Learning Match Quality *

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Abstract

For many new products or products with multiple attributes, learning the price is often easier than learning one's willingness to pay. We model a market in which consumers face a transportation cost to discover a seller's price, and then have the *option* to pay a learning cost to discover the product's match value before deciding whether to purchase or continue searching. In equilibrium each seller optimally sets either a "regular" price which induces a visiting consumer to learn or a sufficiently low "preemption" price which induces the consumer to accept immediately. In contrast to the common intuition about search frictions, we find that higher learning costs can improve consumer welfare by increasing sellers' incentive to preempt, which lowers prices and increases sales. We also demonstrate that the incentive to preempt is lower in a monopoly than in an oligopoly, and in a uniform example show that welfare and consumer surplus are higher in a monopoly for a range of learning costs. From a platform design perspective, we find that consumers are better off from clear disclosure for products with low learning costs and from obfuscation for products with high learning costs.

*An earlier version of this project had the title "Quality Obfuscation and Competition", but the focus of the present work has shifted substantially.

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1 Introduction

Frictions that reduce consumers' ability to compare products can lead to higher prices. Such is the case in the commonly studied differentiated products framework of Wolinsky (1986) and Anderson and Renault (1999), in which buyers must pay a search cost to learn the price and match quality at each seller. In equilibrium a consumer continues to search only if her current match quality is below a threshold, and the higher the search cost the lower that threshold. Higher search costs thus induce a smaller group of comparison shoppers and lead to higher prices.

The Wolinsky framework has been exceedingly popular in studying search markets (Anderson and Renault (2006), Armstrong et al. (2009), Bar-Isaac et al. (2012), Moraga-González et al. (2015) and many more), in part due to the fact that model predictions seem in line with classic economic intuition.[1] However, we will demonstrate that these predictions are driven by the implicit assumption that a single search cost allows consumers to learn both the price and the match quality of a product, and in reality this is often not the case. For example, when selecting a health insurance plan a consumer immediately observes the premium, but must expend additional energy to determine how well the plan covers her particular medical needs in terms of deductibles, prescription benefits, the size and quality of the network of doctors, and in a myriad of other dimensions. Similarly, to buy a weather app in Apple's App Store one may simply click on the button showing the price to download, or one may first carefully read user reviews to determine whether the app works well in a particular geographic region, and then decide whether to purchase. The Wolinsky framework bundles the costs of learning the price and learning the match value into one, thus leaving unmodeled the *decision* of learning the match value after having seen the price. However the dynamics of this decision are quite interesting and, as we show, the competitive implications of a higher cost to learn the match value are qualitatively different from that of a higher cost of learning the price.

To get some intuition for the decision to learn the match value, consider the case of a monopoly seller as in Wang (2013). A consumer that faces a price below the expected match value may either buy immediately or learn the match value and buy only if the value exceeds the price. Learning is valuable only if the match value is *lower* than the price, else the decision remains the same, and thus the lower the price the lower the expected value of learning. In particular, when learning is costly there is a price $\hat{p}$ such that for all prices below it the consumer buys without learning. The seller must thus decide whether to charge the monopoly price p^i and sell only if the match value is sufficiently high or to charge the lower preemption price $\hat{p}$ and sell for sure. The latter may be more profitable if $\hat{p}$ is sufficiently

[1] In contrast, for instance, with the homogeneous good Diamond (1971) model in which the same monopoly outcome ensues regardless of the (strictly positive) search costs and the number of firms.

high, that is if the learning cost is sufficiently high. In other words, as the learning cost increases the optimal price may *decrease* from p^i to $\hat{p}$.

In this paper we embed the consumer's learning decision into Wolinsky's oligopoly model. A consumer spends a transportation cost t to visit a seller, where she observes the price and has the option either to buy immediately or to spend an additional cost ℓ to learn her match value and then decide whether to buy. The model nests Wolinsky (1986) as a special case with $t > 0$ and $\ell = 0$. We show that the structure of the equilibrium depends on the magnitude of learning costs. For low learning costs the equilibrium is as in Wolinsky's model, in which consumers always learn conditional on arriving and effectively bundle the learning and transportation costs into one. For intermediate learning costs, this equilibrium is no longer supported and instead sellers now mix between two prices: a high price that induces learning and a lower preemption price. For high learning costs the equilibrium is similar to that of Diamond (1971) in which all sellers preempt learning and charge monopoly prices.

Comparative statics with respect to search frictions differ substantially across the three regions. In the region of low learning costs, the effects of increased learning and transportation costs are the same as in Wolinsky's model, inducing consumers to be less selective during search and sellers to set higher prices, and resulting in lower consumer surplus and higher seller profits. For intermediate learning costs however, many of these effects reverse. A higher learning cost induces more sellers to charge the lower preemption price and results in consumers being *more* selective during search and in *lower* prices even at sellers that do not preempt. Higher learning costs within the intermediate region thus increase consumer surplus and reduce seller profits, and it is in fact possible for consumer surplus to be higher at the maximal intermediate learning cost than at a learning cost of zero. In addition, the transportation cost t has no effect on equilibrium payoffs in the intermediate region. While the direct effect of a higher t is to reduce the net benefit of searching, the margin of adjustment here is not a decrease in the consumers' search threshold but rather an increase in the proportion of sellers that charge the lower preemption price. The intuition here is that as the transportation cost rises the market become less competitive, which in turn disproportionately increases the preemption profit and results in more sellers charging the preemption price. Sellers' profits are unaffected since they are indifferent between the informed and preemption prices, and although the average price charged is lower, once the higher transportation cost is accounted for the consumer's payoff remains unchanged.

As learning costs cross into the highest third region there is a dramatic shift in equilibrium as competition unravels. By the logic in Diamond (1971), if sellers have a homogeneous product and all set the same price, any individual seller can deviate to a slightly higher price and not lose any consumers to search, and this upward force results in the monopoly price as the only equilibrium. Indeed this force is present even in the intermediate region in which all preempting sellers strictly exceed the consumers' utility threshold, but they are dissuaded from marginally increasing their price as this would induce learning. However, as the learning costs become sufficiently large this restraint is removed and the Diamond logic takes hold. The result is a jump up in seller profits and a jump down in consumer surplus as the learning cost transitions from the competitive intermediate region to the monopolistic high learning cost region.[2]

Anderson and Renault (1999) also show that the equilibrium in the Wolinsky model transitions into a Diamond equilibrium when products become sufficiently undifferentiated. In a sense this is what happens in our model in that the level of differentiation is endogenous. That is, as the learning cost increases the proportion of sellers that induce learning falls, thus it is increasingly likely that consumers are presented with an undifferentiated product. However, the mechanics of learning are important in our model beyond the differentiation effect. In particular, we have equilibria in the intermediate range of learning costs in which prices are low and products are either almost or even completely undifferentiated, while in Anderson and Renault (1999) a minimal level of differentiation is required to prevent the Diamond equilibrium.

We also investigate the role of competition in the presence of learning costs and show that the incentive to preempt learning is smaller in an oligopoly than in a monopoly. The intuition for this comes from reframing preemption in terms of price discrimination. That is, charging an uninformed consumer her ex-ante expected willingness to pay is equivalent to instead having the consumer learn her willingness to pay and committing to selling to her for this amount. This extracts the consumer's information rents when her willingness to pay is high, but also commits the seller to incur a loss when the willingness to pay is below his cost. The result is that an uninformed consumer is more profitable only if the seller's cost is sufficiently low relative to the distribution of consumers' willingness to pay (Johnson and Myatt (2006)). In the present setting, a duopoly seller differs from a monopoly only in that she faces demand from consumers with strictly positive outside options instead of an outside option of zero. Hence the willingness to pay of consumers in a duopoly is closer to the seller's marginal cost than in a monopoly, and consequently preemption is less attractive. Using this observation, we demonstrate that there is a range of learning costs in which no learning occurs in a monopoly while learning does occur in a duopoly.

