Incentives of Low-Quality Sellers to Disclose Negative Information*

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Abstract

We study incentives of low-quality sellers to disclose negative information when the only way to communicate one's quality is cheap-talk messages. This setting limits ability of high-quality sellers to separate as any communication strategy they pursue can be costlessly imitated by low-quality sellers. Two factors that can incentivize low-quality sellers to communicate their quality are buyers' risk-attitude and competition. Quality disclosure reduces buyers' risk increasing their willingness to pay. It also introduces product differentiation softening the competition. We show that equilibria where low-quality sellers separate exist under monopoly and duopoly. Equilibria where high-quality sellers separate exist under duopoly only.

Keywords: Negative information, product differentiation, cheap talk, lemon markets

JEL classification: D21, L15

1 Introduction

Is honesty the best policy for sellers when it comes to revealing negative aspects of their products? Arguably, in the presence of asymmetric information about product quality, low-quality sellers would be better off concealing information about weaknesses of their products. Many empirical studies also support this view by documenting multiple instances where negative information damages sales and purchases likelihood, through various routes such as publicity, negative customer reviews, or word-of-mouth (Berger, Sorensen & Rasmussen 2010). Therefore, it seems natural for sellers to hide negative information about their products from their customers.

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At the same time, we can find many cases where sellers voluntarily share negative information about their products, contrary to what one would have expected. For example, Chipotle Mexican Grill's website highlighted their drawbacks in its "Room for Improvement" section, in addition to explaining positive aspects of their ingredients.[1] Woot.com is famous for its preemptive revelation of the disadvantages of listed products, stating that they would prefer customers not buying from them to regretting their purchases.[2]

Many online retailers choose to disclose negative information about their products, albeit indirectly, through customers' reviews on their websites. In the early days of online trading, consistent with the view that negative information can only damage sellers' profit, sellers were reluctant to allow customers to freely leave feedbacks on their websites. For example, online retailers working with Amazon were "*initially terrified*" of Amazon's product review policy, worrying about what bad reviews could do to their bottom line. However, according to Craig Berman, Amazon's then vice president of global communications, its product review system turned out to have positive effects on their performances despite the initial fears (Webley 2010). Ghose and Ipeirotis (2011) have analyzed user-generated online product reviews with text-mining techniques and found that negative product reviews are associated with increased product sales when the review text is informative and detailed, so that it reduces customers' uncertainty about the product.

Voluntary disclosure of negative information can also be found in many consumer-to-consumer online marketplaces. Sellers at eBay often voluntarily describe weaknesses of their listed products, both via cheap-talk messages such as "*the product is in fair condition*", and via verifiable information such as pictures of specific damages and scratches. Notably, sellers at Craigslist also often reveal negative information about their listings even though there is no reputation building mechanisms and most sales are one-time interactions with no repeated-game incentives of being honest. Jin and Kato (2006) found that many online sellers of collectible baseball cards revealing low grades were actually honest about their claims, even though buyers could not correctly evaluate the quality prior to purchase.

Several empirical studies have also found some evidences that negative information may not necessarily hurt profits. A frequently observed practice of two-sided advertising, wherein sellers provide certain negative information about their products in conjunction with asserting positive claims, has been shown to help sellers by enhancing credibility and positively affecting consumer choices (Crowley & Hoyer 1994; Eisend 2006, 2007; Settle & Golden 1974). Similarly, the blemishing effect, which is an introduction of small amount of negative information, has been shown to enhance positive image of the product, both in the lab and in the field settings (Ein-Gar, Shiv & Tormala 2012).[3]

Even though the phenomenon of seller honesty is not as uncommon as one would expect, it

[1] https://www.bloomberg.com/news/articles/2013-06-18/the-genetically-modified-burrito-chipotle-tells-all

[2] `http://www.woot.com/faq?ref=ft_wiw_faq`. Archived version: `https://archive.is/b9cxI`

[3] In the literature, the blemishing effect is considered to be different from the two-sided message's effect. While the former is just about highlighting a single minor flaw or downside, the latter is about addressing negatives, offering more complete information, or seeming more balanced (Ein-Gar et al. 2012, p. 856).

has not received much attention in the academic literature especially from theoretical perspective, and the extensive literature on information disclosure has focused primarily on the tension between consumers who want more information on quality and low-quality sellers who would like to hide it (Dranove & Jin, 2010). This paper thus attempts to fill this gap by investigating the incentives of low-quality sellers to disclose, rather than conceal, negative information about their products.

To focus on the incentives of low-quality sellers, we consider a model where buyers cannot evaluate the quality of the products,[4] and standard tools that high-quality sellers can use to signal their quality are not available. More specifically, there are no repeated purchases, no reputation concerns, no warranties, and buyers do not use prices to infer quality. The only tool available to sellers to affect buyers' beliefs about product's quality is cheap-talk messages. The cheap-talk setting limits the ability of high-quality sellers to separate, as any communication strategy they employ can be costlessly imitated by low-quality sellers, thereby shifting the focus to incentives of low-quality sellers' in any information transmission. We identify two factors that can incentivize low-quality sellers to reveal their quality in our model. First, revealing one's quality, whether it is low or high, reduces the risk associated with the purchase, and increases buyers' willingness to pay. The literature has generally agreed that risk has a major negative influence on customers' purchase decisions (Bauer 1960; Dowling 1986; Markin, Jr. 1974; Ross 1975; Stone & Winter 1985; Taylor 1974). In an online setting, Dewally and Ederington (2006) showed empirically that risk reduction increases the valuation of listed products on online auctions. Second, revealing one's quality allows a seller to differentiate one's product from the competitor's, thereby softening the competition and increasing one's profits. Jin and Sorensen (2006) have shown that hospitals' decisions to disclose quality scores, either being positive or negative, are driven by incentives to differentiate themselves from competitors.

To separate the roles of these two factors, we first consider the setting with a monopolistic seller and then extend it by adding the second seller afterwards. The model's setup is as follows. In the case of the monopolistic setting, the seller observes his type and sends a cheap-talk message. Buyers update their beliefs about the product's quality based on the received message, and then buyers' demand is determined by the updated beliefs. The seller sets the profit-maximizing price given the demand, and buyers decide whether to purchase the product. The duopoly setting is similar, except that there are two sellers and their prices are determined by an equilibrium of a pricing subgame. Sellers are risk-neutral profit-maximizers and buyers are not risk-neutral and dislike uncertainty about product quality. We model buyers' risk-attitudes using two alternative frameworks: risk-aversion and loss-aversion. Buyers differ in their degree of loss- or risk-aversion. Other things being equal, buyers with higher (lower) loss/risk-aversion are more (less) likely to prefer the product with certain but lower quality over the product with uncertain but higher expected quality.

In our setting, sellers, when deciding whether to reveal their types or not, face a potential trade-off between a *quality advantage* of pretending to be of a higher quality, and an *information advantage* of revealing their (lower) quality and removing uncertainty. The strength of these two

[4] In other words, there exists information asymmetry about quality, and the products are high in experience or credence attributes.

effects determine whether revealing one's quality can occur in equilibrium. In the monopoly setting we show that low-quality and only low-quality sellers can separate. Were high-quality sellers able to separate, low-quality sellers would always find it optimal to imitate them. A low-quality seller, on the other hand, can choose to separate as long as the benefits of information disclosure are higher than the benefits of a high-quality claim. Furthermore, since benefits of separation for low-quality sellers are not as high, imitating the low-quality type who chooses to reveal its quality is not as attractive.

In the second part of the paper, we study how incentives and opportunities to disclose quality information change in the duopoly environment with two sellers. It turns out that equilibria under the duopoly setting include outcomes that were not possible under monopoly setting. First, there are equilibria where the highest-quality type separates, and the low-quality type chooses not to imitate it. Second, in the case of risk-averse buyers, there are equilibria where the seller with the lowest quality separates. Both types of equilibria become possible because of the effect that competition has on the quality advantage. Different from the monopoly setting, sellers' profit depends not only on their expected quality but also on the competition intensity. If a message associated with higher quality is likely to result in a more intensive competition, it weakens the quality advantage, which in turn weakens the incentives of low-quality sellers to pool with high-quality types. That allows for a possibility of high-quality sellers to separate in equilibrium and, in the case of risk-averse buyers, allows for an equilibrium where the lowest-quality seller chooses separation over pooling with high-quality types.

Overall, this study contributes to the literature on information disclosure by focusing on the incentives of low-quality sellers to reveal negative information. First, we show that even when the ability to communicate one's quality is limited to cheap-talk messages, and when there are no market frictions, such as search or matching, information transmission is nonetheless possible. Second, due to the limited ability of high-type sellers to communicate their quality, the information transmission is driven by incentives of low-quality sellers. Third, we identify two factors—buyers' risk-attitude and increased product differentiation—that can incentivize low-quality sellers to separate. We analyze the role of each factor on information disclosure and show how they interact with each other. Finally, despite the cheap-talk nature of information transmission, we show that under certain conditions low-quality sellers might choose not to imitate high-quality sellers, allowing for cheap-talk equilibria where high-quality sellers separate.

The paper is organized as follows. In Section 2 we review related literature. In Section 3 we introduce our benchmark model assuming monopoly, and in Section 4 we consider the effect on competition by examining the duopoly situation. Section 5 provides an extension of the model by showing what happens when price is used as a signal of quality. We summarize and discuss the findings of this paper in Section 6. All proofs are provided in the appendix.

2 Literature Review

The literature on information disclosure and information asymmetry has been primarily centered on the analysis of conflicting interests of high-quality sellers who want to credibly communicate their quality information to customers, and low-quality sellers who want to hide it. Studies on unraveling, for example, have argued that full disclosure naturally begins from the seller with the highest quality and advances to the sellers with lower quality (Grossman 1981; Milgrom 1981; Viscusi 1978), without considering how low-quality sellers might initiate information disclosure. Empirical literature has mainly examined how negative information hurts sellers, suggesting little basis for low-quality sellers' information disclosure. For example, many studies have shown that negative information decreases sales and purchase likelihood through publicity (Tybout, Calder & Sternthal 1981; Wyatt & Badger 1984), word-of-mouth (Arndt 1967; Engel, Kegerreis & Blackwell 1969; Haywood 1989; Laczniak, DeCarlo & Ramaswami 2001; Mizerski 1982; Wright 1974), and customer reviews (Basuroy, Chatterjee & Ravid 2003; Chevalier & Mayzlin 2006; Clemons, Gao & Hitt 2006; Dellarocas, Zhang & Awad 2007; Reinstein & Snyder 2005).

Our paper differs from that strand of literature and belongs to a smaller group of papers that study incentives of low-quality sellers to disclose quality information. Board (2009) has shown that in the framework with risk-neutral buyers where information disclosure is costless, credible, and verifiable, low-quality sellers may disclose their types if the loss from lower perceived value is smaller than the gain from decreased competition with high-quality sellers. Guo and Zhao (2009) have considered a duopoly setting where private information can be credibly and truthfully, though not costlessly, disclosed. In their setting, duopoly sellers consider voluntarily disclosing quality information even when the quality is not the highest, in order to achieve differentiation and avoid direct competition. Our paper has a similar trade-off between information disclosure and product differentiation, but in a different setting, where private information cannot be credibly communicated and buyers are not risk-neutral. In the setting where quality information is not verifiable, Gardete (2013) has applied a cheap-talk model to a market with a search good, where customers know the true quality before purchase. He shows that if customers differ in their marginal valuations for quality, then low-quality firms may want to reveal their types to attract those customers with low marginal valuations for quality. Kim (2012) has shown that, in the presence of search and matching frictions and when it is buyers who make offers, low-quality sellers can use cheap-talk messages to reveal their types in order to attract more buyers and intensify competition among them. Similar to Gardete (2013) and Kim (2012), we assume that quality information is communicated via cheap-talk messages. However, in our paper, buyers cannot learn quality prior to purchase, and there are no search and matching frictions. Finally, Tadelis and Zettelmeyer (2015) have performed a large-scale field experiment in wholesale automobile auctions and proved that disclosure of negative information can increase the revenue of sellers through matching buyers with different quality preferences to appropriate markets.

Our paper is also related to the literature on cheap-talk communication between customers and sellers when their incentives do not align, information disclosure is payoff-irrelevant, and there is no credibility cost (Aumann & Hart 2003; Gardete 2013; Li 2005; Yi Zhu & Dukes 2015). Various instruments, such as properly termed revenue-sharing contracts (Li 2005) and advertising (Gardete 2013), have been found to match the incentives of sellers and buyers, thereby making the cheap-talk communication credible. In our setting, the information transmission also occurs via cheap-talk messages, and the incentives of sellers and buyers are originally misaligned as sellers want to hide weaknesses of their products while buyers would like to have that information. However, the consideration of risk and risk-attitudes of customers play the role of matching buyers' and sellers' incentives and making credible information disclosure mutually beneficial. Therefore, our paper contributes to this literature by suggesting that consideration of risk can facilitate cheap-talk communications by aligning the incentives of both parties.

