18 October 2016

Recursive CARA Preferences and Contracting under Ambiguity

Abstract

We present a decision-theoretic model of ambiguity in an interpersonal context. Individuals receive information about the world directly, through observation of signals, and indirectly through signals observed by others. Information about one's own signal is unambiguous, while information about another's signal is ambiguous. We model this situation using two-stage recursive constant absolute risk aversion (CARA) preferences. We apply the model to a principal–agent problem to determine the form of optimal contracting in the presence of ambiguity and risk.

JEL Classification: D80, D82

Key words: ambiguity, principal-agent problem, state-contingent versus output contingent contracts

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1 Introduction

The problem of ambiguity in insurance contracts has received a good deal of attention from both economists (beginning with Hogarth and Kunreuther [1989]) and legal scholars (for example Thomas [2006]). Fittingly, however, the term 'ambiguity' is itself ambiguous.

In the legal studies literature, as in ordinary language, the term 'ambiguity' is used to describe statements which are open to multiple interpretations. The key concern is which interpretation should be preferred in construing the provisions of contracts. We will refer to 'linguistic ambiguity' to describe this usage. Grant, Kline and Quiggin (2012,2014) and Halpern and Kets (2015) have developed models of linguistic ambiguity in game theoretic settings. Li (2015) has used empirical methods to elicit ambiguity attitudes when the source of ambiguity is linguistic.

In economics and decision theory, 'ambiguity' refers to decision problems in which the probability distribution over states of the world is itself unknown or uncertain. The term is derived from Ellsberg (1961), who uses it to describe "the nature of one's information concerning the relative likelihood of events.... What is at issue might be called the ambiguity of this information, a quality depending on the amount, type, reliability and 'unanimity' of information and giving rise to one's degree of 'confidence' in an estimate of relative likelihoods.(p 657)"

In subsequent decision-theoretic writing, the referent of the term 'ambiguity' has shifted, from the information used to derive relative likelihoods to the likelihoods themselves. We will therefore use the term 'probabilistic ambiguity' to describe the decision-theoretic usage. A crucial distinction between probabilistic and linguistic ambiguity is that the former is typically interpreted in terms of individual beliefs and preferences, while the latter refers naturally to communication between people. In the insurance setting, probabilistic ambiguity is most commonly treated as a property of risks, as perceived by insurers or insurees, while linguistic ambiguity is a property of contracts between insurers and insurees.

In this paper, we present a decision-theoretic model of ambiguity in an interpersonal context. Individuals receive information about the world directly, through observation of signals, and indirectly through signals observed by others. Information about one's own signal is unambiguous, while information about another's signal is ambiguous. In general, the outcome of acts will depend on both signals, so that the relevant state space is a Cartesian product of the two signals.

We propose a two-stage recursive constant absolute risk aversion (CARA) model in which preferences are determined first by the outcome of one's own signal and then, conditional on each possible realization of one's own signal, by the realization of the other's signal. In this model, preferences over unambiguous acts (those measurable with respect to one's own signal) satisfy the usual subjective expected utility properties but preferences over general (that is, ambiguous) acts do not.

The link between this representation of probabilistic ambiguity and the linguistic ambiguity inherent in contracts is illustrated by an application to insurance, considered as a principal–agent problem. In this setting, both the insurer (the principal) and insuree (the agent) receive signals regarding an output produced by the agent, but the signals may disagree, leading to disputes.

We model the agent as undertaking state-contingent production, as in Chambers and Quiggin (2000). The optimal output is lower in unfavorable states of nature, as perceived by the agent. An insurance contract transfers risk from the agent to the principal (insurer) by way of a payment that may depend either on the observed output or the state of nature.

We consider both statement-contingent contracts and output-contingent contracts. Statement-contingent contracts depend on the signal received. Since the principal and agent may receive different signals, these contracts are subject to disputes. Output-contingent contracts, as modeled in Quiggin and Chambers (2006), preclude disputes, since the output is assumed to be observable by both parties. However, since the principal's perception of the state of nature differs from that of the agent, the payoff from the contract is, from the principal's viewpoint ambiguous.

For statement-contingent contracts we consider various types of dispute resolution. Disputes may be resolved as a 'war of attrition' as in Grant, Kline and Quiggin (2012, 2014). In this case, the potential loss from ambiguity is maximized, and output-contingent contracts may be preferred. Under the standard legal doctrine of *contra proferentum*, however, disputes are resolved against the party who drew up the contract (in this case, the principal).[1,2] A third alternative, increasingly preferred by those drafting contracts, is to require disputes to be resolved by arbitration panels, which are generally seen as more favorable to principals (in this case, insurers). We refer to this as mandatory arbitration which we model under the polar assumption that ambiguity is always resolved in favor of the principal.

These are not the only possibilities. One might also envisage contracts that mix the dispute resolution regime. In particular, taking the legal doctrine of contra proferentum as the default, the contract could specify the event in which the agent has agreed to set aside the contra proferentem doctrine and instead have any dispute resolved by mandatory arbitration. In order to avoid the possibility of a meta-dispute about which dispute resolution regime should operate, we argue that such an event should be *unambiguous* in a sense we define formally in the sequel.

In the next section we introduce and axiomatize a model of recursive CARA preferences. In the first stage, the individual sees the realization of her own signal. In the second stage, the realization of the other signal is revealed to her. When restricted to random variables that are measurable with respect to the realization of her own signal, her preferences are consistent to subjective expected utility theory and display no income effects. That is, they admit a representation from the class of CARA functionals. In addition, we require her preferences, conditional on the realization of her own signal, to satisfy the sure thing principle and translation invariance over random variables that are measurable with respect to the other signal. Together these results imply that her unconditional preferences can be represented by

[1] We thank Daniel Quiggin for bringing this doctrine to our attention.

[2] Board and Chung (2009) discuss this doctrine in terms of their object-based model of differential awareness.

what we refer to as a *recursive CARA functional.*[3] Building on the insights of Skiadas (2013) with regard to his source-dependent constant *relative* risk aversion expected-utility model, we see that, for certain natural parameter configurations, her conditional preferences may be construed as being more ambiguous averse than her preferences over random variables measurable with respect to the realization of her own signal.

In section 3 we employ this model of preferences in a principal–agent problem to explore what form optimal contracts might take in the setting referred to above. We characterize various cases in which different forms of contract are optimally chosen. In section 4 we apply the taxonomy developed in this paper to a variety of crop insurance programs. Finally, we offer some concluding comments.

2 Two-stage Recursive CARA Preferences

2.1 The Set-up.

An individual i faces two sources of uncertainty, corresponding to the signal she receives from the space S^i (her "own signal") and the signal another individual receives from the space S^{-i} (the "other signal"). Thus the state space she faces is given by $\Omega = S^i \times S^{-i}$. We take the information flow to be as follows: the individual is first informed about the realization of her own signal followed by the realization of the other signal.

Let f denote a random variable defined on Ω. We write $\mathcal{F}$ for the set of all random variables. Let f^i (respectively, f^{-i}) denote a random variable that is measurable with respect to S^i (respectively, S^{-i}) and let $\mathcal{F}^i$ (respectively, $\mathcal{F}^{-i}$) denote the corresponding set of such random variables. That is, for any f^i in $\mathcal{F}^i$ and any f^{-i} in $\mathcal{F}^{-i}$, we have $f^i(s, s') = f^i(s, \hat{s}')$ and $f^{-i}(s, s') = f^{-i}(\hat{s}, s')$, for all $s, \hat{s}$ in S^i and all $s', \hat{s}'$ in S^{-i}. With slight abuse of notation,

[3] Skiadas (2013) introduces and axiomatizes a two-stage recursive constant *relative* risk aversion expected utility model in which a homotheticity property of preferences replaces translation invariance. However, in his model, the conditional preferences over random variables measurable with respect to the other's-signal are *invariant* to the realization of the individual's own-signal. That is, the degree of ambiguity aversion regarding the realization of the other's-signal is independent of the realization of her own-signal.

To the best of our knowledge, this feature of Skiadas's model is present in all other source-dependent expected utilty models in the extant literature such as Nau (2006), Chew & Sagi (2008), Ergin & Gul (2009) and Grant, Polak & Strzalecki (2009).

any consequence $c \in \mathbb{R}$ will also denote the constant random variable that yields c no matter which state $(s, s') \in \Omega$ obtains. Let $\mathcal{C}$ denote the set of constant random variables. Notice that $\mathcal{C} = \mathcal{F}^i \cap \mathcal{F}^{-i}$.

For any two random variables f and f' in $\mathcal{F}$ and any event $E \subset \Omega$, let $f_E f'$ denote the random variable defined as:

$$f_E f'(\omega) = \begin{cases} f(\omega) & \text{if } \omega \in E \\ f'(\omega) & \text{if } \omega \notin E \end{cases}.$$

In order to avoid overly cumbersome subscripts, for any $A \subseteq S^i$ (respectively, any $B \subseteq S^{-i}$), we will abuse our notation and identify $f_A f'$ (respectively, $f_B f'$) with the random variable $f_{A \times S^{-i}} f'$ (respectively, $f_{S^i \times B} f'$).

An individual will be associated with a preference relation $\succsim$ defined on $\mathcal{F}$. As usual $\succ$ and $\sim$ will denote its asymmetric and symmetric parts, respectively.

An event $E \subset \Omega$ is *null* (with respect to the relation $\succsim$) if $f'_E f \sim f$, for all $f, f' \in \mathcal{F}$. Let $\mathcal{N}$ denote the set of null events. Throughout we shall assume that for each signal realization $s \in S^i$, the event $\{s\} \times S^{-i}$ is not null. We shall refer to this property as *every own-signal realization is essential.*

For each $s \in S^i$, set $S_s^{-i} := \{s' : \{(s, s')\} \notin \mathcal{N}\}$. That is, S_s^{-i} is the set of realizations of the other signal that the individual thinks possible given the realization s of her own signal. Notice that this set is never empty since $\{s\} \times S^{-i}$ is not null.

2.2 Axioms and the characterization

In addition to requiring every own-signal realization to be essential, we consider only preferences that admit a continuous certainty equivalent representation. That is, for each random variable, there is a unique constant random variable with respect to which the individual is indifferent. Equivalently, this can be formally stated as follows.

Definition 1 *A preference relation $\succsim$ on $\mathcal{F}$ admits a* continuous certainty equivalent representation *if there exists a continuous (in the topology of pointwise convergence) function*

$e : \mathcal{F} \to \mathbb{R}$, such that for all pairs of random variables $f, f' \in \mathcal{F}$,

$$e(f) \geq e(f') \text{ if and only if } f \succsim f'.$$

We will interpret in the sequel the restriction of an individual's preferences to random variables measurable with respect to her own signal as reflecting her attitudes toward risk. Furthermore, given the realization of her own signal, her (conditional) preferences over random variables measurable with respect to the other signal will be interpreted as reflecting her perception of the ambiguity embodied in the resolution of the other signal *given* that realization of her own signal.

