CONVENTIONS UNDER HETEROGENEOUS CHOICE RULES[1]

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Strategies of players in a population are updated according to the choice rules of agents, where each agent is a player or a coalition of players. It is known that classic results on the stochastic stability of conventions are due to an asymmetry property of the strategy updating process. We show that asymmetry can be defined at the level of the choice rule and that asymmetric rules can be mixed and matched whilst retaining asymmetry of the aggregate process. Specifically, we show robustness of asymmetry to heterogeneity within an agent (Alice follows different rules at different times); heterogeneity between agents (Alice and Bob follow different rules); and heterogeneity in the timing of strategy updating. These results greatly expand and convexify the domain of choice rules for which results on the stochastic stability of conventions are known.

Keywords: evolution, conventions, heterogeneity, representative agent.

1. INTRODUCTION

Lewis (1969) argued that conventions, regularities in the behavior of members of a population when faced with a coordination problem, might arise from processes in which individuals in a population follow simple, adaptive choice rules. Young (1993) and Kandori et al. (1993) formulated these ideas mathematically using the theory of Markov chains and showed, using the ideas of Freidlin and Wentzell (1984), that conventions can be ranked by their stability properties under given models of choice behavior. Since then, the stability of conventions under many particular rules has been considered (see Sandholm, 2010; Newton, 2018).

Methodologically, a given choice rule that is followed by members of a population leads to a Markov chain, the transition probabilities of which can be summarized by a *cost function*. Typically, the cost function then provides the input to a graph theoretic problem, the

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solution to which tells us the stability of our conventions, the most stable conventions being known as *stochastically stable* (Foster and Young, 1990). Peski (2010) showed that, if the cost function satisfies an asymmetry condition with respect to one of the conventions, then that convention is stochastically stable. Considering an environment with two strategies, A and B, *asymmetry* (towards A) roughly corresponds to the requirement that for any two strategy profiles σ, $\tilde{\sigma}$ such that all players who play B at σ play A at $\tilde{\sigma}$, switches to strategy B from σ are weakly less likely than switches to strategy A from $\tilde{\sigma}$.[1,2]

Here, instead of considering asymmetry of the process as a whole, we consider asymmetry in the choice rule of each agent (an individual or coalition of players). This allows us to consider three dimensions of heterogeneity. Firstly, we consider heterogeneity within an agent. It turns out that the set of asymmetric choice rules is convex. If two choice rules are asymmetric, then a compound rule that sometimes follows one of the rules and sometimes follows the other is also asymmetric (Theorem 1). Secondly, we consider heterogeneity between agents. If every agent follows an asymmetric choice rule, then the process as a whole is asymmetric (Theorem 2). Finally, we consider heterogeneity in the timing of strategy updating. Asymmetry of the process as a whole does not depend on whether agents update their strategies at the same time or at different times (Theorem 3).

Consequently, when every agent follows an identical choice rule, we can obtain results on stochastic stability by showing asymmetry for a single representative agent. Many results from the literature can be recovered in this manner and results for many alternative choice rules can be derived.[3] Even better, we can mix and match agents who follow different choice rules, and if the agent-specific conditions for asymmetry are satisfied in each case, we are done. In summary, we can treat the choice rules of agents in the population like Lego bricks. Firstly, if every brick (choice rule) used in constructing the process is the same, then we can say something about the entire process by analyzing a single brick (a representative agent). Secondly, if our bricks (choice rules) are heterogeneous but they all

[1] This result finally provided an affirmative answer to the long unanswered question of whether the strategy profile at which every player plays a risk dominant strategy is stochastically stable under the best response with uniform deviations choice rule for any network of interactions.

[2] To define asymmetry (towards A) for more than two strategies (see Peski, 2010) and to extend our general results, all that is required is to replace strategy B with "strategies other than A" in the definitions and analysis. For clarity and unity of exposition, we present the two strategy case throughout.

[3] In particular, we consider choice rules, recover results and extend results from Alós-Ferrer and Schlag (2009); Axelrod (1984); Bilancini and Boncinelli (2019); Blume (1993, 2003); Dokumaci and Sandholm (2011); Ellison (1993, 2000); Ellison and Fudenberg (1995); Kandori et al. (1993); Kreindler and Young (2013); Malawski (1989); Maruta (2002); Newton (2012); Newton and Angus (2015); Norman (2009a,b); Ohtsuki et al. (2006); Peski (2010); Sawa (2014); Schlag (1998); Young (1993, 2011).

satisfy asymmetry, then we can combine them arbitrarily to construct processes that also satisfy asymmetry.

To give an example, Alice and Bob may update their strategies according to best response rules (Section 4), perhaps occasionally collaborating to play a coalitional best response (Section 6). Alice may be a caring person who takes Bob's welfare into account in her decision making (also Section 4). Bob may take his moral philosophy seriously so that his choices have a Kantian (Alger and Weibull, 2013, 2016; Bergstrom, 1995) aspect (Section 7.1). Their friend Colm may follow an imitative rule, perhaps copying the strategy of whichever player currently has the highest payoff (Section 5). For each of these rules, we give conditions under which asymmetry holds. These include conditions on relative incentives such as risk dominance (Harsanyi and Selten, 1988) and an altruistic variant of risk dominance (Maruta, 2002), as well as ordinal conditions such as payoff dominance, maximin and the 'Lewis conditions' that relate to a debate between Lewis and Gilbert (1981) over which games are appropriate to the study of conventions.

Sections 4-6 consider broad classes of choice rules. While previous studies have also considered classes of rules (e.g. Blume, 2003), our approach stands out with respect to the variety of choice rules that it considers. Furthermore, convexity of the set of asymmetric choice rules makes a huge number of hybrid rules accessible to study. This is important, as evidence suggests that human behavior can be a mixed bag, with empirical studies of evolutionary dynamics finding aspects of both best response and imitation (Cason et al., 2013; Friedman et al., 2015; Selten and Apesteguia, 2005). In particular, studies of evolution in coordination games have found support for best response plus deviations with an intentional component (Hwang et al., 2018; Lim and Neary, 2016; Mäs and Nax, 2016).

Our results suggest that when faced with a problems of conventions, we should first check whether the choice rules of agents are asymmetric. For example, if A is risk dominant, then both the logit choice rule and best response with uniform deviations are asymmetric towards A (Section 4). Hence, if some players follow the logit choice rule and the remainder follow best response with uniform deviations, then it follows from Theorems 2 and 3 that the process as a whole is asymmetric. Consequently, the convention at which every player plays A is stochastically stable. We know this without having to consider basins of attraction, escape trajectories, potential functions, spanning trees or any of the other methodology that usually surrounds such results.

The paper is organized as follows. Section 2 gives the model. Section 3 gives our main theoretical results. Section 4 applies these results to payoff-difference based choice rules, a class that includes the most popular best response rules. Section 5 does similarly for imitative rules. Section 6 considers coalitional rules. Section 7 considers Kantian and altruistic payoff transformations, discusses simplified conditions for Sections 4-6 and concludes. Proofs are relegated to the appendix.

2. MODEL

Let V be a finite set of players and $\{A, B\}$ the set of strategies available to each player. A strategy profile $\sigma \in \Sigma := \{A, B\}^V$ is a function $\sigma : V \to \{A, B\}$ that associates each player with one of the two strategies. Let σ^A, σ^B be the homogeneous strategy profiles such that for all $i \in V$, $\sigma^A(i) = A$, $\sigma^B(i) = B$. Let σ_S denote σ restricted to the domain $S \subseteq V$. Denote by $V_A(\sigma) \subseteq V$ the set of players who play strategy A at profile σ and by $V_B(\sigma) \subseteq V$ the set of players who play strategy B at profile σ.

Each player $i \in V$ has a payoff function $U_i : \Sigma \to \mathbb{R}$ such that $U_i(\sigma)$ gives the payoff of player i at strategy profile σ. When we consider specific choice rules (Section 4 onwards), we shall assume

$$U_i(\sigma) = \sum_{j \in V \setminus \{i\}} u_{ij}\big(\sigma(i), \sigma(j)\big), \qquad \text{(Additive Separability)} \tag{2.1}$$

where, for all $j \neq i$, $u_{ij} : \{A, B\}^2 \to \mathbb{R}$ gives the payoff of player i from his interaction with player j. If u_{ij} is constant, then the payoff of player i is unaffected by the strategy of player j. In addition, we shall assume

$$u_{ij}(A, A) \geq u_{ij}(B, A), \; u_{ij}(B, B) \geq u_{ij}(A, B). \qquad \text{(Coordination)} \tag{2.2}$$

A special case of this specification is when each player plays some given coordination game against every other player (e.g. Kandori et al., 1993; Young, 1993) or against some subset of players (e.g. Ellison, 1993, 2000).

Assume that the strategy profile evolves according to a discrete time Markov process on Σ. Specifically, we define a family of Markov processes $P = \{P^\varepsilon\}_\varepsilon$ indexed by $\varepsilon \in [0, 1)$, where higher values of ε correspond to a greater frequency of perturbations from the *unperturbed process* P^0. Let the state at time t be σ^t. Let P^ε be determined by the following steps. At time $t + 1$, select a subset $S \subseteq V$ of updating players according to a probability measure π on the power set of V. Then, let σ^{t+1} be randomly determined according to a probability measure $P^\varepsilon_S(\sigma^t, \cdot)$ satisfying $P^\varepsilon_S(\sigma^t, \sigma) = 0$ if $\sigma_{V \setminus S} \neq \sigma^t_{V \setminus S}$. We refer to $\{P^\varepsilon_S\}_\varepsilon$

as a *choice rule* for S. In summary, the two step strategy updating process selects a set S of updating players before (possibly) updating their strategies, leaving the strategies of players outside of S unchanged. The relationship between P^ε and $\{P^\varepsilon_S\}_{S\subseteq V}$ is given by

$$P^\varepsilon(\sigma,\cdot) = \sum_{S:\pi(S)>0} \pi(S)\, P^\varepsilon_S(\sigma,\cdot). \tag{2.3}$$

We consider *regular* choice rules (Young, 1993; see also Sandholm, 2010). Let P^ε_S be continuous in ε. If $P^0_S(\sigma,\sigma') = 0$ and $P^{\hat\varepsilon}_S(\sigma,\sigma') > 0$ for some $\hat\varepsilon > 0$, let $\{P^\varepsilon_S(\sigma,\sigma')\}_\varepsilon$ satisfy

$$P^\varepsilon_S(\sigma,\sigma') = \big(a + o(1)\big)\,\varepsilon^k \quad \text{for some } a > 0,\ k > 0, \tag{2.4}$$

where a, k may depend on σ, σ', S, but not on ε; and $o(1)$ represents a term that vanishes as $\varepsilon \to 0$. This class of rules includes popular rules such as the logit choice rule and best response with uniform deviations. Finally, assume that there is strictly positive probability of the players in S retaining their current strategies. That is, $P^\varepsilon_S(\sigma,\sigma) > 0$ for all σ, ε. Given that the ultimate object of our analysis is the long run behavior of the process as summarized by its invariant measure, this is without loss of generality.[4]

For $\varepsilon > 0$, assume that any state can be reached with positive probability from any other state in some finite number of steps, therefore the process is irreducible and has a unique invariant probability measure μ^ε on the state space Σ. By standard arguments, the limit of μ^ε as $\varepsilon \to 0$ exists. For small ε, the process will spend most of the time at states which have positive probability under this limiting measure. These are known as *stochastically stable* states (Foster and Young, 1990).

Definition 1 $\sigma \in \Sigma$ is *stochastically stable* under $P = \{P^\varepsilon\}_\varepsilon$ if $\lim_{\varepsilon\to 0}\mu^\varepsilon(\sigma) > 0$.

Define the cost $c_S(\sigma,\sigma')$ of a transition by S from σ to σ' as the exponential rate of decay of the probability of such a transition as $\varepsilon \to 0$. That is, for each family of processes $P_S = \{P^\varepsilon_S\}_\varepsilon$, $S \subseteq V$, $\pi(S) > 0$,

$$c_S(\sigma,\sigma') := \begin{cases} \lim_{\varepsilon\to 0} \frac{\log P^\varepsilon_S(\sigma,\sigma')}{\log\varepsilon} & \text{if } P^{\hat\varepsilon}_S(\sigma,\sigma') > 0 \text{ for some } \hat\varepsilon > 0,\\ \infty & \text{otherwise}. \end{cases} \tag{2.5}$$

[4]If $P^\varepsilon_{S^*}(\sigma^*,\sigma^*) = 0$ for some S^*, σ^*, we can define $\bar P^\varepsilon$ such that, for all S such that $\pi(S) > 0$, for all σ, σ', $\sigma \neq \sigma'$, $\bar P^\varepsilon_S(\sigma,\sigma) = q + (1-q)P^\varepsilon_S(\sigma,\sigma)$, $\bar P^\varepsilon_S(\sigma,\sigma') = (1-q)P^\varepsilon_S(\sigma,\sigma')$, for some arbitrary $q \in (0,1)$. $\bar P^\varepsilon$ then has the same invariant measure as P^ε, but, for all S such that $\pi(S) > 0$, for all σ, $\bar P^\varepsilon_S(\sigma,\sigma) > 0$.

Cost functions measure the order of magnitude of transition probabilities for low values of ε. Transitions with a high cost are less likely than transitions with a low cost. From (2.5), we see that if $P^0_S(\sigma, \sigma') > 0$, then $c_S(\sigma, \sigma') = 0$. That is, transitions that can occur under the unperturbed dynamic have zero cost. In contrast, if a transition is only possible for $\varepsilon > 0$, then $c_S(\sigma, \sigma') = k$, where k is the k from expression (2.4).

Define a cost function $c(\cdot, \cdot)$ for the overall process P^ε by dropping the S subscripts in (2.5). To relate this to $c_S(\cdot, \cdot)$, observe that if there exist distinct $S, T \subseteq V$ such that $P^\varepsilon_S(\sigma, \sigma') > 0$ and $P^\varepsilon_T(\sigma, \sigma') > 0$, then it is the most likely of these transitions (i.e. the lowest cost) which determines the overall likelihood of the transition. Specifically, we derive

Lemma 1 $\quad c(\sigma, \sigma') = \min_{S:\pi(S)>0} c_S(\sigma, \sigma')$.

Peski (2010) considers processes that satisfy a certain type of asymmetry. Asymmetry means, roughly speaking, that if $\sigma, \tilde{\sigma}$ are such that all players who play B at σ play A at $\tilde{\sigma}$, then switches to strategy B from state σ are weakly less likely than switches to strategy A from state $\tilde{\sigma}$.

Definition 2 $\quad c(\cdot, \cdot)$ is *asymmetric* (towards A) if, for any $\sigma, \sigma', \tilde{\sigma} \in \Sigma$, such that $V_B(\sigma) \subseteq V_A(\tilde{\sigma})$, there exists $\tilde{\sigma}' \in \Sigma$ such that $V_A(\tilde{\sigma}) \subseteq V_A(\tilde{\sigma}')$, $V_B(\sigma') \subseteq V_A(\tilde{\sigma}')$ and $c(\sigma, \sigma') \geq c(\tilde{\sigma}, \tilde{\sigma}')$.

The concept can be extended from cost functions to underlying processes in the obvious way.

Definition 3 $\quad P = \{P^\varepsilon\}_\varepsilon$ is *asymmetric* (towards A) if its cost function is asymmetric.

It turns out that asymmetry is a sufficient condition for the stochastic stability of σ^A.

Theorem P (Peski, 2010) $\quad$ If $c(\cdot, \cdot)$ is asymmetric, then σ^A is stochastically stable.[5]

In the cited paper, Theorem P is used to show that risk dominance of strategy A implies stochastic stability of σ^A under best response with either uniform or payoff dependent deviations for any network of interaction.[6] In the next section, we give results that allow us to apply Theorem P to processes that admit a great deal of heterogeneity in choice rules.