3 Monopoly

3.1 Basic setup

We first consider a model with a seller that sells a product with exogenously given quality that is unobserved by buyers. The quality is distributed with a cdf $F(v)$ on an interval $[v_L, v_H]$ and the distribution can be either discrete, or continuous with a positive density on $[v_L, v_H]$. The marginal cost of the product with quality v is c_v. Unless explicitly stated otherwise, we assume that c_v and $v - c_v$ are strictly increasing functions of v. If the seller of type v serves share s_v of buyers at price p_v, his expected profit is

$$U_v = (p_v - c_v)s_v.$$

There is a continuum of buyers that we normalize to 1. Buyers' utility is determined by the price, p, at which they purchase the product, as well as their beliefs about the product's quality, μ. Buyers are not risk-neutral and dislike uncertainty about product quality. We model buyers' risk-attitude using two alternative frameworks. The first framework assumes that buyers are loss-averse with the references point endogenously determined by the expected quality. Among a variety of reference-dependent models with endogenous reference points (see e.g. Gul, 1991; Shalev, 2000; or Köszegi & Rabin, 2006) the framework that we employ in the paper *"... has proven quite popular in applications, as the reference point is neither stochastic nor recursively defined, but is simply the expected consumption utility of the lottery."* (Masatlioglu and Raymond, 2016, p. 2765).[5] Specifically, if, given beliefs μ and price p, the buyer purchases the product of quality v, then his

[5] To see how relation to Köszegi & Rabin (2006), consider a simple case of two qualities (v_L, v_H) with probabilities (q_L, q_H). Define the reference point as getting the product with quality Ev with probability 1. Ignoring price p, for the sake of example, the agents' utility from getting the product is $Ev + q_L(\lambda - 1)(v_L - Ev)$. Here λ corresponds to notations from Köszegi & Rabin (2006), and is the slope in the domain of losses. According to notations in our paper $\lambda - 1 = b$. Expression $Ev + q_L(\lambda - 1)(v_L - Ev)$ is exactly the equation (2) in our paper. The difference between preferences used in our paper and Köszegi & Rabin (2006) is in the utility of not buying the product. In our paper it is 0. For the KR-agents, utility from not buying is not zero but $-\lambda Ev$. Thus for the KR preferences, it is a personal equilibrium to have reference point "quality Ev with probability 1" and decision "to buy" whenever $Ev + q_L(\lambda - 1)(v_L - Ev) > -\lambda Ev$. In our paper, it's simpler. Buyers buy whenever $Ev + q_L(\lambda - 1)(v_L - Ev) > 0$.

utility is

$$u_b(v,p,\mu) = \begin{cases} v-p & \text{if } v \geq E_\mu v \\ v-p+b(v-E_\mu v) & \text{if } v < E_\mu v \end{cases} \tag{1}$$

$E_\mu v$ is a reference point that determines whether the outcome is viewed as a gain or a loss. Parameter $b \geq 0$ measures the degree of loss-aversion. If the purchased product has the quality below what the buyer had expected, then the buyer experiences the loss. When $b = 0$, the buyer is risk-neutral. Higher b means higher degree of loss-aversion. In what follows, we assume that buyers differ in their degree of loss-aversion. We assume that b is distributed with a positive differentiable log-concave density $\phi(b)$ and support $[0, B]$. At the same time, as equation (1) indicates, all buyers value quality equally and have the same price-quality trade-off.

Taking expectations of (1) over v we get

$$U_b^{LA}(p,\mu) = E_\mu v - p + b\int_{v_L}^{E_\mu v}(v-E_\mu v)f(v)dv = E_\mu v - p - b \cdot ELoss_\mu, \tag{2}$$

where $ELoss_\mu = -E[(v-E_\mu v)\cdot \mathbf{1}_{v<E_\mu v}] > 0$ is the buyer's expected loss, which is defined to be positive.

The second framework is the expected utility framework with risk-averse buyers. Buyers have concave Bernoulli utility function, $u(\cdot)$, with constant absolute risk-aversion. Buyers' utility from purchasing a product with uncertain quality v at price p is

$$U_b^{EU}(p,\mu) = E_\mu u(v-p). \tag{3}$$

Given the CARA assumption, we do not need to specify the initial wealth. Buyers differ in the degree of absolute risk-aversion, γ. With a slight abuse of notations, we will denote the distribution of buyers' risk-aversion as $\Phi(\gamma)$. Density $\phi(\gamma)$ is assumed to be positive, differentiable, and log-concave.[6]

Sellers can communicate their product's quality to buyers using cheap-talk message, $m \in \mathcal{M}$. We assume that $\mathcal{M}$ is rich enough that it includes the support of $F(v)$. The timing is as follows. First, the seller learns his type, v. Second, the seller sends a publicly observable cheap-talk message $m \in \mathcal{M}$. Third, buyers observe the message and form posterior beliefs $\mu(m)$ about the quality distribution, which determines their demand for the seller's product. Fourth, the seller chooses the price p, and, finally, buyers decide whether to purchase the product or not.

[6]Both loss-aversion and expected-utility frameworks capture the idea that buyers receive dis-utility when the product's quality is uncertain. The loss-aversion framework has a simpler functional form, making it more tractable. However, just like many non-EU frameworks such as variance-aversion frameworks, the loss-aversion preferences can violate state-dominance. For example, when b is high, the buyer might prefer a product with known low quality over the product whose quality can be either low or high. It violates state-dominance as the latter option will result in either the same (low) quality as the former option, or in the better (high) quality. Yet, buyers with high b will prefer the former. Our results, however, are not based on the violation of state-dominance, as state-dominance holds in the expected-utility framework.

Definition 1 *An equilibrium is a quadruple* $(m(v), p(m, v), \mu(m), s(\mu, p))$ *where* $m(v)$ *is the seller's messaging strategy,* $p(m, v)$ *is the seller's pricing strategy,* $\mu(m)$ *is the buyers' beliefs about the quality distribution, and* $s(\mu, p)$ *is the share of buyers who purchase the product such that the following conditions hold:*

a) given m *and buyer's demand* $s(\mu(m), p)$*, the seller of type* v *chooses price,* $p(m, v)$*, that solves his profit-maximization problem:*

$$\max_p (p - c_v) s(\mu(m), p);$$

b) the seller of type v *sends message* m *that maximizes his profit given buyers' beliefs and purchasing decisions*

$$\max_{m \in \mathcal{M}} (p(m, v) - c_v) s(\mu(m), p);$$

c) buyers' purchasing decision is optimal, that is

$$s(\mu, p) = \Pr(\{U_b(p, \mu) \geq 0\});$$

d) if message m *is sent with positive probability, buyers beliefs* $\mu(m)$ *are derived from* $m(v)$ *by the Bayes' rule.*

Some remarks are due here. First, we assume that prices are determined after the message, and not jointly. This assumption is not essential in the case of a monopolistic seller. We do impose it for the sake of the similarity with the duopoly setting. Second, buyers' beliefs, μ, depend only on cheap-talk message m. The model is stripped away from standards mechanisms that are identified in the literature as a way for high-quality sellers to credibly signal their quality. There are no repeated purchases, warranties, reputation, and buyers do not use prices to infer a product's quality. Limiting the ability of high-quality sellers to signal their quality puts focus on the incentives of low-quality sellers who, if they choose to, can costlessly imitate any strategy pursued by high-quality sellers. If any information about low-quality product is revealed, it is driven not by high-quality sellers' ability to separate, as is the case in unraveling or education-as-signaling models, but by low-quality sellers' intent not to pool with high-quality sellers. Finally, even though sellers with different qualities will set different profit-maximizing prices given the same beliefs μ, we assume that prices do not have the signaling role, as they do not affect buyers' beliefs.[7] That is the case when, for example, buyers do not have information regarding the underlying sellers' cost structure, or demand conditions to infer quality from sellers' prices. From an empirical point of view, the literature has identified product categories for which there is a weak or no relationship between perceived quality and prices.[8] From a theoretical point of view, it is well-known that under certain

[7] An alternative way to ensure that prices do not serve as signals is by letting an uninformed party — buyers, in our case — make a price offer (see e.g. Kim 2012).

[8] Rao and Monroe (1989) and Lichtenstein and Burton (1989) have found that customers are not capable of predicting quality from price signal for durable, higher-priced, or non-frequently purchased products. In their recent meta-study that covered 23 studies on price-perceived quality relationship, Vöckner and Hofmann (2007) have shown that durable goods as well as services have much weaker price-perceived quality relationships than fast-moving

conditions prices can serve as credible signals of quality resulting in the fully revealing equilibrium. What our setting allows us to demonstrate is that, even if prices are not perceived to be informative about products' quality, information transmission is, nonetheless, possible. In Section 5, we relax this assumption and consider an extension where prices serve as signals of quality.

3.2 Equilibrium Analysis

We now begin analysis of our framework. As a benchmark, we first look at the case of risk-neutral buyers. Proposition 1 shows that there is no equilibrium with any relevant information being transmitted, where by relevant information we mean information that changes buyers' valuation of the product. This proposition is trivial, so no formal proof is provided. Intuitively, if two on-equilibrium messages result in different beliefs about expected quality, then no seller would find it optimal to send the message with lower expected quality.

Proposition 1 *If buyers are risk-neutral, then for any on-equilibrium message m, posterior beliefs, μ, are such that $E_{\mu(m)}v = Ev$. After any equilibrium messages, the seller, regardless of the product's quality, charges price $p(m, v) = Ev$ and serves the whole market.*

Consider now a case when buyers are not risk-neutral. In what follows, we say that message $m_s \in \mathcal{M}$ is a *separating* message if $Pr(v = v_s|m_s) = 1$ for some type v_s. We say that $m_p \in \mathcal{M}$ is a *pooling* message if there is more than one type that sends m_p. We say that a seller *separates* if he sends a separating message with positive probability. Otherwise, we say that a seller *pools.*

It is straightforward to derive the buyers' demand and the seller's optimal price as a function of buyers' beliefs. We will do it for the case of loss-averse buyers, and the case of risk-averse buyers is similar. If a seller with quality v sends a separating message m_s, then buyers no longer face uncertainty about the seller's quality. All buyers know the quality of the product to be v. The seller will set price $p = v$, all buyers will purchase the product, and the seller will earn the profit of $v - c_v$. If a seller with quality v sends a pooling message m_p, then the customer indifferent between purchasing and not has loss-aversion b^0 such that:

$$E_{\mu(m_p)}v - p - b^0 ELoss_{\mu(m_p)} = 0,$$

where $\mu(m_p)$ are buyers' beliefs about the quality distribution conditional on m_p. Only buyers with $b < b^0$ will purchase the product, and therefore the demand function is given by $\Phi\left(\frac{E_{\mu(m_p)}v - p}{ELoss_{\mu(m_p)}}\right)$.

The optimal price for type v is then determined from $\max_p \Phi\left(\frac{E_{\mu(m_p)}v - p}{ELoss_{\mu(m_p)}}\right)(p - c_v)$.

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consumer goods. This is relevant to our paper, given that durable goods, as any other complex products, and services are generally low in search attributes and high in experience and credence attributes. According to Vöckner and Hofmann (2007), with fast-moving goods, consumers are more likely to simplify the decision-making process and rely on readily available cues, such as prices. With services and durable goods, on the other hand, consumers are more motivated to engage in an extensive decision-making process instead of relying on prices.

The next Proposition characterizes properties of equilibrium messaging strategy in the case of loss-averse buyers. There exists threshold $\bar{v}$ such that types with higher quality, $v \geq \bar{v}$, send messages according to the partition $v_L \leq \bar{v} = v_1 \leq \cdots \leq v_{N+1} = v_H$. All sellers in the interval $[v_i, v_{i+1}]$ send the same message m_i, and for sellers with $v \in (v_i, v_{i+1})$ message m_i is strictly optimal. Types with lower quality, those with $v < \bar{v}$, are indifferent between on-equilibrium messages and set the price low enough to serve the whole market. The price has to be the same, regardless of the message the low types send, since otherwise a seller charging a lower price could profitably deviate. When it is not possible to serve the whole market, i.e. when B is sufficiently high, then $\bar{v} = v_L$ meaning that the equilibrium messaging strategy will only have the interval structure. The last part of Proposition 2 is to show that all prices charged by sellers in $[v_{i-1}, v_i]$ are lower than prices charged by sellers in $[v_i, v_{i+1}]$. That is, even though buyers do not infer quality from prices, in equilibrium one cannot have a seller claiming "*my quality is in interval* $[1,2]$" charging a price higher than a seller claiming "*my quality is in interval* $[2,3]$".

The proof of Proposition 2 relies on the fact that consumers' demands given different buyers' beliefs satisfy the single-crossing condition. The proof does not depend on whether the quality distribution is discrete or continuous. That is, nowhere in the proof it is assumed that type $\bar{v}$ or types $\{v_i\}$ belong to the support of the quality's distribution $F(v)$.

Proposition 2 *Assume that buyers are loss-averse. Assume that no two messages m and m' result in beliefs such that $E_{\mu(m)}v = E_{\mu(m')}v$ and $ELoss_{\mu(m)} = ELoss_{\mu(m')}$. Let $\bar{v}$ be the highest quality type that serves the whole market, if it exists, and let $\bar{\mathcal{M}}$ be the set of equilibrium messages sent by types with $v \leq \bar{v}$. Then*

i) all types $v \leq \bar{v}$ charge the same price, serve the whole market, and are indifferent between any message from $\bar{\mathcal{M}}$;

ii) if there are two types v_1 and v_2 such that $\bar{v} \leq v_1 < v_2$ and that weakly prefer message m over any other messages, then any $v \in (v_1, v_2)$ will strictly prefer m over any other messages;

iii) for any v_1 and v_2 such that $\bar{v} \leq v_1 < v_2$, type v_1 will charge a strictly lower price.