As is standard in the analysis of principal–agent problems and other *partial equilibrium* settings, we focus on individuals for whom wealth has no effect on their attitudes toward risk and ambiguity. That is, no matter what the individual's current state-contingent endowment is, adding a state-independent amount of wealth leaves unaffected the set of state-contingent bets she would be willing to accept. In the standard expected utility framework, these are preferences for which risk attitudes are characterized by a single parameter, the constant (Arrow–Pratt) coefficient of absolute risk aversion. For such cases the Bernoulli utility index may be expressed as a member of the following family of functions.

Definition 2 *A (Bernoulli) utility index $v : \mathbb{R} \to \mathbb{R}$ has the* constant absolute risk aversion (CARA) *form with parameter α if it can be expressed as a positive affine transformation of the canonical CARA function*

$$u(c) = \begin{cases} [1 - \exp(-\alpha c)] / \alpha & \text{if } \alpha \neq 0 \\ c & \text{if } \alpha = 0 \end{cases}.$$

Notice that, for the canonical CARA function, and for all α, $u(0) = 0$ and $u'(0) = 1$. Further, if $u(\cdot)$ has the CARA form with parameter α, then the coefficient of absolute risk aversion given by $-u''(c)/u'(c)$ is indeed the same for all c and equal to α.

Our characterization of the family of (two-stage) recursive CARA (RCARA) preferences relies on three axioms: the sure thing principle for her own signal; the sure thing principle for conditional preferences over random variables measurable with respect to the other signal arising from each possible realization of her own signal; and, translation invariance.

Axiom 1 (Own-Signal Sure Thing Principle [OS-STP]) *For any* $A \subseteq S^i$*, and any pair of random variables* f, f' *in* $\mathcal{F}$*:* $f \succsim f'_A f \Rightarrow f_A f' \succsim f'$*.*

The own-signal sure thing principle (OS-STP) may be equivalently interpreted as the requirement that the preference relation is *additively separable* with respect to her own signal S^i. This in turn allows us to view the individual's *conditional* (on her own-signal realization) preferences over random variables that are measurable with respect to the other signal, as *not* depending on what might have happened if another realization of her own signal had obtained. More formally, for any pair of random variables f^{-i} and $\hat{f}^{-i}$ in $\mathcal{F}^{-i}$, any pair of random variables $\hat{f}$ and $\hat{f}'$ in $\mathcal{F}$, and any own signal realization s in S^i, it follows from OS-STP that $f^{-i}_{\{s\}}\hat{f} \succsim \hat{f}^{-i}_{\{s\}}\hat{f}$ implies $f^{-i}_{\{s\}}\hat{f}' \succsim \hat{f}^{-i}_{\{s\}}\hat{f}'$.[4]

In light of this, our next axiom, the conditional sure thing principle (C-STP) can be interpreted as requiring the conditional preferences over random variables measurable with respect to the other signal to be additively separable with respect to the other signal S^{-i}.

Axiom 2 (Conditional Sure Thing Principle [C-STP]) *For any own-signal realization* $s \in S^i$*, any set of other-signal realizations* $B \subseteq S^{-i}$*, and any pair of random variables* $f^{-i}, \hat{f}^{-i}$ *in* $\mathcal{F}^{-i}$*:*

$$f^{-i} \succsim \left(\hat{f}^{-i}_B f^{-i}\right)_{\{s\}} f^{-i} \Rightarrow \left(f^{-i}_B \hat{f}^{-i}\right)_{\{s\}} f^{-i} \succsim \hat{f}^{-i}_{\{s\}} f^{-i}.$$

Furthermore, for any $s \in S^i$ *such that* $\left|S^{-i}_s\right| = 2$*, the following 'hexagon' condition also holds: for any six constant random variables* $c, c', c'', \hat{c}, \hat{c}', \hat{c}''$ *in* C*, any* $s' \in S^{-i}_s$ *and any random variable* $f^{-i} \in \mathcal{F}^{-i}$*;*

$$\left(c_{\{s'\}}\hat{c}'\right)_{\{s\}} f^{-i} \sim \left(c'_{\{s'\}}\hat{c}\right)_{\{s\}} f^{-i}, \text{ and } \left(c''_{\{s'\}}\hat{c}\right)_{\{s\}} f^{-i} \sim \left(c'_{\{s'\}}\hat{c}'\right)_{\{s\}} f^{-i} \sim \left(c_{\{s'\}}\hat{c}''\right)_{\{s\}} f^{-i}$$
$$\text{implies } \left(c'_{\{s'\}}\hat{c}''\right)_{\{s\}} f^{-i} \sim \left(c''_{\{s'\}}\hat{c}'\right)_{\{s\}} f^{-i}.$$

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[4] To apply the axiom take $A = \{s\}$, $f = f^{-i}_{\{s\}}\hat{f}$ and $f' = \hat{f}^{-i}_{\{s\}}\hat{f}'$.

Our third axiom, translation invariance, entails that there are no wealth effects on the agent's attitudes toward risk associated with the realization of her own signal or toward the ambiguity that she perceives there to be associated with the realization of the other signal.

Axiom 3 (Translation Invariance [TI]) *For any pair of random variables $f, f' \in \mathcal{F}$, and any constant random variable $c \in \mathcal{C}$,*

$$f \succsim f' \text{ implies } f + c \succsim f' + c.$$

For the family of preferences that we consider, that is, those for which every realization of her own signal is essential and that admit a continuous certainty equivalent representation, our representation result establishes that these three axioms are necessary and sufficient for the preferences to admit a recursive CARA representation.

Theorem 1 (Representation Result) *Suppose $\left|S^i\right| > 2$. For any preference relation for which every realization $s \in S^i$ of her own signal is essential and for which there exists a continuous certainty equivalent representation, the following are equivalent:*

1. *The preference relation $\succsim$ satisfies the own-signal sure thing principle, the conditional sure thing principle and translation invariance.*

2. *There exist: a probability density $\mu(\cdot,\cdot)$ on Ω, marginal density $\mu_{S^i}(\cdot)$ on S^i, and for each $s \in S^i$, conditional density $\mu_{S^{-i}}(\cdot|s)$ on S^{-i} with support S_s^{-i}; a coefficient of constant (ex ante) absolute risk aversion α_0; and, for each $s \in S^i$, a coefficient of (ex interim) absolute uncertainty aversion α_s, such that the certainty equivalent representation may be expressed as*

$$e(f) = u_0^{-1}\left(\sum_{s\in S^i}\mu_{S^i}(s) \times u_0 \circ v_s^{-1}\left(\sum_{s'\in S_s^{-i}}\mu_{S^{-i}}\left(s'|s\right)v_s\left(f\left(s,s'\right)\right)\right)\right),$$

where $u_0(\cdot)$ takes a CARA form with parameter α_0,

and for each $s \in S^i$, $v_s(\cdot)$ takes a CARA form with parameter α_s.

Moreover, $\mu(\cdot,\cdot)$ and α_0 are unique, as is α_s for every own-signal realization s for which the conditional density $\mu_{S^{-i}}(\cdot|s)$ is not degenerate (that is, for which $\left|S_s^{-i}\right| > 1$.)

Proof. 1. $\Rightarrow$ 2. We begin with the restriction of $\succsim$ to $\mathcal{F}^i$, the set of random variables measurable with respect to her own signal. The fact that $\left|S^i\right| > 2$, and for each $s \in S^i$, $\{s\} \times S^{-i} \notin \mathcal{N}$, means that OS-STP implies that the restriction of $\succsim$ to $\mathcal{F}^i$ is additively separable across S^i. Furthermore, from TI it follows that $e\left(f^i + c\right) = e\left(f^i\right) + c$. As is well-known this means that $e(\cdot)$ restricted to $\mathcal{F}^i$ can be expressed as a CARA functional. That is, there exists a (strictly positive) probability density $\mu_{S^i}(\cdot)$ on S^i and a constant coefficient of absolute risk aversion, $\alpha_0 \in \mathbb{R}$, such that $e\left(f^i\right) = u_0^{-1}\left(\sum_{s\in S^i} \mu_{S^i}(s)\, u_0\left(f^i(s,s')\right)\right)$, where $u_0(c) = 1 - \exp(-\alpha_0 c)/\alpha_0$, if $\alpha_0 \neq 0$ or $u_0(c) = c$ if $\alpha_0 = 0$, represents $\succsim$ restricted to $\mathcal{F}^i$.

To extend this representation to general f, it is useful to define for each $s \in S^i$ the binary relation $\succsim^{\{s\}}$ on $\mathcal{F}^{-i}$ by setting $f^{-i} \succsim^{\{s\}} \hat{f}^{-i}$ whenever $f^{-i} \succsim \hat{f}^{-i}_{\{s\}} f^{-i}$.

Lemma 1 *If $f^{-i} \succsim^{\{s\}} \hat{f}^{-i}$ then $f^{-i}_{\{s\}}\tilde{f} \succsim \hat{f}^{-i}_{\{s\}}\tilde{f}$, for all $\tilde{f} \in \mathcal{F}$.*

Proof. *If $f^{-i} \succsim^{\{s\}} \hat{f}^{-i}$ then by definition $f^{-i} \succsim \hat{f}^{-i}_{\{s\}} f^{-i}$. Thus by applying OS-STP for $A = \{s\}$, $f = f^{-i}$ and $\hat{f} = \hat{f}^{-i}_{\{s\}}\tilde{f}$, we have $f^{-i} = f \succsim \hat{f}_{\{s\}} f = \hat{f}^{-i}_{\{s\}} f^{-i}$, implies $f_{\{s\}}\hat{f} = f^{-i}_{\{s\}}\tilde{f} \succsim \hat{f}^{-i}_{\{s\}}\tilde{f} = \hat{f}$, as required.* ■

An immediate implication of Lemma 1 is that this derived relation $\succsim^{\{s\}}$ inherits the properties of completeness and transitivity from $\succsim$. Furthermore, since $\succsim$ admits a continuous certainty equivalent representation, it follows that its restriction to random variables that agree outside the event $\{s\} \times S^{-i}$ to some random variable $\tilde{f}$ (and by Lemma 1 to *any* random variable $\tilde{f}$) satisfies continuity with respect to pointwise convergence. Thus there exists a function $e_s : \mathcal{F}^{-i} \to \mathbb{R}$, that represents $\succsim^{\{s\}}$. It remains to show that $\succsim^{\{s\}}$ also satisfies TI.

