Directed Search on a Platform: Meet Fewer to Match More?*

Chengsi Wang† Makoto Watanabe‡

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Abstract

This paper explores the relationship between market transparency and efficiency in a directed search market intermediated by a profit-maximizing platform. We first provide necessary and sufficient conditions under which efficiency requires imperfect transparency such that each seller is not observed by all buyers. Full transparency is not optimal for either buyers or sellers. In particular, buyers prefer the minimum level of transparency. We then show that the platform can implement the efficient outcome. Our key insight is robust to the introduction of a second chance for unmatched buyers to search.

Keywords: meeting technology, directed search, platform, intermediation

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†Department of Economics, Monash University, Australia, chengsi.wang@monash.edu.

‡Department of Economics, FEWEB VU University of Amsterdam and Tinbergen Institute, Netherlands, makoto.wtnb@gmail.com.

1 Introduction

Unlike in markets where trading opportunities are fully displayed, e.g., price comparison sites, in some markets buyers often face a strict limit on the number of sellers they can meet before deciding with whom they wish to trade, e.g., job interview scheduling, or housing search. This is not very surprising if high transaction costs prevent them from doing so. But in many of these markets the meetings between buyers and sellers are managed at very low cost by online platforms. More importantly, these platforms are profit-maximizing and it seems that they limit the market transparency for a strategic reason.

The objective of this paper is to explore the relationship between market transparency and efficiency in markets with search frictions. Is it good or bad for overall efficiency if participants' search opportunities are deliberately limited? If a profit-maximizing platform intermediates transactions, what degree of market transparency will it choose and will the resulting allocation be efficient? We provide answers to these questions by using a parsimonious duopoly model with homogeneous goods.

More precisely, based on a directed-search model with generalized meeting technologies,[1] we measure the degree of market transparency by the number of buyers who observe an individual seller. This number is equal to the total number of buyers in the standard directed-search models (see, e.g., Peters, 1984b, Julien, Kennes and King, 2000, and Burdett, Shi and Wright, 2001), where full market transparency is assumed, but we allow it to be any admissible number. Given the possible imperfect market transparency, there might be a situation in which each seller is observed by a buyer who only observes him and has no option of searching the other seller. Indeed, this is a desirable situation for matching efficiency. We show that an unmatched seller exists if and only if (i) every searching buyer observes both sellers; and (ii) all of these buyers select the same seller.

A higher degree of market transparency has a negative effect on the probability of event (ii), increasing matching efficiency, but its effect on the probability of event (i) is non-monotone. This occurs because with an improved transparency, more buyers

[1]The terminology "meeting" has been used in the literature of directed search, e.g., Eeckhout and Kircher (2010). See Section 5.3 for the detailed discussion.

search both sellers rather than one seller, generating externalities on other buyers and decreasing matching efficiency, if the initial degree of market transparency is already high enough. An intermediate degree of market transparency provides a good balance in minimizing the probabilities of events (i) and (ii) and achieves matching efficiency. Intuitively, full market transparency creates too many “fully informed buyers” as trouble makers who generate search externalities. As a direct implication of our findings, policies that promote greater transparency on a marketplace may not always work in the direction of improving efficiency.

Our insight can be generalized as follows. First, by exploring a set of meeting technologies that the platform is possibly equipped with, we provide a necessary and sufficient condition for the efficient market transparency to be imperfect—an imperfect market transparency is efficient if and only if there are sufficient meeting opportunities that are separately scheduled (like in job scheduling) rather than jointly scheduled (like in price comparison sites) by the two sellers. Second, in our model, allowing unmatched buyers to search one more time would increase the relative efficiency of full transparency. However, even with a multiple-period setup, our conclusion that an imperfect transparency is efficient holds as long as the number of buyers is not too small. Third, we find that both the buyer- and seller-optimal degrees of transparency are imperfect. In particular, buyers prefer the minimum level of transparency as it allows them to avoid competition with each other.

We show that the efficient degree of market transparency emerges as the optimal choice of a profit-maximizing platform that adopts a simple fee setting policy. The total number of matches is inelastic in fees, provided that participants are willing to use the platform to trade. Hence, the platform chooses the efficient degree of market transparency to maximize the total transaction volume.

Recent studies support our finding that full market transparency unlikely leads to matching efficiency.[2] Using a field experiment conducted in an online labor market, Horton and Vasserman (2021) find that some jobs receive too many applications and a cap on the application number would significantly reduce such congestion—a worker's hiring rate conditional on applying to a given job, would increase by 17%. Our theory

[2]Li and Netessine (2019) show that on an online peer-to-peer holiday property rental platform, doubling market size leads to a 5.6% *reduction* of matches.

also predicts that online platforms are better off limiting participants' search options. In fact, Oneflare, an online marketplace for service providers (e.g., plumbers, electricians, pet groomers and interior designers), allows a customer who posts a job request to be approached by a maximum of only three service providers.[3]

Our paper is related to two recent branches of the directed search literature. The first set of papers address the issue of market transparency, including Peters (1984a) who allows for sellers to send costless advertisements, Lester (2011) who introduces heterogeneous search costs so that some buyers can observe all posted prices and other buyers can only observe one price, and Gomis-Porqueras, Julien and Wang (2017) who study costly stochastic advertising that generate information heterogeneity among buyers. None of these papers explore the implication of the exact number of meetings each seller can schedule, which is the main focus of the current paper. The second set of papers, including Kennes and Schiff (2008), and Gautier, Hu and Watanabe (2019), consider fee-setting intermediaries but do not study the implication of market transparency.[4]

In different contexts, a few papers also show that limiting one side's access to the information about the other side can improve matching efficiency, e.g., Calvó-Armengol and Zenou (2005) in the context of job network formation, Casadesus-Masanell and Halaburda (2014) for network goods, Halaburda, Piskorski and Yildirim (2018) in the presence of competing dating platforms, and Glebkin, Yueshen, and Shen (2021) for financial intermediaries in over-the-counter (OTC) markets. Our approach points to a novel source for limiting agents' choices—an excessive amount of search externalities and coordination frictions—for the inefficiency/sub-optimality of full market transparency.

Finally, our paper is also related to the burgeoning literature which studies the information design problem of platforms for differentiated goods markets without search frictions, e.g., Armstrong and Zhou (2020), Johnson, Rhodes and Wildenbeest (2020),

[3]Mapping this example to our model, customers are sellers of a job opportunity and service workers are buyers.

[4]Fee-setting intermediaries have been systematically studied in the literature of two-sided markets (Armstrong, 2006, Caillaud and Jullien, 2003, and Rochet and Tirole, 2003, 2006). A recent trend is to incorporate buyer search in the study of intermediaries as we do in this paper. That includes sequential search models (Wolinsky, 1986, Anderson and Renault, 1999) such as in Eliaz and Spiegler (2011), de Corniere (2016), Wang and Wright (2016, 2020), and Teh and Wright (2020).

and Teh (2020).[5] We found that, with homogeneous goods and search frictions, the optimal degree of market transparency in a buyers-sellers equilibrium can be decentralized by a profit-maximizing platform. This finding is new to the literature.

2 Basic setup and benchmark

We consider a platform economy with a mass of symmetric and independent product categories. Each seller belongs to only one product category, and there are two homogeneous sellers in each product category. This market structure captures the fact that, although platforms usually list many items, there is competition between only a few of them within each category.[6] We thereby focus on a representative product category with two sellers, indexed by $i = 1, 2$, and $B = 3, 4, 5, ...$ homogeneous buyers. Each seller has only one unit of the product. The products' consumption value is normalized to one for buyers and zero for sellers. Buyers and sellers can trade only through a platform that charges a per-transaction seller fee f and offers a meeting technology that determines which buyer observes which seller. We shall refer to a buyer who observes both sellers as a *fully informed* buyer, a buyer who observes only one seller as a *partially informed buyer*, and a buyer who does not observe any seller as an *uninformed buyer*.

□ **Trading protocols**. Individual sellers offer a first-price auction to their buyers.[7] Each seller i posts a reserve price, denoted by r_i, $i = 1, 2$. The reserve price r_i is honoured only if exactly one buyer participates in seller i's auction. If more than one buyer participates, the participating buyers bid for trade. If multiple buyers submit the same highest bid then each of them obtains the product with equal probabilities. Auctions with reserve prices capture the idea that sellers only have limited commitment power with respect to the posted prices, which is the case in all the markets that we are interested in, including labor, real estate and digital marketplaces.

When attending an auction, a buyer's bidding strategy depends on the posted reserve price, r_i, and the observed number of participants, denoted by n_i, $i = 1, 2$. Bertrand

[5]See also Armstrong and Vickers (2019), Bergemann, Brooks and Morris (2021), and Shi and Zhang (2020) who study market segmentation in models where oligopolistic sellers selling homogeneous goods can price discriminate between captive and contested buyers.

[6]This modelling approach was also used for example in Karle, Peitz and Reisinger (2020).

