Industrialization and the Evolution of Enforcement Institutions*

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September 2016

Abstract

We analyze the evolution of economic institutions during the process of industrialization. In particular, we focus on the institution of contract enforcement. Empirically, we show that countries tend to shift their manufacturing production towards industries that require more relationship-specific investment during the process of industrialization. Theoretically, we build a dynamic model with incomplete contracts and evolving institutions to account for this pattern. In our model, the incompleteness of contracts leads to two types of misallocations that generate production inefficiency: unbalanced use of inputs and unbalanced production of different goods. In addition to this production inefficiency, the imperfect contract enforcement leads to distortions in factor supplies. These distortions have quantitatively significant effects on the aggregate outcome. The model can replicate the empirical patterns when we allow for the government to invest in enforcement institutions in order to improve the contractual environment. We also analyze how different types of governments, in terms of ability to control physical investment and ability to commit to future policies, choose different patterns of institutional investment over time.

Keywords: Industrialization, Institution, Incomplete contract, Misallocation
JEL Classifications: E02, L14, L16, O14, O43

*We thank Satyajit Chatterjee, Nobu Kiyotaki, and seminar and conference participants at Canadian Economic Association Annual Meetings, Midwest Economic Theory Meetings, Midwest Macro Meetings, Tsinghua Workshop in Macroeconomics, SED Meetings, Econometric Society World Congress meeting, and UVa-Richmond Fed Research Jamboree for useful comments. Ilhan Guner provided valuable research assistance. We also thank Allison Oldham for editorial help. All errors are our own.

1 Introduction

There is an emerging empirical and theoretical literature that attributes the fundamental causes of differences in economic development to institutions. Many researchers emphasize the importance of having good economic institutions in the process of long-run economic growth. In their survey of the recent literature, Acemoglu, Johnson, and Robinson (2005a) summarize the empirical evidence and conclude that "differences in economic institutions appear to be the robust causal factor underlying the differences in income per capita across countries" and that "[i]nstitutions are therefore the fundamental cause of income differences and long-run growth" (p.421). In his book on economic growth, Helpman (2004) writes, "institutions that promote the rule of law, enforce contracts, and limit the power of rulers are important for economic development" (p.112).

As a general concept, economic institutions can be considered as a broad set of rules. For example, in his influential work, North (1990) defines institutions as: "the rules of the game in a society or, more formally, are the humanly devised constraints that shape human interaction" (p.3). Since the pioneering work of De Soto (1989) and (2000), it has been widely documented that such "rules of the game" vary substantially across countries.[1] In this paper, we focus on one particular element of such economic institutions: *enforcement of contracts*. This part of institutions is often viewed as essential in promoting economic development. North (1990) forcefully argues that "the inability of societies to develop effective, low-cost enforcement of contracts is the most important source of both historical stagnation and contemporary underdevelopment in the Third World" (p.54). In our empirical and theoretical analysis, we consider one particular effect of imperfect contract enforcement: underinvestment in specific capital.

A key to our analysis is that the economic institutions evolve over time. In the empirical analysis, we show that countries shift production towards industries that requires more specific investments as they industrialize. In the theoretical model, this shift occurs through the gradual improvement of the contractual environment. Our model involves governmental

[1] See, for example, Djankov et al. (2002) and (2003).

investments that improve the contractual environment, such as better protection of property rights and better-functioning courts. This reflects North's (1990) argument that "creating a system of effective enforcement and of moral constraints on behavior is a long, slow process that requires time to develop if it is to evolve" (p.60). Similar views are expressed in Hough and Grier (2014), who argue that developing effective taxation and law enforcement institutions takes centuries.[2]

The main contributions of our paper are twofold. First, we document that over the course of their industrialization, countries shift their production to industries that require more specific investments, and therefore better contract enforcement. Second, we develop a dynamic general equilibrium model that is consistent with this pattern. In particular, we extend Acemoglu, Antràs, and Helpman's (2007) framework into a dynamic environment and allow the government to engage in investment that improves the contractual environment. We provide novel characterizations of the distortions in an economic environment with incomplete contract enforcement. We also highlight the importance of government commitment in long-run economic performance.

In our empirical analysis, we focus on the sectoral composition of different economies and we utilize the fact that industries benefit from good institutions to different degrees. For this reason, we look at the mix of industries across space and over time. Cross-sectional regressions reveal that institutions have a robust effect on a country's sectoral composition, even after controlling for GDP per capita. Industries that require more specific investments (we call these *contract sensitive industries*) have a higher share in countries with better institutions. In panel specifications, we find that average contract sensitivity and GDP per capita are *dynamically* linked: countries that grow rapidly shift towards more contract sensitive industries.

Our theoretical results are related to several strands of literature. We show that, in the model equilibrium, the inefficiencies due to incompleteness of contracts are attributed

[2]They state, "[t]he immediate reason that the development process is so long is that the length of time needed to develop an effective set of laws and rules and the government on which they [economic actors] depend" (p.383).

to two different elements. The first is the loss of productivity due to *misallocation*. When the investment level is not contractible for some production activities, the investment level of these activities is smaller than the contractible investment, and this leads to inefficiently unbalanced input. When the degree of incompleteness is different across products, the quantity produced becomes unbalanced across products as well. This productivity loss shows up as a loss of Total Factor Productivity (TFP) in the aggregate production function in our model. One can view our results as providing a novel microfoundation for the causes of misallocation emphasized in the recent literature, such as Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). The second inefficiency comes from the gap between private and social returns to factor inputs. Since investors can reap only a fraction of the social returns from investment due to contract incompleteness, the demand of production factors is distorted, and this eventually affects the equilibrium factor supply. Our model can be viewed as providing a microfoundation for distortions affecting factor supply, emphasized by Chari, Kehoe, and McGrattan (1997). Many other authors also emphasized the role of investment-good prices on international income differences; see, for example, Restuccia and Urrutia (2001) and Hsieh and Klenow (2007). In an extended model, we allow for heterogeneous productivity across sectors. We find that there is a positive correlation between an industry's productivity and its sensitivity to the contractual environment. This correlation strengthens the effect of contractual frictions on aggregate productivity, in a similar manner as idiosyncratic distortions work in Restuccia and Rogerson (2008). The quantitative effect of these frictions on the aggregate outcome turns out to be economically significant.

We show that our model exhibits a pattern that is consistent with the empirical results when we allow for the institutions to evolve over time. The changes in economic institutions have been considered as an important engine of economic development by many historians. North and Weingast (1989) discuss how the government was able to commit to growth-promoting economic rules, such as secure property rights, following the Glorious Revolution in 17th-century England. They argue that this change led to factors that became a foundation of England's long-term economic success, such as the development of private capital markets.

Acemoglu, Johnson, and Robinson (2005b) describe the growth of Western Europe from 1500 to 1850 and argue that its dynamics were strongly affected by the development of commercial-friendly economic institutions, such as secure private property rights, which in turn was influenced by the medieval political institutions and the growth of Atlantic trade.

In our particular context, there are several approaches to modeling how the contracting environment can be improved. For example, it is possible that private agents who benefit from better contract enforcement spend their own resources in order to improve the contracting environment.[3] In this paper, as a first step, we assume that the contracting institutions are determined by government actions. This assumption is in line with many episodes documented in the economic history literature. North (1990) argues that "there are immense scale economies in policing and enforcing agreements by a polity that acts as a third party and uses coercion to enforce agreements" (p.58). Acemoglu, Johnson, and Robinson (2005a) argue that "if the legal system functions properly, there is an array of enforceable contracts ... These contracts can be enforced because there is an authority, a third party, with the power to enforce contracts. ... all such power emanates from state" (p.429). North and Thomas (1973) state, "[w]e can, as first approximation, view government simply as an organization that provides protection and justice in return for revenue. That is, we pay government to establish and enforce property rights" (p.6). Thus the natural first attempt would be to model this process as government investment in a type of "institutional capital" that enhances contract enforcement.[4] One example that fits this formulation is an improvement of the legal

[3]Popov (2014) considers a related principal-agent model where the principal chooses the level of enforcement.

[4]We use the term "institutional capital" as an analogy to physical capital stock in production, as is clear in our formulation. North (1981, Chapter 15) is explicit about the analogy to capital stock in describing the incremental nature of institutional change. Some other researchers use a similar term "social capital" with somewhat different connotation—for example, Knack and Keefer (1997) consider "trust" and "civic norms" as the contents of social capital. Hall and Jones (1999) consider a concept similar to our social capital and they call it "social infrastructure." They use an index of government antidiversion policy, originally used by Knack and Keefer (1995), and openness to trade in constructing their social infrastructure measure. They argue that social infrastructure is essential in explaining cross-country income differences. Acemoglu, Johnson, and Robinson (2001) and Rodrik, Subramanian, and Trebbi (2004) also find that measures of property rights and rule of law are important in explaining cross-country income differences using different empirical strategies. La Porta et al. (1999) analyze the determinants of government quality, which include the index of property rights, corruption, and bureaucratic delays, in a cross section of countries.

system and courts, which are an important part of contract enforcement institutions.[5]

After demonstrating that the model with evolving institutions is consistent with the data pattern, we endogenize the process of institution building. We analyze a setting where a benevolent government maximizes the representative consumer's utility. We focus on the *positive* implications of the equilibrium outcome: our setting can, for example, be viewed as the result of a political outcome where the policy is determined by probabilistic voting.[6] We consider three different types of governments: one can control investments in both physical capital and institutional capital, and the other two only determine institutional capital directly and let the private market decide the investment in physical capital. Among the two economies where the physical investment is determined by the market, in one case the government can commit to the future government policy and in the other it cannot. We highlight the difference in the institutional dynamics under different types of government.

A number of recent studies emphasize the importance of the contractual environment in economic outcomes. Our empirical study directly builds on the index of contractual exposure, called *contract sensitivity* here, originally created by Nunn (2007). In Nunn (2007), it is shown that this index helps explain the pattern of international trade. Our theoretical framework extends Acemoglu, Antràs, and Helpman's (2007), who analyze the effect of contract enforcement on technology adoption. Building on their model, we offer new characterizations of the distortions in a dynamic environment with incomplete contract enforcement. Boehm (2015) also analyzes contract enforcement and aggregate productivity. He uses a different index of contract sensitivity, called "enforcement intensity," in his cross-country regressions, and his model is static. In contrast, our focus is the dynamic evolution of contract enforcement institutions over time. This paper is also related to recent literature that emphasizes

[5] In a more specific context of default enforcement, Arellano and Kocherlakota (2014) and Drozd and Serrano-Radial (2015) consider limited capacity of legal system and courts in enforcing financial contract. Drozd and Serrano-Radial (2015) call this *enforcement capacity*, which is a very similar concept as our institutional capital.

[6] In the standard model of probabilistic voting, political competition leads to a policy outcome that maximizes the weighted sum of voters' welfare. See, for example, Persson and Tabellini (2000) for a textbook treatment. With a different political setting, there can be distortions in policy choices. See, for example, Mukoyama and Popov (2014) for a recent example with lobbying.

the importance of input-output linkages in the context of economic development.[7] Our paper focuses on a particular type of friction in the production linkages: contract enforcement frictions in specific investment.

A large recent literature studies the role of enforcement frictions in financial markets on aggregate outcome. Examples include Amaral and Quintin (2010), Buera, Kaboski, and Shin (2011), Erosa and Hidalgo-Cabrillana (2008), Midrigan and Xu (2014), and Moll (2014). In contrast to this literature, we focus on the enforcement friction in production process.

Our model in Section 6 features a benevolent social planner who tries to maximize representative consumer's utility. When a representative agent does not exist, there is an additional consideration on how to transfer the efficiency gains from better institutions across different agents.[8] Another important implication of heterogeneity is the possibility of political conflict. An influential strand of literature (see, for example, Acemoglu (2006) for an overview) models the role of political conflict in the persistence of inefficient institutions. Our model abstracts from these considerations, but rather focuses on the role of government in the investment in different types of capital stock and highlights the interaction between government commitment and institutional inefficiency. In this sense, our paper is complementary to this literature.

The paper is organized as follows. The next section documents empirical regularities that motivate our theoretical model. Section 3 develops a model of production under incomplete contracts. Section 4 extends the basic model and incorporates heterogeneous productivity across industries. Section 5 considers a model setting where institutions evolve over time and shows that the model exhibits a pattern that is consistent with the data. Section 6 analyzes a model where the evolution of the institutions is endogenous. Section 7 concludes.

2 Empirical regularities

The quality of enforcement institutions has pervasive effects throughout the economy, including investment in physical and human capital, entrepreneurship, and relationships along the

[7]See, for example, Jones (2011, 2013) and Bartelme and Gorodnichenko (2015).

[8]See, for example, Koeppl, Monnet, and Quintin (2014).

supply chain. In this paper, our main focus is one particular manifestation of distortions due to enforcement frictions: underinvestment in relationship-specific capital.

We investigate the interaction of relationship specificity and contract enforcement through the window of different countries' industrial composition. Industries that require a larger amount of specific investments in their production process are affected to a larger degree by the extent of enforcement frictions. We call the degree of exposure to enforcement frictions of an industry its *contract sensitivity*. Here, our hypotheses are twofold: first, more severe enforcement frictions tilt industrial composition away from contract-sensitive industries; second, a country's industrial composition changes over time with industrialization, reflecting the change in enforcement frictions.

In order to classify industries according to their their contract sensitivity, we build on the influential work of Nunn (2007). He constructs a measure of exposure to enforcement frictions for a variety of industries.[9] Nunn's (2007) measure is constructed as the fraction of intermediate inputs that are nonstandardized, that is, not traded on an exchange and not having a reference price. These intermediate goods correspond to the notion of specific investments. This measure conforms closely to the way we model contractual incompleteness in the next section.

Focusing on international trade, Nunn (2007) empirically shows that better enforcement quality confers comparative advantage in more contract-sensitive industries. Note that our application differs from his analysis in two important dimensions. First, Nunn (2007) considers exports, while we are interested in production.[10] Second, the focus of Nunn (2007) is on the cross-sectional patterns of comparative advantage, while we investigate the dynamics of industrial composition.

In addition to Nunn's measure, we use data on output and value added of industries from the World KLEMS initiative. The initiative coordinates efforts by national and regional statistical offices to construct comparable data on output and productivity at the industry level. We combine Nunn's score of contract sensitivity with output and value added from the

[9] Nunn (2007) calls this measure "contract intensity."

[10] As a robustness check, we also analyze exports in Appendix A. Our conclusions remain the same.

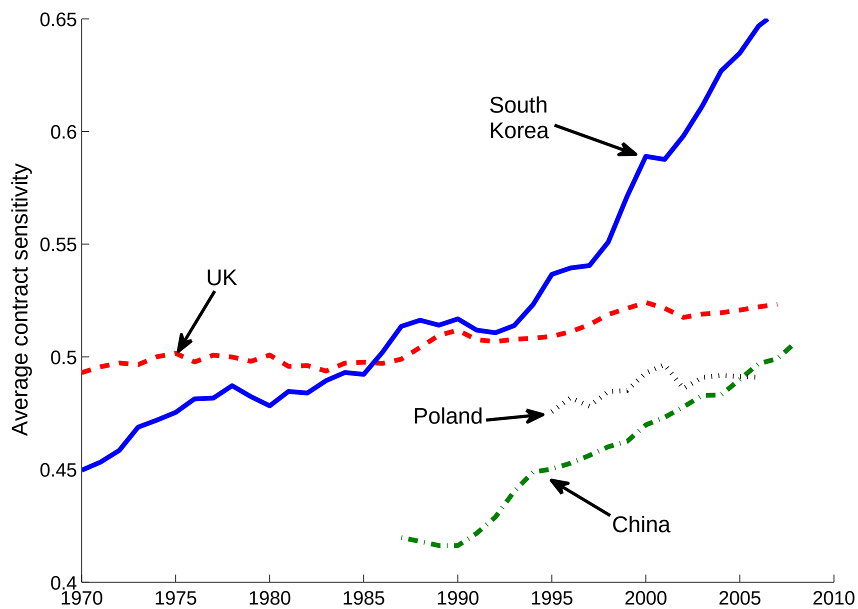

Figure 1: Average contract sensitivity, weighted by value added

KLEMS database to construct weighted scores of contract sensitivity over time and across countries.[11] We also use measures of Purchasing Power Parity GDP per capita from the Penn World Tables version 7. Finally, we use World Governance Indicators from the World Bank as measure of institutions quality. We focus on manufacturing sector, since the structural transformation of an economy (the shift from agriculture to manufacturing to services) is mainly driven by entirely different mechanisms.

Figure 1 plots average contract sensitivity in manufacturing, weighted by value added in different industries for four countries: the United Kingdom (UK), Korea, China, and Poland. We find the following patterns: (1) average contract sensitivity for the already developed countries such as the UK does not exhibit a strong trend; (2) countries that industrialize successfully, such as South Korea and China, experience a marked increase in

[11]KLEMS data is in ISIC 3.1 classification at the two-digit level, while Nunn's measure is in the BEA IO classification. We use a crosswalk between the two classifications using NAICS (North American Industry Classification System) with concordances from the BEA and the Census. For more details on the data construction, see Appendix B.

average contract sensitivity over time. Poland has exhibited moderate growth in recent years, but not as rapid as the industrialization of South Korea and China; its contract sensitivity exhibits an intermediate pattern between the UK and the rapidly-growing economies. Later we construct a dynamic general equilibrium model to interpret this pattern.