Lemma 2 *For any pair of random variables measurable with respect to the other signal $f^{-i}, \hat{f}^{-i} \in \mathcal{F}^{-i}$, and any constant random variable $c \in \mathcal{C}$,*

$$f^{-i} \succsim^{\{s\}} \hat{f}^{-i} \text{ implies } f^{-i} + c \succsim^{\{s\}} \hat{f}^{-i} + c.$$

Proof. *If $f^{-i} \succsim^{\{s\}} \hat{f}^{-i}$ then by definition $f^{-i} \succsim \hat{f}^{-i}_{\{s\}} f^{-i}$. Applying TI yields*

$$\left(f^{-i}+c\right)_{\{s\}}\left(f^{-i}+c\right) \succsim \left(\hat{f}^{-i}+c\right)_{\{s\}}\left(f^{-i}+c\right),$$

which by definition gives us $f^{-i}+c \succsim^{\{s\}} \hat{f}^{-i}+c$, as required. ■

From Lemma 2 it follows that $e_s\left(f^{-i}+c\right) = e_s\left(f^{-i}\right)+c$. Combining this translation invariance with the additive separability across S_s^{-i} that follows from C-STP, leads to a CARA representation of $\succsim^{\{s\}}$ of the form

$$e_s\left(f^{-i}\right) = \begin{cases} v_s^{-1}\left(\sum_{s' \in S_s^{-i}} \mu_{S^{-i}}\left(s'|s\right) v_s\left(f^{-i}\left(s,s'\right)\right)\right) & \text{if } \left|S_s^{-i}\right| > 1 \\ f^{-i}\left(s,\hat{s}'\right), \text{ where } \{\hat{s}'\} = S_s^{-i} & \text{if } \left|S_s^{-i}\right| = 1 \end{cases}$$

To complete the proof that the axioms are sufficient, fix a random variable $f \in \mathcal{F}$. For each realization $s \in S^i$, denote by f_s^{-i} the random variable in $\mathcal{F}^{-i}$ in which $f_s^{-i}\left(\hat{s},s'\right) = f\left(s,s'\right)$ for each $\left(\hat{s},s'\right) \in S^i \times S^{-i}$. It follows from OS-STP and C-STP that for each $s \in S^i$, $\left[e_s\left(f_s^{-i}\right)\right]_{\{s\}} f \sim f$ and hence that $f \sim_{\{s\}} f^i$, where $f^i \in \mathcal{F}^{-i}$ is the random variable (measurable with respect to her own signal) in which $f^i\left(s,s'\right) = e_s\left(f_s^{-i}\right)$ for each $\left(s,s'\right) \in S^i \times S^{-i}$. Combining the representations for each $\succsim^{\{s\}}$ with the representation of $\succsim$ restricted to $\mathcal{F}^{-i}$ yields the expression in statement 2 of the theorem. ■

The necessity of the axioms for the representation is straightforward and so we omit the proof. In the appendix, however, we do provide examples to demonstrate that this characterization is tight.

2.3 Ambiguity aversion and a dual representation

The key feature of the RCARA representation is that the CARA parameter α_s for the conditional other-signal contingent payoffs can be *higher* than the CARA parameter α_0 assigned to own-signal contingent payoffs. This reflects, we would argue, the ambiguity aversion that the individual exhibits with respect to the other signal. This is more apparent in the dual representation for RCARA preferences that we present below.

First, consider a recursive CARA certainty equivalent functional

$$e(f) = u_0^{-1}\left(\sum_{s\in S^i}\mu_{S^i}(s)\left[u_0\circ v_s^{-1}\left(\sum_{s'\in S_s^{-i}}\mu_{S^{-i}}\left(s'|s\right)v_s\left(f\left(s,s'\right)\right)\right)\right]\right),$$

where, without loss of generality, we take

$$u_0(c) = \begin{cases} sign(-\alpha_0)\exp(-\alpha_0 c) & \text{if } \alpha_0 \neq 0 \\ c & \text{if } \alpha_0 = 0 \end{cases}$$

$$\text{and } v_s(c) = \begin{cases} sign(-\alpha_s)\exp(-\alpha_s c) & \text{if } \alpha_s \neq 0 \\ c & \text{if } \alpha_s = 0 \end{cases}, \text{ for each } s\in S.$$

Notice that the certainty equivalent may be re-expressed as follows:

$$e(f) = u_0^{-1}\left(\sum_{s\in S^i}\mu_{S^i}(s)\left[\varphi_s^{-1}\left(\sum_{s'\in S_s^{-i}}\mu_{S^{-i}}\left(s'|s\right)\times\varphi_s\circ u_0\left(f\left(s,s'\right)\right)\right)\right]\right) \quad (1)$$

where,

$$\varphi_s(w) = v_s\circ u_0^{-1}(w) = \begin{cases} sign(-\alpha_s)\left(sign(-\alpha_0)\,w\right)^{\alpha_s/\alpha_0} & \text{if } \alpha_0\neq 0,\ \alpha_s\neq 0 \\ -\frac{1}{\alpha_0}\ln\left(sign(-\alpha_0)\,w\right) & \text{if } \alpha_0\neq 0,\ \alpha_s = 0 \\ v_s(w) & \text{if } \alpha_0 = 0 \end{cases}.$$

In particular, notice that for $\alpha_s > \alpha_0 > 0$, $\varphi_s(\cdot)$ is a strictly increasing and strictly concave *constant relative risk averse (CRRA)* Bernoulli utility index with CRRA parameter

$$-\frac{\varphi_s''(w)}{\varphi_s'(w)}w = \frac{\alpha_s-\alpha_0}{\alpha_0}.$$

Hence for the case where $\alpha_s > \alpha_0 \geq 0$, for all $s\in S$, the certainty equivalent becomes

$$e(f) = \begin{cases} \sum_{s\in S^i}\mu_{S^i}(s)\left[-\frac{1}{\alpha_s}\ln\left(\sum_{s'\in S_s^{-i}}\mu_{S^{-i}}(s'|s)\exp\left(-\alpha_s f(s,s')\right)\right)\right] & \text{if } \alpha_0 = 0 \\ u_0^{-1}\left(\sum_{s\in S^i}\mu_{S^i}(s)\left[-\left(\sum_{s'\in S_s^{-i}}\mu_{S^{-i}}(s'|s)\times\left(-u_0\left(f(s,s')\right)\right)^{\frac{\alpha_s}{\alpha_0}}\right)^{\frac{\alpha_0}{\alpha_s}}\right]\right) & \text{if } \alpha_0 > 0 \end{cases}. \quad (2)$$

For any finite set X, let $\Delta(X)$ denote the set of probability densities defined on X. By applying expressions (9–12) in Skiadas (2013, Theorem 11(ii), p.71) we obtain the following 'dual' representation of the conditional certainty equivalents.

1. For $\alpha = 0 : e(f) =$

$$\sum_{s \in S^i} \mu_{S^i}(s) \left[\min_{p \in \Delta\left(S_s^{-i}\right)} \sum_{s' \in S_s^{-i}} p_{s'} \left(f\left(s, s'\right) + \frac{1}{\alpha_s} \ln \frac{p_{s'}}{\mu_{S^{-i}}\left(s'|s\right)} \right) \right].$$

2. For $\alpha > 0 : e(f) =$

$$u_0^{-1} \left(\sum_{s \in S^i} \mu_{S^i}(s) \left[\min_{p \in \Delta\left(S_s^{-i}\right)} \left(\sum_{s' \in S_s^{-i}} p_{s'} u_0\left(f\left(s, s'\right)\right) \right) \left(\sum_{s' \in S_s^{-i}} \mu_{S^{-i}}\left(s'|s\right) \left(\frac{p_{s'}}{\mu_{S^{-i}}\left(s'|s\right)} \right)^{\frac{\alpha_s}{\alpha_s - \alpha_0}} \right)^{-\frac{(\alpha_s - \alpha_0)}{\alpha_0}} \right] \right).$$

For $\alpha_0 = 0$, it is as if the individual perceives that nature is selecting the conditional probability on S_s^{-i} to minimize the sum of the expected value of the conditional act $f(s, \cdot)$ and a "cost" of selecting a probability on S_s^{-i} that is not $\mu_{S^{-i}}(\cdot|s)$. This cost is proportional to the relative entropy or Kullback–Leibier divergence given by

$$KL\left(p \parallel \mu_{S^{-i}}(\cdot|s)\right) = \sum_{s' \in S_s^{-i}} p_{s'} \ln \frac{p_{s'}}{\mu_{S^{-i}}\left(s'|s\right)}.$$

Thus for $\alpha_0 = 0$, the conditional preferences can be seen to be from the family of multiplier preferences introduced by Hansen and Sargent (2001) and axiomatized by Strzalecki (2011).

For $\alpha_0 > 0$, it is as if the individual perceives nature to be selecting the conditional probability to minimize *the product* of the expected (own-signal) utility of the conditional act $f(s, \cdot)$ and an amount that may be viewed as the "confidence" of that probability relative to the conditional probability $\mu_{S^{-i}}(\cdot|s)$. The confidence is given by

$$C\left(p \parallel \mu_{S^{-i}}(\cdot|s)\right) = \left(\sum_{s' \in S_s^{-i}} \mu_{S^{-i}}\left(s'|s\right) \left(\frac{p_{s'}}{\mu_{S^{-i}}\left(s'|s\right)} \right)^{\frac{\alpha_s}{\alpha_s - \alpha_0}} \right)^{-\frac{(\alpha_s - \alpha_0)}{\alpha_0}}.$$

Its range is $(0, 1]$, with $\max_{p \in \Delta\left(S_s^{-i}\right)} C\left(p | \mu_{S^{-i}}(\cdot|s)\right) = C\left(\mu_{S^{-i}}(\cdot|s) | \mu_{S^{-i}}(\cdot|s)\right) = 1$ and with $1/C\left(p | \mu_{S^{-i}}(\cdot|s)\right)$ convex. Thus for $\alpha_0 > 0$, the conditional preferences can be seen to be related to the multiplicatively variational preference model introduced by Chateauneuf and Faro (2009) and the source-dependent CRRA preference model of Skiadas (2013).

3 Application: Insurance Contracting under Ambiguity

We begin with contracting between a monopolist insurer, referred to as the principal (P), and an insuree, referred to as the agent (A). The parties share a common language for expressing the contingencies or statements that can be included in a contract. We take the most refined set of mutually exclusive and exhaustive contingencies expressible in these statements to correspond to the space S, with $|S| < \infty$. Although they use the same language, the principal and the agent may disagree as to which statement obtains. In particular, when the agent perceives statement s in S to be true, the principal may perceive $s' \neq s$ to have obtained. This leads to the possibility of a dispute. We represent all these possibilities using a state (of the world) space

$$\Omega = S^A \times S^P, \text{ where } S^A = S^P = S.$$

The generic state of the world (s, s') corresponds to an ordered pair of statements, where the agent believes statement s is true and the principal believes statement s' is true.