[7]A second-price auction will yield the same outcome in this environment.

type of reasoning yields the optimal bidding strategy,

$$b(r_i, n_i) = \begin{cases} r_i & \text{if} \;\; n_i = 1; \\ 1 & \text{if} \;\; n_i > 1. \end{cases}$$

Seller i's realized profit is given by

$$\Pi_i(r_i, n_i) = \begin{cases} 0 & \text{if} \;\; n_i = 0; \\ r_i & \text{if} \;\; n_i = 1; \\ 1 & \text{if} \;\; n_i > 1. \end{cases}$$

□ **Buyers search**. A buyer can visit a seller only if she observes this seller, and in case she observes two sellers she needs to decide which seller to visit. Buyers cannot coordinate with each other over which seller to visit so that the number of buyers who visit a seller i, n_i, will be stochastic. The possible mis-coordination among buyers captures the endogenous search (or coordination) friction that has been commonly assumed in the literature of directed search.[8]

□ **Meeting technology**. The meeting process between buyers and sellers is determined by the platform's meeting technology. The key function of the meeting technology is to control *market transparency*, which is summarized by the number of buyers who observe an individual seller, denoted by $N \in [1, B]$. We shall refer to $N = B$ as *full transparency* and $N < B$ as *imperfect transparency*. The parameter N can be interpreted as the number for which a customer can be contacted, the number of interviews an employer can schedule, or the number of advertisements a seller can send. We initially treat N as an exogenous parameter, but later allow the platform to determine N optimally. Assume the meeting technology is such that any two sellers are treated equally in reaching buyers (symmetry); any two buyers observe the same seller with equal probability (anonymity); and there is no waste of meeting opportunities such that the N buyers who observe a seller are all distinct buyers (no waste).[9] In addition, the meeting technology cannot in any way help buyers coordinate.

[8] See Wright, Kircher, Julíen and Guerrieri (2021) for a comprehensive overview of this literature and the rationale for assuming this type of frictions.

[9] Note that this no-waste assumption excludes the advertising technology proposed in Butters (1977) since Butters allows a buyer to receive multiple ads from a single seller.

A meeting technology that obviously satisfies these assumptions is such that the N opportunities for observing each seller are given to buyers randomly and each buyer has a probability N/B to observe each seller. We refer to this meeting technology as a *separate-meeting technology* and will use it throughout the paper as it is arguably the most reasonable meeting technology that satisfies the assumptions above. An example of separate-meeting technology is job interview scheduling. Each job seeker is scheduled, at most, to attend one interview for each vacancy, and the interviews for vacancy 1 are separately scheduled from the interviews for vacancy 2. This meeting technology is the reminiscence of the matching technology where the short side of the market is always cleared (Stevens, 2007).

Under the separate-meeting technology, a buyer can only observe one seller at a time. Another meeting technology that satisfies all the assumptions is what we call *joint-meeting technology*. Under the joint-meeting technology, a meeting opportunity with seller 1 is bundled with a meeting opportunity with seller 2, and the N "bundles of meeting opportunities" are assigned to the B buyers randomly such that each buyer receives a bundle with probability N/B. An example of the joint-meeting technology is price comparison sites which usually provide a search result that contains multiple listings. Hence, buyers who use a price comparison site can observe multiple sellers at once (i.e. the two sellers are always shown together on the same page). In Section 3.2, we generalize these meeting technologies.

□ **Timing of the game**. The timing of the game is as follows. In the first stage, the platform publicly sets a seller fee f and a level of market transparency N. In the second stage, buyers and sellers decide whether to join the platform. Each participating seller i sets a reserve price r_i. Then, participating buyers' information regarding sellers are realized. The fully or partially informed buyers choose a seller to visit. Finally, the chosen sellers and the informed buyers trade using auctions. The equilibrium concept we use is the subgame perfect Nash equilibrium.

3 Efficient market transparency

Before proceeding to fully characterizing the equilibrium, we first address the question of what level of market transparency maximizes the efficiency, or in our context, the total number of matches. We do this under the separate-meeting technologies in Subsection 3.1 and then generalize the findings by providing the necessary and sufficient conditions for the generalized meeting technology in Subsection 3.2.

3.1 Efficiency of imperfect transparency

Consider the problem of a social planner who is also subject to the search/coordination friction and the meeting technology. The planner selects a transparency level N to maximize the expected total surplus, which is equal to the expected total number of matches. Denote by T the expected total number of matches and

$$T = \text{Pr.}[n_1 \geq 1] \cdot \text{Pr.}[n_2 = 0] + \text{Pr.}[n_1 = 0] \cdot \text{Pr.}[n_2 \geq 1] + 2 \cdot \text{Pr.}[n_1 \geq 1] \cdot \text{Pr.}[n_2 \geq 1].$$

To compute these probabilities, we define $\Gamma(k|N, B)$ as the probability of having $k = 0, ..., N$ fully informed buyers when there are B buyers and the transparency level is N.

If there are N fully informed buyers, which occurs with probability $\Gamma(N|N, B)$, the total number of matches is affected by the coordination friction given that fully informed buyers randomize over which seller to visit, i.e., each buyer visits each seller with probability 1/2. There will be only one match with probability $2(1/2)^N = (1/2)^{N-1}$, and two matches with probability $1 - (1/2)^{N-1}$. If there are less than N fully informed buyers, or in other words there exist partially informed buyers, then, there is at least one buyer who observes only seller 1 and another buyer who only observes seller 2. Hence, if partially informed buyers exist, which occurs with probability $1 - \Gamma(N|N, B)$, each seller meets at least one buyer and there will be two matches with probability one. So the expected total number of matches is

$$\begin{aligned} T &= \Gamma(N|N,B)\left[\left(\frac{1}{2}\right)^{N-1} + 2\left(1 - \left(\frac{1}{2}\right)^{N-1}\right)\right] + 2(1 - \Gamma(N|N,B)) \\ &= 2\left[1 - \left(\frac{1}{2}\right)^{N} \Gamma(N|N,B)\right] \end{aligned} \tag{1}$$

When the meeting technology is the one with separate meetings, there are in total C_B^N possible cases in terms of which buyers observe seller $i = 1, 2$. In addition, there are in total C_B^N cases where exactly N buyers are fully informed. Therefore, under the separate-meeting technology, $\Gamma(N|N,B) = C_B^N/(C_B^N)^2 = 1/C_B^N$, and

$$T = 2\left[1 - \left(\frac{1}{2}\right)^N \frac{1}{C_B^N}\right]. \tag{2}$$

Proposition 1 summarizes the efficient market transparency N^e under the separate-meeting technology.

Proposition 1. *Under the separate-meeting technology, an intermediate degree of market transparency is efficient, i.e.,* $N^e \in (1, B)$.

Clearly, search frictions that causes less than two total matches only arise when there are exactly N fully informed buyers. As the probability of having N fully informed buyers, the term $\Gamma(N|N,B)$ in (1), is what we call the *extensive margin of search friction.* The term, $(1/2)^N$, measures the mis-coordination (selecting the same seller) among the N fully informed buyers, and we call it the *intensive margin of search friction.* On the one hand, a higher degree of market transparency, measured by greater N, strictly decreases the intensive margin of search friction, which helps increasing efficiency. On the other hand, the effect of a greater transparency on the extensive margin of search friction is less clear-cut. However, under the separate-meeting technology, such an effect is non-monotonic in that $\Gamma(N|N,B) = 1/C_B^N$ initially decreases and then increases in N. The intuition of the non-monotonicity is clear. When N is relatively small, there are plenty of uninformed buyers, and so, due to the no-waste assumption, an additional meeting opportunity with each seller is unlikely to reach the same buyer. When N is relatively large, most buyers are either fully or partially informed, and an additional meeting opportunity with each seller is very likely to create an additional fully informed buyers. Due to this trade-off, maximizing efficiency requires an intermediate degree of market transparency.

3.2 General meeting technology

How general are the results in Proposition 1? It is worth noting that, under the joint-meeting technology, $\Gamma(N|N,B) = 1$ for any $N = 1, 2, ..., B$. That is, while the intensive margin of search friction strictly decreases in N, the extensive margins stays constant.

As a result, full transparency is efficient, i.e., $N^e = B$, under the joint-meeting technology. When is imperfect transparency efficient as stated in Proposition 1? In this section, we provide necessary and sufficient conditions to this property by generalizing the meeting technology.

The separate-meeting technology and the joint-meeting technologies are the two extreme forms of all the meeting technologies that satisfy our assumptions of symmetry, anonymity and no waste. By exploring the convex combination of the two, we can cover all the meeting technologies that satisfy our assumptions. Specifically, we denote by N_j the number of joint meetings, and N_s the number of separate meetings with a seller. It holds that $N = N_j + N_s$. Define $x_s \equiv N_s/N \in [0, 1]$ as the share of separate meetings, with $x_s = 0$ and $x_s = 1$ corresponding to the joint-meeting technology and separate-meeting technology, respectively. Note that x_s reflects the exogenous constraint on the meeting technology, and is therefore taken as given by all agents.[10]

The planner's problem is to select a transparency level N to maximize the expected total number of matches for a given x_s, denoted by $T(N \mid x_s)$. Define $\Gamma_{N_j}(k|N, B)$ as the probability of having $k = 0, ..., N$ fully informed buyers when there are B buyers and N_j $(= 0, 1, ..., N)$ are joint. By definition, a meeting technology with N_j joint meetings generate at least N_j fully informed buyers. The probability of having N fully informed buyers is then given by

$$\Gamma_{N_j}(N|N, B) = \Gamma_0(N_s|N_s, B - N_j) = \frac{1}{C^{N_s}_{B-N_j}}, \tag{3}$$

where $C^{N_s}_{B-N_j} = (B - N_j)!/(N_s!(B - N)!)$.[11] The expected total number of matches is

$$\begin{aligned} T(N \mid x_s) &= \Gamma_0(N_s|N_s, B - N_j)\left[\left(\frac{1}{2}\right)^{N-1} + 2\left(1 - \left(\frac{1}{2}\right)^{N-1}\right)\right] \\ &= 2\left[1 - \left(\frac{1}{2}\right)^{N} \Gamma_0(N_s|N_s, B - N_j)\right]. \end{aligned} \tag{4}$$

[10]One interpretation of N_s or x_s is to capture the degree of correlation between the two technologies, which is determined by how well the platform's meeting algorithm identifies correlated preferences. A greater N_{nb} or x_s implies the algorithm is less competent in finding out a buyer is interested in both sellers, possibly due to the vagueness of buyers' search queries or sellers' product description.

