Central Bank Digital Currency:
When Price and Bank Stability Collide

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Abstract

We build a stylized, nominal Diamond-and-Dybvig (1983) model, a consolidated central bank conducts maturity transformation, issuing on-demand, nominal liabilities in the form of an account-based central bank digital currency (CBDC) to citizens and investing the funds in a real asset. We show, the central bank's classic role as the guardian of price stability is in fundamental conflict with its role as a financial intermediator. Implementation of the socially optimal allocation requires a commitment to inflation. Commitment to price stability jeopardizes the real return on currency, and causes runs. Central bank runs manifest themselves as a 'run on the price level'.

Keywords: currency crises, monetary policy, bank runs, financial intermediation, central bank digital currency, inflation targeting
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1 Introduction

The preservation of their currency's purchasing power is a main reason why central banks exist. This price stability objective is written explicitly in many central bank's statutes such as the Federal Reserve's 1977 "dual mandate" in the U.S., and Article 127 of the "Treaty on the Functioning of the European Union" regulating the ECB. Money as a store of value, that is, as a way to shift consumption into the future is widely accepted today, even though the intrinsic value of common paper money is zero. Therefore, such monetary trust is fragile and should not be taken for granted. In this paper, we examine how expectations about the goods supply and aggregate spending behavior impact price stability and monetary trust in currency.

At the heart of this paper is the foundational tenet of economics, that people accept currency only because it enables future consumption and that people do not care for money per se. Therefore, we emphasize the intertemporal consumption problem of an agent when evaluating the acceptance of money as a store of value. Financial intermediators play a large role in enabling optimal consumption patterns of agents across time. In this context, Diamond and Dybvig (1983) show that financial intermediators provide value by enabling optimal risk-sharing allocations among agents that are subject to liquidity shocks. When investing in a demand-deposit contract with an intermediating bank rather than investing in a real production technology directly under autarky, all agents may be better off. Nevertheless, such an outcome is fragile since there also exists a self-fulfilling panic equilibrium, where agents run on the bank, enforcing the costly liquidation of the illiquid, real technology so that everyone had been better off when investing directly, under autarky.

Diamond and Dybvig (1983) emphasize the necessity of bank fragility for providing optimal risk-sharing allocations. In the real world, however, real investment is conducted not only by banks but also by firms and governments, and demand-deposit contracts are not real but nominal, where a central bank exists to control the price level via the money supply through the banking system. The real value of a currency, therefore, depends on complex interactions of a central bank, the government, the profit-maximizing banking system and firms, firm production, and agent's consumption behavior, see Allen and Gale (1998), Skeie (2008), and Allen, Carletti, and Gale (2014). Yet, it is foremost the tension between an agent's nominal claim versus expectations on the real value of currency that is crucial for realizing optimal intertemporal consumption bundles and maintenance of monetary trust. To focus on this tension, this paper studies a highly stylized model, a nominal version of Diamond and Dybvig (1983), in which agents interact with the central bank directly, neglecting firms and other financial intermediators in the analysis. Instead, the central bank here also comprises the functionality of firm and government investment in the real economy and the bank's role as a financial intermediator, in addition to the functionality of a traditional central bank. In that way, we follow Velasco (1996), Calvo (1988) and Obstfeld (1996) who also consider a consolidated central bank, however without the financial intermediation role, as modeled here.

These issues become particularly salient when a central bank issues a (nominal) account-based central bank digital currency (CBDC) to its citizens. Many central banks and policymaking institutions are openly debating the implementation of a CBDC.[1] With a CBDC, households will have access to an electronic means of payment and, thus, an attractive alternative to traditional deposit accounts. [2] This raises the question of what the central bank should do regarding its asset side. Investment in highly liquid assets will dry up the resources for loans traditionally funded via retail bank deposits, creating inefficiencies. If, instead, the central bank chooses to funnel the obtained CBDC funds back to long-term borrowers, the central bank enters the business of financial intermediation and maturity transformation. The tension between a central bank's traditional role as guardian of the price level and its new role as financial intermediary is the focus of this paper.[3]

In our model, the CBDC acts as the unique currency and nominal asset, and takes the role of a short-term, on-demand central bank liability akin to demand-deposit contracts with private banks. Therefore, there can no longer be a "withdrawal" of deposits. Instead, agents "spend" their CBDC balances on real goods.[4] Real goods can only be traded against the CBDC, implicitly setting a form of a cash-in-advance constraint in the tradition of Svensson (1985) and Lucas and Stokey (1987). The central bank acts as the unique investor in the real economy and is the monopoly supplier of real goods to citizens in return for nominal currency. We allow for competition with the private banking sector in section 7.2. This modeling simplification allows us to explore and highlight fundamental trade-offs between the central bank's traditional price stability objective, the objective of attaining optimal risk-sharing allocations through financial intermediation, and the maintenance of monetary trust in currency (no 'runs on the central bank'). As the main contribution of the paper, we show an impossibility result which we term the "CBDC trilemma." The central bank can never attain price stability, monetary trust (absence of central bank runs) and the socially optimal allocation at the same time.

But what does a run on the central bank here mean? The central bank is the issuer of currency and can, therefore, always deliver on its nominal obligations.[5] Rationing or limited service of CBDC

[1]See Barrdear and Kumhof, 2016; Bech and Garratt, 2017; Chapman et al., 2017; Lagarde, 2018; Ingves, 2018; Kahn et al., 2019; Davoodalhosseini et al., 2020; Auer and Böhme, 2020; Auer et al., 2020; Group of 30, 2020).

[2]As Fernández-Villaverde et al. (2020) show, a CBDC offered by the central bank may be such an attractive alternative to private bank deposits that the central bank becomes a deposit monopolist, further consolidating its role as a financial intermediator.

[3]Central banks have engaged in large-scale, long-term lending to the economy ("quantitative easing") since the financial crises in 2008-2009. The introduction of a CBDC will considerably enlarge these activities.

[4]Think about the "electronic dollars" that many universities issue to faculty and students in their ID cards for purchases on campus. One can spend "electronic dollars" in different campus locations, such as vending machines and food courts, but one cannot "withdraw" the "electronic dollars" or transform them into other assets.

[5]Notice, that under a run on the price level, issuing additional amounts of CBDC not only does not help, but it makes the inflation worse. The ability of a central bank to issue as much CBDC as desired is a useless tool against runs on its liabilities. See Section 6 for more details.

deposits as inherent in the Diamond and Dybvig (1983) bank run equilibrium can, therefore, not arise. Instead, an attack and run on the central bank will manifest itself as a collective spending spree where agents who have no instantaneous consumption needs, nevertheless, spend their CBDC balances on goods because they expect the real value of currency to decline across time. CBDC forfeits its purpose as the store of value if agents store for future consumption in terms of (non-perishable) consumption goods.[6] Therefore, a run on the central bank is equivalent to *monetary distrust*. The aggregate spending behavior at a given goods supply impacts the price level via market clearing. Therefore, a central bank run will manifest itself as a run on the price level. In section 5 and 6, we also discuss the case of goods rationing and first-come-first-serve style supermarket stockouts, which both implement a form of 'suspension of spending', and are therefore alternative revelations of monetary mistrust when prices may not clear markets.

As the first part of the CBDC Trilemma, we show that the central bank can implement the social optimum in dominant strategies and deter central bank runs ex ante when credibly committing to giving up price stability whenever necessary. As the second and third part of the CBDC trilemma, we show that a central bank policy that is designed to keeping prices stable either fails to implement the socially optimal allocation or gives rise to multiple equilibria, one of which is the central bank run equilibrium.

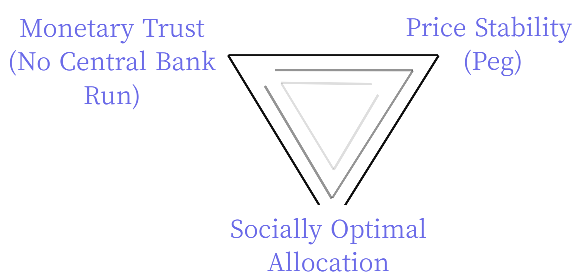


Figure 1: CBDC Trilemma: For the consolidated central bank, it is impossible to attain all three objectives at a time. When one objective is fixed, at least one other objective has to be sacrificed.

For the mechanism behind these results, note first that monetary trust hinges on expectations about output growth and about other agent's spending behavior. As an important feature of our model, output is endogenous and subject to a liquidation externality inherited from Diamond and Dybvig (1983). Investment in the real economy is illiquid and long-term. The central bank can increase output in the short-run via costly liquidation, which implies sacrificing future output

[6]There exists a well-established literature that micro-founds the use of money by noting that consumption goods are perishable while money is not. We are aware of this literature. Our approach allows the possibility that agents can flee into semi-durable goods such as pasta, rice, and tuna tins, as happened during the supermarket stockouts in Europe and the U.S. during the start of the Corona pandemic in 2020.

and its returns.[7] One may think of this externality as costly short-termism such as asset and labor misallocation or underinvestment in future economic prosperity such as supply chain stability, vaccine development, and research. All agents are aware of the externality. They, therefore, pay close attention to the central bank's liquidation policy and its implied goods supply today versus tomorrow, when making their nominal spending decisions. The goods supply is equally shared by all agents that go shopping in that period. Therefore, at a given goods supply, high aggregate spending reduces the purchasing power of CBDC balances since goods prices adjust to clear the market. "Patient" agents who have no instantaneous consumption needs strategically time their spending decision since they can spend CBDC early and store the purchased goods for later consumption. A run on the central bank, i.e., strategic early spending occurs if patient agents expect their CBDC balances to buy fewer goods tomorrow rather than today.

There are various ways how the central bank can react to a run. Unlike a private bank that would need to take the price level as given, since CBDC contracts are nominal while central bank investment is real, the central bank is not constrained to liquidate assets in a particular proportion to realized CBDC spending. Instead, and as an important mechanism in the paper, the central bank can strategically choose the extent of asset liquidation for an observed measure of CBDC spending, thus, simultaneously setting the goods supply and the market-clearing price level. [8] Through the liquidation externality, however, the goods supply today pins down the goods supply tomorrow. Therefore, at a given level of aggregate spending, the central bank's liquidation policy also pins down the real return on CBDC. In particular, the price level and the real return on CBDC are intertwined, and cannot be set independently of one another.

While the central bank controls the goods supply, she does not (directly) control the agent's spending behavior. The central bank can, however, impact the agent's spending behavior via her liquidation policy by steering the real return on CBDC. A 'run-deterring liquidation policy' sufficiently shortens the early goods supply to impose a positive real return on CBDC, whenever strategic early spending occurs. Such a policy renders 'spend early' ex post suboptimal for patient types, and ex ante deters them from spending early if the announcement of such a policy is credible. Therefore, a credible 'run-deterring liquidation policy' deters central bank runs ex ante.

The absence of central bank runs is necessary but not sufficient for attaining the socially optimal allocation. Implementing the socially optimal allocation requires the central bank to supply the optimal amount of goods to patient and impatient agent types at different points in time. Since the central bank cannot observe types, she cannot directly deter patient agents from spending early. Instead, she has to adopt a central bank policy under which self-selection is individually rational

[7]See for instance Radelet, Sachs, Cooper, and Bosworth (1998) on how the liquidation of long-run assets may have caused the East Asian financial crises in a self-fulfilling manner in mid-1997.

[8]Historically, governments have limited spending through alternative policies, from calling loans to the private sector, increasing the policy interest rate to force savings, or rationing in war times, which are just forms of hidden inflation. The details of how a central bank can achieve a limited liquidation of projects are somewhat irrelevant for our main argument, but the historical evidence suggests governments have plenty of levers to do so.

for the different agent types. This is exactly the case, when announcing a run-deterring liquidation policy which additionally finetunes the extent of asset liquidation to match the socially optimal allocation. The deterrence of central bank runs requires the central bank to limit the extent of premature asset liquidation - no matter what. The implementation of the social optimum thus implies that the central bank will not increase the goods supply as a response to high aggregate spending. Such strategic limitation of liquidation, however, interferes with the price level. Because the CBDC price level must clear the market, the occurrence of high (potentially off-equilibrium path) nominal spending at a limited goods supply requires the central bank to tolerate high prices and inflation. If the central bank's commitment to limit asset liquidation is, however, credible, patient agents anticipate the low purchasing power when spending early, and therefore 'roll over.' As a consequence, only agents with genuine, instantaneous consumption needs spend on goods early, and high price levels only occur off-equilibrium. Therefore, the central bank's credible commitment to limit asset liquidation implements the social optimum in dominant strategies, and deters central bank runs ex ante, in the spirit of Kydland and Prescott (1977b) and Barro and Gordon (1983). In contrast to Obstfeld (1996) and Velasco (1996), the liquidation externality here allows the central bank to avoid a self-fulfilling currency crisis using short-term inflation as an off-equilibrium path threat to deter runs.

If the central bank imposes price level stability as the main objective, instead, she would need to counter high nominal spending by liquidating proportionally more real assets. Through the liquidation externality, this may require the real value of CBDC to decline across time. If the target price level is high, then the required asset liquidation for maintaining the price target level is low so that runs on the central bank would not arise. But the socially optimal allocation is then never attained since agents with instantaneous consumption needs, 'impatient agents', consume too little. For attaining the socially optimal allocation, a lower target price level is required. We show that these lower price target levels cannot be supported if high aggregate spending occurs. Instead, via the externality, the required liquidation for maintaining the price level at target reduces the real goods supply for tomorrow by too much, causing the real return on CBDC to become negative. The patient agents can anticipate that the return on CBDC balances will be low when aggregate spending is high. Therefore, if patient agents believe that aggregate spending will be high, they panic and optimally respond by spending early as well, thus, contributing to a self-fulfilling 'run on the central bank.' That is, anticipated inflation tomorrow causes a central bank run and hyperinflation today.[9] Thus, price stability imposes strong restrictions on the central bank's liquidation policy, such that the social optimum may no longer be reached, or central bank runs may reoccur.

[9] It may be tempting to think of the runs on toilet paper and spaghetti, as happened all over the world during the start of the COVID-19 pandemic in 2020. The production technology here, however, yields risk-free real interest. Hence, the comparison to a potato field is more suitable, where potatoes are small in the short run but grow large in the long run. Harvesting small potatoes today means giving up large potatoes tomorrow.