The preferences of the agent and the principal over random variables in $\mathcal{F}$ are assumed to admit RCARA representations. We assume the agent is risk averse with respect to random variables in $\mathcal{F}^A$ (that is, $\alpha_0^A > 0$), and the principal is risk-neutral with respect to random variables in $\mathcal{F}^P$ (that is, $\alpha_0^P = 0$). Furthermore, we take each party to be (weakly) ambiguity averse in the sense that:

$$\alpha_s^A \geq \alpha_0^A > 0 \text{ and } \alpha_s^P \geq \alpha_0^P = 0 \text{ for every } s \in S. \tag{3}$$

In addition, we presume that each party perceives a possibility of agreement at each $s \in S$ in the sense that:

$$s \in S_s^A \cap S_s^P \text{ for all } s \in S. \tag{4}$$

We also presume that the parties perceptions of possibilities of disagreement are coherent in the sense that:

$$s' \in S_s^A \Leftrightarrow s \in S_{s'}^P \text{ for all } s, s' \in S. \tag{5}$$

Finally, we presume that their marginals agree, that is:

$$\mu^A_{S^A}(s) = \mu^A_{S^P}(s) = \mu^P_{S^A}(s) = \mu^P_{S^P}(s) = \mu(s) \quad \text{for all } s \in S. \tag{6}$$

The assumption of (5) amounts to the parties agreeing on what are the non-null states of the world. Conditions (4) and (5) were already explored in Grant et al. (2012) in terms of complementary symmetry and reflexivity, respectively, of the possibility of dispute relations of the players. Finally, (6) may be interpreted as saying that the agent and the principal share a common prior over the realizations of their respective own signals. Under (6), condition (3) implies that each party weakly prefers random variables measurable with respect to his or her own signal.

Adopting the utility index forms of (1) with the above assumptions, the preferences can be expressed by:

1. for the agent,

$$e^A(f) = -\frac{1}{\alpha^A_0} \ln \left(\sum_{s \in S} \mu(s) \left(\sum_{s' \in S^P_s} \mu^A_{S^P}(s'|s) \exp\left(-\alpha^A_s f(s,s')\right) \right)^{\frac{\alpha^A_0}{\alpha^A_s}} \right);$$

2. for the principal,

$$e^P(f) = -\sum_{s \in S} \mu(s) \frac{1}{\alpha^P_s} \ln \left(\sum_{s' \in S^A_s} \mu^P_{S^A}(s'|s) \exp\left(-\alpha^P_s f(s',s)\right) \right),$$

noting, by l'Hôpital's rule, that

$$\lim_{\alpha^P_s \to 0} \frac{-\ln \left(\sum_{s' \in S^A_s} \mu^P_{S^A}(s'|s) \exp\left(-\alpha^P_s f(s',s)\right) \right)}{\alpha^P_s} = \sum_{s' \in S^A_s} \mu_{S^A}(s'|s) f(s',s).$$

For any vector $\mathbf{z} \in \mathbb{R}^{|S|}$, let $f^A_{\mathbf{z}}$ (respectively, $f^P_{\mathbf{z}}$) denote the random variable generated from $\mathbf{z}$ that is measurable with respect to S^A (respectively, S^P). That is, $f^A_{\mathbf{z}}(s,s') = \mathbf{z}_s$ (respectively, $f^P_{\mathbf{z}}(s,s') = \mathbf{z}_{s'}$) for every $(s,s') \in \Omega$.

Definition 3 *A vector* $\mathbf{z} \in \mathbb{R}^{|S|}$ entails a non-null possibility of dispute at $(s,s') \in \Omega$ *if* (s,s') *is non-null and* $z_s \neq z_{s'}$.

3.1 Production technology

The agent has access to a production technology that uses an input $x \in X \subset \mathbb{R}_+$ to produce a statement contingent output vector $\mathbf{z} \in Z \subset \mathbb{R}_+^{|S|}$. The agent, having committed to produce $\mathbf{z}$ can describe this by saying, 'if statement s is true, I will produce $\mathbf{z}_s$.' The technology is characterized by the input requirement function $x(\mathbf{z})$, which we take to be equal to the agent's cost of effort. For compactness, we take $Z = [0, \overline{z}]^{|S|}$ for some $\overline{z} > 0$. Following Chambers and Quiggin (2000), we presume that $x(\mathbf{z})$ is twice-differentiable, strictly increasing and strictly convex.

3.2 Contracts

We consider two polar forms of contracts in which the principal may choose to make his payment to the agent contingent either on which statement in S obtains or on the output she produces. The advantage of the statement-contingent contract is that the agent's reward need not be made to depend on the output, reducing the risk to which the agent is exposed. However, this comes at the cost of potential disputes arising should the statements the agent and the principal perceive to have obtained not agree. Who bears the ambiguity depends on how and in whose favor such disputes are resolved. The advantage of the output-contingent contract is that it avoids any possibility of dispute, albeit at the cost of having to impose risk on the agent to induce her to commit the input required to generate the statement-contingent output that the contract is designed to deliver, and of exposing the principal to ambiguity since production and payments are measurable with respect to the agent's signal.

For an agent operating in our setting with (certainty equivalent) outside option $\underline{c}$, we shall consider optimal contracts of each type. We give results on the domination of one contract type over another.

In the absence of wealth effects, the form of the optimal contract does not depend on bargaining power. The quasi-linearity of certainty-equivalent utility with respect to non-state-contingent transfers implies that the locus of efficient contracts just differ by a non-state-contingent transfer between the two parties. Hence there is no loss of generality in focusing on a principal-agent problem with the principal offering a take-it-or-leave-it contract.

3.2.1 Statement-contingent contracts

A statement-contingent contract is characterized by a pair of vectors $(\mathbf{y}, \mathbf{z}) \in Z \times Z$, where $\mathbf{z}$ is the statement-contingent output that the contract is designed to implement and $\mathbf{y}$ is the corresponding statement-contingent payment made by the principal to the agent. How these statement-contingent vectors are converted to the corresponding random variables faced by the two parties depends on how disputes are resolved.

War of attrition A dispute may be modeled as a 'war of attrition' as in Grant et al. (2012, 2014). Recall that in the stationary mixed strategy Nash equilibrium of a war of attrition, at each stage a party must be indifferent between disputing the other party's interpretation or conceding. Hence for the agent (respectively, principal), her (respectively, his) state-contingent payoff is $f^A_{\mathbf{z}} - f^P_{\mathbf{z}} + f^P_{\mathbf{y}}$ (respectively, $f^A_{\mathbf{z}} - f^A_{\mathbf{y}}$).

The optimal statement-contingent contract for our principal in this setting is the solution to the following program:

$$\max_{(\mathbf{y},\mathbf{z}) \in Z \times Z} e^P \left(f^A_{\mathbf{z}} - f^A_{\mathbf{y}} \right)$$

subject to

$$e^A \left(f^A_{\mathbf{z}} - f^P_{\mathbf{z}} + f^P_{\mathbf{y}} \right) \geq \underline{c} + x(\mathbf{z}) \qquad \text{(PC\_WA)}$$

We shall refer to a solution to this program as an optimal war-of-attrition contract. We presume that there is a $(\mathbf{y}, \mathbf{z}) \in Z \times Z$ that satisfies the participation constraint $e^A \left(f^A_{\mathbf{z}} - f^P_{\mathbf{z}} + f^P_{\mathbf{y}} \right) \geq \underline{c} + x(\mathbf{z})$. Since $Z \times Z$ is compact and $e^A(\cdot)$, $e^P(\cdot)$ and $x(\mathbf{z})$ are continuous, the war-of-attrition program has a solution.[5]

[5] Since $Z \times Z$ is compact and there is a $(\mathbf{y}, \mathbf{z}) \in Z \times Z$ that satisfies the participation constraint $e^A \left(f^A_{\mathbf{z}} - f^P_{\mathbf{z}} + f^P_{\mathbf{y}} \right) \geq \underline{c} + x(\mathbf{z})$, it follows by the continuity of $e^A(\cdot)$ and $x(\mathbf{z})$ that the constraint set $\left\{ (\mathbf{y}, \mathbf{z}) \in Z \times Z : e^A \left(f^A_{\mathbf{z}} - f^P_{\mathbf{z}} + f^P_{\mathbf{y}} \right) \geq \underline{c} + x(\mathbf{z}) \right\}$ is compact. Then, since $e^P(\cdot)$ is continuous over the compact constraint set, the WA program has a solution by the Weierstrass Theorem.

Being able to make the payment to the agent conditional on the state eliminates the need to satisfy an incentive compatibility constraint. However, since the parties may disagree on which state occurred, the parties are now subject to ambiguity and the associated costs.

There are other possible equilibrium outcomes for a dispute modeled as a war of attrition. Rather than exploring these we shall instead consider two legal procedures that result in polar opposite resolutions. The first always resolves disputes in accordance with the interpretation of the agent while the second does so in accordance with that of the principal.

"Contra proferentem" doctrine The contra proferentem contract, is based on the doctrine of *"verba fortius accipiuntur contra proferentem"* (literally, "words are to be taken most strongly against him who uses them"), which is a rule of contractual interpretation which states that ambiguities in a contract should be construed against the party who drafted the contract. Since it is the principal who is choosing (that is, drafting) the contract, application of this doctrine entails any dispute being resolved in favor of the agent. Hence for the agent (respectively, principal), her (respectively, his) state-contingent payoff is $f_{\mathbf{y}}^A$ (respectively, $f_{\mathbf{z}}^A - f_{\mathbf{y}}^A$).

The optimal statement-contingent contract for our principal in this setting is the solution to the following program:

$$\max_{\langle(\mathbf{y},\mathbf{z})\in Z\times Z\rangle} e^P\left(f_{\mathbf{z}}^A - f_{\mathbf{y}}^A\right)$$

subject to

$$e^A\left(f_{\mathbf{y}}^A\right) \geq \underline{c} + x(\mathbf{z}) \tag{PC\_CP}$$

We shall refer to a solution to this program as an optimal contra proferentem contract. Under the presumption that there is a $(\mathbf{y},\mathbf{z}) \in Z\times Z$ that satisfies the participation constraint $e^A\left(f_{\mathbf{y}}^A\right) \geq \underline{c} + x(\mathbf{z})$, the contra proferentem program has a solution.