[5] Peski (2010) also gives a strict version of asymmetry that ensures that σ^A is *uniquely* stochastically stable. The theorems in the current paper can also be stated for strict asymmetry. We choose to present results for (non strict) asymmetry as the definition is cleaner and it admits broader classes of choice rules.

[6] To give some context, stochastic stability of σ^A under best response plus uniform deviations under uniform interaction (i.e. u_{ij} independent of i, j) was proved by Young (1993) and Kandori et al. (1993);

3. COMBINING ASYMMETRIES

We now present a lemma upon which the theorems of this section build. Asymmetry of cost functions is preserved under minima.

LEMMA 2 *If cost functions c_1 and c_2 are asymmetric, then* $\min\{c_1, c_2\}$ *is also asymmetric.*

It follows from Lemma 2 that if we combine two asymmetric processes in such a way that the resulting process has a cost function which is a minimum of the two original cost functions, then the resulting process will also be asymmetric. This result allows us to consider two types of heterogeneity in choice rules. These are heterogeneity within agents (Alice sometimes follows one rule and sometimes follows another rule) and heterogeneity between agents (Alice and Bob follow different rules). The first of these considers a set S of updating players that sometimes chooses according to one rule and sometimes according to another.

THEOREM 1 (Heterogeneity within agents)
If $\tilde{P}_S$ and $\bar{P}_S$ are asymmetric, then P_S defined by $P_S^\varepsilon = \lambda \tilde{P}_S^\varepsilon + (1-\lambda)\bar{P}_S^\varepsilon$, $\lambda \in (0,1)$, is asymmetric.

For example, Alice may sometimes follow a best response rule (see Section 4) and sometimes follow an imitative rule (see Section 5), but as long as both rules are asymmetric, Theorem 1 tells us that a process which combines them will also be asymmetric.[7]

The next theorem considers heterogeneity across updating sets of players. Each S that updates with positive probability may do so according to a different choice rule.

THEOREM 2 (Heterogeneity between agents)
If P_S is asymmetric for all $S \subseteq V$ such that $\pi(S) > 0$, then P is asymmetric.

For example, Alice may try to maximize Bob's payoff (see Section 4), Bob may follow Homo Moralis preferences (see Section 7.1), and sometimes Alice and Bob may even

possible multiplicity of stochastically stable states under best response plus uniform deviations for general networks of interaction is described in Blume (1996); stochastic stability of σ^A under logit choice (a form of best response plus payoff dependent deviations) for general networks of interaction was proved by Blume (1993).

[7]Note that the examples following Theorems 1, 2 and 3 are simple and illustrative. More complicated examples are easy to construct. For example, Alice might follow one rule when she updates at the same time as Bob and another rule when she updates at the same time as Colm. Alternatively, it could be that when Alice and Bob update at the same time, their rules are perfectly correlated so that exactly one of them follows an imitative rule and exactly one of them follows a best response rule.

form a coalition for their mutual benefit (see Section 6), but as long as all three rules are asymmetric, Theorem 2 tells us that the aggregate process will also be asymmetric.

As well as heterogeneity, Theorem 2 can help us to understand homogeneity. Consider a situation in which the only S selected with positive probability are singleton players, each of whom follows the same choice rule. If we can give some general conditions under which this choice rule is asymmetric for any given *representative agent*, then Theorem 2 implies that the aggregate process must be asymmetric under the same conditions. Later in the paper (Corollary 1), we show that the most famous results in this literature can be recovered by this method.

The next theorem allows us to consider disjoint sets of players S and T that simultaneously and independently follow asymmetric choice rules. When this is the case, the joint choice rule for $S \cup T$ is also asymmetric.

THEOREM 3 (Heterogeneity in timing)
Let $S, T \subseteq V$, $S \cap T = \emptyset$, P_S and P_T be asymmetric. If $P_{S\cup T}$ satisfies, for all ε, σ, $\sigma'_{S\cup T}$,

$$P^{\varepsilon}_{S\cup T}\Big(\sigma, (\sigma'_{S\cup T}, \sigma_{V\setminus(S\cup T)})\Big) = P^{\varepsilon}_{S}\Big(\sigma, (\sigma'_{S}, \sigma_{V\setminus S})\Big)\; P^{\varepsilon}_{T}\Big(\sigma, (\sigma'_{T}, \sigma_{V\setminus T})\Big),$$

then $P_{S\cup T}$ is asymmetric.

For example, if Alice follows an asymmetric choice rule and Bob follows an asymmetric choice rule, then the aggregation of these choice rules is asymmetric regardless of whether Alice and Bob adjust their strategies at different times or at the same time. In general, the possibility of simultaneous strategic updating can be important. Alós-Ferrer and Netzer (2015) define a robustness concept based on the possibility of the identity of stochastically stable states being affected by simultaneity in updating. Arieli and Young (2016) need a particular combination of simultaneity and non-simultaneity in strategy updating in order to obtain rapid convergence to Nash equilibrium in a class of learning models. Theorem 3 shows that, when it comes to asymmetry, we do not have to worry.

Of course, the above three theorems are only useful if the class of asymmetric processes includes interesting and common choice rules. This does indeed turn out to be the case and Sections 4, 5 and 6 give a wide variety of such rules. First, however, we give some results that will help with considering asymmetry in the most common type of rules, those that involve choice by a single player.

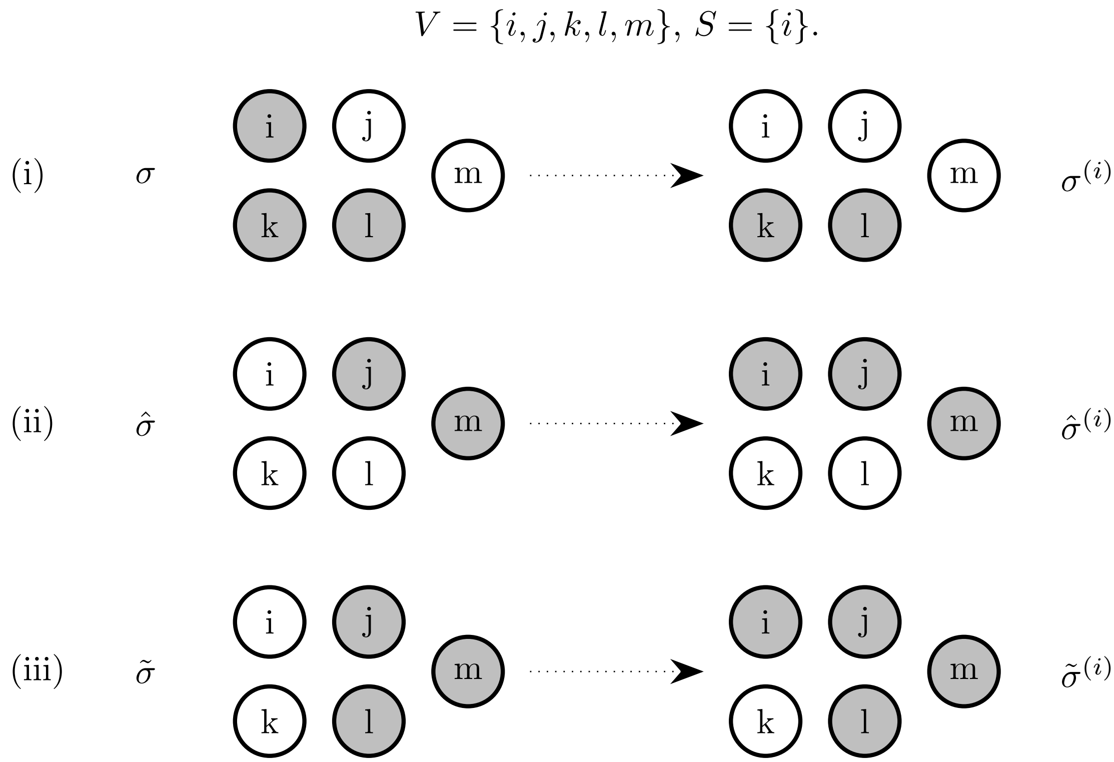

FIGURE 1.— **asymmetry at the individual level.** Vertices shaded grey play A. Unshaded vertices play B. Note that $V_B(\sigma) = V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$, so that asymmetry (Definition 2 and Lemma 3) implies that a transition from σ to $\sigma^{(i)}$ (**Panel [i]**) is less likely than the transitions in **Panels [ii,iii]**. Weak asymmetry (Definition 4) implies that the transition in **Panel [i]** is less likely than the transition in **Panel [ii]**. Supermodularity (Definition 5) implies that the transition in **Panel [ii]** is less likely than the transition in **Panel [iii]**.

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3.1. *Asymmetry at the level of the individual*

For $S = \{i\}$, that is when only a single player updates his strategy (e.g. player i in Figure 1), we can simplify Definition 2. Given a strategy profile σ, let $\sigma^{(i)}$ denote the strategy profile which is identical to σ except for the strategy of player i. That is, $\sigma^{(i)}(j) = \sigma(j)$ for all $j \neq i$, and $\sigma^{(i)}(i) \neq \sigma(i)$.

LEMMA 3 *$c_{\{i\}}(\cdot, \cdot)$ is asymmetric if and only if, for all $\sigma, \tilde{\sigma} \in \Sigma$ such that $V_B(\sigma) \subseteq V_A(\tilde{\sigma})$, if $i \in V_A(\sigma)$ and $i \in V_B(\tilde{\sigma})$, then $c_{\{i\}}(\sigma, \sigma^{(i)}) \geq c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)})$.*

When $S = \{i\}$, it will help to consider asymmetry as an implication of two other properties: weak asymmetry and supermodularity.

Definition 4 $c_{\{i\}}(\cdot,\cdot)$ is *weakly asymmetric* (towards A) if, for all $\sigma, \hat{\sigma} \in \Sigma$ such that $V_B(\sigma) = V_A(\hat{\sigma})$, if $i \in V_A(\sigma)$, then $c_{\{i\}}(\sigma, \sigma^{(i)}) \geq c_{\{i\}}(\hat{\sigma}, \hat{\sigma}^{(i)})$.

States σ and $\hat{\sigma}$ in Definition 4 mirror each other in that players who play A at σ, play B at $\hat{\sigma}$, and players who play B at σ, play A at $\hat{\sigma}$ (see Figure 1[i,ii]). Weak asymmetry means that a switch from A to B by player i from state σ is weakly less probable than a switch from B to A by player i from state $\hat{\sigma}$.

Definition 5 $c_{\{i\}}(\cdot,\cdot)$ is *supermodular* (towards A) if, for all $\hat{\sigma}, \tilde{\sigma} \in \Sigma$ such that $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$, if $i \in V_B(\tilde{\sigma})$, then $c_{\{i\}}(\hat{\sigma}, \hat{\sigma}^{(i)}) \geq c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)})$.

States $\hat{\sigma}$, $\tilde{\sigma}$ in Definition 5 are such that all players who play A at $\hat{\sigma}$ also play A at $\tilde{\sigma}$ (see Figure 1[ii,iii]). Let player i be any player who plays B at both states. Supermodularity means that a switch from B to A by player i from state $\hat{\sigma}$ is weakly less probable than a switch from B to A by player i from state $\tilde{\sigma}$. That is, switches by player i from B to A are weakly more probable when more of the other players are playing A.

Lemma 4 *If $c_{\{i\}}(\cdot,\cdot)$ is weakly asymmetric and supermodular, then it is asymmetric.*

The notation chosen for Definitions 4 and 5 has been chosen to facilitate understanding of Lemma 4 in terms of these definitions. Specifically, if we consider σ, $\hat{\sigma}$, $\tilde{\sigma}$ as given in Definitions 4 and 5, we have

$$c_{\{i\}}(\sigma, \sigma^{(i)}) \underbrace{\geq}_{\text{weak asymmetry}} c_{\{i\}}(\hat{\sigma}, \hat{\sigma}^{(i)}) \underbrace{\geq}_{\text{supermodularity}} c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)}), \tag{3.1}$$

which implies the condition $c_{\{i\}}(\sigma, \sigma^{(i)}) \geq c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)})$ for asymmetry given in Lemma 3. As it is possible that $\tilde{\sigma} = \hat{\sigma}$, asymmetry implies weak asymmetry. In contrast, (3.1) tells us that if weak asymmetry holds strictly, then supermodularity can be violated by some amount while retaining asymmetry.

4. CHOICE BASED ON PAYOFF DIFFERENCES

We first consider decision rules according to which the probability of an individual player switching from his current strategy to the alternative strategy decreases in the vector of payoff losses from the switch. An updating player following such a rule acts according to a predisposition to improve things, or at least not make them worse, for some group of players. When this group is the updating player himself, we have the subclass of best/better response rules.[8] As we shall see, this class includes many rules that have been considered in the literature.

4.1. *Definition of payoff-difference based rules*

Consider the vector of differences in payoff for every player when player i changes his strategy so that the strategy profile changes from σ to $\sigma^{(i)}$. That is, consider

$$D_i^\sigma := \left(U_j(\sigma) - U_j(\sigma^{(i)})\right)_{j\in V} \in \mathbb{R}^V.$$

Positive elements of D_i^σ correspond to payoff losses and negative elements of D_i^σ correspond to payoff gains.

Let a *payoff-difference based* choice rule for player i be defined as follows. For non-decreasing *unlikelihood function* $\Upsilon_i(\cdot) : \mathbb{R}^V \to \mathbb{R}_+$ and constant (with respect to ε) $d_i^\sigma \in (0, 1)$, $\sigma \in \Sigma$, let

$$P^\varepsilon_{\{i\}}(\sigma, \sigma) = 1 - d_i^\sigma \varepsilon^{\Upsilon_i(D_i^\sigma)} \quad \text{and} \quad P^\varepsilon_{\{i\}}(\sigma, \sigma^{(i)}) = d_i^\sigma \varepsilon^{\Upsilon_i(D_i^\sigma)}, \tag{4.1}$$

with the convention that $0^0 = 1$ so that $P^\varepsilon_{\{i\}}$ is continuous in ε at $\varepsilon = 0$. Such rules satisfy the restriction on behavior that if a transition is at least as good (measured by changes in payoff) for everybody as another transition, then the first transition should be no less likely to occur than the second.

A strictly positive unlikelihood $\Upsilon_i(D_i^\sigma)$ implies that the probability of a transition from σ to $\sigma^{(i)}$ approaches zero as ε approaches zero. In contrast, $\Upsilon_i(D_i^\sigma) = 0$ implies that the probability of a transition from σ to $\sigma^{(i)}$ is strictly positive even under the unperturbed

[8]Fixed points of such rules define the equilibrium concepts of Cournot (1838) and Nash (1950). In his proofs of the existence of Nash equilibria, Nash uses two best/better response mappings. Most famously (Nash, 1950), the classic best response correspondence that is definitional to Nash equilibrium, but also, in an alternative proof (Nash, 1951), a smoothed better response correspondence that allows the use of Brouwer's rather than Kakutani's fixed point theorem.

process P^0. Substituting (4.1) into the definition of a cost function, we obtain

$$c_{\{i\}}(\sigma,\sigma') := \begin{cases} 0 & \text{if } \sigma' = \sigma, \\ \Upsilon(D_i^\sigma) & \text{if } \sigma' = \sigma^{(i)}, \\ \infty & \text{otherwise.} \end{cases} \tag{4.2}$$

We shall now illustrate the breadth and flexibility of this class of rules by giving some examples. Following this, we give sufficient conditions for the asymmetry of such processes.