Next, we proceed to analyze the data more formally. We explore the relationship between the measure of contract sensitivity, various measures of institutional quality, and output per capita. First, we demonstrate the empirical link between average contract sensitivity in a country and the quality of its economic institutions. We use measures of institutional quality from the World Bank's World Governance Indicators. Our focus is on the variables such as rule of law and government effectiveness, which are most closely related to the notion of contract enforcement. We estimate the following equations:

$$\mathcal{S}_c = \alpha + \beta_1 X_c + \beta_2 \ln(y_c) + \epsilon_c \tag{1}$$

$$\mathcal{S}_c = \alpha + \beta_1 X_c + \beta_2 y_c + \epsilon_c, \tag{2}$$

where c is the country index, $\mathcal{S}_c$ is the average contract sensitivity in country c, X_c are measures of institutional quality in country c, and y_c is the GDP per capita of country c from the Penn World Tables. For the cross-sectional analysis, all variables are for year 2007.

The results for regressions (1) and (2) are displayed in Table 1. Even with a modest sample size, enforcement institutions, interpreted via the rule of law and government effectiveness variables, have a statistically and economically significant effect on the contract sensitivity of industrial composition. In contrast, corruption appears to have little effect. In terms of the magnitude, an improvement in the rule of law variable from the tenth to the ninetieth percentile implies an increase of average contract sensitivity by 0.076.

Next, we consider the link between economic growth and contract sensitivity. We run a panel regression of the change in the contract sensitivity on the change in log real GDP per capita and fixed effects:

$$\Delta\mathcal{S}_{c,t} = \alpha + \beta_1 \Delta \ln(y_{c,t}) + \beta_2 Y_{c,t} + \epsilon_{c,t}, \tag{3}$$

	[1]	[2]	[3]	[4]	[5]	[6]
	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$
Rule of law	0.049*	0.047**				
	(.025)	(.02)				
Government effectiveness			0.049*	0.051**		
			(.027)	(.023)		
Control of corruption					0.028	0.033*
					(.021)	(.018)
ln GDP per capita	-0.036		-0.028		-0.20	
	(.035)		(.033)		(.036)	
GDP per capita ($\times 10^{-6}$)		-1.4		-1.35		-1.28
		(1.08)		(1.06)		(1.15)
N	32	32	32	32	32	32

The dependent variable is average contract sensitivity weighted by value added.
Standard errors in parenthesis.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.
Constant omitted. Institutions measures are on a -2.5: 2.5 scale.

Table 1: Cross-sectional results; value added weights

where $Y_{c,t}$ denotes the fixed effect variables.

The results for regression (3) are summarized in Table 2. The regression results confirm our conclusions from Figure 1: countries that successfully industrialize switch to more contract-sensitive industries. These results suggest that institutional development plays an important role in the process of industrialization.[12]

3 Model

This section constructs a dynamic general equilibrium model where contracts between firms can be incomplete. We show that the incompleteness of contracts affects aggregate productivity through factor misallocation. It also influences the incentives for factor supply through factor prices.

Our model extends Acemoglu, Antràs, and Helpman (2007). We not only embed their model in a dynamic environment, but also provide characterizations of distortions in the

[12] In Appendix A, we repeat the same exercises using the gross output as a weight, instead of value added, and obtain similar results.

	[1] $\Delta\mathcal{S}$	[2] $\Delta\mathcal{S}$	[3] $\Delta\mathcal{S}$	[4] $\Delta\mathcal{S}$
$\Delta \ln$ GDP per capita	0.0339***	0.0317***	0.0278***	0.0277***
	(.007)	(.007)	(.007)	(.008)
Year fixed effect	No	No	Yes	Yes
Country fixed effect	No	Yes	No	Yes
N	945	945	945	945

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Panel regression results; value added weights

economy with incomplete contract enforcement.

There are three layers of production: final goods, intermediate goods, and raw materials. Final goods are introduced mainly for the ease of accounting, and our main focus is on the differentiated intermediate-good sector. Each intermediate good is produced by a monopolist who has to engage in the production activity with many suppliers. The contract between an intermediate-good producer and his suppliers is incomplete, and the degree of incompleteness differ across goods. Therefore, each intermediate good in this model corresponds to an industry in Section 2. Later in Sections 5 and 6, we allow this incompleteness to change over time through the government's activities.

3.1 Environment

The economy consists of a representative consumer and three sectors of production. The final good can be used for consumption and investment and is the numeraire.

3.1.1 Consumers

The representative consumer's utility is

$$\mathbf{U} = \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t),$$

where $\beta \in (0, 1)$ is the discount factor, $u(\cdot, \cdot)$ is the period utility function which is strictly increasing and strictly concave, C_t is consumption at time t, and L_t is the labor supply at time t. The total time endowment is normalized to one.

The household supplies labor, owns a portfolio of shares of all firms in the economy, and owns and accumulates physical capital.[13] Hence the budget constraint for the consumer is

$$K_{t+1} + C_t + T_t = (1 + r_t - \delta_K)K_t + w_t L_t + \Pi_t,$$

where K_t is the physical capital stock, T_t is the lump-sum tax from the government, and δ_K is depreciation rate of physical capital, r_t is the rental rate of physical capital, and w_t is the wage rate. Π_t is the profit from all firm sectors.

3.1.2 Raw-material producers

Raw materials, which are necessary for producing intermediate-goods inputs, are produced from physical capital and labor in a competitive sector. The representative producer has the production function $F(K, L)$ that has constant returns to scale, is strictly concave, and is differentiable.

We also assume standard properties for the marginal product of physical capital:

$$\lim_{K \to 0} F_1(K, L) = \infty,$$

$$\lim_{K \to \infty} F_1(K, L) < \delta_K.$$

Raw materials are used for investment in creating specialized inputs, which are then used for intermediate-good production. One unit of raw material is priced at q_t. Thus, in equilibrium, the equations

$$r_t = q_t F_1(K_t, L_t)$$

and

$$w_t = q_t F_2(K_t, L_t)$$

determine the factor prices, where we use the notational convention of $F_1(\cdot, \cdot)$ and $F_2(\cdot, \cdot)$ to denote the partial derivatives with respect to the first and the second term.

[13] Since there is a representative consumer, it is not necessary to consider trading of shares.

3.1.3 Intermediate-good producers

There is a unit measure of *intermediate goods*, indexed by $z \in [0,1]$. Each intermediate good is produced by a monopolist, and thus we use the index z to also represent each intermediate-good producer. Intermediate goods are made from specialized input $X(j)$, where $j \in [0,1]$, each of which is provided by the supplier j. The production function for the intermediate good z is:

$$y_t(z) = \left[\int_0^1 X_t(j)^\alpha dj \right]^{1/\alpha}, \tag{4}$$

where $\alpha \in (0,1)$. Each supplier provides the specialized input $X(j)$ which is made of $x(i,j)$ specific investments, where $i \in [0,1]$ is a type of investment. The input $X(j)$ is produced from

$$X_t(j) = \exp \left[\int_0^1 \ln x_t(i,j) di \right].$$

The specific investment is done by purchasing the corresponding amount of raw materials, whose unit price is q_t. Thus, in order to invest x_t units, the supplier has to pay $q_t x_t$.

3.1.4 Final-good producers

Final goods are assembled from the intermediate goods under perfect competition. The production function is

$$Y_t = \left[\int_0^1 y_t(z)^\phi dz \right]^{1/\phi},$$

where $y_t(z)$ is the quantity of intermediate good z. We assume that $\phi \in (0,1)$. In equilibrium, the output is used for consumption, investment in physical capital, and government spending:

$$K_{t+1} - (1-\delta)K_t + C_t + T_t = Y_t.$$

The price of each intermediate good z at time t is $p_t(z)$. Because of the constant-returns-to-scale production function and perfect competition, there is no profit in the production of final goods, and we can also assume a representative producer.

3.2 The economy with complete contracts

For the purpose of comparison, we first consider the situation where the contract between the intermediate-good producers and the suppliers can be fully specified. We further assume that the intermediate-good producers can offer a take-it-or-leave-it contract to the suppliers. Since the suppliers' outside option is zero, the intermediate-good producer can extract all surplus.

Then, from symmetry, the intermediate-good producer z can produce $y_t(z)$ units of output with the cost $q_t y_t(z)$. From the profit maximization in the final-good sector, the demand for intermediate good z is

$$y_t(z) = p_t(z)^{\frac{1}{\phi-1}} Y_t.$$

Thus the profit-maximization problem for the intermediate-good producer z is

$$\max_{p_t(z)} \quad \pi_t(z) \equiv (p_t(z) - q_t) p_t(z)^{\frac{1}{\phi-1}} Y_t$$

and the solution is

$$p_t(z) = \frac{q_t}{\phi}.$$

Since the final good's price is assumed to be 1, $p_t(z) = 1$ and therefore $q_t = \phi$. The factor prices are determined by

$$r_t = \phi F_1(K_t, L_t)$$

and

$$w_t = \phi F_2(K_t, L_t).$$

The quantity of raw material produced is $F(K_t, L_t)$ and in equilibrium, this is equal to $x_t(i,j) = X_t(j) = y_t(z) = Y_t$. The total profit for the intermediate-good producers is $\int_0^1 \pi(z)dz = (1-\phi)Y_t$. Since there is no profit for the suppliers, this is the total profit Π_t.

The representative consumer's Euler equation is

$$u_1(C_{t+1}, 1 - L_{t+1}) = \beta(1 + r_{t+1} - \delta_K) u_1(C_t, 1 - L_t),$$

which can be rewritten as

$$u_1(C_{t+1}, 1 - L_{t+1}) = \beta(1 + \phi F_1(K_{t+1}, L) - \delta_K)u_1(C_t, 1 - L_t).$$

Similarly, the first-order condition for labor supply is

$$u_2(C_t, 1 - L_t) = w_t u_1(C_t, 1 - L_t),$$

which implies that

$$u_2(C_t, 1 - L_t) = \phi F_2(K_t, L_t)u_1(C_t, 1 - L_t).$$

Because $T_t = 0$ and $r_t K_t + w_t L_t + \Pi_t = F(K_t, L_t)$, the resource constraint is

$$K_{t+1} + C_t = (1 - \delta_K)K_t + F(K_t, L_t).$$

3.3 The economy with incomplete contracts

Now we turn to the economy with incomplete contracts. We first solve the within-period problem. As will become clear, the decisions of intermediate- and final-good producers are static in nature. It will be shown that, within a period, all quantities and prices can be determined as a function of the physical capital, labor, and by the current contractual environment.

3.3.1 The intermediate good producer problem

First, we tackle the problem of an intermediate-good producer. Suppose that the contract between the intermediate-good producers and the suppliers is incomplete. For product z, we assume that (for every supplier j) the contract is complete for the investments $0 \leq i \leq \mu(z)$ and incomplete for $\mu(z) < i \leq 1$. That is, for the first $\mu(z)$ investments, the level of investments $x_t(i, j)$, where t denotes time t, is contractible, but for the rest of the investments, $x_t(i, j)$ is determined by suppliers. The departure from Acemoglu, Antràs, and Helpman's (2007) framework is that we allow $\mu(z)$ to be heterogeneous across z. Note that $(1 - \mu(z))$ corresponds to the *contract sensitivity* that we measured in Section 2. In this section, we take $\mu(z)$ as given and constant. In Sections 5 and 6, we allow $\mu(z)$ to change over time. We assume that $\mu(z)$ is nondecreasing and Borel-measurable.

Following Acemoglu, Antràs, and Helpman (2007), we adopt the following timing in the game between the intermediate-good producer and the specialized input supplier.

1. The intermediate-good producer z offers a contract $[\{x_{c,t}(i,j)\}_{i=0}^{\mu(z)}, \tau_t(j)]$ for every $j \in [0,1]$. Here, $x_{c,t}(i,j)$ is the contractible investment level and $\tau_t(j)$ is the upfront payment to supplier j.

2. Specialized input suppliers decide whether to accept the contract.

3. For $0 \leq i \leq \mu(z)$, the suppliers invest $x_t(i,j) = x_{c,t}(i,j)$ that is specified in the contract. For $\mu(z) < i \leq 1$, they *decide* $x_t(i,j)$.

4. The suppliers and the intermediate-good producer bargain over the division of the revenue.

5. Output is produced, sold, and the revenue is distributed following the bargaining agreement.

The symmetric subgame perfect equilibrium (SSPE) of this game can be defined and characterized in a similar manner as Acemoglu, Antràs, and Helpman (2007). Below, for simplicity, we suppress the dependence on variables in t and z in this subsection.

In order to solve for the SSPE, we will move backwards, starting from the bargaining stage. As in Acemoglu, Antràs, and Helpman (2007), we use the Shapley value as the bargaining solution among the suppliers and the intermediate good producer. Following the same steps as Acemoglu, Antràs, and Helpman (2007), we obtain that supplier j receives

$$s_j = (1-\gamma) Y^{1-\phi} \left(\frac{x_n(j)}{x_n} \right)^{(1-\mu)\phi} x_c^{\phi\mu} x_n^{\phi(1-\mu)} \tag{5}$$

where $\gamma \equiv \alpha/(\alpha + \phi)$, when it makes $x_n(j)$ units of noncontractible investment, the other suppliers make x_n units of noncontractible investment, and x_c is the amount of contractible investment.[14] The intermediate-good producer i receives

$$s_i = \gamma Y^{1-\phi} x_c^{\phi\mu} x_n^{\phi(1-\mu)}. \tag{6}$$

Foreseeing the bargaining outcome, each supplier decides the noncontractible investment to maximize its profit. Thus, in a symmetric equilibrium,

$$x_n = \arg\max_{x_n(j)} \quad (1-\gamma)Y^{1-\phi}\left(\frac{x_n(j)}{x_n}\right)^{(1-\mu)\phi} x_c^{\phi\mu} x_n^{\phi(1-\mu)} - q(1-\mu)x_n(j).$$

Solving this problem, the optimal noncontractible investment by the supplier for a given x_c is

$$x_n(x_c) = \left[\frac{\alpha(1-\gamma)}{q} x_c^{\mu\phi} Y^{1-\phi}\right]^{\frac{1}{1-\phi(1-\mu)}}. \tag{7}$$

The intermediate-good producer i solves the problem

$$\max_{x_c,\tau} \quad s_i - \tau$$

subject to (7) and

$$s_j + \tau \geq \mu q x_c + (1-\mu) q x_n, \tag{8}$$

where s_i is the surplus received by the intermediate-good producer i, solved in (6), and s_j is the surplus for a supplier, solved in (5). Since (8) holds with equality, the problem can be rewritten, using (5) and (6):

$$\max_{x_c} \quad Y^{1-\phi}\left[x_c^{\mu} x_n(x_c)^{1-\mu}\right]^{\phi} - \mu q x_c - (1-\mu) q x_n(x_c),$$

where $x_n(x_c)$ is the function (7). The solution to this problem is

$$x_c = Y q^{-\frac{1}{1-\phi}} [\alpha(1-\gamma)]^{\frac{\phi(1-\mu)}{1-\phi}} B(\mu)^{1-\phi(1-\mu)},$$

where

$$B(\mu) \equiv \left\{\frac{\phi}{1-\phi(1-\mu)}\left[1-(1-\mu)\alpha(1-\gamma)\right]\right\}^{\frac{1}{1-\phi}}. \tag{9}$$

[14]Appendix C describes the derivation of this solution.

By substituting the above x_c in (7), x_n can be solved as

$$x_n = Yq^{-\frac{1}{1-\phi}} [\alpha(1-\gamma)]^{\frac{1-\phi\mu}{1-\phi}} B(\mu)^{\mu\phi}.$$

The output is

$$y = x_c^{\mu} x_n^{1-\mu} = Yq^{-\frac{1}{1-\phi}} D(\mu), \tag{10}$$

where

$$D(\mu) \equiv [\alpha(1-\gamma)]^{\frac{1-\mu}{1-\phi}} B(\mu)^{\mu}. \tag{11}$$

A notable property of this solution is that, up to the multiplicative factor $Yq^{-\frac{1}{1-\phi}}$, the intermediate-good producer's choice of inputs (x_c, x_n) and output depend only on its own parameter μ. Also, the firm's profit similarly depends on a function of its μ and on the multiplicative factor $Yq^{-\frac{\phi}{1-\phi}}$. This property allows us to derive some properties of the economy, independent of physical capital and labor.

3.3.2 General equilibrium

Finally, we turn to characterize the aggregate outcome. From (10),

$$\left[\int_0^1 y_t(\mu(z))^{\phi} dz\right]^{\frac{1}{\phi}} = Y_t q_t^{-\frac{1}{1-\phi}} \left[\int_0^1 D(\mu(z))^{\phi} dz\right]^{\frac{1}{\phi}}.$$

Since the left-hand side is equal to Y_t, the price of raw materials is given by

$$q_t = \left[\int_0^1 D(\mu(z))^{\phi} dz\right]^{\frac{1-\phi}{\phi}}. \tag{12}$$

Notice that the price of raw materials depends only on the distribution of $\mu(z)$. If this distribution is unchanged, the value of q_t stays constant.

Next, we turn to aggregate output. The total demand for the raw materials is

$$\int_0^1 [\mu(z) x_{c,t}(\mu(z)) + (1-\mu(z)) x_{n,t}(\mu(z))] dz = Y_t q^{-\frac{1}{1-\phi}} \int_0^1 H(\mu(z)) dz,$$

where

$$H(\mu(z)) \equiv \mu(z)[\alpha(1-\gamma)]^{\frac{\phi(1-\mu(z))}{1-\phi}} B(\mu(z))^{1-\phi(1-\mu(z))} + (1-\mu(z))[\alpha(1-\gamma)]^{\frac{1-\phi\mu(z)}{1-\phi}} B(\mu(z))^{\mu(z)\phi}. \tag{13}$$

Here, $H(\mu)$ is the normalized demand for inputs by a firm with enforceability μ.