[11]To randomly introduce seller 1 to N_s buyers out of $B - N_j$ buyers, there are in total $C^{N_s}_{B-N_j}$ cases. Similarly, to randomly introduce seller 2 to N_s buyers, there are in total $C^{N_s}_{B-N_j}$ cases. On the other hand, to randomly introduce both sellers 1 and 2 to N_s buyers, there are in total $C^{N_s}_{B-N_j}$ cases. Hence, the probability of having N_s fully informed buyers is $C^{N_s}_{B-N_j}/(C^{N_s}_{B-N_j})^2 = 1/C^{N_s}_{B-N_j}$.

Proposition 2 provides the necessary and sufficient conditions on the meeting technologies under which the matching efficiency requires an intermediate level of market transparency as in Proposition 3.

Proposition 2. *Consider a general meeting technology which generates N_s $(= 0, 1, .., N)$ separate meetings and $N_j = N - N_s$ joint meetings. There exists a unique critical share of separate meetings $\hat{x}_s \in (0, 1)$ such that full transparency is efficient, $N^e = B$, if and only if $x_s \leq \hat{x}_s$, and an intermediate degree of transparency is efficient, $N^e \in (1, B)$, if and only if $x_s > \hat{x}_s$.*

The conditions in Proposition 2 clearly indicate that full transparency is efficient under the joint-meeting technology as $x_s = 0 < \hat{x}_s$ and an intermediate level of transparency is optimal under the separate-meeting technology as $x_s = 1 > \hat{x}_s$. In general, an efficient matching market requires an intermediate level of market transparency if a sufficient number of meetings is separate.

It is important to notice that the comparison of transparency levels in Proposition 2 is based on an exogenous x_s. If we instead fix N, a meeting technology with more joint meetings, and thus a smaller x_s, decreases efficiency as it automatically creates more fully informed buyers. This provides a justification for why the separate-meeting technology can be the preferred technology by participants. We will focus on the separate-meeting technology in the subsequent discussion, although extending the analysis to allow for the general meeting technology is straightforward.[12]

4 Equilibrium characterization

We provide the full characterization of the equilibrium in this section. It turns out that the equilibrium involves sellers using symmetric mixed strategy when $N = 1$. For the sake of exhibition, we solve the buyer-seller games when $N \geq 2$ in Section 4.1, and when $N = 1$ in Section 4.2 by assuming $f = 0$. We derive the equilibrium platform fee and market transparency in Section 4.3.

[12]Under the general meeting technology with $N_j > 0$ joint meetings, any expression written in terms of $\Gamma_{N_j}(\cdot|\cdot)$ can be transformed to one in terms of $\Gamma_0(\cdot|\cdot) = \Gamma(\cdot)$ as in (3), and our analysis with the separate-meeting technology readily follows.

4.1 Buyer-seller games when $N \geq 2$

We work backward and start with buyers' search problem. Except for the extreme case of full transparency, note that buyers' information is dispersed: potentially, there exist buyers who observe no seller, one seller, and two sellers. If a buyer observes no seller, then she has no one to visit and her payoff is zero. If a buyer observes one seller, she is partially informed and can only visit the seller she observes. Her payoff depends on whether the seller in question receives visits from other buyers.

We next describe the visiting strategy of a fully informed buyer who observes both sellers. Let $\sigma_1 \in (0, 1)$ be the symmetric equilibrium probability that a fully informed buyer attends seller 1's auction. She obtains a positive payoff from seller 1 only if she is the only one to visit him, because otherwise the *ex post* competition would shift all buyer surplus to the seller. If partially informed buyers exist, then each seller receives at least one buyer. Hence, a fully informed buyer will be the only visitor of seller 1 only if all other $N - 1$ buyers are fully informed, which happens with probability $\Gamma(N-1 \mid N-1, B-1)$, and none of them select seller 1, which happens with probability $(1 - \sigma_1)^{N-1}$. Her expected payoff for selecting seller 1, who posts a reserve price r_1, is therefore given by

$$u_1(r_1, r_2) = (1 - r_1)\Gamma(N - 1|N - 1, B - 1)(1 - \sigma_1)^{N-1}.$$

Her expected payoff for selecting seller 2 can be similarly derived. The lack of coordination among buyers implies that they must use symmetric strategies in equilibrium. The equilibrium strategy $\sigma_1 = \sigma_1(r_1, r_2)$ is implicitly determined by $u_1 = u_2$.

Given the buyers' search behavior, seller 1's expected profit of choosing r_1 is

$$\pi_1(r_1, r_2) = r_1 \cdot \text{Pr.}[n_1 = 1] + \text{Pr.}[n_1 > 1] = 1 - \text{Pr.}[n_1 = 0] - \text{Pr.}[n_1 = 1] \cdot (1 - r_1).$$

Note that $n_1 = 0$ when there are N fully informed buyers and none of them select seller 1. The probability of this event is $\Gamma(N|N, B)(1 - \sigma_1)^N$. Also, $n_1 = 1$ when (i) there are N fully informed buyers but only one of them selects seller 1, which happens with probability $\Gamma(N|N, B)N\sigma_1(1-\sigma_1)^{N-1}$; or (ii) there are $N-1$ fully informed buyers (and therefore two partially informed buyers) but none of them select seller 1. This happens with probability $\Gamma(N - 1|N, B)(1 - \sigma_1)^{N-1}$, where

$$\Gamma(N - 1|N, B) = \frac{C_B^{N-1} C_{B-N+1}^1 C_{B-N}^1}{(C_B^N)^2} = \frac{N(B - N)}{C_B^N},$$

given that $C_B^{N-1} = NC_B^N/B - N + 1$.[13] Note that in this case, it is a partially informed buyer who only observes seller 1 that participates in the seller 1's auction. Then,

$$\pi_1(r_1, r_2) = 1 - \Gamma(N|N,B)(1-\sigma_1)^N \\ - [\Gamma(N|N,B)N\sigma_1 + \Gamma(N-1|N,B)](1-\sigma_1)^{N-1}(1-r_1)$$

The endogenous search friction implies fully informed buyers use mixed strategies in equilibrium. Then, from $u_1 = u_2$,

$$\frac{d\sigma_1}{dr_1} = \frac{-(1-\sigma_1)^N}{(N-1)(1-r_2)(\sigma_1)^{N-2}}.$$

In what follows, we use the subscript $N = 1, .., B$ for equilibrium variables to index the meeting transparency level. Applying the first-order conditions, we obtain the symmetric equilibrium reserve price, denoted by $r_{N\geq 2}$,

$$\begin{aligned} r_{N\geq 2} &= 1 - \frac{\Gamma(N|N,B)N}{\Gamma(N|N,B)N^2 + 2\Gamma(N-1|N,B)(N-1)} \\ &= = 1 - \frac{1}{N + 2(N-1)(B-N)}, \end{aligned} \tag{5}$$

where in the second equality we use $\Gamma_0(N|N,B) = 1/C_B^N$ and $\Gamma(N-1|N,B) = N(B-N)/C_B^N$. In Appendix, we show that the first-order condition is necessary and sufficient, so (5) is indeed a unique equilibrium.

Theorem 1. *For $N \in [2, B]$, a directed search equilibrium exists and is unique with the symmetric equilibrium reserve price given by (5).*

4.2 Buyer-seller games when $N = 1$

The analysis above does not readily extend to the case of $N = 1$. Indeed, there is no symmetric pure-strategy equilibrium when $N = 1$. If both sellers set some $r_1 = r_2 > 0$, one of them can undercut slightly and get a fully informed buyer with probability one,

[13] The first equality can be derived as follows. To have $N-1$ fully informed buyers, we must introduce both sellers to $N-1$ buyers, and simultaneously have one random buyer who observes seller 1 but not seller 2, and another random buyer who observes seller 2 but not seller 1. There are in total $C_B^{N-1}C_{B-N+1}^1C_{B-N}^1$ such cases. Hence, the probability of having $N-1$ fully informed buyers is given by $(C_B^{N-1}C_{B-N+1}^1C_{B-N}^1)/(C_B^N)^2$.

if there is one. But neither seller will set zero reserve price since they can set a positive reserve price and sell only to a partially informed buyer with a positive probability.

When $N = 1$, there are two possible scenarios regarding buyers' information: (i) a single buyer observes both sellers; and (ii) one buyer observes only seller 1 and another buyer observes only seller 2. If a buyer is fully informed, she will select the seller with the lower reserve price. If a buyer is partially informed, she will select the observed seller provided the reserve price is no greater than 1. Note that the *ex post* bidding never takes place when $N = 1$ as each seller can meet at most one buyer.

The symmetric equilibrium involves a mixed strategy. Denote the symmetric equilibrium mixed strategy by a distribution function with support $[\underline{r}, \overline{r}]$. By the standard argument given in Varian (1980), there is no gap and no mass point in the support of $F(r)$. Note that all reserve prices in $[\underline{r}, \overline{r}]$ should yield the same expected profit. Suppose $\overline{r} < 1$. Then at $\overline{r}$, only a partially informed buyer will buy. However, seller i can instead set $r = 1$ without losing demand and make a strictly higher profit. So we must have $\overline{r} = 1$.