4.2. *Examples of payoff-difference based rules*

4.2.1. *Utilitarian rules*

A player i follows a *utilitarian* rule if, for some nonnegative vector $\lambda \in \mathbb{R}_+^V$, we have that

$$\Upsilon_i(x) = [\lambda \cdot x]_+, \tag{4.3}$$

where $[x]_+ = \max\{0, x\}$. Under this rule, the probability of player i changing his strategy is decreasing in a weighted sum of payoff changes when he does so. A special case of this is when $\lambda_i = 1$ and $\lambda_j = 0$ for $j \neq i$, in which case we have *best response with log-linear deviations*, which for small ε approximates the logit choice rule (Blume, 1993). This rule is self-regarding in the following sense.

Definition 6 A rule Υ_i is *self-regarding* if $\Upsilon_i(x) = f(x_i)$ for some non-decreasing function $f : \mathbb{R} \to \mathbb{R}_+$.

The class of self-regarding payoff-difference based rules is effectively the class of *skew-symmetric* rules considered by Blume (2003) and Norman (2009a). In contrast, if $\lambda_j = 1$ for some $j \neq i$ and $\lambda_k = 0$ for $k \neq j$, then we have a *best friend forever* rule, where player i makes his decisions according to their impact on player j. Clearly, this rule is not self-regarding.

4.2.2. *Own-payoff based rules*

A player i follows an *own-payoff based* best response rule (Peski, 2010) if, for some strictly increasing function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that $f(0) = 0$, we have that

$$\Upsilon_i(x) = f([x_i]_+). \tag{4.4}$$

A special case is $f(z) = z$, which again gives best response with log-linear deviations. Another special case is $f(z) = z^2$, in which case we have *best response with log-quadratic deviations*, which for small ε approximates the probit choice rule in two strategy environments such as the one in the current paper (Dokumaci and Sandholm, 2011).

4.2.3. *Hippocratic rules*

A player i follows a *Hippocratic* rule if, for some nonnegative vector $\lambda \in \mathbb{R}^V_+$,

$$\Upsilon_i(x) = \lambda \cdot [x]_+, \tag{4.5}$$

so that the probability of player i changing his strategy is decreasing in a weighted sum of payoff losses when he does so. Unlike the utilitarian rule, any gains in payoff are disregarded. If $\lambda_i = 1$ and $\lambda_j = 0$ for $j \neq i$, this is once again best response with log-linear deviations.

4.2.4. *Best response with uniform deviations*

A player i follows *best response with uniform deviations* (Kandori et al., 1993; Young, 1993) if

$$\Upsilon_i(x) = [sgn(x_i)]_+, \tag{4.6}$$

so that, for small ε, player i will rarely change his strategy unless his payoff weakly increases as a consequence.

4.2.5. *Best response with switching costs*

A player i follows best response with *switching costs* and uniform deviations (Norman, 2009b) if, for some strictly positive $\delta > 0$,

$$\Upsilon_i(x) = [sgn(x_i + \delta)]_+, \tag{4.7}$$

so that, for small ε, player i will rarely change his strategy unless his payoff increases by at least δ as a consequence.

4.2.6. *Disjunction and conjunction*

Consider rules similar to (4.7) in that Υ_i takes values on $\{0, 1\}$. These Υ_i are *truth functions* that output a value of 1 if a condition is satisfied and output 0 if it is not satisfied.

Another example is

$$\Upsilon'_i(x) = \begin{cases} 1 & \text{if } \sum_{k\in V}[\mathrm{sgn}(x_k)]_+ > 3, \\ 0 & \text{otherwise.} \end{cases} \tag{4.8}$$

which corresponds to a process in which, for small ε, player i will rarely change his strategy unless by doing so he harms no more than three players.

Any two truth functions can be combined through logical conjunction, which corresponds to taking the minimum of the functions, or logical disjunction, which corresponds to taking the maximum of the functions. For example, in the case of Υ_i given by (4.7) and Υ'_i given by (4.8), the truth function

$$\Upsilon^*_i = \max\{\Upsilon_i, \Upsilon'_i\}, \tag{4.9}$$

corresponds to a process in which, for small ε, player i rarely changes his strategy unless, as a consequence, his payoff increases by at least δ and the payoff of no more than three players decreases. Note that Υ^*_i inherits the non-decreasing property from Υ_i, Υ'_i. Furthermore, given any set of primitive truth functions, the set of truth functions that can be constructed in this way has a lattice structure with a maximal and minimal element.

4.3. *Asymmetry of payoff-difference based rules*

Recall that a risk dominant strategy (Harsanyi and Selten, 1988) for player i is a strategy that maximizes his payoff when he faces an opponent who plays each strategy with equal probability. Similarly, we define an altruistically risk dominant strategy for player i against player j to be a strategy that player i should play to maximize the payoff of player j when player j plays each strategy with equal probability. Maruta (2002) refers to this latter condition as dominance with respect to homogeneous externality, as it compares the change in payoff of players of each strategy when an opponent switches to that strategy. However, an interpretation as altruistic risk dominance emphasizes the symmetry with risk dominance that is important to the results of this section.

Definition 7 Strategy A is RD_i (risk dominant for i) if

$$\sum_{j\in V\setminus\{i\}} u_{ij}(A,A) + u_{ij}(A,B) \geq \sum_{j\in V\setminus\{i\}} u_{ij}(B,A) + u_{ij}(B,B);$$

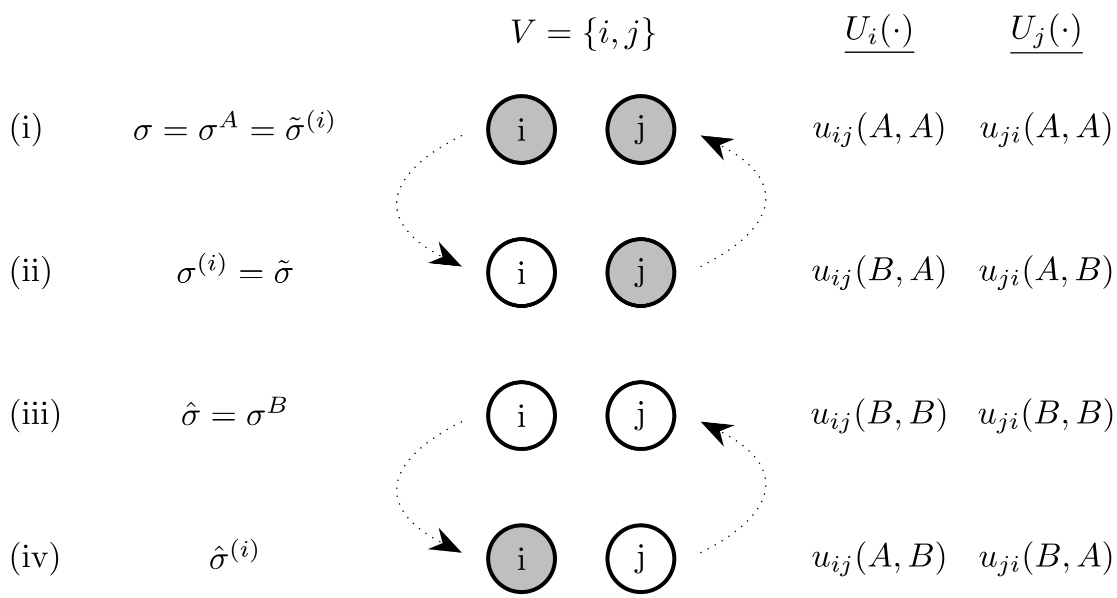

FIGURE 2.— **payoff-difference based choice.** Vertices shaded grey play A. Unshaded vertices play B. Weak asymmetry (Lemma 5) implies that a transition from σ (**Panel [i]**) to $\sigma^{(i)}$ (**Panel [ii]**) is less likely than a transition from $\hat{\sigma}$ (**Panel [iii]**) to $\hat{\sigma}^{(i)}$ (**Panel [iv]**), and supermodularity (Lemma 6) implies that this latter transition is less likely than a transition from $\tilde{\sigma}$ (**Panel [ii]**) to $\tilde{\sigma}^{(i)}$ (**Panel [i]**).

and ARD_{ij} (altruistically risk dominant for i against j) if

$$u_{ji}(A, A) + u_{ji}(B, A) \geq u_{ji}(A, B) + u_{ji}(B, B).$$

These two properties turn out to be exactly what is required to give weak asymmetry of payoff-difference based choice rules.

LEMMA 5 *If player i follows a payoff-difference based choice rule, A is RD_i and*

(i) Υ_i is self-regarding, or

(ii) A is ARD_{ij} for all j,

then $c_{\{i\}}(\cdot, \cdot)$ is weakly asymmetric.

The intuition behind Lemma 5 can be conveyed in an example, which is illustrated in Figure 2. Consider $V = \{i, j\}$ and states $\sigma = \sigma^A$, $\hat{\sigma} = \sigma^B$ that mirror each other as in Definition 4. From state σ, if i switches from A to B so that the state becomes $\sigma^{(i)}$, then changes in payoff for players i and j respectively are

$$(4.10)\quad \begin{aligned}(D_i^{\sigma})_i &= U_i(\sigma) - U_i(\sigma^{(i)}) = u_{ij}(A,A) - u_{ij}(B,A), \quad \text{and}\\ (D_i^{\sigma})_j &= U_j(\sigma) - U_j(\sigma^{(i)}) = u_{ji}(A,A) - u_{ji}(A,B).\end{aligned}$$

From state $\hat{\sigma}$, if i switches from B to A so that the state becomes $\hat{\sigma}^{(i)}$, then changes in payoff are

$$(4.11)\quad \begin{aligned}(D_i^{\hat{\sigma}})_i &= U_i(\hat{\sigma}) - U_i(\hat{\sigma}^{(i)}) = u_{ij}(B,B) - u_{ij}(A,B), \quad \text{and}\\ (D_i^{\hat{\sigma}})_j &= U_j(\hat{\sigma}) - U_j(\hat{\sigma}^{(i)}) = u_{ji}(B,B) - u_{ji}(B,A).\end{aligned}$$

Comparing (4.10) to (4.11) component-wise, it is clear that $D_i^{\sigma} \geq D_i^{\hat{\sigma}}$ if and only if A is RD_i and ARD_{ij}. For payoff-difference based processes, $D_i^{\sigma} \geq D_i^{\hat{\sigma}}$ implies that $\Upsilon_i(D_i^{\sigma}) \geq \Upsilon_i(D_i^{\hat{\sigma}})$ and therefore, by (4.2), $c_{\{i\}}(\sigma, \sigma^i) \geq c_{\{i\}}(\hat{\sigma}, \hat{\sigma}^i)$ as required by Definition 4 (weak asymmetry).

It turns out that payoff-difference based choice rules satisfy supermodularity without the additional conditions required for weak asymmetry.

Lemma 6 *If player i follows a payoff-difference based choice rule, then $c_{\{i\}}(\cdot, \cdot)$ is supermodular.*

To see the intuition behind Lemma 6, we continue with our previous example, which we continue to illustrate in Figure 2. Let $\tilde{\sigma}$ be such that $\tilde{\sigma}(i) = B$, $\tilde{\sigma}(j) = A$. Note that $\hat{\sigma}, \tilde{\sigma}$ are as in Definition 5. From state $\tilde{\sigma}$, if i switches from B to A then changes in payoff are

$$(4.12)\quad \begin{aligned}(D_i^{\tilde{\sigma}})_i &= U_i(\tilde{\sigma}) - U_i(\tilde{\sigma}^{(i)}) = u_{ij}(B,A) - u_{ij}(A,A), \quad \text{and}\\ (D_i^{\tilde{\sigma}})_j &= U_j(\tilde{\sigma}) - U_j(\tilde{\sigma}^{(i)}) = u_{ji}(A,B) - u_{ji}(A,A).\end{aligned}$$

Subtracting (4.12) from (4.11) component-wise, we see that

$$(4.13)\quad \begin{aligned}(D_i^{\hat{\sigma}})_i - (D_i^{\tilde{\sigma}})_i &= u_{ij}(A,A) - u_{ij}(B,A) + u_{ij}(B,B) - u_{ij}(A,B),\\ (D_i^{\hat{\sigma}})_j - (D_i^{\tilde{\sigma}})_j &= u_{ji}(A,A) - u_{ji}(B,A) + u_{ji}(B,B) - u_{ji}(A,B),\end{aligned}$$

which are nonnegative by (2.2). Therefore, $D_i^{\hat{\sigma}} \geq D_i^{\tilde{\sigma}}$. For payoff-difference based processes, $D_i^{\hat{\sigma}} \geq D_i^{\tilde{\sigma}}$ implies that $\Upsilon_i(D_i^{\hat{\sigma}}) \geq \Upsilon_i(D_i^{\tilde{\sigma}})$ and therefore, by (4.2), $c_{\{i\}}(\hat{\sigma}, \hat{\sigma}^i) \geq c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^i)$ as required by Definition 5 (supermodularity).

Lemma 4 tells us that weak asymmetry and supermodularity suffice for asymmetry, so Lemmas 5 and 6 can be combined to give the following proposition.

Proposition 1 *If player i follows a payoff-difference based choice rule, A is* RD_i *and*

(i) Υ_i *is self-regarding, or*

(ii) A is ARD_{ij} *for all j,*

then $c_{\{i\}}(\cdot,\cdot)$ *is asymmetric.*

So, if Alice follows a self-regarding payoff-difference based choice rule such as best response with uniform deviations and A is risk dominant for Alice, then her cost function will be asymmetric (Proposition 1[i]). If Bob follows a utilitarian rule and tries to maximize the total payoff for him and Alice, A is risk dominant for Bob and altruistically risk dominant for Bob against Alice, then his cost function will be asymmetric (Proposition 1[ii]). If Alice and Bob alter their strategies simultaneously, then the resulting cost function for $S = \{\text{Alice}, \text{Bob}\}$ will also be asymmetric (Theorem 3). If P^{ε} is such that sometimes Alice alters her strategy, sometimes Bob alters his strategy and sometimes they alter their strategies simultaneously, then the resulting cost function for the combined process is asymmetric (Theorem 2), hence Theorem P applies and the state at which both Alice and Bob play A is stochastically stable.

4.4. *Relation to the literature*

If A is RD_i and player i follows a self-regarding payoff-difference based rule, then $c_{\{i\}}$ is asymmetric (Proposition 1[i]). If this holds for all $i \in V$ and only individual players update their strategies, then the combined process is asymmetric (Theorem 2), hence Theorem P applies and σ^A is stochastically stable. We have the following corollary.

Corollary 1 *Let* $\pi(\{i\}) > 0$ *for all* $i \in V$, $\pi(S) = 0$ *otherwise. If, for all* $i \in V$, *A is* RD_i *and i follows a self-regarding payoff-difference based choice rule, then* σ^A *is stochastically stable.*

This corollary nests existing results on stochastic stability under best response with uniform deviations and own-payoff based rules (Peski, 2010, Theorems 2 and 3 respectively), special cases of which include best response with uniform deviations and uniform interaction (Kandori et al., 1993; Young, 1993); best response with uniform deviations on specific interaction structures such as the ring network and the two dimensional square lattice with von Neumann neighborhoods (Ellison, 1993, 2000); and best response with log-linear deviations for any interaction structure (Blume, 1993). Considering the full set of self-regarding payoff-difference based choice rules, but again restricting attention to uniform interaction, the Corollary is effectively the result of Blume (2003, Theorem 1). Combining this with Theorem 3 of the current paper then gives us the equivalent result for simultaneous choice Norman (2009a, Theorem 1).[9]

5. IMITATIVE CHOICE

A process is imitative if an updating player is more likely to switch to a strategy that currently obtains high payoffs for those who play it. Formally, let $C \subseteq V$ be player i's *comparison set*. When player i considers changing his strategy, his switching probability will depend on the current payoffs of the players in his comparison set. Define a function $h^C : \{S : S \subseteq C\} \times \mathbb{R}^V \to \mathbb{R}$ such that, for given $S \subseteq C$,

$$h^C(S,x) \text{ is } \begin{cases} \text{non-decreasing in } x_j & \text{if } j \in S, \\ \text{non-increasing in } x_j & \text{if } j \in C \setminus S, \\ \text{constant in } x_j & \text{if } j \notin C. \end{cases}$$

Using this function, we define a statistic Δ_i^σ that measures, at strategy profile σ, how well players in C who play the same strategy as player i perform relative to players who play the alternative strategy.

$$\Delta_i^\sigma := h^C\left(V_{\sigma(i)}(\sigma) \cap C\,,\, \left(U_j(\sigma)\right)_{j \in V}\right).$$

Δ_i^σ is non-decreasing in the payoffs of players in C who play the same strategy as player i, non-increasing in the payoffs of players in C who play a different strategy to player i, and constant in the payoff of players outside of C.