As an application of Proposition 2, consider a setting with three quality levels where $v_L = 0 < v_M < v_H$ and $c_L = 0 < c_M < c_H$. Assume that each quality level has prior probability of $1/3$ and that $v_M < Ev$ so that the medium quality is below average. Buyers are loss-averse with $b \sim U[0, B]$. Consider an equilibrium where v_M separates. For the sake of example, we look at equilibria with only two on-equilibrium messages: a pooling message m_p, and a separating message m_s. Then types v_L and v_H send m_p with probability 1, and type v_M sends m_s with a positive probability and m_p with a complementary probability. Then $\mu(m_p) = (\sigma, \tau, \sigma)$ where $\sigma = \Pr(v = v_L|m_p) = \Pr(v = v_H|m_p)$. Given the prior, $1/3 \leq \sigma \leq 1/2$. For example, if v_M sends m_s with probability 1 then $\sigma = 1/2$; if v_M sends m_s with probability 0 then $\sigma = 1/3$.

Conditional on m_s, type v_M sets optimal price $p_M = v_M$ and serves the whole market, earning profit $v_M - c_M$. To calculate optimal price given m_p, we first calculate buyers' demand, which is determined by $ELoss_\mu$ and Ev_μ where:

$$
\begin{aligned}
ELoss_\mu &= -[\tau(v_M - \tau v_M - \sigma v_H) + \sigma(v_L - \tau v_M - \sigma v_H)] \\
&= -\sigma(v_M\tau - v_H(\tau+\sigma)) = \sigma[(v_H - v_M) - \sigma(v_H - 2v_M)],
\end{aligned}
$$

and $E_\mu v = v_M + \sigma(v_H - 2v_M)$. Given m_p, the demand faced by a seller is

$$
s(\mu, p) = \frac{1}{B}\frac{E_\mu v - p}{ELoss_\mu} = \frac{1}{B}\frac{v_M + \sigma(v_H - 2v_M) - p}{\sigma[(v_H - v_M) - \sigma(v_H - 2v_M)]}. \tag{4}
$$

From Proposition 2.i) follows that in equilibrium two conditions must hold: it is optimal for both v_L and v_M to serve the whole market, and the price they charge must be equal to v_M.

Type v_M has a higher cost, so it is sufficient to check the first condition for v_M only. Given μ, the highest price to serve the whole market is $p_w = E_\mu v - B \cdot ELoss_\mu$. By the second condition $p_w = v_M$. Price $p = v_M$ is optimal for the medium-quality seller given m_p when

$$
\frac{\sigma(v_H - 2v_M) + v_M - c_M}{2\sigma(v_H - 2v_M)} \geq 1. \tag{5}
$$

From $p_w = E_\mu v - B \cdot ELoss_\mu = v_M$, we can solve for σ, which is equal to

$$
\sigma = \frac{v_H - v_M}{v_H - 2v_M} - \frac{1}{B}, \tag{6}
$$

and by plugging it into (5) we get

$$
c_M \leq (v_H - 2v_M)\frac{1-B}{B}. \tag{7}
$$

It is easy to verify that if parameters are such that $\sigma \in [1/3, 1/2]$ and (7) is satisfied then an equilibrium where type v_M separates exists. We already established that v_L and v_M are indifferent between pooling and separating. The only thing left is to show that it is optimal for v_H to pool. If v_H deviates and sends m_s, the optimal price is $p = v_M$ and the profit is $v_M - c_H$. If v_H sends m_p, it can guarantee itself the profit of $v_M - c_H$ by setting $p = v_M$. But this is a lower bound on v_H's profit since its optimal price, given m_p, can differ from $p = v_M$. Thus, v_H weakly prefers m_p over m_s. Note that B can be neither too high nor too low. It cannot be too high as otherwise too many loss-averse buyers would make separation attractive for other types. It cannot be too low since otherwise separation is not profitable.

An example of equilibrium where type v_M separates is given below.

Example 1 *Consider $v_H = 3, v_M = 1$ and $v_L = 0$. The probability of each quality is 1/3. The cost is $c_i = 1/4v_i$. The loss-aversion is distributed with $b \sim U[0, B]$, where $B = 2/3$. v_M sends separating message m_s with probability 1, and v_L and v_H send pooling message m_p with probability 1. Prices are $p_L = 1$, $p_M = 1$, and $p_H = 9/8$. Demand conditional on m_p is given by $\frac{1}{B}\frac{3/2 - p}{3/4}$, where $3/2 = (v_H + v_L)/2$ is the expected quality of the product conditional on m_p, and $3/4$ is the expected loss. Profits are $\pi_L = 1, \pi_M = 3/4, \pi_H = 9/32$.*

3.3 Equilibria with Separation

In the general setting, as defined above, the set of equilibria is very rich. In what follows, we will focus on equilibria which satisfy an additional property that there exists a type that separates with a positive probability, just like in Example 1.

Proposition 3 shows that in any equilibrium at most one type can separate, and if a seller separates, his quality must be below average. The first part is trivial. If two types, $v_1 < v_2$, separate then type v_1 will optimally deviate and imitate v_2. The second part follows from the fact that high- and low-quality sellers differ in their incentives and ability to separate. When low-quality sellers, those with quality below average, separate, they face the trade-off between the *information advantage* of revealing their (lower) quality but removing quality uncertainty and the *quality advantage* of pooling with higher-quality sellers. High-quality sellers, on the other hand, do not face such a trade-off. For them, if they can separate, they get both the information advantage and the quality advantage. But then their separation is impossible in an equilibrium as there will always exist a low-quality type that would find it profitable to imitate the separating high-quality seller.

The next proposition formalizes this argument. It holds for both loss- and risk-averse buyers.

Proposition 3 *In equilibrium, at most one type separates. There is no equilibrium where type $v > Ev$ separate.*

Proposition 3 have shown that types with too high quality cannot separate. The natural question then is whether types with too low quality can separate or not. Clearly, for a product of lower quality, the quality advantage of imitating higher types is greater and can dominate the information advantage of separation.

To answer this question we look at the lowest-quality type v_L and whether it is able to separate or not. It turns out that the answer differs based on whether buyers are loss- or risk-averse. In the case of the loss-averse buyers there exist equilibria where even the lowest quality type separates. In the case of risk-averse buyers such equilibrium do not exist. To illustrate this point, we first present an example of an equilibrium with loss-averse buyers where v_L separates.

Example 2 *Let $v_L = 1, v_M = 2$ and $v_H = 3$. Assume that $c_i = (1/4)v_i$, $Pr(v = v_L) = 1/2$, $Pr(v = v_M) = Pr(v = v_H) = 1/4$, and $b \sim U[0, B]$ where $B = 147/16$. In equilibrium, v_L mixes between separating, m_s, and pooling, m_p, messages with equal probabilities. Other types send m_p. Conditional on m_p, beliefs are $Ev_{m_p} = 2$ and $ELoss_{m_p} = 1/3$. Conditional on m_s, v_L sets price 1, and earns profit of 3/4. Conditional on m_p, v_L sets price 9/8, serves 6/7 of the market and earns profit of 3/4. For other types: $p_M = 5/4$ and $\pi_M = 9/49$; $p_H = 11/8$ and $\pi_H = 25/196$.*

Next, we prove that with risk-averse buyers, type v_L cannot separate in equilibrium. Proof by contradiction. Assume the lowest type separates with message m_L. It then charges price $p = v_L$ and earns profit $v_L - c_L$. Consider another on-equilibrium message, call it m, that generates beliefs μ. Given μ, the product has a positive probability of having quality above v_L and, by definition of v_L, zero probability of having quality below v_L. Thus $E_{\mu(m)}u(w + v - v_L) > u(w)$, where w is initial wealth. It then turns out that type v_L has a profitable deviation from equilibrium message m_L. The seller can send message m and charge the price $v_L + \varepsilon$ where ε is sufficiently small. Just as in the case of sending separating message m_L, the seller will serve the whole market but at a higher price. Indeed, for any risk-averse buyer $E_{\mu(m)}u(w + v - v_L - \varepsilon) > u(w)$ as long as ε is small enough, so that all buyers prefer purchasing the product with uncertain quality at price $v_L + \varepsilon$ over not purchasing it. Then the seller's deviation profit is $v_L + \varepsilon - c_L$ and the deviation is profitable.

This establishes the following proposition.

Proposition 4 *Let v_L be the lowest possible quality. When buyers are loss-averse, there exist parameter values such that there exists an equilibrium where the seller with v_L separates. When buyers are risk-averse, there is no equilibrium where the seller with v_L separates.*

The difference between loss-aversion and risk-aversion cases is that risk-aversion respects state-dominance. Risk-averse buyers will always have a higher willingness to pay for a product whose quality can be either v_L or better than for a product with certain quality of v_L. This is why it is never optimal for v_L to separate. For loss-averse buyers, on the other hand, if degree of loss-aversion is high enough, a buyer can have higher willingness-to-pay for a product of certain quality v_L over a product that can be either v_L or better. That makes separation of v_L possible in equilibrium.

4 Duopoly

4.1 Equilibrium. Risk-neutral benchmark.

In the section we consider how competition affects sellers' incentives to reveal negative information. We extend the framework from the previous section by assuming that there are two sellers on the market. The seller's quality is exogenously given and is pure private information, i.e. it is unobserved by buyers and by the other seller. The quality distribution is the same for both sellers and is given by a cdf $F(v)$. The distribution can be either discrete or continuous with a positive density on $[v_L, v_H]$.

The game has three stages. The first stage is the messaging stage, where both sellers simultaneously send costless messages (m_i, m_j) that are publicly observed. The second stage is the pricing stage. Given (m_i, m_j), sellers simultaneously determine prices for their products (p_i, p_j). The third stage is the purchasing stage. Buyers observe messages and prices of both sellers and choose which product to purchase. Utility of buyers and sellers is the same as in the monopoly case. Sellers are risk-neutral maximizers of their expected profit, and buyers can be either loss- or risk-averse. Furthermore, we assume that buyers' valuation of the product is high enough that they always purchase a product. The equilibrium in this model is defined as follow.

Definition 2 *An equilibrium of the duopoly setting is a set of messaging and pricing strategies, $m_i(v_i)$, $m_j(v_j)$, $p_i(m_i, m_j, v_i)$, $p_j(m_i, m_j, v_j)$; buyers' beliefs $\mu_i(m_i), \mu_j(m_j)$; and buyers' purchasing strategies $s_i(\mu_i, \mu_j, p_i, p_j)$, $s_j(\mu_i, \mu_j, p_i, p_j)$ such that* *a) messaging strategy $m_i(v_i)$ maximizes i's profit given seller j's messaging and pricing strategies, and buyers' beliefs and purchasing strategies:*

$$\max_{m_i \in \mathcal{M}} E_{v_j}(p_i(m_i, m_j, v_i) - c_{v_i}) s_i(\mu_i, \mu_j, p_i, p_j);$$ [9]

b) pricing strategy $p_i(m_i, m_j)$ maximizes i's profit given m_i, m_j, seller j's pricing strategy, and buyers' beliefs and purchasing strategies:

$$\max_{p_i} E_{v_j}(p_i - c_{v_i}) s_i(\mu_i, \mu_j, p_i, p_j);$$

c) buyers purchasing decisions are optimal:

$$s_i(\mu_i, \mu_j, p_i, p_j) = \Pr(\{U_b(p_i, \mu_i) \geq U_b(p_j, \mu_j)\}),$$

and they always purchase a product: $s_i(\mu_i, \mu_j, p_i, p_j) + s_j(\mu_i, \mu_j, p_i, p_j) = 1$;

d) if message m_i is sent with positive probability, buyers' beliefs on the quality of seller i, $\mu_i(m_i)$, are derived from $m_i(v_i)$ by Bayes' rule.

As the definition of equilibrium reveals, we have made several simplifying assumptions. First, we assume that prices are determined after the message, not jointly. The assumption is reasonable as long as it is quicker to adjust pricing strategy so that sellers can react with their pricing decision to the type of information disclosed (Janssen & Teteryatnikova 2016). Second, we ignore participation constraints and assume that buyers' valuation is sufficiently high so that all buyers make purchases. Thus, sellers always directly compete with each other. Finally, as in the monopoly case, buyers do not use prices to update their beliefs about a seller's quality.

The general duopoly setup, as defined above, is quite complicated to analyze as it is a cheap-talk model with multiple senders and with a non-trivial subgame — the Hotelling model with incomplete information — that follows the messaging stage.[10] For our purposes, however, it will be sufficient to use a simpler setting. Throughout this section, we will assume that sellers have two quality types v_L and v_H, where $v_L < v_H$, and costs of both types are zero. The probability of the product being high-quality is q. There are two possible messages $\mathcal{M} = \{L, H\}$. We will use words "*low*" or "*high*" when referring to the actual quality; and labels L or H when referring to cheap-talk messages. For example, expression an H-seller will refer to a seller who sends message H, regardless of the actual product's quality.

4.2 Equilibria with Separation.

In the monopoly setting, we identified two effects that affect benefits of separation. The first effect comes from removing buyers' uncertainty. It is always positive, as it increases buyers' willingness-to-pay. Earlier, we referred to it as the information advantage. The second effect comes from a

[9] For the sake of brevity, we omit arguments of pricing and messaging strategies.