We now derive the expected profit of an individual seller (say that of seller 1's) and the lower bound of the equilibrium price distribution, $\underline{r}$. There is a fully informed buyer with probability $\Gamma(1|1, B)$ and, given that seller 2 mixes using the price distribution $F(\cdot)$, seller 2's price is higher than a price r_1 with probability $1 - F(r_1)$, in which case the fully informed buyer will buy from seller 1. On the other hand, with probability $\Gamma(0|1, B)$, there is a partially informed buyer who can only buy from seller 1. To sum up, seller 1's expected profit with a reserve price r_1 is

$$\pi_1(r_1, F(r)) = r_1[\Gamma(1|1, B)(1 - F(r_1)) + \Gamma(0|1, B)].$$

Sellers must be indifferent between any $r \in [\underline{r}, 1)$ and $r = 1$, which yields an expected profit $\Gamma(0|1, B)$. The indifference condition is then

$$r[\Gamma(1|1, B)(1 - F(r)) + \Gamma(0|1, B)] = \Gamma(0|1, B)$$

This condition generates the equilibrium price distribution,

$$F(r) = 1 - \frac{\Gamma(0|1, B)}{\Gamma(1|1, B)} \left(\frac{1}{r} - 1\right) = 1 - (B - 1) \left(\frac{1}{r} - 1\right). \tag{6}$$

Further, from $F(\underline{r}) = 0$, we can derive the lower bound $\underline{r} = \Gamma_0(0|1, B) = 1 - (1/B)$, which is also equal to each firm's equilibrium expected profit.

Theorem 2. *For $N = 1$, a directed search equilibrium exists and is unique, characterized by a non-degenerate distribution of reserve prices (6) on* $[1 - (1/B), 1]$.

4.3 Profit-maximizing platform

The platform's objective is to set a transaction fee f and a transparency level N to maximize its own profit, which is given by $\Pi = f \cdot T(N)$, subject to the buyers' and the sellers' participation, where $T(N)$ is the total expected number of trades in equilibrium given in (4) conditioned on the value of $N = 1, ..., B$. Following the platform's choice of f and N, sellers play the same game as before except that their profit margins will reduce by f.

With the transaction fee, the fully informed buyers' problem of choosing a seller is the same as before. With $N \geq 2$, seller 1's expected profit is

$$\begin{aligned}\pi_1(r_1, r_2, f) &= (r_1 - f) \cdot \text{Pr.}[n_1 = 1] + (1 - f) \cdot (1 - \text{Pr.}[n_1 = 0] - \text{Pr.}[n_1 = 1]) \\ &= (1 - f)(1 - \text{Pr.}[n_1 = 0]) - (1 - r_1) \cdot \text{Pr.}[n_1 = 1].\end{aligned}$$

Seller 2's profit can be similarly derived. Following the same steps as in Section 4.1, we can derive the symmetric equilibrium reserve price for given values of f as

$$r(f) = 1 - \frac{1 - f}{N + 2(N - 1)(B - N + 1)}.$$

Observe that $r(f) = 1$ when $f = 1$, which is the highest possible fee the platform can charge without causing sellers and buyers to withdraw.

It is important to note that the fee does not influence the total number of matches in any symmetric equilibrium where fully informed buyers select each seller with probability 1/2. Hence, the platform optimally selects $f^* = 1$ for any given $N \geq 2$. Given $f^* = 1$, as for the selection of N, the platform cares only about the total expected number of matches. We know from Proposition 2 that it is maximized at $N = N^e \in (1, B)$.

By setting $f^* = 1$ and $N^* = N^e \in (0, B)$, the platform obtains the maximum profit, $\Pi = T_{N^e}$, and fully extracts the expected surplus, leaving both the buyers and sellers zero utility/profit. The solution is indeed optimal, i.e., it dominates any other $f < 1$ and $N \neq N^e$ (including $N = 1$).

Proposition 3. *A profit-maximizing platform sets the efficient level of market transparency, i.e., $N^* = N^e$ and charges the maximum transaction fee $f^* = 1$ to extract all the surplus.*

Despite the potential complexity introduced by imperfect transparency, the efficient meetings, which maximize the expected total number of matches or surplus, can be implemented by a profit-maximizing platform that adopts a simple fee-setting policy for its intermediation service. In other words, the efficient matching outcome can be decentralized by a profit-maximizing platform that introduces imperfect transparency to its participants. It is worth noting that, from the platform's perspective, transaction fees weakly outperforms all other types of fees (transaction fee, percentage fee, etc) as transaction fees induce full-surplus extraction.

5 Extension and discussion

We derive the market transparency levels that are optimal for individual sellers and buyers in Section 5.1. In Section 5.2, the analysis is extended to allow unmatched buyers and sellers to rematch, and we show that our key insight is robust for a wide range of parameters. Finally, we discuss the relationship between our meeting technologies to those having been proposed in the literature.

5.1 Buyer- and seller-optimal market transparency

The seller-optimal transparency level, denoted by N^s, maximizes sellers' expected profit. For $N = 2$, by applying $r_{N=2}$ in (5) to π_1, we get

$$\pi_{N\geq 2} = 1 - \left(\frac{1}{2}\right)^{N-1} \frac{1}{C_B^N} \left[1 + \frac{1}{\frac{N}{B-N} + 2(N-1)}\right]. \tag{7}$$

For $N = 1$, it is immediate that $\pi_{N=1} = 1 - \frac{1}{B}$. By comparing these profits, we characterize N^s below.

Proposition 4. *The seller-optimal market transparency is at an intermediate level that is greater than the efficient level, i.e., $N^s \in (N^e, B)$.*

The logic for the interior optimum, i.e., $N^s \in (1, B)$, is similar to the one for the efficient meeting transparency. A seller cannot sell if there are N fully informed buyers and all of them select the rival seller. When N is relatively large, the probability of having N fully informed buyer increases with N, which makes $N = B$ suboptimal. When N is relatively small, increasing N lowers both the probability of having N fully informed buyers and the probability that all these fully informed buyers choose the rival seller. As a result, $N^s < B$. Further, by comparing (2) and (7), it is clear that the only difference is the term related to competition,

$$2\left[1 + \frac{1}{\frac{N}{B-N} + 2(N-1)}\right],$$

which is strictly decreasing in N. Hence, the optimal meeting transparency required for maximizing a seller's profit should be higher than the one required for maximizing the total number of matches, i.e., $N^s > N^e$. From a seller's point of view, an even higher transparency level could be beneficial as he does not take into account the increased risk of the other seller being unmatched (caused by the more severe mis-coordination problem among buyers).

Next, denote N^b as the buyer-optimal transparency level that maximizes a buyer's expected payoff. To write down a buyer's ex ante expected payoff, we need to consider not only the fully informed buyers (as already described above) but also the partially informed buyers. In either case, note that, as before, a buyer can get a positive payoff from visiting a seller only if all other $N-1$ buyers are fully informed and none of them select the seller. Each buyer observes a seller with probability N/B, so the probability of becoming partially fully informed is $2(N/B)\,(1-(N/B))$ and the probability of becoming fully informed is $(N/B)^2$. To derive a partially informed buyer's expected utility, we need to compute the probability of having $N-1$ fully informed buyers conditional on the buyer being partially informed, which is given by[14]

$$\frac{(B-N)C_{B-1}^{N-1}}{C_{B-1}^{N}C_{B-1}^{N-1}} = \frac{B-N}{C_{B-1}^{N}}.$$

A buyer's ex-ante expected utility, when $N \in [2, B-1]$, is given by

$$u_{N\geq 2} = (1 - r_{N\geq 2})\left[2\frac{N}{B}\left(1-\frac{N}{B}\right)\frac{B-N}{C_{B-1}^{N}} + \left(\frac{N}{B}\right)^2 \frac{1}{C_{B-1}^{N-1}}\right]\left(\frac{1}{2}\right)^{N-1}, \tag{8}$$

where the equilibrium reserve price $r_{N\geq 2}$ is given by (5). When all buyers are fully informed, the expected utility becomes the one obtained by applying $N = B$ to (8) (note that $C_{B-1}^{B} = 0$).

When $N = 1$, a buyer who observes any seller can trade with probability one since there will be no competitors between buyers. So a buyer's equilibrium expected payoff is

$$u_{N=1} = (1 - \mathbb{E}[r_{N=1}])\left[1 - \left(1 - \frac{1}{B}\right)^2\right], \tag{9}$$

where the equilibrium expected reserve price is defined over the equilibrium price distribution (6) and is given by

$$\mathbb{E}(r_{N=1}) = \int_{1-\frac{1}{B}}^{1} r dF(r) = (B-1)\ln\left(\frac{B}{B-1}\right).$$

By comparing these payoffs, we characterize the buyer-optimal transparency level below.

Proposition 5. *The buyer-optimal market transparency is at the minimum level, i.e.* $N^b = 1$.

