = 2\left(\beta + \nu + \theta\gamma\nu\xi\right) + \nu\left(1 + \beta - \gamma\xi\left(1 + \theta\left(1 - \beta\right)\right)\right) = 2\beta + 3\nu + \beta\nu + (\theta - 1)\gamma\nu\xi + \theta\beta\gamma\nu\xi > 0$ so it holds since $\theta \geq 0$. Note that at $\theta = 0$, $2\beta + 3\nu + \beta\nu - \gamma\nu\xi > 0$.

Second, $\left|-b - \sqrt{b^2 - 4ac}\right| < 2a$. If $-b - \sqrt{b^2 - 4ac} < 0$ then this is equivalent to show that $b + \sqrt{b^2 - 4ac} < 2a$, or that $\sqrt{b^2 - 4ac} < 2a - b$, and given that $2a - b = 2\left(\beta + \nu + \theta\gamma\nu\xi\right) + \nu\left(1 + \beta - \gamma\xi\left(1 + \theta\left(1 - \beta\right)\right)\right) = 2\beta + 3\nu + \beta\nu + (\theta - 1)\gamma\xi\nu + \theta\beta\gamma\nu\xi > 0$ then this is equivalent to show that $b^2 - 4ac < (2a - b)^2$, $b^2 - 4ac < (2a - b)^2 = 4a^2 + b^2 - 4ab$ or $-c < a - b$, or $0 < a - b + c = \nu(1 + \beta)(2 - \gamma\xi) > 0$. If $-b - \sqrt{b^2 - 4ac} > 0$ then this is equivalent to show that $-b - \sqrt{b^2 - 4ac} < 2a$, or that $0 < 2a + b$, but since we know that $a + b = \beta(1 - \nu) + \gamma\nu\xi + (2 - \beta)\theta\gamma\nu\xi > 0$, then $0 < 2a + b$. *Q.E.D.*

The two additional eigenvalues are zero and $\rho_m + \rho_l < 1$.

It follows that the balanced growth path of the detrended equilibrium with idea flows from migration and trade is unique up to scale.

We now proceed to prove the existence and uniqueness of the steady state equilibrium of the detrended model at the balanced growth path in different versions of the model.

D.1 Case 1: Model with No Idea Flows

Without idea flows, the steady state equilibrium is characterized by the following set of equilibrium conditions:

$$\bar{w}_i^{1+\xi\gamma\theta}\bar{P}_i^{\theta(1-\xi\gamma)}\bar{L}_i\bar{A}_i^{-1} = (\Psi T)^{-\theta}\sum_{n=1}^{N}\bar{\kappa}_{ni}^{-\theta}\bar{w}_n\bar{P}_n^{\theta}\bar{L}_n, \tag{D.6}$$

$$\bar{P}_i^{-\theta} = (\Psi T)^{-\theta}\sum_{n=1}^{N}\bar{\kappa}_{in}^{-\theta}\bar{w}_n^{-\theta\xi\gamma}\bar{P}_n^{-\theta(1-\xi\gamma)}\bar{A}_n, \tag{D.7}$$

$$\bar{w}_i^{-\frac{\beta}{\nu}}\bar{P}_i^{\frac{\beta}{\nu}}\bar{L}_i\bar{\phi}_i^{-\beta} = \sum_{n=1}^{N}\widetilde{m}_{ni}\bar{L}_n\bar{\phi}_n^{-1}, \tag{D.8}$$

$$\bar{\phi}_i = \sum_{n=1}^{N}\widetilde{m}_{in}\bar{w}_n^{\frac{\beta}{\nu}}\bar{P}_n^{-\frac{\beta}{\nu}}\bar{\phi}_n^{\beta}. \tag{D.9}$$

We can write the matrices Λ and Γ representing the exponents of $\{\bar{w}_i, \bar{P}_i, \bar{L}_i, \bar{\phi}_i\}$ on the left-hand side and right-hand side of the system of equations, respectively. These matrices are given by

$$\Lambda = \begin{pmatrix} 1+\theta\xi\gamma & \theta(1-\xi\gamma) & 1 & 0 \\ 0 & -\theta & 0 & 0 \\ -\frac{\beta}{\nu} & \frac{\beta}{\nu} & 1 & -\beta \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 1 & \theta & 1 & 0 \\ -\theta\xi\gamma & -\theta(1-\xi\gamma) & 0 & 0 \\ 0 & 0 & 1 & -1 \\ \frac{\beta}{\nu} & -\frac{\beta}{\nu} & 0 & \beta \end{pmatrix}.$$

We then define the matrix $\Omega = \Gamma\Lambda^{-1}$. Following Kleinman et al. (2023) and Allen et al. (2020), we show that if the spectral radius of Ω is equal to one ($\rho(\Omega) = 1$) and if Ω is invertible, then the solution must be unique up to scale. Evaluating the eigenvalues of Ω, we have

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \frac{-b+\sqrt{b^2-4ac}}{2a} \\ \frac{-b-\sqrt{b^2-4ac}}{2a} \end{bmatrix},$$

where

$$
\begin{aligned}
a &= \beta + \nu + \theta\gamma\nu\xi, && \text{(D.10)}\\
b &= -\nu\left(1 + \beta - \gamma\xi\left(1 + \theta\left(1 - \beta\right)\right)\right), && \text{(D.11)}\\
c &= \beta\left(\nu - 1 - \gamma\xi\nu\left(1 + \theta\right)\right). && \text{(D.12)}
\end{aligned}
$$

To prove existence and uniqueness, we need show again that $|\zeta_3| < 1$ and $|\zeta_4| < 1$.

First, $\left|-b + \sqrt{b^2 - 4ac}\right| < 2a$. If $-b + \sqrt{b^2 - 4ac} > 0$, then this is equivalent to show that $b^2 - 4ac < (2a + b)^2$ or that $b^2 - 4ac < (2a + b)^2 = 4a^2 + 4ab + b^2$, or equivalently that $0 < a + b + c$ which holds since $a + b + c = \gamma\nu\xi(1 - \beta)(1 + 2\theta) > 0$.

Otherwise, if $-b + \sqrt{b^2 - 4ac} < 0$, we need to show that $b - \sqrt{b^2 - 4ac} < 2a$, or that $-\sqrt{b^2 - 4ac} < 2a - b$ but note that $2a - b = 2\left(\beta + \nu + \theta\gamma\nu\xi\right) + \nu\left(1 + \beta - \gamma\xi\left(1 + \theta\left(1 - \beta\right)\right)\right) = 2\beta + 3\nu + \beta\nu + \left(\theta - 1\right)\gamma\nu\xi + \theta\beta\gamma\nu\xi > 0$ so it holds since $\theta \geq 0$. Note that at $\theta = 0$, $2\beta + 3\nu + \beta\nu - \gamma\nu\xi > 0$.

Second, $\left|-b - \sqrt{b^2 - 4ac}\right| < 2a$. If $-b - \sqrt{b^2 - 4ac} < 0$ then this is equivalent to show that $b + \sqrt{b^2 - 4ac} < 2a$, or that $\sqrt{b^2 - 4ac} < 2a - b$, and given that $2a - b = 2\left(\beta + \nu + \theta\gamma\nu\xi\right) + \nu\left(1 + \beta - \gamma\xi\left(1 + \theta\left(1 - \beta\right)\right)\right) = 2\beta + 3\nu + \beta\nu + \left(\theta - 1\right)\gamma\xi\nu + \theta\beta\gamma\nu\xi > 0$ then this is equivalent to show that $b^2 - 4ac < (2a - b)^2$, $b^2 - 4ac < (2a - b)^2 = 4a^2 + b^2 - 4ab$ or $-c < a - b$, or $0 < a - b + c = \nu(1 + \beta)(2 - \gamma\xi) > 0$. If $-b - \sqrt{b^2 - 4ac} > 0$ then this is equivalent to show that $-b - \sqrt{b^2 - 4ac} < 2a$, or that $0 < 2a + b$, but since we know that $a + b = \beta(1 - \nu) + \gamma\nu\xi + (2 - \beta)\theta\gamma\nu\xi > 0$, then $0 < 2a + b$. *Q.E.D.*

D.2 Case 2: Model with Idea Flows from Migration

With idea flows from migration, the balanced growth path in the detrended equilibrium is characterized by the following set of equilibrium conditions:

$$
\bar{w}_i^{1+\xi\gamma\theta} \bar{P}_i^{\theta(1-\xi\gamma)} \bar{L}_i \bar{A}_i^{-1} = (\Psi T)^{-\theta} \sum_{n=1}^{N} \bar{\kappa}_{ni}^{-\theta} \bar{w}_n \bar{P}_n^{\theta} \bar{L}_n, \tag{D.13}
$$

$$
\bar{P}_i^{-\theta} = (\Psi T)^{-\theta} \sum_{n=1}^{N} \bar{\kappa}_{in}^{-\theta} \bar{w}_n^{-\theta\xi\gamma} \bar{P}_n^{-\theta(1-\xi\gamma)} \bar{A}_n, \tag{D.14}
$$

$$
\bar{w}_i^{-\frac{\beta}{\nu}} \bar{P}_i^{\frac{\beta}{\nu}} \bar{L}_i \bar{\phi}_i^{-\beta} \zeta^{-\frac{\beta}{\nu}} = \sum_{n=1}^{N} \widetilde{m}_{ni} \bar{L}_n \bar{\phi}_n^{-1}, \tag{D.15}
$$

$$\bar{\phi}_i = \sum_{n=1}^{N} \widetilde{m}_{in} \zeta^{\frac{\beta}{\nu}} \bar{w}_n^{\frac{\beta}{\nu}} \bar{P}_n^{-\frac{\beta}{\nu}} \bar{\phi}_n^{\beta}, \tag{D.16}$$

$$\bar{w}_i^{-\frac{\beta}{\nu}} \bar{P}_i^{\frac{\beta}{\nu}} \bar{L}_i \bar{\phi}_i^{-\beta} \bar{A}_i \zeta^{-\frac{\beta}{\nu}} = \frac{\alpha_0 \Gamma_\rho}{g_A} \sum_{n=1}^{N} \widetilde{m}_{ni} \bar{L}_n \bar{\phi}_n^{-1} \left(\bar{A}_n\right)^{\rho_l}. \tag{D.17}$$

Analogous to the previous case, we can write the matrices Λ and Γ representing the exponents of $\{\bar{w}_i, \bar{P}_i, \bar{L}_i, \bar{\phi}_i, \bar{A}_i\}$ on the left-hand side and right-hand side of the system of equations, respectively. These matrices are given by

$$\Lambda = \begin{pmatrix} 1+\theta\xi\gamma & \theta\left(1-\xi\gamma\right) & 1 & 0 & -1 \\ 0 & -\theta & 0 & 0 & 0 \\ -\frac{\beta}{\nu} & \frac{\beta}{\nu} & 1 & -\beta & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\frac{\beta}{\nu} & \frac{\beta}{\nu} & 1 & -\beta & 1 \end{pmatrix},$$

$$\Gamma = \begin{pmatrix} 1 & \theta & 1 & 0 & 0 \\ -\theta\xi\gamma & -\theta\left(1-\xi\gamma\right) & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ \dfrac{\beta}{\nu} & -\dfrac{\beta}{\nu} & 0 & \beta & 0 \\ 0 & 0 & 1 & -1 & \rho_l \end{pmatrix}.$$

As in the previous case, evaluating the eigenvalues of Ω, we have

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \frac{-b+\sqrt{b^2-4ac}}{2a} \\ \frac{-b-\sqrt{b^2-4ac}}{2a} \\ \rho_l \end{bmatrix},$$

where

$$\begin{aligned} a &= \beta + \nu + \theta\gamma\nu\xi, \\ b &= -\nu\left(1+\beta-\gamma\xi\left(1+\theta\left(1-\beta\right)\right)\right), \\ c &= \beta\left(\nu-1-\gamma\xi\nu\left(1+\theta\right)\right). \end{aligned}$$

Hence, the first four eigenvalues are the same as in the previous case, and the additional eigenvalue is given by $\rho_l < 1$. It follows that the balanced growth path equilibrium of the detrended model with idea flows from migration is unique up to scale.

D.3 Case 3: Model with Idea Flows from Trade

With idea flows from trade, the equilibrium is characterized by the following set of equilibrium conditions:

$$\bar{w}_i^{1+\xi\gamma\theta} \bar{P}_i^{\theta(1-\xi\gamma)} \bar{L}_i \bar{A}_i^{-1} = (\Psi T)^{-\theta} \sum_{n=1}^{N} \bar{\kappa}_{ni}^{-\theta} \bar{w}_n \bar{P}_n^{\theta} \bar{L}_n, \tag{D.18}$$

$$\bar{P}_i^{-\theta} = (\Psi T)^{-\theta} \sum_{n=1}^{N} \bar{\kappa}_{in}^{-\theta} \bar{w}_n^{-\theta\xi\gamma} \bar{P}_n^{-\theta(1-\xi\gamma)} \bar{A}_n, \tag{D.19}$$

$$\bar{w}_i^{-\frac{\beta}{\nu}} \bar{P}_i^{\frac{\beta}{\nu}} \bar{L}_i \bar{\phi}_i^{-\beta} \zeta^{-\frac{\beta}{\nu}} = \sum_{n=1}^{N} \widetilde{m}_{ni} \bar{L}_n \bar{\phi}_n^{-1}, \tag{D.20}$$

$$\bar{\phi}_i = \sum_{n=1}^{N} \widetilde{m}_{in} \zeta^{\frac{\beta}{\nu}} \bar{w}_n^{\frac{\beta}{\nu}} \bar{P}_n^{-\frac{\beta}{\nu}} \bar{\phi}_n^{\beta}, \tag{D.21}$$

$$\bar{P}_i^{-(1-\rho_m)\theta} \bar{A}_i = \frac{\alpha_0 \Gamma_\rho (T\Psi)^{-(1-\rho_m)\theta}}{g_A} \sum_{n=1}^{N} \bar{\kappa}_{in}^{-(1-\rho_m)\theta} \bar{w}_n^{-(1-\rho_m)\theta\xi\gamma} \bar{P}_n^{-(1-\rho_m)\theta(1-\xi\gamma)} \bar{A}_n. \tag{D.22}$$

Analogous to the case of idea flows from migration, we can write the matrices Λ and Γ representing the exponents of $\{\bar{w}_i, \bar{P}_i, \bar{L}_i, \bar{\phi}_i, \bar{A}_i\}$ on the left-hand side and right-hand side of the system of equations, respectively. These matrices are given by

$$\Lambda = \begin{pmatrix} 1+\theta\xi\gamma & \theta(1-\xi\gamma) & 1 & 0 & -1 \\ 0 & -\theta & 0 & 0 & 0 \\ -\frac{\beta}{\nu} & \frac{\beta}{\nu} & 1 & -\beta & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -(1-\rho_m)\theta & 0 & 0 & 1 \end{pmatrix},$$

$$\Gamma = \begin{pmatrix} 1 & \theta & 1 & 0 & 0 \\ -\theta\xi\gamma & -\theta\left(1-\xi\gamma\right) & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ \dfrac{\beta}{\nu} & -\dfrac{\beta}{\nu} & 0 & \beta & 0 \\ -\left(1-\rho_m\right)\theta\xi\gamma & -\left(1-\rho_m\right)\theta\left(1-\xi\gamma\right) & 0 & 0 & 1 \end{pmatrix}.$$

As before, evaluating the eigenvalues of Ω, we have

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \frac{-b+\sqrt{b^2-4ac}}{2a} \\ \frac{-b-\sqrt{b^2-4ac}}{2a} \\ \rho_m \end{bmatrix},$$

where

$$\begin{aligned} a &= \beta+\nu+\theta\gamma\nu\xi, \\ b &= -\nu\left(1+\beta-\gamma\xi\left(1+\theta\left(1-\beta\right)\right)\right), \\ c &= \beta\left(\nu-1-\gamma\xi\nu\left(1+\theta\right)\right). \end{aligned}$$

Hence, the five eigenvalues are the same as in the case with no idea flows (Case 1) except that now the fifth eigenvalue is given by $\rho_{m.}$ Since $\rho_m < 1$, it follows that the balanced growth path of the detrended equilibrium with idea flows from trade is unique up to scale.