Mandatory arbitration A mandatory arbitration clause requires disputes to be resolved by an arbitration panel selected by the principal. We model this under the polar assumption that any dispute is resolved in favor of the principal. Hence for the agent (respectively, principal), her (respectively, his) state-contingent payoff is $f^A_{\mathbf{z}} - f^P_{\mathbf{z}} + f^P_{\mathbf{y}}$ (respectively, $f^P_{\mathbf{z}} - f^P_{\mathbf{y}}$). One way to interpret this, is that there is a "spot" market where the agent can buy and sell as much of the output at the (normalized) price 1 to deliver to the principal what he expects, given his signal's realization.[6]

The optimal statement-contingent contract for our principal in this setting is the solution to:

$$\max_{\langle(\mathbf{y},\mathbf{z})\in Z\times Z\rangle} e^P\left(f^P_{\mathbf{z}} - f^P_{\mathbf{y}}\right)$$

subject to

$$e^A\left(f^A_{\mathbf{z}} - f^P_{\mathbf{z}} + f^P_{\mathbf{y}}\right) \geq \underline{c} + x(\mathbf{z}) \qquad \text{(PC\_MA)}$$

We shall refer to a solution to this program as an optimal mandatory-arbitration contract. Under the presumption that there is a $(\mathbf{y},\mathbf{z}) \in Z\times Z$ that satisfies the participation constraint $e^A\left(f^A_{\mathbf{z}} - f^P_{\mathbf{z}} + f^P_{\mathbf{y}}\right) \geq \underline{c} + x(\mathbf{z})$, this program also has a solution.

3.2.2 Output-contingent contracts

We now consider the case when the parties are unwilling or unable to make the contract contingent on statements in the common language S. Instead they contract on the basis of output which is assumed to be costlessly verifiable and hence unambiguous. This is the form of the standard moral hazard problem.[7]

An output-contingent contract is characterized by a function $y : [0, \overline{z}] \to \mathbb{R}$ and a vector $\mathbf{z} \in Z$, where $\mathbf{z}$ is the statement-contingent output that the contract is designed to implement. The vector $y \circ \mathbf{z} \in Z$ is the associated statement-contingent payment vector in which for each

[6] Alternatively, one might model the situation as generating a relationship-specific product. In this case arbitration leads the agent to pay "damages" that makes the principal as well off as he would have been under his interpretation that statement s' has obtained. Furthermore, for a state (s, s') in which $\mathbf{z}_{s'} < \mathbf{z}_s$, the firm would only accept the output $\mathbf{z}_{s'}$, with the excess $\mathbf{z}_s - \mathbf{z}_{s'}$ 'lost'. In this case the agent's net payment would be $f^P_{\mathbf{y}} - \left[f^P_{\mathbf{z}} - f^A_{\mathbf{z}}\right]^+$, where $\left[f^P_{\mathbf{z}} - f^A_{\mathbf{z}}\right]^+(\omega) = \max\left\{f^P_{\mathbf{z}}(\omega) - f^A_{\mathbf{z}}(\omega), 0\right\}$.

[7] For the representation of the technology used here, the problem is analyzed by Quiggin and Chambers (1998).

$s \in S$, $y(\mathbf{z}_s)$ is the payment that will be made by the principal to the agent in return for output $\mathbf{z}_s$. The optimal output-contingent contract is thus the solution to the following program:

$$\max_{(y,\mathbf{z})} e^P \left(f^A_{\mathbf{z}} - f^A_{y \circ \mathbf{z}} \right) \tag{7}$$

subject to

$$e^A \left(f^A_{y \circ \mathbf{z}} \right) \geq \underline{c} + x(\mathbf{z}) \tag{PC\_OC}$$

$$\mathbf{z} \in \text{argmax}_{\hat{\mathbf{z}} \in Z} \; e^A \left(f^A_{y \circ \hat{\mathbf{z}}} \right) - x(\hat{\mathbf{z}}). \tag{IC\_OC}$$

We shall refer to the solution to this program as an optimal output-contingent contract. (PC_OC) is the standard participation constraint and (IC_OC) is the incentive-compatibility constraint. For the agent to be willing to accept the contract, (PC_OC) says that the agent's certainty equivalent of the statement-contingent payment vector $y \circ \mathbf{z}$ must be at least as great as the sum of her outside option $\underline{c}$ and the input cost of the production schedule $\mathbf{z}$. In addition, (IC_OC) notes that for the principal to expect the agent to plan to produce $\mathbf{z}$, the payment schedule y must be structured so as to make $\mathbf{z}$ a best choice for the agent from those technologically feasible production plans. Racionero and Quiggin (2006) characterized optimal output-contingent contracts for a continuous state space.

3.3 Results

In this section, we present results comparing optimal contracts of various types in the presence of risk and ambiguity. Let C and C' denote two contracts that may be statement-contingent or output contingent. We say that contract C *weakly dominates* contract C' if each party weakly prefers C to C'. Similarly, we say C *strictly dominates* C' if C weakly dominates C' and at least one party strictly prefers C to C'. Our results on domination are summarized as follows. First, we show in Proposition 1 that when contra proferentem contracts are available, output contracts will never be optimally chosen. When contra proferentem contracts are not available, however, output contracts may become viable alternatives to mandatory-arbitration or war-of-attrition contracts. Next, we dispense with war-of-attrition contracts that are weakly dominated by contra proferentem and mandatory-arbitration contracts (Proposition 2). This leads us to focus on the optimality of contra proferentem versus mandatory-arbitration contracts. In Proposition 3 we show that for the polar case of an ambiguity neutral principal, the optimal contra proferentem contract fully ensures the agent and dominates the optimal mandatory-arbitration contract. For any positive ambiguity aversion of the principal, however, we find that there are parameter configurations for the agent, namely those where he is not too ambiguity or risk averse, such that the optimal mandatory-arbitration contract dominates the optimal contra proferentem contract (Proposition 4). These results motivate mixed contracts which are considered in Section 3.3.1. These contracts allow the parties to condition the choice of mandatory-arbitration and contra proferentem contracts on an unambiguous event.

First, we show that contra proferentem contracts weakly dominate output-contingent contracts.

Proposition 1 *The optimal contra proferentem contract weakly dominates the optimal output-contingent contract.*

Proof. Let $(y', \mathbf{z}')$ be an optimal output-contingent contract. Consider the contra proferentem contract $(\mathbf{y}, \mathbf{z})$ in which $y_s = y(\mathbf{z}_s)$ and $\mathbf{z} = \mathbf{z}'$. This contract implements the optimal output-contingent contract and thus generates the same welfare to the agent and the principal. Since, under the contra proferentem doctrine, the principal no longer has to satisfy the IC constraint, the optimal contra proferentem contract weakly dominates the optimal output-contingent contract. ■

The next proposition suggests that war-of-attrition contracts can be dispensed with if optimal contra proferentem or mandatory-arbitration contracts are available. This is perhaps not surprising since both parties are subject to ambiguity under wars of attrition, while one party is spared under contra proferentem and mandatory arbitration.

Proposition 2 *The optimal contra proferentem contract and the optimal mandatory-arbitration contract each weakly dominate the optimal war-of-attrition contract.*

Proof. Let $(\mathbf{y}', \mathbf{z}')$ be the optimal war-of-attrition contract. Consider the contra proferentem contract $(\mathbf{y}, \mathbf{z})$ with $\mathbf{y} = \mathbf{y}'$ and $\mathbf{z} = \mathbf{z}'$. By construction, the principal is indifferent between the war-of-attrition contract $(\mathbf{y}', \mathbf{z}')$ and the contra proferentem contract. By ambiguity aversion, the agent weakly prefers the contra proferentem contract $(\mathbf{y}, \mathbf{z})$. Thus, the optimal contra proferentem contract is seen to weakly dominate the optimal war-of-attrition contract. In the same way, we can show that the optimal mandatory-arbitration contract dominates the optimal war-of-attrition contract. Take the mandatory-arbitration contract $(\mathbf{y}, \mathbf{z})$ with $\mathbf{y} = \mathbf{y}'$ and $\mathbf{z} = \mathbf{z}'$ and observe that the agent is indifferent between this contract and the war-of-attrition contract $(\mathbf{y}', \mathbf{z}')$, while the principal, who now faces no ambiguity, is weakly better off. ■

Based on these findings, we focus attention in what follows on the contra proferentem and mandatory-arbitration contracts. The contra proferentem contract has the advantage of putting no ambiguity on the agent. If the principal is ambiguity neutral in the sense that $\alpha_s^P = \alpha_0^P = 0$ for all $s \in S$, then applying the contra proferentem doctrine will dominate requiring mandatory arbitration as we show in the next proposition. In part (4) of the proposition, we give a sufficient condition for strict domination of mandatory arbitration by contra proferentem. The sufficient condition is that the optimal mandatory-arbitration contract entails a possibility of dispute at a non-null state. Moving from the optimal mandatory-arbitration contract to a contra proferentem contract allows the principal to eliminate this ambiguity cost to the agent at no additional cost.

Proposition 3 *Suppose the principal is ambiguity neutral in the sense that $\alpha_s^P = \alpha_0^P = 0$ for all $s \in S$. (1) The optimal mandatory-arbitration and war-of-attrition contracts are equivalent; (2) The optimal contra proferentem contract fully insures the agent; (3) The optimal contra proferentem contract weakly dominates any optimal mandatory-arbitration contract; (4) The optimal contra proferentem contract strictly dominates any optimal mandatory-*

arbitration contract $(\mathbf{y}', \mathbf{z}')$ *for which* $\mathbf{z}'$ *entails a non-null possibility of dispute at some* $(s, s') \in \Omega$.

Proof. (1) Since the principal is ambiguity neutral, it follows that $e^P\left(f^P_{\mathbf{z}} - f^P_{\mathbf{y}}\right) = e^P\left(f^A_{\mathbf{z}} - f^A_{\mathbf{y}}\right)$ for any contract $(\mathbf{y}, \mathbf{z})$. Consequently, the war-of-attrition program and the mandatory-arbitration program are equivalent.

(2) Consider any contra proferentem contract $(\mathbf{y}, \mathbf{z})$ that does not fully insure the agent, that is, $y_s \neq y_{s'}$ for some states $s, s' \in S$. By the risk aversion of the agent, it follows that $\sum_{s\in S} \mu(s) y_s = \sum_{s\in S} \mu^A_{S^A} y_s > e^A\left(f^A_{\mathbf{y}}\right)$. Consider the contra proferentem contract $(\mathbf{y}', \mathbf{z}')$ with $y'_s = e^A\left(f^A_{\mathbf{y}}\right)$ for all $s \in S$ and with $\mathbf{z}' = \mathbf{z}$. The agent will be indifferent between $(\mathbf{y}, \mathbf{z})$ and $(\mathbf{y}', \mathbf{z}')$. However, since $(\mathbf{y}', \mathbf{z}')$ involves a smaller expected wage payment to the agent, the principal is strictly better off. Consequently, $(\mathbf{y}, \mathbf{z})$ cannot be an optimal contra proferentem contract.