The equilibrium construction in the intermediate region of learning costs is of independent interest. First, in contrast to models in the price dispersion literature that either rely on seller heterogeneity (Reinganum (1979), Benabou and Gertner (1993)), ex-ante buyer heterogeneity (Varian (1980), Stahl (1989), Rob (1985)), or interim buyer heterogeneity (Butters (1977), Burdett and Judd (1983)), we generate price dispersion without any heterogeneity.

[2]To be precise equilibrium payoffs are upper-hemicontinuous, with multiple possible equilibria at the boundary between the intermediate and high learning cost regions.

The only other work of which we are aware that accomplishes this is Garcia et al. (2015), where upstream competition among manufacturers results in mixed strategies in wholesale prices, and thus dispersion in retail prices. Both their model and ours predict a bimodal retail price distribution, but the settings are quite different and so too is the rationale for the dispersion. The authors note that the bimodal price distribution is quite appealing as it captures a common dynamic of regular and sales prices often observed at stores. Our model provides a different explanation driven by demand rather than supply considerations. A sale price is not just meant to capture those consumers that are willing to pay less, but rather those who know less. In addition, our work is related to that of Bar-Isaac et al. (2012) in describing markets for niche versus broad products. One interpretation of that model is that firms choose the extent to which match value information is available to buyers and consequently some products have broad appeal while others are sold only to a small number of consumers with high willingness to pay. We show that the same dynamic emerges even if sellers have no direct control over the match value information available to consumers and instead it is consumers that decide about whether to become informed.

The rest of this article is organized as follows. In Section 2 we describe the model and solve for the set of equilibria. We then compute comparative statics, showing the pro-competitive effects of learning costs in the intermediate region and the strong anti-competitive effect in the high region. Then in Section 3 we compare the outcomes of the oligopoly to that of a monopoly, and show that consumer surplus can be higher in the latter since for some learning costs preemption occurs in the monopoly but not the oligopoly. Section 4 then extends the model to allow for sellers to actively obfuscate or disclose match values, and shows that the equilibrium structure is similar to when learning costs are exogenous. Section 5 then concludes.

2 Model

The setup is that of Wolinsky (1986) with the addition of a learning decision. There is a unit mass of consumers with unit demand for a product from one of N sellers with constant marginal cost c. Sellers simultaneously set prices p_i but these are not immediately observed by consumers, who instead must undertake costly sequential search. A consumer faces a transportation cost t to visit any particular seller i, where she observes the price p_i and has the option to immediately purchase the product. Alternatively, she can then spend an additional cost ℓ to learn her idiosyncratic match value ε_i for i's product, drawn from log-concave distribution F with density f on full support on $[\underline{\varepsilon}, \bar{\varepsilon}] \in \mathbb{R}_+$, and average value $E[\varepsilon] = \mu$. Having learned the value the consumer decides whether to purchase from i, to visit a new seller, or to return to a previously visited seller at no additional cost. A consumer j who has purchased a product from seller i after visiting M sellers and learning $m \leq M$

match values obtains a payoff of

$$U_j = \alpha_i + \varepsilon_{ij} - p_i - tM - \ell m,$$

where α_i denotes the commonly known quality of seller i's product. To begin we will assume that all products have the same quality and suppress the subscript. It can be seen that Wolinsky's model is a special case in which $t > 0$ and $\ell = 0$. We will focus on transportation costs that are small enough to accommodate the possibility of search in Wolinsky's model.[3] As a simplification we assume that the consumer does not incur a transportation cost for her first visit.[4] For tractability we assume $N = \infty$ to ensure that consumers never return in equilibrium, which means the probability of selling to any visiting consumer equals the probability of exceeding her threshold.

Consumer Strategy

Upon visiting a seller the consumer makes a learning decision and then a search decision. For the search decision a threshold policy is optimal because in equilibrium the distribution of future offers is independent of history. Thus there is a utility u such that the consumer buys whenever her current offer exceeds u and searches or exits otherwise.

For the learning decision, the consumer compares her maximized payoff without the information to her maximized payoff if she learns. Learning the match value is thus only valuable when it alters the consumer's search decision. Specifically, if $\alpha + \mu - p \geq u$ then without learning the consumer accepts, and learning is beneficial only when the match realization is low enough to induce search. In this case the value of learning is

$$w(p + u - \alpha) \equiv \int_{\underline{\varepsilon}}^{p+u-\alpha} (u - (\alpha + \varepsilon - p)) \; dF(\varepsilon) \;\; = \;\; \int_{\underline{\varepsilon}}^{p+u-\alpha} F(\varepsilon) \; d\varepsilon.$$

Conversely, if $\alpha+\mu-p \leq u$ then without learning the consumer searches or exits, and learning is beneficial only when the match realization is high enough to induce buying. Then the value of learning is

$$W(p + u - \alpha) \equiv \int_{p+u-\alpha}^{\bar{\varepsilon}} ((\alpha + \varepsilon - p) - u) \; dF(\varepsilon) \;\; = \;\; \int_{p+u}^{\bar{\varepsilon}} (1 - F(\varepsilon)) \; d\varepsilon.$$

[3]In particular, that $t \leq \bar{t} \equiv \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} F(\varepsilon) d\varepsilon$.

[4]This assumption is typically made in the literature to ensure that the market does not collapse. In our case it will only bind in the space of parameters in which no learning occurs in equilibrium. Else, as in Wolinsky (1986) the assumption is not necessary whenever the distribution of values F is sufficiently diffuse to ensure that the consumers' expected information rents outweigh the transportation cost.

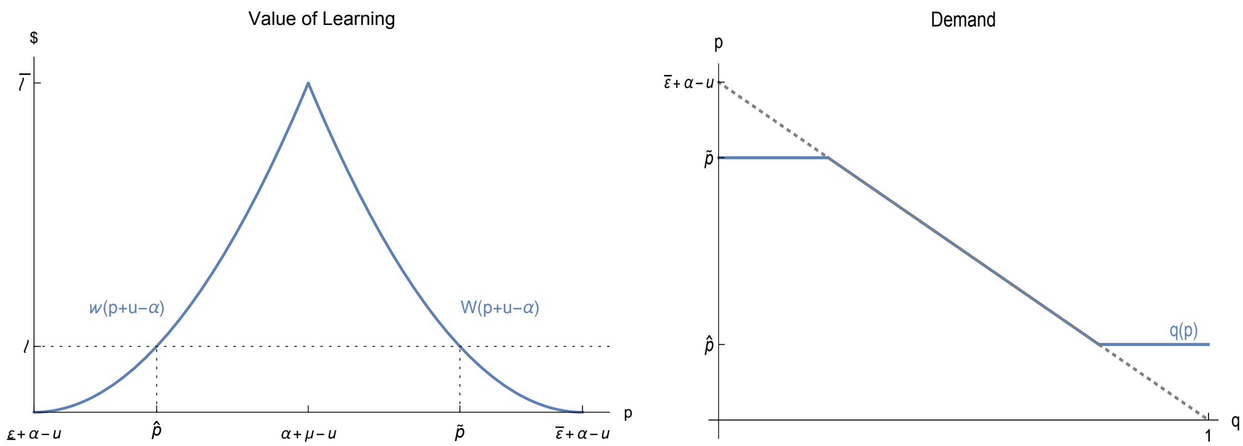

Figure 1: The value of learning in the left panel defines the range of prices $(\hat{p}, \tilde{p})$ over which consumers learn, which then yields the censored demand in the right panel.

It can be verified that $w(\underline{\varepsilon}) = W(\bar{\varepsilon}) = 0$, $w(\mu) = W(\mu) \equiv \bar{\ell}$, and $w(\cdot)$ is increasing while $W(\cdot)$ is decreasing. Thus for any $\ell \in [0, \bar{\ell}]$ we can define a preemption price

$$\hat{p}(u) \equiv w^{-1}(\ell) - u + \alpha$$

as the highest price at which consumers buy without learning and an inclusion price

$$\tilde{p}(u) \equiv W^{-1}(\ell) - u + \alpha$$

as the highest price at which consumers learn rather than immediately searching or exiting,[5] as depicted in Figure 1. For costs above $\bar{\ell}$ the consumer never learns and either buys if $\alpha + \mu - p \geq u$ or searches or exits otherwise.