E Dynamic-Hat Algebra

Proposition 4. Dynamic-Hat Algebra. Define the term $\hat{y}_{t+1}$ as the relative time difference of the detrended endogenous variable $\tilde{y}$; namely, $\hat{y}_{t+1} = (\tilde{y}_{t+1}/\tilde{y}_t)$. *Given an initial observed allocation* $\left\{\left\{\lambda_{in,0}\right\}_{i=1,n=1}^{N,N}, \left\{\mu_{in,0}\right\}_{i=1,n=1}^{N,N}, \left\{w_{i,0}L_{i,0}\right\}_{i=1}^{N}, \left\{K_{i,0}\right\}_{i=1}^{N}, \left\{L_{i,0}\right\}_{i=1}^{N}\right\}$, *the parameters and elasticities* $(\rho_\ell, \rho_m, \theta, \nu, \gamma, \xi, \beta)$, *the initial rate and growth rate in the arrival of ideas* (α_0, g_α) *and a convergent sequence of future changes in fundamentals under perfect foresight* $\left\{\hat{\kappa}_{in,t}, \hat{m}_{in,t}\right\}_{i=1,n=1,t=1}^{N,N,\infty}$, *the solution for the sequence of changes in the model's endogenous variables in the detrended model* $\left\{\hat{y}_{t+1}\right\}_{t=1}^{\infty}$ *does not require information on the level of fundamentals (trade and migration costs).*

Proof. Let us define the variable $\hat{y}_{t+1}$ as the time difference in the detrended variable $\tilde{y}$;

namely, $\hat{y}_{t+1} = (\tilde{y}_{t+1}/\tilde{y}_t)$. The equilibrium conditions in time differences of the detrended system are given by

$$\log(\hat{u}_{i,t+1}) = \log\left(\hat{w}_{i,t+1}/\hat{P}_{i,t+1}\right) + \nu \log\left(\sum_{n=1}^{N} \mu_{in,t} \left(\hat{u}_{n,t+2}\right)^{\beta/\nu} \left(\hat{m}_{in,t+1}\right)^{-1/\nu}\right), \tag{E.1}$$

$$\mu_{in,t+1} = \frac{\mu_{in,t} \left(\hat{u}_{n,t+2}\right)^{\beta/\nu} \left(\hat{m}_{in,t+1}\right)^{-1/\nu}}{\sum_{h=1}^{N} \mu_{ih,t} \left(\hat{u}_{h,t+2}\right)^{\beta/\nu} \left(\hat{m}_{ih,t+1}\right)^{-1/\nu}}, \tag{E.2}$$

$$L_{i,t+1} = \sum_{n=1}^{N} \mu_{ni,t} L_{n,t}, \tag{E.3}$$

$$\hat{x}_{i,t} = \left(\hat{w}_{i,t}^{\xi} \hat{r}_{i,t}^{1-\xi}\right)^{\gamma} \hat{P}_{i,t}^{1-\gamma}, \tag{E.4}$$

$$\hat{P}_{i,t+1} = \left(\sum_{n=1}^{N} \lambda_{in,t} \hat{A}_{n,t+1} \left(\hat{\kappa}_{in,t+1} \hat{x}_{n,t+1}\right)^{-\theta}\right)^{-1/\theta}, \tag{E.5}$$

$$\lambda_{in,t+1} = \lambda_{in,t} \hat{A}_{n,t+1} \left(\frac{\hat{\kappa}_{in,t+1} \hat{x}_{n,t+1}}{\hat{P}_{i,t+1}}\right)^{-\theta}, \tag{E.6}$$

$$\hat{w}_{i,t+1} \hat{L}_{i,t+1} = \frac{1}{\tilde{w}_{i,t} L_{i,t}} \sum_{n=1}^{N} \lambda_{ni,t+1} \hat{w}_{n,t+1} \hat{L}_{n,t+1} \tilde{w}_{n,t} L_{n,t}, \tag{E.7}$$

$$\tilde{K}_{i,t+1} = \frac{\beta}{(1+g_k)} \tilde{R}_{i,t} \tilde{K}_{i,t}, \tag{E.8}$$

$$\tilde{R}_{i,t+1} = 1 - \delta + \frac{\hat{w}_{i,t+1} \hat{L}_{i,t+1}}{\hat{P}_{i,t+1} \hat{K}_{i,t+1}} \left[\tilde{R}_{i,t} - (1-\delta)\right], \tag{E.9}$$

$$\hat{A}_{n,t+1} = \frac{1}{(1+g_A)} + \frac{\alpha_0 \Gamma_\rho}{\tilde{A}_{n,t}(1+g_A)} \sum_{i=1}^{N} s_{in,t} \left(\tilde{A}_{i,t}\right)^{\rho_\ell} \left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{\tilde{A}_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}\right], \tag{E.10}$$

where $\hat{u}_{i,t+1} = \exp(\tilde{V}_{i,t+1} - \tilde{V}_{i,t})$, $\hat{m}_{in,t+1} = \exp(m_{in,t+1} - m_{in,t})$, $\tilde{R}_{i,t} = \tilde{r}_{i,t}/\tilde{P}_{i,t} + (1-\delta)$. Note we use the fact that $L_{n,t} = \tilde{L}_{n,t}$, $\mu_{ni,t} = \tilde{\mu}_{ni,t}$, and $\lambda_{in,t} = \tilde{\lambda}_{in,t}$.

In what follows we provide the algebra to arrive in the system of equilibrium conditions in changes. As the system of equations in time differences shows, solving the model in relative time differences requires conditioning the model on the initial observable allocations $\lambda_{in,0}$, $\tilde{w}_{i,0} L_{i,0} + \tilde{r}_{i,t} \tilde{K}_{i,0}$, $L_{i,0}$, $\mu_{in,0}$, and $\tilde{K}_{i,0}$, and elasticities θ, ν, β, δ, ρ_ℓ, ρ_m, and α_0, which contains information on the initial level of fundamentals as the model inversion shows.

To derive the system of equations in time differences, we first reproduce the equilibrium conditions of the detrended model derived in Appendix C,

$$\tilde{V}_{i,t} = \beta log\left(1+g_v\right) + log\left(\frac{\tilde{w}_{i,t}}{\tilde{P}_{i,t}}\right) + \nu log\left(\sum_{n=1}^{N} exp\left(\beta \tilde{V}_{n,t+1} - m_{in,t}\right)^{1/\nu}\right), \tag{E.11}$$

$$\tilde{P}_{i,t} = T\left(\sum_{n=1}^{N} \tilde{A}_{n,t}\left(\kappa_{in,t}\tilde{x}_{n,t}\right)^{-\theta}\right)^{-1/\theta}, \tag{E.12}$$

$$\tilde{w}_{i,t}L_{i,t} = \sum_{n=1}^{N} \tilde{A}_{i,t}\left(\frac{\kappa_{ni,t}\tilde{x}_{i,t}}{\tilde{P}_{n,t}/T}\right)^{-\theta} \tilde{w}_{n,t}L_{n,t}, \tag{E.13}$$

$$\tilde{r}_{i,t}\tilde{K}_{i,t} = \sum_{n=1}^{N} \tilde{A}_{i,t}\left(\frac{\kappa_{ni,t}\tilde{x}_{i,t}}{\tilde{P}_{n,t}/T}\right)^{-\theta} \tilde{r}_{n,t}\tilde{K}_{n,t}, \tag{E.14}$$

$$L_{i,t+1} = \sum_{n=1}^{N} \mu_{ni,t}L_{n,t}, \tag{E.15}$$

$$\tilde{K}_{i,t+1} = \frac{\beta}{(1+g_k)}\left(\tilde{r}_{i,t}/\tilde{P}_{i,t} + (1-\delta)\right)\tilde{K}_{i,t}, \tag{E.16}$$

$$\tilde{A}_{n,t+1} - \frac{\tilde{A}_{n,t}}{(1+g_A)} = \frac{\alpha_0\Gamma_{\rho_\ell,\rho_m}}{(1+g_A)}\sum_{i=1}^{N} s_{in,t}\left(\tilde{A}_{i,t}\right)^{\rho_l}\sum_{i=1}^{N}\lambda_{ni,t}\left(\frac{\tilde{A}_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}, \tag{E.17}$$

Recall first that the share of workers moving from location i to n at time $t+1$ is given by

$$\mu_{in,t+1} = \frac{exp\left(\beta V_{n,t+2} - m_{in,t+1}\right)^{1/\nu}}{\sum_{h=1}^{N} exp\left(\beta V_{h,t+2} - m_{ih,t+1}\right)^{1/\nu}} = \frac{exp\left(\beta \tilde{V}_{n,t+2} - m_{in,t+1}\right)^{1/\nu}}{\sum_{h=1}^{N} exp\left(\beta \tilde{V}_{h,t+2} - m_{ih,t+1}\right)^{1/\nu}},$$

where for the second equality we use the definition $e^{V_{i,t}} = e^{\tilde{V}_{i,t}}(1+g_v)^t$ for all i and t.

By multiplying and dividing $\mu_{in,t}$ in the numerator and $\mu_{ih,t}$ for each term in the summation in the denominator, we have

$$\mu_{in,t+1} = \frac{\mu_{in,t} exp\left(\beta\tilde{V}_{n,t+2} - \beta\tilde{V}_{n,t+1} + m_{in,t+1} - m_{in,t}\right)^{1/\nu}}{\sum_{h=1}^{N}\mu_{ih,t} exp\left(\beta\tilde{V}_{h,t+2} - \beta\tilde{V}_{h,t+1} + m_{ih,t+1} - m_{ih,t}\right)^{1/\nu}}$$
$$= \frac{\mu_{in,t}\left(\hat{u}_{n,t+2}\right)^{\beta/\nu}\left(\hat{m}_{in,t+1}\right)^{-1/\nu}}{\sum_{h=1}^{N}\mu_{ih,t}\left(\hat{u}_{h,t+2}\right)^{\beta/\nu}\left(\hat{m}_{ih,t+1}\right)^{-1/\nu}},$$

which is equation (E.2).

To obtain equilibrium condition (E.1), we take the time difference using (E.11). We obtain

$$\log(\hat{u}_{i,t+1}) = \tilde{V}_{i,t+1} - \tilde{V}_{i,t}$$
$$= \log\left(\frac{\tilde{w}_{i,t+1}/\tilde{P}_{i,t+1}}{\tilde{w}_{i,t}/\tilde{P}_{i,t}}\right) + \nu\log\left(\sum_{n=1}^{N}\frac{\exp\left(\beta\tilde{V}_{n,t+2} - m_{in,t+1}\right)^{1/\nu}}{\sum_{h=1}^{N}\exp\left(\beta\tilde{V}_{h,t+1} - m_{ih,t}\right)^{1/\nu}}\right)$$
$$= \log\left(\hat{w}_{i,t+1}/\hat{P}_{i,t+1}\right) + \nu\log\left(\sum_{n=1}^{N}\frac{\exp\left(\beta\tilde{V}_{n,t+1} - m_{in,t}\right)^{1/\nu}\frac{\exp\left(\beta\tilde{V}_{n,t+2}-m_{in,t+1}\right)^{1/\nu}}{\exp\left(\beta\tilde{V}_{n,t+1}-m_{in,t}\right)^{1/\nu}}}{\sum_{h=1}^{N}\exp\left(\beta\tilde{V}_{h,t+1} - m_{ih,t}\right)^{1/\nu}}\right)$$
$$= \log\left(\hat{w}_{i,t+1}/\hat{P}_{i,t+1}\right) + \nu\log\left(\sum_{n=1}^{N}\mu_{in,t}\exp\left(\tilde{V}_{n,t+2} - \tilde{V}_{n,t+1}\right)^{\beta/\nu}\exp\left(m_{in,t+1} - m_{in,t}\right)^{-1/\nu}\right)$$
$$= \log\left(\hat{w}_{i,t+1}/\hat{P}_{i,t+1}\right) + \nu\log\left(\sum_{n=1}^{N}\mu_{in,t}\left(\hat{u}_{n,t+2}\right)^{\beta/\nu}\left(\hat{m}_{in,t+1}\right)^{-1/\nu}\right),$$

where for the third equality we use the expression of $\mu_{in,t}$ previously derived.

Since labor in each location is constant in the long run, we immediately obtain (E.3) from the law of motion (E.15).

To obtain equation (E.4), note that

$$\tilde{x}_{i,t} = B\left(\tilde{w}_{i,t}^{\xi}\tilde{r}_{i,t}^{1-\xi}\right)^{\gamma}\tilde{P}_{i,t}^{1-\gamma}.$$

Taking the time difference yields

$$\hat{x}_{i,t} \equiv \frac{\tilde{x}_{i,t+1}}{\tilde{x}_{i,t}} = \frac{\left(\tilde{w}_{i,t+1}^{\xi}\tilde{r}_{i,t+1}^{1-\xi}\right)^{\gamma}\tilde{P}_{i,t+1}^{1-\gamma}}{\left(\tilde{w}_{i,t}^{\xi}\tilde{r}_{i,t}^{1-\xi}\right)^{\gamma}\tilde{P}_{i,t}^{1-\gamma}} = \left(\hat{w}_{i,t}^{\xi}\hat{r}_{i,t}^{1-\xi}\right)^{\gamma}\hat{P}_{i,t}^{1-\gamma}.$$

Recall that in the detrended version of the model, the trade flow share from location n

to location i at time t is

$$\lambda_{in,t} = \frac{T^{-\theta} \tilde{A}_{n,t} \left(\kappa_{in,t} \tilde{x}_{n,t}\right)^{-\theta}}{\tilde{P}_{i,t}^{-\theta}},$$

where T is some constant. Taking the time difference yields

$$\frac{\lambda_{in,t+1}}{\lambda_{in,t}} = \hat{A}_{n,t+1} \left(\frac{\hat{\kappa}_{in,t+1} \hat{x}_{n,t+1}}{\hat{P}_{i,t+1}} \right)^{-\theta},$$

which leads to equilibrium condition (E.6).

Note that the detrended price index in location i is

$$\tilde{P}_{i,t} = T \left(\sum_{n=1}^{N} \tilde{A}_{n,t} \left(\kappa_{in,t} \tilde{x}_{n,t}\right)^{-\theta} \right)^{-1/\theta}.$$

Taking the time difference, we have

$$\begin{aligned}
\hat{P}_{i,t+1} &= \left(\sum_{n=1}^{N} \frac{\tilde{A}_{n,t+1} \left(\kappa_{in,t+1} \tilde{x}_{n,t+1}\right)^{-\theta}}{\sum_{h=1}^{N} \tilde{A}_{h,t} \left(\kappa_{ih,t} \tilde{x}_{h,t}\right)^{-\theta}} \right)^{-1/\theta} \\
&= \left(\sum_{n=1}^{N} \frac{\tilde{A}_{n,t} \left(\kappa_{in} \tilde{x}_{n,t}\right)^{-\theta} \tilde{A}_{n,t+1} \left(\kappa_{in,t+1} \tilde{x}_{n,t+1}\right)^{-\theta} / \tilde{A}_{n,t} \left(\kappa_{in,t} \tilde{x}_{n,t}\right)^{-\theta}}{\sum_{h=1}^{N} \tilde{A}_{h,t} \left(\kappa_{ih,t} \tilde{x}_{h,t}\right)^{-\theta}} \right)^{-1/\theta} \\
&= \left(\sum_{n=1}^{N} \lambda_{in,t} \hat{A}_{n,t+1} \left(\hat{\kappa}_{in,t+1} \hat{x}_{n,t+1}\right)^{-\theta} \right)^{-1/\theta},
\end{aligned}$$

where we use $\lambda_{in,t} = \frac{\tilde{A}_{n,t}(\kappa_{in}\tilde{x}_{n,t})^{-\theta}}{\sum_{h=1}^{N} \tilde{A}_{h,t}\left(\kappa_{ih}\tilde{x}_{h,t}\right)^{-\theta}}$ and which gives equilibrium condition (E.5).