[10] Even in the simplest case of risk-neutral buyers the pricing subgame is a rather complex model of Bertrand competition with unknown costs. It is similar to the standard auction setting, and the symmetric case has been analyzed by Spulber (1995). The asymmetric case, which in our setting naturally arises after different messages, is similar to auctions with asymmetric bidders, where with few exceptions the explicit solution does not exist.

change in buyers' beliefs about the product's quality. For high-quality sellers the second effect is likely to be positive, for low-quality sellers the second effect is likely to be negative.

It turns out that competition introduces another factor that affects seller's incentives to separate. In addition to removing buyers' uncertainty about the product, separation affects the intensity of competition. This changes the relative strength of both the quality and information effects, as sellers' profit depends not only on buyers' beliefs about the product's quality but also on the competition intensity. Consider, for example, an equilibrium where all types send the same messages. In this equilibrium, buyers' information about both products' quality is identical and they will purchase the cheapest product, resulting in intense Bertrand competition. In contrast, if seller i sends a separating message while seller j sends a pooling message, buyers have different beliefs about those two products, introducing product differentiation. Some buyers with a low degree of loss/risk-aversion have stronger preferences for higher expected quality; other buyers with a high degree of loss/risk-aversion have stronger preferences for better information about the quality. Therefore, buyers from the latter group are more likely to purchase seller i's product, even if it has lower quality or is more expensive. This product differentiation softens the competition, benefiting both sellers.

The duopoly setting allows for two outcomes that were not possible under the monopoly setting. First, in the case of risk-averse buyers, there are equilibria where the lowest-quality seller can separate. Second, there are equilibria where the highest quality seller can separate. In both cases, low-quality sellers choose not to imitate high-quality sellers and instead disclose negative information. Both outcomes can happen in equilibria because now imitating higher-quality sellers can be less attractive if a likelihood of intense competition is high. But then if pooling with high-quality sellers is less attractive, it becomes possible for high-quality types to separate, and, in the risk-averse case, for the lowest-quality type to choose separation over pooling.

4.2.1 Risk-Averse Buyers. Separation of the Lowest-Quality Type

Consider the following messaging strategy $m^*(v)$: a high-quality seller sends message H with probability 1, and a low-quality seller mixes between messages L and H with probabilities λ and $1-\lambda$. Given $m^*(v)$, message L is the separating message which is sent by the lowest-quality type, v_L. We will determine conditions when $m^*(v)$ is a messaging strategy in a symmetric equilibrium.

We solve for equilibrium using backward induction. First, we look at the purchasing stage. If both sellers send the same messages (L, L) or (H, H) then, from buyers' point of view, the two sellers are identical. Therefore, buyers will purchase the product with the lowest price. Consider now the purchasing decision given messages (L, H), and prices (p_L, p_H). The L-product has a certain quality of v_L. The H-product may be of low quality with probability $q_{LH} = Pr(v = v_L|H)$, or of high quality with probability $1 - q_{LH}$. Given m^*:

$$q_{LH} = \frac{(1-\lambda)(1-q)}{(1-\lambda)(1-q)+q}.$$

A buyer is indifferent between L- and H-products if

$$u(v_L - p_L) = q_{LH}u(v_L - p_H) + (1 - q_{LH})u(v_H - p_H),$$

or equivalently

$$e^{-\gamma^0(p_H - p_L)} - q_{LH} - (1 - q_{LH})e^{-\gamma^0(v_H - v_L)} = 0, \tag{8}$$

where γ^0 is the risk-aversion degree of an indifferent buyer. Given the CARA utility function, a buyer's initial wealth does not affect the indifference condition, so we do not specify it here.

Proposition 5 *The indifference condition*

$$e^{-\gamma^0(p_H - p_L)} = q_{LH} + (1 - q_{LH})e^{-\gamma^0(v_H - v_L)},$$

has at most one solution $\gamma^0 > 0$. When the solution exists, all buyers with $\gamma > \gamma^0$ prefer an L-product while all buyers with $\gamma < \gamma^0$ prefer H-product.

Next, we consider the pricing stage. Suppose at the messaging stage both sellers sent the same messages (L, L) or (H, H). Then, as established earlier, buyers will purchase the cheapest product. Thus, the pricing equilibrium is for both sellers to charge $p_L = p_H = 0$. Consider now the (L, H)-subgame. From Proposition 5 follows that demand for the L-product is $(1 - \Phi(\gamma^0(p_L, p_H))$, and demand for the H-product is $\Phi(\gamma^0(p_L, p_H))$. The L-seller chooses price p_L to maximize $\max_{p_L}(1 - \Phi(\gamma^0(p_L, p_H)) \cdot p_L$, and the H-seller chooses price p_H to maximize $\max_{p_H} \Phi(\gamma^0(p_L, p_H)) \cdot p_H$. The corresponding first-order conditions are

$$-\phi(\gamma^0)\frac{\partial\gamma^0(p_L, p_H)}{\partial p_L}p_L + (1 - \Phi(\gamma^0)) = 0, \tag{9}$$

and

$$\phi(\gamma^0)\frac{\partial\gamma^0(p_L, p_H)}{\partial p_H}p_H + \Phi(\gamma^0) = 0. \tag{10}$$

Finally, at the messaging stage, a low-quality seller should be indifferent between L and H when the competitor plays the equilibrium strategy. This condition is

$$\lambda(1 - q)\pi_{LL} + (1 - \lambda(1 - q))\pi_{LH} = \lambda(1 - q)\pi_{HL} + (1 - \lambda(1 - q))\pi_{HH}. \tag{11}$$

Here, the LHS is the expected profit of the low-quality seller from sending message L and the RHS is the expected profit from sending message H; $\lambda(1 - q)$ is the probability that the competitor sends message L, and $(1 - \lambda(1 - q))$ is the probability that the competitor sends message H. As we established earlier, $\pi_{LL} = \pi_{HH} = 0$. Notice that since both types have the same cost, the high-quality seller is also indifferent between L and H; therefore, message H is optimal for high-quality type. Combining equations (8), (9), (10) and (11), an equilibrium is determined by the following system of equations:

$$\begin{cases} e^{-\gamma^0(p_H - p_L)} = q_{LH} + (1 - q_{LH})e^{-\gamma^0(v_H - v_L)}. \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L, p_H)}{\partial p_L}p_L + (1 - \Phi(\gamma^0)) = 0 \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L, p_H)}{\partial p_L}p_H + \Phi(\gamma^0) = 0 \\ (1 - \lambda(1 - q))(1 - \Phi(\gamma^0))p_L = \lambda(1 - q)\Phi(\gamma^0)p_H. \end{cases} \tag{12}$$

As the next proposition shows, whether the solution to system (12) exists depends on the distribution of γ.

Proposition 6 *Assume that there are two quality types, and buyers are risk-averse with CARA utility function. Then the equilibrium where the lowest-quality type separates does not exist if*

i) $\Phi(\gamma)$ *is a uniform distribution; or*

ii) $\Phi(\gamma)$ *is a convex function.*

The equilibrium where the lowest-quality type separates exists if

iii) $\Phi(\gamma)$ *has infinite support.*

Furthermore,

iv) for any concave $\Phi(\gamma)$ *there exists* $\alpha^0 > 0$ *such that for any* $\alpha \in (0, \alpha^0)$*, if risk-aversion is distributed with cdf* $\Phi(\alpha\gamma)$*, the equilibrium where the lowest-quality type separates exists.*

The intuition is as follows. Consider the pricing stage after messages (L, H). In terms of quality, the H-product is superior to the L-product, The L-product is guaranteed to be of low quality, while the H-product can be of either low or high quality. Regardless of γ, all risk-averse buyers have higher willingness-to-pay for the H-product. However, for those who are more risk-averse, the difference in willingness-to-pay between the H- and the L-products is smaller. Thus, the only way the L-seller can get a positive share of the market is by competing with the H-seller for risk-averse buyers with high γ (see also Proposition 5). The H-seller's willingness to compete for those buyers depends on two factors: a) how high γ can get; and b) how large the share of buyers with high γ is. If Γ is low or if there are too many buyers with high γ, then it is optimal for the H-seller to simply outprice the L-seller away, and serve the whole the market. In the first case, when Γ is low, even the most risk-averse buyers are not too concerned about quality uncertainty. In the second case, when there are sufficiently many buyers with high risk-aversion, it is suboptimal for H to choose not to serve them. But then from v_L's point of view, if either of the two conditions is satisfied, it is not optimal to send message L, as every pricing subgame results in zero profits. Thus, only when there are sufficiently risk-averse buyers *but* their share is sufficiently low, it is possible to have an equilibrium where v_L separates.

Proposition 6 captures that notion of having "*sufficiently risk-averse buyers but their share is sufficiently low*" using convexity and support of $\Phi(\gamma)$. For any $\Phi(\gamma)$ with infinite support, the equilibrium with the lowest-type separation exists. First, there are sufficiently risk-averse buyers. Second, there exists $\tilde{\gamma}$ high enough that $\phi(\gamma)$ can be made arbitrarily small for any $\gamma > \tilde{\gamma}$. The marginal benefit of decreasing p_H to serve buyers with $\gamma > \tilde{\gamma}$ then will be too small due to low increase in the market share. In equilibrium L and H will split the market.

For uniform or convex cdfs, no matter how high Γ is, $\phi(\gamma)$ never approaches to zero for high values of γ. Marginal decrease in p_H to attract buyers with high-risk aversion is always optimal, and seller L will be priced out of the market. With concave distributions, more buyers have a low degree of risk-aversion. Concavity alone, however, is not enough to guarantee the existence. One also needs to have $\phi(\gamma)$ to be sufficiently small when γ is close to Γ. One way to do this is to stretch $\Phi(\gamma)$ to a larger support, and Proposition 6 offers one way how it can be achieved. Notably, as the next example shows, one does not need unrealistic levels of risk-aversion for the equilibrium to exist.

Example 3 *Let the degree of risk-aversion* γ *be distributed with* $\Phi(\gamma) = \sqrt{\gamma}$ *on* $[0, 1]$*. Let* $v_H - v_L = 3$ *and* $q \approx 0.484$.[11] *One can verify that the following is equilibrium:* $\gamma^0 \approx 0.713$, $p_L \approx 0.184$, $p_H \approx 1$, $\lambda \approx 0.063$ *and* $q_{LH} = 1/2$.

In this example, the probability of buying from a low-quality seller is slightly above 1/2, $1 - q \approx 0.52$*. The low-quality seller reveals the negative information with probability 6.3%. If one seller announces L and the competitor announces H then both prices are above the marginal cost and both sellers make positive profits. The indifferent buyer is located at* $\gamma^0 \approx 0.713$*. The market share served by the L-seller is* $1 - \sqrt{\gamma^0} \approx 15.6\%$*, and the market shares served by the H-seller is 84.4%. In the* (L, H) *subgame, sellers profits are* $\pi_H \approx 0.84$ *and* $\pi_L \approx 0.03$*.*

It is worth highlighting that it is the competitive environment that makes it possible for the lowest-quality seller to separate. As argued earlier, in the monopoly case with risk-averse buyers, it is never optimal for v_L to separate, as he is guaranteed to get higher profit by pooling with the high-quality sellers. In a symmetric equilibrium of duopoly case, if types v_L of sellers i and j pool with v_H with probability 1, they earn zero Bertrand profit. This makes pooling with high-quality sellers less attractive. By separating with positive probability, type v_L creates product differentiation, which allows it to earn a positive profit.

4.2.2 Separation of Highest-Quality Type

In this section, we assume that buyers are loss-averse. Consider the following messaging strategy $m^*(v)$: a high-quality seller randomizes between sending messages L and H with probabilities λ and $1 - \lambda$, and a low-quality seller sends message L with probability 1. Given $m^*(v)$, message H is the separating message which is sent by the highest-quality type v_H only. In what follows we will determine conditions when $m^*(v)$ is a messaging strategy in a symmetric equilibrium.

As before, buyers will purchase the product with the lowest price after (L, L) or (H, H) messages, so that both sellers charge prices equal to the marginal cost and earn zero profit. Consider now the purchasing decision given message profile (L, H) and prices (p_L, p_H). The H-product has a certain quality of v_H. The L-product may be of low quality with probability $q_{LL} = Pr(v = v_L|L)$, or of high quality. Given m^*, q_{LL} is equal to $\dfrac{1-q}{1-q+q\lambda}$. The loss-aversion of the indifferent buyer is given by

$$v_H - p_H = q_{LL}v_L + (1 - q_{LL})v_H - p_L + b^0 q_{LL}(v_L - q_{LL}v_L - (1 - q_{LL})v_H).$$

Buyers with $b > b^0$ will purchase from the H-seller and buyers with $b < b^0$ will purchase from the L-seller, so that the profit of the H-seller is $(1-\Phi(b^0))p_H$, and of the L-seller is $\Phi(b^0)p_L$. Combining the buyers' indifference condition and the FOCs for L and H-sellers, we get

[11] Numerically, it is simpler to start with q_{LH} instead of q. The equilibrium in this example was calculated based on $q_{LH} = 1/2$. Values of q and λ were then calculated from (γ^0, p_L, p_H) and q_{LH} using the fourth equation of (12).

$$
\begin{cases}
b^0 = \dfrac{p_H - p_L}{q_{LL}(1-q_{LL})\Delta v} - \dfrac{1}{1-q_{LL}} \\
\phi(b^0)\dfrac{\partial b^0(p_L, p_H)}{\partial p_L} p_H + (1-\Phi(b^0)) = 0 \\
\phi(b^0)\dfrac{\partial b^0(p_L, p_H)}{\partial p_L} p_L + \Phi(b^0) = 0
\end{cases}
\tag{13}
$$

We do not specify the indifference condition, as one can use the equivalent of Lemma 2 in the appendix to show that, if a solution to (13) exists, the equilibrium exists.