Seller Strategy

Because searchers never return in equilibrium, a firm either sells to a consumer upon her arrival or not at all. Taking as given the consumer's search threshold u and corresponding preemption price $\hat{p}(u)$ and inclusion price $\tilde{p}(u)$, the seller faces demand

$$q(p) = \begin{cases} 1 & \text{if } \; p \leq \hat{p}(u) \\ 1 - F(p + u - \alpha) & \text{if } \; p \in (\hat{p}(u), \tilde{p}(u)] \\ 0 & \text{if } \; p > \tilde{p}(u) \end{cases},$$

[5]We assume for simplicity that at $\hat{p}(u)$ the consumer does not learn and that at $\tilde{p}(u)$ she learns, although she is indifferent in both cases. It can instead be argued more formally that any other behavior is inconsistent with equilibrium because it leads to non-existence of best responses. For example, if the consumer still learns at $\hat{p}(u)$ with positive probability then a preempting seller wishes to charge the highest price below $\hat{p}(u)$, which does not exist.

with corresponding profit $\pi(p) = (p-c)q(p)$. The profit increases linearly in price up to $\hat{p}$, then jumps down and is quasi-concave (because F is log-concave) in the region $(\hat{p}(u), \tilde{p}(u)]$, and then jumps to zero for prices above $\tilde{p}$. Letting $p^i(u)$ be the solution to the seller's first order condition

$$0 = 1 - F(p^i + u - \alpha) - (p^i - c)f(p^i + u - \alpha), \tag{1}$$

it is optimal for a seller either to set the preemption price $\hat{p}(u)$ or the best informed price $\min(p^i(u), \tilde{p}(u))$.

Equilibrium

We solve for a Perfect Bayesian equilibrium, in which consumers correctly anticipate the distribution of seller prices and optimally select threshold u for the search decision and corresponding preemption price $\hat{p}(u)$ and inclusion price $\tilde{p}(u)$ for the learning decision, while sellers correctly anticipate the consumers' strategy and optimally charge either $\hat{p}(u)$ or $\min(p^i(u), \tilde{p}(u))$. The equilibrium structure depends on the magnitude of the learning cost ℓ. We will demonstrate that when ℓ is small consumers always learn and the equilibrium is that of Wolinsky (1986) while when ℓ is high consumers do not learn and instead a Diamond (1971) equilibrium ensues in which all sellers charge a single high price and still preempt search. For intermediate learning costs we find a different equilibrium in which sellers mix between a high price that induces consumers to learn and a low price which preempts learning.

Definition 1 *In a Wolinsky equilibrium all consumers learn and use search threshold u and all sellers charge the informed price p^i, satisfying*

(i) $t = -\ell + W(p^i + u - \alpha)$ *(search threshold)*

(ii) $0 = 1 - F(p^i + u - \alpha) - (p^i - c)f(p^i + u - \alpha)$ *(informed price)*

(iii) $\underline{\varepsilon} \leq p^i + u - \alpha$ *(search occurs)*

(iv) $\hat{p}(u) \leq p^i \leq \tilde{p}(u)$ *(learning incentive)*

(v) $\hat{p}(u) - c \leq (p^i - c)(1 - F(p^i + u - \alpha))$. *(preemption incentive)*

Conditions (i), (ii), and (iii) all presume that consumers learn and are the exact analog of Wolinsky's equilibrium. Condition (i) describes the threshold u at which the cost of search t equals the expected net improvement $-\ell + W(p^i + u - \alpha)$ and condition (ii) describes the optimal price p^i as the solution to the first order condition. Condition (iii) then verifies that the marginal match value $\varepsilon = p^i + u - \alpha$ at which a consumer begins searching is interior, else the first order condition in (ii) is not well-defined. In contrast to Wolinsky's model, here we must also account for the fact that learning is a decision. For this, condition (iv) verifies that it is indeed optimal for the consumer to search at p^i, and condition (v) verifies that the highest price $\hat{p}(u)$ which preempts search is less profitable than the informed price p^i.

As an illustration consider the case when $\ell = t = 0$. Plugging (i) into (ii) obtains

$$p^i = c + \frac{1 - F(W^{-1}(t+\ell))}{f(W^{-1}(t+\ell))} = c + \frac{1 - F(\bar{\varepsilon})}{f(\bar{\varepsilon})} = c,$$

which then yields $u = \alpha + W^{-1}(t+\ell) - p_i = \alpha + \bar{\varepsilon} - c$. In other words, with no frictions and infinite sellers the result is Bertrand competition in which consumers search for the maximal match value and zero markup. Condition (iii) is satisfied because $p^i + u - \alpha = \bar{\varepsilon} > \underline{\varepsilon}$. In addition, the preemption price is $\hat{p}(u) = w^{-1}(\ell) - u + \alpha = c - (\bar{\varepsilon} - \underline{\varepsilon})$ and the inclusion price is $\tilde{p}(u) = W^{-1}(\ell) - u + \alpha = c$, therefore condition (iv) is satisfied. Furthermore, condition (v) holds with slack because the informed profit equals zero and the preemption profit is strictly negative, with the preemption price below marginal cost.

As ℓ begins to grow the search threshold u consistent with (i) and (ii) begins to fall, both directly because search is now costlier and indirectly because sellers expect less search and increase prices. The resulting decrease in u increases both the informed profit and the preemption profit, but as we show in the Appendix the preemption profit increases more quickly. The slack in (iv) diminishes as ℓ grows, and if there is a learning cost ℓ_1 at which the two profits are equal, then for $\ell > \ell_1$ a Wolinsky equilibrium can no longer be supported and instead the Preemption equilibrium ensues, as described below.

Definition 2 *In a Preemption equilibrium, a proportion $1-\gamma$ of sellers charge the lower of the informed or inclusion price $\min(p^i, \tilde{p}(u))$, a proportion γ charge the preemption price $\hat{p}(u)$, consumers use search threshold u, learn upon observing $\min(p^i, \tilde{p}(u))$, and buy immediately upon observing $\hat{p}(u)$, where u, p^i, and γ satisfy*

(i) $t = (1-\gamma)(-\ell + W(\min(p^i, \tilde{p}(u)) + u - \alpha)) \; + \; \gamma(\alpha + \mu - \hat{p}(u) - u)$ *(search threshold)*

(ii) $0 = 1 - F(p^i + u - \alpha) - (p^i - c) f(p^i + u - \alpha)$ *(informed price)*

(iii) $\underline{\varepsilon} \leq p^i + u - \alpha$ *(search occurs)*

(iv) $\hat{p}(u) \leq p^i$ *(learning incentive)*

(v) $\hat{p}(u) - c = (\min(p^i, \tilde{p}(u) - c)(1 - F(\min(p^i, \tilde{p}(u)) + u - \alpha))$ *(preemption incentive).*

To get intuition for how the Preemption equilibrium is constructed, consider the boundary ℓ_1 between the Wolinsky and Preemption regions. Here, consumers optimally choose search threshold u anticipating that all sellers charge p^i, and this threshold in turn makes sellers indifferent between p^i and the preemption price $\hat{p}$. At a slightly higher ℓ this type of equilibrium cannot be supported – the search threshold u consistent with all sellers charging the informed price p^i is low enough so that an individual seller strictly improves by switching to the preemption price $\hat{p}$. But it is also not an equilibrium for all sellers to charge $\hat{p}$, because in expecting this low price consumers use a substantially higher search threshold u, and in this case each seller strictly benefits by deviating to p^i. Instead, in equilibrium a proportion γ of sellers preempt while $1-\gamma$ charge the informed price, and γ is calibrated to induce the search threshold u for which the indifference in condition (v) obtains.