To obtain equilibrium condition (E.7), we use labor market clearing condition (E.13),

$$\tilde{w}_{i,t+1} L_{i,t+1} = \sum_{n=1}^{N} \lambda_{ni,t+1} \tilde{w}_{n,t+1} L_{n,t+1},$$

and divide by $\tilde{w}_{i,t} L_{i,t}$ on both sides, to obtain

$$\begin{aligned}
\hat{w}_{i,t+1} \hat{L}_{i,t+1} &= \frac{1}{\tilde{w}_{i,t} L_{i,t}} \sum_{n=1}^{N} \lambda_{ni,t+1} \tilde{w}_{n,t+1} L_{n,t+1} \\
&= \frac{1}{\tilde{w}_{i,t+1} L_{i,t+1}} \sum_{n=1}^{N} \lambda_{ni,t+1} \hat{w}_{n,t+1} \hat{L}_{n,t+1} \tilde{w}_{n,t} L_{n,t},
\end{aligned}$$

where as before we use $\tilde{L}_{n,t} = L_{n,t}$.

Equation (E.8) is exactly the detrended law of motion of capital as in equation (E.16). To obtain equation (E.9), we use the equilibrium condition:

$$\frac{\tilde{w}_{i,t}\tilde{L}_{i,t}}{\left[\tilde{R}_{i,t}-(1-\delta)\right]\tilde{P}_{i,t}\tilde{K}_{i,t}}=\frac{\xi}{1-\xi}.$$

Taking the time difference and rearranging this expression yields the desired result.

Finally, to obtain the law of motion of knowledge in relative time changes (E.10), note that equation (E.17) gives the detrended law of motion of knowledge:

$$\tilde{A}_{n,t+1}-\frac{\tilde{A}_{n,t}}{(1+g_A)}=\frac{\alpha_0\Gamma_\rho}{(1+g_A)}\sum_{i=1}^{N}s_{in,t}\left(\tilde{A}_{i,t}\right)^{\rho_l}\sum_{i=1}^{N}\lambda_{ni,t}\left(\frac{\tilde{A}_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}.$$

Divided by $\tilde{A}_{n,t}$ on both sides, we have

$$\hat{A}_{n,t+1}=\frac{1}{(1+g_A)}+\frac{\alpha_0\Gamma_\rho}{\tilde{A}_{n,t}(1+g_A)}\sum_{i=1}^{N}s_{in,t}\left(\tilde{A}_{i,t}\right)^{\rho_\ell}\left[\sum_{i=1}^{N}\lambda_{ni,t}\left(\frac{\tilde{A}_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}\right].$$

F Solution Algorithm

In this section we describe the algorithm used to compute the dynamic spatial growth model.

F.1 Algorithm to Solve for the Sequential Equilibrium Given Initial Conditions

In what follows, we describe the algorithm to solve the detrended model given an initial allocation of the economy, $\left(\{L_{i,0}\}_{i=1}^N, \left\{\tilde{K}_{i,0}\right\}_{i=1}^N, \{\lambda_{in,0}\}_{i,n=1}^N, \{\mu_{in,-1}\}_{i,n=1}^N, \left\{\tilde{A}_{i,0}\right\}_{i=1}^N\right)$, and given an unanticipated convergent sequence of changes in fundamentals, $\left\{\{\hat{m}_{in,t}\}_{i,n=1}^N, \{\hat{\kappa}_{in,t}\}_{i,n=1}^n\right\}_{t=1}^\infty$. We first describe the algorithm to solve the model under the given initial conditions and constant fundamentals going forward; namely, with $\left\{\{\hat{m}_{in,t}=1\}_{i,n=1}^N, \{\hat{\kappa}_{in,t}=1\}_{i,n=1}^n\right\}_{t=1}^\infty$. We then describe how to solve the model under a change in fundamentals.

1. Initiate the algorithm at $t=0$ with a guess for the path of $\left\{\hat{u}_{i,t+1}^{(0)}\right\}_{t=0}^T$, where the superscript (0) indicates that it is a guess. The path should converge to $\hat{u}_{i,T+1}^{(0)}=1$ for sufficiently large T.

2. For all $t \geq 0$, use $\left\{\hat{u}^{(0)}_{i,t+1}\right\}_{t=0}^{T}$ and $\{\mu_{in,-1}\}_{i,n=1}^{N}$ to solve for the path of $\left\{\{\mu_{in,t}\}_{i,n=1}^{N}\right\}_{t=0}^{T}$ using equation (E.2).

3. Use the path for $\left\{\{\mu_{in,t}\}_{i,n=1}^{N}\right\}_{t=0}^{T}$ and $\{L_{i,0}\}_{i=1}^{N}$ to obtain the path for $\left\{\{L_{i,t+1}\}_{i=1}^{N}\right\}_{t=0}^{T}$ using equation (E.3).

4. Solve for the trade equilibrium:

   (a) For each $t \geq 0$, given $\hat{L}_{i,t+1}$, define the term $\hat{\omega}_{i,t} = \tilde{w}^{\xi}_{i,t}\tilde{r}^{1-\xi}_{i,t}$. Guess a value for $\hat{\omega}_{i,t+1}$.

   (b) Obtain $\hat{x}_{i,t+1}$, $\hat{P}_{i,t+1}$, $\lambda_{in,t+1}$, $\tilde{R}_{i,t+1}$, $\tilde{K}_{i,t+1}$, and $\hat{A}_{i,t+1}$ using equations (E.4), (E.5), (E.6), (E.8) and (E.9). Use the fact that $\hat{r}_{i,t+1} = \hat{w}_{i,t+1}\hat{L}_{i,t+1}/\hat{K}_{i,t+1}$ and $\hat{w}_{i,t+1} = \hat{\omega}_{i,t+1}\left(\hat{K}_{i,t+1}/\hat{L}_{i,t+1}\right)^{1-\xi}$.

   (c) Check if the market clearing condition (E.7) holds using $\hat{w}_{i,t+1} = \hat{\omega}_{i,t+1}\left(\hat{K}_{i,t+1}/\hat{L}_{i,t+1}\right)^{1-\xi}$. If it does not, go back to step (a) and adjust the initial guess for $\hat{\omega}_{i,t+1}$ until labor markets clear.

   (d) Repeat steps (a) through (d) for each period t and obtain paths for $\left\{\hat{w}_{i,t+1}, \hat{P}_{i,t+1}\right\}_{t=0}^{T}$ for all i.

5. For each t, use $\mu_{in,t}$, $\hat{w}_{i,t+1}$, $\hat{P}_{i,t+1}$, and $\hat{u}^{(0)}_{n,t+2}$ to solve backwards for $\hat{u}^{(1)}_{i,t+1}$ using equation (E.1). This solution delivers a new path for $\left\{\left\{\hat{u}^{(1)}_{i,t+1}\right\}_{i=1}^{N}\right\}_{t=0}^{T}$, where the superscript 1 indicates an updated value for $\hat{u}$.

6. Check whether $\left\{\left\{\hat{u}^{(1)}_{i,t+1}\right\}_{i=1}^{N}\right\}_{t=0}^{T} \approx \left\{\left\{\hat{u}^{(0)}_{i,t+1}\right\}_{i=1}^{N}\right\}_{t=0}^{T}$. If it does not, go back to step 1 and update the initial guess with $\left\{\left\{\hat{u}^{(1)}_{i,t+1}\right\}_{i=1}^{N}\right\}_{t=0}^{T}$.

F.2 Solving for Counterfactual Changes in Fundamentals

We now describe how to solve the dynamic spatial growth model given an unanticipated convergent sequence of changes in fundamentals, $\hat{\Theta}_{t+1} = \left\{\{\hat{m}_{in,t}\}_{i,n=1}^{N}, \{\hat{\kappa}_{in,t}\}_{i,n=1}^{n}\right\}_{t=1}^{\infty}$.

The algorithm used to solve for a change in fundamentals follows the same steps described in the previous section, but the sequence of changes in fundamentals is fed into the model. The main difference from the previous section is that we now consider the fact that agents are surprised in the first period by the changes in fundamentals. The surprise in the changes in fundamentals is captured in the initial gross flow equation. That is, in the first period we now use the following equilibrium condition:

$$\mu_{in,1}(\hat{\Theta}) = \frac{\vartheta_{in,0}\left(\hat{u}_{n,2}(\hat{\Theta})\right)^{\beta/\nu}\left(\hat{m}_{in,1}\right)^{-1/\nu}}{\sum_{i=1}^{N}\vartheta_{ih,0}\left(\hat{u}_{h,2}(\hat{\Theta})\right)^{\beta/\nu}\left(\hat{m}_{ih,1}\right)^{-1/\nu}},$$

where $\vartheta_{in,0} = \mu_{in,0}\exp\left(V_{n,1}(\hat{\Theta}) - V_{n,1}\right)^{\beta/\nu}$. Therefore, we also use the equilibrium condition,

$$\log(\hat{u}_{i,1}) = \log\left(\hat{w}_{i,1}/\hat{P}_{i,1}\right) + \nu\log\left(\sum_{n=1}^{N}\vartheta_{in,0}\left(\hat{u}_{n,2}\right)^{\beta/\nu}\left(\hat{m}_{in,1}\right)^{-1/\nu}\right).$$

G Data Sources and Empirical Moments

In this section of the appendix, we describe in more detail the data sources and construction used in the quantitative analysis.

List of Provinces. The geographic units used in the quantitative analysis are Chinese provinces and the rest of the world. Strictly speaking, the province-level administrative divisions in China include provinces, autonomous regions, and municipalities under the direct jurisdiction of the central government. For simplicity, we refer to these highest-level administrative divisions of China as provinces. These provinces are Beijing, Tianjin, Hebei, Shanxi, Inner Mongolia, Liaoning, Jilin, Heilongjiang, Shanghai, Jiangsu, Zhejiang, Anhui, Fujian, Jiangxi, Shandong, Henan, Hubei, Hunan, Guangdong, Guangxi, Hainan, Sichuan, Guizhou, Yunnan, Tibet, Shaanxi, Gansu, Qinghai, Ningxia, and Xinjiang.

Province-Level Data and National Accounts. We obtain China's GDP, employment, from the China Compendium of Statistics, 1949-2008, and for the export, and import data we also use the China's Statistics Yearbook, as we describe below.[1] The China Compendium of Statistics consists of three main parts. The first part contains data at the national level compiled by the National Bureau of Statistics. The second part presents data from provinces, autonomous regions, and municipalities under the direct jurisdiction of the central government; the data are complied by local statistical bureaus. The third part provides data from the Special Administrative Regions (SARs) of Hong Kong and Macao that have been edited by the National Bureau of Statistics. The national GDP, employment, and trade data do not include those of the Hong Kong SAR, Macao SAR, or Taiwan Province.

[1] The digitized data can be extracted from the China statistical yearbooks available at https://data.cnki.net/yearBook/single?id=N2010042091.

We make several adjustments to the data. First, the Chinese national accounts are based on data provided by local governments to the National Bureau of Statistics (Bai, Hsieh, and Qian (2006), and Chen, Chen, Hsieh, and Song (2019)). Given the incentive of local governments to overstate the local GDP and other measurement discrepancies, the National Bureau of Statistics adjusts the data reported from the local governments to calculate the national-level GDP using independent data sources. Consequently, the reported aggregate GDP is generally lower than the sum of reported province-level GDP. We address this issue by scaling down province-level GDP by the same proportion for each Chinese province to match the reported GDP at the national level. We follow the same strategy to adjust province-level employment, export, and import data to match their reported national aggregates.

Second, we account for the changing status of Chongqing. Before 1997, Chongqing was not considered a municipality under the direct jurisdiction of the central government. One of the focuses of this paper is to understand the rise of China in 1990s. For most of this period, Chongqing was still part of Sichuan, we thus treat Chongqing and Sichuan as an integrated province, Sichuan-Chongqing, throughout our paper. We aggregate relevant variables for the two regions.

Third, for some provinces, the measurement units are not consistent with the those of the national aggregates. For example, the export and import data of Guangdong Province are inaccurately reported by the local statistical bureau in units of 100 million Chinese Yuan, although the indicated unit is still 10,000 Chinese Yuan. We carefully checked and addressed this type of issues in the data.

International Trade Data. We obtain province-level international exports and imports data from two data sources. For the years 1995, 2000, 2005, and 2008, we rely on the data from China's Statistics Yearbook 1996, 2001, 2006, and 2009. We obtain export and import values by firm location for each province. For earlier years in 1990s, China's Statistics Yearbook does not provide exports and imports data at the province level. Therefore, we obtain province-level international trade data in 1990 from the China Compendium of Statistics, 1949-2008.

One potential concern about the international trade data in the 1990s is that there were two government agencies that reported exports and imports data in the last four decades in China: the first one is the Ministry of Trade, and the second one is the Customs. The names of the Ministry of Trade's provincial counterparts can be different; for example, in Shanghai, it was called the Foreign Trade and Economic Cooperation Commission, which was in charge of collecting exports and imports data in Shanghai before 1999. For historical reasons, in the 1980s and 1990s, the trade data for many provinces were reported by the Ministry of Trade, while in the recent two decades, almost all provinces' trade data has been reported by their customs.

The province-level international trade data from the China Compendium of Statistics are consistent with those from China's Statistics Yearbook for year 2000 onwards. For those years, we rely on the data from the China's Statistics Yearbook, as it clearly states the definition and the methodology to report the province-level international trade flows between provinces and the rest of the world. For the year 1995, due to the mixed sources providing province-level exports and imports data, the trade data from some provinces in the Compendium have some discrepancies from China's Statistics Yearbook. For the year 1995 the international trade data in the Compendium for the provinces of Beijing, Fujian, Guangdong, Guizhou, Hebei, Heilongjiang, Henan, Hunan, Jiangxi, Liaoning, Shaanxi, Shandong, Shanxi, Sichuan and Zhejiang match with those provided by the Yearbook, with a discrepancy smaller than 3 percent, while the discrepancy is larger for the rest of the provinces. We use the deviation of the Compendium data from the Yearbook data in the year 1995 to adjust the years 1985 and 1990. One concern might be that the data sources for 1985, 1990 and 1995 in the Compendium might differ, so this adjustment might not be plausible. Assuringly, the international trade data for all provinces but Gansu are reported by the same government agency in all these years.[2]

Internal Trade Data. As mentioned in the main text, there is no inter-provincial trade data compiled as official statistics for 1990s, although there are some related data sources that we describe in what follows that could be useful for imputation. The earliest multi-region input-output table available is by Ichimura and Wang (2003), where the authors construct a seven-regions and nine-sectors input-output table for the year 1987. However, the first official statistics on interregional trade is the eight-regions and eight-sectors input-output table released for the year 1997. Another useful data on internal trade is the provincial input-output tables for 1987, 1992, and 1997 used by Poncet (2003). In this data, eleven provinces in 1992 and seven provinces in 1997 separate trade inflows and outflows into domestic and foreign flows, but not bilaterally across provinces. The author uses the data to input provinces' trade inflows from and outflows to the rest of China using a gravity structure. For the rest of the provinces, one can not distinguish the trade inflows from (outflows to) the rest of the world or the rest of the country. An alternative is to use the railroad shipment data between provinces. The available digitized version ranges from 1997 to 2007 and contains inter-province shipments by railroads. To input inter-province trade

[2]The international trade data for Gansu in 1985 and 1990 are reported by the Ministry of Foreign Trade, while those in 1995 are from Customs Statistics. Gansu represents a very minor fraction of international trade in China; its exports in 1990 accounted for 0.24 percent of the national total exports, while its imports accounted for 0.18 percent of the national total imports in the same year.

flows, it would require making an assumption about the relationship between aggregate bilateral trade flows and shipment flows or about trade costs.

GDP Data. We use the GDP deflator from the World Development Indicators compiled by the World Bank, to compute the real GDP of each province at 1990 prices. We rely on the Penn World Table 10.0 (PWT 10.0) to construct data for the rest of the world. The PWT 10.0 reports real GDP at constant 2017 national prices (rgdpna) and employment (emp). We first keep all countries but China. Second, we drop countries with missing data for either GDP or employment. We aggregate all countries in our sample to obtain GDP and employment for the rest of the world. The World Development Indicators database reports the world GDP deflator from 1985 to 2017. Combining the two data sources, we compute GDP for the rest of the world at current year prices and real GDP at 1990 prices. We express GDP, exports, and imports in 100 million USD, while employment is measured in units of 10,000 people.