Since the supply of the raw materials is $F(K_t, L_t)$, this implies (using (12))

$$Y_t = \Theta F(K_t, L_t),$$

where

$$\Theta \equiv \frac{\left[\int_0^1 D(\mu(z))^{\phi} dz\right]^{\frac{1}{\phi}}}{\int_0^1 H(\mu(z))dz}. \tag{14}$$

Note that Θ depends only on the distribution of $\mu(z)$, and thus the existence of $\mu(z) < 1$ acts as the change in Hicks-neutral technology in this framework. The source of this change in efficiency is misallocation: the raw material, which is the only productive resource here, is not allocated optimally across different investments—recall x_c and x_n are different—because of the contractual incompleteness.

Since $r_t = qF_1(K_t, L_t)$ and $w_t = qF_2(K_t, L_t)$, the optimal solution of the representative consumer's problem satisfies the following first order conditions:

$$u_1(C_t, 1-L_t) = \beta(1 + qF_1(K_{t+1}, L_{t+1}) - \delta_K)u_1(C_{t+1}, 1-L_{t+1}), \tag{15}$$

$$u_2(C_t, 1-L_t) = qF_2(K_t, L_t)u_1(C_t, 1-L_t), \tag{16}$$

where q is given in (12). The resource constraint is

$$K_{t+1} + C_t = (1-\delta)K_t + \Theta F(K_t, L_t). \tag{17}$$

3.4 Characterizing the economy with incomplete contracts

In this subsection, we characterize the properties of the incomplete-contracts model discussed above. We first develop several general properties of the model. Then we solve a version of the model analytically to gain further insights.

First, we look at the distribution of output across intermediate good producers. This is directly related to our empirical observations in Section 2.

Proposition 1 *The relative output of intermediate goods, $y(\mu(z'))/y(\mu(z))$, is independent of K_t and L_t.*

Proof. This is straightforward from the fact that

$$\frac{y(\mu(z'))}{y(\mu(z))} = \frac{D(\mu(z'))}{D(\mu(z))}$$

and that $D(\mu)$ only depends on μ and parameters. ■

Therefore, as long as the distribution of μ is invariant over time, the normalized distribution of y is also invariant over time. In a dynamic setting of our model, the aggregate output changes over time as K_t changes, as in the standard neoclassical growth model. However, if the institutions, and hence the firms' enforceability μ, do not change over time, the relative output will be constant. This is inconsistent with the empirical regularities presented in Section 2. Of course, this outcome is a consequence of the particular formulation we adopted here, but this shows that replicating the pattern in Section 2 along the growth path is not a trivial task.

We can further characterize the cross section of output as the following.

Proposition 2 *If $\mu(z) > \mu(z')$, then $y_t(\mu(z)) > y_t(\mu(z'))$.*

Proof. See Appendix F. ■

An intermediate firm z with a larger value of $\mu(z)$ faces less problems with imperfect contractability with the supplier. Proposition 2 shows that their output is larger.

Next, we turn to the aggregate consequences of the contract incompleteness.

Proposition 3 *If $\mu(z) \in (0, 1)$ for a strictly positive measure of z, then $\Theta < 1$.*

Proof. See Appendix F. ■

Intuitively, there are two reasons for Θ to be less than one. The first is an *unbalanced input* for a given intermediate good z whose $\mu(z) \in (0,1)$. When $\mu(z) \in (0,1)$, the contractible investment x_c and noncontractible investment x_n are different, since they are governed by different incentives. Since the cost-minimizing combination of investments is to make all investment equal, this lack of balance in the inputs leads to inefficiency in production. The second is *unbalanced production* across intermediate goods. If $\mu(z)$ for some z is smaller than others, these intermediate goods are produced at different amounts. This is another cause of inefficiency, since the final goods are produced most effectively with equal amounts of intermediate goods.

We have seen that, at each point in time, the production side of the economy has an equilibrium such that all prices and all quantities (up to a multiplicative factor $F(K,L)$) depend only on the distribution of enforceability $\mu(z)$. The coefficient Θ describes the effect of these distortions in production. However, enforceability problems affect the demand for physical capital and labor and therefore their prices. As a result, the supply of labor and the accumulation of physical capital can be distorted due to the incompleteness of the contracts.

Note that the system of equations above nests several different possibilities. First, when $\Theta = 1$ and $q = 1$, the economy is Pareto efficient. This is the case, for example, when the contract is complete and $\phi = 1$. Second, when $\phi < 1$ and contracts are complete, $\Theta = 1$ and $q = \phi < 1$. In this case, even with complete contracts, there are distortions that are reflected to factor income. This is due to imperfect competition in the intermediate-good sector. Third, if the contractual enforcement is imperfect and $\phi < 1$, $\Theta < 1$, and $q < 1$ hold, while their exact values depend on the whole distribution of enforceabilities $\mu(z)$. For the second and third cases, the allocation is not Pareto efficient.

In our model, both Θ and q are influenced by the distribution of $\mu(z)$ and therefore affected by the contractual incompleteness. A small value of Θ means that there is a large production inefficiency originating from the two misallocations we discussed earlier. A value of q less than Θ means that the factor suppliers are not able to receive their marginal product. The effect of μ on Θ and q can be distinctive. For example, take an extreme case where $\mu(z) = 0$

for all z. In this case $\Theta = 1$ holds, because for given K and L, all factors are used in a balanced manner across tasks and products. Thus there are no misallocations that reduce Θ. However, the value of $q = \alpha\phi/(\alpha + \phi)$ is lower than ϕ (the value of q in the complete contract case) in this case, because the factor demand is affected by the incompleteness of the contract.

The discussion above implies that the effect of the friction on factor supply is distinct from its effect on production efficiency. To see this, consider a social planner that takes the economy's TFP Θ as fixed, but can directly choose labor supply and investment in capital. Then her optimality conditions will be the same as that for the agent, (15) and (16), but with Θ, instead of q. Thus the ratio Θ/q measures the divergence between social and private returns to factors. The following proposition implies that in the economy with contractual incompleteness this discrepancy is larger than in the complete economy $(1/\phi)$ and that it increases in the degree of contractual incompleteness. Therefore, the enforcement friction reduces factor supply even below the level optimal for the lower productivity.

Proposition 4 *If $\mu(z) \in [0, 1)$ for a strictly positive measure of z, then $\Theta/q > 1/\phi$. Let $\Theta(\mu)$ be the value of Θ in an economy where all firms have enforceability μ. Define $q(\mu)$ similarly. Then $\Theta(\mu)/q(\mu)$ is a strictly decreasing function of μ.*

Proof. See Appendix F. ■

The first part of the proposition implies that when contract enforcement is imperfect, the gap between social and private returns is larger than in the perfect enforcement case. Here the imperfection in contract enforcement worsens the factor supply distortions. The second part of the proposition deals with a special case where all industries have the same μ. In this special case, we can obtain a sharper characterization: the gap between the social and private returns widens monotonically as μ becomes smaller. Thus, the factor supply is more distorted in the economy where contract enforcement is more difficult.

Now we consider a special case where we can obtain an analytical solution. Let $F(K, L) = K^{\psi} L^{1-\psi}$ where $\psi \in (0, 1)$, and $u(C, 1-L) = (C^{\eta}(1-L)^{1-\eta})^{\sigma}/(1-\sigma)$ where $\eta \in (0, 1)$, $\sigma > 0$, and $\sigma \neq 1$. From (15), in the steady state, the following holds:

$$\frac{K}{L} = \left[\frac{\psi q}{1/\beta - 1 + \delta_K}\right]^{\frac{1}{1-\psi}}.$$

Recall that in the first best outcome, $q = 1$. When $q < 1$, the capital-labor ratio is smaller than the first best level, because the return to physical capital is lower than in the case of the first best outcome. Combining this with (16) and (17), we obtain the steady-state level of labor[15]

$$L = \left[\frac{1-\eta}{\eta(1-\psi)}\left(\frac{\Theta}{q} - \frac{\psi\delta_K}{1/\beta - 1 + \delta_K}\right) + 1\right]^{-1}$$

and therefore the steady-state level of physical capital stock is

$$K = \left[\frac{\psi q}{1/\beta - 1 + \delta_K}\right]^{\frac{1}{1-\psi}} \left[\frac{1-\eta}{\eta(1-\psi)}\left(\frac{\Theta}{q} - \frac{\psi\delta_K}{1/\beta - 1 + \delta_K}\right) + 1\right]^{-1}.$$

The level of L is a function of Θ/q, which we characterized in Proposition 4. In particular, a large value of Θ/q, which can result from a small μ in Proposition 4, leads to a small value of L. For K, the level of q has an effect in addition to Θ/q. Note, once again, that these distortions on K and L are in addition to the production inefficiency, that is, $\Theta < 1$, that we highlighted in Section 3.3.

The inefficiency due to the misallocation is closely related to the recently evolving literature on misallocation, such as Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). In fact, our Θ and q each correspond to what Hsieh and Klenow (2009) call "output distortion" and "capital distortion." A similar distortion to q is also emphasized by Chari, Kehoe, and McGrattan (1997). Our model can be viewed as providing a microfoundation for these distortions. The discrepancy between private and social returns due to the incompleteness of contracts also echoes a traditional view of institutional inefficiencies. For example, North and Thomas (1973) argue "Growth will simply not occur unless the existing economic organization is efficient. ... Some mechanism must be devised to bring social and private rates of return to closer parity." Sections 5 and 6 describe how the evolution of such a mechanism over time can affect the economic outcome.

[15]See Appendix D for details of derivation.

Restuccia and Rogerson (2008) emphasize that the correlation between the productivity of individual production units and the distortion that induces misallocation is an important determinant of the aggregate consequences of misallocation. In our context, the corresponding correlation is the one between productivity of industries (or intermediate-good firms) and contract sensitivity. In Section 4, we calculates this correlation using the U.S. data, and find that the correlation between the productivity of industries and their contract sensitivity is positive. This implies that the aggregate productivity loss from misallocation is more pronounced compared to the case of no correlation. Our baseline model does not feature heterogeneous productivity among intermediate-good firms, but Section 4 extends our model to the case of heterogeneous productivity and finds that this is indeed the case.

4 Extension: heterogeneous productivity

In this section, we extend our model in order to quantify the effects of contract incompleteness in the presence of specific investments. In our baseline model, industries (and firms) differ only by their contract sensitivity measure, $(1-\mu(z))$; however, in reality, there are large differences in productivity across industries and this heterogeneity is important in, for example, analyzing misallocation of inputs across industries. To accommodate this, we extend the model to allow heterogeneity in productivity as well as in contract sensitivity.

In the extended model, we find that heterogeneity in productivity does not interact with contracting frictions if the enforcement frictions $(1 - \mu(z))$ and firm productivity are uncorrelated. However, since the distortions in our model affect firms proportionally to their undistorted optimal size, the same degree of enforcement friction will lead to larger output loss if it is applied to a more productive firm. Quantitatively, we find that the correlation between productivity and potential enforcement frictions is large and has sizable implications, a finding consistent with the rapidly expanding literature on misallocation.

4.1 A simple model with productivity dispersion

First, for the purpose of illustration, we introduce heterogeneity in productivity in a simple manner: in addition to a different enforceability parameter $\mu(z)$ every firm has a different productivity parameter $A(z)$. The demand for the differentiated product and the relationship between the firm and its suppliers is assumed to be the same as in the baseline model. The intermediate output is now given by:

$$y(z) = A(z)\left[\int_0^1 X_j(z)^\alpha dj\right]^{1/\alpha} \tag{18}$$

instead of (4).

It is straightforward to show that the Shapley value for the firm and each supplier is equal to the values in the baseline model multiplied by $A(z)^\phi$. It can also be shown that the game and the firm's problem are isomorphic to a firm with productivity one and demand parameter $YA(z)^{\frac{\phi}{1-\phi}}$ instead of Y. Therefore, the equilibrium $x_{c,t}(z)$ and $x_{n,t}(z)$ can be derived as:

$$x_{c,t}(z) = Y_t A(z)^{\frac{\phi}{1-\phi}} q_t^{-\frac{1}{1-\phi}} [\alpha(1-\gamma)]^{\frac{\phi(1-\mu(z))}{1-\phi}} B(\mu(z))^{1-\phi(1-\mu(z))}$$

and

$$x_{n,t}(z) = Y_t A(z)^{\frac{\phi}{1-\phi}} q_t^{-\frac{1}{1-\phi}} [\alpha(1-\gamma)]^{\frac{1-\phi\mu(z)}{1-\phi}} B(\mu(z))^{\mu(z)\phi},$$

where $B(\cdot)$ is given as (9).

Plugging the expression above into the production function, we obtain

$$y_t(z) = Y_t A(z)^{\frac{1}{1-\phi}} q_t^{-\frac{1}{1-\phi}} D(\mu(z)),$$

where $D(\cdot)$ is given as (11).

We can find the general equilibrium following the same steps as in Section 3.3.2. As a result, we obtain

$$q_t = \left[\int_0^1 A(z)^{\frac{\phi}{1-\phi}} D(\mu(z))^\phi dz\right]^{\frac{1-\phi}{\phi}}$$

and

$$\Theta = \frac{\left[\int_0^1 A(z)^{\frac{\phi}{1-\phi}} D(\mu(z))^{\phi} dz\right]^{\frac{1}{\phi}}}{\int_0^1 A(z)^{\frac{\phi}{1-\phi}} H(\mu(z)) dz},$$

where $H(\cdot)$ is given as (13).

It is straightforward to see that if $\mu(z) = 1$ for all z,

$$\Theta = \left[\int_0^1 A(z)^{\frac{\phi}{1-\phi}} dz\right]^{\frac{1-\phi}{\phi}} \tag{19}$$

holds. This is the TFP of the frictionless economy.

By interpreting $Z = [0, 1]$ as a sample space with the usual measure, we can think of $A(z)$ and $\mu(z)$ as random variables. If they are independent,

$$\Theta = \left[\int_0^1 A(z)^{\frac{\phi}{1-\phi}} dz\right]^{\frac{1-\phi}{\phi}} \frac{\left[\int_0^1 D(\mu(z))^{\phi} dz\right]^{\frac{1}{\phi}}}{\int_0^1 H(\mu(z)) dz}.$$

This is simply a multiplication of the TFP in the frictionless economy in (19) and Θ in the homogeneous productivity case in (14). Therefore, the productivity dispersion does not interact with the misallocation problem if productivity and contract sensitivity are independent. Similarly, the factor supply distortion Θ/q is unaffected by the distribution of productivity and depends only on the distribution of enforcement frictions $\mu(z)$. Heterogeneity in productivity and the misallocation problem from contracting frictions interact only if these two are correlated. In the following, we investigate whether we observe correlations across these two factors in the U.S. industries.

4.2 Measurement

We use the NBER-CES Manufacturing Industry database to map the model to the data. The database contains 473 industries in the 1997 NAICS classification. After data cleaning, we are left with 464 industries. Nunn's (2007) contract sensitivity measure is in the 1997 IO classification, so we use a concordance (provided by the BEA) between the IO classification and the NAICS.

We assume that there are no enforcement frictions in the U.S.[16] In order to map the model to the data, we have to modify the intermediate-good production function slightly. The production function (18) can be viewed as $y(z) = A(z)W(z)$, where $W(z)$ is the quantity of input: $W(z) = [\int_0^1 X_j(z)^\alpha dj]^{1/\alpha}$. Now, assume that there are m types of inputs, with each quantity $W_i(z)$, where $i = 1, ..., m$:

$$y(z) = A(z) \prod_{i=1}^{m} W_i(z)^{\eta_i},$$

where η_i, $i = 1, ..., m$ are parameters. Denote p_i as the price of input i.

Then the problem for firm z in the frictionless economy is:

$$\max_{\{W_1(z),...,W_m(z)\}} Y^{1-\phi}[A(z) \prod_{i=1}^{m} W_i(z)^{\eta_i}]^\phi - \sum_{i=1}^{m} p_i W_i(z).$$

The first order conditions imply that

$$\eta_i \phi = \frac{p_i W_i(z)}{R(z)},$$

where $R(z)$ is revenue of firm z: $R(z) \equiv Y^{1-\phi}[A(z) \prod_{i=1}^{m} W_i(z)^{\eta_i}]^\phi$. Denote the *observed cost share of input* i by $\tilde{\eta}_i \equiv \eta_i \phi$. This can be calculated for each industry in our dataset.

Taking logs of the definition of the revenue, we obtain

$$\log R(z) = (1 - \phi) \log Y + \phi \log A(z) + \sum_i \tilde{\eta}_i \log W_i(z).$$

Therefore, $A(z)$ can be measured using

$$A(z) \propto \exp\left(\frac{\log R(z) - \sum_i \tilde{\eta}_i \log W_i(z)}{\phi}\right). \tag{20}$$

We conduct this measurement assuming that an industry z in the data corresponds to a continuum of intermediate good firms with measure $\chi(z)$. We assume that the elasticity of substitution between any two intermediate goods is the same, regardless of whether they belong to the same industry or not.

[16] Recall that $1-\mu$ is a measure of potential, not actual distortions in an industry. The underlying assumption is that effective enforcement in the US solves the potential contractibility issues. This assumption is consistent with the model we develop in Section 5.