The price $\min(\tilde{p}, p^i)$ is the most profitable price at which consumers choose to learn. We will show that $p^i < \tilde{p}$ at ℓ_1, but as ℓ grows the two prices move closer together. It is possible that when ℓ is large enough $p^i > \tilde{p}$, and in this case sellers mix between $\hat{p}$ and $\tilde{p}$ in equilibrium.

We will show that the higher is the learning cost ℓ, the larger is the required proportion γ of preempting sellers to keep preemption profits equal to informed profits. Eventually ℓ is large enough so that every seller charges the preemption price, and as the learning cost increases further the Preemption equilibrium cannot be supported. Instead, we enter into Diamond region described below.

Definition 3 *In a Diamond equilibrium, all sellers charge preemption price*

$$p = \min(\alpha + w^{-1}(\ell), \alpha + \mu)$$

and consumers buy at the first seller without learning or searching.

A logic similar to Diamond's emerges here, but as before the learning decision introduces a new deviation that must be ruled out. In particular, on the equilibrium path consumers anticipate not learning when visiting the next seller, thus there is no value to searching if all sellers set the same price. Because consumers *strictly* prefer not to search, in Diamond's model a seller can slightly increase the price and not induce search, thus the only equilibrium price is the monopoly price. Presently, although charging a slightly higher price cannot induce search it may induce learning, and thereby cause a discrete drop in demand. Therefore, sellers charge the highest price $p = \min(\alpha + w^{-1}(\ell), \alpha + u)$ that preempts learning. That said, prices in the Diamond region are essentially monopoly prices and thus substantially higher than in the Preemption region.

With the structure of each of the three possible equilibria defined above, we now describe when each of the equilibria occurs.

Proposition 1 *There exist* $0 < \ell_1 < \ell_2$ *so that*

(i) if $\ell \in [0, \ell_1)$ *then there is a Wolinsky equilibrium and it is unique,*

(ii) if $\ell \in (\ell_1, \ell_2)$ then there is a Preemption equilibrium and it is unique,

(iii) if $\ell \in (\ell_2, \infty)$ then there is a Diamond equilibrium and it is unique.

That a Wolinsky equilibrium with full search and learning ensues for low ℓ and a Diamond equilibrium with no search or learning ensues for high ℓ is quite intuitive, but it is perhaps less intuitive that a preemption region (ℓ_1, ℓ_2) with some search and learning must exist. We demonstrate this to be the case in the proof in the Appendix, along with the fact that the equilibrium in every region is unique. To get a better sense of the mechanics of these equilibria we provide the following example.

Uniform example

To fix ideas we describe the set of equilibria when $\alpha = c = 0$ and $\varepsilon \sim U[1,2]$.[6] The value of learning when avoiding low outcomes is

$$w(\varepsilon) = \frac{1}{2}(\varepsilon - 1)^2,$$

and the value of learning when taking advantage of high outcomes is

$$W(\varepsilon) = \frac{1}{2}(2 - \varepsilon)^2,$$

implying preemption and inclusion prices

$$\hat{p}(u) = \sqrt{2\ell} - u, \quad \tilde{p}(u) = 2 - \sqrt{2\ell} - u.$$

Wolinsky equilibrium

The Wolinsky equilibrium is characterized by the system of conditions (i) and (ii) of Definition 1, the solution to which is

$$u = 2 - 2\sqrt{2(t+\ell)}, \quad p^i = \sqrt{2(t+\ell)}. \tag{2}$$

To find the boundary of the Wolinsky region where the preemption profit becomes equal to the informed profit, we define

$$\Delta\pi(\ell) \equiv \pi(\hat{p}(\ell)) - \pi(p^i(\ell)) = 2\left(\ell + t - \frac{1}{2}\left(1 - (2\ell)^{\frac{1}{4}}\right)^2\right). \tag{3}$$

We observe that $\Delta\pi(0) < 0$, which implies that for small values of ℓ we are in the Wolinsky region, that $\Delta\pi(\ell)$ is increasing, and that there exists an ℓ_1 at which $\Delta\pi(\ell_1) = 0$.[7]

[6] Observe this satisfies the assumption that $\underline{\varepsilon} - \frac{1}{f(\underline{\varepsilon})} \geq 0$.

[7] Furthermore, we can confirm that at ℓ_1 search is still supported in equilibrium. For this, note that at the maximal learning cost at which search can be supported in a Wolinsky equilibrium, i.e. at $\bar{\ell} \equiv W(\underline{\varepsilon}) - t = \frac{1}{2} - t$, we obtain $\Delta\pi(\bar{\ell}) > 0$, and because $\Delta\pi$ is increasing it must be that $\ell_1 < \bar{\ell}$.

Note that in the Wolinsky region the inclusion price always exceeds the informed price, i.e. $\tilde{p} - p^i = \sqrt{2(t+\ell)} - \sqrt{2\ell} > 0$, and thus it is optimal for consumers to learn at p^i.

The Wolinsky region is depicted in the example in Figure 2, where it is assumed that $t = 0.05$. The region's right boundary is $\ell_1 \approx 0.05$, and in this region as ℓ increases the seller's price increases, the consumer's search threshold falls, and total surplus falls as in the standard Wolinsky model.

Preemption equilibrium

Next, the Preemption equilibrium is characterized by the system of conditions (i), (ii), and (v) of Definition 2. To deal with the min operator in (i) and (v), we first assume that consumers would search at the informed price (i.e. $p^i < \tilde{p}$), in which case the three equilibrium conditions yield

$$u = 2(2\ell)^{\frac{1}{4}}, \quad p^i = 1 - (2\ell)^{\frac{1}{4}}, \quad \gamma = \frac{t + \ell - \frac{1}{2}\left(1 - (2\ell)^{\frac{1}{4}}\right)^2}{(\frac{1}{2} - \sqrt{2\ell}) + \ell - \frac{1}{2}\left(1 - (2\ell)^{\frac{1}{4}}\right)^2}. \tag{4}$$

Observe that at the right endpoint of the Wolinsky region ℓ_1, exactly $\gamma = 0$ sellers charge the preemption price. As ℓ grows so too does the proportion of preempting sellers, until $\ell_2 \equiv \frac{1}{2}\left(\frac{1}{2} - t\right)^2$ at which point $\gamma = 1$ and every seller preempts.

Returning to check the assumption that $p^i < \tilde{p}$, the difference between two prices is $\tilde{p}(\ell) - p^i(\ell) = 1 - (2\ell)^{\frac{1}{2}} - (2\ell)^{\frac{1}{4}}$. This difference is strictly positive at ℓ_1 but decreases as ℓ increases, and at $\hat{\ell} \equiv \frac{1}{4}(7 - 3\sqrt{5})$ the informed price p^i begins to exceed the inclusion price $\tilde{p}$. If $\hat{\ell} > \ell_2$ (i.e. if $t \geq \frac{\sqrt{5}}{2} - 1$) then the equilibrium is as described above, else for $\ell \in [\hat{\ell}, \ell_2)$ sellers mix between the inclusion and preemption price and the equilibrium is described as follows:

$$u = 1 + \frac{2\ell}{1 - \sqrt{2\ell}}, \quad \tilde{p} = 2 - \frac{1}{1 - \sqrt{2\ell}}, \quad \gamma = \frac{2t}{1 - 2\sqrt{2\ell}}. \tag{5}$$

Observe that even when there is a regime change from the informed price p^i to the inclusion price $\tilde{p}$, the right boundary at which $\gamma = 1$ is the same value ℓ_2 as before. In addition, it can be shown that $\ell_2 > \ell_1$ and therefore that the Preemption region is nonempty.[8]

Returning to Figure 2, the Preemption region covers the interval $(\ell_1, \ell_2) \approx (0.05, 0.1)$. Starting at the left boundary ℓ_1, some sellers begin to charge the preemption price and consequently the average price paid falls, as depicted by the dotted line in the top panel. In addition, both the informed price p^i and the preemption price $\hat{p}$ fall with ℓ, while the consumer's search threshold (and consequently their payoff) increases. In the middle of the

[8] This follows because $0 = \Delta\pi(\ell_1) < \Delta\pi(\ell_2)$ and $\Delta\pi$ is increasing.

region at $\hat{\ell} \approx 0.07$ the inclusion price $\tilde{p}$ falls below the informed price p^i, and thereafter the inclusion price and preemption price are charged as ℓ increases, with both falling as we approach the region boundary. The switch from p^i to $\hat{p}$ accounts for the observed kink at $\hat{\ell}$ in the graph of the informed price.