By subtracting the second equation from the third equation, and using the expression for b^0 to calculate its derivative with respect to prices, we get

$$
\frac{1-\Phi(b^0)}{\phi(b^0)} - \frac{\Phi(b^0)}{\phi(b^0)} - b^0 = \frac{1}{1-q_{LL}}.
$$

For a given q_{LL}, the solution to (13) exists if and only if

$$
\max_b \left\{ \frac{1-\Phi(b)}{\phi(b)} - \frac{\Phi(b)}{\phi(b)} - b \right\} > \frac{1}{1-q_{LL}}.
$$

For distributions with log-concave densities, the expression inside the parenthesis is a decreasing function of b^0. By Theorem 1 of Bergstrom and Banoli (2006), if $\phi(b)$ is log-concave, then so is $\Phi(b)$. Term $(1-\Phi(b))/\phi(b)$ is a decreasing function of b by Corollary 2 in Bergstrom and Banoli (2006). Term $\Phi(b)/\phi(b)$ is an increasing function of b by definition of log-concavity, and is decreasing when multiplied by minus one. Therefore, the maximum of the expression in the parenthesis is reached at $b = 0$. Thus, for a given q_{LL}, the equilibrium exists if and only if

$$
\frac{1}{\phi(0)} > \frac{1}{1-q_{LL}}. \tag{14}
$$

Since $q_{LL} \geq 1-q$, a necessary condition for the equilibrium where high type separates to exist is

$$
\frac{1}{\phi(0)} > \frac{1}{q}. \tag{15}
$$

Whether (15) puts any restriction on B or not depends on the underlying distribution. For example, when $\Phi(b)$ is uniform, condition (15) becomes $B > 1/q$. When $\Phi(b) = (b/B)^2$, condition (15) is satisfied for any B.

The intuition is as follows. The H-product is superior to the L-product in that its quality is both higher *and* certain. In order for the L-seller to be able to have positive profit, two conditions have to be satisfied: the L-product should be sufficiently differentiated from the H-product, and there should be sufficiently few buyers with low loss-aversion to make it suboptimal for the H-seller to compete for them and price the L-seller out of the market. The first condition requires q to be sufficiently high. When q is low, the L-product is very likely to have low quality, in which case there is not enough product differentiation between H- and L-sellers to generate positive profit for the L-seller. Indeed, the product differentiation comes from the difference in riskiness of the H- and L- products, and when the riskiness is similar the product differentiation is not sufficient for the L-seller to compete against the H-seller.[12] The second condition requires $\phi(0)$ to be sufficiently low. The intuition is similar to that behind Proposition 6 from the previous section. The only difference is that in the previous case the seller of the inferior product, L-seller, was competing for buyers with high-degree of risk-aversion, whereas now it competes for buyers with a low degree of loss-aversion. In order for the H-seller to find it suboptimal to serve the whole market, value of $\phi(0)$ should be sufficiently small.

Figure 1 depicts equilibrium parameters in equilibria where high-quality seller separates. It is plotted for b distributed uniformly on $[0, 3]$, and $\Delta v = 3$. On the horizontal axis is q_{LL}, which is the probability of getting low-quality product from the L-seller. q_{LL} varies from 0 to 2/3, where 2/3 is the highest value of q_{LL} that satisfies condition (14). We use q_{LL}, not q, for the horizontal axis because, as one can see from Figure 1, q has to be extremely close to 1 in order for an equilibrium to exist. Not surprisingly, the price of the H-product in the (H, L)-subgame is much higher than that of the L-product as the the H-product is superior to the L-product. However, in equilibrium, the high profit of the H-seller in the (H, L)-subgame comes at a cost of very high likelihood of the (H, H)-subgame and zero profit. For the L-seller, a low profit in the (H, L)-subgame comes with a benefit of having very low likelihood of being in the (L, L)-subgame and earning zero profit.

[INSERT FIGURE 1 HERE]

5 Extension: Prices Used as Signals

Up to this point we have assumed away the possibility that buyers can infer products' quality from prices, following the findings from the empirical literature. In this section, we discuss what happens when buyers use prices to update their beliefs. We will focus on the case of loss-averse buyers and a monopolistic seller. All other assumptions are the same as before. The new equilibrium is defined as follows.

Definition 3 *An equilibrium is a triple $(p(v), \mu(p), s(\mu, p))$, where $p(v)$ is the seller's pricing strategy, $\mu(p)$ is the buyers' beliefs about the quality distribution given p, and $s(\mu, p)$ is the share of buyers who purchase the product, such that the following conditions hold:*

a) the seller of type v chooses price p that maximizes his profit given buyers' beliefs $\mu(p)$ and buyers' purchasing decision:

$$\max_p (p - c_v) s(\mu(p), p);$$

[12] With risk-averse buyers, as in the previous subsection, Proposition 6 did not depend on q. Since the risk-averse model is harder, conditions required for existence were only established for sufficiently large Γ. For a given q, one can always find Γ large enough so that the equilibrium exists, which is, effectively, what Proposition 6 did. With loss-averse buyers, a closed-form solution is easy to derive. The existence of equilibrium can be established without requiring B to be arbitrarily large, which makes it possible to highlight the role of q and the role of the product differentiation motive. The fact that this subsection looks at the case when the highest-quality type separates is not crucial. An earlier version of the paper showed that with loss-averse buyers there is an equilibrium where the lowest-quality type separates iff $B + \dfrac{1}{\phi(B)} > \dfrac{1}{1-q}$. Therefore, again, the existence of the equilibrium depends not only on B and $\phi(B)$, but also on q.

b) buyers' purchasing decision is optimal:

$$s(\mu, p) = \Phi(\{b|U_b(p, \mu) \geq 0\});$$

c) if a given price p *is chosen with positive probability, buyers' beliefs* $\mu(p)$ *are derived from* $p(v)$ *by Bayes' rule.*

The case when c_v is an increasing function of v is well understood: when the single-crossing condition is satisfied and it is the informed party that makes an offer, sellers can separate even in the case of risk-neutral buyers. Unlike the cheap-talk setting considered above, neither loss/risk-aversion nor competition is necessary for separation. A simple example below illustrates this point.

Example 4 *Let's assume that there are two types:* $\{v_L, v_H\}$ *with* $v_L = 1, c_L = 0$ *and* $v_H = 2, c_H = 1/2$*. The separating prices are* $p_L = 1$ *and* $p_H = 2$*. The corresponding sales are* $q_L = 1$ *and* $q_H = 1/2$*. Buyers are indifferent between purchasing or not, so the purchasing decisions are optimal for both prices. The corresponding profits are* $\pi_H = 3/4$ *and* $\pi_L = 1$*. If type* v_H *imitates type* v_L *its profit is strictly lower,* $1/2$*. If type* v_L *imitates type* v_H *its profit is the same, 1. The off-equilibrium beliefs that support this equilibrium are:* $\Pr(v_H|p) = 1$ *for any* $p \geq 2$ *and* $\Pr(v_H|p) = 0$ *for any* $p < 2$*.*

Thus, in this section we consider the case when the single-crossing condition is not satisfied. To make our result as clean as possible, we consider the case where the cost is a *decreasing* function of quality.[13] It turns out that, when the single-crossing condition is not satisfied, the setting with loss-averse buyers allows for more information transmission than the setting with the risk-neutral buyers.

It is straightforward to characterize equilibria in this setting. All equilibria must have an interval structure, whereby if any two types choose the same price then all types in between must choose the same price. Furthermore, since costs are decreasing, it has to be the case that low-quality/high-cost sellers charge higher prices and serve lower share of the market. This holds regardless of buyers' risk-attitude as it is fully driven by firms' incentives. What buyers' risk-attitude affects is informativeness of equilibiria. When buyers are risk-neutral, at most two on-equilibrium prices can be charged. Intuitively, let p_1 be the price charged by v_L. It has to be the case that $p_1 \leq E_{p_1} v$ in order for v_L to make any positive sales. Since higher-quality types have to charge lower prices, we then have $p_i < E_{p_i} v$ for any on-equilibrium p_i. It implies that for any on-equilibrium price, $p \neq p_1$, sellers would serve the whole market. But then there can be at most one on-equilibrium price which differs from p_1. When buyers are loss-averse, on the other hand, one can sustain any finite number of on-equilibrium prices, as demonstrated in Proposition 8.

[13]Needless to say, assuming that cost is a decreasing function of quality is an unrealistically extreme assumption. It is employed merely to illustrate our point, which is that loss/risk-aversion allows for more information transmission when the single-crossing condition fails. A less extreme example of the single-crossing condition failure is the assumption that cost is not a monotonically increasing function of quality. This assumption is more realistic. A textbook (Besanko et al. 2009) example is Miller and Friesen (1986)'s study of consumer durable industries. Miller and Friesen (1986) have shown that firms which appeared to have achieved cost advantage also scored highly on measures related to benefit advantage, such as product quality.

Proposition 7 *In any equilibrium with risk-neutral buyers, there can be at most two different on-equilibrium prices. An equilibrium with two on-equilibrium prices exists.*

Proposition 8 *Fix integer $N > 0$. When buyers are loss-averse, there exists an equilibrium with N on-equilibrium prices.*

6 Conclusion

Although we often observe the cases where sellers voluntarily disclose negative information to buyers, the literature has mostly focused on the incentives and possibility to credibly reveal positive information. Therefore, this study attempts to fill this gap by examining sellers' incentives to reveal negative information regarding their products.

We thus focus on those markets where customers cannot assess the quality of the products and there is no reputation concern. Unlike previous literature, our model limits the ability of high-quality sellers to separate themselves, based on the assumption that buyers only update their beliefs through cheap-talk messages. As low-quality sellers can costlessly imitate any messages sent by high-quality sellers, the quality is only credibly revealed when low-quality sellers voluntarily choose not to imitate high-quality sellers. This is different from classical information asymmetry models such as unraveling or education-as-signaling, where the separation is driven by high-quality types.

We identify two factors that can allow low-quality sellers to prefer separation over pooling with high-quality sellers: buyers' aversion to quality uncertainty and product differentiation. When choosing to reveal one's type, low-quality sellers face the trade-off between a *quality advantage* of pretending to be of a higher-quality type by pooling with high-quality sellers, and an *information advantage* of revealing one's low quality but removing quality uncertainty. In the monopoly setting, where there is no competition, as long as there are sufficient loss- or risk-averse buyers, the information advantage can dominate the quality advantage, making separation optimal for the low-quality seller. In the duopoly setting, the trade-off between quality and information advantages is further affected by the fact that different information about competitors' products results in product differentiation and weaker competition. This leads to two new outcomes that were impossible in the case of the monopolistic sellers: high-quality types can separate and, in the case of risk-averse buyers, the lowest-quality seller can also choose to separate. Both can happen in equilibrium if the competition makes quality advantage too small so that it is not optimal for the low-quality sellers to pretend to be of high quality.

Overall, this paper shows how honesty may work as an effective communication strategy. We believe our finding can be widely applied to various market settings where information asymmetry is high, providing valuable insights to both the field and the academia. For managers, this paper can provide helpful strategic implications on how to wisely consider buyers' uncertainty aversion while communicating the information about the weaknesses of their products. For academic researchers, this paper provides a theoretical explanation about an important market phenomenon that has been somewhat neglected in the literature. More specifically, this paper suggests that the consideration of risk might establish the incentive of low-quality sellers to voluntarily reveal their types.

On a final note, we hope that this study provides motivations for further empirical and theoretical research regarding the effect of revealing negative information under various other settings, which can lead to a better understanding about information asymmetry in markets.

7 Appendix: Proofs

Proof of Proposition 2: Proof of i): First, we show that if type v_0 serves the whole market then any types $\nu < v_0$ also serves the whole market. Let p_0 denote the price charged by type v_0, and c_0 its marginal cost. Let s_ν denote the share of the market served by type ν, p_ν its price and c_ν its marginal cost. Proof by contradiction. Assume that $s_\nu < 1$. Then,

$$\begin{aligned}(p_\nu - c_\nu)s_\nu &\geq (p_0 - c_\nu)\cdot 1\\ (p_\nu - c_0)s_\nu + (c_0 - c_\nu)s_\nu &\geq (p_0 - c_0)\cdot 1 + (c_0 - c_\nu)\cdot 1\\ (p_\nu - c_0)s_\nu &\geq (p_0 - c_0)\cdot 1 + (c_0 - c_\nu)(1 - s_\nu) > (p_0 - c_0)\cdot 1.\end{aligned}$$

Here the first inequality comes from the fact that type ν prefers its equilibrium strategy over imitating type v_0 and charging price p_0. The last inequality is strict because $s_\nu < 1$, and it implies that type v_0 has a profitable deviation of sending the same message as type ν and charging price p_ν. Contradiction.