(3) This follows from (1) and Proposition 2.

(4) Let $(\mathbf{y}', \mathbf{z}')$ be an optimal mandatory-arbitration contract for which $\mathbf{z}'$ entails a possibility of dispute at a non-null state $(s, s') \in \Omega$. Consider the contra proferentem contract $(\mathbf{y}, \mathbf{z})$ with $\mathbf{z} = \mathbf{z}'$ and $y_s = \sum_{s\in S} \mu(s) y'_s$ for all $s \in S$. Under (6), this contra proferentem contract gives the agent the expected value $\sum_{s\in S} \mu(s) y'_s$ of the random variable $f^A_{\mathbf{z}'} - f^P_{\mathbf{z}'} + f^P_{\mathbf{y}'}$ of the mandatory-arbitration contract $(\mathbf{y}', \mathbf{z}')$ at every state $\omega \in \Omega$. Since $\mathbf{z}'$ entails a non-null possibility of dispute at $(s, s') \in \Omega$, the agent must be strictly better off with the contra proferentem contract $(\mathbf{y}, \mathbf{z})$. To see this, observe that both (s, s) and (s', s') are non-null by (4). To have equality of net payments at these states, we need $y_s = y_{s'}$. But then, since $z_s \neq z_{s'}$, it follows that $z_s - z_{s'} + y_{s'} \neq y_s$, that is, the payments at (s, s') and (s, s) must differ. The risk aversion and ambiguity aversion of the agent imply that she strictly prefers the contra proferentem contract $(\mathbf{y}, \mathbf{z})$. Since the ambiguity neutral principal is indifferent between the two contracts by (6), the optimal contra proferentem contract strictly dominates $(\mathbf{y}', \mathbf{z}')$. ■

When the ambiguity of the principal is non-neutral, things get more interesting. One might wonder if we will reverse result (3) from Proposition 3 and find that mandatory-arbitration contracts dominate contra proferentem contracts whenever the agent is ambiguity neutral in the face of an ambiguity averse principal. Let's look at this situation more closely to see why it is not so straightforward. It is true that, by using the mandatory-arbitration rule, an ambiguity averse principal is freed from ambiguity. This suggests that there are then mutual gains to be had from fully insuring the agent after choosing mandatory arbitration. If the parties could contract on $\Omega = S \times S$, the optimal mandatory-arbitration contract would involve such insurance. However, since the parties can only contract on S, fully insuring the agent might not be desirable. Under mandatory arbitration, the agent faces the random variable $f^A_{\mathbf{z}} - f^P_{\mathbf{z}} + f^P_{\mathbf{y}}$. Full insurance might require constant output across states and thus involve inefficient production plans.

In short, for an ambiguity averse principal, there are trade-offs between mandatory-arbitration and contra proferentem contracts which depend upon the ambiguity and risk aversions of the parties as well as the production technology. contra proferentem allows the principal to fully insure the agent without imposing any efficiency loss in production. However, contra proferentem entails ambiguity costs born by the principal. mandatory-arbitration contracts, on the other hand, involve no ambiguity cost to the principal, but do involve costly ambiguity and risk to the agent. These costs can be reduced, but may come at the cost of sub-optimal output plans.

As we have seen in Proposition 3, for an ambiguity neutral principal, the optimal contra proferentem contract strictly dominates any optimal mandatory-arbitration contract that involves a possibility of dispute. From the continuity of the certainty equivalent representations, it follows that the optimal contra proferentem contract will be preferred to the optimal mandatory-arbitration contract provided the ambiguity aversion of the principal is sufficiently small, that is $\max_{s \in S} \alpha^P_s$ is sufficiently close to zero.

In the other direction, if the agent has sufficiently low levels of ambiguity and risk aversion, then the optimal mandatory-arbitration contract will strictly dominate any optimal contra proferentem contract that entails a possibility of dispute for a principal who is sufficiently ambiguity averse.

Proposition 4 *If the agent's ambiguity and risk aversion is sufficiently low, the optimal mandatory-arbitration contract will strictly dominate any optimal contra proferentem contract* $(\mathbf{y}, \mathbf{z})$ *for which the random variable* $\mathbf{z} - \mathbf{y}$ *entails a non-null possibility of dispute at some* $(s, s') \in \Omega$ *where* $\alpha_{s'}^{P} > 0$*.*

Proof. Let $(\mathbf{y}, \mathbf{z})$ be any optimal contra proferentem contract for which the random variable $\mathbf{z} - \mathbf{y}$ entails a possibility of dispute at a non-null state $(s, s') \in \Omega$. Consider the same contract under mandatory arbitration. Since $\alpha_{s'}^{P} > 0$, it follows from (3) and (6) that $e^{P}\left(f_{\mathbf{z}}^{P} - f_{\mathbf{y}}^{P}\right) - e^{P}\left(f_{\mathbf{z}}^{A} - f_{\mathbf{y}}^{A}\right) > \varepsilon$ for some $\varepsilon > 0$. As we reduce $\alpha_{s''}^{A}$ to zero for all $s'' \in S$, the difference $e^{A}\left(f_{\mathbf{y}}^{A}\right) - e^{A}\left(f_{\mathbf{z}}^{A} - f_{\mathbf{z}}^{P} + f_{\mathbf{y}}^{P}\right)$ goes to zero. In particular, there exists $\delta > 0$ such that $e^{A}\left(f_{\mathbf{y}}^{A}\right) - e^{A}\left(f_{\mathbf{z}}^{A} - f_{\mathbf{z}}^{P} + f_{\mathbf{y}}^{P}\right) < \varepsilon$ whenever $\alpha_{s''}^{A} < \delta$ for all $s'' \in S$. Thus, if the agent's ambiguity and risk aversion is sufficiently low, the optimal mandatory arbitration contract will strictly dominate any optimal contra proferentem contract $(\mathbf{y}, \mathbf{z})$ for which the random variable $\mathbf{z} - \mathbf{y}$ entails a possibility of dispute at a non-null state $(s, s') \in \Omega$ where $\alpha_{s'}^{P} > 0$. ■

3.3.1 "Mixed" contra proferentem/mandatory-arbitration contract

The results of the previous subsection, in particular, Propositions 3 and 4 indicate that the optimal choice of mandatory arbitration or contra proferentem as a means of managing dispute resolution depends upon the risk and ambiguity parameters of the parties to the contract. Since the ambiguity parameters are statement and party dependent, it is plausible that allowing a contract to condition the type of dispute resolution (mandatory arbitration or contra proferentem) on statements, or subsets of statements, could potentially improve the welfare of both parties. However, to be able to implement such contracts, there would need to be no ambiguity as to whether or not the subset of statements holds.

It seems natural for a subset $E \subset S$ of statements to be deemed an *unambiguous event* whenever each party understands that if the realization of their signal is in E then so will the realization of the other party's signal. Hence, there can be no dispute between the parties about that event obtaining. Adapting Grant et al. (2012), we formally define the set of unambiguous events as follows.

Definition 4 *The set of* unambiguous *events* $\mathcal{E}_{\text{U}} \subseteq 2^S$ *is given by*

$$\mathcal{E}_{\text{U}} = \left\{ E \subseteq S : \bigcup_{s \in E} S_s^A = \bigcup_{s \in E} S_s^P = E \right\}.$$

In Grant et al. (2012, Lemma 1), we established that $\mathcal{E}_{\text{U}}$ is an algebra under conditions that correspond here to (4) and (5). We thus allow the two parties to condition which dispute resolution regime operates on any unambiguous event. We refer to such a contract as a *mixed dispute-resolution-regime contract.* A mixed dispute-resolution-regime contract is characterized by a pair $(\mathbf{y}, \mathbf{z}) \in \mathbb{R}_+^{|S|} \times Z$, and an event $E \in \mathcal{E}_{\text{U}}$. The vectors $\mathbf{z}$ and $\mathbf{y}$ are as before, while E denotes the event in which the agent has agreed to set aside the *contra preferentem* doctrine and instead have any dispute resolved by *mandatory arbitration*.

To specify the program for a principal choosing a mixed dispute-resolution-regime contract it is convenient to introduce the following notation. For any vector $\mathbf{z} \in Z$ and any $E \in \mathcal{E}_{\text{U}}$ let $\left(f_{\mathbf{z}}^P\right)_E \left(f_{\mathbf{z}}^A\right)$ denote the random variable

$$\left(f_{\mathbf{z}}^P\right)_E \left(f_{\mathbf{z}}^A\right)(s, s') = \begin{cases} \mathbf{z}_{s'} & \text{if } s \in E \\ \mathbf{z}_s & \text{if } s \notin E \end{cases}.$$

This random variable agrees with $f_{\mathbf{z}}^P$ on $E \times S$ and with $f_{\mathbf{z}}^A$ on $S \backslash E \times S$. Alternatively, we might consider the random variable defined as:

$$\left(g_{\mathbf{z}}^P\right)_E \left(g_{\mathbf{z}}^A\right)(s, s') = \begin{cases} \mathbf{z}_{s'} & \text{if } s' \in E \\ \mathbf{z}_s & \text{if } s' \notin E \end{cases}.$$

Notice that $\left(g_{\mathbf{z}}^P\right)_E \left(g_{\mathbf{z}}^A\right)$ agrees with $\left(f_{\mathbf{z}}^P\right)_E \left(f_{\mathbf{z}}^A\right)$ on $E \times E$ and on $S \backslash E \times S \backslash E$. Now, since $E \in \mathcal{E}_{\text{U}}$, it follows that $E \times S \backslash E$ and $S \backslash E \times E$ are null for both parties. Hence we have $e^A \left(\left(f_{\mathbf{z}}^P\right)_E \left(f_{\mathbf{z}}^A\right)\right) = e^A \left(\left(g_{\mathbf{z}}^P\right)_E \left(g_{\mathbf{z}}^A\right)\right)$ and $e^P \left(\left(f_{\mathbf{z}}^P\right)_E \left(f_{\mathbf{z}}^A\right)\right) = e^P \left(\left(g_{\mathbf{z}}^P\right)_E \left(g_{\mathbf{z}}^A\right)\right)$, for all $\mathbf{z} \in$

$\mathbb{R}^{|S|}$. So, without loss of generality, we shall associate with a mixed dispute-resolution-regime contract $(\mathbf{y}, \mathbf{z}, E)$ the random variable $\left(f_{\mathbf{z}}^{P}\right)_{E}\left(f_{\mathbf{z}}^{A}\right)$.