Capital Stock. We follow Shan (2008) to estimate province-level capital stock from 1952 to 2010. We use the perpetual inventory method to estimate the time series of capital stock. For capital stock at the base year, we follow Young (2003), using 10 percent of the gross capital formation in 1952. As Young (2003) and Bai et al. (2006) argue, the most appropriate measure of investment in China is fixed capital formation. We obtain this measure from the China Compendium of Statistics. The investment price deflator is constructed by Shan (2008) based on official statistics. We follow Shan (2008) to choose the value for the depreciation rate.

For the rest of the world, we obtain capital stock at constant 2017 national prices from the PWT. We deflate country-level capital stock to reflect 1990 national prices using the GDP deflator. We further adjust the capital stock of the rest of the world by matching the percentage gross fixed capital formation in GDP compiled by the World Bank. We start from the aggregate capital stock of all countries (including China) in 1985 according to the PWT. We adjust for the aggregate capital stock in the years 1990, 1995, and 2000 to match the average gross capital formation (percentage of GDP) in 1985-1990, 1990-1995, and 1995-2000, respectively. Afterward, by excluding the capital stock of China, we obtain the capital stock for the rest of the world.

Input Shares. We compute the values of $\gamma = 0.38$ and $\xi = 0.54$, which correspond to the parameter values for the year 1990 from world's aggregates in the Eora multi-region input-output table.

Gross Migration Flows. We use the Chinese census data from IPUMS to construct the migration flow matrix. Our constructed migration flow matrix matches the employment share of each province reported in the China Compendium of Statistics. We leverage the 1% samples of the 1990 and 2000 censuses from IPUMS as our data source to calculate migration flows for 1985-1990, 1990-1995, and 1995-2000.

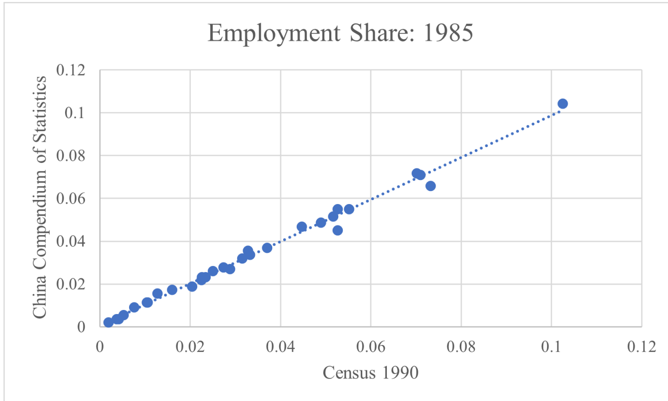
To construct the migration flows for 1985-1990, we proceed as follows. With the 1% sample of the 1990 census, we keep any census respondent who is actively employed in 1990 with age between 15 and 64. We put a weight on each province to match exactly the provincial employment share shown in the census with that of each province reported by the China Compendium of Statistics. For each individual, we determine the Hukou registration location as follows. For the 1990 census, the status and nature of registration was asked. If the person chose "(1) residing and registered here", we use the person's location in 1990 as the registration location; if the person chose "(2) residing here over 1 year, but registered elsewhere", "(3) living here less than 1 year absent from registration place over 1 year" or "(4) living here with registration unsettled", we use the person's location in 1985 as the registration location.

For a person whose Hukou registration is in category (2)-(4) but who lived in the same province (stayer) in 1985 and 1990, we assign a Hukou place to them as follows. We first construct a sample of migrants who switched their habitant province between 1985 and 1990, as measured in the data. Then, for each destination province, we compute the share of migrants coming from different origin provinces. We assign the Hukou place to the aforementioned stayer according to this share.[3] For each Hukou location (province-level), we construct a five-year migration flow matrix from origin province to destination province. Combining the migration matrix and the data in 1990, we can check whether the employment share of each province out of the nationwide total employment is consistent with the data from the China Compendium of Statistics (see Figure G.1). Using the same method, we calculate migration flow between 1995 and 2000 using the 1% sample of the 2000 census from IPUMS. For the migration flow between 2000 and 2005, we use the 2005 Mini Census.[4] To the best of our knowledge, there are no publicly available micro-level people census data from 1995. When necessary, we thus use as proxy the migration flows from 1985 to 1990 for the flows from 1990 to 1995.

[3] A potential concern is step migration, i.e., a person does not directly migrate from her registration location to the current location. We cannot check this using the 1990 census. Imbert, Seror, Zhang, and Zylberberg (2022) uses 2005 mini census data to show that step migration was negligible in 2000-2005. We do not expect this to be any different for the period 1985 to 1990.

[4] We thank Jingting Fan for sharing the 2005 Mini Census data.

Figure G.1: Data validation



H Initial Stock of Knowledge, Empirical Moments, and TFP Residuals

In this appendix, we derive the model inversion to compute the initial local stock of knowledge. We then describe the moment conditions used to estimate the diffusion parameters, and provide evidence of assumptions in our empirical approach.

H.1 Initial Stock of Knowledge

In this section we derive the model inversion used to estimate the initial local stock of knowledge. We start from the domestic expenditure $\lambda_{nn,0} = A_{n,0}\left(\frac{x_{n,0}}{P_{n,0}/T}\right)^{-\theta}$. Using this equation, we obtain

$$A_{n,0} = \left(\frac{B\left(w_{n,0}^{\xi} r_{n,0}^{1-\xi}\right)^{\gamma} P_{n,0}^{1-\gamma}}{P_{n,0}/T}\right)^{\theta} \lambda_{nn,0}.$$

Using the first-order condition of the firm's problem, $\frac{w_{n0}L_{n0}}{r_{n0}K_{n0}} = \frac{\xi}{1-\xi}$, we obtain

$$A_{n,0} = (BT)^{\theta}\left(\frac{1-\xi}{\xi}\right)^{(1-\xi)\gamma\theta}\left(\frac{\frac{w_{n,0}L_{n0}}{P_{n,0}}}{(K_{n,0})^{1-\xi}(L_{n,0})^{\xi}}\right)^{\gamma\theta}\lambda_{nn,0}.$$

Finally, using the fact that $w_{n,0}L_{n0} = \xi\left(w_{n,0}L_{n,0} + r_{n,0}K_{n,0}\right)$, we find that the initial stock of knowledge across locations is given by

$$A_{n,0} = \Upsilon\left(\frac{Real\ GDP_{n,0}}{(K_{n,0})^{1-\xi}(L_{n,0})^{\xi}}\right)^{\gamma\theta}\lambda_{nn,0},$$

where $\Upsilon = (BT)^{\theta} (1-\xi)^{(1-\xi)\gamma\theta} (\xi)^{\xi\gamma\theta} = \Gamma\left(1 - \frac{\eta-1}{\theta}\right)^{\frac{\theta}{1-\eta}} \left[\gamma^{-\gamma} (1-\gamma)^{\gamma-1}\right]^{\theta}$.

H.2 Empirical Moments

Table H.1 presents the empirical moment conditions targeted to discipline the elasticities that govern innovation and idea diffusion $(\alpha_0, \rho_l, \rho_m)$, and the model-implied moments predicted by the evolution of fundamental productivity using equation (9).

Table H.1: Moment Conditions

Moment	Data	Model
Moment 1	34.5	24.1
Moment 2	0.84	0.60
Moment 3	0.81	0.12
Moment 4	0.23	0.31
Moment 5	-1.7	-1.9

Note: Moment 1 is the average change in fundamental productivity levels across locations. Moment 2 is the average growth rate in fundamental productivities. Moment 3 is the variance in the time changes in fundamental productivity levels (in thousands). Moment 4 is the covariance between the initial fundamental productivities and the change in fundamental productivity levels (in thousands). Moment 5 is the covariance between the initial fundamental productivities and the growth rate in fundamental productivities.

Notice that in the GMM estimation, g_α depends itself on ρ_l and ρ_m. From our balanced growth path formulas, we established that

$$g_\alpha = (1 + g_{TFP})^{\gamma\theta(1-\rho_l-\rho_m)} - 1. \tag{H.1}$$

As explained in the main text, we compute g_{TFP} by assuming the U.S. economy was on the balanced growth path in the 1990, and obtain $g_{TFP} = 0.03974$.[5]

Hence, in the GMM estimation, we obtain g_α in the following steps.

Step 0: We first guess a value of $g_\alpha^{(1)}$. Let $g_\alpha = g_\alpha^{(1)}$.

Step 1: We get our GMM estimates $\left(\alpha_0^{(1)}, \rho_l^{(1)}, \rho_m^{(1)}\right)$.

Step 2: With $\left(\alpha_0^{(1)}, \rho_l^{(1)}, \rho_m^{(1)}\right)$ and g_{TFP} from the data, we get $g_\alpha^{(2)}$ using (H.1).

Step 3: If $g_\alpha^{(1)}$and $g_\alpha^{(2)}$are close enough, our estimates will be $\left(\alpha_0^{(1)}, \rho_l^{(1)}, \rho_m^{(1)}\right)$. Otherwise, we update $g_\alpha = g_\alpha^{(2)}$ and repeat step 1-3.

[5]The U.S. TFP data in the 1990s is extracted from https://fred.stlouisfed.org/series/RTFPNAUSA632NRUG. We use data between 1990 and 2000 to calculate the five-year growth rate of TFP.

H.3 Unobserved TFP Residuals

As we discussed in the main text, in our empirical strategy we allow for an unobserved residual to explain the differences in observed and model-implied TFP. Our assumption is that the unobserved TFP shocks across locations during the estimation are uncorrelated across time. To provide support to this assumption, we write the empirical counterpart of equation (9) as follows:

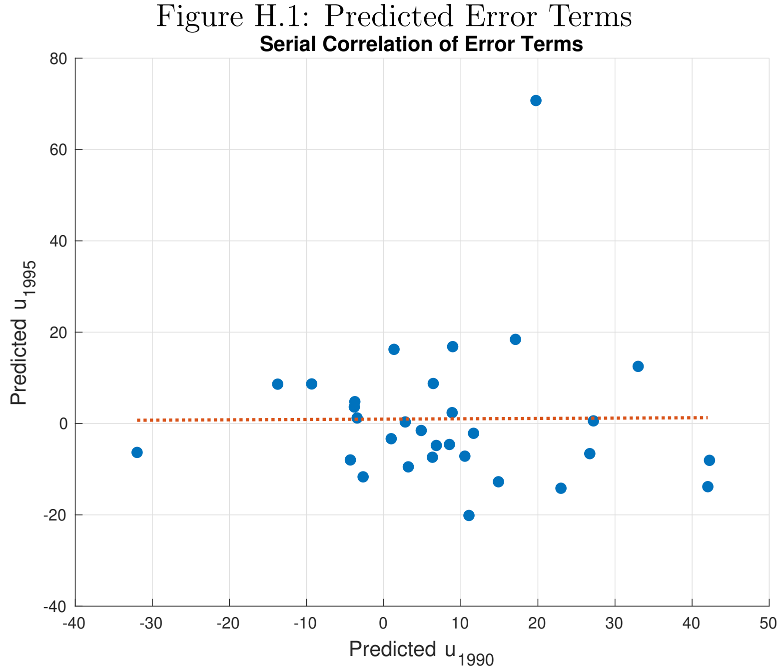
$$A_{n,t+1} - A_{n,t} = \alpha_t \Gamma_{\rho_\ell,\rho_m} \left[\sum_{i=1}^{N} s_{in,t} \left(A_{i,t} \right)^{\rho_\ell} \right] \left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}} \right)^{\rho_m} \right] + u_{n,t}.$$

With our estimates $\hat{\alpha}_t, \hat{\rho}_l, \hat{\rho}_m$, we obtain the predicted residuals,

$$\hat{u}_{n,t} = \left(A_{n,t+1} - A_{n,t} \right) - \hat{\alpha}_t \Gamma_{\hat{\rho}_\ell,\hat{\rho}_m} \left[\sum_{i=1}^{N} s_{in,t} \left(A_{i,t} \right)^{\hat{\rho}_\ell} \right] \left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}} \right)^{\hat{\rho}_m} \right],$$

and we then test whether $\hat{u}_{n,t}$ are serially correlated. We fail to reject the null hypothesis that they serially uncorrelated. Figure H.1 presents a scatter plot of the predicted $\hat{u}_{n,1995}$ against the predicted $\hat{u}_{n,1990}$.

Figure H.1: Predicted Error Terms

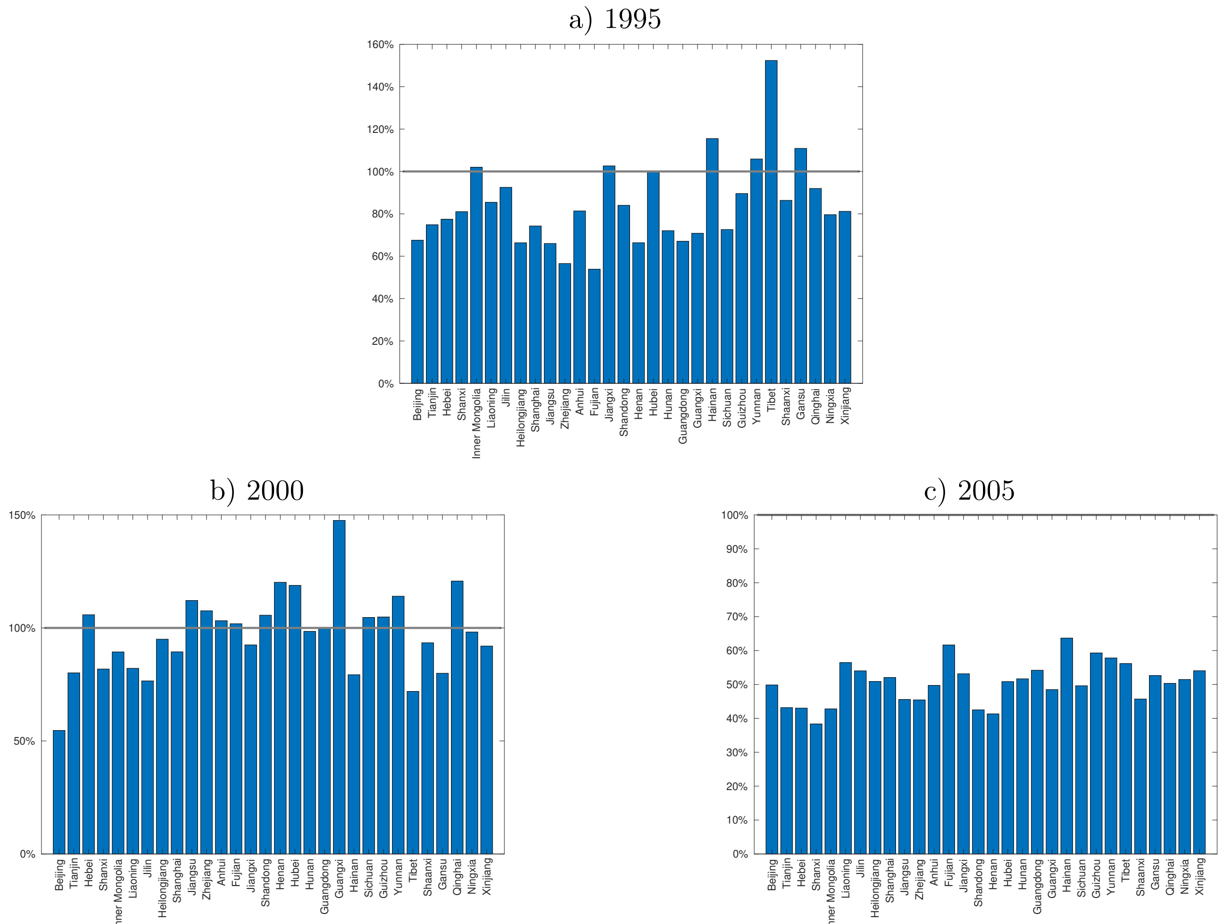


Note: The figure presents a scatter plot of the predicted residuals $\hat{u}_{n,1995}$ against the predicted residuals $\hat{u}_{n,1990}$.