Diamond equilibrium
For $\ell > \ell_2$ the Diamond equilibrium ensues with all sellers charging the price

$$p = \min\left(1 + \sqrt{2\ell}, \frac{3}{2}\right) \tag{6}$$

and consumers buying on their first visit without learning. At $\ell = \ell_2$ a continuum of equilibria with no search or learning can be supported, with search thresholds between $u = 0$ and $u = 2(2\ell_2)^{\frac{1}{4}}$, the limiting threshold in the Preemption region. This is a knife-edge case in which, when all sellers charge $\hat{p}$, the value of learning equals $w(\hat{p}+u) = w\left(\hat{p} + \left(\frac{1}{2} - t - \hat{p}\right)\right) = w\left(\frac{1}{2} - t\right) = \frac{1}{2}\left(\frac{1}{2} - t\right)^2 = \ell_2$, and thus is not a function of the level of $\hat{p}$. At ℓ_2 the value of learning thus exactly equals the cost, which allows for a continuum of $(u, \hat{p})$ combinations consistent with equilibrium. The highest supportable search threshold u_2 is the one at which the profit indifference from condition (iv) in the Preemption equilibrium holds (any higher threshold strictly favors charging the informed price), and lowest supportable search threshold is $u = 0$, beyond which the price charged by the next seller becomes irrelevant for the continuation value. The case of $\ell = \ell_2$ thus connects the equilibria between the Preemption and Diamond regions, making the set of equilibria upper-hemicontinuous across all parameters.

The transition from the Preemption to the Diamond region is depicted in Figure 2, with a continuum of preemption prices supporting an equilibrium exactly at the boundary, and then unique high preemption prices for higher learning costs. In the region just above ℓ_2 the preemption price continues to rise and the consumer's payoff continues to all, with the highest price $\hat{p}$ which preempts learning still below the ex-ante expected value of the product of 1.5. However, once ℓ grows sufficiently the equilibrium simply becomes that all sellers charge $p = 1.5$ and consumers buy immediately.

Comparative Statics

The example in Figure 2 suggests some novel comparative statics with respect to search frictions, here we establish these generally. In the low learning cost region we show equilibrium outcomes are those found in Wolinsky (1986), with higher learning and transportation costs having precisely the same anticompetitive effect as the search cost in Wolinsky's model. However, for intermediate learning costs the mechanics of the preemption equilibrium are quite different. We demonstrate that increasing learning costs is pro-competitive, resulting

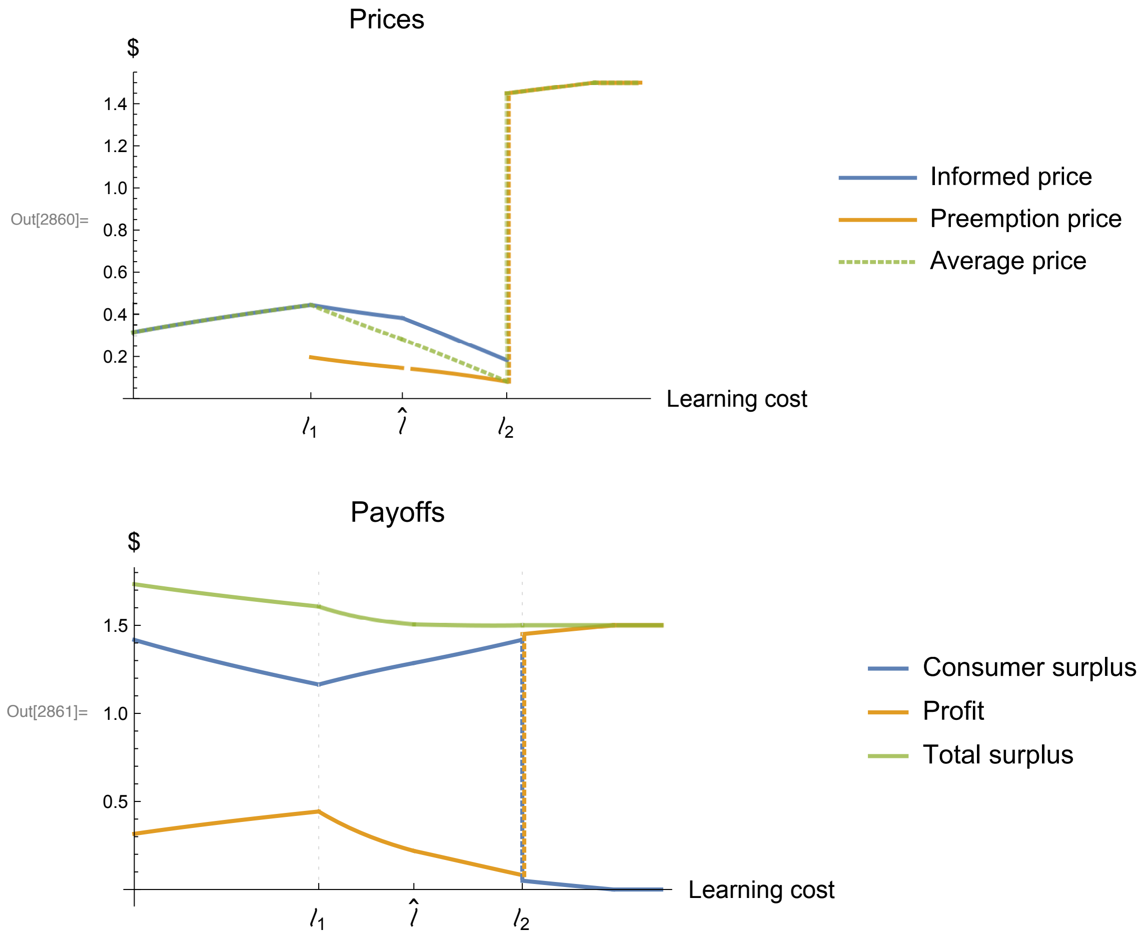

Figure 2: Equilibrium prices and payoffs when $\alpha = c = 0$, $t = 0.05$, and $\varepsilon \sim U[1,2]$. In the Wolinsky region $(0, \ell_1)$ only the informed price is charged, in the Preemption region (ℓ_1, ℓ_2) sellers mix between the preemption price $\hat{p}$ and the either the informed price p^i or the inclusion price $\tilde{p}$, whichever is lower, and in the Diamond region all sellers charge the preemption price $\hat{p}$.

in lower prices, higher consumer surplus, and lower profits. Interestingly, we also find that an increase in transportation costs lowers prices on average but does not affect the set of prices that are charged. Finally, we show that the transition from the Preemption equilibrium to the Diamond equilibrium corresponds to substantial increase in price, resulting in a much higher seller profit and a much lower consumer surplus, with total surplus slightly higher as well.

Lemma 1 *If $\ell \in [0, \ell_1]$ then a marginal increase in learning cost ℓ or transportation cost t result in the same increase in profit, decrease in consumer surplus, and decrease in total surplus.*

In the Wolinksy region $\ell \in [0, \ell_1]$ conditions (i) and (ii) of Definition 1 provide a closed-form solution for equilibrium values of p^i and u, and the comparative statics follow directly from the solution. The intuition is exactly as in Wolinsky's model – the direct effect of an increase in $t + \ell$ is to reduce u, which then leads to the indirect effect of an increased p^i and a further reduced u. For total surplus, observe that prices are simply a transfer and only match values and transportation and learning costs are relevant. In the Wolinsky region consumers expect the same price at every seller, and when deciding to search compare the search cost $t + \ell$ with the expected improvement in match value, which coincides with the incentives of the social planner. As ℓ increases in the Wolinsky region, search decisions thus remain socially optimal but total surplus falls both because search is now more expensive and because consumers settle for lower match values. Next we consider comparative statics for the Preemption equilibrium.