The assumption that real production is non-random and previsible, depending only on the extent of liquidation, is essential for our mechanism. There is no import of goods, no exogenous shocks, and monetary spending does not translate into increased real production. There are no explicit nominal rigidities. But in the second part of the trilemma, we impose price stability, which can be interpreted as an extreme form of price stickiness. Under aggregate risk in output fundamentals, such as considered in Obstfeld (1996) and Velasco (1996), the deterrence of runs may fail.

In Section 6, we investigate the obvious alternative to strategic asset liquidation, namely how the classic central bank intervention of changing the money supply affects the agent's incentives to run. We show that since the run-deterrence mechanism works via the aggregate supply of real goods, nominal fiscal backing, changes in the money supply, and higher nominal interest rates cannot prevent agents from running on the central bank. In Section 7, we show that our results extend to a setting where the central bank directly competes with private retail banks for deposits. If the goods market is centralized, large-scale liquidation by the central bank can cause runs on private banks and vice versa. Consequently, there is a strong motive for the central bank and private banks to coordinate real asset liquidation which, as we show, can deter runs. If coordination is not possible, e.g., by regulating private banks, the central bank can only deter runs if it controls a sufficiently large share of the deposit market. In Section 8, we discuss the connection of our run-deterrence mechanism to Jacklin (1987). There, a financial intermediary can always attain the socially optimal allocation when offering the possibility of interim trade in equity shares to agents instead of enabling risk-sharing via demand deposits. We demonstrate that Jacklin (1987)'s dividend policy is a special case of a run-deterring policy. Moreover, we transfer Jacklin (1987) to a nominal world, and discuss the differences from our nominal setting. In Section 10.2, we discuss the case of hidden competition between the central bank and private banks in the form of a synthetic CBDC.

Our mechanism is not specific to a CBDC, and already exists in the current financial system when assuming that money is neutral and that output is subject to the described externality. We outline the parallels in section 9.

Related literature

Building on the seminal Diamond and Dybvig (1983) model, we contribute to the literature on financial intermediation and bank fragility by stressing the central bank's role in liquidity transformation when issuing a CBDC that allows depositors to share idiosyncratic liquidity risk. Similar to Diamond and Dybvig (1983) and Ennis and Keister (2009), we study the micro incentives of depositors to spend from the bank. But unlike them, we employ nominal instead of real demand-deposit contracts, giving "the bank" an additional tool –the price level– to prevent runs. We share with Ennis and Keister (2009), that our run deterrence mechanism crucially relies on the central bank's knowledge of the measure of agents with instantaneous consumption needs ('impatient' types).

Proneness of nominal demand-deposit contracts to runs has previously been considered by Allen and Gale (1998), Diamond and Rajan (2006), Skeie (2008), Allen, Carletti, and Gale (2014), and Leiva and Mendizábal (2019), amongst others. Unlike in all these papers, in our framework, not only the agents but also the central bank is a strategic player, endowed with direct power over the price level and the goods supply. This allows her to steer the agent's incentives to not run. Moreover, we abstract from explicitly modeling firms and banks, focusing on the interaction between depositors and the consolidated central bank. As a consequence, we attain implementation of the socially optimal risk-sharing allocation in dominant strategies, when using the price level as a strategic tool.

This article is related to the first and second generation literature on self-fulfilling currency crises. Similar to Krugman (1979), a currency crisis is caused due to expectations of rationally behaving agents. Similar to Obstfeld (1984, 1988, 1996), multiple equilibria can arise due to self-fulfilling expectations. In Obstfeld (1996), a government holds foreign reserves to defend an exchange rate peg. The amount of foreign reserves and domestic currency holdings by agents determine how resilient the government is against speculative currency attacks. High reserves can deter attacks completely, while lower reserve holdings give rise to self-fulfilling currency attacks. In a different section of the paper, the government targets output and exchange rate stability subject to exogenous output shocks. The government can respond to shocks and maintain output high by devaluing its currency, i.e. giving up the peg. Similarly, Obstfeld (1984) features exogenous shocks to domestic credit. Here instead, there is no exogenous randomness. Output shocks are endogenous and only occur if the central bank is willing to stabilize prices by liquidating real assets following high, endogenous spending behavior (run on currency). Moreover, here, the central bank can deter the run on currency by credibly committing to abandon the peg whenever output is threatened in the short-run, see also Velasco (1996). In Calvo (1988), the government cannot commit to the real value of public debt, and can repudiate either via taxation or inflating debt. The agents anticipate the government's repudiation, which may cause a self-fulfilling debt crisis. Unlike there, here, it is not the government but the spending agents who cannot commit. And the central bank takes action, using repudiation as a threat to patient agents which deters them from running on the central bank, but requires currency to lose value in the short-run. As the main difference to Calvo (1988), Obstfeld (1984, 1996), and Velasco (1996) our model emphasizes the maturity transforming role of the central bank for enabling optimal allocations via CBDC contracts , similar to Diamond and Dybvig (1983). Due to a liquidation externality, output is an endogenous function of both the agent's actions and the central bank's commitment to either price stability or the implementation of socially optimal allocations. Because premature liquidation increases output in the short-run only at the expense of reducing output in the long-run, price stabilization via liquidation is costly. Due to this liquidation externality, short-term inflation can be socially optimal since it acts as an off-equilibrium path threat to deter speculation against the real value of currency.

Last, we contribute to a growing literature on the macroeconomic implications of introducing a

CBDC (Andolfatto, 2021; Berentsen, 1998; Böser and Gersbach, 2019a,b; Brunnermeier and Niepelt, 2019; Chiu et al., 2019; Fernández-Villaverde et al., 2020; Ferrari et al., 2020; Keister and Sanches, 2019; Skeie, 2019; Williamson, 2019). We differ from this literature by pointing out the central bank's trade-off between optimal financial intermediation and the price stability objective when issuing a CBDC.

For our analysis, we abstract from the existence of competing national or digital currencies (Benigno, 2019; Benigno, Schilling, and Uhlig, 2019; Fernández-Villaverde and Sanches, 2019; Schilling and Uhlig, 2019) and assume full functionality of the CBDC account and ledger system.

2 The basic framework

Our framework builds on the classic Diamond and Dybvig (1983) model of banking. Time is discrete with three points in time $t = 0, 1, 2$, and no discounting. There is a $[0, 1]$-continuum of agents, each endowed with 1 unit of a real consumption good in period $t = 0$. Agents are symmetric in the initial period, but can be of two types in period 1: patient and impatient. An agent is impatient with probability $\lambda \in (0, 1)$ and otherwise is patient. The agent's type is randomly drawn at the beginning of period 1 and independently across agents. Types are private information. Since we have a continuum of agents, there is no aggregate uncertainty about the measure of patient and impatient types in the economy. Thus, λ also denotes the share of impatient agents. Impatient agents value consumption only in period 1. In contrast, patient agents value consumption in period $t = 2$. To make this precise, consider some agent $j \in [0, 1]$ and let c_t represent goods consumed by an agent j at period t. Preferences for agent j are then given by

$$U(c_1, c_2) = \begin{cases} u(c_1), & \text{if } j \text{ is impatient} \\ u(c_2), & \text{if } j \text{ is patient} \end{cases}$$

where $u(\cdot) \in \mathbb{R}$ is a strictly increasing, strictly concave, and continuously differentiable utility function over consumption $c \in \mathbb{R}_+$. We further assume a relative risk aversion, $-x \cdot u''(x)/u'(x) > 1$, for all consumption levels $x > 1$.

There exists a long-term production technology in the economy. For each unit of the good invested in $t = 0$, the technology yields either 1 unit at $t = 1$ or $R > 1$ units at $t = 2$. Additionally, there is a goods storage technology between periods 1 and 2, yielding 1 unit of the good in $t = 2$ for each unit invested in $t = 1$. Let $x_1 \geq 0$ denote the agent's real consumption when deciding to spend at $t = 1$, and let $x_2 \geq 0$ denote the agent's consumption when spending at $t = 2$.

2.1 Optimal risk sharing

Following Diamond and Dybvig (1983), we derive, first, the optimal allocation. The social planner collects and invests the aggregate endowment in the long technology. Given that all agents behave according to their type, the social planner maximizes *ex-ante* welfare

$$W = \lambda u(x_1) + (1-\lambda)u(x_2) \tag{1}$$

by choosing (x_1, x_2), subject to the feasibility constraint $\lambda x_1 \leq 1$, and the resource constraint $(1-\lambda)x_2 \leq R(1-\lambda x_1)$. The interior first-order condition for this problem implies that the optimal allocation (x_1^*, x_2^*) satisfies:

$$u'(x_1^*) = Ru'(x_2^*). \tag{2}$$

Given our assumptions, the resource constraint binds in the optimum

$$R(1-\lambda x_1^*) = (1-\lambda)x_2^*. \tag{3}$$

This condition, together with equation (2), uniquely pins down (x_1^*, x_2^*) and delivers the familiar optimal deposit contract in Diamond and Dybvig (1983). Together with $R > 1$ and the concavity of $u(\cdot)$, equation (2) implies that the optimal consumption of patient agents is higher than the consumption of impatient ones: $x_1^* < x_2^*$.

Moreover, the depositors' relative risk-aversion exceeding unity and the resource constraint yield $x_1^* > 1$ and $x_2^* < R$.[10]

Diamond and Dybvig (1983) show that a demand-deposit contract can implement the efficient allocation. A key feature of their analysis is the use of a "real" demand deposit contract (i.e., a contract that promises to pay out goods in future periods). Due to a maturity mismatch between real long-term investment and real deposit liabilities, the Diamond and Dybvig (1983) environment, however, also features a bank run equilibrium under which the social optimum is not implemented. Our main contribution is to show that a nominal contract can lead to the implementation of the efficient allocation in dominant strategies. In other words, runs do not occur along the equilibrium path. The key mechanism is that the central bank can set the price level, thereby controlling the wedge between real long-term investment and nominal deposit liabilities. The implementation in dominant strategies comes at a price, requiring flexibility of the price level.

[10] Following the proof in Diamond and Dybvig (1983),

$$Ru'(R) = u'(1) + \int_1^R \frac{\partial}{\partial x}(x \cdot u'(x))\,dx = u'(1) + \int_1^R (x \cdot u''(x) + u'(x))\,dx < u'(1) \tag{4}$$

by $-x \cdot u''(x)/u'(x) > 1$ for all x.

3 A nominal economy

Consider now an economy with a social planner that uses nominal contracts to implement the efficient allocation.

Nominal contracts. The planner offers contracts in a unit of account for which it is the sole issuer. Because central banks have a monopoly on currency, the planner in our analysis can be equated with the central bank or any other monetary authority with the ability to issue currency. In this paper, we refer to the unit of account as a central bank digital currency (CBDC) or digital euros. Agents who sign a contract with the central bank hand over their real goods endowment and receive CBDC balances in return. The most straightforward interpretation of our environment is to think of a CBDC as an account-based electronic currency in the sense of Barrdear and Kumhof (2016) and Bordo and Levin (2017), i.e., to think of a CBDC as being akin to a deposit account at the central bank. In Section 10, we show that the results of our paper largely carry over to a token-based system or hybrid systems. Agents can spend their CBDC balances by redeeming them at the central bank in exchange for goods. Spending therefore reduces the CBDC supply. As with physical euros, we impose the constraint that agents cannot hold negative amounts of a CBDC.

Timing. At $t = 0$, the central bank creates an empty account, i.e., a zero-balance CBDC account, for each agent in the economy. In the benchmark model, we assume that in $t = 0$, all agents sell their unit endowment of the good to the central bank in exchange for $M > 0$ units of digital euros, credited to that agent's account. The central bank then invests all goods in the long-term technology. We consider voluntary participation of the agents in central bank contracts in section 7.

In $t = 1$, agents learn their type and decide whether to spend their CBDC balances, M, or to 'roll them over'. In $t = 1$, agents also have access to the goods storage technology between $t = 1$ and $t = 2$.[11] The central bank contract imposes the constraint that an agent either spends all of her balances or none at all. Because types are unobservable, the central bank cannot discriminate between patient and impatient agents to deny a patient agent access to her balances. Let $n \in [0, 1]$ denote the share and measure of agents who decide to spend in $t = 1$. The central bank observes n and then decides on the fraction $y = y(n)$ of the technology to liquidate, supplying that according quantity in the goods market at the market-clearing unit price P_1. Notice that through the resource constraint, early liquidation of the technology reduces the remaining investment and, hence, the supply of goods in $t = 2$. That is, there is a real payoff externality, and the central bank's liquidation choice in $t = 1$ determines the real supply of goods for both of the periods $t = 1$ and $t = 2$. There is no free disposal, thus, all returns that accrue to the technology in $t = 2$ are offered in the goods market for purchase against CBDC. Given n, the central bank also chooses a nominal interest rate $i = i(n)$ to be paid in period 2 on the remaining CBDC balances. Each digital euro held at the end

[11] Our model is equivalent to Diamond and Dybvig (1983), where storage between $t = 1$ and $t = 2$ does not exist, but where patient agents can also consume in $t = 1$.

of $t = 1$ turns into $1 + i(n)$ digital euros at the beginning of $t = 2$. Notice that $i(n) \geq -1$, given that agents cannot hold negative amounts of digital euros.

In $t = 2$, the remaining investment in the technology matures so that the central bank supplies $R(1 - y(n))$ units of goods in exchange for the remaining money balances. The measure of depositors $1 - n$ who rolled over each have $(1 + i)M$ digital euros to spend on goods at a market-clearing price P_2. Figure 2 summarizes this timing.