At first glance, this result is somewhat counter-intuitive. Why would buyers prefer being given the minimum access to sellers in a matching market? The key to understanding this result lies in the competition among buyers. When $N = 1$, a buyer is either partially informed of a seller and no other buyers know this seller, or fully informed and there is no other informed buyer. Therefore, buyers do not face any ex post

[14] Suppose that a buyer observes seller 1 but not seller 2. Among the other $B-1$ buyers, $N-1$ buyers should observe seller 1 and N $(\leq B-1)$ buyers should observe seller 2. There are in total $C_{B-1}^{N}C_{B-1}^{N-1}$ such cases. On the other hand, for $N-1$ of them to be fully informed, both of the sellers must be introduced to $N-1$ buyers, which has C_{B-1}^{N-1} cases, and seller 2 should be introduced to one of the $(B-1)-(N-1) = B-N$ remaining buyers, which has $C_{B-N}^{1} = B-N$. To sum up, the probability of having $N-1$ fully informed buyers when there is already a partially informed buyer is $((B-N)C_{B-1}^{N-1})/(C_{B-1}^{N}C_{B-1}^{N-1}) = B-N/C_{B-1}^{N}$.

competition after being informed and thus always obtain a positive utility. The benefits from avoiding competition with other buyers outweigh the cost of being less frequently informed, and buyers' utility is maximized at the minimum transparency level.

5.2 A second chance to search

We have so far followed the standard approach in the canonical directed-search models by assuming that the buyers who fail to match cannot search again. It is natural to wonder whether the imperfect market transparency result continues to hold if we allow unmatched buyers to use the information on sellers one more time in their shopping life. This is possible if and only if they have a second chance to search. In this section, we study a variant of our model by incorporating an additional period for buyers and sellers to interact, and we show that our main insight is robust.

There are two periods, 1 and 2. To avoid unnecessary complication, we assume that supply of goods is constant across periods, i.e., each seller has a unit to sell in both periods. Unlike sellers, buyers exit the market upon successful trade. The meeting parameter N is chosen at the beginning of period 1 and the information structure will be unchanged in period 2, i.e., buyers who observe a seller in period 1 will know him in period 2 as well and buyers who do not observe a seller in period 1 do not know him in period 2 as well. To highlight the additional benefit of full transparency, we assume away the issue of strategic delay of buyers' search.[15] Under this assumption, the equilibrium in each period is the same as in our original setting except that there are fewer buyers in period 2. Assume a common discount factor $\delta \in [0, 1]$ for the value derived in period 2.

The intuition can be best understood by considering a simple case with only three buyers. We will generalize the results to arbitrary B at the end of the section. Suppose $N = 1$. With a slight abuse of notation, we use T_N to denote the discounted total expected values generated from the matches formed in both periods. There will be a fully informed buyer with probability 1/3 and two partially informed buyers with

[15] If buyers are sophisticated, they want to delay their participation if everyone else does not delay. This is because the number of participating buyers is always smaller in period 2 than in period 1, which causes sellers to set a lower equilibrium reserve price in period 2. However, if everyone else delays their participation, sellers will set a high reserve price in period 2 and an informed buyer's optimal response might be not to delay.

probability 2/3. There will be one match in the former case and two matches in the latter case. All matches are formed in period 1. So the discounted total value is

$$T_{N=1} = \frac{1}{3} + 2 \times \frac{2}{3} + \delta \times 0 = \frac{5}{3}. \tag{10}$$

Suppose $N = 3$ and therefore all buyers are fully informed. There is one match with probability 1/4 and two matches with probability 3/4 in period 1. So the expected value in period 1 is 7/4. If there is only one match in period 1, there is one match with probability 1/2 and two matches with probability 1/2 in period 2. If there are two matches in period 1, the likelihood of a match in period 2 will occur at probability one. So the expected value generated in period 2 is $(1/4)\times(1/2+2\times(1/2))+(3/4)\times 1 = 9/8$. So the discounted total value is

$$T_{N=3} = \frac{7}{4} + \delta\left(\frac{9}{8}\right). \tag{11}$$

Suppose $N = 2$. With probability 1/3, there are two fully informed buyers. With probability 2/3, there is one fully informed buyer and there are two partially informed buyers. In the former case, there will be two matches with probability 1/2 and only one match with probability 1/2 in period 1. In the latter case, there are two matches in period 1. The expected value generated in period 1 is $(1/3)\times((1/2)+2\times(1/2))+(2/3)\times 2 = 11/6$. If there is only one match in period 1, there will be one match in period 2. If there are two matches in period 1, there will be an additional match in period 2 if there exist partially informed buyers, and no match in period 2 otherwise. The expected value generated in period 2 is $(1/3) \times (1/2) + (2/3) = 5/6$. So the discounted total value is

$$T_{N=2} = \frac{11}{6} + \delta\left(\frac{5}{6}\right). \tag{12}$$

Let us compare (10), (11), and (12). It is clear that $N = 1$ generates the lowest value. When $N = 1$, having a second chance to search does not add any value to matching as all the informed buyers are matched in period 1. According to our results in the static setting, $N = 2$ always generates a higher expected number of matches in period 1. The question is then whether $N = 3$ can generate a higher expected number of matches in period 2. The comparison between the second terms in (12) and (11) gives a positive answer. Full transparency, $N = 3$, indeed dominates in period 2. So we can conclude that full transparency is efficient if participants are patient enough, i.e., $\delta > 2/7$, and partial transparency is efficient otherwise.

The reason why, unlike in the static case, $N = 2$ is less efficient relative to full transparency in this example is because it happens with strictly positive probability there exist uninformed buyers who cannot participate. This is a serious problem for efficiency given that there are in total only three buyers. This issue will be significantly mitigated when there is a large number of buyers because enough informed buyers will be available for matching in both periods (despite the presence of some uninformed buyers). The next proposition summarizes the general result.

Proposition 6. *Suppose the market operates in both periods 1 and 2 and participants have the common discount factor $\delta \in [0, 1]$. Then, there exists $\hat{B} > 3$ and $\hat{\delta} \in (0, 1)$ such that*

- $N^e = B$ *if* $B \leq \hat{B}$ *and* $\delta > \hat{\delta}$*;*
- $N^e \in (1, B)$ *if* $B \leq \hat{B}$ *and* $\delta < \hat{\delta}$ *or* $B > \hat{B}$*.*

The intuition for this result is clear. The disadvantage of imperfect transparency is that some consumers might be uninformed and therefore cannot participate the matching market in both periods. This disadvantage is severe when the number of buyers is small. If there is a large number of buyers, this disadvantage will be mitigated as long as N is not too small relative to B. This is because a sufficient number of buyers will be informed, partially or fully, to match with the two sellers, and the existence of uninformed buyers becomes irrelevant for the total number of matches. Together with the fact that imperfect transparency always dominates in period 1, we can conclude that imperfect transparency is optimal in generating matches even in the dynamic setting provided that B is sufficiently large.

Let T_N^2 be the expected number of matches generated in period 2 given the transparency level N. Figure 1 plots T_B^2 (the red dots) and T_{B-1}^2 (the blue dots) for all $B \in [3, 10]$. Clearly, the number of matches in period 2 is maximized by full transparency only when $B = 3$ or 4, but will be maximized by imperfect transparency when B becomes larger. Therefore, our main insight derived in the static setting continues to hold with a large range of parameters even in the presence of additional search opportunities.

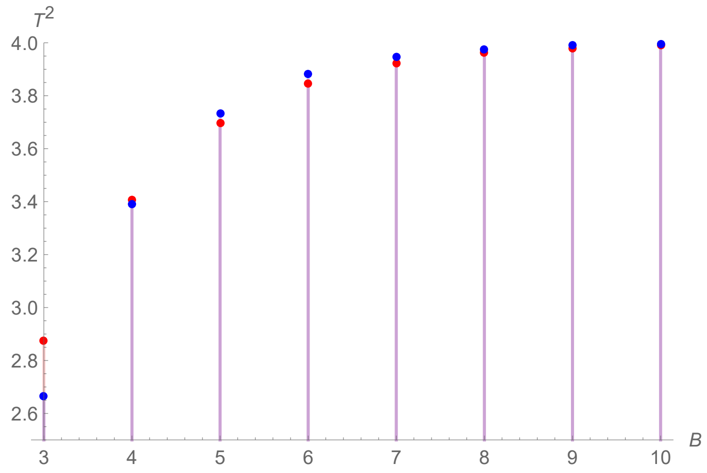

Figure 1: Comparison of T_B^2 and T_{B-1}^2

5.3 Meeting technologies

Eeckhout and Kircher (2010) distinguished meeting from matching in the directed search framework. The meeting considered in their model is ex post in the sense that buyers first search submarkets where individual sellers post prices and then meeting takes place in the submarkets according to the meeting technology. In contrast, the meeting technology proposed in our model generates submarkets or a network of contacts between buyers and sellers within which buyers can search individual sellers. As in Eeckhout and Kircher (2010), our meeting technologies, in particular the separate-meeting technology, can capture various types of rivalry in meeting, including rival, non-rival, and partially rival meetings. Namely, the meeting technology is rival when $N = 1$ where a meeting schedule between a seller and a buyer implies that other buyers are not allowed to meet the seller. The meeting technology is non-rival when $N = B$ where a meeting between a seller and a buyer does not affect the meeting opportunity between this seller and any other buyers. Finally, the meeting technology is partially rival when $N \in (1, B)$, where a meeting opportunity between a seller and a buyer reduces the opportunity for other buyers to meet this seller but it is not completely eliminated. Unlike in Eeckhout and Kircher (2010), we use auction as the trading mechanism, and explore the optimal meeting technology for the society, sellers and buyers respectively. Further, we endogenize the meeting technology with a profit maximizing platform and show that the social optimum can be decentralized.