H.4 Contribution of Idea Diffusion to TFP Growth

As explained in the main text, in our estimation strategy we confront the fact that part of the observed TFP evolution is shaped by determinants other than idea diffusion. In this section of the appendix, we look at the contribution of idea diffusion to the accumulation of knowledge stocks across provinces in China by using equation (9) and our estimates of diffusion parameters. To do so, we start from the year 1990 and use our estimates of ρ's and α, as well as our calibrated A_{1990} and the migration and trade shares $s_{in,1990}, \lambda_{in,1990}$ to calculate the model implied $A_{1995,model}$ from idea diffusion. We then plot the ratio between $A_{1995,model}$ and the observed $A_{1995,data}$ for each province. Figure H.2 shows the results for the years 1995, 2000, and 2005. We can see from the figures that idea diffusion does not match the observed TFP growth, which reinforce the fact that our empirical approach does not attribute all TFP growth to idea diffusion.

Figure H.2: Contribution of idea diffusion to TFP growth



Note: The figure presents the contribution of idea diffusion to the observed TFP growth for different time frames.

I Additional Quantitative Results

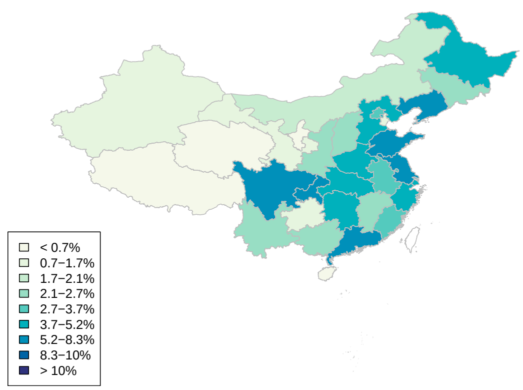
In this section of the appendix we describe additional results from our quantitative analysis.

I.1 Regional Distribution of Economic Activity

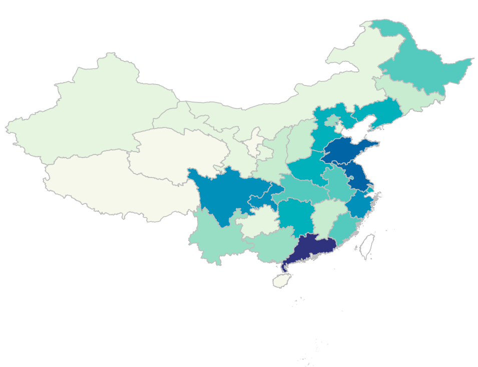
Figures I.1, and I.2 display the evolution of actual GDP shares in China and their evolution under initial 1990 conditions. The figure presents the GDP shares across provinces in China every five years during the period 1990-2010.

Figure I.1: Regional distribution of economic activity (GDP shares)

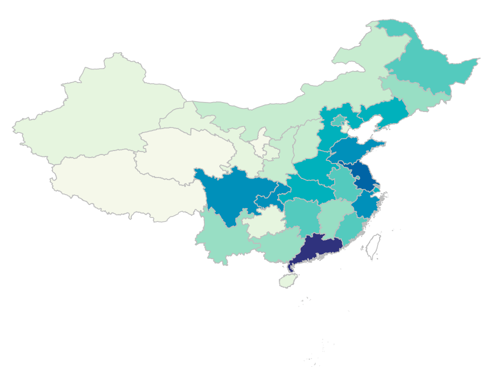
a) 1990



b) Actual
1995

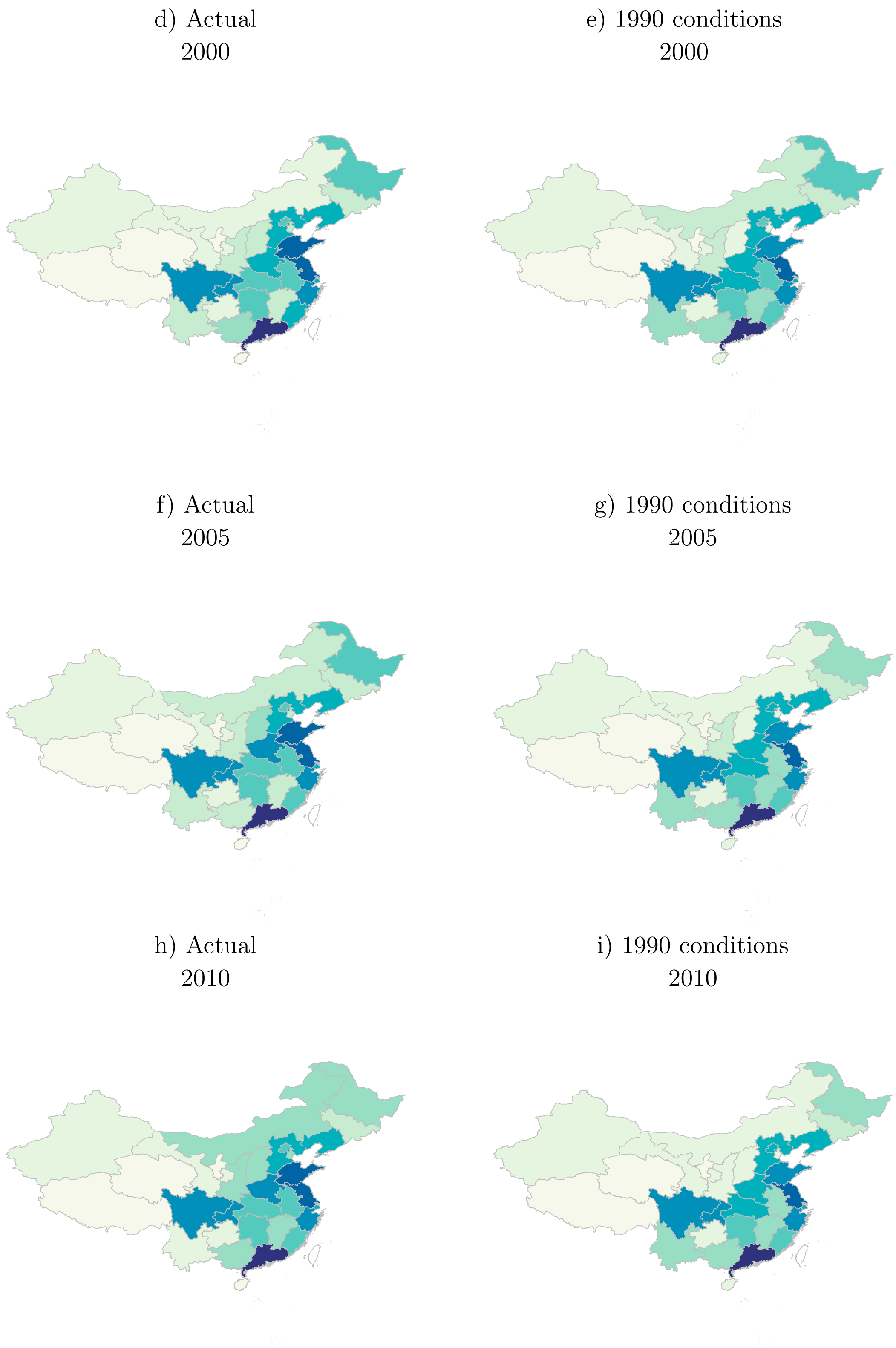


c) 1990 conditions
1995



Note: The figures show the distribution of economic activity across provinces in China, measured in GDP shares, in the data and with 1990 fundamentals over the period 1990-2010.

Figure I.2: Regional distribution of economic activity (GDP shares, continued)

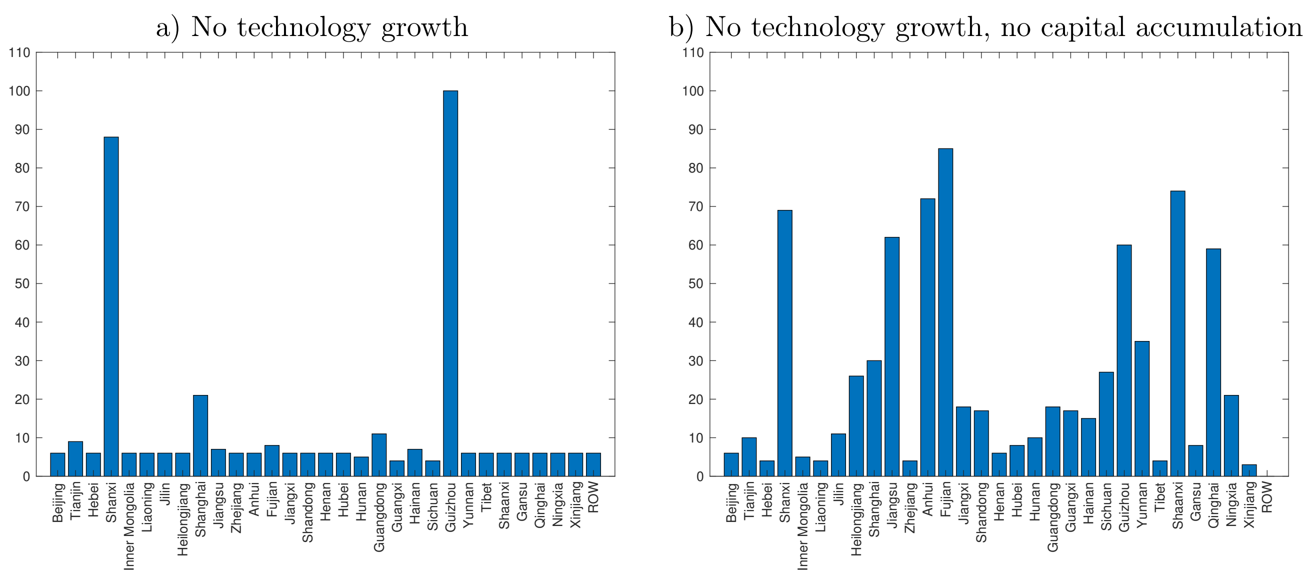


Note: The figures show the distribution of economic activity across provinces in China, measured in GDP shares, in the data and with 1990 fundamentals over the period 1990-2010.

I.2 Speed of Convergence

In this section, we report the half-life of real GDP convergence to the detrended steady-state across provinces to understand how different margins in the model affect the speed of transition. To do so, we compare different versions of the model. We start by computing the speed of convergence in a model with no technology growth, and compare versions of the model with and without capital accumulation. Figure I.3 illustrates the half-life of GDP convergence in each case. We find that the economy with capital accumulation converges, on average, at a faster rate to the detrended steady state. However, we also find spatial heterogeneity in the speed of convergence. In fact, when we take a closer look at the provinces, we see that the longer half-life in Shanxi and Guizhou makes the convergence of the economy as a whole to take longer in the version of the model with capital accumulation. These two provinces have much larger initial labor force than in steady state so that the marginal productivity of capital is high, and capital accumulates relatively fast as a result. During the transition, as labor moves out, capital eventually goes down to the detrended steady state. This overshooting results in slower convergence in the model with capital accumulation. The intuition of these results is in line with Kleinman et al. (2023), who find that convergence is slower when the gaps of capital and labor from steady-state are positively correlated across locations.

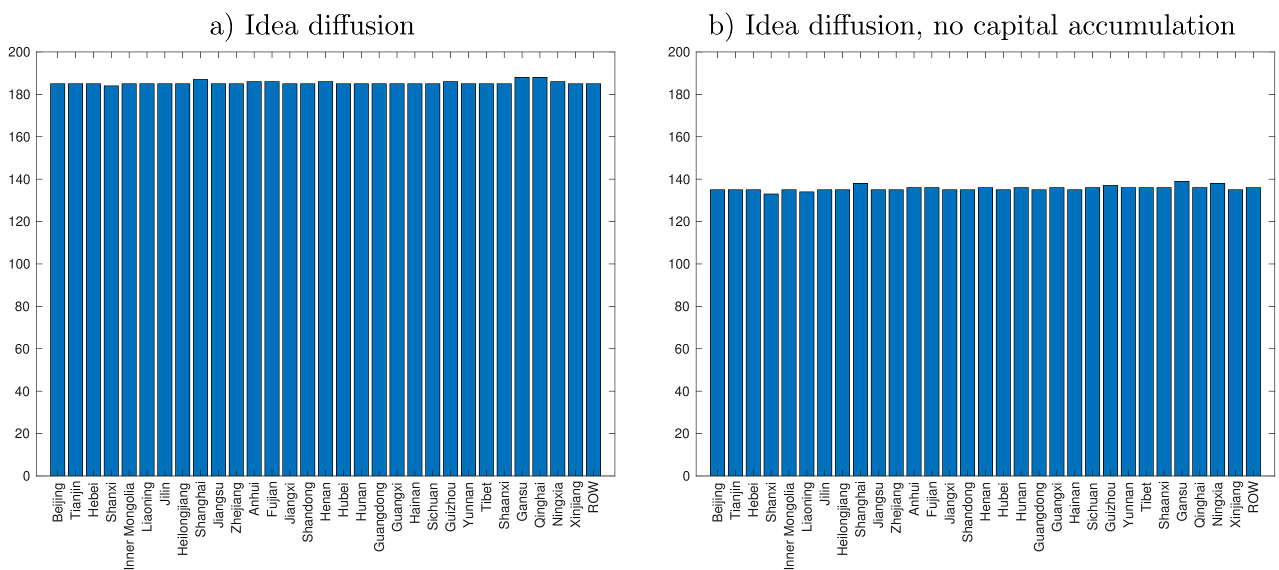
Figure I.3: Speed of convergence without idea diffusion



Note: The figures show the half life of real GDP convergence to the detrended steady-state. Panel (a) shows the speed of convergence in a model with no growth in the stock of knowledge, and Panel (b) presents the speed of convergence in a model with no technology growth and no capital accumulation.

We next look at the model with idea diffusion and compare the speed of convergence in the versions of the model with and without capital accumulation. By contrasting Figure I.3 and Figure I.4, we can see that convergence to the detrended steady state takes longer. It is not surprising that with idea diffusion, the economy converges to a different detrended steady state with a higher stock of knowledge, and the transition takes longer as a result. Similar to our previous finding, in the model with idea diffusion, capital accumulation makes the transition for the whole economy longer, but mainly through its interaction with the accumulation of knowledge stock. As the stock of knowledge increases, landlords always have an incentive to accumulate capital, which takes more time for the economy to converge to the detrended steady state.

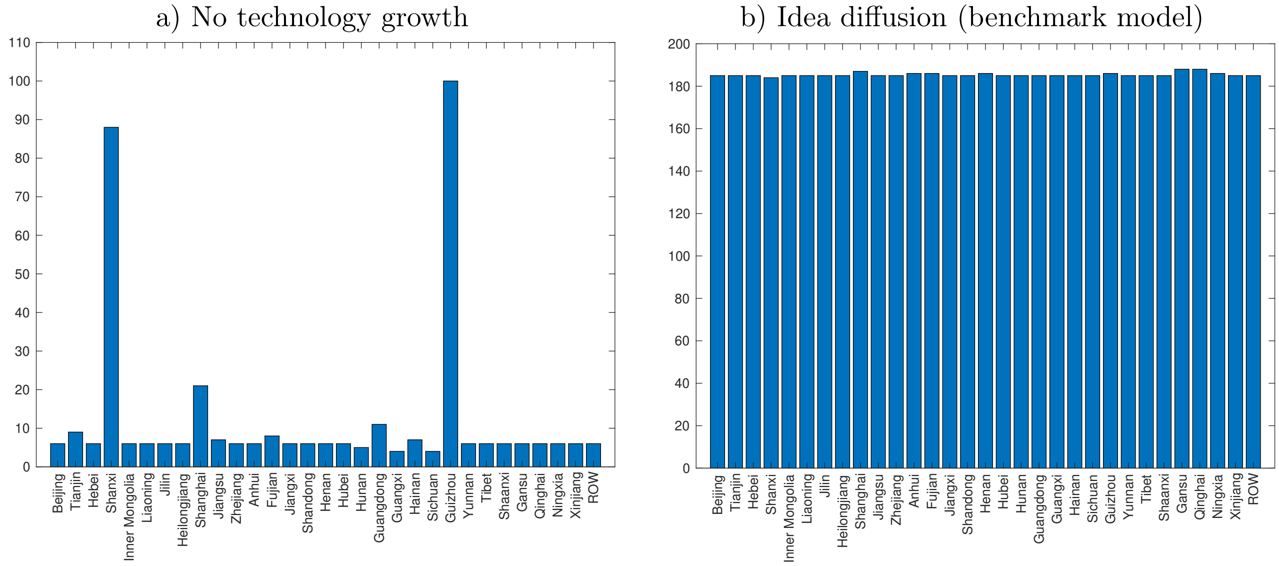
Figure I.4: Speed of convergence with idea diffusion



Note: The figures show the half life of real GDP convergence to detrended steady-state. Panel (a) shows the speed of convergence in a model with growth from idea diffusion and capital accumulation, and Panel (b) presents the speed of convergence in a model with technology growth but without capital accumulation.