Let an *imitative* choice rule for player i be defined as follows. For non-decreasing unlikelihood function $\Upsilon_i^{Im} : \mathbb{R} \to \mathbb{R}_+$ and constant $d_i^\sigma \in (0,1)$, $\sigma \in \Sigma$, let

$$P^\varepsilon_{\{i\}}(\sigma,\sigma) = 1 - d_i^\sigma \varepsilon^{\Upsilon_i^{Im}(\Delta_i^\sigma)} \quad \text{and} \quad P^\varepsilon_{\{i\}}(\sigma,\sigma^{(i)}) = d_i^\sigma \varepsilon^{\Upsilon_i^{Im}(\Delta_i^\sigma)}, \tag{5.1}$$

[9] The qualifier 'effectively' here refers to the fact that both Blume (2003) and Norman (2009a) deal with strict risk dominance and unique stochastic stability for large populations, whereas here we deal with (not necessarily strict) risk dominance and (not necessarily unique) stochastic stability, without any restriction on population size.

with the convention that $0^0 = 1$ so that $P^{\varepsilon}_{\{i\}}$ is continuous in ε at $\varepsilon = 0$. Such rules satisfy the restriction on behavior that the probability that a strategy is chosen is non-decreasing in the payoffs of those who currently play that strategy.

A variety of imitative rules have been studied in the literature. For example, player i may sample some player j in his comparison set and adopt j's strategy if j obtains a higher payoff than i, that is if $U_j(\sigma) > U_i(\sigma)$ (Malawski, 1989). A smoothed version of this rule has i switching to j's strategy with a probability proportional to $U_j(\sigma) - U_i(\sigma)$ (Schlag, 1998). Alternatively, player i may simultaneously consider the payoffs of all of the players in his comparison set and adopt the strategy associated with the highest average payoff (Ellison and Fudenberg, 1995) or the strategy of whichever player currently obtains the highest payoff (Axelrod, 1984). In the biology literature (see Ohtsuki et al., 2006) it is common to assume that the strategy of each player in the comparison set is adopted with a probability proportional to that player's payoff – a *death-birth Moran* process. If every player simultaneously follows such a process (a possible application of Theorem 3), then we have a *Wright-Fisher* process (see Lehmann et al., 2007). For a survey of imitative rules, the reader is referred to Alós-Ferrer and Schlag (2009).

5.1. *Weak asymmetry of imitative rules*

Definition 8 Strategy A is PD_{ij} (payoff dominant for i against j) if

$$u_{ij}(A, A) \geq u_{ij}(B, B);$$

and MM_{ij} (maximin for i against j) if

$$u_{ij}(A, B) \geq u_{ij}(B, A).$$

These two properties turn out to be exactly what is required to give weak asymmetry of imitative rules.

Let $\sigma, \hat{\sigma}$ be such that $V_B(\sigma) = V_A(\hat{\sigma})$ and $\sigma(i) = A$ as in the definition of weak asymmetry. Note that, by definition, the set of players who play the same strategy as player i at σ is the same as the set of players who play the same strategy as player i at $\hat{\sigma}$.

Consider the payoff of some player j, $\sigma(j) = A$, from interaction with an opponent k whose strategy at σ is the same as his. This opponent causes the payoff of player j at σ to differ from his payoff at $\hat{\sigma}$ by

$$u_{jk}(A,A) - u_{jk}(B,B). \tag{5.2}$$

The same reasoning applies to the payoffs of all other players, with the sign of the difference reversed for players who play strategy B at σ.

Next, consider the payoff of player j from interaction with an opponent k whose strategy at σ is different to his. This opponent causes the payoff of player j at σ to differ from his payoff at $\hat{\sigma}$ by

$$u_{jk}(A,B) - u_{jk}(B,A). \tag{5.3}$$

Again, this applies to the payoffs of all other players, with the sign of the difference reversed for players who play strategy B at σ.

If payoff differences such as (5.2) and (5.3) give players who play A at σ weakly higher payoff at σ than at $\hat{\sigma}$, and the opposite holds for players who play B at σ, then player i will be less likely to switch to strategy B from σ than he is to switch to strategy A from $\hat{\sigma}$ and weak asymmetry will hold. For this to be the case, both (5.2) and (5.3) should be weakly positive. That is, we require PD_{jk} and MM_{jk}.

Lemma 7 *If player i follows an imitative choice rule, A is PD_{jk} and MM_{jk} for all j,k, then $c_{\{i\}}(\cdot,\cdot)$ is weakly asymmetric.*

5.2. *Supermodularity of imitative rules*

Unlike payoff-difference based processes, imitative processes can violate supermodularity. For example, if Δ_i^{σ} equals the average payoff of players who play strategy $\sigma(i)$ minus the average payoff of players who play the alternative strategy, then adding to the set of players who play A may reduce the average payoff of players who play A and thus reduce the probability of switches to strategy A. However, some popular imitative rules do satisfy supermodularity, as we shall now see.

5.2.1. *Condition dependence*

If $C = \{i\}$, then the switching probability for a player i decreases in his current payoff $U_i(\sigma)$ and is independent of the payoffs of the other players. This is known as *condition* *dependence* (Bilancini and Boncinelli, 2019) after the biology literature.[10] The justification for the use of such a rule is simple: if one is obtaining a low payoff, it makes sense to try something else.

Let $\hat{\sigma}, \tilde{\sigma}$ be such that $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$ and $\tilde{\sigma}(i) = B$ as in the definition of supermodularity. The set of players who play B is weakly larger at $\hat{\sigma}$ than at $\tilde{\sigma}$, so the only players that do not play the same strategy at $\hat{\sigma}$ and $\tilde{\sigma}$ will be those who play B at $\hat{\sigma}$ and A at $\tilde{\sigma}$. Such an opponent j causes the payoff of player i at $\hat{\sigma}$ to differ from his payoff at $\tilde{\sigma}$ by

$$u_{ij}(B, B) - u_{ij}(B, A) \tag{5.4}$$

If payoff differences such as (5.4) give player i weakly higher payoff at $\hat{\sigma}$ than at $\tilde{\sigma}$, then player i will be less likely to switch to strategy A from $\hat{\sigma}$ than he is from $\tilde{\sigma}$ and supermodularity will hold. For this to be the case, (5.4) should be weakly positive.

Lemma 8 *If player i follows a condition dependent choice rule and* $u_{ij}(B, B) \geq u_{ij}(B, A)$ *for all* j*, then* $c_{\{i\}}(\cdot, \cdot)$ *is supermodular.*

Note that if A is PD_{ij} and MM_{ij}, then $u_{ij}(B, B) \geq u_{ij}(B, A)$, otherwise (2.2) would be violated. Therefore, the conditions of Lemma 7, suitably weakened for condition dependence, imply the condition of Lemma 8. By Lemma 4, weak asymmetry and supermodularity suffice for asymmetry, so we have the following proposition.

Proposition 2 *If player i follows a condition dependent choice rule, A is* PD_{ij} *and* MM_{ij} *for all* j*, then* $c_{\{i\}}(\cdot, \cdot)$ *is asymmetric.*

5.2.2. *Imitate the best*

Consider a player i whose choice probabilities are a function of the highest payoff obtained amongst all of the players who play A and the highest payoff obtained amongst all of the players who play B. To pick out the highest payoff obtained by some player in a set

[10]The actual choice rule of Bilancini and Boncinelli (2019) is a mixture of condition dependence and best response. One very flexible family of such rules is that considered by Maruta (2002). The author of the current paper has worked on establishing sufficient conditions for weak asymmetry, supermodularity and hence asymmetry of such rules, and can recover the results of the cited paper in this way. Perhaps unsurprisingly, the conditions derived are a mixture of those for payoff-difference based processes and those for imitative processes. In order to keep this paper of a tolerable length, these results are not included in the current exposition.

of players S, define, for $S \subseteq C$, functions $M^S : \mathbb{R}^V \to \mathbb{R}$,

$$M^S(x) = \max \{\underline{h}\} \cup \{x_j : j \in S\},$$

where $\underline{h} \in \mathbb{R}$ is a constant that is independent of S. That is, $M^S(x)$ equals the maximum value of x_j for $j \in S$, except in the cases when this maximum is less than $\underline{h}$, or S is empty, in which case $M^S(x)$ equals $\underline{h}$.

If h^C is such that $h^C(S, x)$ can be written as

$$h^C(S, x) = f(M^S(x), M^{C \setminus S}(x)),$$

for a function f that is non-decreasing in its first argument and non-increasing in its second argument, we say the rule is an *imitate-the-best* rule.

Let $\hat{\sigma}, \tilde{\sigma}$ be such that $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$ and $\tilde{\sigma}(i) = B$ as in the definition of supermodularity. By similar arguments to the case of condition dependence, to ensure that the maximum payoff amongst players who play B is at least as high at $\hat{\sigma}$ as at $\tilde{\sigma}$, we require that $u_{jk}(B, B) \geq u_{jk}(B, A)$ for all j, k. Similarly, to ensure that the maximum payoff amongst players who play A is no higher at $\hat{\sigma}$ than at $\tilde{\sigma}$, we require that $u_{jk}(A, A) \geq u_{jk}(A, B)$ for all j, k.

Lemma 9 *If player i follows an imitate-the-best choice rule,* $u_{jk}(B, B) \geq u_{jk}(B, A)$ *and* $u_{jk}(A, A) \geq u_{jk}(A, B)$ *for all* j, k, *then* $c_{\{i\}}(\cdot, \cdot)$ *is supermodular.*

Note that if A is PD_{jk} and MM_{jk}, then $u_{jk}(B, B) \geq u_{jk}(B, A)$ and $u_{jk}(A, A) \geq u_{jk}(A, B)$, otherwise (2.2) would be violated. So, under the conditions of Lemma 7, Lemma 9 also applies. By Lemma 4, weak asymmetry and supermodularity suffice for asymmetry, so we have the following proposition.

Proposition 3 *If player i follows an imitate-the-best choice rule, A is* PD_{jk} *and* MM_{jk} *for all* j, k, *then* $c_{\{i\}}(\cdot, \cdot)$ *is asymmetric.*

Interestingly, the $u_{jk}(B, B) \geq u_{jk}(B, A)$ and $u_{jk}(A, A) \geq u_{jk}(A, B)$ conditions that guarantee supermodularity correspond to conditions that Lewis (1969) imposes on the games he considers in his philosophical theory of conventions. Gilbert (1981) later argued that these conditions were too stringent. Indeed, as we saw in Section 4, they are not directly relevant to the class of payoff-difference based rules that has predominated in game theoretic forays into this territory. However, as we have just determined, they are of direct relevance to imitative choice.

Conditions PD_{jk} and MM_{jk} may seem strong when compared with conditions in prior studies (Alós-Ferrer and Weidenholzer, 2008; Khan, 2014; Robson and Vega-Redondo, 1996) that give stochastic stability of σ^A under an imitate-the-best rule when A is PD_{jk} but not MM_{jk}. These conditions involve interaction structures and comparison sets set up in such a way that the payoff dominance assumption can be leveraged so that, from nearly everywhere in the state space, there exists a path of zero cost transitions that leads to σ^A. The assumptions that permit this are not innocuous, but here is not the place to debate their plausibility. Suffice to say, when we consider asymmetry across the whole state space, stronger conditions are required.

6. COALITIONAL CHOICE

From Theorem 3, we know that asymmetric c_S can arise from independent, simultaneous choice by $i \in S$ who follow rules with asymmetric $c_{\{i\}}$. In this section, we consider choice by S as a coalition and study a coalitional variant of payoff-difference based rules. Let

$$E_S^\sigma = \left(U_j(\sigma_S^A, \sigma_{V\setminus S}) - U_j(\sigma_S^B, \sigma_{V\setminus S})\right)_{j\in V} \in \mathbb{R}^V.$$

That is, starting from profile σ and keeping the strategies of players in $V \setminus S$ constant, $(E_S^\sigma)_j$ equals the difference between the payoff of player j when S plays σ_S^A and the payoff of player j when S plays σ_S^B.

Let a *coalitional payoff-difference based* choice rule for S be a rule that gives the following cost function. For non-decreasing unlikelihood function $\Upsilon_S^C(\cdot) : \mathbb{R}^V \to \mathbb{R}_+$,

$$c_S(\sigma, \sigma') = \begin{cases} 0, & \text{if } \sigma' = \sigma, \\ \Upsilon_S^C(-E_S^\sigma), & \text{if } \sigma' = (\sigma_S^A, \sigma_{V\setminus S}) \neq \sigma, \\ \Upsilon_S^C(E_S^\sigma), & \text{if } \sigma' = (\sigma_S^B, \sigma_{V\setminus S}) \neq \sigma, \\ \infty, & \text{otherwise.} \end{cases} \tag{6.1}$$

That is, greater values of E_S^σ make it more likely that S will choose σ_S^A and less likely that S will choose σ_S^B. Note that if $S = \{i\}$, then the cost function (6.1) reduces to the cost function (4.2).[11] That is, the individualistic payoff-difference based models of Section 4 are a special case of the models of this section.

[11] To see this, observe that if $S = \{i\}$, then when $\sigma(i) = A$, $D_i^\sigma = E_S^\sigma$ and when $\sigma(i) = B$, $D_i^\sigma = -E_S^\sigma$.

6.1. *Examples of coalitional rules*

Coalitional versions of the rules in Section 4.2 can be considered. For example, S follows a Hippocratic rule if, for some nonnegative $\lambda \in \mathbb{R}^V_+$,

$$\Upsilon_S(x) = \lambda \cdot [x]_+. \tag{6.2}$$

Under this rule, the probability of S switching to σ^A_S depends on a weighted sum of payoff losses relative to when S switches to strategy σ^B_S. If $\lambda_i = 1$ for all $i \in S$ and $\lambda_j = 0$ for $j \notin S$, then we have a *coalitional logit* rule (Sawa, 2014), which can be understood as the rule that arises when each member of S votes for S to switch to σ^A_S or σ^B_S according to the (individualistic) logit choice rule based on payoffs at $(\sigma^A_S, \sigma_{V\setminus S})$ and $(\sigma^B_S, \sigma_{V\setminus S})$, with a switch being implemented only if the vote is unanimously in favor. This rule is self-regarding in the following sense.

Definition 9 A rule Υ_S is *self-regarding* if $\Upsilon_S(x) = f(x_S)$ for some non-decreasing function $f : \mathbb{R}^S \to \mathbb{R}_+$.