Proof of ii):[14] Consider the two types v_1 and v_2 such that $\bar{v} \leq v_1 < v_2$, and that weakly prefer message m to all other messages. Let μ denote buyers' beliefs given message m. Consider now a different message m' and let μ' be buyers' beliefs given m'. Let $\pi(\mu, v) = (p_v^* - c_v)s(\mu, p_v^*)$, where p_v^* is optimal price for type v given μ, and $s(\mu, p_v^*) = \Phi\left(\dfrac{E_\mu v - p_v^*}{ELoss_\mu}\right)$.

Lemma 1 *Take beliefs μ and μ' and assume that either $E_\mu v \neq E_{\mu'} v$ or $ELoss_\mu \neq ELoss_{\mu'}$, or both. Then there exist at most one type with quality above $\bar{v}$ such that $\partial\pi(\mu, v)/\partial v = \partial\pi(\mu', v)/\partial v$.*

Proof. By the envelope theorem, $\partial\pi(\mu, v)/\partial v = -(c_v)' s(\mu(m), p_v^*) = -(c_v)'\Phi\left(\dfrac{E_\mu v - p_v^*}{ELoss_\mu}\right)$. When for a given quality type, profit derivatives are equal it implies that $\Phi\left(\dfrac{E_\mu v - p_v^*}{ELoss_\mu}\right) = \Phi\left(\dfrac{E_{\mu'} v - p_v'^*}{ELoss_{\mu'}}\right)$. From monotonicity of $\Phi(\cdot)$ then

$$\frac{E_\mu v - p_v^*}{ELoss_\mu} = \frac{E_{\mu'} v - p_v'^*}{ELoss_{\mu'}}. \tag{16}$$

[14] As mentioned in the main body of the paper, the proof does not depend on whether the quality distribution is discrete or continuous. In equilibrium, the only thing that quality distribution affects is buyers' posterior beliefs $\mu(m)$. Given $\mu(m)$, one can then calculate optimal sellers' behavior for any quality v regardless of whether the quality belongs to the support of $F(v)$ or not. In particular, nowhere in the proof it is required that variables v_1, v_2, v_0 or $\tilde{v}$ that we introduce during the proof belong to the support of the quality distribution.

Since p_v^* and $p_v'^*$ are optimal prices and the quality is above $\bar{v}$, they must satisfy the FOCs

$$\begin{aligned}\Phi\left(\frac{E_\mu v - p_v^*}{ELoss_\mu}\right) - \frac{1}{ELoss_\mu}(p_v^* - c_v)\phi\left(\frac{E_\mu v - p_v^*}{ELoss_\mu}\right) &= 0 \\ \Phi\left(\frac{E_{\mu'} v - p_v'^*}{ELoss_\mu}\right) - \frac{1}{ELoss_{\mu'}}(p_v'^* - c_v)\phi\left(\frac{E_{\mu'} v - p_v'^*}{ELoss_{\mu'}}\right) &= 0.\end{aligned}$$

Combined with (16) it follows from the FOCs that

$$\frac{p_v^* - c_v}{ELoss_\mu} = \frac{p_v'^* - c_v}{ELoss_{\mu'}}. \tag{17}$$

Adding equations (16) and (17) we have

$$\frac{E_\mu v - c_v}{ELoss_\mu} = \frac{E_{\mu'} v - c_v}{ELoss_{\mu'}}. \tag{18}$$

Notice that functions $\dfrac{E_\mu v - x}{ELoss_\mu}$ and $\dfrac{E_{\mu'} v - x}{ELoss_{\mu'}}$ intersect at most ones, which means that there can exist at most one type where $\Phi\left(\dfrac{E_\mu v - p^*}{ELoss_\mu}\right) = \Phi\left(\dfrac{E_{\mu'} v - p'^*}{ELoss_{\mu'}}\right)$, and that's the type which cost satisfies (18). ■

Take types v_1 and v_2 that weakly prefer m to any message and assume that there is type $v_0 \in (v_1, v_2)$ that weakly prefers message m' over m. There are two possibilities: either there is more than one type that weakly prefers m' or such type is unique. Consider the first possibility. By continuity then, there exist at least two types that are indifferent between m and m'. With a slight abuse of notation denote them as v_1 and v_2. By the mean value theorem there exists type $v \in (v_1, v_2)$ such that $\pi'(\mu, v) = \pi'(\mu', v)$ and by Lemma 1 such type is unique. Furthermore, by the mean value theorem and Lemma 1, type v strictly prefers m' over m. Since derivatives of $\pi(\mu, \cdot)$ and $\pi(\mu', \cdot)$ are equal to each other at most once there cannot be three different types that are indifferent between m and m'.

From, $\pi(\mu, v) < \pi(\mu', v)$ and $s(\mu, p_v^*) = s(\mu', p_v'^*)$ follows that $p_v^* < p_v'^*$. Consider now type v_2. By its definition, $\pi(\mu, v_2) = \pi(\mu', v_2)$. From the fact that $v_2 > v$ and Lemma 1 follows that $\pi'(\mu, v_2) > \pi'(\mu', v_2)$. By the envelope theorem then $s(\mu, p_{v_2}^*) < s(\mu', p_{v_2}'^*)$. Since profits at v_2 are equal it means that $p_{v_2}^* > p_{v_2}'^*$. By continuity there exists $\tilde{v} \in (v, v_2)$ such that $p_{\tilde{v}}^* = p_{\tilde{v}}'^*$, which we will denote simply as $\tilde{p}^*$. Since $\tilde{v} > v$ we know that $s(\mu, \tilde{p}^*) < s(\mu', \tilde{p}^*)$ and, therefore,

$$\frac{E_\mu v - \tilde{p}^*}{ELoss_\mu} < \frac{E_{\mu'} v - \tilde{p}^*}{ELoss_{\mu'}}. \tag{19}$$

From the FOCs (17) it follows that when the prices are equal then

$$ELoss_\mu \cdot \frac{\Phi\left(\dfrac{E_\mu v - \tilde{p}^*}{ELoss_\mu}\right)}{\phi\left(\dfrac{E_\mu v - \tilde{p}^*}{ELoss_\mu}\right)} = ELoss_{\mu'} \cdot \frac{\Phi\left(\dfrac{E_{\mu'} v - \tilde{p}^*}{ELoss_{\mu'}}\right)}{\phi\left(\dfrac{E_{\mu'} v - \tilde{p}^*}{ELoss_{\mu'}}\right)}. \tag{20}$$

By log-concavity of $\Phi(\cdot)$ term Φ/ϕ is an increasing function. Then from (19) and (20) follows that $ELoss_\mu > ELoss_{\mu'}$. We reached a contradiction. When $ELoss_\mu > ELoss_{\mu'}$ then equality (18) and inequality (19) cannot be satisfied simultaneously. This is because $\tilde{p}^*$ is an optimal price charged by type with a cost higher than c_v. Therefore, $\tilde{p}^* > c_v$. But then given $ELoss_\mu > ELoss_{\mu'}$, the inequality in (19) must be reversed.

Now consider the second case where type $v_0 \in (v_1, v_2)$ is unique. Then, $\pi(\mu, v_0) = \pi(\mu', v_0)$ and $\pi'(\mu, v_0) = \pi'(\mu', v_0)$. By the envelope theorem, the latter implies that $s(\mu, p^*_{v_0}) = s(\mu', p'^*_{v_0})$ and, therefore, $p^*_{v_0} = p'^*_{v_0}$. Then $p^*_{v_0}$ satisfies Equation (20) which would imply that $ELoss_\mu = ELoss_{\mu'}$. But then $s(\mu, p^*_{v_0}) = s(\mu', p'^*_{v_0})$ would imply that $E_\mu v = E_{\mu'} v$, which is a contradiction.

Proof of iii): Take any $v_1 < v_2$. If they both belong to the same interval, say $[v_i, v_{i+1}]$, then both types face the same demand function and, by a standard argument, type v_2 will charge a higher price because it has higher cost. Take type v_{i+1}. It is indifferent between messages m_i and m_{i+1}. But types slightly higher than v_{i+1} strictly prefer m_{i+1}. Then, $\pi'(\mu(m_i), v_{i+1}) < \pi'(\mu(m_{i+1}, v_{i+1})$ which by the envelope theorem implies that $s(\mu(m_i), p^*_{v_{i+1}}) > s(\mu(m_{i+1}, p'^*_{v_{i+1}})$. Since v_{i+1} is indifferent between m_i and m_{i+1} we then have that $p^*_{v_{i+1}} < p'^*_{v_{i+1}}$. This means that all types in $[v_i, v_{i+1}]$ will charge price less than $p'^*_{v_{i+1}}$ and all types on $[v_{i+1}, v_{i+2}]$ will charge prices higher than $p'^*_{v_{i+1}}$. By applying the argument inductively, one can establish that price is a strictly increasing function on $[\bar{v}, v_H]$. ■

Proof of Proposition 3: Proof by contradiction. Assume that there are two types that separate: $v_1 < v_2$ with messages $m_1 \neq m_2$. Then these types will set prices $p_1 = v_1$ and $p_2 = v_2$ and will serve the whole market. But that cannot happen since type v_1 would deviate and send the message m_2. Thus at most one type separates. Label this type v_0, and let m_0 be its separating message. After m_0, it will charge the price $p_0 = v_0$ and will serve the whole market. By Proposition 2, all types below v_0 will also serve the whole market.

Consider first the case of loss-averse buyers. If v_0 separates and $v_0 = v_L$ then we are done as $v_L < Ev$. Otherwise, take type $\nu < v_0$. Let $m(\nu)$ be the message sent by type ν and $\mu(\nu)$ be corresponding beliefs about the quality. Since all sellers with lower quality, including seller ν, serve the whole market, they will charge price $p_\nu = E_{\mu(\nu)} v - B \cdot ELoss_{\mu(\nu)}$, which is the highest price that allows a seller to serve the whole market. Given that neither type v_0 nor type ν wants to deviate and imitate each other, it must be the case that $p_\nu = v_0$, for any $\nu < v_0$. Thus $E_{\mu(\nu)} v - B \cdot ELoss_{\mu(\nu)} = v_0$ and, therefore, $E_{\mu(\nu)} v \geq v_0$. We can reach the same conclusion for the case of risk-averse buyers. Any seller with $\nu < v_0$ serves the whole market. It will then choose the highest price that allows doing so: $E_{\mu(\nu)} u_\Gamma(w + v - p_\nu) = u(w)$, where u_Γ is the Bernoully utility function of the most risk-averse buyer. Since type ν does not imitate type v_0, and vice versa, it has to be the case that $p_\nu = v_0$. But then by Jensen's inequality $Ev_{\mu(\nu)} > v_0$. The rest of the proof goes regardless of whether buyers are risk- or loss-averse.

As established above any $\nu < v_0$ will send an on-equilibrium message that will result in buyers having beliefs $Ev_{\mu(\nu)} > v_0$. We will use it to show that $v_0 \leq Ev$. Let $\mathcal{M}$ be the set of messages sent by sellers with $\nu < v_0$. Let $\mathcal{V}$ be the set of all types that send messages in $\mathcal{M}$. As we established,

$E_{\mu(m)}v > v_0$ for any $m \in \mathcal{M}$. Let $\mathcal{V}^c = [v_L, v_H]/(\mathcal{V} \cup \{v_0\})$ be the set of all types that are not in $\mathcal{V}$ or v_0, and $\mathcal{M}^c$ be the set of messages they send. Since all types in $\mathcal{V}^c$ have quality higher than v_0, we have that $E_{\mu(m)}v > v_0$ for any $m \in \mathcal{M}^c$. Thus any equilibrium message results in beliefs about expected quality that are either equal to v_0, which is v_0's separation message, or strictly greater than v_0. Given that on-equilibrium beliefs are correct, it has to be the case that $v_0 < Ev$. ■

Proof of Proposition 5: Notice that the indifference condition (8) is always satisfied when $\gamma = 0$. This is because $u(x) \equiv 1$ when $\gamma = 0$ and the indifference condition becomes trivial.

Let y denote $e^{-\gamma^0}$ so that (8) is

$$y^{p_H - p_L} - q_{LH} - (1 - q_{LH})y^{v_H - v_L} = 0. \tag{21}$$

As γ varies between 0 and $+\infty$, variable y varies between 1 and 0. There is one solution $y = 1$, which corresponds to $\gamma = 0$. In order to prove the proposition, we need to show that on interval $0 \leq y < 1$ there is at most one solution. Given the root $y = 1$, it is equivalent to showing that there are at most two solutions of (21) on $0 \leq y \leq 1$.

Assume not. If function (21) has three or more roots on $[0, 1]$, then its derivative should have two or more roots on $[0, 1]$. Taking derivative of the LHS of (21) with respect to y and setting it equal to zero we get:

$$(p_H - p_L)y^{p_H - p_L - 1} - (1 - q_{LH})(v_H - v_L)y^{v_H - v_L - 1} = 0,$$

so that

$$1 - (1 - q_{LH})\frac{v_H - v_L}{p_H - p_L}y^{(v_H - v_L) - (p_H - p_L)} = 0.$$

Clearly, the equation above has at most one solution on interval $[0, 1]$. Therefore, equation (21) has at most two solutions on $[0, 1]$. As one solution is $y = 1$, it implies that there is at most one solution when $0 < y < 1$.