Then the formulas for q and Θ can be modified as

$$q = \left[\sum_z \chi(z) A(z)^{\frac{\phi}{1-\phi}} D(\mu(z))^{\phi}\right]^{\frac{1-\phi}{\phi}} \tag{21}$$

and

$$\Theta = \frac{\left[\sum_z \chi(z) A(z)^{\frac{\phi}{1-\phi}} D(\mu(z))^{\phi}\right]^{\frac{1}{\phi}}}{\sum_z \chi(z) A(z)^{\frac{\phi}{1-\phi}} H(\mu(z))}. \tag{22}$$

Total sales of an industry in an economy without frictions are proportional to $\chi(z)A(z)^{\frac{\phi}{1-\phi}}$. Therefore, we set $\chi(z)$ so that the ratio of sales between two industries in the model economy is equal to the corresponding ratio we observe in the data:

$$\frac{\chi(z)}{\chi(z')} = \frac{R(z)/R(z')}{(A(z)/A(z'))^{\frac{\phi}{1-\phi}}}.$$

The ratio $A(z)/A(z')$ can be calculated from the equation (20). This condition and the normalization $\sum_z \chi(z) = 1$ pin down the values of $\chi(z)$.

Since q and Θ in (21) and (22) are homogeneous of degree one in $\{A(z)\}$, we can normalize the measured $A(z)$. Therefore we calculate the productivity $A(z)$ from

$$A(z) = C \exp\left(\frac{\log R(j) - \sum_i \tilde{\eta}_i \log W_i(j)}{\phi}\right),$$

where we can observe $R(z)$, $\tilde{\eta}_i$, and $W_i(z)$ in the data. The value of ϕ is set at 0.8, and the constant C is chosen so that the aggregate Θ is one when $\mu(z) = 1$ for all z.

Figure 2 presents a scatterplot of this measured productivity against contract sensitivity for the final year in the sample, 2009. The data reveals that more productive industries tend to be more contract sensitive, with a correlation coefficient of 0.38 when industries are not weighted and 0.45 when we use the computed weights χ.[17]

4.3 Model results

Next, we turn to a model-based evaluation of the importance of the correlation between productivity and contract sensitivity. This correlation is emphasized by Restuccia and Rogerson

[17] This relationship is valid for all years in the sample, with a minimum for the correlation of 0.27.

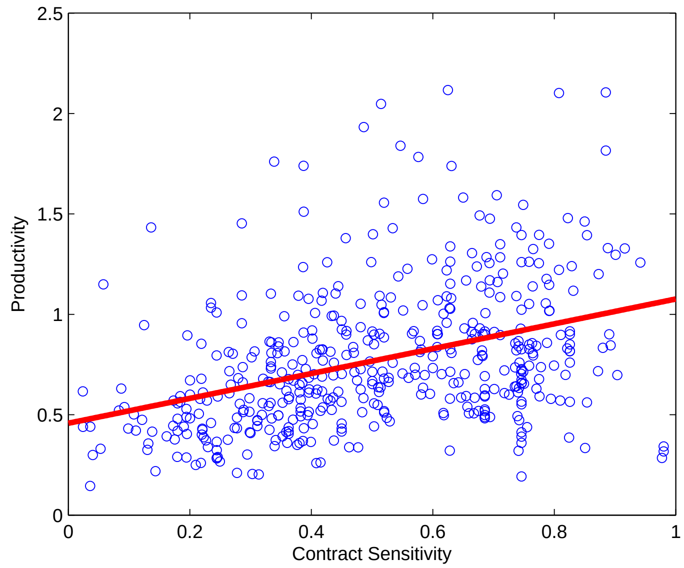

Figure 2: Contract sensitivity and productivity. The red line is a linear fit.

	Θ	q	Θ/q
(i) Perfect enforcement	1.00	0.80	1.25
(ii) Imperfect enforcement, no correlation	0.83	0.62	1.34
(iii) Imperfect enforcement, correlation	0.76	0.52	1.46

Table 3: The values of Θ and q, for different correlations between contract sensitivity and productivity

(2008). For every industry, we set $\mu(z)$ to be one minus the measure of contract sensitivity. Then given the parameter values (as in the next section, we use $\phi = 0.8$ and $\alpha = 0.5$), we compute Θ (TFP) and q (price of raw materials) for three cases: (i) an economy with perfect enforcement ($\mu(z) = 1$ for all z); (ii) the economy with imperfect enforcement ($\mu(z)$ values set from the observed contract senstitivity) but no correlation between productivity and contract sensitivity; and (iii) the economy with imperfect enforcement and the observed correlation between productivity and contract sensitivity (0.45).

Table 3 describes the results. The mechanism in the paper generates quantitatively significant misallocation and hence TFP loss. Moreover, the correlation between productivity and contract sensitivity is an important amplification mechanism for our model. As a result

	Θ/Θ^*	K/K^*	$Y/Y^* = \frac{\Theta}{\Theta^*}(K/K^*)^\psi$
(i) Perfect enforcement	1.00	1.00	1.00
(ii) Imperfect enforcement, no correlation	0.83	0.69	0.74
(iii) Imperfect enforcement, correlation	0.76	0.54	0.63

Table 4: Steady state value of K and Y for different correlations between contract sensitivity and productivity

of the positive correlation, TFP drops by an additional 7% and misalignment between private and social marginal returns worsens by 9%.

In addition to the inefficient allocation of inputs, the enforcement friction leads to suboptimal labor supply and capital accumulation decisions, driven by the ratio of private to social factor returns, Θ/q, which we display in the last column of the table. The enforcement friction worsens this disparity (which is present even in the full enforcement economy due to imperfect competition). As we do in the rest of the paper, we assume that labor is supplied inelastically and that $F(K,1) = K^\psi$ with $\psi = 0.3$. Given this structure, we can compare the steady states of the three scenarios without specifying other parameters (β, δ and preference parameters). The results are described in Table 4 (* denotes variables in the complete contracts case). In total, the barriers to capital accumulation leads to an additional 13% drop (0.76 versus 0.63) in output at the steady state.

5 Economy with evolving institutions

In our baseline model of Section 3, the distribution of intermediate outputs is constant over time if the distribution of μ is constant. This is at odds with the empirical pattern shown in Section 2. As we discussed in the Introduction, historical studies indicate that the evolution of institutions played an important role in the process of industrialization. Here we focus on the institutional evolution led by government actions. In this and the next section, we use the baseline model of Section 3 and abstract from endogenous labor supply.

Here we introduce *institutional capital* G_t and assume that it influences μ in the following manner:

$$\mu(z, G_t) = \underline{\mu}(z) + h(G_t)[1 - \underline{\mu}(z)], \tag{23}$$

where we assume that the function $h(\cdot)$ is differentiable, strictly increasing, strictly concave, and $0 = h(0) < h(G) < 1$ for any $G > 0$. The value $\underline{\mu}(z)$ is heterogeneous across z. This is fixed over time for a given z, and in the context of the model in this section and the next section, $1 - \underline{\mu}(z)$ corresponds to Nunn's (2007) contract sensitivity measure we used in Section 2. One interpretation of (23) is that there are $1 - \underline{\mu}(z)$ amount of activities that require certain contract enforcement action by the government, and $h(G_t)$ is the probability that this enforcement action is successful.

We model institutions to be increasing the probability of successful enforcement. A natural interpretation is that the government's investment in the court system increases the number of disputes that can be resolved. Intermediate goods producers try to enforce their contracts with their suppliers, but they may not have the opportunity to use the court since the number of disputes that can be tried is limited. A similar setup has been used by Arellano and Kocherlakota (2014) and Drozd and Serrano-Radial (2015) to study default externality, caused by crowding out of enforcement capacity when more agents default.

The institutional capital can be accumulated by government investment I_t^G.

$$G_{t+1} = (1 - \delta_G)G_t + I_t^G - \tau(G_t, G_{t+1}),$$

where δ_G is the depreciation rate of the institutional capital. We assume that the investment I_t^G is made in final goods and that institutional investment involves adjustment cost $\tau(G_t, G_{t+1}) \geq 0$.

In our model, institutions, represented by G_t, affect economic outcomes by changing the enforceability characteristics of firms $\mu(z, G_t)$. As a result, institutions determine the distribution of μ and hence the economy's productivity Θ. In the following, we denote this dependence as $\Theta(G_t)$. Similarly, the price of raw materials is also a function of G_t and expressed as $q(G_t)$.

In this section, we assume that the institution evolves exogenously:[18]

$$G_t = \left(1 - \rho e^{-\zeta t}\right) \bar{G},$$

[18]This process will become endogenous in the next section.

β	δ_K	δ_G	ψ	ϕ	α	κ
0.95	0.1	0.1	0.3	0.8	0.5	0.3

Table 5: Parameter values

where $\rho \in (0,1)$ and $\zeta > 0$. This implies that

$$G_{t+1} = e^{-\zeta}(G_t - \bar{G}) + \bar{G},$$

and this implicitly determines I_t^G as

$$I_t^G = (e^{-\zeta} + \delta_G - 1)G_t + (1 - e^{-\zeta})\bar{G} + \tau(G_t, G_{t+1}).$$

The representative consumer's utility is specified as

$$\mathbf{U} = \sum_{t=0}^{\infty} \beta^t u(C_t),$$

where $u(\cdot)$ is a strictly increasing and strictly concave function. The consumer's budget constraint is

$$K_{t+1} + C_t + T_t = (1 + r_t - \delta_K)K_t + w_t + \Pi_t.$$

Here we normalized the labor supply to 1. The production structure is the same as in Section 3.3. The tax T_t is used for financing the government investment, that is, the government budget constraint is $T_t = I_t^G$.

In the following, we characterize the equilibrium of this economy numerically. The outcome should be viewed as a numerical example rather than a carefully calibrated quantitative exercise, as many of parameters in this model are difficult to pin down from the data. We specify the utility function as $u(C_t) = \log(C_t)$, the production function as $f(K_t) \equiv F(K_t, 1) = K_t^{\psi}$, and the adjustment cost function as $\tau(G_t, G_{t+1}) = \kappa[(G_{t+1} - G_t)/G_t]^2 G_t$.

The parameter values are set as in Table 5. We consider one period to be one year, and set $\beta = 0.95$, $\delta_K = 0.1$, and $\delta_G = 0.1$. For production technologies, we set $\psi = 0.3$, $\phi = 0.8$, and $\alpha = 0.5$. The adjustment cost parameter $\kappa = 0.3$. The function $h(G)$ is assumed to be $G/(\xi + G)$, where ξ is set to be 1.0. The contract sensitivity of each industry, $1 - \underline{\mu}(z)$ is

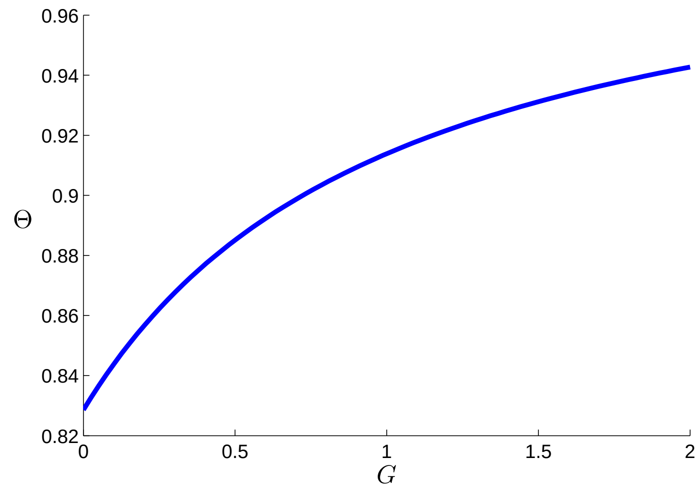

Figure 3: The relationship between Θ and G

assumed to be distributed uniformly on $[0, 1]$. The computational details of this model and the models in the next section are in Appendix G.

Figure 3 plots the relationship between Θ and G. With our parametrization, Θ is monotonically increasing in G. Although this looks like a natural outcome, given that a higher G improves the contractual environment, this monotonic relationship does not always hold. With other sets of parameter values, there are cases where Θ exhibits a U-shape relationship with G. The reason is that the misallocation stems from the imbalance among factor inputs and among the production of different intermediate goods. If contract enforcement is uniformly poor in all industries, this misallocation is not very severe, and therefore the value of Θ can be high. Figure 4 plots the relationship between q and G. As with Θ, here q is monotonically increasing in G. In sum, with our parametrization, a high G always improves the two distortions.

To highlight the role of improving institutions, we compare two cases: $\zeta = 0.01$ and $\zeta = 0$. In the first case G_t increases over time, while in the second case G_t is constant. Figures 5 and 6 plots the equilibrium paths for K, G, Θ, q, Y, and the weighted average of the contract sensitivity $1 - \underline{\mu}(z)$ for both cases, starting from $K_0 = 1.0$ and $G_0 = 0.05$. The most notable result is that the average contract sensitivity goes up with income only when G_t improves over

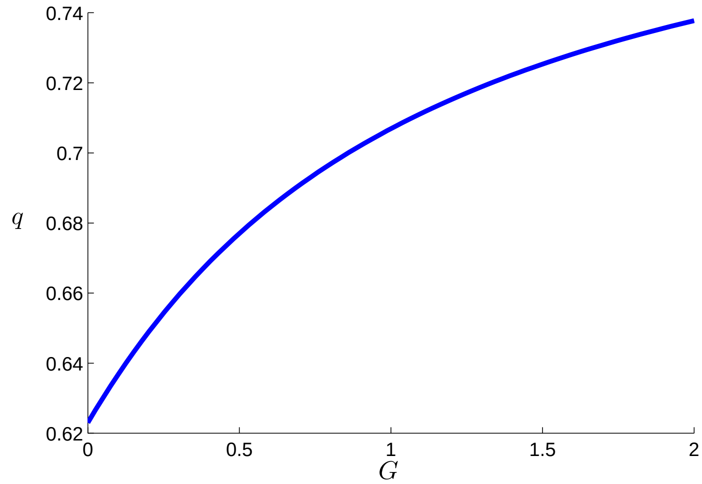

Figure 4: The relationship between q and G

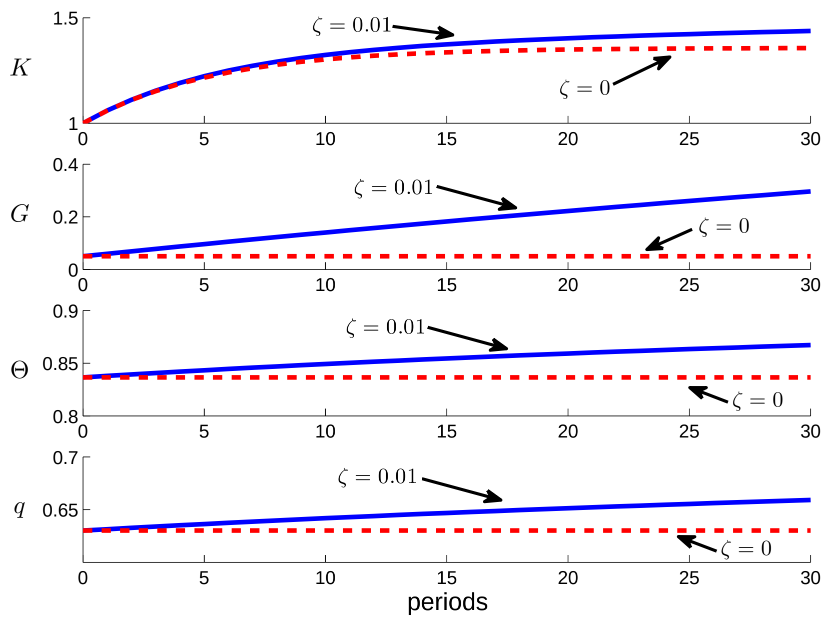

Figure 5: Results with evolving institutions: K, G, Θ, and q

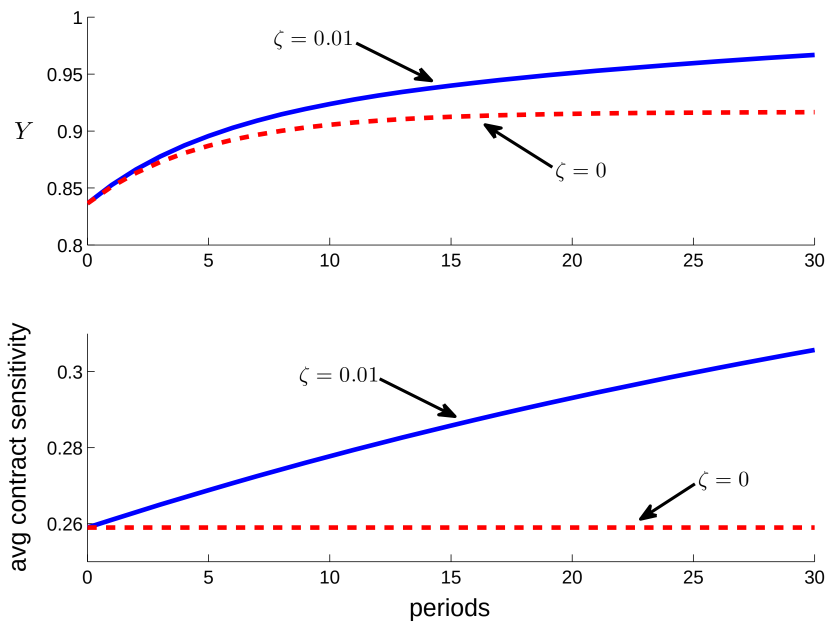

Figure 6: Results with evolving institutions: Y and the weighted average of contract sensitivity $(1 - \underline{\mu}(z))$

time. The average contract sensitivity corresponds directly to $\mathcal{S}$ in Section 2. In particular, we have seen that, in Figure 1, a rapidly-growing country tends to experience an increase in average contract sensitivity. The model with $\xi = 0.01$ is qualitatively consistent with this empirical observation.