To further understand the role of idea diffusion in the speed of convergence, Figure I.5 shows the speed of convergence in the versions of the model with and without growth in the stock of knowledge. It is clear from the figures that with idea diffusion, the convergence to the detrended steady state takes longer. The intuition is as follows. With technology growth, the detrended steady state is farther away from the initial allocations because the stock of knowledge is higher as a result of the idea diffusion. Additionally, in the economy with labor dynamics, capital accumulation, and growth in the stock of knowledge, all three allocations need to converge to their respective detrended steady state for the GDP convergence to be completed, making the transition longer.

Figure I.5: Speed of convergence with idea diffusion

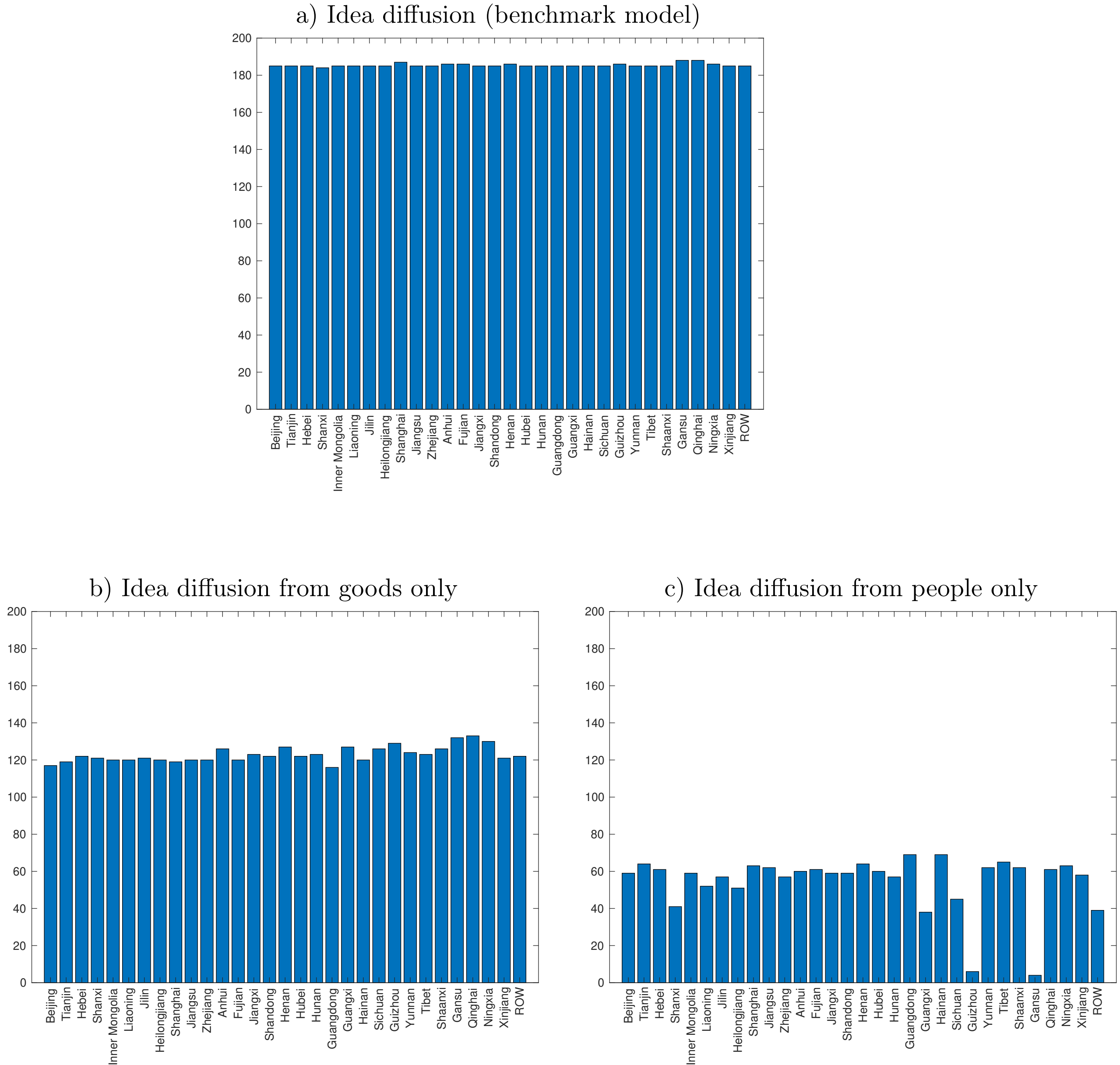


Note: The figures show the half life of real GDP convergence to detrended steady-state. Panel (a) shows the speed of convergence in a model with no growth in the stock of knowledge, and Panel (b) presents the speed of convergence in a model with technology growth from idea diffusion (the benchmark model).

Finally, Figure I.6 compares the speed of convergence with only idea diffusion from goods and only idea diffusion from people. Convergence takes longer in the benchmark model with both idea diffusion from goods and people. The main reason for this is that, with diffusion from both sources, the stock of knowledge in the detrended steady state is higher than with either one. Hence, the detrended steady state is farther away from the initial condition, which makes the transition longer.

We can also see that it takes longer for the economy to converge to the steady state with idea diffusion from goods only than with idea diffusion from people only. With idea diffusion from goods, provinces in China benefit from the global insights from the rest of the world, which has a higher stock of knowledge. Hence, the stock of knowledge in the detrended steady state is higher, and the transition is longer. With idea diffusion from people, provinces in China learn from each other through migration. As explained in the main text, the evolution of the stock of knowledge across provinces depends on how good the insights from migrants are, so the stock of knowledge in the detrended steady state is not as high.

Figure I.6: Speed of convergence with idea diffusion from goods or people

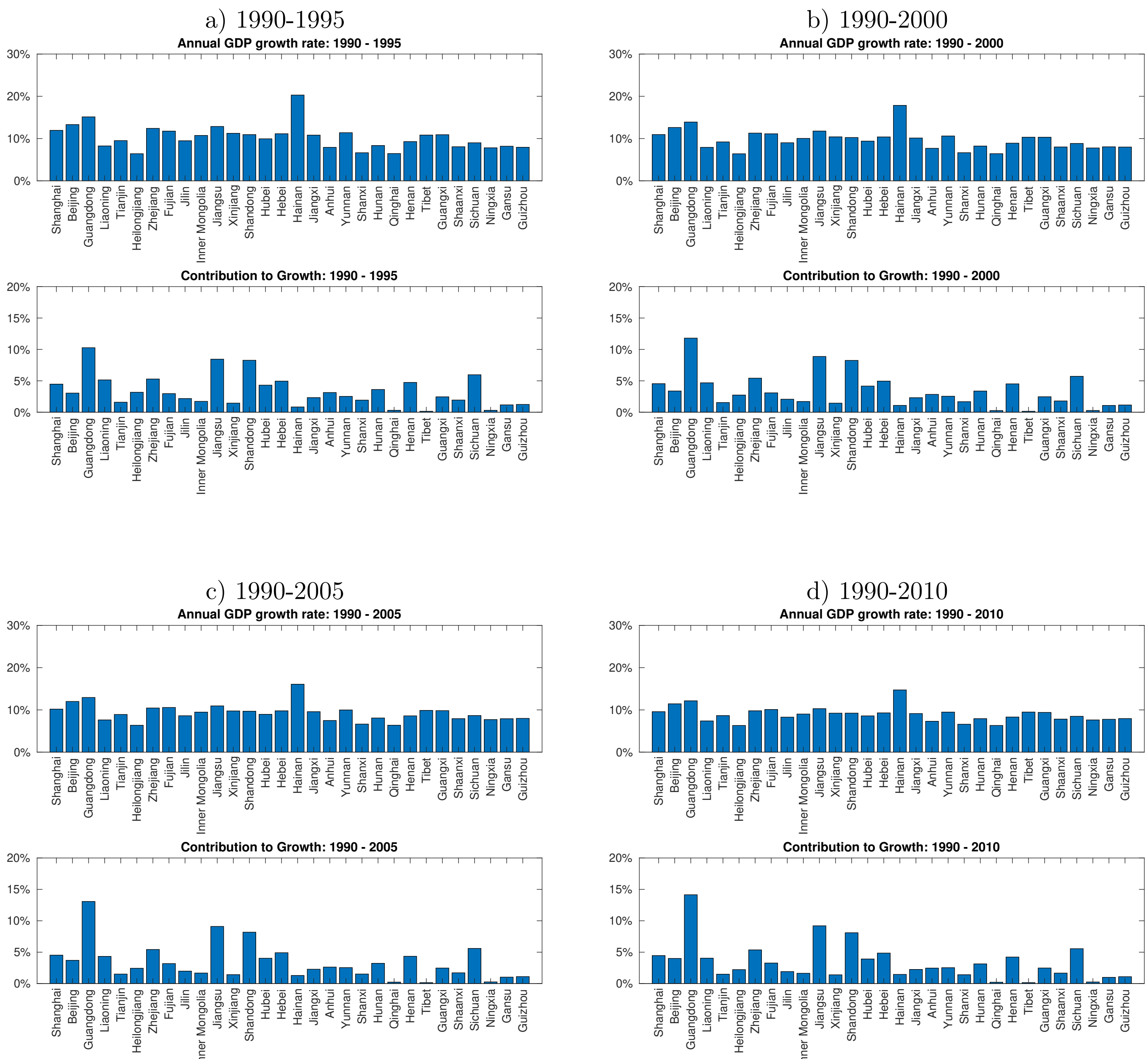


Note: The figures show the half life of real GDP convergence to detrended steady-state. Panel (a) shows the speed of convergence in a model with growth in the stock of knowledge, Panel (b) presents the speed of convergence in a model with idea diffusion from goods only, and Panel (c) presents the speed of convergence in a model with idea diffusion from people only.

I.3 Initial Conditions and Spatial Growth

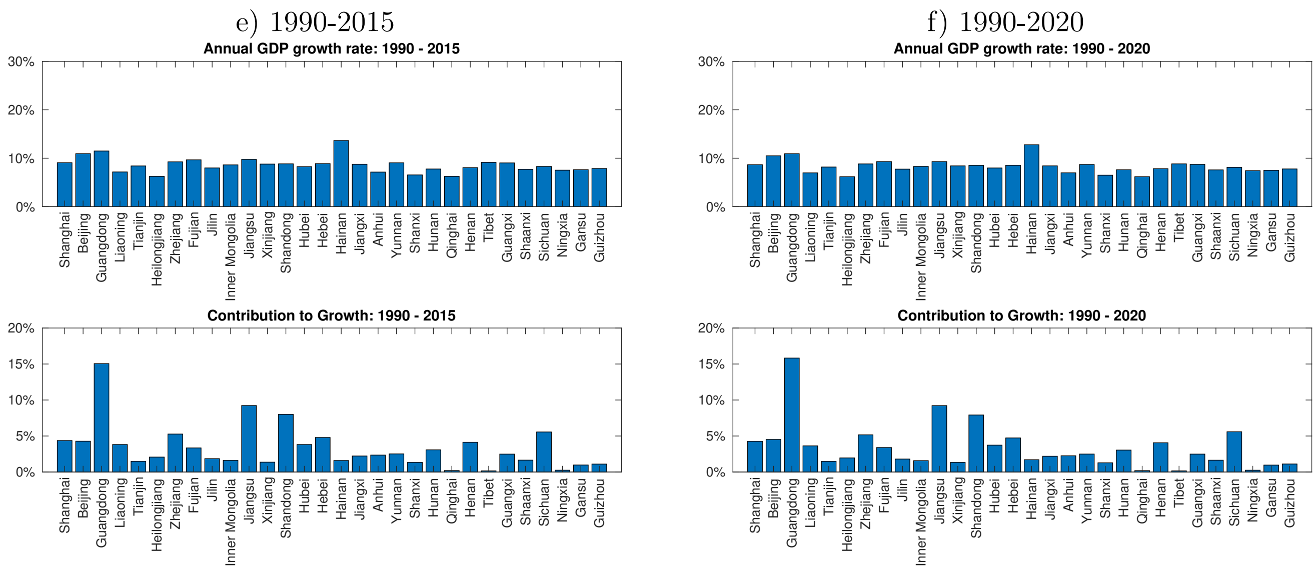
Figures I.7 and I.8 display spatial growth in China during different time frames under the initial conditions in 1990. In each panel, the upper figure presents the annual real GDP growth and the lower panel displays the contribution of each province to the aggregate real GDP growth in China during that period.

Figure I.7: Spatial growth (annual, percent)



Note: The figures show the annual real GDP growth across provinces and the contribution of each province to the aggregate growth in China in different time frames over the period 1990-2020. Spatial growth in each figure is computed in the model under the initial 1990 conditions.

Figure I.8: Spatial growth across provinces (annual, percent)



Note: The figures show the annual real GDP growth across provinces and the contribution of each province to the aggregate growth in China in different time frames over the period 1990-2020. Spatial growth in each figure is computed in the model under the initial 1990 conditions.

I.4 Spatial Growth from Idea Diffusion

In this section of the appendix we present the relative contribution of idea diffusion from goods and idea diffusion from people to spatial growth across provinces in China. The results illustrate the spatial heterogeneity in the contribution of idea diffusion to growth that shape their aggregate effects described in Table 1 in the main text.

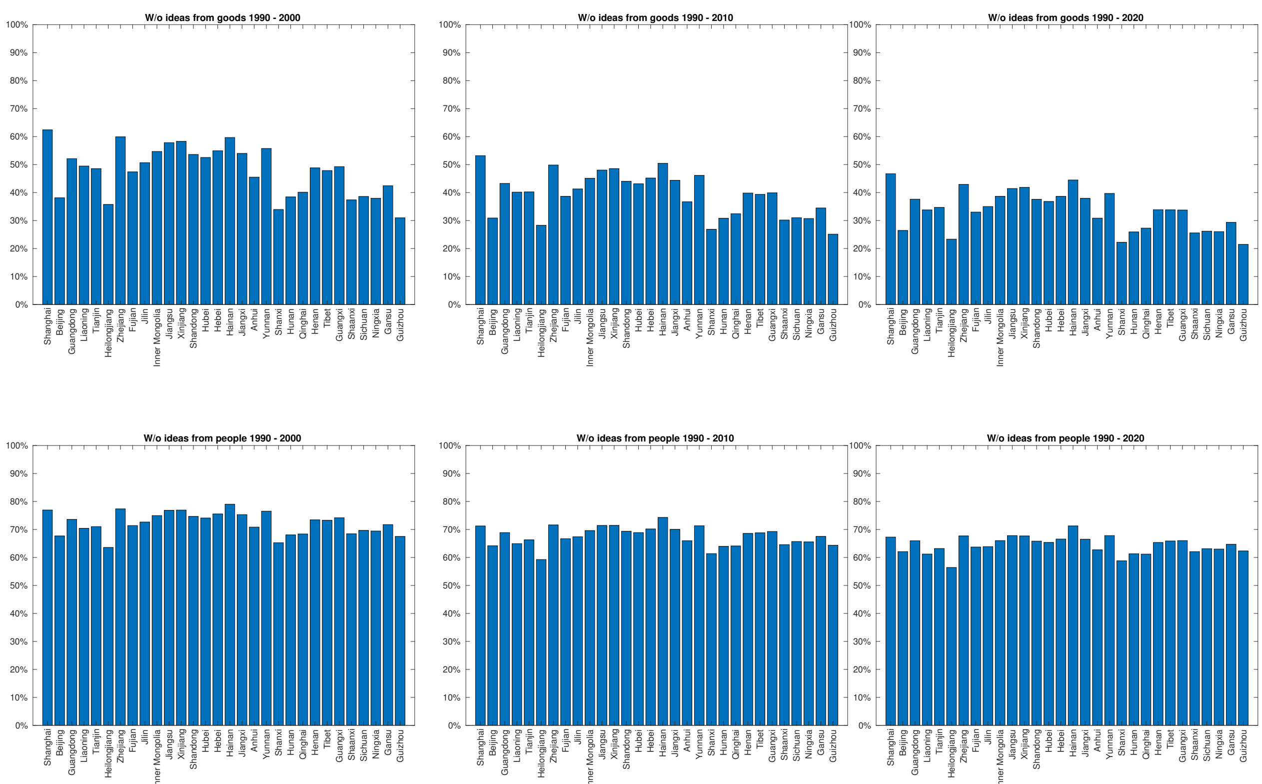
Figure I.9 presents the relative contribution of ideas from people and ideas from goods to spatial GDP growth across provinces in China for different time frames. To do this, we start with the benchmark model computed using the 1990 fundamentals (trade and migration costs). We then quantify the contribution of idea diffusion from goods by computing the model with $\rho_m = 0$, meaning that only idea flows from people contribute to the evolution of productivity, and compare the GDP growth in each province to the benchmark.