Lemma 2 *If $\ell \in [\ell_1, \ell_2)$ then a marginal increase in learning cost ℓ results in a decrease in profit, an increase in consumer surplus, and a decrease in total surplus. Meanwhile a marginal increase in transportation cost t does not affect consumer surplus and decreases producer surplus and total surplus.*

The driving force behind these results is that when the learning cost increases so does the profitability of preemption. In the preceding Wolinsky region this was not relevant because the preemption profit was strictly lower than the informed profit, but in the Preemption region an increase in ℓ now manifests in a higher proportion γ of sellers that charge the preemption price in equilibrium. This makes consumers better off by increasing their continuation value of searching, and by making consumers more selective reduces seller profits.

The effect on total surplus rests on the fact that preemption prices create a distortion in the consumers' search decision from the perspective of the social planner. In contrast to the Wolinsky region, consumers now search both to improve the match value but also to find a lower price, and thus from the social planner's perspective search too much. Because an increase in ℓ induces more search in the Preemption region, total surplus falls both because of the direct effect and the increased distortion. When the transportation cost t increases, we show that a higher proportion γ of sellers charge the preemption price, but otherwise that the consumer's search threshold, the preemption price, and the informed price all remain fixed. Consumer surplus thus remains unchanged, producer surplus falls because it simply equals the average price paid by consumers, and therefore total surplus also falls.

Finally we consider high learning costs where there is a sharp change in equilibrium outcomes.

Lemma 3 *Consumer surplus discontinuously jumps down and seller profits discontinuously jump up as ℓ moves from the region (ℓ_1, ℓ_2) to $(\ell_2, \infty]$. That is,*

$$\lim_{\ell \to^- \ell_2} \hat{p} < \lim_{\ell \to^+ \ell_2} \hat{p} \quad \textit{and} \quad \lim_{\ell \to^- \ell_2} u > \lim_{\ell \to^+ \ell_2} u.$$

This intuition behind this jump owes to a variant of the argument in Diamond (1971). Approaching ℓ_2 from the left in the Preemption region, a consumer visiting a seller setting preemption price $\hat{p}$ knows that almost surely he will visit another seller setting the same price, or with a small chance a seller with an even worse price. Diamond's logic would suggest that the current seller can slightly increase the price without losing the consumer to search, which is still true here, but instead he would lose the consumer to learning. This keeps the preemption price low even though essentially all sellers preempt learning, and thus sell an undifferentiated product to a consumer with a strictly positive transportation cost. However, there is a threshold value $w(\mu - t)$, which denotes the return to learning when all sellers charge the same price, and when the learning costs exceeds this threshold a seller can set a slightly higher price than his competitors and not induce learning. By this logic the low preemption price equilibrium "unravels" and what obtains is essentially a monopoly price equilibrium as in Lemma 3, with high seller profit and low consumer surplus.

It should be noted that the total surplus effect of the regime change from the Preemption to the Diamond equilibrium is neutral. While there is a large transfer from consumers to sellers due to the price change, on the equilibrium path consumers still do not search or learn and have the same expected match value μ as in the limit of the Preemption equilibria as the learning costs approaches ℓ_2.

3 Comparison with Monopoly

In the previous section we showed that in an oligopoly preempting learning may improve consumer welfare by reducing prices. Here we investigate whether the motive to preempt is strengthened by competition. We demonstrate that in fact a monopolist has a stronger incentive to preempt learning than an oligopoly seller, and that consumer surplus may be higher in a monopoly.

Consider a seller facing consumers with match values drawn from $F(\varepsilon)$ and continuation value u. From the seller's perspective, the only difference between monopoly and oligopoly is in the former case $u = 0$ while in the latter $u \geq 0$. The increase in u can be visualized as a downward shift of the seller's demand function,[9] and we will show the shift reduces preemption profit faster than informed profit.

Lemma 4 *When a seller faces a consumer with outside option u, the net benefit of preemption $\pi(\hat{p}) - \pi(p^i)$ falls in u.*

The preemption price satisfies $\ell = w(\hat{p} + u) \;\; \Rightarrow \;\; \hat{p} = w^{-1}(\ell) - u$ and thus $\frac{d\pi(\hat{p})}{du} = -1$. Put differently, in the preemption region demand is perfectly elastic and a downward shift of the demand curve at a rate of one causes a reduction in profit at a rate of one since quantity remains unchanged. Meanwhile, at the optimal informed price p^i, profit is $\pi(p^i) = (p^i - c)(1 - F(p + u))$ and changes at a rate

$$\frac{d\pi(p^i)}{du} = \left(\frac{dp^i}{du}\right)(1 - F(p^i + u)) - (p^i - c)f(p^i + u)\left(\frac{dp^i}{du} + 1\right).$$

The rate of change of the maximized profit when $\frac{dp^i}{du}$ is chosen optimally is weakly higher than if $\frac{dp^i}{du}$ is chosen specifically to equal -1. Thus

$$\frac{d\pi(p^i)}{du} \geq (-1)(1 - F(p^i + u)) > -1.$$

That is, the demand curve also shifts down at a rate of one and the seller is at least as well off as if he kept fixed his quantity by shifting the price down by one, which would result in a lost profit of $1 - F(p^i + u) < 1$. Thus the preemption profit falls faster than the informed profit, which leads to the following Lemma.

Lemma 5 *Let $\tilde{\ell}$ be the lowest learning cost at which a monopolist ($u = 0$) prefers to preempt. Then there exists a nonempty interval $[\tilde{\ell}, \ell_1)$ in which there is no preemption in an oligopoly ($u > 0$).*

Lemma 5 demonstrates that competition dampens the incentive to preempt, but this does not necessarily imply that consumers prefer a monopoly, since the informed price in an oligopoly is already low relative to the monopoly price, and may potentially be low relative to the monopoly preemption price. However, we can indeed find examples in which consumers welfare is higher under a monopoly than an oligopoly.

Lemma 6 *Consumers can be better off in a monopoly than in an oligopoly.*

We look for an example where the above statement holds and for this we return to the uniform distribution and assume cost $c = 0$. The preemption price is $\hat{p} = \sqrt{2\ell}$, which with

[9] The demand function is $q = 1 - F(p + u)$ and thus the inverse demand function is $p = F^{-1}(1 - q) - u$.

zero cost is also the profit, while the monopoly price is $p^i = \frac{1}{2}$ with ensuing profit of $\frac{1}{4}$. Preemption thus becomes profitable at learning cost $\tilde{\ell} = \frac{1}{32}$, at which the preemption price is $\hat{p}(\tilde{\ell}) = \frac{1}{4}$ and the consumer's payoff is $u = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$.

By Lemma 5 at $\tilde{\ell}$ the oligopoly equilibrium is in the Wolinsky region, which by equation (**??**) gives the consumer a payoff $u = 1 - 2\sqrt{2(t + \tilde{\ell})} = 1 - \frac{1}{2}\sqrt{1 + 32t}$. Then for all $t \geq \frac{5}{128} \approx 0.04$ the consumer's payoff is higher in a monopoly than in a duopoly. That is, if the transportation cost is sufficiently high then the informed oligopoly price is high enough to make the consumer worse off than the monopoly preemption price.