Another notable result is that there is an interaction between the accumulation of G and accumulation of K. An increase in G increases q, which increases the return to physical capital and enhances the private incentive for capital accumulation. It demonstrates the point we argued earlier—an increase in G alleviates two channels of misallocation, one through an increase in Θ and one through an increase in q.

6 Endogenous evolution of institutions

In reality, the path of G_t is (at least partially) *chosen* by the government. In this section, we extend the previous section's model by endogenizing the process of institutional building.[19] Throughout the section, we assume that the government is benevolent and maximizes the representative consumer's utility. The purpose of this section is to theoretically examine how different *types* of governments make choices regarding institutional dynamics. In this sense, our analysis is *positive* rather than normative.[20]

Our model of institutional determination is based on resource cost: if perfect enforcement institutions are costless, then they will be chosen under any of the different scenarios that we consider. An alternative view focuses on the political process that leads to choices of public policies or institutions. When the gains of better institutions are heterogenous, inefficient institutions may be preferred by the majority of the the agents, even though total output will be increased by better institutions. For example, Koeppel, Monnet, and Quintin (2014) show that the optimal level of loan enforcement capacity can be obtained only if an endowment redistribution scheme can be implemented.

An influential line of research is based on the fact that economic advantage leads to political advantage, as forcefully expounded by Acemoglu and Robinson (2012). In a formal model, Acemoglu (2006) shows that a ruling elite may be hurt by better institutions in a variety of ways: (i) better institutions bid up factor prices and hence cut into the elite's income; (ii) if a competing group benefits disproportionately from better institutions then the elite loses its economic advantage and hence may lose its political supremacy; (iii) a variety of inefficient institutions lead to a direct transfer of resources to the elite from the rest of society. Sonin (2003) explores the operation of these mechanisms in the context of protection of property rights. Acemoglu, Johnson, and Robinson (2005b) model how exogenous events that increase the de facto power of the group desiring better institutions

[19] Our model bears some resemblance to a model of investment in public capital. See, for example, Glomm and Ravikumar (1997) and Azzimonti, Sarte, and Soares (2009).

[20] As is argued in the Introduction, it is also possible to cast our model in a political economy context using a probabilistic voting model, since the policy choice maximizes social welfare in the standard formulation.

(opening up transatlantic trade) will break the persistent equilibrium and lead to institutional change. Finally, weak economic and political institutions may interact: institutional change may require concentration of power in one of the member of the elite, who will no longer have an incentive to implement changes.[21] Our approach focuses on a different tradeoff and is largely complementary to the political economy view in that we consider the process of institutional building once a such a decision has been made. We also show that even with benevolent government, commitment is key to achieving positive institutional change.

We consider three different types of governments. First, we consider a government that can control both investment in physical capital I_t^K and investment in institutional capital I_t^G. The government can coerce private agents to save and invest in physical capital, in addition to deciding the institutional investment. Thus we call it a *coercive government*. We additionally assume that it can commit to its future actions. There have been many examples of governments that conducted "forced saving" policies—recent examples for this type of policies are found in China and Singapore. The coercive government here tries to emulate this type of government.

The second and third governments let the market decide its physical capital. Thus we call these cases *market economies*. For the second, we assume that the government can commit to future I_t^G. For the third, we assume that the government cannot commit, and there are non-trivial interactions between the government and the private agents. We analyze the Markov Perfect Equilibrium of the game between the government and the private agents.

6.1 Coercive government

The coercive government's optimization problem is

$$v_c(K, G) = \max_{C,K',G'} u(C) + \beta v_c(K', G')$$

[21] See, for example, Guriev and Sonin (2009). Acemoglu, Egorov, and Sonin (2012) explore the problem of stability of institutions more generally.

subject to the resource constraint

$$C + K' + G' = \Theta(G)f(K) + (1-\delta_K)K + (1-\delta_G)G - \tau(G, G').$$

The problem can be solved by the standard value function iteration method.

6.2 Market economy: commitment

In the cases with a market economy, the government has to respect the private agents' decision about I_t^K. As in the classic problem of finding optimal distortionary taxation, future government policies on I_t^G matter for the current decision of private agents, and therefore the ability of government to commit to future policies has important consequences. First, we set up the problem for the case where the government can commit to the future policies. In the next section, we consider the case without commitment.

The sequential problem for the benevolent government is

$$v^*(K_0, G_0) = \max_{\{C_t, K_{t+1}, G_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{SP1}$$

subject to

$$C_t + G_{t+1} + K_{t+1} \leq \Theta(G_t)f(K_t) + (1-\delta_K)K_t + (1-\delta_G)G_t, \tag{24}$$

$$C_t > 0, \tag{25}$$

$$K_{t+1} > 0, \tag{26}$$

and

$$u'(C_t) = \beta(1 + q(G_{t+1})f'(K_{t+1}) - \delta_K)u'(C_{t+1}). \tag{27}$$

The constraint (24) is the resource constraint, and C_{t+1} and K_{t+1} are assumed to be positive in (25) and (26). The constraint (27) is the Euler equation for consumers.

We transform this problem into a recursive formulation by using the technique pioneered by Kydland and Prescott (1980). In particular, we introduce an additional state variable λ, which is equal to the right-hand side of equation (27). The problem becomes:

$$v(K,G,\lambda) = \sup_{C>0,K'>0,G'\geq 0,\lambda'} u(C) + \beta v(K',G',\lambda'), \tag{RP1}$$

subject to

$$C + K' + G' \leq \Theta(G)f(K) + (1-\delta_K)K + (1-\delta_G)G, \tag{28}$$

$$u'(C) = \lambda', \tag{29}$$

$$\beta[1-\delta_K + q(G)f'(K)]u'(C) = \lambda, \tag{30}$$

and

$$\lambda' \in \Omega(K',G'). \tag{31}$$

We call this problem (RP1). Here, $\Omega(K',G')$ is called the "admissible set." The admissible set provides the feasible values of λ' to choose from. In particular, if we choose λ' that is too low, the future consumption that is dictated by (30) may not be feasible. In choosing λ', we have to guarantee that the sequence of (K_t, G_t) that satisfies all future constraints is nonempty. We state the formal definition of $\Omega(K',G')$ in Appendix E. Appendix E also describes further technical aspects of this problem.

In order to solve this problem, we first solve for $\Omega(K',G')$. Appendix E, in particular Proposition 6, describes the iterative method that we can use to find $\Omega(K',G')$. The constraint (30) pins down C uniquely by

$$C = u'^{-1}\left[\frac{\lambda}{\beta(1-\delta_K + q(G)f'(K))}\right].$$

Similarly (29) and (30) imply

$$\lambda' = \frac{\lambda}{\beta(1-\delta_K + q(G)f'(K))}.$$

Then the planner's problem is reduced to the choice of (K',G') subject to the resource and admissibility constraints.

6.3 Market economy: non-commitment

The solution in the previous section is not time consistent. A social planner invests in institutions to improve economy-wide productivity and to reduce the gap between private and social returns to physical capital. It has an incentive to announce a high I_t^G in future in order to facilitate the private investment in physical capital. However, after private agents accumulate physical capital, the planner has an incentive to cut institutional investment compared to the announcement and save resources.

In this section, we analyze the relationship between the government and the private agents as a game. In each period, the government chooses institutional investment taking into account the response of the private agents; similarly consumers making investment decisions choose optimally given their beliefs of the government reaction function.

We use the concept of Markov Perfect Equilibria (MPE): subgame perfect equilibria in which strategies are conditioned only on payoff-relevant variables.[22] Let $C(K, G, K', G')$ denote consumption if the current state of the economy is (K, G) and the government and the private sector choose (K', G'). It is given by:

$$C(K, G, K', G') = \Theta(G)f(K) + (1 - \delta_K)K + (1 - \delta_G)G - G' - \tau(G, G') - K'. \tag{32}$$

Now, in order to find an MPE, we consider the government's one-period deviation for G'. In doing so, the government needs to take into account the private agents' responses. Let $\nu(K, G)$ be the continuation government policy function for G' and let $M(K, G)$ be the private agents' decision of physical capital K'. Given the government's choice of G', the private agents makes a choice of K' that satisfies their Euler equation:

$$u'(C(K, G, K', G')) = \beta(1 - \delta_K + q(G')f'(K'))u'(C(K', G', M(K', G'), \nu(K', G')). \tag{33}$$

We denote the solution of the equation as $\tilde{M}(K, G, G')$.

Let the value from following the policy functions ν and M be denoted by v. Then v solves

[22] See Klein, Krusell, and Ríos-Rull (2008) for the details and properties of the MPE in this type of dynamic game. Our computational algorithm largely follows Azzimonti, Sarte, and Soares (2009).

the Bellman equation:

$$v(K,G) = u(C(K,G,M(K,G),\nu(K,G)) + \beta v(M(K,G),\nu(K,G)). \tag{34}$$

Finally, given the value function v and the private agents' response function, the government solves:

$$\max_{G'} \quad u(C(K,G,\tilde{M}(K,G,G'),G'))) + \beta v(\tilde{M}(K,G,G'),G').$$

In choosing G', the government takes into account the direct effect of increasing future productivity and the effect on the choices of consumers and future governments.

Definition 1 *An MPE is the collection of functions $(\nu, M, \tilde{M}, v)$, such that:*

1. *The perceived government policy function is optimal*

$$\nu(K,G) \in \arg\max_{G'} \quad u(C(K,G,\tilde{M}(K,G,G'),G')) + \beta v(\tilde{M}(K,G,G'),G').$$

2. *The private Euler equation is satisfied: $\tilde{M}(K,G,G')$ solves equation (33).*

3. *The value function v satisfies equation (34).*

4. *The law of motion for physical capital is consistent: $M(K,G) = \tilde{M}(K,G,\nu(K,G))$.*

The Generalized Euler Equation (GEE) has been commonly used to develop intuitions for the working of an MPE in this type of environment.[23] We derive the GEE in our environment in the following proposition. First, we introduce some notation; let u_1 be the first derivative of the utility function, C_i be the partial derivative of $C(K,G,K',G')$ function with respect to the ith term, and $\tilde{M}_i$ be the partial derivative of $\tilde{M}(K,G,G')$ function with respect to the ith term. Also, let $'$ denote the value in the next period and $''$ denote the value two periods ahead.

Proposition 5 *Define $\Delta_K \equiv u_1 C_3 + \beta u_1' C_1'$ and similarly $\Delta_G \equiv u_1 C_4 + \beta u_1' C_2'$. If the*

[23]See, for example, Klein, Krusell, and Ríos-Rull (2008).

equilibrium policy functions $\tilde{M}$ and ν are differentiable, then the following GEE holds:

$$\Delta_G + \tilde{M}_3 \Delta_K + \beta(\tilde{M}_2' + \tilde{M}_3 \tilde{M}_1')\Delta_K' - \frac{\beta(\tilde{M}_2' + \tilde{M}_3 \tilde{M}_1')\tilde{M}_1''}{\tilde{M}_2'' + \tilde{M}_3' \tilde{M}_1''}[\Delta_G' + \tilde{M}_3' \Delta_K'] = 0.$$

Proof. In Appendix F. ■

Recall that, from (32), $C_3 = -1$ and $C_1' = \Theta(G')f'(K') + (1 - \delta_K)$. Thus $\Delta_K = 0$ in the coercive planner's solution. In the MPE, social and private returns diverge ($q < \Theta$), and Δ_K is always positive. Similarly, $\Delta_G = 0$ under the coercive planner's solution. Thus, Δ_K and Δ_G represent utility losses caused by inefficient capital accumulation and inefficient institutional investment. The GEE above shows how the government balances the effect of its actions on various distortions. A change in G' has a direct effect through changes in Θ, an effect on private agents' savings decisions, and an effect on future governments' decisions. At the margin, the sum of all these effects is zero when the policy is set optimally from the point of view of the current government.

6.4 Results

The functional forms and the parameter values we use here are the same as the ones in Section 5. The results of the three cases are depicted in Figure 7.

A coercive government faces the social return to physical capital accumulation $\Theta(G)f'(K)$, instead of the private return $q(G)f'(K)$. As we have seen in Proposition 4, when contract enforcement is imperfect, $\Theta(G) > q(G)$ holds, and there is a discrepancy between social return and private return. The physical capital stock K of the coercive government case is larger than in the both market economy cases for that reason.

Comparing the institutional capital accumulation G, the market economy case with commitment has a higher rate of investment in G. This is because in order to facilitate the investment in K, knowing that the social return is high, the government has to induce private agents to invest in K. For that purpose, it is necessary to increase $q(G)$ and therefore G. The path of G is reflected in Θ, q, and the average contract sensitivity, which are all functions of G. An interesting implication is that the government with many policy means

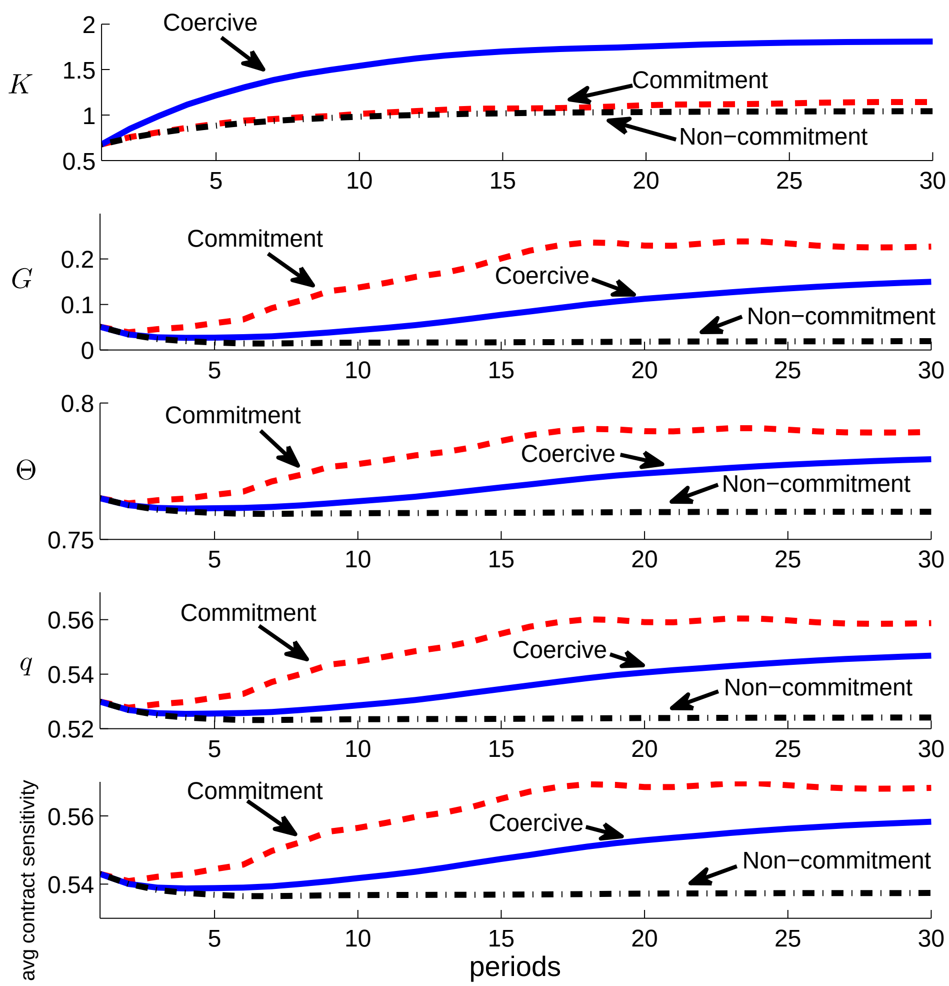

Figure 7: Results with endogenously evolving institutions

does not necessarily have the largest incentive to build institutions. If the government has direct means of influencing the economic outcome, rather than through the institutions, they may resort to these direct methods rather than conducting costly institution building.

Comparing the commitment case and non-commitment case, the effect of the time-consistent problem is apparent in the results. In the case of non-commitment, the government cannot credibly promise that the future $q(G)$ will be high, because once K is accumulated it has an incentive not to follow through on the promise of building costly institution. In our case, the institutional capital G in the MPE of the non-commitment case collapses to zero. This underlines the importance of commitment in institutional building.[24]

7 Conclusion

This paper examines the relationship between industrialization and contract enforcement. Empirically, we find that throughout the process of industrialization, a country tends to shift its production to more contract-sensitive sectors.

Then we build a dynamic general equilibrium model where production features contract incompleteness. We show that the contract incompleteness affects aggregate productivity through misallocation in production. In addition, it affects the factor supply behavior through factor prices. We offer simple characterizations of these distortions in the model economy.

The model with evolving institutions exhibits dynamics that are consistent with the pattern in the data. We also analyze endogenous institutional investment by the government, and found that when the government can control the accumulation of both physical capital and institutional capital, the incentive of the government to improve institution can be smaller than the case where physical capital is decided by the private agents. We also find that the possibility of commitment has an important impact on the dynamic accumulation of institutional capital.

[24] Acemoglu (2003) emphasizes the importance of commitment in the context of inefficient institutions. Our results are also consistent with North and Weingast's (1989) emphasis on commitment in the Glorious Revolution, discussed in the Introduction.