A class of rules that only makes sense in a non-individualistic setup is the class of *coalitional stochastic stability* rules (Newton, 2012), where the likelihood of strategic change by coalition S depends on the size of S. For example, if, for some constant $\kappa \in \mathbb{R}_{++}$, nonnegative $\lambda \in \mathbb{R}^V_+$,

$$\Upsilon_S(x) = \kappa\,|S| + [\lambda \cdot x]_+, \tag{6.3}$$

then we have an augmented utilitarian rule in which the larger a coalition is, the less likely it is to change its strategies.

6.2. *Asymmetry of coalitional rules*

When it comes to conditions for asymmetry, the differences between coalitional and individualistic payoff-difference based choice can be concisely explained. First, consider $i \in S$, $j \notin S$. Note that the strategy of player i affects the payoff of players i and j in exactly the same way as it would if player i were updating his strategy as an individual. This creates the need for risk dominance and altruistic risk dominance conditions similar to those of Proposition 1.

Definition 10 Strategy A is RD_{iT} (risk dominant for i against T) if

$$\sum_{j\in T\setminus\{i\}} u_{ij}(A,A)+u_{ij}(A,B)\geq \sum_{j\in T\setminus\{i\}} u_{ij}(B,A)+u_{ij}(B,B);$$

ARD_{Sj} (altruistically risk dominant for S against j) if

$$\sum_{i\in S\setminus\{j\}} u_{ji}(A,A)+u_{ji}(B,A)\geq \sum_{i\in S\setminus\{j\}} u_{ji}(A,B)+u_{ji}(B,B).$$

Our previous risk dominance condition summed over all $j \neq i$. Now, the relevant summation is over players outside of S, that is $T = V \setminus S$ in the definition above. ARD_{Sj} simply aggregates ARD_{ij} over all $i \in S$.

Second, note that there is an additional consideration present in the coalitional case, which is the payoff that players in S obtain from interacting with one another. When players in S all play A, interaction between $i, j \in S$ will generate payoff of $u_{ij}(A,A)$ for player i and $u_{ji}(A,A)$ for player j. When players in S all play B, these payoffs will be $u_{ij}(B,B)$ and $u_{ji}(B,B)$ respectively. Consequently, to ensure that within-coalition incentives to play A outweigh within-coalition incentives to play B, we require a payoff dominance condition.

Definition 11 Strategy A is PD_{iS} (payoff dominant for i against S) if

$$\sum_{j\in S\setminus\{i\}} u_{ij}(A,A)\geq \sum_{j\in S\setminus\{i\}} u_{ij}(B,B).$$

Combining the above arguments, we obtain the following proposition.

Proposition 4 *If S follows a coalitional payoff-difference based choice rule, A is $RD_{i(V\setminus S)}$ and PD_{iS} for all $i \in S$, and*

(i) Υ_S is self-regarding, or

(ii) A is ARD_{Sj} for all $j \notin S$,

then $c_S(\cdot,\cdot)$ is asymmetric.

Finally, note that the coalitional rules we have considered in this section involve coalition S comparing $(\sigma_S^A, \sigma_{V\setminus S})$ to $(\sigma_S^B, \sigma_{V\setminus S})$. Another possibility is that a coalition would compare an alternative profile to the status quo σ. This leads to difficulties similar to violations of supermodularity discussed in Section 5. Given the constraints of space, this is not pursued further here, although a detailed study of the intricacies of such rules would certainly be an interesting topic for further study.

7. DISCUSSION

7.1. *Payoff transformations*

Sometimes a transformation of payoffs can carry conceptual weight. In such cases, it can be instructive to consider the implications of the transformation with respect to conditions on the underlying payoffs. For example, we can subject the payoffs of player i to a *Homo Moralis* transformation (Alger and Weibull, 2013, 2016; Bergstrom, 1995),

$$u_{ij}^{HM}\big(\sigma(i),\sigma(j)\big) = (1-\sigma)\,u_{ij}\big(\sigma(i),\sigma(j)\big) + \sigma\,u_{ij}\big(\sigma(i),\sigma(i)\big), \tag{7.1}$$

where $\sigma \in [0,1]$ is a parameter that weighs the payoff maximizing first term against the *Kantian* second term.

Consider a player i who follows a self-regarding payoff-difference based choice rule according to the transformed payoffs. For this rule to be asymmetric, we require risk dominance of A under the transformed payoffs. Using payoffs u_{ij}^{HM} in the definition of RD_i and substituting, we obtain

$$\begin{aligned}(1-\sigma)\underbrace{\sum_{j\in V\setminus\{i\}} u_{ij}(A,A)+u_{ij}(A,B)-u_{ij}(B,A)-u_{ij}(B,B)}_{\geq 0 \text{ if and only if } A \text{ is } \mathrm{RD}_i} \\ +\,2\sigma\underbrace{\sum_{j\in V\setminus\{i\}} u_{ij}(A,A)-u_{ij}(B,B)}_{\geq 0 \text{ if and only if } A \text{ is } \mathrm{PD}_{iV}} \geq 0.\end{aligned} \tag{7.2}$$

If $\sigma = 0$, then (7.2) is the risk dominance condition of Proposition 1[i]. If $\sigma = 1$, then (7.2) is the component of the payoff dominance condition of Proposition 4[i] that relates to the incentives of player i under coalitional choice by the entire player set V. If both terms under the summations are greater than zero, then the condition holds regardless of the value of σ and so asymmetry will continue to hold even when σ changes (see Nax and Rigos, 2016; Newton, 2017; Wu, 2017).

It is similarly possible to subject the payoffs of player i to an altruistic transformation,

$$u_{ij}^{A}(\sigma(i),\sigma(j)) = (1-\alpha)\,u_{ij}(\sigma(i),\sigma(j)) + \alpha\,u_{ji}(\sigma(j),\sigma(i)),$$

where $\alpha \in [0,1]$ is a parameter that weights the payoff maximizing first term against the altruistic second term. This approach to altruistic choice is less flexible than the approach taken in Section 4. However, it is common, so it is worth noting that it can easily fit into our framework.

Again consider a player i who follows a self-regarding payoff-difference based choice rule according to the transformed payoffs. For this rule to be asymmetric, we require risk dominance of A under the transformed payoffs. Using payoffs u^A_{ij} in the definition of RD_i and substituting, we obtain a convex combination of risk dominance and altruistic risk dominance,

$$(7.3)\qquad (1-\alpha)\underbrace{\sum_{j\in V\setminus\{i\}} u_{ij}(A,A)+u_{ij}(A,B)-u_{ij}(B,A)-u_{ij}(B,B)}_{\geq 0 \text{ if and only if } A \text{ is } \mathrm{RD}_i} \\ +\alpha\sum_{j\in V\setminus\{i\}}\underbrace{u_{ji}(A,A)+u_{ji}(B,A)-u_{ji}(A,B)-u_{ji}(B,B)}_{\geq 0 \text{ if and only if } A \text{ is } \mathrm{ARD}_{ij}}\geq 0.$$

7.2. *The curse of the subscript*

A considerable number of subscripts and associated quantifiers have their origins in the arbitrary dependence of u_{ij} on both i and j. This helps to clarify cause and effect in our discussion of asymmetric rules, but comes at the cost of simple statements. Such simple statements can be obtained in the following manner. First, note that most of the prior literature considers the case of a coordination game that is played across pairs on some network of interactions. Even allowing for directed and weighted networks, this restricts each u_{ij} to the linear form

$$(7.4)\qquad u_{ij}\big(\sigma(i),\sigma(j)\big)=\lambda_{ij}\,u\big(\sigma(i),\sigma(j)\big),\qquad \lambda_{ij}\in\mathbb{R}_+.$$

When this is the case, all of our conditions for asymmetry in Sections 4-6 simplify and can be stated without player-specific subscripts.

$$(7.5)\qquad u(A,A)+u(A,B)\geq u(B,A)+u(B,B)$$

implies that A is RD_i for all i and RD_{iT} for all i,T.

$$(7.6)\qquad u(A,A)+u(B,A)\geq u(A,B)+u(B,B)$$

implies that A is ARD_{ij} for all i, j and ARD_{Sj} for all S, j.

$$u(A, A) \geq u(B, B) \tag{7.7}$$

implies that A is PD_{ij} for all i, j and PD_{iS} for all i, S.

$$u(A, B) \geq u(B, A) \tag{7.8}$$

implies that A is MM_{ij} for all i, j.

7.3. *Afterword*

In the first part of this paper (Section 3), we showed how choice rules can be combined whilst retaining asymmetry. We considered heterogeneity within agents' choice rules (Theorem 1), heterogeneity between agents' choice rules (Theorem 2), and heterogeneity in the timing of strategy updating (Theorem 3). In the second part of the paper (Sections 4-6), we discussed choice rules to which our theorems apply. Taken as a whole, this analysis vastly expands the set of choice rules under which we know that certain conventions are stochastically stable. In particular, many important results in the literature follow as corollaries.

It will be apparent to the reader that this by no means exhausts what can be said on this subject. Important avenues for future research would seem to include (i) the study of more choice rules; (ii) the study of different payoff specifications; (iii) applications to specific economic problems that admit heterogeneity in behavior.

APPENDIX A: PROOFS OF GENERAL RESULTS

Proof of Lemma 1: Keep in mind that, as $\varepsilon < 1$, $\log \varepsilon < 0$, and let $\frac{\log 0}{\log \varepsilon} := \infty$. Then, for all $\varepsilon > 0$,

$$\begin{aligned}
&\underbrace{\max_{S:\pi(S)>0} \frac{\log \pi(S)}{\log \varepsilon}}_{\to 0 \text{ as } \varepsilon \to 0} + \min_{S:\pi(S)>0} \underbrace{\frac{\log P_S^\varepsilon(\sigma, \sigma')}{\log \varepsilon}}_{\to c_S(\sigma,\sigma') \text{ as } \varepsilon \to 0} \\
&\geq \min_{S:\pi(S)>0} \frac{\log \pi(S) + \log P_S^\varepsilon(\sigma, \sigma')}{\log \varepsilon} = \min_{S:\pi(S)>0} \frac{\log \left(\pi(S) P_S^\varepsilon(\sigma, \sigma')\right)}{\log \varepsilon} \\
&= \frac{\log \left(\max_{S:\pi(S)>0} \pi(S) P_S^\varepsilon(\sigma, \sigma')\right)}{\log \varepsilon} \geq \frac{\log \left(\sum_{S:\pi(S)>0} \pi(S) P_S^\varepsilon(\sigma, \sigma')\right)}{\log \varepsilon} \\
&= \underbrace{\frac{\log P^\varepsilon(\sigma, \sigma')}{\log \varepsilon}}_{\to c(\sigma,\sigma') \text{ as } \varepsilon \to 0} = \frac{\log \left(\sum_{S:\pi(S)>0} \pi(S) P_S^\varepsilon(\sigma, \sigma')\right)}{\log \varepsilon}
\end{aligned} \tag{A.1}$$

$$\geq \frac{\log\left(2^{|V|}\max_{S:\pi(S)>0} P^{\varepsilon}_S(\sigma,\sigma')\right)}{\log\varepsilon} = \min_{S:\pi(S)>0}\frac{\log\left(2^{|V|}P^{\varepsilon}_S(\sigma,\sigma')\right)}{\log\varepsilon}$$
$$= \min_{S:\pi(S)>0}\frac{\log 2^{|V|} + \log P^{\varepsilon}_S(\sigma,\sigma')}{\log\varepsilon} = \underbrace{\frac{\log 2^{|V|}}{\log\varepsilon}}_{\to 0 \text{ as } \varepsilon\to 0} + \min_{S:\pi(S)>0}\underbrace{\frac{\log P^{\varepsilon}_S(\sigma,\sigma')}{\log\varepsilon}}_{\to c_S(\sigma,\sigma') \text{ as } \varepsilon\to 0}.$$

Taking limits of (A.1) as $\varepsilon \to 0$, we obtain

$$\min_{S:\pi(S)>0} c_S(\sigma,\sigma') \geq c(\sigma,\sigma') \geq \min_{S:\pi(S)>0} c_S(\sigma,\sigma'),$$

and therefore $c(\sigma,\sigma') = \min_{S:\pi(S)>0} c_S(\sigma,\sigma')$.

Q.E.D.

Proof of Lemma 2: Consider $\sigma,\sigma',\tilde\sigma \in \Sigma$, such that $V_B(\sigma) \subseteq V_A(\tilde\sigma)$.
As $c_1(\cdot,\cdot)$ is asymmetric, there exists $\bar\sigma \in \Sigma$ such that $V_A(\tilde\sigma) \subseteq V_A(\bar\sigma)$, $V_B(\sigma') \subseteq V_A(\bar\sigma)$ and

(A.2) $$c_1(\sigma,\sigma') \geq c_1(\tilde\sigma,\bar\sigma).$$

As $c_2(\cdot,\cdot)$ is asymmetric, there exists $\bar{\bar\sigma} \in \Sigma$ such that $V_A(\tilde\sigma) \subseteq V_A(\bar{\bar\sigma})$, $V_B(\sigma') \subseteq V_A(\bar{\bar\sigma})$ and

(A.3) $$c_2(\sigma,\sigma') \geq c_2(\tilde\sigma,\bar{\bar\sigma}).$$

Consequently, we have that

$$\begin{aligned} c(\sigma,\sigma') &\underbrace{=}_{\text{by defn of } c} \min\{c_1(\sigma,\sigma'), c_2(\sigma,\sigma')\} \\ &\underbrace{\geq}_{\text{by (A.2) and (A.3)}} \min\{c_1(\tilde\sigma,\bar\sigma), c_2(\tilde\sigma,\bar{\bar\sigma})\} \\ &\geq \min\left\{\min\{c_1(\tilde\sigma,\bar\sigma), c_2(\tilde\sigma,\bar\sigma)\}, \min\{c_1(\tilde\sigma,\bar{\bar\sigma}), c_2(\tilde\sigma,\bar{\bar\sigma})\}\right\} \\ &\underbrace{=}_{\text{by defn of } c} \min\left\{c(\tilde\sigma,\bar\sigma), c(\tilde\sigma,\bar{\bar\sigma})\right\}, \end{aligned}$$

so $c(\sigma,\sigma') \geq c(\tilde\sigma,\bar\sigma)$ or $c(\sigma,\sigma') \geq c(\tilde\sigma,\bar{\bar\sigma})$, and the condition for c to be asymmetric is satisfied.

Q.E.D.