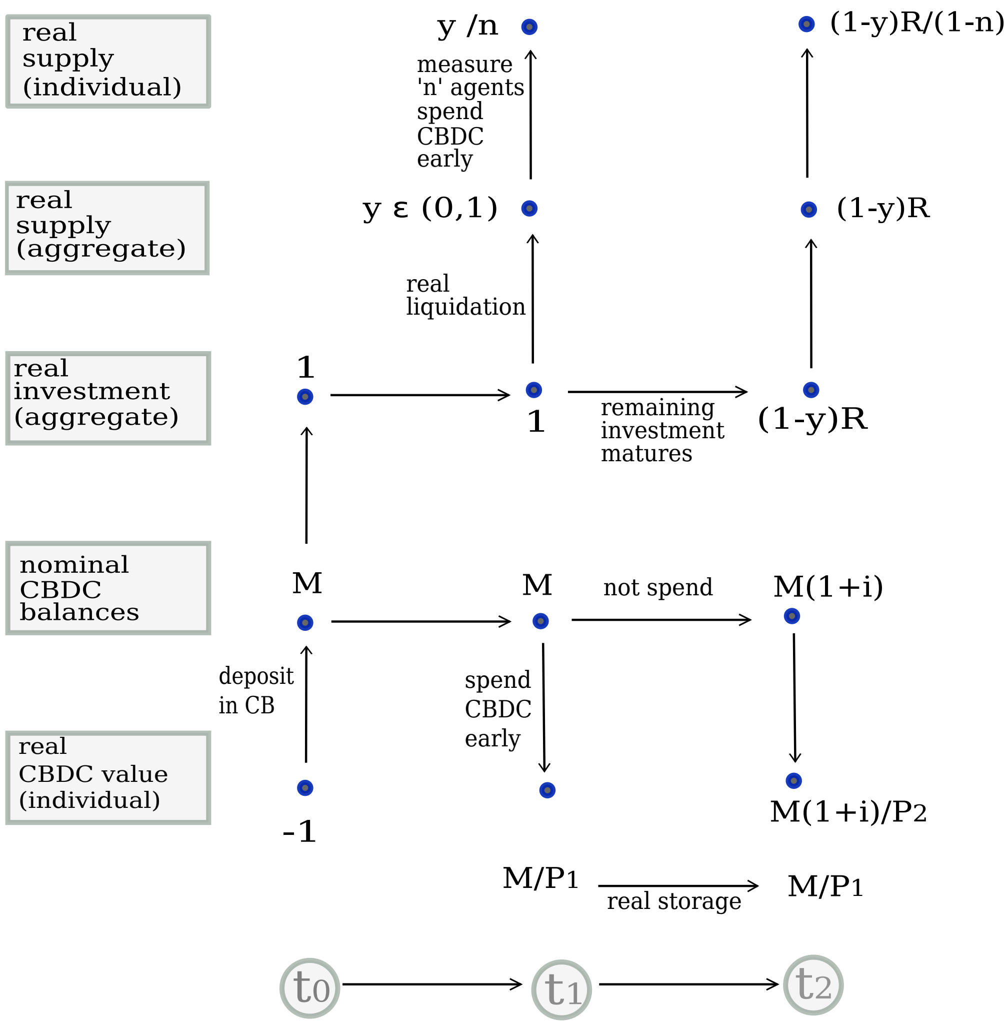


Figure 2: Nominal and real investment and contracts

Definition 1. *A central bank policy is a triple $(M, y(\cdot), i(\cdot))$, where $y : [0, 1] \to (0, 1]$ is the central bank's liquidation policy and $i : [0, 1] \to [-1, \infty)$ is the interest rate policy for every possible spending level $n \in [0, 1]$.*

Notice that M is not state-contingent. The logic here is that, traditionally, 1 dollar today is 1 dollar tomorrow. In Section 6, we discuss an extension where we allow M to be state-contingent.

We restrict attention to strictly positive liquidation policies $y(\cdot) > 0$ to rule out equilibria where impatient agents do not spend CBDC early since no goods are supplied in the economy.

Market clearing. In periods 1 and 2, agents spend their money balances for goods in a Walrasian market. The market-clearing conditions are:

$$\underbrace{nM}_{\substack{\text{nominal CBDC}\\ \text{supply in } t_1}} = P_1 \cdot \underbrace{y(n)}_{\substack{\text{real goods}\\ \text{supply in } t_1}} \tag{5}$$

$$\underbrace{(1-n)(1+i(n))M}_{\substack{\text{nominal CBDC}\\ \text{supply in } t_2}} = P_2 \underbrace{R(1-y(n))}_{\substack{\text{real goods}\\ \text{supply in } t_2}}, \tag{6}$$

which take the form of the quantity theory equation in each period. As these equations reveal, a higher interest rate $i(n)$ results only in a higher price level P_2, when n and $y(n)$ remain unchanged. This is the standard Fisher relationship between nominal interest rates and inflation. Quantity theory then implies a higher nominal CBDC supply in t_2. Given aggregate spending n in $t = 1$, and the central bank's policy, these conditions determine the price level, $P_1 = P_1(n)$ and $P_2 = P_2(n)$, in each period:

$$P_1(n) = \frac{nM}{y(n)} \tag{7}$$

$$P_2(n) = \begin{cases} \frac{(1-n)(1+i(n))M}{R(1-y(n))}, & y(n) < 1 \\ \infty, & y(n) = 1,\ n < 1 \\ \in [0, \infty], & y(n) = 1,\ n = 1 \end{cases} \tag{8}$$

The special case $y(n) = 1$, $n < 1$ denotes the incidence where the goods supply in $t = 2$ equals zero while a demand for goods exists. The special case $y(n) = 1$, $n = 1$ denotes the incidence where both the goods supply and the goods demand in $t = 2$ equal zero. So far, we have not imposed price stability. Instead, the price levels flexibly adjust in aggregate spending and the central bank's liquidation policy. The central bank chooses the initial money supply before learning the measure of spending in the intermediate period. The central bank, however, controls the supply of goods, which is chosen conditional on the measure of spending. As a result, the central bank simultaneously, and interdependently controls the price level in period 1 and the real value of CBDC at time one versus time two.[12] The nominal interest rate allows the central bank to control the price level in period 2 independently of the price level in period 1. Because investment is real and since the intermediary is the central bank with a monopoly on the unit of account in which contracts are denominated, the

[12] A private bank, in contrast, would need to take P_1, P_2 as given, which together with the observation n implies a unique liquidation $y(n, P_1)$. In a more detailed model, the central bank could determine the supply of goods by different instruments, such as calling loans to private banks or by moving the policy interest rate (as in New Keynesian models). The details of how that happens are not central to our argument.

liquidation policy is flexible. An additional CBDC euro spent does not necessarily translate into a specific, proportional raise in asset liquidation. Rather, liquidation is strategically directed to serve as a monetary policy tool.

Implied real contract. Patient agents have no instantaneous consumption needs in $t = 1$. Because storage of consumption goods is possible between $t = 1$ and $t = 2$, patient agents strategically spend their CBDC early or late. The individual real allocation that a patient agent can afford with her CBDC balances when spending early versus late is all that matters. The real value of the CBDC balances in $t = 1$ equals

$$x_1 = \frac{M}{P_1}, \tag{9}$$

while the real value of CBDC balances in $t = 2$ equals

$$x_2 = \begin{cases} \frac{(1+i(n))M}{P_2}, & P_2 < \infty \\ 0, & P_2 = \infty \end{cases} \tag{10}$$

Aggregate spending n and the liquidation policy $y\,(n)$ jointly determine the allocation of goods via the market-clearing conditions. The real allocations when spending in $t = 1$ versus $t = 2$ can therefore be rewritten via (7) and (8) as

$$x_1(n) = \begin{cases} \frac{y(n)}{n}, & n > 0 \\ \infty, & n = 0 \end{cases} \tag{11}$$

$$x_2(n) = \begin{cases} \frac{1-y(n)}{1-n}R, & n < 1 \\ 0, & n = 1, y(n) = 1 \\ \infty, & n = 1, y(n) < 1 \end{cases} \tag{12}$$

That is, for given aggregate spending, via her liquidation policy, the central bank directly sets the real value of CBDC in $t = 1$ *and* $t = 2$. Because all agents that spend CBDC in the same period have the same nominal expenses, and since the goods market is centralized, the real goods supply $y(n)$ is equally distributed across all spending agents in period 1, and the supply $R(1 - y(n))$ is equally allocated to all spending agents in period 2.[13]

Given an aggregate spending level $n \in [0, 1]$, for a patient agent $j \in [0, 1]$ it is optimal to 'spend' CBDC money balances M in $t = 1$ if $x_1(n) \geq x_2(n)$ while it is optimal to 'not spend' if $x_1(n) \leq x_2(n)$. Since $y(n) > 0$ for all $n \in [0, 1]$, and thus $x_1(n) > 0$ for all $n \in [0, 1]$ 'spend' is always optimal for an impatient agent. We restrict attention to pure strategy Nash equilibria with regard to the depositors' coordination game. Therefore, in the case $x_1(n) = x_2(n)$ and $\lambda < n < 1$, a

[13] These equations remain intuitive even if $y(n) = 0$ or $y(n) = 1$. Therefore, we assume that they continue to hold, despite one of the price levels being potentially ill-defined or infinite.

mass $n - \lambda$ of patient agents spends their CBDC money balances in $t = 1$ and the remaining mass of agents $1 - n$ does not. This is consistent with optimal behavior. Our analysis can be extended to allow mixed strategy equilibria via the law of large numbers applied to the continuum of agents, see (Uhlig, 1996).

To summarize: in $t = 0$, the central bank announces and commits to a policy $(M, y(\cdot), i(\cdot))$, pinning down a spending-contingent real goods supply and an offer of a nominal contract $(M, M(1+i(\cdot)))$ in exchange for 1 unit of the good. All consumers accept the contract and the policy, meaning they have the option to spend either M digital euros in period 1 or $M(1 + i(n))$ digital euros in period 2, for every possible level of aggregate spending $n \in [0, 1]$. We discuss voluntary participation in contracts in Section 7.

In $t = 1$, the aggregate spending level n is realized. Finally, the central bank's policy, together with the market-clearing conditions, result in the real consumption amounts $(x_1(n), x_2(n)) = (\frac{M}{P_1}, \frac{M(1+i(n))}{P_2}) = \left(\frac{y(n)}{n}, \frac{1-y(n)}{1-n}R\right)$. Notice that the central bank is fully committed to carry through with its policy (M, y, i), regardless of which n obtains and independently of the implications for the price levels (P_1, P_2). We, therefore, define

Definition 2. *A* **commitment equilibrium** *consists of a central bank policy $(M, y(\cdot), i(\cdot))$, aggregate spending behavior $n \in [0, 1]$ and price levels (P_1, P_2) such that:*

(i) The spending decision of each individual consumer is optimal given aggregate spending decisions n, the announced policy $(M, y(\cdot), i(\cdot))$, and price levels (P_1, P_2).

(ii) Given aggregate spending n, the central bank provides $y(n)$ goods and sets the nominal interest rate $i(n)$.

(iii) Given $(n, y(n), M)$, the price level P_1 clears the market in $t = 1$.
Given $(n, y(n), i(n), M)$, the price level P_2 clears the market in $t = 2$.

As a particular consequence of this equilibrium concept, the price levels (P_1, P_2) flexibly adjust to the aggregate spending realization and the announced central bank policy.

4 Implementation of socially optimal allocation

Given the preferences and technology that we postulated above, only the real allocation of goods matters to the two types of agents. If the central bank acts to enable optimal financial intermediation as in (Diamond and Dybvig, 1983), the implementation of the optimal risk-sharing arrangement (x_1^*, x_2^*) is the central bank's key objective when determining her policy. There is, consequently, no additional motive for the monetary authority to keep prices stable.

However, focusing only on real allocations is a narrow perspective. There is a vast literature arguing in favor of central banks keeping prices stable or setting a goal of low and stable inflation for reasons that are absent from our model.[14] Having to hold cash to accomplish transactions, such as in cash-in-advance or money-in-utility models, creates a whole range of distortions that can be minimized by deft management of the price level (think about the logic behind the Friedman rule). Rather than extending the model to include these considerations, for simplicity, we shall proceed by discussing the tradeoffs between achieving the optimal real allocation of consumption and the implications of such an effort for the stability of prices. We return to the price stability objective in section 5.

Runs on the central bank. A nominal contract, *per se*, does not rule out the possibility of a run on the central bank. Since impatient agents only care for consumption in $t = 1$, every equilibrium will exhibit aggregate spending behavior of at least λ, implying $n \geq \lambda$.[15] Patient agents, on the other hand, spend their CBDC balances strategically in $t = 1$ or $t = 2$. They spend in $t = 1$ if they believe that the central bank's policy implies a higher real value of CBDC balances in $t = 1$ rather than $t = 2$, $x_1 > x_2$. In that case, patient agents will use the storage technology to consume x_1 in period 2. Otherwise, patient agents will find it optimal to wait until the final period. We say,

Definition 3 (Central Bank Run)**.** *A run on the central bank occurs if not only impatient but also patient agents spend in* $t = 1$*,* $n > \lambda$*.*

In a bank run, the central bank is not running out of the item that it can produce freely (i.e., it is not running out of digital money). This feature distinguishes the run equilibrium here from the bank run equilibrium in Diamond and Dybvig (1983), in which a commercial bank prematurely liquidates all of its assets to satisfy the demand for withdrawals in period 1, therefore, ultimately running out of resources. Yet, the real consequences of a run on the central bank with nominal contracts can be similar to its counterpart in the model with real contracts. Importantly, by equations (11) and (12), a patient agent's optimal decision whether to run on the central bank, to spend or not, depends on the central bank's policy choices only through the liquidation policy $y(\cdot)$ and not via the nominal elements M and $i(n)$. By our equilibrium definition, the aggregate spending behavior n has to be consistent with optimal individual choices. These considerations imply the following lemma.

Lemma 4.1. *Given the central bank policy* $(M, y(\cdot), i(\cdot))$*,*

(i) The absence of a run, $n = \lambda$*, is an equilibrium only if* $x_1(\lambda) \leq x_2(\lambda)$*.*

(ii) A central bank run, $n = 1$*, is an equilibrium if and only if* $x_1(1) \geq x_2(1)$*.*

[14] For instance, stable prices minimize the misallocations created by nominal rigidities as in Woodford (2003).

[15] When $y(n) = 0$, impatient agents are indifferent between spending and not spending. To break ties, we assume that they spend their CBDC balances in $t = 1$.

(iii) A partial run, $n \in (\lambda, 1)$, occurs in equilibrium if and only if patient agents are indifferent between either action, requiring $x_1(n) = x_2(n)$.

Given this equilibrium characterization for a given policy-implied real allocation, how can central bank policy attain the first-best allocation?

4.1 Implementation of optimal risk sharing via liquidation policy

By $(x_1^*, x_2^*) = \left(\frac{y^*}{\lambda}, \frac{R(1-y^*)}{1-\lambda}\right)$, the feasibility constraint $y \in [0, 1]$, and the optimality conditions in Section 2.1, the implementation of optimal risk sharing requires a liquidation policy to satisfy

$$y^*(\lambda) = x_1^* \lambda \in (\lambda, 1]. \tag{13}$$

That is, given that only impatient types spend, the central bank needs to liquidate enough of the technology to provide the optimal x_1^*. Similarly to Diamond and Dybvig (1983), the resource constraint $y \in [0, 1]$ and $x_1^* > 1$ imply that optimal risk sharing is not feasible when all agents spend: If $n = 1$, then the goods provision would need to exceed one, $1 \cdot x_1^* > 1$ but the central bank cannot liquidate a share larger than one of the entire investment. Combining the previous derivation with Lemma 4.1, we arrive at the following lemma.