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Appendix

A Additional empirical results

In Table 6, we report the estimates of regression equations (1) and (2) when the average contract sensitivity is computed using industry gross output as weights. Compared to Table 1, the magnitude and significance of the coefficients for institutions is virtually unchanged, with the exception that government effectiveness is no longer significant.

Table 7 reports the estimates of regression equation (3) when the average contract sensitivity is computed using industry gross output as weights. Compared to Table 2, the coefficients remain highly significant and their magnitude increases markedly. This is consistent with findings of Jones (2011, 2013), Bartelme and Gorodnichenko (2015), and Boehm (2015) that frictions in intermediate goods usage are significantly more severe in developing countries. Thus contract sensitive industries are impacted even more once we take into account the amount of purchased intermediate goods. Figure 8 corresponds to Figure 1 in the main text. The two graphs look very similar to one another.

As a further robustness check, we estimate the regression equation (1) when $\mathcal{S}_c$ (average contract sensitivity) is computed using exports as weights and report the results in Table 8. There is a strong positive relationship between quality of institutions and contract sensitivity of a country, partly due to a large sample size. The details of data construction can be found in Appendix B.

Table 6: Cross-sectional results; gross output weights

	[1] $\mathcal{S}$	[2] $\mathcal{S}$	[3] $\mathcal{S}$	[4] $\mathcal{S}$	[5] $\mathcal{S}$	[6] $\mathcal{S}$
Rule of law	0.049**	0.043**				
	(.023)	(.018)				
Government effectiveness			0.040	0.040*		
			(.025)	(.021)		
Control of corruption					0.019	0.023
					(.019)	(.016)
ln GDP per capita	−0.057		−0.040		−0.28	
	(.032)		(.031)		(.033)	
GDP per capita ($\times 10^{-6}$)		−2.03**		−1.73*		−1.58
		(0.96)		(0.97)		(1.05)
N	32	32	32	32	32	32

The dependent variable is average contract sensitivity weighted by gross output.
Standard errors in parenthesis.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.
Constant omitted. Institutions measures are on a -2.5: 2.5 scale.

Table 7: Panel regression results; gross output weights

	[1] $\Delta\mathcal{S}$	[2] $\Delta\mathcal{S}$	[3] $\Delta\mathcal{S}$	[4] $\Delta\mathcal{S}$
Δ ln GDP per capita	0.0485***	0.0426***	0.0444***	0.0390***
	(.006)	(.006)	(.006)	(.007)
Year fixed effect	No	No	Yes	Yes
Country fixed effect	No	Yes	No	Yes
N	952	952	952	952

The dependent variable is average contract sensitivity weighted by gross output.
Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 8: Cross sectional results; export weights

	[1]	[2]	[3]	[4]	[5]	[6]
	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$	$\mathcal{S}$
Law	0.0992***					
	(.021)					
Effectiveness		0.106***				
		(.021)				
Stability			0.0558***			
			(.02)			
Regulation				0.0975***		
				(.018)		
Accountability					0.0997***	
					(.017)	
Corruption						0.0796***
						(.02)
ln GDP per capita	-0.0436***	-0.0472***	-0.0148	-0.0360***	-0.0333***	-0.0307**
	(.016)	(.016)	(.015)	(.013)	(.012)	(.015)
N	154	154	152	154	154	154

The dependent variable is average contract sensitivity weighted by exports.
Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

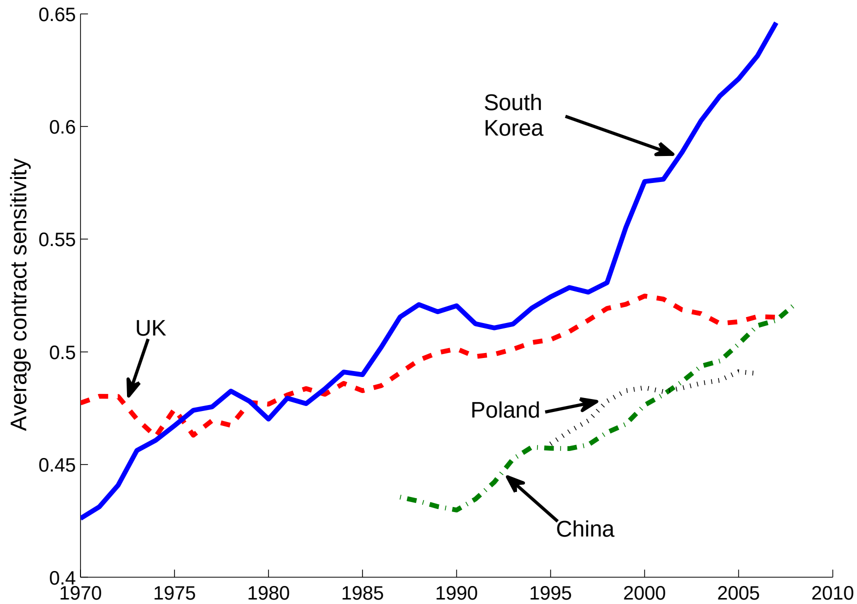

Figure 8: Average contract sensitivity, gross output weights

B Details on data sources

We use the measure of contract sensitivity developed by Nunn (2007). It is based on data from the input-output tables of the United States in 1997. Nunn (2007) assigns a contract sensitivity score to 386 out of 486 categories in the 1997 IO classification. We focus the analysis on manufacturing, partly because there are missing categories in the service sector while the data is complete for manufacturing. Another reason we use the manufacturing sector is to stay away from the issue of structural transformation, which can be driven by other forces, such as the change in demand composition. We use data on production from the World KLEMS initiative (`http://www.worldklems.net/`). KLEMS uses ISIC 3.1. industry classifications at the two digit level (60 industries). We employ the following procedure to assign a contract sensitivity measure on each of these industries. The BEA provides a concordance between 1997 IO and the 1997 version of the North American Industry Classification System (NAICS). The Census provides a concordance between 2002 version of NAICS and

ISIC 3.1. Finally, we use the concordance between the 1997 and 2002 versions of NAICS, provided by the BEA.

Unfortunately, this procedure does not yield a one-to-one mapping: many IO codes are mapped onto multiple ISIC 3.1 categories. For category i at the two-digit ISIC 3.1 classification, we assign the following contract sensitivity measure:

$$\hat{s}_i = \frac{\sum_j x_{i,j} s_j}{\sum_j x_{i,j}},$$

where j spans the 1997 IO categories, $x_{i,j}$ is the number of NAICS codes that are mapped to both the i category in ISIC and the j category in the IO classification; s_j is the contract sensitivity measure of Nunn (2007).

For a few countries, some of the 60 ISIC 3.1 categories are aggregated. In this case, we compute a weighted average of the scores of constituent categories and use it as a measure. We use the average share of each industry in the grouping from the UK as weights.

In Appendix A, we perform a robustness exercise and we focus on exports. We use data from Feenstra et al (2005), which is in SITC Rev. 2 classification. The reported values are for the last year in our sample, 2000, but the results are virtually unchanged for all other years. We assign a contract sensitivity measure to each SITC category in a similar manner to Nunn (2007). We use the SITC-HS10 concordance provided by Feenstra and the HS10-IO category concordance provided by the BEA. We use the same procedure to compute the measure as for the KLEMS data.

C Derivation of the Shapley value

We follow the "heuristic derivation" of Acemoglu, Antràs and Helpman (2007). To compute the Shapley value, we first consider all feasible permutations of all players. Then consider the marginal contribution of a particular player to the coalition ordered below her. The Shapley value is the average of these marginal contributions.

Now, consider the marginal contribution of a particular supplier. The price of intermediate good z is

$$p(z) = Y^{1-\phi} y(z)^{\phi-1},$$

and thus the revenue is

$$R(z) = Y^{1-\phi} y(z)^{\phi}.$$

Using the production function, the revenue generated by the coalition of the intermediate good firm and n suppliers ($n \in [0,1]$) is

$$R_n = Y^{1-\phi} \left[\int_0^n \left(\exp \left(\int_0^1 \ln x(i,j) di \right) \right)^{\alpha} dj \right]^{\phi/\alpha}$$

The marginal contribution, evaluated with $x(i,j) = x_c$ for $i \in [0,\mu]$ and $j \in [0,n]$, $x(i,j) = x_n$ for $i \in (\mu, 1]$ and $j \in [0,n)$, and $x(i,j) = x_n(j)$ for $i \in (\mu, 1]$ and $j = n$ is

$$m(j,n) \equiv \frac{\partial R_n}{\partial n} = \frac{\phi}{\alpha} Y^{1-\phi} \left[\frac{x_n(j)}{x_n} \right]^{(1-\mu)\phi} x_c^{\phi\mu} x_n^{\phi(1-\mu)} n^{(\phi-\alpha)/\alpha}.$$

The marginal contribution of supplier j is zero if the intermediate good firm is not included in the coalition. Thus the marginal contribution is $m(j,n)$ with probability n and zero with probability $1-n$. Considering all possible orderings, the Shapley value for supplier j is

$$s_j = \int_0^1 n m(j,n) dn = (1-\gamma) Y^{1-\phi} \left[\frac{x_n(j)}{x_n} \right]^{(1-\mu)\phi} x_c^{\phi\mu} x_n^{\phi(1-\mu)}.$$

The intermediate firm receives the leftover of the revenue in the symmetric equilibrium:

$$s_i = \gamma Y^{1-\phi} x_c^{\phi\mu} x_n^{\phi(1-\mu)}.$$

D Derivation of the steady state expressions in Section 3.4

The first-order conditions at any date are:

$$\frac{w_t}{c_t} = \frac{(1-\eta)/\eta}{1-L_t}$$

$$c_t^{\eta(1-\sigma)-1} (1-L_t)^{(1-\eta)(1-\sigma)} = \beta(1-\delta_K + r_{t+1}) c_{t+1}^{\eta(1-\sigma)-1} (1-L_{t+1})^{(1-\eta)(1-\sigma)}$$

Consider the steady state values and omit the time subscripts. Then,

$$\frac{w}{Y - \delta_K K} = \frac{(1-\eta)/\eta}{1-L} \tag{35}$$

and

$$r = \frac{1}{\beta} - 1 + \delta_K \tag{36}$$

hold.

Let $X = K^{\psi} L^{1-\psi}$ be the steady-state production of raw materials. Final output is $Y = \Theta X$. Then the steady state factor prices are:

$$w = \frac{(1-\psi)X}{L} q$$

and

$$r = \frac{\psi X}{K} q. \tag{37}$$

Plugging r in (36), we obtain

$$\frac{X}{K} = \frac{m}{q}, \tag{38}$$

where $m = (1/\beta - 1 + \delta_K)/\psi$ and m is independent of the distribution of μ.

Using the expression for w, we rewrite (35) as

$$\frac{\frac{(1-\psi)X}{L} q}{\Theta X - \delta_k K} = \frac{(1-\eta)/\eta}{1-n}$$

or

$$\frac{\frac{(1-\psi)X}{K} q \frac{1}{L}}{\Theta \frac{X}{K} - \delta_K} = \frac{(1-\eta)/\eta}{1-L}.$$

Using (38),

$$\frac{\frac{(1-\psi)m}{q} q \frac{1}{L}}{\Theta \frac{m}{q} - \delta_K} = \frac{(1-\eta)/\eta}{1-L}.$$

Then solving for L, and substituting m back, we obtain

$$L = \left[\frac{1-\eta}{\eta(1-\psi)} \left(\frac{\Theta}{q} - \frac{\delta_K \psi}{1/\beta - 1 + \delta_k} \right) + 1 \right]^{-1},$$

the expression in the main text. The expression of K/L can easily be derived from (36) and

(37).

E Characterization of market economy with commitment in Section 6.2

This section provides characterizations of the model in Section 6.2. The results will also serve as a basis for computing the equilibrium of the model.

First, one issue with the problem (SP1) is that it is not obvious whether the resource constraint is always binding. It is easy to see that for fixed λ and G, consumption is decreasing in physical capital; furthermore λ' must increase if K increases, which may lead to lower consumption in future periods and so on. Therefore, it could be that $v(K, G, \lambda)$ is decreasing in physical capital, so the planner may find it optimal to destroy resources, that is, choose an allocation such that the resource constraint is not binding. The next lemma shows that without loss of generality, in the sequential problem we may impose the additional constraint that all resources are used up.

Lemma 1 *Suppose that $\Theta(G)$ is weakly increasing in G. Let $\{c_t, k_{t+1}, g_{t+1}\}_{t=0}^{\infty}$ be a sequence that satisfies all the constraints for* (SP1). *There exists another sequence $\{c'_t, k'_{t+1}, g'_{t+1}\}_{t=0}^{\infty}$ that also satisfies all the constraints for* (SP1)*, $c'_t = c_t$ for all t, and the resource constraint is binding for all t.*

Proof. See Appendix F. ■

Thus below we can assume that the resource constraint is binding.

Second, it will be convenient to establish some properties of the admissible set $\Omega(K, G)$ in order to be able to compute it. The formal definition of $\Omega(K, G)$ is the following. Let $\Delta \equiv \{\{J_t\}_{t=0}^{\infty}, J_t \in \mathbf{R}_+^3\}$ be the space of sequences of nonnegative triplets of real numbers. Let $\Gamma : \mathbf{R}_{++} \times \mathbf{R}_+ \rightrightarrows \Delta$ be the correspondence that maps from the pair (K, G) into the set of all sequences that satisfy (24), (25), (26), and (27), where the first component of the triplet is understood as C, the second is K', and the third is G'. Define the mapping

$m : \Delta \times \mathbf{R}_{++} \times \mathbf{R}_{+} \to \mathbf{R}$ by

$$m((J_t), K, G) \equiv \beta(1 - \delta_K + q(G)f'(K))u'(J_0(1)),$$

where (J_t) is an element of Δ and $J_0(1)$ represents the first component of J_0 (that is, the consumption at time 0). Then the admissible set is formally given by

$$\Omega(K, G) = \{m((J_t), K, G) : (J_t) \in \Gamma(K, G)\}. \tag{39}$$

Now, the following proposition shows that $\Omega(K, G)$ can be characterized by its lower bound.

Proposition 6 *For every $(K, G) \in \mathbf{R}_{++} \times \mathbf{R}_{+}$, $\Omega(K, G) = [\omega(K, G), \infty)$, where ω is a continuous function which is strictly decreasing in K.*

Proof. See Appendix F. ■

Proposition 6 implies that $\Omega(K, G)$ is convex-valued and closed. The admissibility set $\Omega(K, G)$ has an obvious recursive structure: for a given admissible triplet (λ', K', G') that satisfies (31), the constraints (28), (29), and (30) provide restrictions on all combinations of (λ, K, G) that makes (λ', K', G') feasible. In turn, these combinations of (λ, K, G) have to be consistent with the admissible set of (λ', K', G') postulated initially.[25]

The special structure of our problem allows us to find Ω by an iterative procedure. Let $\mathbf{w} : \mathbf{R}_{++} \times \mathbf{R}_{+} \to \mathbf{R}_{+}$ be a continuous function. This is the (initial) guess for ω. Define the operator $\mathbf{T_w}$ by:

$$\mathbf{T_w}\mathbf{w}(K, G) = \min_{C>0, K'>0, G'\geq 0, \lambda'} \lambda = \beta(1 - \delta_K + q(G)f'(K))u'(C)$$

subject to

$$C + K' + G' \leq \Theta(G)f(K) + (1 - \delta_K)K + (1 - \delta_G)G,$$

$$u'(C) = \lambda',$$

[25]This idea was first demonstrated by Kydland and Prescott (1980) and later formalized by Abreu, Pearce and Stacchetti (1990).

and

$$\lambda' \geq \mathbf{w}(K', G').$$

This problem is well defined for a particular subset of functions, as is shown below.

Lemma 2 *Let* $\mathcal{A} \equiv \{\mathbf{w} : \mathbf{R}_{++} \times \mathbf{R}_{+} \to \mathbf{R}_{++}$, *continuous, increasing, and* $\lim_{K \to 0} \mathbf{w}(K, G) = \infty$ *for all* $G\}$. *Then* $\mathbf{T_w}$ *is defined on* $\mathcal{A}$ *and* $\mathbf{T_w}\mathcal{A} \subseteq \mathcal{A}$.

Proof. See Appendix F. ■

Since the function ω is in the class $\mathcal{A}$, one systematic method of finding it turns out to be to start with some $\mathbf{w} \leq \omega$ and iterate on the operator $\mathbf{T}$ until convergence.

Lemma 3 *Let* $\mathbf{w} \in \mathcal{A}$, $\mathbf{w} \leq \omega$, *and* $\mathbf{T_w w} \geq \mathbf{w}$. *Then for every* (K, G), *the sequence* $(\mathbf{T_w}^n \mathbf{w})(K, G)$ *converges and* $\lim_{n \to \infty} (\mathbf{T_w}^n \mathbf{w})(K, G) = \omega(K, G)$.

Proof. See Appendix F. ■

One initial guess that satisfies the conditions of Lemma 2 and Lemma 3 (with Inada-like conditions on $f(K)$) is

$$\mathbf{w}(K, G) = \beta(1 - \delta_K + q(G) f'(K)) u'(\Theta(G) f(K) + (1 - \delta_K) K + (1 - \delta_G) G).$$

This gives the upper bound on consumption C to the consumer in the current period, because this consumption is derived from setting $K' = 0$ and $G' = 0$ (in fact, this consumption is just unattainable because of the constraint $K' > 0$). Thus both $\mathbf{w}$ and $\mathbf{T_w w}$ cannot be larger than this $\mathbf{w}$ and therefore $\mathbf{w} \leq \omega$ and $\mathbf{w} \leq \mathbf{T_w w}$ are satisfied.