The optimal statement-contingent contract for our principal in the mixed dispute-resolution-regime setting is the solution to the following program:

$$\max_{\langle(\mathbf{y},\mathbf{z})\in\mathbb{R}^{2S}, E\in\mathcal{E}_{\mathrm{U}}\rangle} e^{P}\left(\left(f_{\mathbf{z}}^{P}\right)_{E}\left(f_{\mathbf{z}}^{A}\right)-\left(f_{\mathbf{y}}^{P}\right)_{E}\left(f_{\mathbf{y}}^{A}\right)\right)-y$$

subject to

$$e^{A}\left(f_{\mathbf{z}}^{A}-\left(f_{\mathbf{z}}^{P}\right)_{E}\left(f_{\mathbf{z}}^{A}\right)+\left(f_{\mathbf{y}}^{P}\right)_{E}\left(f_{\mathbf{y}}^{A}\right)\right)\geq \underline{c}+x(\mathbf{z}). \tag{PC\_MDRR}$$

Optimal mixed dispute-resolution-regime contracts will always weakly dominate optimal mandatory-arbitration and optimal contra proferentem contracts since S is always an unambiguous event. The insights of Propositions 3 and 4 can be applied to an unambiguous event $E \neq S$ to find situations where mixed contracts strictly dominate. The following corollary follows directly from part (4) of Proposition 3.

Corollary 1 *Let $E \neq S$ be an unambiguous event over which the principal is ambiguity neutral. If the optimal mandatory arbitration contract $(\mathbf{y}, \mathbf{z})$ weakly dominates the optimal contra proferentem contract and $\mathbf{z}$ entails a possibility of dispute at a non-null state $(s, s') \in E \times E$, then the optimal mixed dispute-resolution-regime contract strictly dominates both the optimal mandatory-arbitration and the optimal contra proferentem contracts.*

4 Crop Insurance – An illustration

The design of crop insurance programs has been the subject of considerable analysis and debate in the United States and elsewhere. Many of the main issues are discussed in Hueth and Furtan (1992). Sharecropping systems, which have been the subject of considerable analysis ever since the critical assessment of Marshall (1890), may also be interpreted as forms of insurance (Cheung 1968, Newbery 1977).

Two main classes of crop insurance have been offered in different countries around the world. The first, which includes sharecropping systems, consists of output-contingent contracts providing insurance based on observed yields. The second class insures against losses arising from specific events, such as hail or drought. Depending on the design, these may be modelled either as statement-contingent insurance (if the payment conditional on a given state is fixed) or as mixed contracts (if the payment is conditional both on the stated event and on observed crop loss).

Most crop insurance in the United States is provided through the Federal Crop Insurance Corporation (FCIC) , operated by the Risk Management Agency (RMA) of the US Department of Agriculture (USDA) and is publicly subsidized. The operating budget of RMA for fiscal year 2016 was $74.8 million. RMA managed more than $102 billion worth of insurance liability in 2015, the most recent completed crop year(USDA RMA, 2016a and b).

Historically, the main form of insurance offered was called Multiple Peril Crop Insurance (MPCI). This description is slightly misleading since insurance is not offered against any specific peril or perils, but against any shortfall in crop yields below an agreed value based on historical averages. Hence, MPCI is essentially an output-contingent contract.

Proposition 1 shows that the optimal contra proferentem contract will dominate the optimal output-contingent contract because of the problem of incentive compatibility. Empirical analysis of the performance of the US MPCI scheme supports the conclusion that incentive compatibility is a major problem. Quiggin, Karagiannis and Stanton (1993) estimate that, other things equal, farmers insured under the program experience yields around 20 per cent lower than uninsured farmers.[8]

Quiggin et al. (1993) suggest that a statement-contingent scheme may be preferable. A preferred approach will typically involve a payment which is defined by the observed yield loss, in combination with some deductible, as in the standard multiple risk crop insurance schemes, but is contingent on the occurrence of some insured event. The insured event could be the failure of the regional average yield to reach some predefined value, or a predefined climatic event, as in the rainfall insurance literature.

[8] This outcome could also arise as a result of adverse selection. However, Quiggin et al (1993) argue that the traditional distinction between moral hazard and adverse selection is not properly applicable in this case.

Recent changes in policy have allowed the introduction of a variety of new insurance contracts, initially on an experimental basis. (USDA RMA, 2016c) A number of the new products offer insurance against specified states of the world. Important examples include:

- Rainfall Index based on weather data collected and maintained by the National Oceanic and Atmospheric Administration's Climate Prediction Center. The index reflects how much precipitation is received relative to the long-term average for a specified area and time frame. A payment is made if the index falls below some predetermined level.

- Group Risk Protection is designed as a risk management tool to insure against widespread loss of production of the insured crop in a county. GRP policies use a county yield index as the basis for determining a loss.

- Group Risk Income Protection is designed as a risk management tool to insure against widespread loss of revenue from the insured crop in a county. GRIP policies use a county revenue index as the basis for determining a loss by using the estimated county yield for the insured crop, as determined by National Agricultural Statistics Service, multiplied by the harvest price. If the county revenue falls below the trigger revenue level chosen by the producer, an indemnity is paid. Unlike Group Risk Protection, it is not necessary to have a decline in yield to be indemnified, as long as the combination of price and yield results in a county revenue that is less than the trigger revenue. *Payments are not based on individual producer's crop yields and revenues.* (emphasis added)

It remains to be seen whether these products will displace the more familiar MPCI contract over time. An important consideration is whether the subsidy component of the insurance program is allocated equally across products or whether the MPCI contract continues to be heavily subsidized while other products are offered on an actuarially fair basis.

It is also important to consider how disputes arising from insurance contracts are resolved. Under the RMA procedures, a mediation option is available. However, if mediation fails, the resolution is through arbitration. (USDA RMA, 2016d) Thus, at least in formal terms, the choice is between output-contingent and mandatory arbitration contracts. In view of Propositions 3 and 4, an obvious question is whether a public provider of insurance should be regarded as ambiguity neutral, in which case a contra proferentem contract would be preferable. Considerations of bureaucratic process suggest that rationally designed bureaucracies should be risk neutral, at least with regard to idiosyncratic risks but ambiguity averse. In a rule-governed bureaucracy, an unambiguous demonstration that the rules have been followed is an essential protection in the case of an adverse outcome. On the other hand, given a large portfolio and a long time horizon, aggregate risk should be close to zero, implying that insurance systems should be based on risk neutrality.

It is also important to consider whether our assumption that mandatory arbitration always favors the insurer is applicable in this case. It is arguable that a public insurer, operated by an agency whose goal is to promote agriculture, may be less likely to seek arbitration rules that would resolve disputes in its favor than would a profit-maximizing private insurer.

5 Concluding comments

Ambiguity is most naturally considered in an interpersonal context. The standard theory of subjective probability and the associated consistency requirements seem most convincing when applied to beliefs about states of nature derived from direct observation and introspection. By contrast, information derived from the statements of others is typically ambiguous. Even in the absence of any intent to deceive, we can never be sure that we have taken the meaning intended by others, or communicated the message we intended to send.

In the present paper, we have shown how this problem may be represented, using a two-stage model of preferences. More precisely, in Section 2, we gave representation and axiomatization of two stage CARA preferences which embody risk and ambiguity.

We have then applied the model, in Section 3, to a simple contracting problem involving risky production, with ambiguity regarding the state of nature. We considered trade-offs between statement-contingent contracts and output-contingent contracts. The main advantage of output-contingent contracts is they are ambiguity free. A cost, however, is that the risk averse agent must be subjected to risk in order to induce effort to produce the desired statement-contingent output. On the other hand, statement-contingent contracts avoid the need to subject the agent to risk, but have the potential for disputes that impose ambiguity costs on the parties. The worst type of statement-contingent contract is the War of Attrition contract since both sides face ambiguity. Any well designed dispute resolution will avoid placing ambiguity on one party. We have considered the polar extremes of mandatory arbitration contracts, which place all ambiguity on the agent, and contra proferentem contracts, which place all ambiguity on the principal, as well as mixed MDR contracts, which determine which party is subjected to ambiguity based on an unambiguous event.

In our setting, we have shown that contra proferentem statement-contingent contracts weakly dominate output contracts (Proposition 1) and thus focused attention on the alternative forms of statement-contingent contracts.

For contra proferentem contracts, the agent faces no risk or ambiguity, so the trade-off for the principal is between ambiguity and production efficiency. Here, greater production efficiency (with greater variability of state-contingent production) will be more prone to ambiguity.

For mandatory arbitration contracts, the principal faces no ambiguity. Variable payments to the agent may now serve as a (partial) hedge against the ambiguity he faces. That is, a specific own-signal realization subject to a lot of potential disputes may involve a larger payment to the agent than the other own-signal realizations less prone to disputes. In addition, 'flatter' less efficient production may be used to curb ambiguity costs to the agent. Taking into account these various trade-offs will determine whether mandatory arbitration or contra proferentem dominates. We have given circumstances for domination in Propositions 3 and 4 Exploiting these insights, MDR contracts use unambiguous events to exploit differences in ambiguity perceptions across the state space, so more efficient outcomes become possible.

The practical importance of this analysis is illustrated by the application to publiclyfunded crop insurance programs.

Appendix: Examples demonstrating characterization is tight

The following examples demonstrate that the characterization is tight. For all three examples we shall take both S^i and S^{-i} to be the three element sets $\{s_1, s_2, s_3\}$ and $\{t_1, t_2, t_3\}$, respectively.