Lemma 4.2. *The central bank policy $(M, y(\cdot), i(\cdot))$ implements optimal risk sharing (x_1^*, x_2^*) in dominant strategies if the central bank*

(i) sets $y(\lambda) = y^*$ for any $n \leq \lambda$.

(ii) sets a liquidation policy that implies $x_1(n) < x_2(n)$ for all $n > \lambda$.

Given that only impatient agents are spending, $n = \lambda$, then a policy choice with $y(\lambda) = y^*$ implements the social optimum. That is, there is a multiplicity of monetary policies that implement the first-best since the pair $(M, i(\cdot))$ is not uniquely pinned down. While the pair $(M, i(\cdot))$ does not affect depositors' incentives, it has an impact on prices via equations (7) and (8). In the second part of Proposition 4.2, the central bank steers the incentives of the patient agents. Patient agents can but do not have to spend their CBDC balances at $t = 1$, and spend at $t = 2$ for sure only if for every possible spending level n the real allocation at $t = 2$ exceeds the allocation at $t = 1$. The central bank internalizes her depositors' decision making. It observes aggregate spending behavior n *before* it liquidates any asset. The central bank can, therefore, liquidate in a spending-contingent way, and is not committed to liquidating y^* if also patient agents are spending. Condition (ii) of this lemma corresponds to the classic incentive-compatibility constraint in the bank run literature: since the depositors' and the central bank's expectations are rational, and since the central bank policy is announced in $t = 0$, the depositors correctly anticipate the real value of their CBDC balances that would follow every aggregate spending behavior n. To deter patient agents from spending, the central bank can threaten to implement a liquidation policy $y(\cdot)$ that makes spending early sub-optimal *ex-post*, i.e., so that $x_1(n) < x_2(n)$ for every $n \in (\lambda, 1]$. If the monetary authority can credibly threaten patient agents by announcing such a liquidation policy, it deters them from spending ex-ante, and a central bank run does not occur in equilibrium. Therefore, in the unique equilibrium, only impatient agents spend, all patient agents roll over, and the social optimum is always attained.

The central bank implements "spending late" as the dominant equilibrium strategy for patient agents by fine-tuning the real goods supply via its liquidation policy, i.e., by making real asset liquidation spending-contingent.

Definition 4. *We call a central bank's liquidation policy $y(\cdot)$ "run-deterring" if it satisfies*

$$y^d(n) < \frac{nR}{1+n(R-1)}, \quad \textit{for all } n \in (\lambda, 1]. \tag{14}$$

Such a liquidation policy implies that "roll over" is ex-post optimal $x_1(n) < x_2(n)$ whenever patient agents are spending early $n \in (\lambda, 1]$.

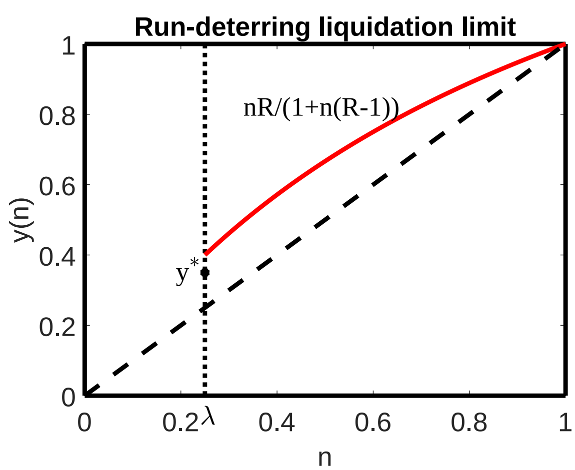


Figure 4: The upper bound of a "run-deterring" liquidation policy as a function of n is plotted in red. The bound starts at λ (for illustration purposes, here 0.25) because "impatient agents" will always spend. Note the social optimum, y^*, which is at λ in the n-axis and below the upper bound in the $y(n)$-axis and, to make interpretation easier, the 45-degree line in discontinuous segments.

The implementation of a run-deterring policy is only possible because the contracts between the central bank and the agents are nominal. The liquidation of investments in the real technology is at the central bank's discretion, thereby controlling the real goods supply and, for a given spending level, the real allocation in either time period. A spending-contingent liquidation policy implies a spending-contingent price level. In the case of real contracts between a private bank and depositors such as in Diamond and Dybvig (1983), in contrast, the real claims of the agents are fixed already in $t = 0$, thus pinning down a liquidation policy for every measure of aggregate spending n. In the case of high spending, rationing must occur. Similarly, in the case of nominal contracts between a private bank and depositors, the private bank has to take the price level as given, which then again pins down the liquidation policy. Alternatively, the price level adjusts via market clearing to high aggregate nominal spending (Skeie, 2008), while here it can serve as a strategic control variable.

As the main result of this paper,

Corollary 5 (Trilemma part I - No price stability)**.** *Every central bank policy* $(M, y(\cdot), i(\cdot))$, $n \in [0, 1]$ *with*

$$y(\lambda) = y^* \text{ and } y^d(n) < \frac{nR}{1 + n(R-1)}, \quad \text{for all } n \in (\lambda, 1], \tag{15}$$

deters central bank runs and implements the social optimum in dominant strategies. Such a deterence policy choice requires the interim price level $P_1(n)$ *to exceed the spending-dependent bound:*

$$P_1(n) > \frac{M}{R}(1 + n(R-1)), \quad \text{for all } n \in (\lambda, 1]. \tag{16}$$

Under a credible liquidation policy (15) all agents have a dominant strategy to spend if and only if the agent is impatient; otherwise they wait. Thus, under rational behavior, runs do not occur, and by $y(\lambda) = y^*$ the social optimum always obtains. That is, a strategic real supply shock enforced by the central bank *causes* a demand shock to CBDC spending that deters runs. The implementation, however, comes at a price. Feasibility of a run-deterring policy $y(\cdot)$ requires sacrificing price stability. By condition (16), the more agents spend, the larger the required price level threat to deter runs. Intuitively, to deter high levels of early CBDC spending, a high CBDC supply must meet a low supply of goods, so that, via market clearing, each good must have an exorbitantly high price. The threat has to be credible to deter runs *ex-ante*. Agents have to believe that *ex-post* the central bank will give up price stability whenever realized spending behavior is excessive. Only then do runs and inflation not occur on the equilibrium path. In that case, inflation arises via (16) only off the equilibrium path. It is not possible to avoid inflation as in (16) by introducing a nominal interest rate between $t = 0$ and $t = 1$, unless the interest rate is spending-contingent and thus random in $t = 0$. A random nominal interst rate brings new challenges, see the discussion in section 6.

In Diamond and Dybvig (1983), we learned the dilemma that offering the optimal amount of risk sharing via demand-deposit contracts requires private banks to be prone to runs. Thus, a bad bank run equilibrium also exists. Our result brings this dilemma to the next level. If the bank is a central bank equipped with the power to set price levels and control the real goods supply, then optimal risk sharing can be implemented in dominant strategies such that a bank run never occurs, but only at the expense of price stability.

Observe that by the optimality conditions and the resource constraint, $y^* < \frac{\lambda R}{1+\lambda(R-1)}$ holds and that the upper bound for $y^d(n)$ is increasing in n. Therefore, the constant liquidation policy

$$y(n) \equiv y^*, \text{ for all } n \in [0, 1] \tag{17}$$

implements optimal risk sharing in dominant strategies. There, however, exist infinitely many other run-deterring liquidation policies, see Figure 4.

Besides its simplicity, policy (17) is particularly interesting, since it is equivalent to the run-proof dividend policy in Jacklin (1987), which implements the social allocation via interim trade in equity shares. Section 8 discusses the connection of this result to our model and argues that Jacklin (1987) features a special case of a run-deterring policy. The policy (17) also implements the same allocation as the classic suspension-of-convertibility option, which is known to exclude bank runs in the Diamond-Dybvig world.

There is a subtle but essential difference, though, between suspension and our liquidation policy. Suspension of convertibility requires the bank to stop paying customers who arrive after a fraction λ of agents have withdrawn. By contrast, in our environment, there is no restriction on agents to spend their digital euros in period 1, and there is no suspension of accounts. Instead, it is the supply of goods offered for trade against those digital euros and the resulting change in the price level that generate the incentives for patient agents to rather prefer 'rolling over'. This reasoning also implies that, in our model, (nominal) deposit insurance will not deter agents from running on the central bank. Only a true commitment to a run-deterring policy is a guarantee or insurance of a positive real return on CBDC.

More concretely, low liquidation and thus a low goods supply push the price level P_1 above an upper bound that is increasing in the aggregate spending.[16] The low liquidation policy, on the other hand, deters large spending *ex-ante*, such that the high price level (16) is a threat that is realized only off-equilibrium. But each time we have an off-equilibrium threat, we should worry about the possibility of time inconsistency. In comparison with the classic treatment of time inconsistency in Kydland and Prescott (1977a), the concern here is not that the central bank will be tempted to inflate too much, but that it would be tempted to inflate too little. The central bank can avoid suboptimal allocations by committing to let inflation grow whenever necessary. A similar concern appears in models with a zero lower bound on nominal interest rates. Eggertsson and Woodford (2003) have shown that a central bank then wants to commit to keeping interest rates sufficiently low for sufficiently long, even after the economy is out of recession, to get the economy off the zero lower bound (see also Krugman, 1998, for an early version of this idea). But once the economy is away from the zero lower bound, there is an incentive to renege on the commitment to lower interest

[16]Our result resembles Theorem 4 in Allen and Gale (1998) and has a similar intuition. In Allen and Gale (1998), a central bank lends to a representative bank an interest-free line of credit to dilute the claims of the early consumers so that they bear a share of the low returns to the risky asset. In their environment, private bank runs are required to achieve the first-best risk allocation.

rates and avoid an increase in the price level.

In our model, we assume that the central bank fully commits such that the threat is credible. But what if the central bank is concerned with price stability and, therefore, refuses to induce a high price level?

5 The classic policy goal: Price level targeting

There are many possible reasons why central banks view the stabilization of price levels or, more generally, inflation rates as one of their prime objectives. The model here should be viewed as part of a larger macroeconomic environment, where the objective of price stability must be taken into account. That objective could arise out of concerns regarding nominal rigidities or legal mandates, and they may be socially optimal, requiring an appropriate modification of (1). The other way around, exogenous price stability can be interpreted as an extreme form of price stickiness. The task at hand, then, is to examine how price stability imposes constraints on central bank policy. In particular, we will document the existence of deep tensions between the three objectives of attaining the first-best outcome, deterring central bank runs, and maintaining price stability. Adressing price stability as a central bank objective requires defining a notion of price stability first. We shall distinguish between two versions of the objective of price stability: full price stability and partial price stability. Let us start by analyzing the former.

5.1 Full price stability

Definition 6. *We call a central bank policy*
(i) P_1***-stable at level*** $\overline{P}$*, if it achieves* $P_1(n) \equiv \overline{P}$ *for the* ***price level target*** $\overline{P}$*, for all spending behavior* $n \in [\lambda, 1]$*.*
(ii) ***price-stable at level*** $\overline{P}$*, if it is* P_1*-stable at level* $\overline{P}$ *and if it achieves* $P_2(n) \equiv \overline{P}$ *for all spending behavior* $n \in [\lambda, 1)$*.*

For the definition of a *price-stable* policy, we exclude the total run $n = 1$, by absence of a demand for goods in $t = 2$, see definition 8. In our definition, price stability here is treated as a mandate and commitment to the price level $\overline{P}$ even for off-equilibrium realizations of n. From the definition, price stability at some level $\overline{P}$ implies P_1-stability at $\overline{P}$. Hence, the second price stability criterion is stronger.

Definition 7. *Given a price goal* $\overline{P}$*, we call a commitment equilibrium a*
(i) P_1**-price-commitment equilibrium***, if the central bank policy is* P_1*-stable at level* $\overline{P}$
(ii) **price-commitment equilibrium***, if the central bank policy is price-stable at level* $\overline{P}$

What constraints does the price stability objective impose on central bank policy?

Proposition 8 (Policy under Full Price Stability)**.** *A central bank policy is:*

(i) P_1*-stable at level* $\overline{P}$*, if and only if its liquidation policy satisfies:*

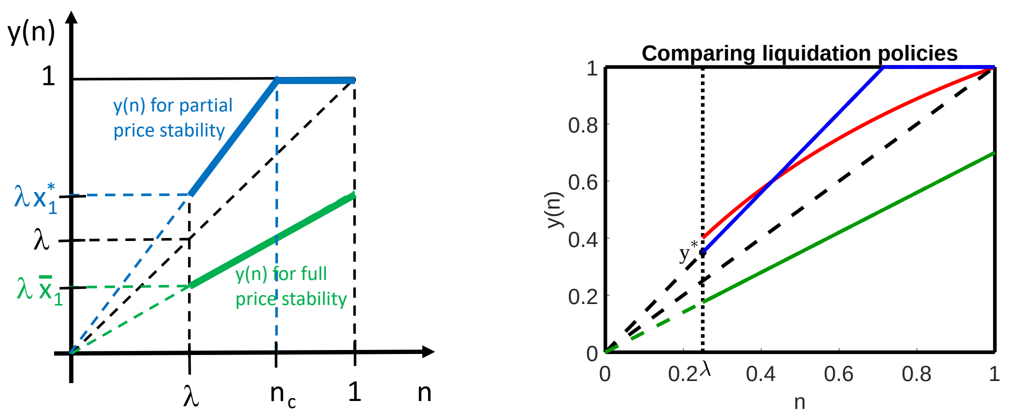
$$y(n) = \frac{M}{\overline{P}} n, \;\; \textit{for all } n \in [0, 1] \tag{18}$$

implying a real interim allocation:

$$x_1(n) \equiv \overline{x}_1 = \frac{M}{\overline{P}} \leq 1. \tag{19}$$

(ii) A central bank policy is price-stable at level $\overline{P}$*, if and only if its liquidation policy satisfies equation* (18)*, its price level satisfies* (19)*, and its interest policy satisfies:*

$$i(n) = \frac{\frac{\overline{P}}{M} - n}{1 - n} R - 1, \quad \textit{for } n < 1 \tag{20}$$



(a) Partial vs. full price-stable liquidation policies

(b) Price-stable versus run-deterring policy

Figure 5: Fully price-stable policies are run-deterring (below the red line) but do not reach the social optimum y^*. Partially price stable policies (which are not fully price stable) are not run-deterring but can reach the social optimum. Run-deterring policies cannot be fully price stable while reaching the social optimum, since all fully price stable policies must be linear in the spending level n while having a slope below or equal to one.