To prove the second part of the proposition, we observe that buyers prefer the L-product if

$$1 - e^{-\gamma(v_L - p_L)} \geq q_{LH}(1 - e^{-\gamma(v_L - p_H)}) + (1 - q_{LH})(1 - e^{-\gamma(v_H - p_H)}),$$

which is equivalent to

$$e^{-\gamma(p_H - p_L)} \leq q_{LH} + (1 - q_{LH})e^{-\gamma(v_H - v_L)}. \tag{22}$$

When $\gamma = +\infty$ then (22) is satisfied as its LHS is zero, and the RHS is positive. In other words, extremely risk-averse buyers will always purchase the L-product. Thus if $\gamma^0 > 0$ is the solution to (8), then all types with $\gamma < \gamma^0$ purchase the H-product and all types with $\gamma > \gamma^0$ purchase the L-product. ■

Proof of Proposition 6: The proof of the proposition consists of two parts. In the first part we reduce the equilibrium system (12) to one equation with one unknown, γ^0. In the second part, we analyze that equation and develop sufficient conditions on $\Phi(\gamma)$ stated in Proposition 6. We begin the first part of the proof proving Lemma 2, which says, for the purpose of proving the existence, one can ignore the last equation in (12) and an unknown variable λ.

Lemma 2 *If for a given $q_{LH} \in [0,1]$ the solution $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ to the reduced system exists, then there exists $q \in [0,1]$ and $\lambda \in [0,1]$ such that $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H, \lambda)$ is a solution to the equilibrium system (12).*

Proof. Plug values $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ into the last equation of (12) to recover $\lambda(1-q)$. Notice that for any $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ one can find $\lambda(1-q)$ such that it is between 0 and 1 and the last equation is satisfied. One can then uniquely recover q and λ: $q = (1-q_{LH})(1-\lambda(1-q))$ and $\lambda = \lambda(1-q)/(1-q)$. The only thing one has to check is that $\lambda, q \in [0,1]$ and that $q_{LH} \leq 1-q$. The last inequality states that the share of low-quality sellers who announce high quality, q_{LH}, cannot be higher than the total share of low-quality sellers, $1-q$. This is straightforward. That $q_{LH} \leq 1-q$ is trivial:

$$q_{LH} \leq 1-q = 1-(1-q_{LH})(1-\lambda(1-q)) = q_{LH} + \lambda(1-q)(1-q_{LH}).$$

Given that $q = (1-q_{LH})(1-\lambda(1-q))$, it is between 0 and 1. Finally, $\lambda < 1$ is equivalent to $\lambda(1-q) < 1-q$, which is also true.

$$\lambda(1-q) \leq 1-q = 1-(1-q_{LH})(1-\lambda(1-q)) = \lambda(1-q) + q_{LH}(1-\lambda(1-q))$$

This completes the proof of the lemma. ■

In what follows, we will call (12) without the last equation the reduced system. The reduced system has three equations and three unknowns (γ^0, p_L, p_H), and it treats q_{LH} as a given parameter. The three equations of the reduced system are the indifference condition

$$e^{-\gamma^0(p_H-p_L)} = q_{LH} + (1-q_{LH})e^{-\gamma^0(v_H-v_L)}, \tag{23}$$

and two FOCs that determine prices:

$$\begin{cases} -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L,p_H)}{\partial p_L}p_L + (1-\Phi(\gamma^0)) = 0 \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L,p_H)}{\partial p_L}p_H + \Phi(\gamma^0) = 0 \end{cases} \tag{24}$$

Take the second equation of (24) and subtract from it the first equation of (24). We get

$$\frac{\Phi(\gamma^0)}{\phi(\gamma^0)} - \frac{1-\Phi(\gamma^0)}{\phi(\gamma^0)} = \frac{\partial\gamma^0(p_L,p_H)}{\partial p_L}(p_H-p_L).$$

It will be convenient to denote $\dfrac{\Phi(\gamma^0)}{\phi(\gamma^0)} - \dfrac{1-\Phi(\gamma^0)}{\phi(\gamma^0)}$ as $A(\gamma^0)$. From (23) we get that

$$\begin{aligned} \frac{\partial\gamma^0}{\partial p_L} = -\frac{\partial\gamma^0}{\partial p_H} &= -\frac{\gamma^0 e^{-\gamma^0(p_H-p_L)}}{-(p_H-p_L)e^{-\gamma^0(p_H-p_L)} + (1-q_{LH})(v_H-v_L)e^{-\gamma^0(v_H-v_L)}} = \\ &= \frac{\gamma^0 e^{-\gamma^0(p_H-p_L)}}{(p_H-p_L)e^{-\gamma^0(p_H-p_L)} - (1-q_{LH})(v_H-v_L)e^{-\gamma^0(v_H-v_L)}}. \end{aligned}$$

Let $\Delta p = p_H - p_L$ and $\Delta v = v_H - v_L$. Thus the reduced system becomes a system of two equations and two unknowns:

$$\begin{cases} e^{-\gamma^0\Delta p} = q_{LH} + (1-q_{LH})e^{-\gamma^0\Delta v} \\ A(\gamma^0) = \dfrac{\gamma^0 e^{-\gamma^0\Delta p}}{\Delta p e^{-\gamma^0\Delta p} - (1-q_{LH})\Delta v e^{-\gamma^0\Delta v}}\Delta p \end{cases} \tag{25}$$

Next we solve for Δp from the second equation of (25):

$$A(\gamma^0) = \frac{\gamma^0 e^{-\gamma^0 \Delta p}}{\Delta p e^{-\gamma^0 \Delta p} - (1 - q_{LH}) \Delta v e^{-\gamma^0 \Delta v}} \Delta p.$$

We can re-write it as

$$\begin{aligned}
A(\gamma^0)\Delta p e^{-\gamma^0 \Delta p} - A(\gamma^0)(1 - q_{LH})\Delta v e^{-\gamma^0 \Delta v} &= \gamma^0 \Delta p e^{-\gamma^0 \Delta p} \\
A(\gamma^0)\Delta p - A(\gamma^0)(1 - q_{LH})\Delta v e^{-\gamma^0 \Delta v} e^{\gamma^0 \Delta p} &= \gamma^0 \Delta p \\
A(\gamma^0)(1 - q_{LH})\Delta v e^{-\gamma^0 \Delta v} \frac{e^{\gamma^0 \Delta p}}{\Delta p} &= A(\gamma^0) - \gamma^0 \\
\frac{\gamma^0 A(\gamma^0)(1 - q_{LH})\Delta v e^{-\gamma^0 \Delta v}}{A(\gamma^0) - \gamma^0} &= \gamma^0 \Delta p e^{-\gamma^0 \Delta p}. \qquad (26)
\end{aligned}$$

It is straightforward to show that in equilibrium $\Delta p > 0$. Indeed, consider the indifference condition

$$e^{-\gamma^0 (p_H - p_L)} = q_{LH} + (1 - q_{LH}) e^{-\gamma^0 (v_H - v_L)},$$

and take natural logarithm of both sides:

$$-\gamma^0 (p_H - p_L) = \ln(q_{LH} + (1 - q_{LH}) e^{-\gamma^0 (v_H - v_L)}).$$

The expression inside the logarithm is less than one. Then $\ln(q_{LH} + (1 - q_{LH}) e^{-\gamma^0 (v_H - v_L)}) < 0$, which implies that $\Delta p > 0$. From the second and third equations of (12), it follows that $\dfrac{\Phi(\gamma^0)}{\phi(\gamma^0)} > \dfrac{1 - \Phi(\gamma^0)}{\phi(\gamma^0)}$ and, therefore, in equilibrium $A(\gamma^0) > 0$. Furthermore, since $\Delta p > 0$, the LHS of (26) must be positive. Since, in equilibrium, $A(\gamma^0) > 0$, it then implies that $A(\gamma^0) > \gamma^0$. If such γ^0 does not exist, then (26) cannot be satisfied and no equilibrium with the disclosure of negative information can exist.

We can now eliminate Δp from (26). From the indifference condition we get that

$$\gamma^0 \Delta p = -\ln(q_{LH} + (1 - q_{LH}) e^{-\gamma^0 \Delta v}),$$

and then

$$\gamma^0 \Delta p e^{-\gamma^0 \Delta p} = -\ln(q_{LH} + (1 - q_{LH}) e^{-\gamma^0 \Delta v})(q_{LH} + (1 - q_{LH}) e^{-\gamma^0 \Delta v}).$$

Plugging it into (26) we get that the equilibrium value of γ^0 is determined by

$$-\frac{\gamma^0 A(\gamma^0)(1 - q_{LH})\Delta v e^{-\gamma^0 \Delta v}}{A(\gamma^0) - \gamma^0} = (q_{LH} + (1 - q_{LH}) e^{-\gamma^0 \Delta v}) \ln(q_{LH} + (1 - q_{LH}) e^{-\gamma^0 \Delta v}). \qquad (27)$$

This completes the first part of the proof. In the second part of the proof, we will analyze equation (27) and derive condition that determines whether the solution exists or not. In what follows we will refer to the RHS and LHS of (27) as simple RHS and LHS without referring to the equation's number.

Proof of i and ii: As we have established earlier, if $A(\gamma) < \gamma$ for every γ, then the solution to (26) and, therefore, to (27) does not exist. We will show that this is the case for uniform and convex distributions. Let the support of $\Phi(\gamma)$ be $[0, \Gamma]$. For uniform and convex distributions it is finite. Inequality $A(\gamma) < \gamma$ is equivalent to $2\Phi(\gamma) - 1 < \gamma\phi(\gamma)$. We will write it as $\Phi(\gamma) - 1 < \gamma\phi(\gamma) - \Phi(\gamma)$. Function $\Phi(\gamma)$ is a weakly convex function such that $\Phi(0) = 0$. By a standard property of convex functions $\gamma\phi(\gamma) \geq \Phi(\gamma) - \Phi(0)$ and since $\Phi(0) = 0$ we have $\gamma\phi(\gamma) \geq \Phi(\gamma)$. Thus in the inequality $\Phi(\gamma) - 1 < \gamma\phi(\gamma) - \Phi(\gamma)$ the left-hand side is negative and the right-hand side is non-negative, which means that it is satisfied for any $\gamma \in [0, \Gamma)$. When $\gamma = \Gamma$ and $\Phi(\gamma)$ is linear, then $A(\Gamma) = \Gamma$, and in all other cases, $A(\Gamma) < \Gamma$. Thus, for the case of convex and uniform distribution functions there is no equilibrium where both firms split the market.

Proof of iii: The RHS is a continuous function of γ. It is negative for any $\gamma > 0$. When $\gamma = 0$ it is equal to zero. When $\gamma \to \infty$, its limit is equal to $q_{LH} \cdot \ln(q_{LH}) < 0$.

The LHS is discontinuous when $A(\gamma) = \gamma$. Let $\hat{\gamma}$ denote the largest root such that $A(\gamma) = \gamma$. We can show that it exists. First, $A(0) = -1/\phi(0) < 0$. Second, $\lim_{\gamma\to\infty} \gamma\phi(\gamma) = 0$. If the limit is positive, say $z > 0$, then it means that for all sufficiently large γ^0, say for all $\gamma > \Gamma^0$, it has to be the case that $\phi(\gamma) > \frac{1}{2}\frac{z}{\gamma}$. But then

$$\int_{\Gamma^0}^{\infty} \phi(s)ds > \frac{1}{2}\int_{\Gamma^0}^{\infty} \frac{z}{\gamma} d\gamma = \infty,$$

which is a contradiction since it has to be less than or equal to 1. Third,

$$\lim_{\gamma\to\infty} (A(\gamma) - \gamma) = \lim_{\gamma\to\infty} \frac{2\Phi(\gamma) - 1 - \gamma\phi(\gamma)}{\phi(\gamma)} = \frac{1}{0} = \infty.$$

Given that $A(\gamma) - \gamma$ is continuous, we can conclude now that it has roots and that there exists the largest root. In other words, there exists $\hat{\gamma}$ such that $A(\hat{\gamma}) = \hat{\gamma}$ and $A(\gamma) > \gamma$ for every $\gamma > \hat{\gamma}$. Therefore, the LHS is a continuous function for any $\gamma > \hat{\gamma}$.

We can now prove the equilibrium existence. Since $\hat{\gamma}$ is the largest root, it means that for any $\gamma > \hat{\gamma}$ it must be the case that $A(\gamma) > \gamma$, and in a sufficiently small right neighborhood of $\hat{\gamma}$ fraction $A(\gamma)/(A(\gamma) - \gamma)$ is close to plus infinity. Then the LHS is close to $-\infty$ and, therefore, is less than the RHS. When γ is close to infinity, the LHS gets arbitrarily close to zero. This is because all terms of the LHS, including $A(\gamma)/(A(\gamma) - \gamma)$, are bounded and the term $e^{-\gamma\Delta v}$ converges to zero. That $A(\gamma)/(A(\gamma) - \gamma)$ is bounded follows from

$$\lim_{\gamma\to\infty} \frac{A(\gamma)}{A(\gamma) - \gamma} = \frac{2\Phi(\gamma) - 1}{2\Phi(\gamma) - 1 - \gamma\phi(\gamma)} = 1.$$

Therefore, for sufficiently large γ the LHS of (27) is less than the RHS. By continuity, the solution to (27) exists.