4 Obfuscation and Disclosure

We have thus far imposed that a seller cannot directly affect the information obtained by the consumer, but rather she can affect only whether that information is *useful* through price setting. In practice however, sellers can often affect how easy or difficult it is for consumers to learn their match value for their product. Examples of this abound, in particular for multi-attribute goods like health insurance plans and financial products, for which sellers decide which attributes to highlight and which to make more difficult to discover. Consider for instance the way in which the website Hotwire.com returns results for hotel searches. Some hotels are listed with their full set of attributes while others appear as "mystery" hotels, where the hotel's name, exact location, and other attributes are not revealed until after the purchase is confirmed. Each individual hotel decides whether to fully disclose its product or to obfuscate, and we wish to study this decision in conjunction with the pricing decision in our oligopoly search setting.

We operationalize the notions of disclosure and obfuscation by endowing each seller with the ability to set the cost of learning about her product to be equal to any value from zero to infinity.[10] Augmenting the original model in this way, the following proposition describes equilibria for different levels of transportation cost t.

Proposition 2 *If each seller i sets own learning cost ℓ_i then there exist $0 < t_1 < t_2$ so that*

i. if $t \in (0, t_1)$ then there is a Wolinsky equilibrium in which all sellers do not obfuscate and charge the informed price,

ii. if $t \in (t_1, t_2)$ then there is a Preemption equilibrium in which some sellers maximally obfuscate and charge the preemption price and some sellers do not obfuscate and charge the informed price,

[10] An alternative specification is a natural learning cost ℓ, and some obfuscation/disclosure cost to the seller to change the learning cost to some other ℓ'. As we will argue, this alternative specification is unlikely to change the qualitative description of the equilibrium.

iii. if $t \in (t_2, \bar{t})$ then there is a Diamond equilibrium in which all sellers maximally obfuscate and charge the preemption price.

The idea behind the equilibrium construction follows from understanding the obfuscation (disclosure) incentives of each individual seller. As before every seller takes the consumer's search threshold u as given and her two candidate optimal prices are either the preemption price $\hat{p}(u)$ or the informed price $p^i(u) = \min\left(\tilde{p}(u), \arg\max_p \ (p-c)(1-F(p+u))\right)$, which is the lower of the inclusion price or the "monopoly" price. By increasing ℓ the seller increases $\hat{p}(u)$ and thus the preemption profit, and either keeps unchanged or reduces the informed profit (the inclusion price $\tilde{p}(u)$ falls with ℓ and the "monopoly" price $\arg\max_p \ (p-c)(1-F(p+u))$ does not change with ℓ). This implies the optimum occurs at one of the two extremes: either a low enough ℓ at which the consumer learns at the "monopoly" price, or a high enough ℓ at which the preemption price is at its maximal value $\mu - u$. Consequently, for any given u one need only check whether the "monopoly" profit $\max_p \ (p-c)(1-F(p-u))$ or the preemption profit $\mu - u$ is optimal. As demonstrated previously, the preemption profit falls faster in u than the monopoly profit, and there exists a threshold $\bar{u}$ so that full obfuscation is optimal if $u < \bar{u}$ and full disclosure is optimal if $u > \bar{u}$.

With the incentives of each individual seller thus explained, the logic behind the equilibrium is straightforward. When the transportation cost t is low, the continuation value of search u is high, and therefore for each seller the informed profit is higher than the preemption profit. Thus for the range $(0, t_1)$ the Wolinsky equilibrium is supported, with all sellers charging informed prices. At $t = t_1$ the continuation value of search is the value $\bar{u}$ that makes sellers indifferent between informed and preemption profits. As t increase beyond t_1, a Wolinsky equilibrium can no longer be supported because if all sellers continued to charge the informed price then the continuation value of search would fall below $\bar{u}$, thus making the informed price no longer optimal. At the same time, if all sellers switched to the lower preemption price then the continuation value of search would jump back up above $\bar{u}$, thus making the preemption price not optimal. Instead, as t grows above t_1 a proportion γ of sellers charges the preemption price, where γ is calibrated so that the continuation value of search is exactly $\bar{u}$. This preemption region (t_1, t_2) extends until the proportion of preempting sellers γ equals one, at which point we enter the Diamond region in which all sellers charge the preemption price.

Similar to when learning costs are exogenous, we again obtain that an increase in search frictions increases prices when those frictions are small, but then decreases prices when they become intermediate. We also generate the coexistence of sellers that obfuscate and sellers that disclose, as in the example with Hotwire.com, in a model with homogeneous sellers. Furthermore, if we introduce seller heterogeneity, for instance with respect to the production cost c, then by the above logic it follows that in equilibrium sellers follow a threshold strategy, with high cost sellers following a transparent policy and low cost sellers obfuscating. Returning again to the Hotwire.com example, suppose that the cost of selling a room at a date in the future is the lost opportunity to charge a higher price if substantial demand arrives in the interim. Hotels with a higher proportion of vacant rooms at a particular future date thus have a lower cost of selling one of those rooms, and our model predicts that these hotels are more likely to be the "obfuscating" mystery hotels, while hotels with lower vacancy rates are likely to provide full listings.

5 Conclusion

Buyers in differentiated product markets can be uncertain both about prices and match values when searching. But while the price is often easily discovered upon arriving at a store, further costly effort is typically necessary to learn the product's match quality. For example, a consumer visiting an Apple store finds the price of a MacBook Pro at once, but her value for the particular combination of RAM, screen quality, weight, battery life, and the myriad other dimensions of the product takes longer to learn and internalize. A consumer is often faced with the decision of whether to think more deeply about her private value of the vector of attributes of a product, or to simply buy it and to hope that on average the product is a good match.

Typically the cost of discovering the product's price (transportation cost) and the cost of learning the match quality (learning cost) have been lumped into a single search cost, as is the case in the well-known Wolinsky (1986) framework. In this paper we unbundle these costs, explicitly embedding the learning decision in Wolinsky's model, and find that the effects of the two search frictions are quite different. In particular, the learning cost introduces an incentive for sellers to set low prices to preempt consumers from learning their match values that is not present in the standard model.

When learning costs are low consumers always discover their match values and the equilibrium and its comparative statics resemble Wolinsky's. However, for intermediate values of the learning cost the Wolinsky equilibrium is no longer supported and instead an equilibrium emerges in which some sellers set a high price that induces learning and others a low price that preempts learning. In this region higher learning costs result in lower prices and higher consumer surplus, and higher transportations costs have no impact on payoffs. When learning costs become sufficiently large however, there is a shift to essentially monopoly pricing due to the Diamond (1971) logic.

Be explicitly modeling the learning decision we demonstrate that search frictions are not by default anticompetitive, and instead a more nuanced story emerges. In particular, the preemption effect can be so strong as to overpower the classic effect of competition, as we show in an example in which for a given learning cost consumers are better off facing a monopolist, who chooses to preempt learning, rather than an oligopoly in which sellers induce consumers to become informed. The motive to preempt learning can thus lead to lower prices, but also induces the market to become more homogeneous which, due to the Diamond argument, can result in dramatically higher prices.

Our model predicts that in equilibrium there are high priced products which consumers research and low priced priced products about which consumers do not learn, and indeed this is often borne out in online markets. For instance, a hotel search on Hotwire.com returns a list of regularly priced hotels interspersed with cheaper mystery hotels, which are of the same quality as the regularly priced hotels but about which consumers cannot learn specific details, such as the hotel's name and exact location. Our results can provide guidance for these platforms in terms of design. In particular, the model suggests that for products with already low learning costs imposing a design in which learning is made even easier would help consumers. On the other hand, if learning costs are substantial then consumers may benefit instead when sellers are allowed to further obfuscate. The opposite holds for the payoffs to the sellers, and insofar as a platform weighs the benefits of increasing producer versus consumer surplus, our model can be used to determine what kind of design is optimal.

The equilibria that we find in the intermediate range are also of interest in that price dispersion is generated without any ex-ante heterogeneity on either the buyer or seller side, a result that to the best of our knowledge has only been obtained in Garcia et al. (2015) in a vertical market. In addition, in the spirit of Bar-Isaac et al. (2012) we can interpret the high priced products that induce learning as niche products and the low priced products that preempt learning as broad products. The departure in the current model is that the products are actually all the same, and they are made niche and broad by their consumers' decisions about whether to become informed, rather than by the seller's explicit design of the product toward a narrower or broader audience.