A price-stable liquidation policy (18) requires asset liquidation in constant proportion to aggregate spending for all $n \in [0, 1]$; see the green line in Figure 5a, where we plot $y(n)$ for partial versus full price-stable liquidation policies. As a consequence, the individual real consumption x_1, and therefore the real value of CBDC balances are constant, regardless of aggregate spending behavior. The real allocation, however, undercuts 1 due to the resource constraint, since the central bank cannot liquidate more than the entire investment. As a consequence, a fully price stable policy can never implement the social optimum. By equation (19) and again due to the resource constraint, for a given money supply M, only price levels $\bar{P} \geq M$ can be P_1- stable or price-stable. The slope of the liquidation policy is, thus, equal to or below 1. In other words, the rationing problem shows up indirectly through an upper bound on all possible price-stable central bank policies, imposing a low goods provision per realized spending level.

There is a caveat here. Should agents be able to operate the production technology on their own, then they can always assure themselves a real payoff of 1 in period $t = 1$ for every good stored in period $t = 0$. Thus, the only CBDC contract that prevails under voluntary participation would be a "green line" coinciding with the 45-degree line and a slope of 1, i.e. $\bar{P} = M$. Slopes below 1 are agreeable, however, if the central bank is the only entity capable of operating the real production technology or the only entity capable of intermediation with operators of that technology. The case $\bar{P} = M$ is further special since it is the only P_1- stable price level target at which the run equilibrium occurs since spending by all agents implies a total asset liquidation $y(1) = 1 = y^d(1)$.

This previous argument provides the second part of our trilemma:

Corollary 9 (Trilemma part II - No optimal risk sharing)**.** *If the central bank commits to a P_1-stable policy, then:*
(i) Optimal risk sharing is never implemented.
(ii) If $\bar{P} > M$, then the no-run equilibrium is implemented in dominant strategies. There is a unique equilibrium in which only impatient agents spend, $n^* = \lambda$. There are no central bank run equilibria.
(iii) If the central bank commits to a price-stable policy, then the nominal interest rate increases in n and is non-negative $i(n) \geq 0$ for all $n \in [\lambda, 1]$.

Intuitively, no runs take place under a P_1-stable policy since the real allocation in $t = 1$ is too low, causing all patient agents to prefer spending late.

5.2 Partial price stability

While price stability and the absence of central bank runs are desirable, the slope constraint (19) and the consequent failure to implement optimal risk sharing allocations is not. The implementation of the social optimum is impossible under full price stability. Recall that optimal risk sharing at $x_1^* > 1$ triggers potential bank runs in models of the Diamond-Dybvig variety: thus, part (ii) of the proposition above should not be a surprise. Demanding price stability for all possible spending realizations of n is thus too stringent. For attaining the social optimum, we therefore examine a more modest goal: a central bank may still wish to ensure price stability, but may deviate from its goal in times of crises. We capture this with the following definition.

Definition 10. *A central bank policy is*
(i) ***partially P_1-stable at level $\overline{P}$****, if for all spending behavior $n \in [\lambda, 1]$, either the policy attains*

the target $P_1(n) = \overline{P}$ for some ***price level target*** $\overline{P}$*, or aggregate spending satisfies $n > \bar{P}/M$. In the latter case, we require full liquidation, $y(n) = 1$.*
(ii) ***partially price-stable at level*** $\overline{P}$*, if for all spending behavior $n \in [\lambda, 1]$, either the policy achieves $P_1(n) = P_2(n) = \overline{P}$ for some* ***price level target*** $\overline{P}$*, or aggregate spending satisfies $n > \bar{P}/M$. In the latter case, we require $y(n) = 1$.*

The idea of this definition is, for a given spending realization the central bank tries to attain the target price level whenever possible. When spending is, however, too high, the price target can no longer be reached in which case the central bank liquidates all assets. For a graphical illustration, see the blue line in Figure 5a. Obviously, P_1-stable central bank policies are also partially P_1-stable, and price-stable central bank policies are also partially price-stable.

Definition 11. *Given a price goal $\overline{P}$, we call a commitment equilibrium a*
(i) **partial P_1-price-commitment equilibrium**, *if the central bank policy is partially P_1-stable at level $\overline{P}$*
(ii) **partial price-commitment equilibrium**, *if the central bank policy is partially price-stable at level $\overline{P}$.*

Recall that only price levels above the money supply $\overline{P} \geq M$ can attain full price stability. We therefore now concentrate on lower price levels $M > \overline{P}$, since attaining optimality requires $1 < x_1^* = M/\bar{P}$. We additionally encounter a (weaker) feasibility constraint for partially price-stable policies. Since the central bank cannot liquidate more than the entire asset, $y(n) = x_1 n \in [0, 1]$ for all $n \in [\lambda, 1]$, it faces the constraint $\lambda x_1 \leq 1$. Feasibility, therefore, implies a lower bound on all possible partially stable price levels, $\overline{P} \geq \lambda M$. Partial price stability restricts central bank policies the following way:

Proposition 12 (Policy under Partial Price-Stability)**.** *Suppose that $M > \overline{P} \geq \lambda M$.*

(i) A central bank policy is partially P_1-stable at level $\overline{P}$, if and only if its liquidation policy satisfies:

$$y(n) = \min\left\{\frac{M}{\overline{P}}n, 1\right\}. \tag{21}$$

(ii) For every partially P_1-stable central bank policy at level $\overline{P}$, there exists a critical aggregate spending level $n_c \equiv \frac{\overline{P}}{M} \in (0, 1)$ such that

(ii.a) For all $n \leq n_c$, the price level is stable at $P_1(n) = \overline{P}$ and the real goods purchased per agent in period $t = 1$ equal $x_1(n) = \overline{x}_1 = \frac{M}{\overline{P}} > 1$ while real goods purchased per agent in period $t = 2$ equal $x_2(n) = R(1 - \overline{x}_1 n)/(1 - n)$.

(ii.b) For spending $n > n_c$, the real goods purchased per agent in period $t = 1$ equal $x_1(n) = 1/n$ while $x_2(n) = 0$ and the price level $P_1(n)$ proportionally increases with total spending n: $P_1(n) = Mn$.

(iii) A central bank policy is partially price-stable at $\overline{P}$, if and only if its liquidation policy satisfies equation (21) and its interest rate policy satisfies:

$$i(n) = \frac{\frac{\overline{P}}{M} - n}{1-n} R - 1, \qquad \textit{for all } n \leq n_c. \tag{22}$$

For $n > n_c$, there is no supply of real goods in $t = 2$. Thus, $P_2 = \infty$ and $i(n)$ is irrelevant.

(iv) For a partially price-stable central bank policy at $\overline{P}$, there exists a spending level

$$n_0 = \frac{R\frac{\overline{P}}{M} - 1}{R-1} = \frac{Rn_c - 1}{R-1} \in [0, n_c), \tag{23}$$

such that the nominal interest rate turns negative for all $n \in (n_0, n_c)$. For $R < M/\overline{P}$, the nominal interest rate is negative for all $n \in [0, n_c)$.

Proposition 12 reflects the central bank's capacity to keep the price level and the real interim allocation x_1 stable for spending behavior below the critical level n_c. The stabilization of the price level requires the liquidation of real investment proportionally to aggregate spending by factor $M/\overline{P}$. At the critical spending level n_c, the central bank is forced to liquidate the entire asset to maintain the price level P_1 at the target. Since the central bank cannot liquidate more than its entire investment, price level stabilization via asset liquidation becomes impossible as spending exceeds the critical level n_c. For all spending behavior $n > n_c$, the real allocation to late spending agents is thus zero. Since liquidation can no longer increase, rationing of real goods occurs in $t = 1$, meaning that the price level has to rise in aggregate spending. Since the goods supply in $t = 2$ is zero, the price level in $t = 2$ explodes. One could argue here, that the price level in $t = 2$ can be maintained when setting a negative nominal interest rate at $i(n) = -1$. That would imply that zero CBDC balances meet zero goods in the market. But that would just be window dressing.

The spending level $n_0 < n_c$ is the level at which the real allocation to early and late spenders is just equal

$$x_1(n_0) = x_2(n_0) = \bar{x}_1. \tag{24}$$

Therefore, n_0 is the spending level at which the red and the blue line in Figure 5b intersect, and thus a partial run equilibrium exists. Notice that $x_2(n)$ declines in n for $n \in [0, n_c]$. Thus, if fewer than measure n_0 of agents spend, then not spending early, i.e. 'roll over' is optimal for patient agents. But for all spending realizations $n > n_0$, the allocation at $t = 2$ undercuts the allocation at $t = 1$: $x_2(n) < x_1(n)$, turning the real interest rate on the CBDC negative, and causing "spend early" to be a patient agent's optimal response to an aggregate spending behavior in excess of n_0. Consequently, self-fulfilling runs are possible as in Diamond and Dybvig (1983), and we obtain the following result as a corollary of Proposition 12:

Corollary 13 (Trilemma part III- Runs on the Central Bank (Fragility))**.** *Under every partially P_1-stable central bank policy with $M > \overline{P} \geq \lambda M$, there is multiplicity of equilibria:*

(i) There exists a good equilibrium in which only impatient agents spend, $n^* = \lambda$. In that case, there is no run on the central bank, the social optimum is attained and the price level target is attained, $P_1 = \overline{P}$.

(ii) There also exists a bad equilibrium in which a central bank run occurs, $n^* = 1$, the social optimum is not attained, and the price level target is missed.

Proposition 12 is in marked contrast to Proposition 8. One could argue that when banking is interesting, i.e., $x_1^* > 1$, then the goal of price stability induces the possibility of runs on the central bank, the necessity for negative nominal interest rates, and the abolishment of the price stability goal, if a run indeed occurs.

6 Money supply policy or suspension of spending

It is natural to ask why the central bank cannot resort to a more classic monetary policy to resolve the trilemma and attain price stability: expansion or reduction of the money supply. In this section, let us then allow for the possibility that M is state-contingent, i.e., M is chosen as a function of aggregate spending $M = M(n)$ at $t = 1$. Therefore, a central bank policy consists of three functions $(M(\cdot), y(\cdot), i(\cdot))$.

The analysis is now straightforward and easiest to explain for the case where the liquidation policy is not state-contingent, $y(n) \equiv y^*$. To maintain price stability at some level $\overline{P}$, market clearing demands

$$nM(n) = \overline{P}y^*. \tag{25}$$

As a result, the total money balances spent in $t = 1$ stay constant in n, implying

$$n\,M(n) \equiv \lambda\,M(\lambda), \quad \text{for all } n \in [\lambda, 1]. \tag{26}$$

But spending per agent alters, as does the total money supply $M(n)$. That is, the central bank would have to commit itself to **reducing** the quantity of money in circulation in response to a demand shock encapsulated in n: the more people go shopping, the lower are individual money balances. With policy (25), $y(n) \equiv y^*$ and $i(n) \equiv i^*$ chosen so that $P_2 = \overline{P}$, the central bank can now achieve full price stability, efficiency, and financial stability. The CBDC trilemma appears to be resolved. There are several ways of thinking about this.

State-contingent money supply. A first approach is to make the amount of CBDC balances available for shopping in $t = 1$ state-contingent. Having such CBDC accounts with random balances is an intriguing possibility: it is quite impossible with paper money but fairly straightforward with electronic forms of currency. A different interpretation of this approach is to think in terms of a state-contingent nominal interest rate paid on CBDC accounts between $t = 0$ and $t = 1$. One should recognize that both of these routes are a bit odd, and contrary to how we usually treat money and interest rates. As for money, a dollar today is a dollar tomorrow: changing that amount in a state-contingent fashion probably risks severely undermining the trust in the monetary system, and trust is key for maintaining a fiat currency. As for interest rates, it is commonly understood that interest rates are agreed upon before events are realized in the future. A state-contingent interest rate turns accounts into risky and equity-like contracts, likewise undermining trust in the safety of the system (see, nonetheless, Section 8 for trade in equity).

Helicopter drops. A third way to think about the state-contingent nature of M corresponds to a classic monetary injection in the form of state-contingent lump-sum payments ("helicopter drops") $M(n) - \bar{M}$ (or taxes, if negative), compared to some original baseline $\bar{M}$. If one wishes to insist that $M(n) - \bar{M} \geq 0$, i.e., only allowing helicopter drops, then the central bank would choose $\bar{M} \leq M(1)$ as payment for goods in period $t = 0$ and thus always distribute additional helicopter money in the "normal" case $n = \lambda$ in period 1. Notice that distributional issues would arise in richer models, where agents are not coordinating on the same action, thereby distorting savings incentives.

Suspension of spending. With an account-based CBDC, there is an additional and rather drastic policy tool at the disposal of the central bank: the central bank can simply disallow agents to spend (i.e., transfer to others) more than a certain amount of their account. In other words, the bank can impose a "corralito" and suspend spending. This policy is different from the standard suspension of liquidation, as the amount of goods made available is a policy-induced choice that still exists separately from the suspension-of-spending policy. Notice also that "suspension of spending" should perhaps not be called "suspension of withdrawal." Since there are only CBDC accounts and they cannot be converted into something else, the amounts can only be transferred to another account. With the suspension-of-spending policy, the central bank could arrange matters in such a way that not more than the initially intended amount of money $\lambda M(\lambda)$ will be spent in period 1; see equation (26). In practice, the central bank would then either take all spending requests at once and, if the total spending requests exceeded the overall threshold, impose a pro-rata spending limit. Alternatively, it could arrange and work through the spending requests in some sequence (first-come-first-served), thereby possibly imposing different limits depending on the position of a request in that queue.

Monetary neutrality. Last but not least, a state-contingent money supply cannot replace the central bank's liquidation policy as the active policy variable. Not only price targeting, but also the deterrence of runs is an objective of the central bank for attaining optimal risk sharing.