Proof of iv: Let support of $\Phi(\gamma)$ be $[0, \Gamma]$ where $\Gamma < \infty$. Then $A(\Gamma) > \Gamma$. Indeed, $A(\Gamma) > \Gamma$ is equivalent to $1 > \Gamma\phi(\Gamma)$. Assume that it is not satisfied so that $\phi(\Gamma) > 1/\Gamma$. Then, since $\phi(\gamma)$ is strictly decreasing we have

$$1 = \int_0^{\Gamma} \phi(s)ds > \int_0^{\Gamma} \frac{1}{\Gamma} ds = 1,$$

which is a contradiction. Also, one can show that $A(\gamma) < \gamma$ when γ is sufficiently close to zero, or more precisely for any γ such that $\Phi(\gamma) < 1/2$. Thus there exists γ such that $A(\gamma) = \gamma$ and let $\hat{\gamma}$ be the largest such γ. Then the LHS of (27) is continuous when $\gamma \in (\hat{\gamma}, \Gamma]$. As in case iii, one could try to use continuity to establish that the solution to (27) exists. However, it might not work with the original distribution because unless Γ is sufficiently large, the LHS will not be close enough to zero to guarantee that the solution exists.

Consider now a cdf function Φ_α defined as $\Phi(\alpha\gamma)$. It is a concave function with support $[0, \Gamma/\alpha]$. Now the largest value of γ is Γ/α. By taking α sufficiently small we can make the support $[0, \Gamma/\alpha]$ large enough so that $\gamma e^{-\gamma \Delta v}$ can be made sufficiently close to zero within the support.

Term $A(\gamma)/(A(\gamma) - \gamma)$ on the other hand will not change. Let $A_\alpha(\gamma)$ be defined similarly to $A(\gamma)$ but with a cdf Φ_α. Then for any $\gamma \in [0, \Gamma]$,

$$\frac{A_\alpha(\gamma/\alpha)}{A_\alpha(\gamma/\alpha) - \gamma/\alpha} = \frac{A(\gamma)}{A(\gamma) - \gamma}.$$

Indeed,

$$\frac{A_\alpha(\gamma/\alpha)}{A_\alpha(\gamma/\alpha) - \gamma/\alpha} = \frac{2\Phi_\alpha(\gamma/\alpha) - 1}{2\Phi_\alpha(\gamma/\alpha) - 1 - (\gamma/\alpha)\phi_\alpha(\gamma/\alpha)} = \frac{2\Phi(\gamma) - 1}{2\Phi(\gamma) - 1 - (\gamma/\alpha)\alpha\phi(\gamma)} = \frac{A(\gamma)}{A(\gamma) - \gamma}.$$

Thus, when α is sufficiently small we can apply the reasoning of case iii) to function $\Phi_\alpha(\gamma)$ to show that the solution exists. ■

Proof of Proposition 7: We will use two lemmas to prove Propositions 7 and 8.

Lemma 3 *Assume that price p results in beliefs $\mu(p)$ and sales s, while price p' results in beliefs μ' and sales s'. Assume that type v weakly prefers price p to price p'. Then*

i) if $s < s'$, then all higher-cost types ($\nu < v$) will strictly prefer p to p';

ii) if $s > s'$, then all lower-cost types ($\nu > v$) will strictly prefer p to p'.

Proof. Prove part i first. Let $\nu < v$. Since type v weakly prefers p to p' it follows that

$$\begin{aligned}
(p - c_v)s &\geq (p' - c_v)s' \\
(p - c_\nu)s + (c_\nu - c_v)s &\geq (p' - c_\nu)s' + (c_\nu - c_v)s' \\
(p - c_\nu)s &\geq (p' - c_\nu)s' + (c_\nu - c_v)(s' - s) > (p' - c_\nu)s',
\end{aligned}$$

where the last inequality follows from the fact that $c_\nu > c_v$ and $s' > s$.

Part ii is similar. Let $\nu > v$ and then

$$\begin{aligned}
(p - c_v)s &\geq (p' - c_v)s' \\
(p - c_\nu)s + (c_\nu - c_v)s &\geq (p' - c_\nu)s' + (c_\nu - c_v)s' \\
(p - c_\nu)s &\geq (p' - c_\nu)s' + (c_\nu - c_v)(s' - s) > (p' - c_\nu)s'.
\end{aligned}$$

It completes the proof. ■

Lemma 4 *For any equilibrium:*

i) there exist $v_1, \dots, v_{N+1}$ such that $v_L = v_1 \leq v_2 \leq \dots \leq v_N \leq v_{N+1} = v_H$, where N can be infinite; all sellers with $v \in (v_i, v_{i+1})$ choose the same price, which we denote as p_i, with probability 1. If type distribution is continuous and $1 < i < N+1$, then type $v = v_i$ is indifferent between p_i and p_{i-1};

ii) $p_1 > \dots > p_N$. Let s_i be the equilibrium sales of sellers with $v \in [v_i, v_{i+1}]$. Then $s_1 < \dots < s_N$.

Proof. First, we show that if there are two types $v_1 < v_2$ that on-equilibrium choose the same price p with a positive probability then all types in (v_1, v_2) will strictly prefer p over any other price and, therefore, will choose it with probability 1. Assume not. Assume type $v' \in (v_1, v_2)$ prefers price $p' \neq p$. If price p' results in higher sales, then by the first part of Lemma 3 type v_2 would also strictly prefer p' to p and, therefore, would not choose p with positive probability. Similarly, if price p' results in lower sales, then by the second part of Lemma 3 type v_1 would strictly prefer p' to p and, therefore, would not choose p with positive probability. Finally, consider the case when price p' results in the same sales as p. Then, if $p' > p$, both v_1 and v_2 would strictly prefer p' over p and would not choose p with positive probability. Similarly, if $p' < p$, then no type would choose price p', including type v'. This proves the interval structure of the equilibrium as defined in i. To show indifference in the case of continuous quality distribution, consider type v_i where $1 < i < N+1$. If type v_i strictly prefers message p_i to p_{i-1} then all types in a sufficiently small neighborhood of v_i also strictly prefer p_i. Similarly, if type v_i strictly prefers p_{i-1} to p_i, then all all types in a sufficiently small neighborhood of v_i will strictly prefer p_{i-1}. Contradiction.

Next we show the monotonicity of prices and sales. Consider type v_i that is indifferent between (p_{i-1}, s_{i-1}) and (p_i, s_i). If the quality distribution is discrete, then v_i does not necessarily belong to its support. However, it exists, and this will suffice for our purpose. It can't be the case that $s_i = s_{i-1}$ because $p_i \neq p_{i-1}$. It cannot be that $s_i < s_{i-1}$ either. Were it the case then by Lemma 3 all types with $v < v_i$ would strictly prefer p_i to p_{i-1}, which contradicts the definition of v_i. Thus, $s_i > s_{i-1}$. But then $p_i < p_{i-1}$ because, were it not the case, all the types in $[v_{i-1}, v_i]$ would strictly prefer price p_i and enjoy both higher price and higher sales. ∎

Now we can prove Proposition 7. Proof by contradiction. Assume there are at least three different on-equilibrium prices: $p_1 > p_2 > p_3 > \dots$. Let p_1 be the price set by seller v_L. Then $p_1 \leq E_{p_1} v$, as otherwise seller v_L would not make any sales. By Lemma 4, $p_i < E_{p_i} v$ for any i, which means that the corresponding sales must be equal to 1 for every $i \neq 1$. When buyers are risk-neutral, it is strictly optimal for each buyer to purchase a product when its price is strictly below its expected quality. But, as we showed in Lemma 4, one cannot have an equilibrium where two different prices result in the same sales. Thus, there are at most two different equilibrium prices. One is the price chosen by low-quality types, including v_L, and that results in $s_1 < 1$. The second price is chosen by all other types, which results in $s_2 = 1$.

It is straightforward to construct an equilibrium with two on-equilibrium prices. Assume there are two types: $\{L, H\}$ with $v_L = 1, c_L = 1$ and $v_H = 2, c_H = 1/2$. The probability of type H is $2/3$.

Two equilibrium prices are $p_1 = 3/2$ and $p_2 = 1$. Type L chooses p_1 with probability 1 and type H randomizes between p_1 and p_2 with equal probabilities. Buyers' beliefs about expected quality are $\mu(p_1) = 3/2$ and $\mu(p_2) = 2$. The corresponding sales are $q_1 = 1/2$ and $q_2 = 1$. One can see that H sellers are indifferent between p_1 and p_2 and seller L strictly prefers p_1. Off-equilibrium beliefs to support this equilibrium are: $\mu(p) = 1$ if $p \neq 1$ and $p \neq 3/2$.■

Proof of Proposition 8: Fix N. Assume that quality is distributed uniformly on $[1, N+1]$. We will specify sellers' costs as well as the distribution of buyers' preferences so that there exists an equilibrium where for any integer i types in $[i, i+1]$ pool. When types $[i, i+1]$ pool and set price p_i the expected loss is equal to

$$\int_i^{i+1} (v - Ev)dv = \int_i^{i+1} \left(v - \frac{i+i+1}{2}\right) dv = \frac{1}{8},$$

and the corresponding demand is given by the location of the indifferent buyer, $Ev - p - b^0\frac{1}{8} = 0$, so that

$$q(p) = \frac{1}{B} 8 \left(\frac{i+i+1}{2} - p\right).$$

Take p_1 as any number between 1 and 1.5 and $p_N = 1$. Here 1.5 is the expected quality of sellers who pool using price p_1, and 1 is the lowest possible quality. Let $\epsilon = (p_1 - p_N)/(N-1)$ and $p_i = p_1 - (i-1)\epsilon$. Type $i+1$ should be indifferent between p_i and p_{i+1}. We will use this indifference to determine c_{i+1}:

$$8((2i+3)/2 - p_{i+1})(p_{i+1} - c_{i+1}) = 8((2i+1)/2 - p_i)(p_i - c_{i+1}).$$

Taking into account that $p_i - p_{i+1} = \epsilon$ we get

$$c_{i+1} = \frac{p_{i+1} - \frac{2i+1}{2}\epsilon + \epsilon(p_{i+1} + p_i)}{1+\epsilon}.$$

Note that, since prices decrease with i, cost decreases as well so that $c_i > c_{i+1}$. The expression above only determines the costs for types with integer quality. For types in between integer points one can use any strictly decreasing function $c(v)$ as long as $c(i) = c_i$ and $c(i+1) = c_{i+1}$. Finally, the corresponding sales are

$$s_{i+1} = \frac{1}{B}\left(\frac{2i+3}{2} - (p_1 - (i-1)\epsilon)\right),$$

and they are increasing function of i. Set B so that $s_N = 1$.

Finally, off-equilibrium beliefs are such that if $p \neq p_i$ for some i, then buyers believe that the seller has the lowest quality of 1.

We can now verify that it's an equilibrium. Any deviation to an off-equilibrium price $p > 1$ will result in zero sales and zero profit. Any deviation to $p < 1$ will result in sales of 1. However, price $p_N = 1$ also results in sales of 1 but at a higher price and, therefore, is more profitable. Deviations to on-equilibrium prices are not profitable by Lemma 4. Consider, for example, sellers with $v \in (i, i+1)$. By construction, type $i+1$ is indifferent between p_{i+1} and p_i. From Lemma 4 it follows that $s_{i+1} > s_i$ and that all types with $v > i+1$ strictly prefer p_{i+1} to p_i while all types with $v < i+1$ strictly prefer p_i. By applying same logic for all types with integer j one can establish that all types with $v \in (i, i+1)$ strictly prefer price p_i over any other equilibrium price.■

8 Tables and Figures

Figure 1: Equilibrium parameters when the high-quality seller separates.

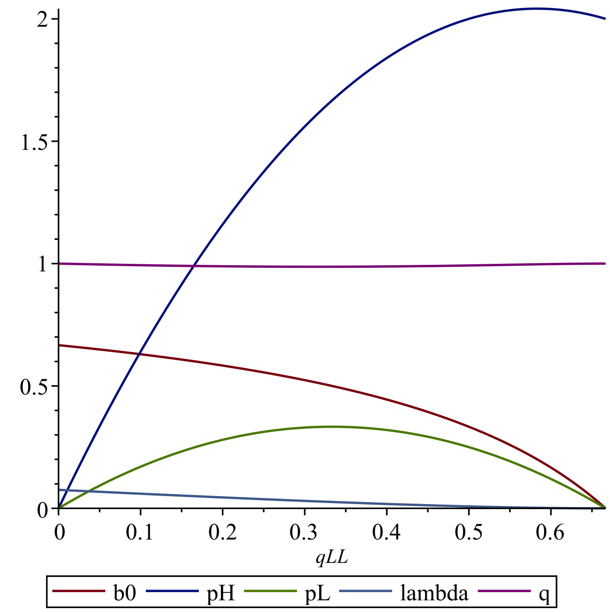

Figure 1: Equilibrium parameters when high-quality seller separates. We assume $b \sim U[0,3]$ and $v_H - v_L = 3$. We use q_{LL}, which is probability of buying low-quality product conditional on message L, instead of q for the horizontal axis because there is not much variation in q as it is very close to 1. b^0 is the loss-aversion of the buyer indifferent between buying the L- and H-products, those with $b < b^0$ will buy the L-product. p_H is the price of the H-seller, and p_L is the price of the L-seller in the subgame after (H, L) messages, λ is the probability that high-quality seller sends message L.

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