Next, we turn to solving the recursive problem (RP1) itself. Define the operator $\mathbf{T_v}$ by

$$\mathbf{T_v} v(K, G, \lambda) = \sup_{C > 0, K' > 0, G' \geq 0, \lambda'} u(C) + \beta v(K', G', \lambda'),$$

subject to (28), (29), (30), and (31). We aim for our model to be consistent with usual parametrization of the standard growth model, so we do not want to impose boundedness constraints on the utility function. This implies the operator $\mathbf{T_v}$ is not necessarily a contraction.

We utilize a different strategy to solving the Bellman equation. We have two tools: first, the neoclassical growth model is extensively studied and, since it is an upper bound on the planner's problem, we can use standard existence results from it; second, consumption is bounded from above by feasibility and strict concavity of the production function. Our approach is similar to the one we use to find Ω: we start with a suitable upper bound for the value function, and iteratively apply the operator $\mathbf{T_v}$. This generates a decreasing sequence that converges to the true value.

Proposition 7 *Let the real-valued function $\hat{v}(K, G, \lambda)$ be continuous, constant in G and λ, strictly increasing in K, and $\hat{v}(K, G, \lambda) \geq v^*(K, G, \lambda)$. In addition, suppose that $\mathbf{T_v}\hat{v} \leq \hat{v}$. Then for any (K, G, λ) such that $\lambda \in \Omega(K, G)$, the sequence $(\mathbf{T_v}{}^n \hat{v})(K, G, \lambda)$ has a limit. Denote the limit function $v(K, G, \lambda)$. Then $v(K, G, \lambda) = v^*(K, G, \lambda)$.*

Proof. See Appendix F. ■

An obvious upper bound for v^* will be the value of the competitive allocation in an economy without enforcement frictions ($\mu(z) = 1$ for all z), which implies $\Theta(G) = q(G) = 1$, and without any constraints on marginal utility in the first period.

F Proofs

Proof of Proposition 2. It is sufficient to show that $D(\mu)$ is strictly increasing. From the expression in the main body of the paper,

$$D(\mu) = [\alpha(1-\gamma)]^{\frac{1-\mu}{1-\phi}} B(\mu)^{\mu}.$$

Define $p \equiv \alpha(1-\gamma)$. Note that $p = \alpha\phi/(\alpha + \phi) < \phi$. Then we have

$$D(\mu) = p^{\frac{1}{1-\phi}} \left(\frac{B(\mu)}{p^{1/(1-\phi)}} \right)^{\mu}.$$

Substitute the expression for $B(\mu)$ to get

$$D(\mu) = p^{\frac{1}{1-\phi}} \left(\frac{\phi}{p} \frac{1-p+p\mu}{1-\phi+\phi\mu} \right)^{\frac{\mu}{1-\phi}} \equiv p^{\frac{1}{1-\phi}} f(\mu)^{\frac{\mu}{1-\phi}}$$

$$\begin{aligned} D'(\mu) &= p^{\frac{1}{1-\phi}} \left[\frac{\mu}{1-\phi} f(\mu)^{\frac{\mu}{1-\phi}-1} f'(\mu) + f(\mu)^{\frac{\mu}{1-\phi}} \frac{1}{1-\phi} \ln(f(\mu)) \right] \\ &= \frac{1}{1-\phi} p^{\frac{1}{1-\phi}} f(\mu)^{\frac{\mu}{1-\phi}} \left[\mu \frac{f'(\mu)}{f(\mu)} + \ln(f(\mu)) \right] \end{aligned}$$

Let $g(\mu) \equiv \mu f'(\mu)/f(\mu) + \ln(f(\mu))$. Then $D'(\mu) > 0$ is equivalent to $g(\mu) > 0$. We will show that $g'(\mu) < 0$ and $g(1) > 0$. First, we take derivatives to see that

$$\begin{aligned} g'(\mu) &= \frac{p-\phi}{(1-p+p\mu)(1-\phi+\phi\mu)} \left[(1-p+p\mu)(1-\phi+\phi\mu) - \mu\phi(1-p+p\mu) - \mu p(1-\phi+\phi\mu) + 1 \right] \\ &= \frac{p-\phi}{(1-p+p\mu)(1-\phi+\phi\mu)} \left[(1-\phi)(1-p+p\mu) + 1 - \mu p(1-\phi+\phi\mu) \right] < 0, \end{aligned}$$

where the last inequality follows from the fact that $p < \phi$ and all the other terms are positive.

Finally $g(1) = \ln(\phi) - \ln(p) + p - \phi$. Since $\phi > p$ and ln is strictly concave, $\ln(\phi) > \ln(p) + \frac{1}{\phi}(\phi - p)$. Then $g(1) > \left(\frac{1}{\phi} - 1\right)(\phi - p) > 0$. ∎

Proof of Proposition 3.

Define

$$\eta(\mu) \equiv \frac{\mu x_c(\mu) + (1-\mu) x_n(\mu)}{y(\mu)}.$$

(Here, we are explicit about the dependence of x_c, x_n, and y on μ.) From the production function (10),

$$\eta(\mu) = \frac{\mu + (1-\mu) x_n(\mu)/x_c(\mu)}{(x_n(\mu)/x_c(\mu))^{1-\mu}}.$$

It is straightforward to show that $\eta(\mu) > 1$ when $x_n(\mu)/x_c(\mu) \neq 1$.

From the expressions on $x_n(\mu)$ and $x_c(\mu)$ in the main text,

$$x_n(\mu)/x_c(\mu) = \alpha(1-\gamma) B(\mu)^{-(1-\phi)}.$$

Substituting the expression for $B(\mu)$ in the main text, we can see that

$$\frac{d}{d\mu}\frac{x_n(\mu)}{x_c(\mu)} = \frac{\alpha(1-\gamma)}{\phi}\frac{\phi-\alpha(1-\gamma)}{(1-(1-\mu)\alpha(1-\gamma))^2} > 0.$$

Then for all $\mu < 1$, $x_n(\mu)/x_c(\mu) < x_n(1)/x_c(1) = \alpha/(\alpha+\phi) < 1$. Therefore $\eta(\mu) > 1$ for any $\mu < 1$.

Since the market for raw materials clears in equilibrium,

$$\begin{aligned}
F(K,L) &= \int_0^1 [\mu(z)x_c(\mu(z)) + (1-\mu(z))x_n(\mu(z))]dz \\
&= \int_0^1 \eta(\mu(z))y(\mu(z))dz \\
&> \int_0^1 y(\mu(z))dz \\
&= \left[\int_0^1 y(\mu(z))dz\right]^{\frac{\phi}{\phi}} \\
&\geq \left[\int_0^1 y(\mu(z))^{\phi}dz\right]^{\frac{1}{\phi}} = Y,
\end{aligned}$$

where the third line follows from the fact that $y(\mu(z)) > 0$ for all z and $\eta(\mu(z)) > 1$ for a positive measure of z. The fifth line follows from Jensen's inequality. Thus, the chain of inequalities implies $Y < F(K,L)$, and therefore $\Theta < 1$. ■

Proof of Proposition 4.

We show the second statement first. From the derivation in the main text, we know that

$$\frac{\Theta}{q} = \frac{\frac{\left[\int_0^1 D(\mu(z))^{\phi}dz\right]^{\frac{1}{\phi}}}{\int_0^1 H(\mu(z))dz}}{\left[\int_0^1 D(\mu(z))^{\phi}dz\right]^{\frac{1-\phi}{\phi}}} = \frac{\int_0^1 D(\mu(z))^{\phi}dz}{\int_0^1 H(\mu(z))dz}.$$

Then in the case when all intermediate goods producers have the same μ, we get:

$$\frac{\Theta(\mu)}{q(\mu)} = \frac{D(\mu)^{\phi}}{H(\mu)} = D(\mu)^{\phi-1}\frac{D(\mu)}{H(\mu)}.$$

From the derivation in the main text,

$$D(\mu)^{\phi-1} = [\alpha(1-\gamma)]^{-(1-\mu)}B(\mu)^{-\mu(1-\phi)} = p^{-(1-\mu)}B(\mu)^{-\mu(1-\phi)},$$

where we defined $p \equiv \alpha(1-\gamma) = \alpha\phi/(\alpha+\phi)$. Note that $p \in (0,1)$.

From the definitions of H and D, it follows that

$$\frac{D(\mu)}{H(\mu)} = \frac{p^{1-\mu}B(\mu)^{-(1-\mu)(1-\phi)}}{\mu + (1-\mu)pB(\mu)^{-(1-\phi)}}$$

Combining the two expressions above,

$$\frac{\Theta(\mu)}{q(\mu)} = \frac{p^{-(1-\mu)}B(\mu)^{-\mu(1-\phi)}p^{1-\mu}B(\mu)^{-(1-\mu)(1-\phi)}}{\mu + (1-\mu)pB(\mu)^{-(1-\phi)}} = \frac{B(\mu)^{-(1-\phi)}}{\mu + (1-\mu)pB(\mu)^{-(1-\phi)}} \equiv \frac{f(\mu)}{g(\mu)},$$

where f and g represent the numerator and denominator respectively. Since

$$\frac{d}{d\mu}\frac{\Theta(\mu)}{q(\mu)} = \frac{1}{f(\mu)g(\mu)}\left[\frac{f'(\mu)}{f(\mu)} - \frac{g'(\mu)}{g(\mu)}\right],$$

it is clearly sufficient to show that $\frac{f'(\mu)}{f(\mu)} - \frac{g'(\mu)}{g(\mu)} < 0$ for all μ.

First, we derive f'/f. Substituting the expression for $B(\mu)$, we get that

$$f(\mu) = B(\mu)^{-(1-\phi)} = \frac{1-\phi+\phi\mu}{\phi[1-p+p\mu]}.$$

Taking derivatives, we see that:

$$\frac{f'(\mu)}{f(\mu)} = \frac{\phi - p}{(1-p+p\mu)(1-\phi+\phi\mu)}.$$

Next, we turn to g'/g. Again, we substitute the expression for $B(\mu)$ to show that

$$g(\mu) = \mu + (1-\mu)\frac{p}{\phi}\frac{1-\phi+\phi\mu}{1-p+p\mu}.$$

Taking derivatives yields,

$$\frac{g'(\mu)}{g(\mu)} = \frac{1 - \frac{p}{\phi}\frac{1-\phi+\phi\mu}{1-p+p\mu} + (1-\mu)\frac{p}{\phi}\frac{\phi-p}{(1-p+p\mu)^2}}{\mu + (1-\mu)\frac{p}{\phi}\frac{1-\phi+\phi\mu}{1-p+p\mu}}.$$

Simplifying,

$$\frac{g'(\mu)}{g(\mu)} = \frac{1}{1-p+p\mu}\frac{\phi-p}{\mu(\phi-p+p\phi)+p-p\phi}.$$

This implies that

$$\frac{f'(\mu)}{f(\mu)} - \frac{g'(\mu)}{g(\mu)} = \frac{\phi - p}{1 - p + p\mu}\left[\frac{1}{1 - \phi + \phi\mu} - \frac{1}{\mu(\phi - p + p\phi) + p - p\phi}\right]$$
$$= -\frac{(\phi - p)(\mu p(1-\phi) + (1-p)(1-\phi))}{(1 - p + p\mu)(1 - \phi + \phi\mu)[\mu(\phi - p + p\phi) + p - p\phi]} < 0$$

(note that $\phi - p > 0$ from the definition of p), which concludes the proof that $\Theta(\mu)/q(\mu)$ is decreasing.

Next, we shall prove the first statement of the proposition. It will be convenient to restate it as $q/\Theta < \phi$ if $\mu(z) \in [0, 1)$ for a strictly positive measure of z, or

$$\frac{\int_0^1 H(\mu(z))dz}{\int_0^1 D(\mu(z))^{\phi}dz} < \frac{H(1)}{D(1)^{\phi}}.$$

This statement is equivalent to

$$\frac{\int H(\mu)d\nu(\mu)}{\int D(\mu)^{\phi}d\nu(\mu)} < \frac{H(1)}{D(1)^{\phi}},$$

where ν is a probability measure on the Borel sigma-algebra such that $\nu(\{1\}) < 1$. Let $h(\mu) \equiv H(\mu)/D(\mu)^{\phi}$. Then the statement can be rewritten as:

$$\frac{\int h(\mu)D(\mu)^{\phi}d\nu(\mu)}{\int D(\mu)^{\phi}d\nu(\mu)} < h(1).$$

Define the Borel probability measure ν_1 by $\nu_1(A) = \int_A D(\mu)^{\phi}d\nu(\mu)/\int D(\mu)^{\phi}d\nu(\mu)$. By the change of variables theorem, the statement is equivalent to:

$$\int h(\mu)d\nu_1(\mu) < \int h(\mu)d\nu_2(\mu),$$

where ν_2 is the probability measure such that $\nu_2(\{1\}) = 1$. Since h is strictly increasing (from the first part of the proof), it is sufficient to show that ν_2 first order stochastically dominates ν_1 strictly. By construction $\nu_1([0, a]) \geq 0 = \nu_2([0, a])$ for all $a < 1$ and $\nu_1([0, 1]) = \nu_2([0, 1]) = 1$. Since $D(\mu) > 0$ and $\nu([0, a]) > 0$ for some $a \in [0, 1)$, $\nu_1([0, a]) > 0 = \nu_2([0, a])$ for some $a \in [0, 1)$, which shows that ν_2 first order stochastically dominates ν_1 strictly, which concludes the proof. ■

We introduce a technical lemma, which will be useful in the proofs of Lemma 1, Proposition 6, Lemma 2 and Lemma 3.

Lemma F.1 *The functions $\Theta(G)$ and $q(G)$ are continuous and $\Theta(G) > 0$, $q(G) > 0$, $\forall G$. Also $q(G)$ is strictly increasing in G.*

Proof. Let $\underline{\nu}$ be the probability measure on μ induced by $\underline{\mu}$. For any Borel set $A \subseteq [0,1]$, define $A' = \{z \in [0,1] : \underline{\mu}(z) \in A\}$. Then $\underline{\nu}(A)$ is equal to the Lebesgue measure of A' (which is a Borel set). Then the probability measure $\nu(\cdot, G)$ on μ in an economy with institutions G is given by:

$$\nu([0,a], G) = \underline{\nu}\left(\left[0, \max\left\{0, \frac{a - h(G)}{1 - h(G)}\right\}\right]\right).$$

Since $[0,a]$ generates all the Borel sets, we have uniquely determined $\nu(\cdot, G)$. Moreover, the CDF that $\nu(\cdot, G)$ defines is continuous in G, hence for any continuous function $g(\mu)$, the integral $\int_{[0,1]} g(\mu)\nu(d\mu, G)$ is continuous in G.

Then since

$$q(G) = \left[\int_{[0,1]} D(\mu)^{\phi}\nu(d\mu, G)\right]^{\frac{1-\phi}{\phi}},$$

$q(G)$ is continuous since $D(\mu)$ is continuous. Furthermore, $q(G) > 0$ since for all μ, $D(\mu) > 0$. Also $q(G)$ is increasing in G since (i) $D(\mu)$ is increasing and (ii) if $G_2 > G_1$ then $\nu(\cdot, G_2)$ first-order stochastically dominates $\nu(\cdot, G_1)$.

Next,

$$\Theta(G) = \frac{\left[\int_{[0,1]} D(\mu)^{\phi}\nu(d\mu, G)\right]^{\frac{1}{\phi}}}{\int_{[0,1]} H(\mu)\nu(d\mu, G)}$$

is continuous in G since $H(\mu) > 0$, $D(\mu) > 0$ for all μ. Similarly $\Theta(G) > 0$. ■

Proof of Lemma 1. The proof is by construction. First, set $c'_t = c_t$ for all t.

Let $M_t \equiv \Theta(g_t)f(k_t) + (1-\delta_K)k_t + (1-\delta_G)g_t - c_t$. If $k_{t+1} + g_{t+1} = M_t$ for all t then the original sequence satisfies all the conditions of the lemma.

Let $N(g'; M, c_a, c_b) \equiv \beta(1 - \delta_K + q(g')f'(M - g'))u'(c_a) - u'(c_b)$. Let n be the earliest period for which $M_t > k_{t+1} + g_{t+1}$. For all $t \leq n$, set $k'_t = k_t, g'_t = g_t$.

$$N(g_{n+1}; M_n, c_n, c_{n+1}) = \beta(1 - \delta_K + q(g_{n+1})f'(M_n - g_{n+1}))u'(c_{n+1}) - u'(c_n)$$

$$< \beta(1 - \delta_K + q(g_{n+1})f'(k_{n+1}))u'(c_{n+1}) - u'(c_n) = 0.$$

Similarly

$$N(M_n - k_{n+1}; M_n, c_n, c_{n+1}) = \beta(1 - \delta_k + q(M_n - k_{n+1})f'(k_{n+1}))u'(c_{n+1}) - u'(c_n)$$

$$> \beta(1 - \delta_K + q(g_{n+1})f'(k_{n+1}))u'(c_{n+1}) - u'(c_n) = 0.$$

Then by continuity, for some $g' \in (g_{n+1}, M_n - k_{n+1})$, $N(g'; M_n, c_n, c_{n+1}) = 0$. Then set $g'_{n+1} = g'$, $k'_{n+1} = M_n - g'$. The resource constraint (24) for period n binds, and the Euler condition (27) also still holds. Since $k'_{n+1} > k_{n+1}$ and $g'_{n+1} > g_{n+1}$, the resource constraint (24) for period $n + 1$ is now slack. Then now we modify the allocation for period $n + 1$ in the same way and so on. ■

Proof of Proposition 6. We first show that that if $\lambda \in \Omega(K_0, G_0)$ and $\lambda' > \lambda$, then $\lambda' \in \Omega(K_0, G_0)$.