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6 Appendix (Under Construction)

Proof of Proposition 1

We will first demonstrate that there is a Wolinsky equilibrium for $\ell \in [0, \ell_1)$, a Preemption equilibrium for $\ell \in (\ell_1, \ell_2)$, and a Diamond equilibrium for $\ell \in (\ell_2, \infty)$, and that in all these ranges the equilibria are unique. We begin with a lemma that will be useful in these proofs, and then proceed to each region.

Lemma 7 *Let $\hat{\pi}(u) \equiv \hat{p}(u) - c$ and $\pi^i(u) \equiv \max_p (p-c)(1 - F(p + u - \alpha))$ denote the preemption and informed profits. Then $\Delta\pi(u) \equiv \hat{\pi}(u) - \pi^i(u)$ decreases in u.*

Proof of Lemma: We can write the maximized informed profit as $\pi^i(u, p(u)) = (p(u) - c)(1 - F(p(u) + u - \alpha))$, where $p(u)$ solves the first order condition $0 = 1 - F(p + u - \alpha) - (p - c)f(p + u - \alpha)$. Also, recall that $\hat{\pi}(u) = \hat{p}(u) - c = w^{-1}(\ell) - u - c$. Then,

$$\frac{d\Delta\pi}{du} = -1 + (p(u) - c)f(p(u) + u - \alpha) = -F(p(u) + u - \alpha),$$

where the first equality applies the envelope theorem to the derivative of π^i, and the second equality applies the first order condition. ■

Lemma 8 *There exists a learning cost $\ell_1 > 0$ so that a Wolinsky equilibrium exists only if $\ell \in [0, \ell_1]$, and this equilibrium is unique.*

Proof of Lemma First we compute the threshold u and informed price p^i consistent with Wolinsky's equilibrium. Solving the system of conditions (i) and (ii) in Definition 1 yields

$$u = \alpha - c + W^{-1}(t + \ell) - \frac{1 - F(W^{-1}(t + \ell))}{f(W^{-1}(t + \ell))}, \qquad p^i = c + \frac{1 - F(W^{-1}(t + \ell))}{f(W^{-1}(t + \ell))}. \tag{7}$$

To verify these values constitute an equilibrium we must meet conditions (iii), (iv), and (v) from Definition 1. Beginning with (v), the difference in profit between charging the informed price and the preemption price is

$$\begin{aligned}
\Delta\pi(\ell) &\equiv (\hat{p} - c) - (p^i - c)(1 - F(u + p^i - c) \\
&= \left(w^{-1}(\ell) - \left(W^{-1}(t + \ell) - \frac{1 - F(W^{-1}(t + \ell))}{f(W^{-1}(t + \ell))}\right)\right) - \left(\frac{\left(1 - F(W^{-1}(t + \ell))\right)^2}{f(W^{-1}(t + \ell))}\right) \\
&= w^{-1}(\ell) - \varphi(\varepsilon(t + \ell)),
\end{aligned} \tag{8}$$

where in the final line we group terms and let $\varepsilon(t + \ell)$ denote the solution to $W(\varepsilon) = t + \ell$. Note that $\varphi(\underline{\varepsilon}) = \underline{\varepsilon}$, that $\varphi(\bar{\varepsilon}) = \bar{\varepsilon}$, and that taking a derivative and simplifying yields

$$\varphi'(\varepsilon) = F(\varepsilon)\left(2 + \frac{f'(\varepsilon)(1 - F(\varepsilon))}{f^2(\varepsilon)}\right) \geq F(\varepsilon)\,(2 - 1) > 0,$$

with the initial inequality following from the assumption that $\frac{f(\varepsilon)}{1-F(\varepsilon)}$ is increasing. Then because $w^{-1}(\cdot)$ is increasing and $W^{-1}(\cdot)$ is decreasing, it follows that $\Delta\pi(\ell)$ is increasing. Defining $\bar{\ell} = W(\underline{\varepsilon}) - t$, then

$$\Delta\pi(0) = w^{-1}(0) - \varphi(\varepsilon(t)) > w^{-1}(0) - \varphi(\underline{\varepsilon}) = 0$$

and

$$\Delta\pi(\bar{\ell}) = w^{-1}(\ell_2) - \underline{\varepsilon} > 0,$$

and therefore there exists $\ell_1 \in (0, \bar{\ell})$ so that $\Delta\pi(\ell) < 0$ if and only if $\ell \in [0, \ell_1)$.

Now to check the remaining conditions of Definition 1, condition (iii) is satisfied because $p^i + u - \alpha = W^{-1}(t + \ell) \geq \underline{\varepsilon}$, with the inequality following by assumption that $t + \ell$ is small enough to satisfy this exact condition. Finally, for condition (iv) it follows from $\Delta\pi(\ell) < 0$ that $\hat{p} < p^i$, and because $W(p^i + u - \alpha) = t + \ell > 0$ while by definition $W(\tilde{p} + u - \alpha) = 0$, it follows that $\tilde{p} > p^i$. With all five conditions of Definition 1 satisfied, this concludes the proof of equilibrium existence.

To demonstrate uniqueness, we conjecture toward a contradiction that the equilibrium the search threshold u' is higher than the value u from (7). By Lemma 7 the informed profit is higher than the preemption profit, thus only the informed price may be charged in equilibrium. But only the value u from (7) and not u' is consistent with an equilibrium in which only the informed price is charged, thus there is a contradiction.

Suppose instead that u' is lower than the value u from (7). From condition (i) of Definition 1 we know that if all sellers charged $p^i(u')$ then the optimal search threshold $u^*(p^i(u')) > u'$. Furthermore, if some positive proportion of sellers charges $\hat{p}(u') < p^i(u')$ then the return to searching is even higher. Therefore, no equilibrium at this lower u' can be supported.

We have thus shown that when $\ell \in [0, \ell_1)$ then the Wolinsky equilibrium exists and is unique. ∎

Now we describe the equilibrium in the Preemption region (ℓ_1, ℓ_2) with $\ell_2 \equiv w^{-1}(W(\underline{\varepsilon}) - t)$. There are two possible sub-regions here – near ℓ_1 the informed price is below the inclusion price and sellers mix between the informed price and the preemption price. As ℓ increases the inclusion price falls more quickly than then informed price, and there is a cost $\hat{\ell}$ at which the two prices intersect. If transportation cost t is sufficiently large then $\hat{\ell} > \ell_2$ and the inclusion price never binds. However, if the transportation cost is below a threshold value then $\hat{\ell} < \ell_2$ and the end of the preemption region is characterized by sellers mixing between the inclusion price and the preemption price.

Lemma 9 *There exists $\hat{\ell} \in (\ell_1, \ell_2)$ so that for $\ell \in (\ell_1, \hat{\ell})$ there is a Preemption equilibrium* *with* $\min(p^i, \tilde{p}) = p^i$ *and for* $\ell \in (\hat{\ell}, \ell_2)$ *there is a Preemption equilibrium with* $\min(p^i, \tilde{p}) = p^i$. *Furthermore, for each* $\ell \in (\ell_1, \ell_2)$ *there is a unique equilibrium.*

Proof of Lemma Fix $\ell \in (\ell_1, \ell_2)$. Recall from Lemma 8 that if $\ell > \ell_1$ then a Wolinsky equilibrium in which all sellers charge the informed price cannot be supported. In particular, the (u, p^i) that solve (7) are such that the preemption price $\hat{p}(u)$ is more profitable than then informed price p^i.

It is also the case that no equilibrium can be supported in which all sellers charge the preemption price. In such an equilibrium the continuation value of searching when expecting the next seller to charge $\hat{p}(u) = w^{-1}(\ell)+u$ is $-t+\alpha+\mu-w^{-1}(\ell)+u < -t+\alpha+\mu-w^{-1}(\ell_2)+u = 0$. Therefore there is no $u > 0$ for which this an equilibrium at ℓ.