A state-contingent money supply, however, does not impact the agent's spending behavior: the individual agents exclusively care for their individual real allocation at $t = 1$, y/n, versus $t = 2$, $R(1-y)/(1-n)$. These allocations are independent of nominal quantities (M, P_1). That is, money is neutral. Given a realization of an individual real allocation y/n, the identity:

$$\frac{y}{n} = \frac{M(n)}{P_1} \tag{27}$$

pins down a relationship that needs to hold between the money supply and the price level that clears the market. The central bank can implement all money supplies and price level pairs (M, P_1) that satisfy equation (27). And as soon as the price level goal P_1 is pinned down, contingent on the realization $\frac{y}{n}$, the money supply that solves equation (27) is unique. But in equation (27) the classic dichotomy holds, and the choice of the right-hand side (M, P_1) cannot alter the left-hand side, i.e., cannot alter incentives to run. Consequently, if the central bank wants to impact consumers' behavior to run on the central bank to implement the social optimum, it can only do so by altering the real goods supply y through adjustment of its liquidation policy.

In summary. Given the previous discussion, a state-contingent money supply strikes us as odd monetary policy. First, the usual inclination for central banks is to accommodate an increase in demand with a rise, rather than a decline in the money supply. A central bank that reacts to an increase in demand by making money scarce may undermine trust in the monetary system. In particular, and needless to say, a spending suspension might create considerable havoc; the experience in Argentina at the end of 2001 provides ample proof. Even if this was not the case, monetary neutrality implies that adjusting the money supply does not affect the run decisions of agents. Therefore, we think that this particular escape route from the CBDC trilemma needs to be treated with considerable caution.

7 Voluntary participation in CBDC and competition by private banks

The main model assumes that all consumers invest in a CBDC. It remains to clarify whether agents may be better off using the investment technology on their own, rather than relying on the central bank. This is an important question: if agents were to decide to stay in autarky and invest in the investment technology directly, they might have incentives to supply goods at the interim stage, thus, potentially undermining the central bank's liquidation policy. Similarly, if the outside option is not autarky but investing in deposits with a different, private bank, then the liquidation policy of that private bank has implications for the aggregate real goods supply at the interim stage, again impairing the effectiveness of the central bank's policy. We now discuss both.

7.1 Autarky and voluntary participation in a CBDC

Assume all but one agent invest in a CBDC. Assume that this single agent invests in the real technology at $t = 0$, yielding storage between $t = 0$ and $t = 1$, and yielding $R > 1$ when held between $t = 0$ and $t = 2$. Then, at $t = 1$, she would learn her type. If she is impatient, she will liquidate the technology, yielding 1 unit of the real good, and she would consume her good. She would not sell the good against nominal CBDC deposits, since she only cares about consumption at $t = 1$. In the case where she is impatient, she is worse off in comparison to an agent who invested in CBDCs with the central bank if the central bank offers optimal risk sharing and manages to implement a run-deterring policy. This is so, since under the latter, an individual impatient agent gets $x_1^* > 1$ real goods.

If the individual agent is patient, she will stay invested in the technology until time two. There, the technology yields $R > 1$ units of the good. The agent will, thus, be better off than under investment in a CBDC since $x_2^* < R$; see Section 2.1. But, in particular, also in the patient case, the individual agent will not offer goods for sale in the interim period, since liquidation and selling against a CBDC will only yield x_2^* in $t = 2$. Thus, in either case, patient or impatient, the agent who invests in autarky will not have an incentive to undermine the central bank's policy by increasing the goods supply in the interim period.

Does the agent prefer to remain in autarky rather than participating in the CBDC? *Ex-ante*, the risk-averse agent cannot know whether she will turn out to be patient or impatient. Diamond and Dybvig (1983) show that pooling of resources via banking can attain the social optimum under an absence of runs, while investment under autarky cannot. That is, the single agent is always better off investing in the CBDC account if the central bank offers optimal risk sharing and implements a run-deterring policy. Thus, participation in the CBDC account is individually rational.

What if the central bank runs a policy of full price stability at goal $\bar{P}$? In that case, our second main result, Corollary 9, shows that runs on the central bank do not occur but $x_1 \leq 1$. Thus, for all $x_1 < 1$, investing in a CBDC is dominated by investing in autarky. Voluntary participation thus requires $x_1 = 1$ or $M = \bar{P}$, implying $x_2 = R$. The agent is then indifferent between investing in a CBDC and staying in autarky. Yet, if she stayed in autarky, she will not undermine the central bank's liquidation policy for the reasons above.

In the case of a partial price-stable policy, the situation is as in Diamond and Dybvig (1983). *Ex-ante*, the agent cannot know whether a run occurs or not. Conditional on the no-run equilibrium, we implement the social optimum and the agent is better off investing in a CBDC. But conditional on the run equilibrium, she was better off in autarky. From within the model, it is not possible to attach likelihoods for each equilibrium.

7.2 Can private banks undermine the central bank's policy?

The question of under what circumstances consumers prefer investing in a CBDC account with the central bank rather than investing in demand deposits with private banks, with implications for how both types of banks can coexist, is addressed in Fernández-Villaverde et al. (2020). In this section, we will analyze private banks' incentives to provide goods at the interim stage, *conditional* *on the coexistence of private banks with the central bank.*

Goods supply. If the central bank coexists with private banks, it controls the market of goods only partially, with the remainder of the real goods being supplied by commercial banks. As before, the measure of agents is normalized to one, divided between a share $\alpha \in (0,1)$ of agents who are CBDC customers at the central bank and a share $1-\alpha$ who are customers at private banks. Assume that all agents invest their 1 unit endowment in their corresponding bank and that the private banks invest in the same asset as the central bank does. Then, at $t = 1$, the central bank can supply up to α goods via liquidation, while private banks can supply up to $1 - \alpha$ goods.

Assume that there is one centralized goods market to which customers and banks have access. That is, CBDC depositors can spend CBDC balances on goods supplied by private banks and private bank customers can spend their private deposit balances on goods supplied by the central bank. Let n denote the total measure of spending agents across both customer groups at the central bank and private banks, given by

$$n = \alpha\, n_{CB} + (1-\alpha)\, n_P, \tag{28}$$

where n_{CB} is the total share of consumers at the central bank who spend, while n_P is the total share of consumers at the private bank who spend. Given total spending n in period $t = 1$, let $y_P(n)$ be the share of assets liquidated by private banks. In contrast, let $y_{CB}(n)$ be the central bank's liquidation policy, i.e., the share of assets liquidated by the central bank. The total goods supply y in the centralized market at the interim stage is then:

$$y(n) = \alpha\, y_{CB}(n) + (1-\alpha)\, y_P(n). \tag{29}$$

Private deposit making. To collect investment in $t = 0$, the private banks offer a nominal demand-deposit account in return for 1 unit of the real good. The private nominal accounts are denominated in units of the CBDC. Due to competition with the central bank, the private contract also offers M units of the CBDC in $t = 1$ or $M(1 + i(n))$ units in $t = 2$.

To service withdrawals in terms of the CBDC, private banks first observe their customers' CBDC withdrawal needs n_P, and borrow the required amount $(1-\alpha)n_P M$ of the CBDC from the central bank at the beginning of period $t = 1$. The central bank creates the CBDC quantity $(1-\alpha)n_P M$ on demand for the private banks. Private banks observe CBDC spending at the central bank n_{CB}, yielding aggregate spending n. During period one, the private banks sell the share $y_P(n)$ of their real goods investment at price P_1 in the centralized market to all consumers, thus receiving proceeds of $P_1 y_P(n)(1-\alpha)$ units of the CBDC in return, where P_1 satisfies market clearing:

$$M\left((1-\alpha)n_P + \alpha n_{CB}\right) = P_1\Big(y_P(n)(1-\alpha) + y_{CB}(n)\,\alpha\Big). \tag{30}$$

The private banks use these CBDC proceeds to (partially) repay their loan to the central bank at zero interest within period one. Since the central bank retains only partial control over the goods market, it generically no longer holds $n_{CB}M = P_1 y_{CB}(n)$. As a consequence, the private banks can hold positive or negative CBDC balances $(1-\alpha)(P_1 y_P(n) - n_P M)$ with the central bank between $t = 1$ and $t = 2$.

We seek to examine a range of possibilities for the private bank withdrawals n_P as well as liquidation choices y_P. Thus, it is useful to impose the condition that private banks make zero profits, regardless of the "circumstances" n_P or their choice for y_P. This requires some careful calculation, which we provide in Appendix 13, and only summarize here.

We assume that the central bank charges or pays the nominal interest rate $z = (RP_2/P_1) - 1$ on the excess or deficit CBDC balances of private banks, to be settled at the end of $t = 2$. Note that $z > i$, if $x_1 > 1$ and equals the internal nominal shadow interest rate regarding private bank liquidation choices. Moreover, we impose a market share tax at the end of period $t = 2$ in order to compensate for profits or losses due to circumstances n_P.

At $t = 2$, the remaining private customers spend the quantity $(1-\alpha)(1-n_P)M(1+i(n))$ of private CBDC accounts that the private banks borrow from the central bank at the beginning of $t = 2$. The private banks sell their returns on the remaining investment $R(1 - y_P(n))(1-\alpha)$ at price P_2, where P_2 satisfies market clearing

$$M(1+i(n))\Big((1-\alpha)(1-n_P) + \alpha(1-n_{CB})\Big) = \\ P_2 R\Big((1-y_P(n))(1-\alpha) + (1-y_{CB}(n))\,\alpha\Big). \quad (31)$$

At the end of $t = 2$, the private banks settle their accounts with the central bank, taking into account the remaining balances at $t = 1$ adjusted for interest, the end-of-period tax compensating for circumstances n_P, the loan at the beginning of $t = 2$, and the sales proceeds at $t = 2$.

Joint liquidation policies. The actions of private banks and the central bank may not be perfectly aligned when it comes to the liquidation of assets and the supply of goods at the interim stage. Private banks can have various objectives depending on their ownership structure and may be subject to regulation of their liquidation policy, both shaping y_P. Independently of whether private banks maximize depositor welfare as in Diamond and Dybvig (1983), or pursue some other objective, the prevention of runs is key. We have seen above that runs occur if the provision of real goods at the interim stage is high. Since the market is centralized, for the spending incentives of bank customers it is irrelevant whether these goods are provided by the central bank's or the private bank's liquidation of assets.

Hence, as before, a run-deterring liquidation policy $y(\cdot)$ is a function of aggregate spending n such that the real allocation at $t = 1$ undercuts the real allocation at $t = 2$:

$$\frac{y(n)}{n} < R\frac{(1-y(n))}{1-n}, \qquad \text{for all } n \in [\lambda, 1]. \quad (32)$$

Thus, again, a run-deterring policy satisfies

$$y(n) < \frac{nR}{1 + n(R-1)}, \qquad \text{for all } n \in [\lambda, 1]. \tag{33}$$

Perfect coordination. If the central bank and the private banks coordinate perfectly, i.e., act as one entity, and have full control over the asset liquidation, then all run-deterring policies are possible, as in the case where the central bank is a monopolist. But why would they coordinate perfectly? By the market's centralization, the destiny of the central bank is intertwined with the destiny of the private banks and both types of banks have an interest in deterring runs. In particular, the private bank will, therefore, not undermine a central bank's run-deterring policy by supplying additional goods when, for instance, prices are high, since this might cause a run not only on the central bank but also on the private bank. Coordination is therefore among the equilibrium outcomes.

Runs under imperfect coordination. The following example shows how, for general liquidation policies y_P of private banks, runs can occur. Assume that the private bank, for some reason, follows a liquidation rule $y_P(n) \in [0,1]$ where $y_P(n_b) = 1$ for all $n \geq n_b$ where $n_b \in (0,1)$. For instance, $n_b = 1 - \alpha$, i.e., the private bank is subject to regulation and has to liquidate all assets if a fraction of its customers equal to its market share spends. In that case, as we show next, the central bank's capacity to deter runs depends on the size of the private banking sector, i.e., its market power α. Since the central bank can only control the liquidation of its own investment y_{CP}, via (32) and (29), a run-deterring policy y_{CB} needs to satisfy:

$$y_{CB}(n) < \frac{Rn - (1-\alpha)y_P(n)(Rn + 1 - n)}{\alpha(Rn + 1 - n)}, \qquad \text{for all } n \in [\lambda, 1]. \tag{34}$$

Now assume $n > n_b$, such that $y_P(n) = 1$. If in addition the central bank has a small market share $\alpha \to 0$, then the numerator converges to $-(1-n)$, while the denominator goes to zero, $\alpha(1 + (R-1)n) \to 0$. That is, for $n_b < n < 1$, the right-hand side in (34) goes to minus infinity such that (34) cannot hold. This implies that the run equilibrium exists.

A sufficient condition: Run-deterrence under imperfect coordination. The example above makes clear that the central bank's share in the deposit market needs to be large enough in order to prevent runs. The following proposition provides the appropriate bound under which the central bank can ensure the absence of a run, regardless of the private bank's liquidation schedule $y_P : [\lambda, 1] \to [0,1]$.

Proposition 14. *Suppose that the central bank's share in the deposit market satisfies*

$$\alpha > \frac{1-\lambda}{(1-\lambda+R\lambda)}. \tag{35}$$

Then the central bank can always find a run-deterring liquidation policy $y_{CB} : [\lambda, 1] \to [0, 1]$*, regardless of the private bank's liquidation policy* $y_P : [\lambda, 1] \to [0, 1]$.

Such an $\alpha \in (0, 1)$ exists since $\frac{1-\lambda}{(1-\lambda+R\lambda)} \in (0, 1)$. Thus, the right-hand side $\frac{1-\lambda}{(1-\lambda+R\lambda)}$ of equation (35) imposes a lower bound on the balance-sheet size of the central bank as a percentage of the total demand deposit market, such that run-deterring policies remain possible despite coexisting private banks that are subject to liquidation restrictions.