The upper panels of Figure I.9 present the results. We observe that if there were only idea diffusion from people, GDP growth in all provinces would have been lower than in the benchmark model. This sheds light on the importance of idea diffusion from goods to GDP growth. The contribution of idea diffusion from goods is also heterogeneous across space. Specifically, if there were only idea diffusion from people, GDP growth would have fallen in the range of 31 percent in Guizhou to 62 percent in Shanghai between 1990 and 2000.

We next conduct a similar analysis, but set $\rho_l = 0$, meaning that only idea flows from goods affect the accumulation of knowledge stocks. The second row of Figure I.9 presents the results. We observe that GDP growth in all provinces would have been lower, with spatial heterogeneity. Specifically, if there were only idea diffusion from goods, GDP growth would have fallen in the range of 20 percent to 40 percent across provinces in China.

Over time, as the stock of knowledge across provinces in China and the rest of the world increases, trade openness and better insights lead to a higher contribution of idea diffusion, notably from goods, to GDP growth. This results in a larger drop in GDP growth when the mechanism is shut down.

Figure I.9: Contribution of ideas from people and goods to growth (percentage points)

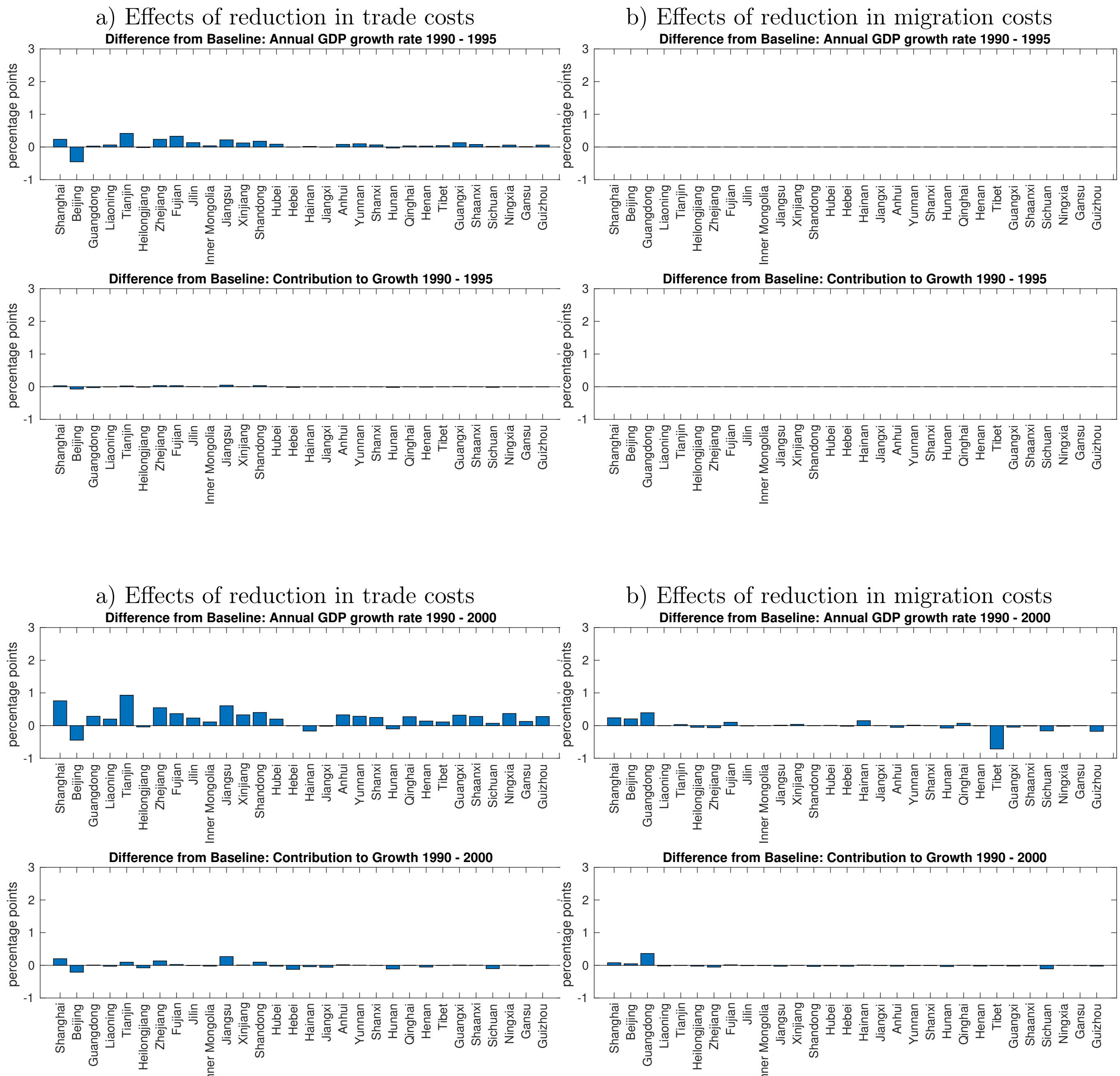


Note: The figures show the relative contribution of ideas from goods and ideas from people provinces with the 1990 initial conditions. For each province, we plot the ratio of GDP growth rate to the benchmark. For example, the first graph reads that without idea flows from goods, the GDP growth rate in Shanghai from 1990 and 2000 would have been about 38% lower than in the benchmark model.

I.5 Spatial Growth Effects of Trade and Migration Costs

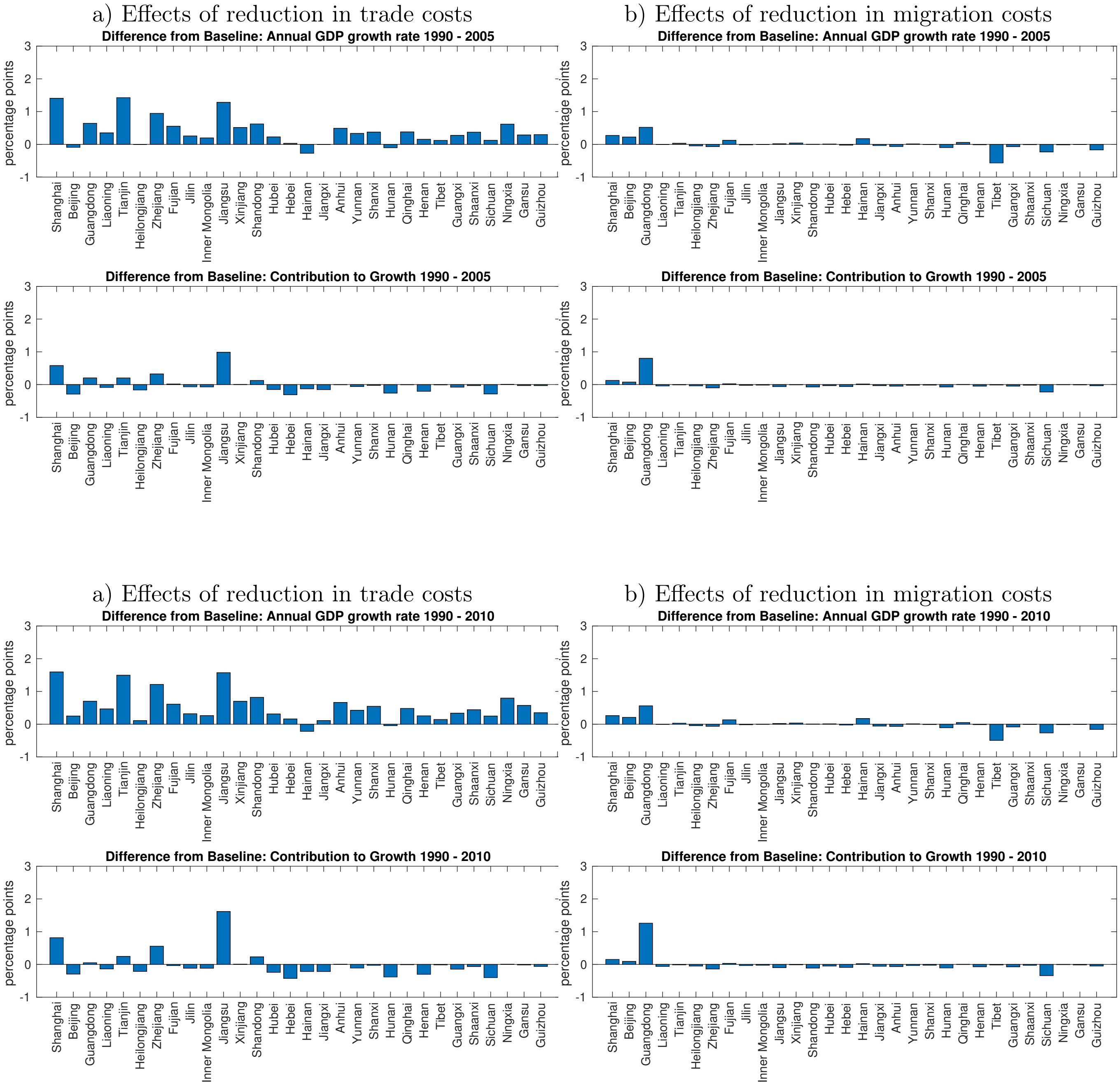
Figures I.10, I.11, and I.12 present the effects of changes in international trade costs and migration restrictions relative to the baseline economy with initial conditions in 1990 for alternative time windows.

Figure I.10: Effects of trade and migration costs on spatial growth (percentage points)



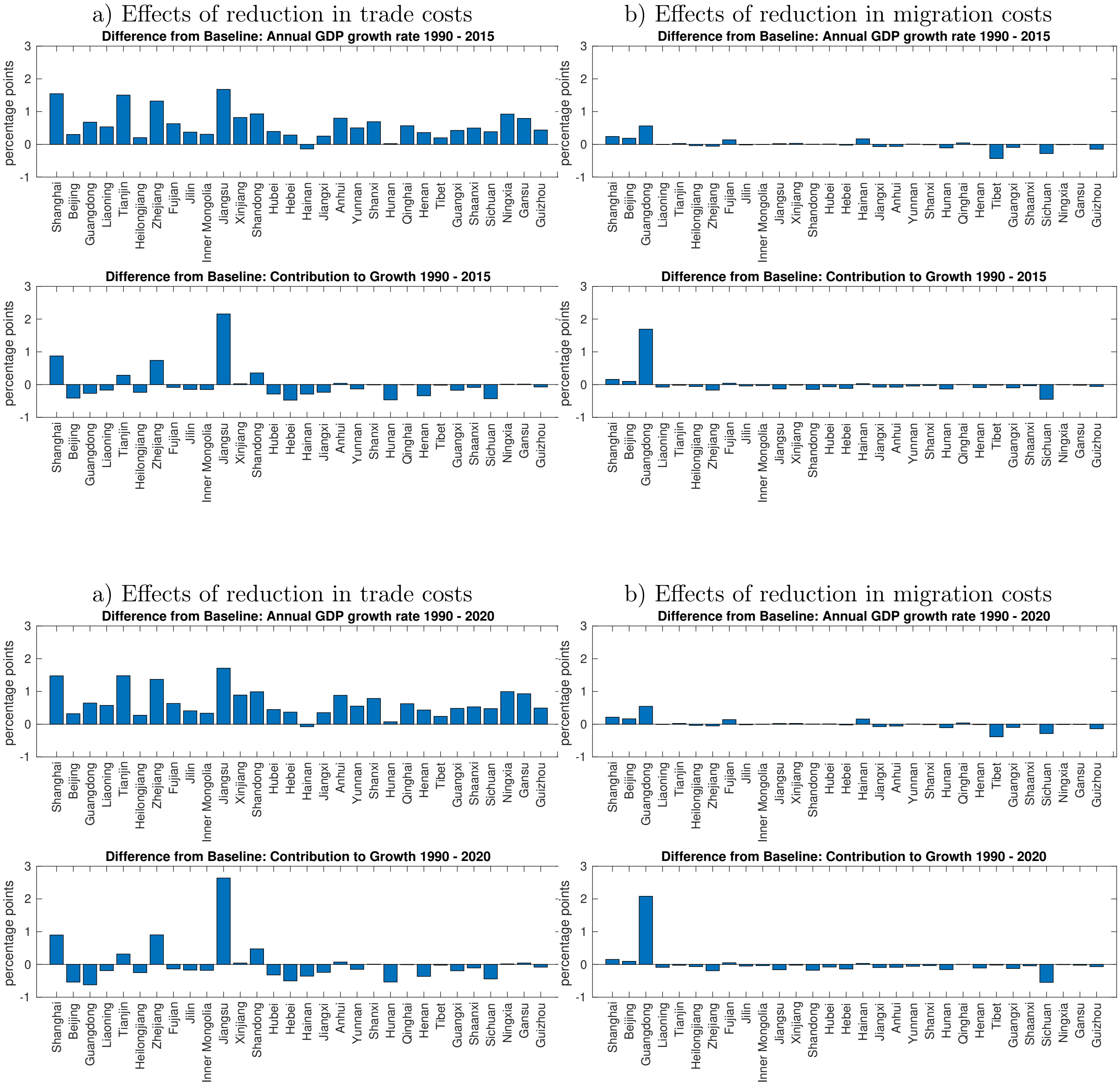
Note: The figures show the percentage point change in real GDP growth across provinces due to the trade and migration restrictions in different time frames. The left-hand panels present the effects of changes in trade costs and the right-hand panels show the effects of migration restrictions. All effects are computed relative to the baseline economy with 1990 trade and migration costs.

Figure I.11: Effects of trade and migration costs on spatial growth (percentage points)



Note: The figures show the percentage point change in real GDP growth across provinces due to the trade and migration restrictions in different time frames. The left-hand panels present the effects of changes in trade costs and the right-hand panels show the effects of migration restrictions. All effects are computed relative to the baseline economy with 1990 trade and migration costs.