Proof of Theorem 1: Keep in mind that, as $\varepsilon < 1$, $\log\varepsilon < 0$, and let $\frac{\log 0}{\log\varepsilon} := \infty$. Then, for all $\varepsilon > 0$,

(A.4) $$\min\left\{\underbrace{\frac{\log\lambda}{\log\varepsilon}}_{\to 0 \text{ as } \varepsilon\to 0} + \underbrace{\frac{\log\tilde P^{\varepsilon}_S(\sigma,\sigma')}{\log\varepsilon}}_{\to \tilde c_S(\sigma,\sigma') \text{ as } \varepsilon\to 0}, \underbrace{\frac{\log(1-\lambda)}{\log\varepsilon}}_{\to 0 \text{ as } \varepsilon\to 0} + \underbrace{\frac{\log\bar P^{\varepsilon}_S(\sigma,\sigma')}{\log\varepsilon}}_{\to \bar c_S(\sigma,\sigma') \text{ as } \varepsilon\to 0}\right\}$$
$$= \min\left\{\frac{\log\left(\lambda\,\tilde P^{\varepsilon}_S(\sigma,\sigma')\right)}{\log\varepsilon}, \frac{\log\left((1-\lambda)\,\bar P^{\varepsilon}_S(\sigma,\sigma')\right)}{\log\varepsilon}\right\}$$
$$= \frac{\log\left(\max\left\{\lambda\,\tilde P^{\varepsilon}_S(\sigma,\sigma'), (1-\lambda)\,\bar P^{\varepsilon}_S(\sigma,\sigma')\right\}\right)}{\log\varepsilon}$$

$$\geq \frac{\log\left(\lambda\,\tilde{P}^{\varepsilon}_S(\sigma,\sigma') + (1-\lambda)\,\bar{P}^{\varepsilon}_S(\sigma,\sigma')\right)}{\log\varepsilon}$$

$$= \underbrace{\frac{\log P^{\varepsilon}_S(\sigma,\sigma')}{\log\varepsilon}}_{\to c(\sigma,\sigma')\text{ as }\varepsilon\to 0} = \frac{\log\left(\lambda\,\tilde{P}^{\varepsilon}_S(\sigma,\sigma') + (1-\lambda)\,\bar{P}^{\varepsilon}_S(\sigma,\sigma')\right)}{\log\varepsilon}$$

$$\geq \frac{\log\left(2\max\left\{\lambda\,\tilde{P}^{\varepsilon}_S(\sigma,\sigma'),(1-\lambda)\,\bar{P}^{\varepsilon}_S(\sigma,\sigma')\right\}\right)}{\log\varepsilon}$$

$$= \min\left\{\frac{\log\left(2\,\lambda\,\tilde{P}^{\varepsilon}_S(\sigma,\sigma')\right)}{\log\varepsilon}, \frac{\log\left(2\,(1-\lambda)\,\bar{P}^{\varepsilon}_S(\sigma,\sigma')\right)}{\log\varepsilon}\right\}$$

$$= \min\left\{\underbrace{\frac{\log(2\,\lambda)}{\log\varepsilon}}_{\to 0\text{ as }\varepsilon\to 0} + \underbrace{\frac{\log\tilde{P}^{\varepsilon}_S(\sigma,\sigma')}{\log\varepsilon}}_{\to\tilde{c}_S(\sigma,\sigma')\text{ as }\varepsilon\to 0}, \underbrace{\frac{\log(2\,(1-\lambda))}{\log\varepsilon}}_{\to 0\text{ as }\varepsilon\to 0} + \underbrace{\frac{\log\bar{P}^{\varepsilon}_S(\sigma,\sigma')}{\log\varepsilon}}_{\to\bar{c}_S(\sigma,\sigma')\text{ as }\varepsilon\to 0}\right\}.$$

Taking limits of (A.4) as $\varepsilon \to 0$, we obtain

$$\min\{\tilde{c}(\sigma,\sigma'),\bar{c}(\sigma,\sigma')\} \geq c(\sigma,\sigma') \geq \min\{\tilde{c}(\sigma,\sigma'),\bar{c}(\sigma,\sigma')\},$$

and therefore $c(\sigma,\sigma') = \min\{\tilde{c}(\sigma,\sigma'),\bar{c}(\sigma,\sigma')\}$.

As $\tilde{P}$ and $\bar{P}$ are asymmetric, $\tilde{c}$ and $\bar{c}$ are asymmetric.

Lemma 2 then implies that c is asymmetric, therefore P is asymmetric. *Q.E.D.*

Proof of Theorem 2: By assumption, for all S such that $\pi(S) > 0$, P_S is asymmetric, so c_S is asymmetric.

By Lemma 1, $c = \min_{S:\pi(S)>0} c_S$. We shall show that c is asymmetric, therefore P is asymmetric.

Let $\{S : \pi(S) > 0\} = \{S_1, S_2, \ldots, S_n\}$ and define cost functions $\hat{c}_1 = c_{S_1}$, $\hat{c}_m := \min\{\hat{c}_{m-1}, c_{S_m}\} = \min\{c_{S_1},\ldots,c_{S_m}\}$ for $m = 2,\ldots,n$. In particular,

$$\hat{c}_n \underbrace{=}_{\substack{\text{by defn}\\ \text{of } c_n}} \min\{c_{S_1},\ldots,c_{S_n}\} \underbrace{=}_{\substack{\text{by defn}\\ \text{of }\{S_1,\ldots,S_n\}}} \min_{S:\pi(S)>0} c_S \underbrace{=}_{\text{by Lemma 1}} c,$$

We complete the proof by showing, by induction, that $\hat{c}_m$ is asymmetric for $m = 2,\ldots,n$. By assumption, $\hat{c}_1 = c_{S_1}$ is asymmetric. Assume $\hat{c}_{m-1}$ is asymmetric for some $m \leq n$. Then $\hat{c}_m = \min\{\hat{c}_{m-1}, c_{S_m}\}$ is asymmetric by Lemma 2. *Q.E.D.*

Proof of Theorem 3: Consider $\sigma, \sigma', \tilde{\sigma} \in \Sigma$, such that $V_B(\sigma) \subseteq V_A(\tilde{\sigma})$. Let $c_{S\cup T}$ be the cost function of $P_{S\cup T}$.

If $c_{S\cup T}(\sigma,\sigma') = \infty$, we are done as then $c_{S\cup T}(\sigma,\sigma') \geq c_{S\cup T}(\tilde{\sigma},\bar{\sigma})$ for any $\bar{\sigma}$.

If $c_{S\cup T}(\sigma,\sigma') < \infty$, then $\sigma' = (\sigma'_S, \sigma'_T, \sigma_{V\setminus(S\cup T)})$.

As cost functions are defined using logs of transition probabilities, it follows from the definition of $P_{S\cup T}$ that

(A.5) $$c_{S\cup T}(\sigma,\sigma') = c_S(\sigma,(\sigma'_S,\sigma_{V\setminus S})) + c_T(\sigma,(\sigma'_T,\sigma_{V\setminus T})).$$

As (A.5) is finite, each of its terms are finite, so

$$c_S(\sigma, (\sigma'_S, \sigma_{V\setminus S})) < \infty, \qquad c_T(\sigma, (\sigma'_T, \sigma_{V\setminus T})) < \infty. \tag{A.6}$$

By asymmetry of c_S, there exists $\bar{\bar{\sigma}}$ such that $V_A(\tilde{\sigma}) \subseteq V_A(\bar{\bar{\sigma}})$, $V_B((\sigma'_S, \sigma_{V\setminus S})) \subseteq V_A(\bar{\bar{\sigma}})$, and

$$c_S(\sigma, (\sigma'_S, \sigma_{V\setminus S})) \geq c_S(\tilde{\sigma}, \bar{\bar{\sigma}}) \tag{A.7}$$

Inequalities (A.6) and (A.7) imply that $c_S(\tilde{\sigma}, \bar{\bar{\sigma}}) < \infty$, which implies that $\bar{\bar{\sigma}} = (\bar{\sigma}_S, \tilde{\sigma}_{V\setminus S})$ for some $\bar{\sigma}_S$. Therefore,

$$c_S(\sigma, (\sigma'_S, \sigma_{V\setminus S})) \geq c_S(\tilde{\sigma}, (\bar{\sigma}_S, \tilde{\sigma}_{V\setminus S})). \tag{A.8}$$

Similarly, by asymmetry of c_T, we obtain $(\bar{\sigma}_T, \tilde{\sigma}_{V\setminus T})$ such that $V_A(\tilde{\sigma}) \subseteq V_A((\bar{\sigma}_T, \tilde{\sigma}_{V\setminus T}))$, $V_B((\sigma'_T, \sigma_{V\setminus T})) \subseteq V_A((\bar{\sigma}_T, \tilde{\sigma}_{V\setminus T}))$, and

$$c_T(\sigma, (\sigma'_T, \sigma_{V\setminus T})) \geq c_T(\tilde{\sigma}, (\bar{\sigma}_T, \tilde{\sigma}_{V\setminus T})). \tag{A.9}$$

Let $\bar{\sigma} = (\bar{\sigma}_S, \bar{\sigma}_T, \tilde{\sigma}_{V\setminus(S\cup T)})$.

As $V_B((\sigma'_S, \sigma_{V\setminus S})) \subseteq V_A(\bar{\bar{\sigma}}) = V_A((\bar{\sigma}_S, \tilde{\sigma}_{V\setminus S}))$ and $V_B((\sigma'_T, \sigma_{V\setminus T})) \subseteq V_A((\bar{\sigma}_T, \tilde{\sigma}_{V\setminus T}))$, it must be that $V_B(\sigma') = V_B((\sigma'_S, \sigma'_T, \sigma_{V\setminus(S\cup T)})) \subseteq V_A((\bar{\sigma}_S, \bar{\sigma}_T, \tilde{\sigma}_{V\setminus(S\cup T)})) = V_A(\bar{\sigma})$.

Similarly, as $V_A(\tilde{\sigma}) \subseteq V_A(\bar{\bar{\sigma}}) = V_A((\bar{\sigma}_S, \tilde{\sigma}_{V\setminus S}))$ and $V_A(\tilde{\sigma}) \subseteq V_A((\bar{\sigma}_T, \tilde{\sigma}_{V\setminus T}))$, it must be that $V_A(\tilde{\sigma}) \subseteq V_A((\bar{\sigma}_S, \bar{\sigma}_T, \tilde{\sigma}_{V\setminus(S\cup T)})) = V_A(\bar{\sigma})$.

Finally,

$$\begin{aligned} c_{S\cup T}(\sigma, \sigma') &= c_S(\sigma, (\sigma'_S, \sigma_{V\setminus S})) + c_T(\sigma, (\sigma'_T, \sigma_{V\setminus T})) \\ &\underbrace{\geq}_{\text{by (A.8) and (A.9)}} c_S(\tilde{\sigma}, (\bar{\sigma}_S, \tilde{\sigma}_{V\setminus S})) + c_T(\tilde{\sigma}, (\bar{\sigma}_T, \tilde{\sigma}_{V\setminus T})) = c_{S\cup T}(\tilde{\sigma}, \bar{\sigma}), \end{aligned}$$

therefore, $c_{S\cup T}$ is asymmetric. *Q.E.D.*

Proof of Lemma 3: Consider $\sigma, \sigma', \tilde{\sigma} \in \Sigma$, such that $V_B(\sigma) \subseteq V_A(\tilde{\sigma})$.

If $c_{\{i\}}(\sigma, \sigma') = \infty$, then letting $\bar{\sigma} = \sigma^A$, we have $V_A(\tilde{\sigma}) \subseteq V_A(\bar{\sigma})$, $V_B(\sigma') \subseteq V_A(\bar{\sigma})$, and $c_{\{i\}}(\sigma, \sigma') \geq c_{\{i\}}(\tilde{\sigma}, \bar{\sigma})$. The condition for asymmetry is satisfied.

If $c_{\{i\}}(\sigma, \sigma') < \infty$, then either $\sigma' = \sigma$ or $\sigma' = \sigma^{(i)}$ for some $i \in V$.

If $\sigma' = \sigma$ or $\sigma' = \sigma^{(i)}$ for $i \in V_A(\tilde{\sigma})$, then let $\bar{\sigma} = \tilde{\sigma}$. We have $V_A(\tilde{\sigma}) \subseteq V_A(\bar{\sigma})$, $V_B(\sigma') \subseteq V_A(\bar{\sigma})$, and $c_{\{i\}}(\sigma, \sigma') \geq 0 = c_{\{i\}}(\tilde{\sigma}, \bar{\sigma})$. The condition for asymmetry is satisfied.

Noting that $i \in V_B(\sigma)$ implies that $i \in V_A(\tilde{\sigma})$ (so the preceding case would apply), we have one remaining case, $\sigma' = \sigma^{(i)}$ for $i \in V_A(\sigma)$, $i \in V_B(\tilde{\sigma})$. This implies that $i \in V_B(\sigma')$, so if we are to have $V_B(\sigma') \subseteq V_A(\bar{\sigma})$, it must be the case that $i \in V_A(\bar{\sigma})$. However, the only $\bar{\sigma}$ that could possibly satisfy both this condition and

$c_{\{i\}}(\tilde{\sigma}, \bar{\sigma}) < \infty$ is $\bar{\sigma} = \tilde{\sigma}^{(i)}$. Therefore, if $c_{\{i\}}(\sigma, \sigma^{(i)}) \geq c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)})$, then the condition for asymmetry is satisfied, and if $c_{\{i\}}(\sigma, \sigma^{(i)}) < c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)})$, then the condition for asymmetry does not hold. *Q.E.D.*

Proof of Lemma 4: Consider $\sigma, \sigma', \tilde{\sigma}$ such that $V_B(\sigma) \subseteq V_A(\tilde{\sigma})$, $i \in V_A(\sigma)$, $i \in V_B(\tilde{\sigma})$.

Let $\hat{\sigma}$ be such that $V_B(\sigma) = V_A(\hat{\sigma})$. Then, as $c_{\{i\}}$ is weakly asymmetric, by Definition 4, we have that $c_{\{i\}}(\sigma, \sigma^{(i)}) \geq c_{\{i\}}(\hat{\sigma}, \hat{\sigma}^{(i)})$.

Note that $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$. Then, as $c_{\{i\}}$ is supermodular, by Definition 5, we have that $c_{\{i\}}(\hat{\sigma}, \hat{\sigma}^{(i)}) \geq c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)})$.

Combining the inequalities above, we have $c_{\{i\}}(\sigma, \sigma^{(i)}) \geq c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)})$, satisfying the condition for asymmetry of Lemma 3.

Q.E.D.

APPENDIX B: PROOFS FOR PAYOFF-DIFFERENCE BASED CHOICE

Proof of Lemma 5: Consider the elements of D_i^{σ},

$$(D_i^{\sigma})_j = U_j(\sigma) - U_j(\sigma^{(i)}) = \begin{cases} u_{ji}(A,A) - u_{ji}(A,B) & \text{if } j \neq i,\ \sigma(j) = A, \\ -\left(u_{ji}(B,B) - u_{ji}(B,A)\right) & \text{if } j \neq i,\ \sigma(j) = B, \\ \sum_{k \in V_A(\sigma)\setminus\{i\}} \left(u_{ik}(A,A) - u_{ik}(B,A)\right) & \\ \quad - \sum_{k \in V_B(\sigma)\setminus\{i\}} \left(u_{ik}(B,B) - u_{ik}(A,B)\right) & \text{if } j = i, \end{cases} \tag{B.1}$$

and the elements of $D_i^{\hat{\sigma}}$,

$$(D_i^{\hat{\sigma}})_j = U_j(\hat{\sigma}) - U_j(\hat{\sigma}^{(i)}) = \begin{cases} -\left(u_{ji}(A,A) - u_{ji}(A,B)\right) & \text{if } j \neq i,\ \hat{\sigma}(j) = A, \\ u_{ji}(B,B) - u_{ji}(B,A) & \text{if } j \neq i,\ \hat{\sigma}(j) = B, \\ - \sum_{k \in V_A(\hat{\sigma})\setminus\{i\}} \left(u_{ik}(A,A) - u_{ik}(B,A)\right) & \\ \quad + \sum_{k \in V_B(\hat{\sigma})\setminus\{i\}} \left(u_{ik}(B,B) - u_{ik}(A,B)\right) & \text{if } j = i. \end{cases} \tag{B.2}$$

Noting that $\sigma(j) = A$ if and only if $\hat{\sigma}(j) = B$, $\sigma(j) = B$ if and only if $\hat{\sigma}(j) = A$, and that, consequently, $V_A(\hat{\sigma}) = V_B(\sigma)$ and $V_B(\hat{\sigma}) = V_A(\sigma)$, we can subtract (B.2) from (B.1) to get

$$(D_i^{\sigma} - D_i^{\hat{\sigma}})_j = \begin{cases} \left(u_{ji}(A,A) - u_{ji}(A,B)\right) - \left(u_{ji}(B,B) - u_{ji}(B,A)\right) & \text{if } j \neq i,\ \sigma(j) = A, \\ \left(u_{ji}(A,A) - u_{ji}(A,B)\right) - \left(u_{ji}(B,B) - u_{ji}(B,A)\right) & \text{if } j \neq i,\ \sigma(j) = B, \\ \sum_{k \in V\setminus\{i\}} \left(\left(u_{ik}(A,A) - u_{ik}(B,A)\right) - \left(u_{ik}(B,B) - u_{ik}(A,B)\right)\right) & \text{if } j = i. \end{cases} \tag{B.3}$$

If A is RD_i, then, from the third case of (B.3), we have that $(D_i^{\sigma} - D_i^{\hat{\sigma}})_i \geq 0$, so $(D_i^{\sigma})_i \geq (D_i^{\hat{\sigma}})_i$.