Proof. [Proposition 14] We need to show that for any private bank liquidation policy $y_P : [\lambda, 1] \to [0, 1]$, there is a central bank liquidation policy $y_{CB} : [\lambda, 1] \to [0, 1]$ so that (34) is satisfied. To derive a sufficient condition on the central bank's market share α under which it can nevertheless implement a run-deterring policy, note that by $R > 1$, the right-hand side in (34) declines in the value y_p for all $\alpha \in (0, 1)$. Thus, if a central bank policy y_{CP} is run-deterring for $y_P = 1$ for all $n \in [0, 1]$, then y_{CP} is also run-deterring for a private bank policy $y_P(n) \leq 1$ for all $n \in [0, 1]$. Thus, assume $y_P = 1$ for all $n \in [0, 1]$. Then, a sufficient condition for a run-deterring policy against all private bank policies y_P is:

$$y_{CB}(n) < \frac{Rn - (1-\alpha)(Rn + (1-n))}{\alpha(1 + (R-1)n)} = 1 - \frac{1-n}{\alpha(1 + (R-1)n)}, \quad \text{for all } n \in [\lambda, 1]. \tag{36}$$

The right-hand side is increasing in n and $y_{CB}(n)$ cannot undercut zero. Thus, a sufficient condition for the existence of a policy $y_{CB} \in [0, 1]$ that satisfies (36) is an α such that:

$$0 < 1 - \frac{1-\lambda}{\alpha(1 + (R-1)\lambda)}. \tag{37}$$

□

8 Trade in equity shares

Diamond and Dybvig (1983) show that banks can offer the socially optimal risk-sharing allocation via demand deposits at the cost of being prone to runs. Jacklin (1987) demonstrates that a run-proof, optimal risk sharing can be implemented when banks offer shares in equity instead of demand deposits. For banks to do so, the real dividend payments $D = \lambda c_1^*$ in $t = 1$ and $R(1 - D)$ in $t = 2$ must be predetermined in $t = 0$ and there must exist a market in which to trade claims on dividends in $t = 1$. The dividends accrue to all investors, patient and impatient. When the equity market opens in $t = 1$, patient investors purchase the impatient agent's late dividend payments in return for the lower early dividend payments. This trade is incentive-compatible once types are revealed. Moreover, before learning their types in $t = 0$, all agents are willing to agree to the predetermined dividend payments. Since in $t = 1$, the equity contract does not allow impatient types to demand an additional share of their late dividend payment, runs that would enforce excess asset liquidation cannot occur.

Would the Jacklin (1987) environment also work in our nominal banking model to prevent runs on the central bank? The answer is not only yes, but in fact, that the dividend policy proposed in Jacklin (1987) is a special case of a run-deterring liquidation policy with a dividend payment equal to:

$$D = \lambda c_1^* = y^*, \quad \text{for all } n \in [0,1] \tag{38}$$

That is, the liquidation policy discussed around equation (17), which implements the social optimum in dominant strategies via CBDC demand deposits, *is* the real allocation that is implemented in Jacklin (1987) via equity shares and trade in dividends.

In both Jacklin (1987) and our special case (17), the total asset liquidation in $t = 1$ is predetermined at $t = 0$. In Jacklin (1987), per default, all agents receive a real dividend in $t = 1$ and then can trade claims on dividends in $t = 2$ for or against claims on dividends in $t = 1$. After this trade, the patient agents will have given up on their early real dividend, while the impatient types will have given up on their late real dividend. In our setting instead, agents are not allocated real goods per default in $t = 1$. Instead, the agreement is, if an agent spends her CBDC balance on goods in $t = 1$, she foregoes her right to spend her CBDC balance on goods in $t = 2$. Moreover, she will share the supply of goods with all agents that spend with her in $t = 1$, where the total share of spending agents will be unknown to her as she makes the spending decision. While in Jacklin (1987), the market-clearing price of dividends induces the optimal spending, in our model, the fixed supply of goods deters patient types from spending.

But the space of run-deterring policies that we give here is much richer than policy (17). In particular, a run-deterring policy can allow for spending-contingent liquidation $y(n)$, $n \in [\lambda, 1]$, where liquidation is not constant in n. With such a policy, liquidation is not predetermined in $t = 0$, yet, runs will not occur, and the social optimum is implemented in dominant strategies if $y(\lambda) = y^*$.

8.1 Jacklin (1987) with nominal contracts

Notice, however, that our banking model features nominal contracts, while in Jacklin (1987), dividends are denominated in real terms. By our main result (5), a run-deterring policy requires an inflation threat (16). What if the dividend payments were nominal? Does inflation necessarily arise there too for deterring runs? And what is a run on a bank under trade in equity shares?

To answer these questions, assume the extreme case where agents hand over their real goods endowment in return for nominal equity shares in the central bank.[17] The total measure of all

[17] Recall that, historically, many central banks sold shares to the public at large that paid dividends. Even today, one can buy shares of the central banks of Japan and Switzerland.

agents remains at one. The central bank pools the real goods for investment in the real technology and commits to a central bank dividend policy (D_1, D_2). The $t = 0$ agreement is that all agents receive a nominal CBDC dividend D_1 in $t = 1$ and another nominal dividend D_2 by the central bank in $t = 2$, irrespective of their type. The central bank follows a liquidation policy $y(n)$, where –as before– $n \in [0, 1]$ denotes the measure of agents who go shopping with CBDC in $t = 1$. Since dividends are paid to all shareholders, the total nominal CBDC supply equals D_1 in $t = 1$ and equals D_2 in $t = 2$. The central bank sets a price level P_1 at time $t = 1$ and P_2 in $t = 2$ that clears the goods market. In $t = 1$, types realize and impatient types want to consume as much as possible in $t = 1$. Impatient types can can sell their claims on a nominal dividend D_2 in $t = 2$ in return for nominal dividends D_1 in $t = 1$ to purchase consumption goods provided by the central bank.

In Jacklin (1987), dividends are real and, thus, promise consumption in a one-to-one relation. With nominal dividends, this is no longer true. Crucially, the central bank sees shareholders and shoppers as two different agent groups.

Let $n \in [0, 1]$, the measure of agents who go shopping with their CBDC in $t = 1$ to spend dividends $\tilde{D}_1 \geq D_1$, after trade in nominal dividends has taken place. Assume that there is no storage technology for nominal dividends. That is, either an agent trades D_1 for consumption goods with the central bank directly or trades D_1 in return for a claim on a larger nominal dividend $\tilde{D}_2 \geq D_2$ in $t = 2$. Otherwise, D_1 would expire. For example, D_1 can be considered a nominal claim on $t = 1$ consumption goods with a flexible exchange rate (food stamps with an expiration date of "1 meal," where the size of the meal decreases with demand). After trade has occurred, the aggregate nominal CBDC supply equals D_1 and is supplied by measure n agents who demand $y(n)$ goods at a market-clearing price P_1. In particular, the total nominal dividend supply D_1 is independent of trade.

We define a run on nominal equity shares as the incidence where patient types are unwilling to trade their early dividends for late dividends with impatient types, meaning $n > \lambda$. That is, patient types also go shopping for real goods early by spending their nominal dividends D_1, and the dividends trade between the agent groups partially collapses. After observing the total measure of shoppers n who jointly supply dividends D_1, the central bank supplies $y(n)$ goods according to its policy. The market-clearing price P_1 satisfies $n \cdot \frac{D_1}{n} = P_1 \, y(n)$, respectively:

$$D_1 = P_1 \, y(n) \tag{39}$$

Likewise in $t = 2$

$$D_2 = P_2 R(1 - y(n)) \tag{40}$$

The real allocations per agent equal in $t = 1$,

$$x_1 = \frac{y(n)}{n} = \frac{D_1}{P_1 n} \tag{41}$$

and in $t = 2$

$$x_2 = \frac{R(1-y)}{1-n} = \frac{D_2}{P_2(1-n)} \tag{42}$$

An important difference between the nominal CBDC demand-deposit contract we discussed in previous sections and the model with nominal equity shares is that, if the liquidation policy is constant in the measure of shopping agents n, $y(n) = const$ for all $n \in [0,1]$, then the price level must be stable in both $t = 1$ and $t = 2$; see (39) and (40). This result holds because the total supply of nominal dividends that are traded for goods in $t = 1$ is constant at D_1. With a nominal CBDC demand-deposit contract, in contrast, the price level varies in n even if total liquidation is constant, because the nominal supply of CBDCs in $t = 1$ depends on the share of spending agents.

8.2 Runs with nominal contracts

Despite the stable price level, in this nominal version of Jacklin (1987) runs can occur: patient types might not be willing to trade their early nominal dividends for late nominal dividends. That is, the key mechanism in the "real" Jacklin (1987) is not the determination of equity shares and dividends in $t = 0$ but rather that the backing of the dividend payments, the real supply of goods, is predetermined in $t = 0$.

To see that, keep the nominal dividend payments D_1, $D_2 > D_1$ strictly positive. Set $y(n) = 1$ for all $n \in [0,1]$. That is, the central bank liquidates all real technology at the interim stage so that the goods supply in $t = 2$ is zero, $R(1 - y(n)) = 0$. Consequently, late dividend payments D_2 have zero real value, $P_2 \to \infty$, and all agents, patient and impatient, go shopping for goods in $t = 1$, implying $n = 1$ and trade in nominal equity shares collapses.

The central bank can, however, implement the social optimum by setting a liquidation policy with $y(n) = y^*$ for $n = \lambda$ that simultaneously deters patient types from shopping early so that nominal equity shares are traded. The latter happens when the individual real allocation in $t = 1$ undercuts the real allocation in $t = 2$, $x_1 < x_2$. Via equations (41) and (42), early shopping is deterred for patient types if $\frac{y(n)}{n} < \frac{R(1-y(n))}{1-n}$ for all $n \in (\lambda, 1]$, respectively, when $y(n) < \frac{nR}{1+n(R-1)}$. This familiar constraint imposes the condition that liquidation policy be "run-deterring," as in equation (14). The requirement of run-proofness implies a particular design on the real value of the aggregate dividends via equation (39):

$$\frac{D_1}{P_1} < \frac{nR}{1+n(R-1)}, \quad \text{for all } n \in (\lambda, 1] \tag{43}$$

Since the nominal dividend payments are predetermined in $t = 0$, they cannot depend on the share of shoppers n. The right-hand side of (43) is increasing in n, and therefore reaches its minimum in $n = \lambda$. If the central bank wants to follow a fixed price level path, $P_1 = \bar{P}$, then the dividends have to satisfy

$$D_1 < \bar{P} \frac{\lambda R}{1 + \lambda(R-1)} \tag{44}$$

so that patient types have no incentive to shop early. By $\frac{\lambda R}{1+\lambda(R-1)} =: \hat{y} \in (0, 1)$, the constant liquidation policy $\hat{y}$ is feasible, and run-proof and implements the price level $\bar{P}$ irrespective of the measure of shoppers $n \in [\lambda, 1]$. For a spending-flexible liquidation policy $y(n)$ that varies in n, the price level will have to adjust for keeping the predetermined dividend constant at D_1, akin to the case of the nominal CBDC demand-deposit contract.

To conclude, generically, the nominal version of Jacklin (1987) is prone to runs, and a central bank faces a trade-off between implementing the social optimum in a run-proof way and keeping prices stable. When setting the specific run-deterring policy $y(n) = y^*$ for all $n \in [\lambda, 1]$, then the nominal-equity-share setting yields price stability for sure. In contrast, the nominal CBDC demand-deposit contract has varying prices, but only off the equilibrium path. Yet, the nominal equity share setting requires the existence of an interim market to trade dividends. The existence of such a market is not required under nominal CBDC contracts.

9 The financial system

Our model abstracts from many features of the financial system. In our baseline setting, we only have households and the central bank interacting with each other, dropping the financial intermediary sector entirely. This can appear as rather different from the institutional framework seen in practice and the risk-sharing framework in place.

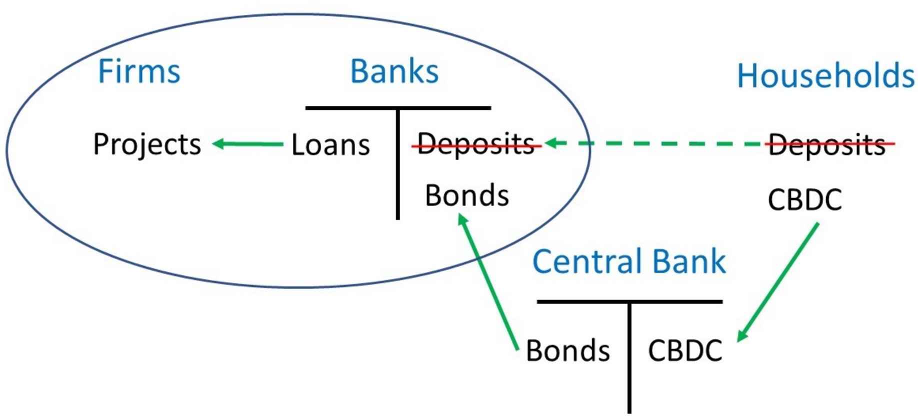


Figure 6: The Financial System: Households, firms, and banks.

Consider then the financial system as depicted in figure 6, containing firms, banks, households and a central bank. Before the introduction of a central bank digital currency, households hold deposits at banks. Banks use these deposits to provide loans to firms, who in turn use them to finance investment projects. These projects are as described in our model above.

With the introduction of a CBDC, households may become inclined to hold CBDC rather than deposits, given the rather similar functionality. Without further action, this would then lead to a disintermediation of the banks and impair their ability to make loans to firms. This issue does not disappear in a hybrid system either, where banks handle the "front end" of the CBDC accounts: in order to assure that the nature of the money does not depend on the handling bank, these cannot be treated as deposit accounts. However, the disintermediation can be avoided if the central bank engages in "pass through," funneling the funds deposited by households back to the retail banks, as Brunnermeier and Niepelt (2019) have argued. In figure 6, this is indicated by the central banks refunding the banks with loans in the form of bank-issued bonds. If done properly, the financing of firms remains unchanged.

With this new structure, however, the central bank is exposed to the intermediation risks inherent in banking and firm financing. In the environment as envision in figure 6, the central bank becomes the main source of bank funding: deposit finance has disappeared. In particular, the central bank can encourage or discourage production by increasing or decreasing the amount of bank bonds it holds. One could enrich this structure by assuming that households may also hold bank bonds or bank equity. What is key to our considerations, however, is that CBDC rather than bank bond holdings or bank equity holdings of households will be the substitute for their original deposits, and that deposits originally are the lion share of stable bank funding. With that and with the introduction of a CBDC, the central bank will now provide the lion share of stable bank funding.

One way to think through the consequences is to model firms, banks, households and central banks as well as their contractual interplay explicitly. It is quite common in the banking literature to assume that banks run these projects directly, rather than explicitly model the relationship between banks and firms. Here and in analogy, we go a step further, and now assume that it is the central banks running these projects directly.

It should be clear that we do not mean to imply that we envision the central bank to run the entire economy. Rather, this is meant to be a useful abstraction of a richer environment as envisioned in figure 6, with the aim of presenting our analysis as clearly as possible.

10 Extensions

10.1 Token-based CBDC

With a token-based CBDC, a central bank issues anonymous electronic tokens to agents in period 1, rather than accounts.[18] These electronic tokens are more akin to traditional banknotes than to

[18]This can be done with or without a blockchain. In the second case, a centralized ledger to record transactions can be kept by a third party that is separate from the central bank. That third party could also potentially pay interest or impose a suspension of spending. For the purpose of this paper, we do not need to worry about the operational

deposit accounts. Trading with tokens only requires trust in the authenticity of the token rather than knowledge of the identity of the token holder. Thus, token-based transactions can be made without the knowledge of the central bank.