We now discuss other assumptions of our meeting technologies. The assumption of "anonymity" rules out the trivial case that the buyers' probability of receiving meeting requests from each seller can depend on their identity (which opens a room for removing search frictions in the first place). If the assumption of "symmetry" is violated, sellers could fully extract all the surplus and the platform can transfer that surplus to itself by setting a listing fee. Consider, for example, a case of two sellers and two buyers. Allow seller 1 to send one meeting request but seller 2 to send two. Then, seller 2 knows that there exists one buyer who only receives request from him so he can set $r_2 = 1$ to fully extract surplus from that buyer. Knowing this, seller 1 can also set $r_1 = 1$. The buyer who observes both sellers is indifferent to which seller he chooses to visit. So we assume she selects seller 1. Then, seller 1 also obtains a profit equal to 1. The platform can set each seller a listing fee equal to 1 and extract the whole surplus.

The "no waste" assumption is important for our analysis. If this property is violated, the analysis becomes less straightforward. In particular, there can be an unmatched seller even when there exists a partially informed buyer. Consider, for example, a case of three buyers and $N = 3$. Then, it is possible that: buyer 1 receives only one meeting request from seller 1; buyer 2 receives two meeting requests from seller 2 and one meeting request of seller 1; and buyer 3 receives one meeting request from each seller. In this case, it is possible that all buyers end up selecting seller 1, leaving seller 2 unmatched. So, when the "no waste" assumption does not hold, the total number of matches does not take the simple form as in (4). A related analysis of advertising in the directed search environment which violates the "no waste" assumption can be found in Gomis-Porqueras, Julien and Wang (2017).

6 Conclusion

This paper studies how a change in market transparency can affect matching efficiency, seller profits and buyer surplus in a directed search equilibrium. Using a model with a continuum of duopoly product categories, we are able to identify the optimal market transparency. In particular, we show that full transparency typically leads to a less desirable outcome, not only for efficiency but also for each side of the market. We further consider a profit-maximizing platform that can manipulate market transparency and charge fees for its intermediation service. We show that the platform can implement the efficient allocation and fully extract surplus by choosing an intermediate level of market transparency.

Various market characteristics, such as entry and exit on both sides, ex ante heterogeneity among participants, and idiosyncratic match values, are excluded from the current analysis. For future research, it would be interesting to incorporate the heterogeneous outside options of buyers. In such an extension, a change in market transparency would affect not only the existing buyers' information sets and search strategies, but also the total number of buyers who participate in the market. While we expect an imperfect market transparency continues to yield the highest efficiency, a profit-maximizing platform is unlikely to implement the full-efficiency outcome, at least with the current simple transaction fee policy.

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Appendix.

Proof of Proposition 1. First, we note that $N = 1$ is weakly dominated by $N = B$. To compare $T_{N=1} = 2 - \frac{1}{B}$ and $T_{N=B} = 2 - \left(\frac{1}{2}\right)^{B-1}$ for any $B \geq 2$, we can instead compare $\ln\left(\frac{1}{B}\right)$ and $\ln\left(\left(\frac{1}{2}\right)^{B-1}\right)$ given that the log function is a monotone transformation. The difference of the two terms is $-\ln(B) + (B-1)\ln(2)$, which is equal to 0 when $B = 2$. The derivative of the difference is $\ln(2) - \frac{1}{2} > 0$. We can conclude that $\frac{1}{B} \geq \left(\frac{1}{2}\right)^{B-1}$ for any $B \geq 2$. Therefore, $T_{N=1} \leq T_{N=B}$ for any $B \geq 2$.

Second, for $B \geq 3$, observe that

$$\begin{aligned} T_{N=B} - T_{N=B-1} &= -\left(\frac{1}{2}\right)^{B-1}\left[\Gamma(B \mid B, B) - 2\Gamma(B-1 \mid B-1, B)\right] \\ &= -\left(\frac{1}{2}\right)^{B-1}\left(1 - \frac{2}{B}\right) < 0. \end{aligned}$$

Therefore, $N = B$ cannot be optimal, and so we must have $N^T \in (1, B)$. ■

Proof of Proposition 2. First, fix a value of N_j with $N_j = 0, 1, .., N-1$. Then, we can write

$$\begin{aligned} T(N \mid x_s) - T(N-1 \mid x_s) &= -\left(\frac{1}{2}\right)^{N-1} \frac{1}{C_{B-N_j}^{N-N_j}} + \left(\frac{1}{2}\right)^{N-2} \frac{1}{C_{B-N_j}^{N-1-N_j}} \\ &= \left(\frac{1}{2}\right)^{N-2} \frac{1}{C_{B-N_j}^{N-1-N_j}} \left[1 - \frac{N-N_j}{2(B-N+1)}\right] \\ &= \left(\frac{1}{2}\right)^{N-2} \frac{1}{C_{B-N_j}^{N-1-N_j}} \left[1 - \frac{x_s}{2\left(\frac{B+1}{N} - 1\right)}\right], \end{aligned}$$

where in the second equality we use $\frac{C_{B-N_j}^{N-1-N_j}}{C_{B-N_j}^{N-N_j}} = \frac{\frac{B-N_j!}{N-1-N_j!B-N+1!}}{\frac{B-N_j!}{N-N_j!B-N!}} = \frac{N-N_j}{B-N+1}$. Hence,

$$T(N \mid x_s) > T(N-1 \mid x_s) \text{ if and only if } N < \frac{2(B+1)}{2+x_s} \equiv \hat{N}(x_s). \tag{13}$$

Notice that $\hat{N}(x_s)$ is defined on $[1/N, 1]$ as $x_s = (N - N_j)/N$ and $N_j = 0, 1, .., N-1$. At one extreme, $\hat{N}(1) = \frac{2(B+1)}{3} \in (1, B)$. This implies, $T(N \mid 1)$ first increases in N when $N < \hat{N}(1)$ and then decreases in N when $N > \hat{N}(1)$. Thus, $T(N \mid 1)$ is maximized at $N = \hat{N}(1)$ when $x_s = 1$. At the other extreme, $\hat{N}(\frac{1}{N}) = \frac{2(B+1)}{2+\frac{1}{N}} > N$ for all $N = 1, ..., B$. This implies $T(N \mid \frac{1}{N})$ is monotonically increasing in all $N < B$ and so $T(N \mid \frac{1}{N})$ is maximized at $N = B$. Second, the special case with $N_j = N$ corresponds to $x_s = 0$ and induces $N^e = B$ as examined in the main text.

Since $\hat{N}(x_s)$ is strictly decreasing in x_s as shown in (13), we can conclude that there exists a unique critical value $\hat{x}_s \in (0, 1)$ such that $N^e = B$, if and only if $x_s \leq \hat{x}_s$ and $N^e \in (1, B)$, if and only if $x_s > \hat{x}_s$. This completes the proof of Proposition 2. ■

Proof of Theorem 1. We have shown in the main text the equilibrium reserve prices $r_{N\geq 2}$ given in (5) satisfies the first-order condition. Here, we first show that the second-order condition is satisfied. The second-order derivative of firm 1's profit at the symmetric equilibrium price $r_{N\geq 2}$ is equal to

$$-2^{-1-N} \frac{[N^2\Gamma_0(N \mid N, B) + (2N-3)\Gamma_0(N-1|N, B)][N^2\Gamma_0(N|N, B) + (2N-2)\Gamma_0(N-1|N, B)]}{N(N-1)\Gamma_0(N|N, B)},$$

which is negative given that $N \geq 2$.

Next, we check that a unilateral deviation to setting $r = 1$ and only selling to partially informed consumers is not profitable. It is a matter to compare (7) with $\pi^d = 1 \times (1 - \Gamma_0(N|N, B))$. After simplification, choosing $r_{N\geq 2}$ is better if the following condition holds

$$\left(2^N - 1\right) \frac{N^2\Gamma_0(N|N,B) + 2\Gamma_0(N-1|N,B)(N-1)}{N^2\Gamma_0(N|N,B) + 2\Gamma_0(N-1|N,B)N} \geq 1.$$

This inequality always holds for all $N \geq 2$. Combined with the second-order condition shown above, it shows that $r_{N\geq 2}$ is a global maximizer. ■

Proof of Theorem 2.

In text. ■

Proof of Proposition 3.

In text. ■

Proof of Proposition 4. First, $\pi_{N=1} = 1 - \frac{1}{B}$ and $\pi_{N=B} = 1 - \left(\frac{1}{2}\right)^{B-1}$. By using the logarithm transformation, it is easy to show that $\pi_{N=1} < \pi_{N=B}$, and so $N = 1$ is never optimal. Second, observe that

$$\begin{aligned}
\pi_{N=B} - \pi_{N=B-1} &= -\left(\frac{1}{2}\right)^{B-1} + \frac{1}{B}\left(\frac{1}{2}\right)^{B-2}\left[1 + \frac{1}{B-1+2(B-2)}\right] \\
&= -\left(\frac{1}{2}\right)^{B-1}\left[1 - \frac{2}{B}\left(1 + \frac{1}{B-1+2(B-2)}\right)\right] \\
&= -\left(\frac{1}{2}\right)^{B-1}\frac{3B^2 - 11B + 8}{B(3B-5)} < 0
\end{aligned}$$

for any $B \geq 3$. Therefore, $N = B$ cannot be optimal, and so we must have $N^s \in (1, B)$. The reason why $N^s > N^e$ has been discussed in the main text. ■

Proof of Proposition 5. It is immediate to show $u_{N=1} > u_{N\geq 2}$ when $B = 3$, and so here we shall focus on the case when $B \geq 4$. The proof proceeds case by case. We show the claim $u_{N=1} > u_{N\geq 2}$ in separation for (i) $N = 2$; (ii) $N = 3, 4, ..., B-1$; (iii) $N = B$. Since these cases altogether cover all the possible values of $N = 2, 3, ..., B$ for all $B \geq 4$, we prove the claim in the proposition.