If Υ_i is self-regarding, then $(D_i^{\sigma})_i \geq (D_i^{\hat{\sigma}})_i$ implies that $\Upsilon_i(D_i^{\sigma}) \geq \Upsilon_i(D_i^{\hat{\sigma}})$ and therefore, by (4.2), $c_{\{i\}}(\sigma, \sigma^{(i)}) \geq c_{\{i\}}(\hat{\sigma}, \hat{\sigma})$. That is, $c_{\{i\}}(\cdot, \cdot)$ is weakly asymmetric, proving Lemma 5[i].

If A is ARD_{ij} for all j, then, from the first and second cases of (B.3), we have that $(D_i^{\sigma} - D_i^{\hat{\sigma}})_j \geq 0$ and $(D_i^{\sigma})_j \geq (D_i^{\hat{\sigma}})_j$ for all $j \neq i$. Therefore $D_i^{\sigma} \geq D_i^{\hat{\sigma}}$, and as Υ_i is non-decreasing, $\Upsilon_i(D_i^{\sigma}) \geq \Upsilon_i(D_i^{\hat{\sigma}})$ and therefore, by (4.2), $c_{\{i\}}(\sigma, \sigma^{(i)}) \geq c_{\{i\}}(\hat{\sigma}, \hat{\sigma})$. That is, $c_{\{i\}}(\cdot, \cdot)$ is weakly asymmetric, proving Lemma 5[ii].

Q.E.D.

PROOF OF LEMMA 6: Using (B.2) for both $D_i^{\hat{\sigma}}$ and $D_i^{\tilde{\sigma}}$ gives

$$(B.4)\qquad (D_i^{\hat{\sigma}} - D_i^{\tilde{\sigma}})_j$$
$$= \begin{cases} \Big(u_{ji}(A,A) - u_{ji}(A,B)\Big) - \Big(u_{ji}(A,A) - u_{ji}(A,B)\Big) = 0 & \text{if } j \neq i,\ \hat{\sigma}(j) = A, \\ \Big(u_{ji}(B,B) - u_{ji}(B,A)\Big) - \Big(u_{ji}(B,B) - u_{ji}(B,A)\Big) = 0 & \text{if } j \neq i,\ \tilde{\sigma}(j) = B, \\ \Big(u_{ji}(A,A) - u_{ji}(A,B)\Big) + \Big(u_{ji}(B,B) - u_{ji}(B,A)\Big) & \text{if } j \neq i,\ \tilde{\sigma}(j) = A,\ \hat{\sigma}(j) = B, \\ \sum_{k \in V_A(\tilde{\sigma}) \setminus \{i\}} \Big(u_{ik}(A,A) - u_{ik}(B,A)\Big) & \\ \quad - \sum_{k \in V_A(\hat{\sigma}) \setminus \{i\}} \Big(u_{ik}(A,A) - u_{ik}(B,A)\Big) & \\ \quad + \sum_{k \in V_B(\hat{\sigma}) \setminus \{i\}} \Big(u_{ik}(B,B) - u_{ik}(A,B)\Big) & \\ \quad - \sum_{k \in V_B(\tilde{\sigma}) \setminus \{i\}} (\tilde{\sigma}) \Big(u_{ik}(B,B) - u_{ik}(A,B)\Big) & \text{if } j = i. \end{cases}$$

The third case of of (B.4) is nonnegative by (2.2). The sum of the first two lines of the fourth case is nonnegative by $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$ and (2.2). The sum of the final two lines of the fourth case is nonnegative by a similar argument. So every element $(D_i^{\hat{\sigma}} - D_i^{\tilde{\sigma}})_j$ is nonnegative, $D_i^{\hat{\sigma}} \geq D_i^{\tilde{\sigma}}$. As Υ_i is non-decreasing, $\Upsilon_i(D_i^{\hat{\sigma}}) \geq \Upsilon_i(D_i^{\tilde{\sigma}})$ and therefore, by (4.2), $c_{\{i\}}(\hat{\sigma}, \hat{\sigma}) \geq c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma})$. That is, $c_{\{i\}}(\cdot, \cdot)$ is supermodular. *Q.E.D.*

PROOF OF PROPOSITION 1: By Lemmas 5 and 6, $c_{\{i\}}(\cdot, \cdot)$ is weakly asymmetric and supermodular, so by Lemma 4, $c_{\{i\}}(\cdot, \cdot)$ is asymmetric. *Q.E.D.*

PROOF OF COROLLARY 1: As A is RD_i for all $i \in V$ and all $i \in V$ follow self-regarding payoff-difference based rules, Proposition 1[i] implies that $c_{\{i\}}$ is asymmetric for all $i \in V$. As, by assumption, $\pi(S) > 0$ if and only if $S = \{i\}$ for $i \in V$, Theorem 2 then implies that $c = \min_{S:\pi(S)>0} c_S$ is asymmetric. By Theorem P, σ^A is stochastically stable. *Q.E.D.*

APPENDIX C: PROOFS FOR IMITATIVE CHOICE

For readability, in this section we write $U(\sigma) := (U_j(\sigma))_{j \in V}$.

PROOF OF LEMMA 7: Let $\sigma, \hat{\sigma}$ be such that $V_A(\sigma) = V_B(\hat{\sigma})$, $\sigma(i) = A$. Note that

$$(C.1)\qquad V_{\sigma(i)}(\sigma) = V_A(\sigma) = V_B(\hat{\sigma}) = V_{\hat{\sigma}(i)}(\hat{\sigma}).$$

Consider the elements of $U(\sigma)$,

$$(C.2)\qquad U_j(\sigma) = \begin{cases} \sum_{k \in V_A(\sigma) \setminus \{j\}} u_{jk}(A,A) + \sum_{k \in V_B(\sigma) \setminus \{j\}} u_{jk}(A,B) & \text{if } \sigma(j) = A, \\ \sum_{k \in V_A(\sigma) \setminus \{j\}} u_{jk}(B,A) + \sum_{k \in V_B(\sigma) \setminus \{j\}} u_{jk}(B,B) & \text{if } \sigma(j) = B, \end{cases}$$

and the elements of $U(\hat{\sigma})$,

$$(C.3) \qquad U_j(\hat{\sigma}) = \begin{cases} \sum_{k\in V_A(\hat{\sigma})\setminus\{j\}} u_{jk}(A,A) + \sum_{k\in V_B(\hat{\sigma})\setminus\{j\}} u_{jk}(A,B) \\ = \sum_{k\in V_B(\sigma)\setminus\{j\}} u_{jk}(A,A) + \sum_{k\in V_A(\sigma)\setminus\{j\}} u_{jk}(A,B) & \text{if } \hat{\sigma}(j) = A, \\ \sum_{k\in V_A(\hat{\sigma})\setminus\{j\}} u_{jk}(B,A) + \sum_{k\in V_B(\hat{\sigma})\setminus\{j\}} u_{jk}(B,B) \\ = \sum_{k\in V_B(\sigma)\setminus\{j\}} u_{jk}(B,A) + \sum_{k\in V_A(\sigma)\setminus\{j\}} u_{jk}(B,B) & \text{if } \hat{\sigma}(j) = B. \end{cases}$$

By (C.1), if $\sigma(j) = A$, then $\hat{\sigma}(j) = B$, and if $\sigma(j) = B$, then $\hat{\sigma}(j) = A$. Consequently, (C.2) and (C.3), together with PD_{jk} ($u_{jk}(A,A) \geq u_{jk}(B,B)$) and MM_{jk} ($u_{jk}(A,B) \geq u_{jk}(B,A)$) imply

$$(C.4) \qquad \begin{aligned} &\text{For all } j \in V_A(\sigma), \quad U_j(\sigma) \geq U_j(\hat{\sigma}), \\ &\text{For all } j \in V_B(\sigma), \quad U_j(\sigma) \leq U_j(\hat{\sigma}). \end{aligned}$$

Then

$$(C.5) \qquad \begin{aligned} \Delta_i^{\sigma} &= h^C(V_{\sigma(i)}(\sigma) \cap C, U(\sigma)) && \text{[by defn of } \Delta_i^{\sigma}\text{]} \\ &= h^C(V_A(\sigma) \cap C, U(\sigma)) && \text{[by (C.1)]} \\ &\geq h^C(V_A(\sigma) \cap C, U(\hat{\sigma})) && \text{[by (C.4) and defn of } h^C\text{]} \\ &= h^C(V_B(\hat{\sigma}) \cap C, U(\hat{\sigma})) && \text{[by (C.1)]} \\ &= h^C(V_{\hat{\sigma}(i)}(\hat{\sigma}) \cap C, U(\hat{\sigma})) && \text{[by (C.1)]} \\ &= \Delta_i^{\hat{\sigma}}. && \text{[by defn of } \Delta_i^{\hat{\sigma}}\text{]} \end{aligned}$$

As Υ_i^{Im} is non-decreasing, (C.5) implies that $\Upsilon_i^{Im}(\Delta_i^{\sigma}) \geq \Upsilon_i^{Im}(\Delta_i^{\hat{\sigma}})$ and therefore, by (4.2), $c_{\{i\}}(\sigma, \sigma^{(i)}) \geq c_{\{i\}}(\hat{\sigma}, \hat{\sigma}^{(i)})$. That is, $c_{\{i\}}(\cdot,\cdot)$ is weakly asymmetric, proving Lemma 7.

Q.E.D.

Proof of Lemma 8: Let $\hat{\sigma}$, $\tilde{\sigma}$ be such that $\hat{\sigma}(i) = \tilde{\sigma}(i) = B$, $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$. From (C.2), if $u_{ik}(B,B) \geq u_{ik}(B,A)$ for all $k \neq i$, then $U_i(\hat{\sigma}) \geq U_i(\tilde{\sigma})$. Then,

$$(C.6) \qquad \begin{aligned} \Delta_i^{\hat{\sigma}} &= h^C(V_{\hat{\sigma}(i)}(\hat{\sigma}) \cap C, U(\hat{\sigma})) && \text{[by defn of } \Delta_i^{\hat{\sigma}}\text{]} \\ &= h^C(\{i\}, U(\hat{\sigma})) && \text{[by condition dependence, } C = \{i\}\text{]} \\ &\geq h^C(\{i\}, U(\tilde{\sigma})) && \text{[by } U_i(\hat{\sigma}) \geq U_i(\tilde{\sigma}) \text{ and defn of } h^C\text{]} \\ &= h^C(V_{\tilde{\sigma}(i)}(\tilde{\sigma}) \cap C, U(\tilde{\sigma})) && \text{[by condition dependence, } C = \{i\}\text{]} \\ &= \Delta_i^{\tilde{\sigma}}. && \text{[by defn of } \Delta_i^{\tilde{\sigma}}\text{]} \end{aligned}$$

As Υ_i^{Im} is non-decreasing, (C.6) implies that $\Upsilon_i^{Im}(\Delta_i^{\hat{\sigma}}) \geq \Upsilon_i^{Im}(\Delta_i^{\tilde{\sigma}})$ and therefore, by (4.2), $c_{\{i\}}(\hat{\sigma}, \hat{\sigma}^{(i)}) \geq c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)})$. That is, $c_{\{i\}}(\cdot,\cdot)$ is supermodular, proving Lemma 8. *Q.E.D.*

Proof of Proposition 2: By definition of condition dependence, the process is independent of the payoffs of players other than i, therefore PD_{ik} and MM_{ik} for all $k \neq i$ suffices for Lemma 7 to imply that $c_{\{i\}}(\cdot,\cdot)$ is weakly asymmetric. Furthermore, PD_{ik} and MM_{ik} for all $k \neq i$, together with (2.2) implies the payoff ordering

$u_{ik}(A,A) \geq u_{ik}(B,B) \geq u_{ik}(A,B) \geq u_{ik}(B,A)$ for all $k \neq i$. In particular, $u_{ik}(B,B) \geq u_{ik}(B,A)$. Therefore, by Lemma 8, $c_{\{i\}}(\cdot,\cdot)$ is supermodular. Consequently, by Lemma 4, $c_{\{i\}}(\cdot,\cdot)$ is asymmetric. *Q.E.D.*

Proof of Lemma 9: Let $\hat{\sigma}, \tilde{\sigma}$ be such that $\hat{\sigma}(i) = \tilde{\sigma}(i) = B$, $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$. Together with (C.2), $u_{jk}(A,A) \geq u_{jk}(A,B)$, $u_{jk}(B,B) \geq u_{jk}(B,A)$ for all j,k, this implies the following inequalities.

$$\text{(C.7)} \qquad \begin{aligned} &\text{For all } j \in V_A(\hat{\sigma}), \quad U_j(\hat{\sigma}) \leq U_j(\tilde{\sigma}),\\ &\text{For all } j \in V_B(\tilde{\sigma}), \quad U_j(\hat{\sigma}) \geq U_j(\tilde{\sigma}). \end{aligned}$$

Note that, as $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$ and $V_B(\tilde{\sigma}) \subseteq V_B(\hat{\sigma})$, (C.7) only relates to j for whom $\hat{\sigma}(j) = \tilde{\sigma}(j)$. Then,

$$\text{(C.8)} \qquad \begin{aligned}
\Delta_i^{\hat{\sigma}} &= h^C(V_{\hat{\sigma}(i)}(\hat{\sigma}) \cap C, U(\hat{\sigma})) && [\text{by defn of } \Delta_i^{\hat{\sigma}}]\\
&= h^C(V_B(\hat{\sigma}) \cap C, U(\hat{\sigma})) && [\text{as } \hat{\sigma}(i) = B]\\
&= f(M^{V_B(\hat{\sigma})\cap C}(\hat{\sigma}), M^{V_A(\hat{\sigma})\cap C}(\hat{\sigma})) && [\text{by defn of } h^C \text{ under imitate-the-best}]\\
&\geq f(M^{V_B(\tilde{\sigma})\cap C}(\hat{\sigma}), M^{V_A(\hat{\sigma})\cap C}(\hat{\sigma})) && [\text{as } V_B(\tilde{\sigma}) \subseteq V_B(\hat{\sigma}) \text{ and } f \text{ non-decreasing in first argument}]\\
&\geq f(M^{V_B(\tilde{\sigma})\cap C}(\hat{\sigma}), M^{V_A(\hat{\sigma})\cap C}(\tilde{\sigma})) && [\text{by (C.7) and } f \text{ non-increasing in second argument}]\\
&\geq f(M^{V_B(\tilde{\sigma})\cap C}(\hat{\sigma}), M^{V_A(\tilde{\sigma})\cap C}(\tilde{\sigma})) && [\text{as } V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma}) \text{ and } f \text{ non-increasing in second argument}]\\
&\geq f(M^{V_B(\tilde{\sigma})\cap C}(\tilde{\sigma}), M^{V_A(\tilde{\sigma})\cap C}(\tilde{\sigma})) && [\text{by (C.7) and } f \text{ non-decreasing in first argument}]\\
&= h^C(V_B(\tilde{\sigma}) \cap C, U(\tilde{\sigma})) && [\text{by defn of } h^C \text{ under imitate-the-best}]\\
&= h^C(V_{\tilde{\sigma}(i)}(\tilde{\sigma}) \cap C, U(\tilde{\sigma})) && [\text{as } \tilde{\sigma}(i) = B]\\
&= \Delta_i^{\tilde{\sigma}}. && [\text{by defn of } \Delta_i^{\tilde{\sigma}}]
\end{aligned}$$

As Υ_i^{Im} is non-decreasing, (C.8) implies that $\Upsilon_i^{Im}(\Delta_i^{\hat{\sigma}}) \geq \Upsilon_i^{Im}(\Delta_i^{\tilde{\sigma}})$ and therefore, by (4.2), $c_{\{i\}}(\hat{\sigma}, \hat{\sigma}^{(i)}) \geq c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)})$. That is, $c_{\{i\}}(\cdot,\cdot)$ is supermodular, proving Lemma 9. *Q.E.D.*

Proof of Proposition 3: PD_{jk} and MM_{jk} for all j,k, together with (2.2) implies the payoff ordering $u_{jk}(A,A) \geq u_{jk}(B,B) \geq u_{jk}(A,B) \geq u_{jk}(B,A)$ for all j,k. In particular, $u_{jk}(A,A) \geq u_{jk}(A,B)$ and $u_{jk}(B,B) \geq u_{jk}(B,A)$. Therefore, by Lemmas 7 and 9, $c_{\{i\}}(\cdot,\cdot)$ is weakly asymmetric and supermodular, so by Lemma 4, $c_{\{i\}}(\cdot,\cdot)$ is asymmetric. *Q.E.D.*

APPENDIX D: PROOFS FOR COALITIONAL CHOICE

Lemma 10 *Let $\sigma, \hat{\sigma}$ be such that $V_B(\sigma) = V_A(\hat{\sigma})$. If S follows a coalitional payoff-difference based choice rule, A is $RD_{i(V\setminus S)}$ and PD_{iS} for all $i \in S$, and*

(i) Υ_S is self-regarding, or

(ii) A is ARD_{Sj} for all $j \notin S$,

then $c_S(\sigma, (\sigma_S^B, \sigma_{V\setminus S})) \geq c_S(\hat{\sigma}, (\sigma_S^A, \hat{\sigma}_{V\setminus S}))$.