With appropriate software, digital tokens can be designed in such a way that each unit of a token in $t = 1$ turns into a quantity $1 + i$ of tokens in $t = 2$, with i to be determined by the central bank at the beginning of period $t = 2$: even a negative nominal interest rate is possible.[19]

With that, the analysis in the previous sections still holds, since nothing of essence depends on the identity of the spending agents other than total CBDC tokens spent in the goods market. With a token-based CBDC, agents obtain M tokens in period $t = 0$, and decide how much to spend in periods $t = 1$ and $t = 2$. Thus, the same allocations can be implemented except for those that require the suspension of spending, as discussed in Subsection 6.

For the latter, the degree of implementability depends on technical details outside the scope of this paper. Note that even with a token-based system, the transfer of tokens usually needs to be registered somewhere, e.g., on a blockchain. It is technically feasible to limit the total quantity of tokens that can be transferred on-chain in any given period. A pro-rata arrangement can be imposed by taking all the pending transactions waiting to be encoded in the blockchain, taking the sum of all the spending requests, and accordingly dividing each token into a portion that can be transferred and a portion that cannot. It may be that off-chain solutions arise circumventing some of these measures, but their availability depends on the precise technical protocol of the CBDC token-based system. In the case where the token-based CBDC is operated by a centralized third party, such an implementation is even easier.

10.2 Synthetic CBDC and retail banking

With a synthetic CBDC, agents do not hold the central bank's digital money directly. Rather, agents hold accounts at their own retail bank, which in turn holds a CBDC not much different from current central bank reserves. This may be due to tight regulation by the monetary authority. The retail banks undertake the real investments envisioned for the central bank in our analysis above. A synthetic CBDC, therefore, corresponds to the model sketched in Section 7.2 with $\alpha = 0$.

The key difference from the current cash-and-deposit-banking system is that cash does not exist as a separate central bank currency or means of payment. That is, in a synthetic CBDC system, agents can transfer amounts from one account to another, but these transactions are always observable to the banking system and, thereby, the central bank. Likewise, agents (and banks)

details of such a third party or to specify which walls should exist between it and the central bank to guarantee the anonymity of tokens.

[19] Historically, we have examples of banknotes bearing positive interest (for instance, during the U.S. Civil War, the U.S. Treasury issued notes with coupons that could be clipped at regular intervals) and negative interest (demurrage-charged currency, such as the prosperity certificates in Alberta, Canada, during 1936). Thus, an interest-bearing electronic token is novel only in its incarnation, but not in its essence.

cannot circumvent negative nominal interest, while they could do so in a classic cash-and-deposit banking system by withdrawing cash and storing it.

For the purpose of our analysis, observability is key. Our analysis is relevant in the case of a systemic bank run, i.e., if the economy-wide fraction of spending agents exceeds the equilibrium outcome. Much then depends on the interplay between the central bank and the system of private banks. For example, if the liquidation of long-term real projects is up to the retail banks, and these retail banks decide to make the same quantity of real goods available in each period, regardless of the nominal spending requests by their depositors, then the aggregate price level will have to adjust. The central bank may seek to prevent this either by imposing a suspension of spending at retail banks or by forcing banks into higher liquidation of real projects: both would require considerable authority for the central bank. Proposition 14, for instance, says that with $\alpha = 0$, the central bank alone cannot implement a run-deterring policy when offering a synthetic CBDC. Run deterrence then requires retail banks to control liquidation in a particular way.

10.3 Cash

The key difference to a fully cash-based system is that spending decisions can only be observed in the goods market, rather than by also tracing accounts or transactions on the blockchain. In principle, the payment of nominal interest rates on cash is feasible, but is demanding in practice. Excluding nominal interest rates on cash, due to these practical considerations, implies the cash-and-deposit banking system discussed in Section 10.2 and the restrictions discussed there. The tools available to a central bank are now considerably more limited. These limitations may be a good thing, as they may impose a commitment technology and may thus lead to the prevention of an equilibrium systemic bank run in the first place, but the restricted tool set may be viewed as a burden *ex-post*, should such a bank run occur.

11 Conclusion

Diamond and Dybvig (1983) have taught us that the implementation of the social optimum via the financial intermediation of banks comes at the cost of making these banks prone to runs. This dilemma becomes a trilemma when the central bank acts as the intermediary offering a CBDC because central banks are additionally concerned about price stability. As our main result, a central bank that wishes to simultaneously achieve a socially efficient solution, price stability, and financial stability (i.e., absence of runs) will see its desires frustrated. We have shown that a central bank can only realize two of these three goals at a time.

12 Appendix A: Proofs

Proof. [Proposition 8] Proof (i): Via the market clearing condition (7), setting $P_1(n) \equiv \bar{P}$ for all n requires $y(n) = \frac{M}{\bar{P}}n$, for all $n \in [0,1]$. Thus, via (11), $x_1(n) = y(n)/n = \frac{M}{\bar{P}}$ is constant for all n. Last, since the central bank cannot liquidate more than the entire investment in the real technology, $y(n) \in [0,1]$ for all n, together with x_1 constant requires, in particular, $\frac{M}{\bar{P}} = x_1 = x_1(1) = y(1) \leq 1$. Thus, $M \leq \bar{P}$. Proof (ii): When additionally requiring price stability, $P_1(n) = P_2(n) \equiv \bar{P}$, the market clearing condition (8) together with (18) yields (20). □

Proof. [Corollary 9] Proof (i): We know that price stability demands $x_1 \leq 1$ but the social optimum satisfies $x_1^* > 1$. Proof (ii): $\overline{x}_1 \leq 1$ implies $x_2(n) = \frac{1-y(n)}{1-n}R = \frac{1-n\overline{x}_1}{1-n}R \geq R > 1 \geq \overline{x}$. Since the real value of the allocation at $t = 2$ always exceeds the real value of the time one allocation at $t = 1$, patient agents never spend at $t = 1$; thus, there are no runs. Proof (iii): By equation (19), $\frac{\overline{P}}{M} \geq 1$, implies $i(n) = \frac{\frac{\overline{P}}{M} - n}{1-n}R - 1 \geq R - 1 > 0$ for all $n \in [\lambda, 1]$ by $R > 1$. Further, $\frac{\overline{P}}{M} \geq 1$ implies that $i(n)$ increases in n. □

Proof. [Proposition 12] Proof (i): Equation (21) follows immediately from (7) and the constraint $y(n) \leq 1$. Proof (ii): In $n = n_c$, we have $\frac{M}{\bar{P}}n = 1$. Therefore, $n_c > 0$. By assumption $\bar{P} < M$, thus $n_c < 1$, with $n_c \in (0,1)$. Equation (21) implies that $x_1(n) = y(n)/n$ is constant at the level $\overline{x} = M/\overline{P}$, as long as $y(n) < 1$: this is the case for $n < n_c$. For $n \geq n_c$, $y(n) \equiv 1$. All goods are liquidated, so $x_1(n) = 1/n$. Equation $P_1(n) = Mn$ follows from equation (7). Proof (iii): Equation (22) follows from (8) combined with (21). Proof (iv): This is straightforward, when plugging in (21) into $P_2(n)$ and observing that n_0 is positive only for $R > M/\overline{P}$. □

13 Appendix B: Private bank accounting

Consider the collective of private banks with market share $(1-\alpha) \in (0,1)$. For the sake of brevity, we refer to the collective as "the private bank." A fraction n_P of the private bank's customers spend in $t = 1$, while a fraction n_{CB} of the central bank's customers do so, for a total fraction n of all agents $n = (1-\alpha)n_P + \alpha n_{CB}$. Agents are promised M units of the CBDC, when spending in $t = 1$, or $M(1+i)$ units, when spending in $t = 2$. The central bank liquidates y_{CB} goods in period $t = 1$, while the private bank liquidates y_P, for total liquidation $y = (1-\alpha)y_P + \alpha y_{CB}$. For accounting, we introduce some notation. The private bank borrows CBDC L_1 from the central bank to meet withdrawals at the beginning of each period, repaying the loan at the end of the period with the sales proceed S_1 from selling real goods. No interest is charged for the within-period loan.

The difference D_1 at the end of period $t = 1$ is kept on account at the central bank, earning or paying the nominal interest rate z, to be settled at the end of period $t = 2$. Further, the bank has to pay a tax $\tau(1-\alpha)$ denoted in CBDC at the end of period 2 (or receive this as a subsidy, if $\tau < 0$). The interest rate z and the tax τ are chosen by the central bank (CB in the accounting below), and may depend on n_P and choices y_P of the private bank. We seek to calculate x and τ so that the private bank makes zero profits, i.e., is left with zero CBDC balances D_2 at the end of period 2, after having liquidated and sold all its remaining goods at the end of period 2. Then:

Accounting in period $t = 1$:

$$\begin{aligned} \text{Loan from CB:} \quad L_1 &= (1-\alpha)n_P M \\ \text{Sales proceeds:} \quad S_1 &= (1-\alpha)P_1 y_P \\ \text{Difference:} \quad D_1 &= S_1 - L_1 = (1-\alpha)(P_1 y_P - n_P M) \end{aligned}$$

Accounting in period $t = 2$:

$$\begin{aligned} \text{Loan from CB:} \quad L_2 &= (1-\alpha)(1-n_P)(1+i)M \\ \text{Sales proceeds:} \quad S_2 &= (1-\alpha)P_2 R(1-y_P) \\ \text{CB account:} \quad A_2 &= (1+z)D_1 - \tau(1-\alpha) \\ \text{Difference:} \quad D_2 &= A_2 + S_2 - L_2 \\ &= (1-\alpha)\Big(P_2 R + ((1+z)P_1 - P_2 R)y_P - (1+i)M - (z-i)n_P M - \tau\Big) \end{aligned}$$

Market clearing:

$$\begin{aligned} \text{In } t = 1: \quad P_1 y &= nM \\ \text{In } t = 2: \quad P_2 R(1-y) &= (1-n)(1+i)M \end{aligned}$$

Sum $(1 + i)$ times the market clearing equation for P_1 with the equation for P_2 to obtain $P_2 R + ((1 + i)P_1 - P_2 R)y = (1 + i)M$. Use the latter equation to replace $(1 + i)M$ in the last expression for D_2 to find

$$\frac{D_2}{P_1(1-\alpha)} = (i-s)(y_P - y) + (z-i)(y_P - n_P x_1) - \frac{\tau}{P_1} \tag{45}$$

where, as usual, $x_1 = \frac{M}{P_1}$ is the amount of real goods acquired by agents in period $t = 1$ and where we introduce:

$$s = \frac{P_2}{P_1}R - 1 \tag{46}$$

to denote the "shadow" nominal interest rate for private banks, equating liquidating a unit of the good in $t = 1$, selling at P_1 and investing at the shadow nominal return $1 + s$ to keeping the unit of good and thus selling R units at price P_2. Notice that $y = nx_1$ and the market clearing equations imply

$$1+s=(1+i)\frac{1-n}{1-x_1 n}x_1 \tag{47}$$

and, thus, $s>i$, whenever $x_1>1$. In particular, this is the case at the efficient outcome. We note that $s=i$, if and only if $x_1=1$, which is the maximal full price-stable solution as well as the market allocation, when agents engage in self-storage.

Suppose now that the private bank sells exactly as many goods as purchased by its withdrawing customers, i.e., $y_P=n_P x_1$. Absent τ, equation (45) reveals that the private bank will make a loss or profit, if $x_1\neq 1$ and if $y_P\neq y$, i.e., $n_P\neq n$. For example, if the share of private-bank customers who go shopping in $t=1$ is larger than the average share of customers who shop economy-wide, $n_P>n$, and if the allocation achieves $x_1>1$ and thus $s>i$, then the private bank incurs a loss $D_2<0$, absent τ, as the opportunity costs for servicing agents in $t=1$ are high. We shall use these observations to fix the tax τ to compensate for these losses or profits, and assume that

$$\tau=P_1(i-s)(n_P-n)x_1 \tag{48}$$

from here onward. This τ depends on the specifics of the bank only via the "circumstances" n_P and does not depend on the choice y_P. To take care of the case where $y_P\neq n_P x_1$, we use the central bank-account interest rate z. Solving for z per setting $D_2=0$ in (45) and imposing (48) yields the following result, which we formulate as a proposition.

Proposition 15. *Suppose τ satisfies (48). Then*

$$\{D_2=0\}\Leftrightarrow\Big(\{y_P=n_P x_1\}\ \textit{or}\ \{z=s\}\Big). \tag{49}$$

In sum, taxing the "circumstance" profits per (48) and paying an internal interest rate z on central bank balances equal to the shadow nominal interest rate s achieves the objective that private banks make zero profits, regardless of their circumstances n_P and regardless of their liquidation choice y_P.

Lemma 16. *If the private bank sets $y_P\equiv y_{CB}$, then the interest rate for which the private bank's balances with the central bank are zero equals $z=i$ and $\tau=0$.*

That is, if the private bank liquidates the same share of assets as does the central bank, then the interest rate on CBDC balances $z=i$ sets bank profits to zero.

Proof. [Lemma 16] With $\tau=0$, the CBDC balance at the end of $t=2$ equals

$$\begin{aligned} D_2 &= (1-\alpha)\left(P_2R(1-y_p)-(1-n_p)(1+i)M+(1+z)(P_1y_p-n_pM)\right)\\ &= (1-\alpha)M*\begin{pmatrix} (1+i)\left(\frac{(1-y_p)(1-n)}{1-y}-(1-n_p)\right)\\ +(1+z)\left(\frac{ny_p}{y}-n_p\right)\end{pmatrix} \end{aligned} \tag{50}$$

where, at the last equality, we have plugged in P_1 and P_2. Then,

$$\frac{(1-y_p)(1-n)}{1-y} - (1-n_p) = -\left(\frac{ny_p}{y} - n_p\right) \tag{51}$$

if and only if

$$\frac{y(1-y_p) - n(y-y_p)}{y(1-y)} = 1 \tag{52}$$

For $\alpha \in (0,1)$, $y_P \equiv y_{CB}$ implies $y_p = y$. If $y = y_p$, then equations (52) and (51) are true. Thus, for $y = y_p$ the choice $z = i$ puts $D_2 = 0$. □

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