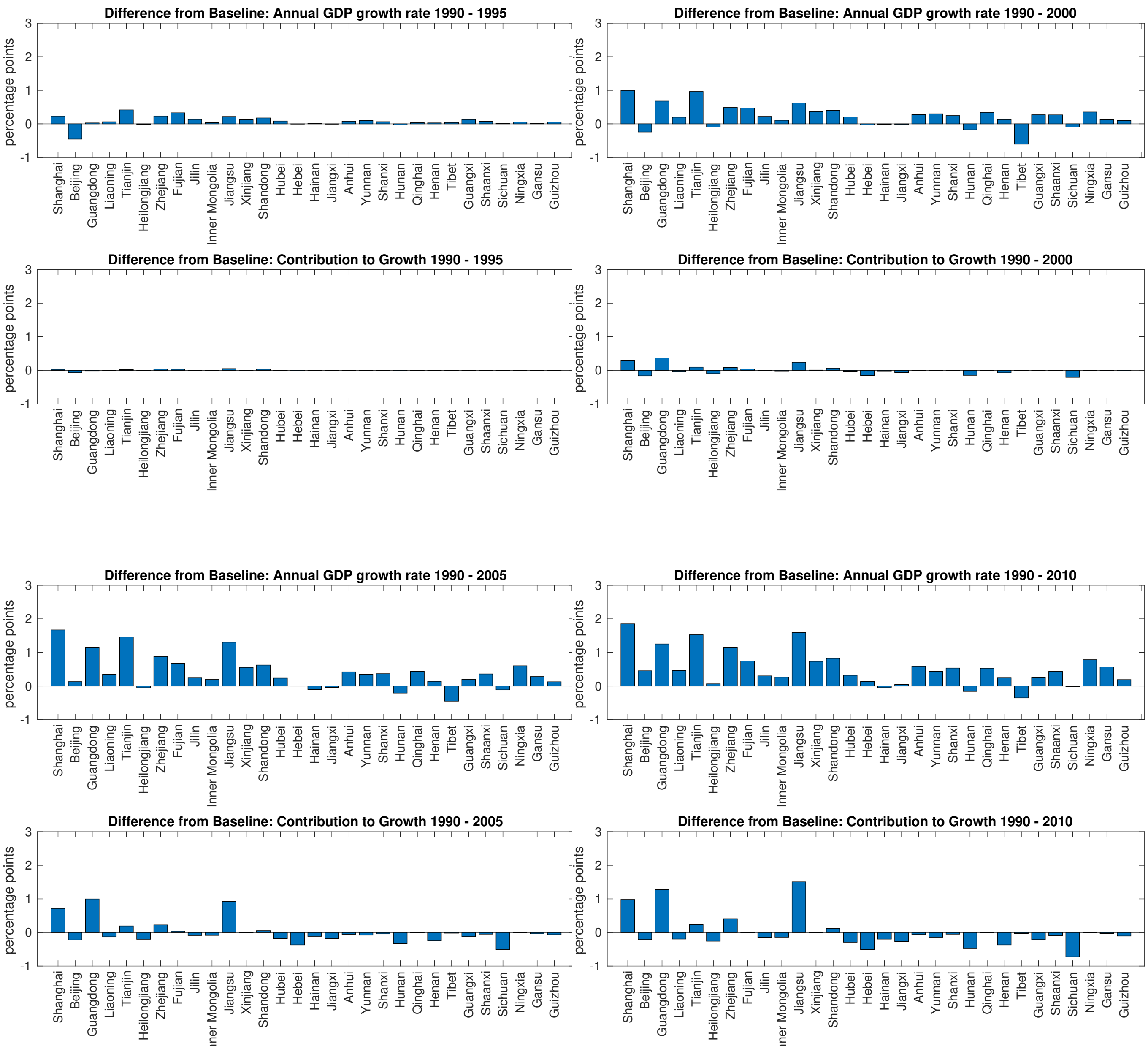
Figure I.12: Effects of trade and migration costs on spatial growth (percentage points)



Note: The figures show the percentage point change in real GDP growth across provinces due to the trade and migration restrictions in different time frames. The left-hand panels present the effects of changes in trade costs and the right-hand panels show the effects of migration restrictions. All effects are computed relative to the baseline economy with 1990 trade and migration costs.

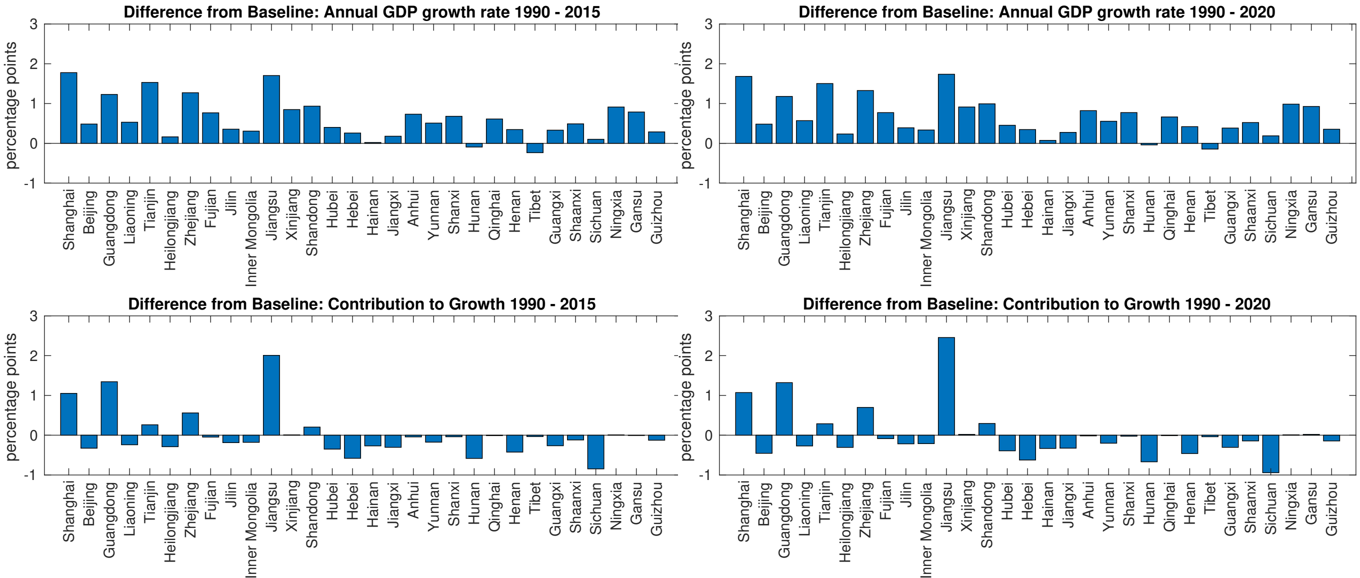
Figures I.13 and I.14 present the combined effects of changes in trade costs and migration restrictions to the baseline economy with 1990 initial conditions. In the upper figures of the panels we display the growth effects, and in the lower figures we present the contribution to aggregate growth relative to the baseline economy. We present results for different time frames.

Figure I.13: Effects of trade and migration costs on spatial growth (percentage points)



Note: The figures show the percentage point change in real GDP growth across provinces as a consequence of the trade and Hukou reforms in different time frames over the period 1990-2020. All effects are computed relative to the baseline economy with 1990 trade and migration costs.

Figure I.14: Effects of trade and migration costs on spatial growth (percentage points)



Note: The figures show the percentage point change in real GDP growth across provinces as a consequence of the trade and Hukou reforms in different time frames over the period 1990-2020. All effects are computed relative to the baseline economy with 1990 trade and migration costs.

J Empirical Evidence of Idea Diffusion

In this section of the appendix, we provide further empirical evidence related to the idea diffusion mechanism in our model. In particular, we use province-level patent data, along with trade and migration data, to support the role played by trade and migration in the diffusion of ideas.

We obtain province-level patent data from the China Statistics Yearbooks. There are three types of patents: innovation, utility, and design. For each type of patent, the yearbook reports the number of applications and number of approved patents in a given year. To proxy the measure of knowledge stock, $A_{n,t}$, we calculate the cumulative approved patents of all three types at the province level for each year, starting from 1985. We then compile the province-level knowledge stock in 1985, 1990, 1995, 2000, 2005, and 2010, with which we calculate the change in the knowledge stock every five years from 1985 to 2010. For the approved patents in the rest of the world, we obtain data from Google Patent from 1985-2010 following Liu and Ma (2021).[6]

In the next section, we document the correlation between the growth in knowledge and trade openness as well as diffusion through migration. In Section J.2, we empirically test

[6] We are grateful to Song Ma for sharing the Google Patent data.

the model-implied law of motion of the knowledge stock (equation 9) by implementing an instrumental variable strategy. The section complements Section 5 in the main text that provides reduced-form evidence of the contribution of idea diffusion from trade and migration to local knowledge.

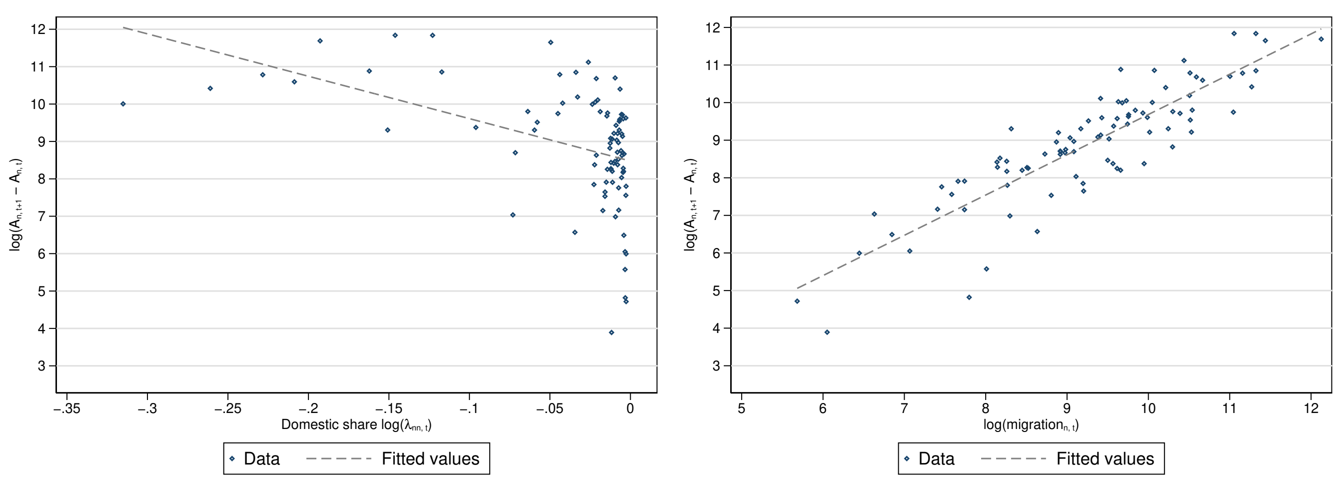
J.1 Simple Correlations

In this section, we compute simple correlations that provide preliminary evidence of knowledge diffusion through trade and migration. In particular, Figure J.1, Panel (a), presents a scatter plot of the change in knowledge stock $\log(A_{n,t+1} - A_{n,t})$, against the domestic expenditure share, $\lambda_{nn,t}$. The negative correlation suggests that the stock of knowledge grows more in locations more open to trade. This correlation is consistent with the mechanism that we highlight in our framework that global ideas diffuse more to provinces that are more exposed to trade. Figure J.1, Panel (b), shows that the change in knowledge stock is positively correlated with idea diffusion through migration, measured as $\log\left(migration_{n,t}\right) = \log\left[\sum_{i=1}^{N} s_{in,t} A_{i,t}\right]$, which is also in line with our model of idea diffusion from people.

Figure J.1: Change in the stock of knowledge and idea diffusion

a) Change in the stock of knowledge and trade openness

b) Change in the stock of knowledge and diffusion from migration



Note: The figures show scatter plots of the change in the stock of knowledge against trade openness (Panel (a)) and against diffusion from migration (Panel (b)). The change in the stock of knowledge is measured using patent data, as described in this appendix; trade openness and diffusion from migration are measured as described in this section.

J.2 Instrumental Variable Regressions

In this section of the appendix, we use the structure of our model to provide further evidence of our spatial mechanisms using our patent, production, and migration data. Recall that the evolution of the stock of knowledge in our model, according to equation (9), is given by

$$A_{n,t+1} - A_{n,t} = \alpha_t \Gamma_{\rho_\ell,\rho_m} \left[\sum_{i=1}^{N} s_{in,t} \left(A_{i,t}\right)^{\rho_\ell} \right] \left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}} \right)^{\rho_m} \right].$$

Taking logs on both sides, we obtain the following estimating equation

$$\log\left(A_{n,t+1} - A_{n,t}\right) = \beta_m \log\left(goods_{n,t}\right) + \beta_l \log\left(people_{n,t}\right) + \tau_t + \tau + \epsilon_{n,t},$$

where we define $\log\left(goods_{n,t}\right) = \log\left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}}\right)^{\rho_m}\right]$ and we also define $\log\left(people_{n,t}\right) = \log\left[\sum_{i=1}^{N} s_{in,t} \left(A_{i,t}\right)^{\rho_\ell}\right]$, τ_t controls for the time fixed effect, which captures the term $\log \alpha_t$; τ is a constant that captures $\log \Gamma_{\rho_\ell,\rho_m}$; and $\epsilon_{n,t}$ follows an $i.i.d.$ standard normal distribution.

It is important to highlight that our model allows for two-way causality in this structural equation due to general equilibrium effects. That being said, in what follows, we still try to establish causality between the growth in knowledge stock and the diffusion through goods and people in order to provide further evidence of our idea diffusion mechanisms. Hence, we try to address potential endogeneity issues in the estimating equation.

As described before, locations more exposed to international trade benefit more from the global diffusion of ideas and experience faster growth in the stock of knowledge. At the same time, fast-growing locations may build up their comparative advantages, impacting international trade as a result. To address potential endogeneity issues, we need an instrument for $\log\left(goods_{n,t}\right)$. We instrument $\lambda_{ni,t}$ by $\lambda_{ni,1985}$, as the growth prospect can hardly affect the trade pattern 15-25 years before.

For the variable $\log\left(people_{n,t}\right)$, the reverse causality concern also holds. Locations where the stock of knowledge grows faster might experience more immigration and less outmigration, which affects the knowledge diffusion through people in those locations. Furthermore, a higher share of immigration might lead to faster or slower growth in the stock of knowledge depending on the relative knowledge level and insights of the locals and the immigrants. Hence, the endogeneity issues might lead to either upward or downward estimation bias. To address this, we need an instrumental variable for $\log\left(people_{n,t}\right)$. We instrument $s_{in,t}$ by $s_{in,1985}$. The rationale is similar to the case of trade; the growth of the stock of knowledge stock can hardly be anticipated by people 15-25 years before, so $s_{in,1985}$ is exogenous from the perspective of location location n in year $t \geq 2000$. In short, the instrument for $\log\left(goods_{n,t}\right)$ is defined as $\log\left[\sum_{i=1}^{N} \lambda_{ni,1985} \left(\frac{A_{i,t}}{\lambda_{ni,1985}}\right)^{\rho_m}\right]$, and the instrument for $\log\left(people_{n,t}\right)$ is defined as $\log\left[\sum_{i=1}^{N} s_{in,1985}\left(A_{i,t}\right)^{\rho_\ell}\right]$, where $\rho_m = 0.61$ and $\rho_l = 0.18$ are taken from our GMM estimation.

The IV regression results are reported in Table J.2. In Column (1), we directly test our model-implied law of motion of the change in knowledge stock, and we do not control for endogeneity. The positive and significant coefficients of the two diffusion variables are consistent with our spatial mechanisms and in line with the reduced-form evidence presented in Section 5. As the patent stock is a proxy for the knowledge stock in the model and is likely a function of knowledge stock and other factors, the magnitudes of the coefficients do not need to be constrained to be close to one. In Columns (2), we report the instrumental variable regression results.[7] The positive effects of idea diffusion through international trade and migration on the growth in knowledge stock are still salient.

Table J.1: Estimates of the effects of knowledge diffusion through trade and migration

Dept. Var: $\log\left(A_{n,t+1} - A_{n,t}\right)$	OLS	IV
	(1)	(2)
$\log\left(goods_{n,t}\right)$	0.530***	0.428**
	(0.127)	(0.164)
$\log\left(people_{n,t}\right)$	5.421***	5.512***
	(0.245)	(0.323)
Constant	-3.558***	-4.272***
	(0.812)	(1.028)
Kleibergen-Paap rk Wald F statistic		79.07
Observations	90	87
R-squared	0.938	0.938
Year FE	✓	✓

Note: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

[7] In 1985 Hainan had not been elevated to the status of a province; therefore, Hainan is dropped from the sample in the instrumental variable regressions.