Proof: If $\sigma_S = \sigma_S^B$, then $\hat{\sigma}_S = \hat{\sigma}_S^A$, so $c_S(\sigma, (\sigma_S^B, \sigma_{V\setminus S})) = c_S(\hat{\sigma}, (\sigma_S^A, \hat{\sigma}_{V\setminus S})) = 0$.

If $\sigma_S \neq \sigma_S^B$, then $\hat{\sigma}_S \neq \hat{\sigma}_S^A$. Consider the elements of E_S^σ,

$$(E_S^\sigma)_j = U_j(\sigma_S^A, \sigma_{V\setminus S}) - U_j(\sigma_S^B, \sigma_{V\setminus S}) \tag{D.1}$$
$$= \begin{cases} \sum_{i\in S} \left(u_{ji}(A,A) - u_{ji}(A,B)\right) & \text{if } j \notin S,\ \sigma(j) = A, \\ -\sum_{i\in S} \left(u_{ji}(B,B) - u_{ji}(B,A)\right) & \text{if } j \notin S,\ \sigma(j) = B, \\ \sum_{\substack{k\notin S \\ \sigma(k)=A}} \left(u_{jk}(A,A) - u_{jk}(B,A)\right) & \\ \quad - \sum_{\substack{k\notin S \\ \sigma(k)=B}} \left(u_{jk}(B,B) - u_{jk}(A,B)\right) & \\ \quad + \sum_{\substack{i\in S \\ i\neq j}} \left(u_{ji}(A,A) - u_{ji}(B,B)\right) & \text{if } j \in S, \end{cases}$$

and the elements of $-E_S^{\hat{\sigma}}$,

$$(-E_S^{\hat{\sigma}})_j = -U_j(\sigma_S^A, \hat{\sigma}_{V\setminus S}) + U_j(\sigma_S^B, \hat{\sigma}_{V\setminus S}) \tag{D.2}$$
$$= \begin{cases} -\sum_{i\in S} \left(u_{ji}(A,A) - u_{ji}(A,B)\right) & \text{if } j \notin S,\ \hat{\sigma}(j) = A, \\ \sum_{i\in S} \left(u_{ji}(B,B) - u_{ji}(B,A)\right) & \text{if } j \notin S,\ \hat{\sigma}(j) = B, \\ -\sum_{\substack{k\notin S \\ \hat{\sigma}(k)=A}} \left(u_{jk}(A,A) - u_{jk}(B,A)\right) & \\ \quad + \sum_{\substack{k\notin S \\ \hat{\sigma}(k)=B}} \left(u_{jk}(B,B) - u_{jk}(A,B)\right) & \\ \quad - \sum_{\substack{i\in S \\ i\neq j}} \left(u_{ji}(A,A) - u_{ji}(B,B)\right) & \text{if } j \in S. \end{cases}$$

Noting that $\sigma(j) = A$ if and only if $\hat{\sigma}(j) = B$, $\sigma(j) = B$ if and only if $\hat{\sigma}(j) = A$, we can subtract (D.2) from (D.1) to get

$$(E_S^\sigma - (-E_S^{\hat{\sigma}}))_j = (E_S^\sigma + E_S^{\hat{\sigma}})_j = \tag{D.3}$$
$$= \begin{cases} \sum_{i\in S} \left(\left(u_{ji}(A,A) - u_{ji}(A,B)\right) - \left(u_{ji}(B,B) - u_{ji}(B,A)\right)\right) & \text{if } j \notin S,\ \sigma(j) = A, \\ \sum_{i\in S} \left(\left(u_{ji}(A,A) - u_{ji}(A,B)\right) - \left(u_{ji}(B,B) - u_{ji}(B,A)\right)\right) & \text{if } j \notin S,\ \sigma(j) = B, \\ \sum_{\substack{k\notin S \\ \sigma(k)=A}} \left(\left(u_{jk}(A,A) - u_{jk}(B,A)\right) - \left(u_{jk}(B,B) - u_{jk}(A,B)\right)\right) & \\ \quad + \sum_{\substack{k\notin S \\ \sigma(k)=B}} \left(u_{jk}(A,A) - u_{jk}(B,A)\right) - \left(u_{jk}(B,B) - u_{jk}(A,B)\right) & \\ \quad + \sum_{\substack{i\in S \\ i\neq j}} \left(\left(u_{ji}(A,A) - u_{ji}(B,B)\right) + \left(u_{ji}(A,A) - u_{ji}(B,B)\right)\right) & \text{if } j \in S, \end{cases}$$

If A is RD_{jS} for $j \in S$, then the sum of the first and second lines of the third case of (D.3) is nonnegative. If A is PD_{jS} for $j \in S$, then the third line of the third case of (D.3) is nonnegative. Therefore, if A is RD_{jS} and PD_{jS}, the third case of (D.3) is nonnegative. That is, if $j \in S$, then $(E_S^\sigma + E_S^{\hat{\sigma}})_j \geq 0$, so $(E_S^\sigma)_j \geq (-E_S^{\hat{\sigma}})_j$.

If Υ_S^C is self-regarding, then $(E_S^\sigma)_j \geq (-E_S^{\hat{\sigma}})_j$ for all $j \in S$ implies that $\Upsilon_S^C(E_S^\sigma) \geq \Upsilon_S^C(-E_S^{\hat{\sigma}})$ and therefore, by (6.1), $c_S(\sigma, (\sigma_S^B, \sigma_{V\setminus S})) \geq c_S(\hat{\sigma}, (\sigma_S^A, \hat{\sigma}_{V\setminus S}))$.

If A is ARD_{Sj} for all $j \notin S$, then the first and second cases of (D.3) are nonnegative. That is, if $j \notin S$, $(E_S^\sigma + E_S^{\hat{\sigma}})_j \geq 0$, so $(E_S^\sigma)_j \geq (-E_S^{\hat{\sigma}})_j$. Therefore $E_S^\sigma \geq E_S^{\hat{\sigma}}$, and as Υ_S^C is non-decreasing, $\Upsilon_S^C(E_S^\sigma) \geq \Upsilon_S^C(-E_S^{\hat{\sigma}})$ and therefore, by (6.1), $c_S(\sigma, (\sigma_S^B, \sigma_{V\setminus S})) \geq c_S(\hat{\sigma}, (\sigma_S^A, \hat{\sigma}_{V\setminus S}))$. *Q.E.D.*

LEMMA 11 *Let $\hat{\sigma}, \tilde{\sigma}$ be such that $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$. If S follows a coalitional payoff-difference based choice rule, then $c_S(\hat{\sigma}, (\sigma_S^A, \hat{\sigma}_{V\setminus S})) \geq c_S(\tilde{\sigma}, (\sigma_S^A, \tilde{\sigma}_{V\setminus S}))$.*

PROOF: If $\tilde{\sigma}_S = \sigma_S^A$, then $c_S(\hat{\sigma}, (\sigma_S^A, \hat{\sigma}_{V\setminus S})) \geq c_S(\tilde{\sigma}, (\sigma_S^A, \tilde{\sigma}_{V\setminus S})) = 0$.

If $\tilde{\sigma}_S \neq \sigma_S^A$, then $\hat{\sigma}_S \neq \sigma_S^A$. Using (D.2) for both $E_S^{\hat{\sigma}}$ and $E_S^{\tilde{\sigma}}$ gives

$$
\begin{aligned}
&((-E_S^{\hat{\sigma}}) - (-E_S^{\tilde{\sigma}}))_j \\
&= \begin{cases}
0 & \text{if } j \notin S,\ \hat{\sigma}(j) = A, \\
0 & \text{if } j \notin S,\ \tilde{\sigma}(j) = B, \\
\sum_{i \in S} \Big(\big(u_{ji}(A,A) - u_{ji}(A,B)\big) + \big(u_{ji}(B,B) - u_{ji}(B,A)\big)\Big) & \text{if } j \notin S,\ \tilde{\sigma}(j) = A,\ \hat{\sigma}(j) = B, \\
\sum_{\substack{k \notin S \\ \tilde{\sigma}(k) = A}} \big(u_{ik}(A,A) - u_{ik}(B,A)\big) & \\
\quad - \sum_{\substack{k \notin S \\ \hat{\sigma}(k) = A}} \big(u_{ik}(A,A) - u_{ik}(B,A)\big) & \\
\quad + \sum_{\substack{k \notin S \\ \hat{\sigma}(k) = B}} \big(u_{ik}(B,B) - u_{ik}(A,B)\big) & \\
\quad - \sum_{\substack{k \notin S \\ \tilde{\sigma}(k) = B}} \big(u_{ik}(B,B) - u_{ik}(A,B)\big) & \text{if } j \in S.
\end{cases}
\end{aligned}
\tag{D.4}
$$

The third case of of (D.4) is nonnegative by (2.2). The first two lines of the fourth case, taken together, are nonnegative as $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$. The final two lines of the fourth case, taken together, are nonnegative as $V_B(\tilde{\sigma}) \subseteq V_B(\hat{\sigma})$. So every element of $((-E_S^{\hat{\sigma}}) - (-E_S^{\tilde{\sigma}}))_j$ is nonnegative and $-E_S^{\hat{\sigma}} \geq -E_S^{\tilde{\sigma}}$. As Υ_S^C is non-decreasing, $\Upsilon_S^C(-E_S^{\hat{\sigma}}) \geq \Upsilon_S^C(-E_S^{\tilde{\sigma}})$ and therefore, by (6.1), $c_S(\hat{\sigma}, (\sigma_S^A, \hat{\sigma}_{V\setminus S})) \geq c_S(\tilde{\sigma}, (\sigma_S^A, \tilde{\sigma}_{V\setminus S}))$. *Q.E.D.*

LEMMA 12 *If S follows a coalitional payoff-difference based choice rule, then c_S is asymmetric if and only if, for all σ, $\tilde{\sigma}$ such that $V_B(\sigma) \subseteq V_A(\tilde{\sigma})$, we have that $c_S(\sigma, (\sigma_S^B, \sigma_{V\setminus S})) \geq c_S(\tilde{\sigma}, (\tilde{\sigma}_S^A, \tilde{\sigma}_{V\setminus S}))$.*

PROOF: Consider σ, σ', $\tilde{\sigma}$ such that $V_B(\sigma) \subseteq V_A(\tilde{\sigma})$.

If σ' is not equal to σ, $(\sigma_S^B, \sigma_{V\setminus S})$ or $(\sigma_S^A, \sigma_{V\setminus S})$, then by (6.1), $c_S(\sigma, \sigma') = \infty$, so setting $\bar{\sigma} = \sigma^A$, we have that $V_B(\sigma') \subseteq V_A(\bar{\sigma})$, $V_A(\tilde{\sigma}) \subseteq V_A(\bar{\sigma})$, and $c_S(\sigma, \sigma') \geq c_S(\tilde{\sigma}, \bar{\sigma})$, satisfying the condition for asymmetry.

If $\sigma = \sigma'$, then, by (6.1), $c_S(\sigma, \sigma') = 0$. Letting $\bar{\sigma} = \tilde{\sigma}$, we have $V_B(\sigma') = V_B(\sigma) \subseteq V_A(\tilde{\sigma}) = V_A(\bar{\sigma})$ and, by (6.1), $c_S(\tilde{\sigma}, \bar{\sigma}) = 0$, so $c_S(\sigma, \sigma') \geq c_S(\tilde{\sigma}, \bar{\sigma}) = 0$, satisfying the condition for asymmetry.

If $\sigma \neq \sigma' = (\sigma_S^A, \sigma_{V\setminus S})$, let $\bar{\sigma} = \tilde{\sigma}$. Then we have $V_B(\sigma') \subset V_B(\sigma) \subseteq V_A(\tilde{\sigma}) = V_A(\bar{\sigma})$ and, by (6.1), $c_S(\tilde{\sigma}, \bar{\sigma}) = 0$, so $c_S(\sigma, \sigma') \geq c_S(\tilde{\sigma}, \bar{\sigma}) = 0$, satisfying the condition for asymmetry.

The only remaining case is $\sigma \neq \sigma' = (\sigma_S^B, \sigma_{V\setminus S})$. For $\bar{\sigma}$ to satisfy $V_B(\sigma') \subseteq V_A(\bar{\sigma})$, it must be that $\bar{\sigma}_S = \sigma_S^A$, and for $c_S(\tilde{\sigma}, \bar{\sigma}) < \infty$, it must be that $\bar{\sigma}_{V\setminus S} = \tilde{\sigma}_{V\setminus S}$. Hence, it only remains to check whether $c_S(\sigma, \sigma') \geq c_S(\tilde{\sigma}, \bar{\sigma}) = c_S(\tilde{\sigma}, (\sigma_S^A, \tilde{\sigma}_{V\setminus S}))$, the condition in the statement of the lemma.

Q.E.D.

PROOF OF PROPOSITION 4: Let σ, $\tilde{\sigma}$ be such that $V_B(\sigma) \subseteq V_A(\tilde{\sigma})$. Define $\hat{\sigma}$ so that $V_B(\sigma) = V_A(\hat{\sigma})$. Note that $V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$. Then,

$$c_S(\sigma,(\sigma_S^B,\sigma_{V\setminus S})) \underbrace{\geq}_{\text{by Lemma 10}} c_S(\hat{\sigma},(\sigma_S^A,\hat{\sigma}_{V\setminus S})) \underbrace{\geq}_{\text{by Lemma 11}} c_S(\tilde{\sigma},(\sigma_S^A,\tilde{\sigma}_{V\setminus S})),$$

satisfying the condition for asymmetry given in Lemma 12. *Q.E.D.*

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