(i) Case 1: $N = 2$. Applying $N = 2$ to (8), we get $u_{N=2} = \frac{2B-3}{B^2(B-1)^2}$. Then,

$$u_{N=1} - u_{N=2} = \frac{1}{B^2(B-1)^2}\left[(2B-1)\left[B(B-2) - (B-1)^3 \ln \frac{B}{B-1}\right] + 2\right].$$

In what follows, we show that the term

$$B(B-2) - (B-1)^3 \ln \frac{B}{B-1}$$

is positive. Since this term can be re-written as

$$(B-1)^2 \left(1 - (B-1) \ln \frac{B}{B-1}\right) - 1,$$

it is sufficient to show that $1 - (B-1) \ln \left(\frac{B}{B-1}\right) > \frac{1}{(B-1^2)}$, or using $\frac{1}{2B} > \frac{1}{(B-1)^2}$,

$$\Upsilon(B) \equiv \frac{2B-1}{2B} - (B-1) \ln \left(\frac{B}{B-1}\right) > 0.$$

Observe that $\Upsilon(4) \approx 0.012 > 0$ and $\Upsilon'(B) = \frac{1}{2B^2} + \frac{1}{B} - \ln\left(\frac{B}{B-1}\right)$. Note that $\Upsilon'(4) = -0.006$ and $\lim_{B\to\infty} \Upsilon'(B) = 0$. Also, $\Upsilon''(B) = \frac{1}{B^3(B-1)} > 0$. Therefore, $\Upsilon'(B) < 0$ for all $B \geq 4$. Since $\Upsilon(4) > 0$ and $\lim_{B\to\infty} \Upsilon = 0$, we then can conclude $\Upsilon(B) > 0$ for all $B \geq 4$. This completes the proof of the case with $N = 2$.

(ii) Case 2: $N \in [3, B-1]$. Using (5) and $C^N_{B-1} = \frac{B-N}{N} C^{N-1}_{B-1}$, we can re-write (8) as

$$u_{N\geq 2} = \left[\frac{1}{N + 2(N-1)(B-N)}\right]\left[\left(\frac{N}{B}\right)^2 \frac{1 + 2(B-N)}{C^{N-1}_{B-1}} \left(\frac{1}{2}\right)^{N-1}\right]. \qquad (14)$$

We separately prove the claim for $B = 4$ and $B \geq 5$.

- **Case 2-1: $B = 4$.** Note that when $B = 4$, the only admissible parameter value is $N = 3$. Applying these values to (9) and (8), we get $u_{N=1} = \left(1 - 3\ln\frac{4}{3}\right)\left(1 - \left(\frac{3}{4}\right)^2\right) \approx 0.0599$ and $u_3 = \frac{1}{7}\left(\frac{3}{4}\right)^2 \frac{3}{C^2_3}\left(\frac{1}{2}\right)^2 \approx 0.0201$. Hence, $u_{N=1} > u_{N=3}$.

- **Case 2-2: $B \geq 5$.** Decomposing (14) into two terms, we will compare the first term in the expression of $u_{N=1}$ in (9) with the first term in (14), and the second term in the expression of $u_{N=1}$ in (9) with the second term in (14). Below, we show that both terms in the expression of $u_{N=1}$ are greater in (9) for $N = 1$ than in (14) for $N \in [3, B-1]$. Note that the term $N + 2(N-1)(B-N)$ takes the minimum when $N = B-1$, which equals $3B - 5$ ($> 2B$) for all $B \geq 5$. Hence, the first term in (14) is strictly smaller than $\frac{1}{2B}$, and so to show $1 - (B-1)\ln\left(\frac{B}{B-1}\right) > \frac{1}{N+2(N-1)(B-N)}$, it is sufficient to show $1-(B-1)\ln\left(\frac{B}{B-1}\right)>\frac{1}{2B}$. But we have already shown it above, i.e., $\Upsilon(B)>0$.

We next show that $1-\left(\frac{B-1}{B}\right)^2>\left(\frac{N}{B}\right)^2\frac{1+2(B-N)}{C_{B-1}^{N-1}}\left(\frac{1}{2}\right)^{N-1}$. Given that $C_{B-1}^{N-1}=C_B^N\frac{N}{B}$, this inequality can be written as

$$\frac{2B-1}{1+2(B-N)}>\frac{BN}{C_B^N}\left(\frac{1}{2}\right)^{N-1}.$$

Note further that $C_B^N>\left(\frac{B}{N}\right)^N$.[16] Thus, to show the inequality in question, it is sufficient to show the following inequality,

$$\Psi(B)\equiv\frac{2B-1}{1+2(B-N)}-\frac{BN}{\left(\frac{B}{N}\right)^N}\left(\frac{1}{2}\right)^{N-1}=\frac{2B-1}{1+2(B-N)}-N^2\left(\frac{N}{2B}\right)^{N-1}>0.$$

Observe that $\Psi(5)=\frac{9}{11-2N}-N^2\left(\frac{N}{10}\right)^{N-1}$. When $N=4$, $\Psi(5)=\frac{9}{3}-4^2\left(\frac{4}{10}\right)^3=3-1.024=1.976$, and when $N=3$, $\Psi(5)=\frac{9}{5}-3^2\left(\frac{3}{10}\right)^2=1.8-0.81=0.99$. Furthermore, $\lim_{B\to\infty}\Psi(B)=1$ and

$$\Psi'(B)=-\frac{4(N-1)}{[1+2(B-N)]^2}+\frac{N^2(N-1)}{B}\left(\frac{N}{2B}\right)^{N-1}.$$

Since $\min\{\Psi(5),\Psi(\infty)\}>0$, if $\Psi(B)$ is monotone in all $B\geq 5$ then $\Psi(B)>0$ for all $B\geq 5$. Suppose otherwise, i.e., $\Psi(B)$ is non-monotone in B. Then, there is a possibility of the existence of a minimum at some interior $\tilde{B}\in(5,\infty)$, satisfying $\Psi'(\tilde{B})=0$. We will check whether the possible minimum attains a positive or negative value. The marginal condition $\Psi'(\tilde{B})=0$ gives $N^2\left(\frac{N}{2\tilde{B}}\right)^{N-1}=\frac{4\tilde{B}}{\left[1+2(\tilde{B}-N)\right]^2}$. Plugging this into $\Psi(B)$, we have

$$\Psi(\tilde{B})=\frac{2\tilde{B}-1}{1+2(\tilde{B}-N)}-\frac{4\tilde{B}}{\left[1+2(\tilde{B}-N)\right]^2}=\frac{4\tilde{B}^2-4(N+1)\tilde{B}+2N-1}{\left[1+2(\tilde{B}-N)\right]^2}>0,$$

where the last inequality follows from the fact that the numerator takes the minimum at $N=B-1$, which equals $2B-3>0$. Hence, we can conclude that $\Psi(B)>0$ for all $B\geq 5$. This completes the proof of the case $N\in[3,B-1]$.

(iii) Case 3: $N=B$**.** Recall that we have $u_{N=B}=\frac{1}{B}\left(\frac{1}{2}\right)^{B-1}=\frac{1}{2B}\left(\frac{1}{2}\right)^{B-2}$. To compare it with $u_{N=1}=\left(1-(B-1)\ln\frac{B}{B-1}\right)\left(1-\left(\frac{B-1}{B}\right)^2\right)$, note first that $1-(B-$

[16] To show $C_B^N\geq\left(\frac{B}{N}\right)^N$, it is sufficient to observe that $C_B^N=\frac{B}{N}\frac{B-1}{N-1}\ldots\frac{B-(N-1)}{1}$, where each of these N terms is no less than $\frac{B}{N}$.

1) $\ln\left(\frac{B}{B-1}\right) > \frac{1}{2B}$, as already shown above, i.e., $\Upsilon(B) > 0$. Hence, what remains to show is that $1 - \left(\frac{B-1}{B}\right)^2 > \left(\frac{1}{2}\right)^{B-2}$ or

$$\Phi(B) \equiv 2B - 1 - B^2 \left(\frac{1}{2}\right)^{B-2} > 0.$$

Note that $\Phi(4) = 7 - 4^2 \left(\frac{1}{2}\right)^2 = 3$ and $\Phi'(B) = 2 - 2B\left(\frac{1}{2}\right)^{B-2} + B^2\left(\frac{1}{2}\right)^{B-2} \ln 2 > 0$, where the last inequality follows from $B\left(\frac{1}{2}\right)^{B-2} < 4\left(\frac{1}{2}\right)^2 = 1$. This completes the proof of the case $N = B$. ■