Example 1 *The following continuous certainty equivalent functional generates preferences that satisfy the own-signal sure thing principle and translation invariance but they do not satisfy the conditional sure thing principle.*

$$\begin{aligned} e(f) = &\frac{1}{6}\left[f(s_1,t_1)+f(s_2,t_2)\right]+\frac{1}{12}\left[f(s_1,t_2)+f(s_2,t_1)\right]+\frac{1}{2}f(s_3,t_3) \\ &-\frac{1}{36}\sqrt{\left(f(s_1,t_1)-f(s_1,t_2)\right)^2}-\frac{1}{36}\sqrt{\left(f(s_2,t_1)-f(s_2,t_2)\right)^2}. \end{aligned}$$

TI is immediate, since

$$\begin{aligned} e(f+c) = &\frac{1}{6}\left[f(1,1)+c+f(2,2)+c\right]+\frac{1}{12}\left[f(1,2)+c+f(2,1)+c\right]+\frac{1}{2}\left[f(3,3)+c\right] \\ &-\frac{1}{36}\sqrt{\left(f(s_1,t_1)+c-f(s_1,t_2)-c\right)^2}-\frac{1}{36}\sqrt{\left(f(s_2,t_1)+c-f(s_2,t_2)-c\right)^2} \\ = &\frac{1}{6}\left[f(s_1,t_1)+f(s_2,t_2)\right]+\frac{1}{12}\left[f(s_1,t_2)+f(s_2,t_1)\right]+\frac{1}{2}f(s_3,t_3) \\ &-\frac{1}{36}\sqrt{\left(f(s_1,t_1)-f(s_1,t_2)\right)^2}-\frac{1}{36}\sqrt{\left(f(s_2,t_1)-f(s_2,t_2)\right)^2}+c \\ = &\, e(z)+c. \end{aligned}$$

Notice that for any random variable $g \in \mathcal{F}^i$ that is measurable with respect to her own signal,

$$e(g) = \frac{1}{4}g(s_1,t_1)+\frac{1}{4}g(s_2,t_2)+\frac{1}{2}g(s_3,t_3),$$

hence OS-STP also holds. To see how C-STP fails to hold, notice that the hexagon condition fails for the following six consequences:

c	c'	c''	$\hat{c}$	$\hat{c}'$	$\hat{c}''$
0	$\frac{2}{5}$	$\frac{24}{25}$	0	1	$\frac{12}{5}$

To see why, notice first that:

$$\begin{aligned} e\left(h_{\{t_1\}}f\right) &= \frac{1}{4}e_1(h) + \frac{1}{12}f(s_2,t_1) + \frac{1}{6}f(s_2,t_2) + \frac{1}{2}f(s_3,t_3) \\ &- \frac{1}{36}\sqrt{\left(f(s_2,t_1) - f(s_2,t_2)\right)^2}, \\ \textit{where } e_1(h) &= \frac{2}{3}h(s_1,t_1) + \frac{1}{3}h(s_1,t_2) - \frac{1}{9}\sqrt{\left(h(s_1,t_1) - h(s_1,t_2)\right)^2}. \end{aligned}$$

So we have

$$\begin{aligned} e_1\left(c_{\{t_1\}}\hat{c}'\right) &= e_1\left(0_{\{t_1\}}1\right) \\ &= \frac{2}{3}\times 0 + \frac{1}{3}\times 1 - \frac{1}{9}\sqrt{1^2} = \frac{2}{9} \end{aligned}$$

$$\begin{aligned} e_1\left(c'_{\{t_1\}}\hat{c}\right) &= e_1\left(\frac{2}{5}{}_{\{t_1\}}0\right) \\ &= \frac{2}{3}\times\frac{2}{5} + \frac{1}{3}\times 0 - \frac{1}{9}\sqrt{\frac{4}{25}} = \frac{2}{9} \end{aligned}$$

$$\begin{aligned} e_1\left(c''_{\{t_1\}}\hat{c}\right) &= e_1\left(\frac{24}{25}{}_{\{t_1\}}0\right) \\ &= \frac{2}{3}\times\frac{24}{25} + \frac{1}{3}\times 0 - \frac{1}{9}\sqrt{\frac{576}{625}} = \frac{8}{15} \end{aligned}$$

$$\begin{aligned} e_1\left(c'_{\{t_1\}}\hat{c}'\right) &= e_1\left(\frac{2}{5}{}_{\{t_1\}}1\right) \\ &= \frac{2}{3}\times\frac{2}{5} + \frac{1}{3}\times 1 - \frac{1}{9}\sqrt{\frac{9}{25}} = \frac{8}{15} \end{aligned}$$

$$\begin{aligned} e_1\left(c_{\{t_1\}}\hat{c}''\right) &= e_1\left(0_{\{t_1\}}\frac{12}{5}\right) \\ &= \frac{2}{3}\times 0 + \frac{1}{3}\times\frac{12}{5} - \frac{1}{9}\sqrt{\frac{144}{25}} = \frac{8}{15} \end{aligned}$$

$$\begin{aligned} e_1\left(c''_{\{t_1\}}\hat{c}'\right) &= e_1\left(\frac{24}{25}{}_{\{t_1\}}1\right) \\ &= \frac{2}{3}\times\frac{24}{25} + \frac{1}{3}\times 1 - \frac{1}{9}\sqrt{\frac{1}{625}} = \frac{218}{225} \end{aligned}$$

$$e_1\left(c'_{\{t_1\}}\hat{c}'\right) = e_1\left(\frac{2}{5}_{\{t_1\}}\frac{12}{5}\right)$$
$$= \frac{2}{3}\times\frac{2}{5}+\frac{1}{3}\times\frac{12}{5}-\frac{1}{9}\sqrt{\frac{100}{25}} = \frac{38}{45}$$

So in summary, we have:

h	$e_1(h)$
$c_{\{t_1\}}\hat{c}' = 0_{\{t_1\}}1$	$\frac{2}{9}$
$c'_{\{t_1\}}\hat{c} = \frac{2}{5}_{\{t_1\}}0$	$\frac{2}{9}$
$c''_{\{t_1\}}\hat{c} = \frac{24}{25}_{\{t_1\}}0$	$\frac{8}{15}$
$c'_{\{t_1\}}\hat{c}' = \frac{2}{5}_{\{t_1\}}1$	$\frac{8}{15}$
$c_{\{t_1\}}\hat{c}'' = 0_{\{t_1\}}\frac{12}{5}$	$\frac{8}{15}$
$c''_{\{t_1\}}\hat{c}' = \frac{24}{25}_{\{t_1\}}1$	$\frac{218}{225}$
$c'_{\{t_1\}}\hat{c}'' = \frac{2}{5}_{\{t_1\}}\frac{12}{5}$	$\frac{38}{45}$

That is, for any $f \in \mathcal{F}$, we have

$$\left(c_{\{t_1\}}\hat{c}'\right)_{\{s_1\}} f \sim \left(c'_{\{t_1\}}\hat{c}\right)_{\{s_1\}} f, \text{ and } \left(c''_{\{t_1\}}\hat{c}\right)_{\{s_1\}} f \sim \left(c'_{\{t_1\}}\hat{c}'\right)_{\{s_1\}} f \sim \left(c_{\{t_1\}}\hat{c}''\right)_{\{s_1\}} f$$
$$\text{but } \left(c''_{\{t_1\}}\hat{c}'\right)_{\{s_1\}} f \succ \left(c'_{\{t_1\}}\hat{c}''\right)_{\{s_1\}} f,$$

a violation of the hexagon condition.

Example 2 *The following continuous certainty equivalent functional generates preferences that satisfy the conditional sure thing principle and translation invariance but they do not satisfy the own-signal sure thing principle.*

$$e(f) = \frac{1}{3}f(s_1,t_1) + \frac{1}{8}\left(\sum_{j=2}^{3}\left[f(s_2,t_j)+f(s_3,t_j)\right]\right) + \frac{1}{12}\min\left\{\sum_{j=2}^{3} f(s_2,t_j), \sum_{j=2}^{3} f(s_3,t_j)\right\}.$$

TI holds since

$$\begin{aligned}
e(f+c) &= \frac{1}{3}[f(s_1,t_1)+c] + \frac{1}{8}\left(\sum_{j=2}^{3}[f(s_2,t_j)+c+f(s_3,t_j)+c]\right) \\
&+ \frac{1}{12}\min\left\{\sum_{j=2}^{3}[f(s_2,t_j)+c], \sum_{j=2}^{3}[f(s_3,t_j)+c]\right\} \\
&= \frac{1}{3}f(s_1,t_1) + \frac{1}{8}\left(\sum_{j=2}^{3}[f(s_2,t_j)+f(s_3,t_j)]\right) + \frac{1}{12}\min\left\{\sum_{j=2}^{3}f(s_2,t_j), \sum_{j=2}^{3}f(s_3,t_j)\right\} \\
&+ \left(\frac{1}{3}+\frac{1}{8}\times 4+\frac{1}{12}\times 2\right)c \\
&= e(f)+c.
\end{aligned}$$

The C-STP holds since we can re-express $e(\cdot)$ *as:*

$$e(z) = \frac{1}{3}e_{s_1}(f^{s_1}) + \frac{1}{4}[e_{s_2}(f^{s_2}) + e_{s_3}(f^{s_3})] + \frac{1}{6}\min\{e_{s_2}(f^{s_2}), e_{s_3}(f^{s_3})\},$$

$$\text{where } e_{s_1}(h) = h(s_1,t_1),\ e_{s_2}(h) = \frac{1}{2}h(s_2,t_2) + \frac{1}{2}h(s_2,t_3),$$
$$\text{and } e_{s_3}(h) = \frac{1}{2}h(s_3,t_2) + \frac{1}{2}h(s_3,t_3).$$

However, the preferences generated by this continuous certainty equivalent do not satisfy OS-STP, since for

$$g = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in \mathcal{F}^i \text{ and } g' = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \in \mathcal{F}^i,$$

we have

$$e(g) = \frac{1}{3} > \frac{1}{4} = e\left(g'_{\{s_1,s_2\}}g\right) \text{ but } e(g') = \frac{2}{3} > \frac{7}{12} = e\left(g_{\{s_1,s_2\}}g'\right),$$

which entails $g \succ g'_{\{s_1,s_2\}}g$ *and* $g' \succ g_{\{s_1,s_2\}}g'$*, a violation of OS-STP.*

Example 3 *The following continuous certainty equivalent functional generates preferences that satisfy the own-signal sure thing principle and the conditional sure thing principle but they do not satisfy translation invariance:*

$$e(f) = \sum_{s\in S}\frac{1}{3}e_s(f^s),$$

where for each $s \in S^i$*,*

$$e_s(h) = u^{-1}\left(\sum_{t\in T}\frac{1}{3}u(h(s,t))\right),\ u(c) = \begin{cases} \sqrt{c} & \text{if } c \geq 0 \\ -\sqrt{-c} & \text{if } c < 0 \end{cases}.$$

It is immediate from the formulation that the preferences generated by this continuous certainty equivalent functional exhibit OS-STP and C-STP. However, they do not satisfy TI. For example, consider

$$f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in \mathcal{F} \text{ and } c = \frac{1}{27} \in \mathcal{C}.$$

We have:

$$e(f) = \frac{1}{3} \times \left(\frac{1}{3}\right)^2 = \frac{1}{27} = e(c),\ \text{but } e(f+1) = \frac{2}{3} + \frac{1}{3}\left(\frac{4}{3}\right)^2 = \frac{22}{9} > \frac{28}{27} = e(c+1),$$

a violation of translation invariance.

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