Proof of Proposition 6

We now consider $B \geq 3$ buyers. Still assume the discount factor $\delta \in [0, 1]$. Assume $N = 1$. All possible matches are formed in period 1, so the discounted total value is

$$T_{N=1} = \frac{5}{3} + \delta \times 0 = \frac{5}{3}.$$

Assume $N = B$. First consider what happens in period 1. This is a market with two sellers and B fully informed buyers. With probability $2 \times (1/2)^B = (1/2)^{B-1}$, there is only one match. With probability $1 - (1/2)^{B-1}$, there are two matches. So, we expect $(1/2)^{B-1} + 2[1 - (1/2)^{B-1}] = 2[1 - (1/2)^B]$ matches. The period-2 market is exactly the same as in period 1 except the number of buyers reduces to $B - 1$ or $B - 2$. With $B - 1$ buyers left, the expected number of matches in period 2 is $2[1 - (1/2)^{B-1}]$. With $B - 2$ buyers left, the expected number of matches in period 2 is $2[1 - (1/2)^{B-2}]$. So the expected value generated in period 2 is $2(1/2)^{B-1}[1 - (1/2)^{B-1}] + 2(1 - (1/2)^{B-1})[1 - (1/2)^{B-2}] = 2[1 - (1/2)^{B-1}]^2$. So the discounted total value is

$$T_{N=B} = 2[1 - (1/2)^B] + 2\delta[1 - (1/2)^{B-1}]^2.$$

Finally, consider $N \in (1, B)$. If there are less than $N - 1$ fully informed buyers, there must exist two partially informed buyers who only know firm 1 and two other partially informed buyers who only know firm 2. Then, there will always be two matches in each period. So if there are less than N fully informed buyers, the discounted total value is

$$2 + 2\delta.$$

With probability $1/C_B^N$ there are N fully informed buyers. In period 1, there are $2[1 - (1/2)^N]$ matches. In period 2, depending on whether there were one (with probability

$(1/2)^{N-1}$) or two matches (with probability $1-(1/2)^{N-1}$) formed in period 1, there are $2[1-(1/2)^{N-1}]$ or $2[1-(1/2)^{N-2}]$ matches respectively. So in case there are $N-1$ fully informed buyers, the discounted total value is

$$\begin{aligned}
&2\left[1-\left(\frac{1}{2}\right)^{N}\right]+\delta\left[\left(\frac{1}{2}\right)^{N-1}\times 2\left(1-\left(\frac{1}{2}\right)^{N-1}\right)+\left(1-\left(\frac{1}{2}\right)^{N-1}\right)\times 2\left[1-\left(\frac{1}{2}\right)^{N-2}\right]\right]\\
=&2\left[1-\left(\frac{1}{2}\right)^{N}\right]+2\delta\left[1-\left(\frac{1}{2}\right)^{N-1}\right]^{2}
\end{aligned}$$

With probability $N(B-N)/C_B^N$ there are $N-1$ fully informed buyers, which means there will be two matches in period 1. In period 2, if both partially informed buyers were matched in period 1, there will be $2[1-(1/2)^{N-1}]$ matches in period 2. The probability that both partially informed buyers are matched in period 1 is

$$\begin{aligned}
&\left(\frac{1}{2}\right)^{N-1}\left[C_{N-1}^{0}\times 1\times\frac{1}{N}+C_{N-1}^{1}\times\frac{1}{2}\times\frac{1}{N-1}+...+C_{N-1}^{N-1}\times\frac{1}{N}\times 1\right]\\
=&\left(\frac{1}{2}\right)^{N-1}(C_{N+1}^{1}+C_{N+1}^{2}+...+C_{N+1}^{N})\frac{1}{N(N+1)}\\
=&\left(\frac{1}{2}\right)^{N-1}\frac{2(2^N-1)}{N(N+1)}.
\end{aligned}$$

To derive the second equality above, we use $C_{N+1}^1+C_{N+1}^2+...+C_{N+1}^N=2^{N+1}-C_{N+1}^0-C_{N+1}^{N+1}$. If only one partially informed buyer was matched in period 1, there is only one match with probability $(1/2)^{N-2}$ and two matches with probability $1-(1/2)^{N-2}$ in period 2. The probability that only one partially informed buyers is matched in period 1 is

$$\begin{aligned}
&2\left(\frac{1}{2}\right)^{N-1}\left[C_{N-1}^{0}1\frac{N-1}{N}+C_{N-1}^{1}\frac{1}{2}\frac{N-2}{N-1}+C_{N-1}^{2}\frac{1}{3}\frac{N-3}{N-2}+...+C_{N-1}^{N-1}\frac{1}{N}0\right]\\
=&2\left(\frac{1}{2}\right)^{N-1}\left[C_{N-1}^{0}1+C_{N-1}^{1}\frac{1}{2}+C_{N-1}^{2}\frac{1}{3}+...+C_{N-1}^{N-2}\frac{1}{N-1}\right]\\
&-2\left(\frac{1}{2}\right)^{N-1}\left[C_{N-1}^{0}\frac{1}{N}+C_{N-1}^{1}\frac{1}{2}\frac{1}{N-1}+C_{N-1}^{2}\frac{1}{3}\frac{1}{N-2}+...+C_{N-1}^{N-2}\frac{1}{N-1}\frac{1}{2}\right]\\
=&2\left(\frac{1}{2}\right)^{N-1}\frac{1}{N}\left[C_N^1+C_N^2+C_N^3+...+C_N^{N-1}\right]\\
&-2\left(\frac{1}{2}\right)^{N-1}\frac{1}{N(N+1)}\left[C_{N+1}^0+C_{N+1}^1+C_{N+1}^2+...+C_{N+1}^{N-2}\right]\\
=&\left(\frac{1}{2}\right)^{N-1}\frac{2(2^N-1)(N-1)}{N(N+1)}.
\end{aligned}$$

To derive the third equality above, we use $C_N^1+C_N^2+C_N^3+...+C_N^{N-1}=2^N-C_N^0-C_N^N$ and $C_{N+1}^0+C_{N+1}^1+C_{N+1}^2+...+C_{N+1}^{N-2}=2^{N+1}-C_{N+1}^0-C_{N+1}^N-C_{N+1}^{N+1}$. If no partially informed buyers were matched in period 1, there will be two matches in period 2. The probability of having no partially informed buyers being matched in period 1 is

$$1-\left(\frac{1}{2}\right)^{N-1}\frac{2(2^N-1)}{N(N+1)}-\left(\frac{1}{2}\right)^{N-1}\frac{2(2^N-1)(N-1)}{N(N+1)}=1-\left(\frac{1}{2}\right)^{N-1}\frac{2N(2^N-1)}{N(N+1)}.$$

Therefore, following having $N-1$ fully informed buyers, the discounted value is

$$\begin{aligned}
&2+\delta\Bigg[\left(\frac{1}{2}\right)^{N-1}\frac{2(2^N-1)}{N(N+1)}2\left[1-\left(\frac{1}{2}\right)^{N-1}\right]\\
&+\left(\frac{1}{2}\right)^{N-1}\frac{2(2^N-1)(N-1)}{N(N+1)}\left[\left(\frac{1}{2}\right)^{N-2}+2\left(1-\left(\frac{1}{2}\right)^{N-2}\right)\right]\\
&+\left(1-\left(\frac{1}{2}\right)^{N-1}\frac{2N(2^N-1)}{N(N+1)}\right)2\Bigg]\\
=&2+\delta\left[2-\frac{16\left[(1/2)^N-(1/2)^{2N}\right]}{N+1}\right]
\end{aligned}$$

Thus, for any $N\in(1,B)$, the generated value is

$$\begin{aligned}
T_N=&2\left[1-\left(\frac{1}{2}\right)^N\frac{1}{C_B^N}\right]+\delta\Bigg[\left(1-\frac{1}{C_B^N}-\frac{N(B-N)}{C_B^N}\right)2\\
&+\frac{2}{C_B^N}\left(1-\left(\frac{1}{2}\right)^{N-1}\right)^2+\frac{N(B-N)}{C_B^N}\left(2-\frac{16\left[\left(\frac{1}{2}\right)^N-\left(\frac{1}{2}\right)^{2N}\right]}{N+1}\right)\Bigg]\\
=&2\left[1-\left(\frac{1}{2}\right)^N\frac{1}{C_B^N}\right]+\delta\left[2-\frac{1}{C_B^N}\left(\left(\frac{1}{2}\right)^{N-2}-\frac{16N(B-N)}{N+1}\left(\left(\frac{1}{2}\right)^N-\left(\frac{1}{2}\right)^{2N}\right)\right)\right]\\
=&2\left[1-\left(\frac{1}{2}\right)^N\frac{1}{C_B^N}\right]+2\delta\left[1-\left(\frac{1}{2}\right)^N\frac{1}{C_B^N}\left(1-\left(\frac{1}{2}\right)^N\right)\frac{4(N+1)+8N(B-N)}{N+1}\right]
\end{aligned}$$

Let T_N^2 be the total matches in period 2. We next compare $T_{N=B}^2$ and $T_{N=B-1}^2$. We have

$$T_{N=B-1}^2=2\left[1-\left(\frac{1}{2}\right)^{B-2}\frac{4B+8(B-1)}{2B^2}+\left(\frac{1}{2}\right)^{2B-2}\frac{4B+8(B-1)}{B^2}\right].$$

Then,

$$T_{N=B}^2-T_{N=B-1}^2=2\left(\frac{1}{2}\right)^{B-2}\left[\frac{4B+8(B-1)}{B^2}\left(\frac{1}{2}-\left(\frac{1}{2}\right)^B\right)-1+\left(\frac{1}{2}\right)^B\right].\qquad(15)$$

Note that in (15), when B becomes large, the term $\frac{4B+8(B-1)}{B^2}$ strictly decreases in B and approaches 0, the term $\frac{1}{2} - \left(\frac{1}{2}\right)^B$ is bounded from above by $\frac{1}{2}$, and the term $\left(\frac{1}{2}\right)^B$ approaches zero. So when B is sufficiently large the sign of (15) is entirely governed by the term -1, which is negative. So we can conclude that $T^2_{N=B} - T^2_{N=B-1} < 0$ when B is sufficiently large. This implies imperfect transparency dominates full transparency even in period 2 if B is large enough. ■