Let $\{C_t, K_{t+1}, G_{t+1}\}_{t=0}^{\infty} \in \Gamma(K_0, G_0)$ such that $\beta(1 - \delta_K + q(G_0)f'(K_0))u'(C_0) = \lambda$. We will construct a modified sequence $\{C'_t, K'_{t+1}, G'_{t+1}\}_{t=0}^{\infty} \in \Gamma(K_0, G_0)$ such that $\beta(1 - \delta_K + q(G_0)f'(K_0))u'(C'_0) = \lambda'$.

Let $G'_t = G_t$ for all t. Let C'_0 be defined by $\beta[1 - \delta_K + q(G_0)f'(K_0)]u'(C'_0) = \lambda'$. By strict concavity and the Inada conditions of u, C'_0 exists and $0 < C'_0 < C_0$. Define K'_1 in such a way to satisfy the resource constraint:

$$K'_1 = \Theta(G_0)f(K_0) + (1 - \delta_K)K_0 + (1 - \delta_G)G_0 - G_1 - C'_0.$$

Clearly, $K'_1 > K_1$. Then, we construct the alternative sequence recursively by (for $t = 1, 2, 3, ...$):

$$C'_t = u'^{-1}\left(\frac{u'(C'_{t-1})}{\beta[1 - \delta_K + q(G_t)f'(K'_t)]}\right)$$

$$K'_{t+1} = \Theta(G_t)f(K'_t) + (1 - \delta_k)K'_t + (1 - \delta_g)G_t - G_{t+1} - C'_t.$$

By induction, $0 < C_t' < C_t$ (because $u'(C_{t-1}') > u'(C_{t-1})$ and $f'(K_t') < f'(K_t)$) and $K_{t+1}' > K_{t+1}$ (because $f(K_t') > f(K_t)$ and $C_t' < C_t$), so the new sequence is feasible, and satisfies all constraints. Therefore $\lambda' \in \Omega(K_0, G_0)$.

Then showing that $\Omega(K_0, G_0) = [\omega(K_0, G_0), \infty)$ is equivalent to proving that the minimization problem:

$$\min\{m((x_t), K_0, G_0) : (x_t) \in \Gamma(K_0, G_0)\}$$

has a well-defined solution. We will use the theorem of the maximum to prove this claim, which will also demonstrate that ω is continuous.

Endow Δ with the metric

$$\rho((x_t), (y_t)) = \sum_{t=0}^{\infty} 2^{-t-1} ||x_t - y_t||_E,$$

where $||.||_E$ is the usual Euclidean metric. Let the t-section of a set A be the set $\{x \in \mathbf{R}_+^3 : x = x_t, (x_t) \in A\}$. Then A is closed if and only if all of its t-sections are closed. Similarly, if all t-sections of a set A are uniformly bounded in the Euclidean metric, then A is totally bounded.

Unfortunately, Γ is not compact-valued. We will show that without loss of generality, we can restrict (x_t) to some $\tilde{\Gamma}(K, G)$ such that $\tilde{\Gamma}(K, G) \subseteq \Gamma(K, G)$ and $\tilde{\Gamma}$ is compact-valued and continuous. The rest of the proofs proceeds by a sequence of claims.

Claim 1: $\Gamma(K_0, G_0)$ is totally bounded.

Let $\delta = \min\{\delta_K, \delta_G\}$. Let $\tilde{K}$ be defined as the solution of $f(\tilde{K}) = \delta \tilde{K}$. $\tilde{K}$ is an upper bound on the maximum sustainable accumulable resources: physical capital and institutional capital. Let $\bar{K}(K_0, G_0) = \max\{K_0 + G_0, \tilde{K_0}\}$ and $\bar{G}(K_0, G_0) = \max\{K_0 + G_0, \tilde{K}\}$. Define $\bar{C}(K_0, G_0) = f(\bar{K}(K_0, G_0)) + (1 - \delta_K)\bar{K}(K_0, G_0) + (1 - \delta_G)\bar{G}(K_0, G_0)$. Then for any feasible path, $0 \leq K_t \leq \bar{K}(K_0, G_0)$, $0 \leq G_t \leq \bar{G}(K_0, G_0)$, and $0 < C_t \leq \bar{C}(K_0, G_0)$ are satisfied. Thus $\Gamma(K_0, G_0)$ is totally bounded.

Claim 2: Define $N(K_0, G_0) \equiv \beta(1 - \delta_K + q(0)f'(\bar{K}(K_0, G_0))$. Then for any feasible K_t and G_t, we have $\beta(1 - \delta_K + q(G_t)f'(K_t)) \geq N(K_0, G_0)$.

For any feasible path, $0 < q(0) \leq q(G_t)$ and $f'(\bar{K}(K_0, G_0)) \leq f'(K_t)$ (the second inequality follows from the strict concavity of f and the fact that $K_t \leq \bar{K}(K_0, G_0)$). Then by substituting, we obtain the inequality.

Claim 3: There exists a sequence $\{\underline{c}_t\}_{t=0}^{\infty}$, $\bar{C}(K_0, G_0) \geq \underline{c}_t > 0$, such that we can impose the additional restriction $c_t \geq \underline{c}_t$ without loss of generality.

Consider a feasible policy $G_{t+1} = (1 - \delta_G)G_t$. Given this sequence of G (and T that balances the government budget), the economy has a unique equilibrium, which is continuous with respect to (K_0, G_0); let period zero consumption of this equilibrium be denoted $\tilde{C}(K, G)$. Then, without loss of generality, we can ignore the elements $(J_t) \in \Delta$ such that $m((J_t), K, G) > \beta(1 - \delta_K + q(G_0)f'(K_0))u'(\tilde{C}(K_0, G_0))$, or $u'(c_0) > u'(\tilde{C}(K, G))$ in the minimization problem. This imposes a restriction on c_0: $u'(c_0) \leq u'(\tilde{C}(K, G))$ or equivalently

$$c_0 \geq \underline{c}_0(K, G) \equiv u'^{-1}(u'(\tilde{C}(K, G))).$$

Next, we turn to finding $\underline{c}_t$ for $t > 0$. Iterating on the Euler equation (27),

$$\begin{aligned} & u'(C_t) \\ & = \frac{u'(c_0)}{\beta(1 - \delta_K + q(G_1)f'(K_1)) \times \beta(1 - \delta_K + q(G_2)f'(K_2)) \times \cdots \times \beta(1 - \delta_K + q(G_t)f'(K_t))} \\ & \leq \frac{u'(c_0)}{N(K_0, G_0)^t} \\ & \leq \frac{u'(\underline{c}_0)}{N(K_0, G_0)^t} \end{aligned}$$

hold, where we used the restriction that $c_0 \geq \underline{c}_0$. Then, without loss of generality, we can impose the restriction on C_t, $t = 1, 2, ...$:

$$c_t \geq \underline{c}_t(K, G) \equiv u'^{-1}\left(\frac{u'(c_0)}{N(K_0, G_0)^t}\right).$$

Claim 4: There exists a sequence $\{\underline{k}_t\}_{t=0}^{\infty}$, $\bar{K}(K_0, G_0) \geq \underline{k}_t > 0$, such that we can impose the additional restriction $k_t \geq \underline{k}_t$.

Iterating on the Euler equation (27) once again,

$$\beta(1-\delta_K+q(G_t)f'(K_t)) = \frac{u'(c_0)}{u'(c_t)\beta(1-\delta_K+q(G_1)f'(K_1))\times\cdots\times\beta(1-\delta_k+q(G_{t-1})f'(K_{t-1}))}.$$

The right-hand side is smaller than

$$\frac{u'(\underline{c}_0)}{u'(\bar{C}(K_0,G_0))N(K_0,G_0)^{t-1}},$$

and therefore

$$q(G_t)f'(K_t) \leq \frac{u'(\underline{c}_0)}{\beta u'(\bar{C}(K_0,G_0))N(K_0,G_0)^{t-1}} - (1-\delta_K).$$

Since $q(G_t)f'(K_t) \geq q(0)f'(K_t)$,

$$f'(K_t) \leq \frac{u'(\underline{c}_0)}{q(0)\beta u'(\bar{C}(K_0,G_0))N(K_0,G_0)^{t-1}} - \frac{1-\delta_K}{q(0)}.$$

This implies that we can define $\underline{k}_t$ as

$$\underline{k}_t = f'^{-1}\left(\frac{u'(\underline{c}_0)}{q(0)\beta u'(\bar{C}(K_0,G_0))N(K_0,G_0)^{t-1}} - \frac{1-\delta_K}{q(0)}\right).$$

Claim 5: There exists a compact-valued and continuous correspondence $\tilde{\Gamma} : \mathbf{R}^2_+ \rightrightarrows \Delta$ such that $\tilde{\Gamma}(K,G) \subset \tilde{\Gamma}(K,G)$ for all $(K,G) \in \mathbf{R}^2_+$ and if $J \in \Gamma(K,G)$, there exists $J' \in \tilde{\Gamma}(K,G)$ such that $m(J';K,G) \leq m(J;K,G)$.

Define

$$\tilde{\Gamma}(K,G) = \{(x_t) \in \Delta : (24),(27), C_t \geq \underline{C}_t(K,G), K_t \geq \underline{K}_t(K,G)\ \forall t \text{ for given } (K,G)\}.$$

Since all the constraint functions are continuous and the inequalities are weak, all t-sections of $\tilde{\Gamma}$ are closed, hence $\tilde{\Gamma}(K,G)$ is closed. Since $\tilde{\Gamma}(K,G) \subseteq \Gamma(K,G)$, it is also totally bounded, hence $\tilde{\Gamma}(K,G)$ is compact-valued. Finally, since all the constraint functions are continuous, $\tilde{\Gamma}(K,G)$ is continuous.

The last part of the claim follows from the properties of $\underline{c}_t, \underline{k}_t$ established in Claims 3 and 4.

Claim 6: $\Omega(K, G) = [\omega(K, G), \infty)$, where $\omega(K, G)$ is a continuous function.

This claim follows directly from Claim 5 and the theorem of the maximum.

Claim 7: The function $\omega(K, G)$ is decreasing in K.

This claim is proven analogously to proving that $\lambda \in \Omega(K, G)$ and $\lambda' > \lambda$ implies that $\lambda' \in \Omega(K, G)$.

Let $\{C_t, K_{t+1}, G_{t+1}\}_{t=0}^{\infty} \in \Gamma(K, G)$ such that $\beta(1 - \delta_K + q(G)f'(K))u'(C_0) = \omega(K, G)$. Let $K' > K$. We will construct a modified sequence $\{C'_t, K'_{t+1}, G'_{t+1}\}_{t=0}^{\infty} \in \Gamma(K', G)$ such that $\beta(1-\delta_K+q(G)f'(K'))u'(C'_0) < \beta(1-\delta_K+q(G)f'(K'))u'(C'_0)$ and hence $\omega(K', G) < \omega(K, G)$.

Let $G'_t = G_t$ for all t. Let $C'_0 = C_0$ Define K'_1 in such a way to satisfy the resource constraint:

$$K'_1 = \Theta(G_0)f(K'_0) + (1 - \delta_K)K_0 + (1 - \delta_G)G_0 - G_1 - C'_0.$$

Clearly, $K'_1 > K_1$. Then, we construct the alternative sequence recursively by (for $t = 1, 2, 3, ...$):

$$C'_t = u'^{-1}\left(\frac{u'(C'_{t-1})}{\beta[1 - \delta_K + q(G_t)f'(K'_t)]}\right)$$

$$K'_{t+1} = \Theta(G_t)f(K'_t) + (1 - \delta_k)K'_t + (1 - \delta_g)G_t - G_{t+1} - C'_t.$$

By induction, $0 < C'_t < C_t$ (because $u'(C'_{t-1}) > u'(C_{t-1})$ and $f'(K'_t) < f'(K_t)$) and $K'_{t+1} > K_{t+1}$ (because $f(K'_t) > f(K_t)$ and $C'_t < C_t$), so the new sequence is feasible, and satisfies all constraints. Moreover, $\beta(1 - \delta_k + q(G)f'(K'))u'(C'_0) < \beta(1 - \delta_k + q(G)f'(K))u'(C_0)$ (because $f'(K') < f'(K)$ and all the other terms are positive and the same). Therefore $\omega(K', G) < \omega(K, G)$. ■

Proof of Lemma 2. Since K and G are fixed, the optimization problem is equivalent to maximizing consumption C subject to the constraints.

Define $\hat{G}'(K, G) \equiv \frac{1}{3}[\Theta(G)f(K) + (1 - \delta_K)K + (1 - \delta_G)G]$ and let $\hat{K}'(K, G)$ be implicitly defined by:

$$u'(\Theta(G)f(K) + (1 - \delta_K)K + (1 - \delta_G)G - \hat{G}'(K, G) - \hat{K}'(K, G)) = \mathbf{w}(\hat{K}'(K, G), \hat{G}'(K, G))).$$

Since $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{K\to 0} \mathbf{w}(K,G) = \infty$, there is at least one solution. Since $u'(c)$ is strictly decreasing and $\mathbf{w}(K,G)$ is decreasing and continuous in K, the solution $\hat{K}'(K,G)$ is unique and continuous. Define $\hat{C}(K,G)$ by

$$\hat{C}(K,G) = \Theta(G)f(K) + (1-\delta_K)K + (1-\delta_G)G - \hat{G}'(K,G) - \hat{K}'(K,G).$$

Then the triplet $(\hat{C}(K,G), \hat{K}'(K,G), \hat{G}'(K,G))$ satisfies the constraints for $\mathbf{T_w}$ and is continuous.

Define the correspondence

$$\hat{\Gamma}(K,G) = \{(C,K',G') : (28), u'(C) \geq \mathbf{w}(K',G'), C \geq \hat{C}(K,G)\}.$$

From this definition, $\mathbf{w}(K',G') \leq u'(\hat{C}(K,G))$ holds for all $(C,K',G') \in \hat{\Gamma}(K,G)$. From this and $\lim_{K\to 0} \mathbf{w}(K,G) = \infty$, the inequality $K' > 0$ is satisfied for all $(C,K',G') \in \hat{\Gamma}(K,G)$. Therefore $\hat{\Gamma}$ respects all the constraints of $\mathbf{T_w}$ and without loss of generality we can perform the minimization subject to $(C,K',G') \in \hat{\Gamma}(K,G)$.

Since $(C,K',G') \in [0, \Theta(G)f(K) + (1-\delta_K)K + (1-\delta_G)G]^3$, clearly $\hat{\Gamma}(G,K)$ is bounded. By continuity of the constraint functions, $\hat{\Gamma}$ is closed, hence compact-valued. Continuity of the constraint functions also ensures continuity of the correspondence. Then $\mathbf{T_w w}$ is continuous by the theorem of the maximum.

Finally, $\mathbf{T_w w}(K,G) \geq \beta(1-\delta_K + q(G)f'(K))u'(\Theta(G)f(K) + (1-\delta_K)K + (1-\delta_G)G)$. Therefore $\lim_{K\to 0} \mathbf{T_w w}(K,G) = \infty$ and $\mathbf{T_w w}(K,G) > 0$ hold. ■

For proving Lemma 3, we use the following results.

Lemma F.2 $\mathbf{T_w}\omega = \omega$.

Proof. The proof is standard. Clearly $\omega \in \mathcal{A}$, so by Lemma 2, $\mathbf{T_w}\omega$ is well-defined.

Fix K and G. Let c_0, k_1, g_1 be an arbitrary triplet satisfying the constraints in problem $\mathbf{T_w}$ for ω. Let $(c_t, k_{t+1}, g_{t+1})_{t=1}^{\infty} \in \Gamma(k_1, g_1)$ such that $u'(c_0) = \beta(1-\delta_K + q(g_1)f'(k_1))u'(c_1)$. Then $\{c_0, k_1, g_1, (c_t, k_{t+1}, g_{t+1})_{t=1}^{\infty}\} \in \Gamma(K,G)$. Therefore, $\beta(1-\delta_K + q(G)f'(K))u'(c_0) \geq \omega(K,G)$.

Since the triplet was arbitrary, $\beta(1-\delta_K+q(G)f'(K))u'(c_0) \geq \omega(K,G)$ for all c_0, k_1, g_1 that satisfy the constraints of problem $\mathbf{T_w}$, so $\mathbf{T_w}\omega(K,G) \geq \omega(K,G)$.

Next, let $(c_t, k_{t+1}, g_{t+1})_{t=0}^{\infty} \in \Gamma(K,G)$ be arbitrary. Then $(c_t, k_{t+1}, g_{t+1})_{t=1}^{\infty} \in \Gamma(k_1, g_1)$, so $\beta(1-\delta_K+q(g_1)f'(k_1))u'(c_1) \geq \omega(k_1,g_1)$. Then $u'(c_0) = \beta(1-\delta_K+q(g_1)f'(k_1))u'(c_1) \geq \omega(k_1,g_1)$; also (c_0,k_1,g_1) satisfies the resource constraint and the non-negativity constraints. Thus (c_0,k_1,g_1) satisfies the constraints for $\mathbf{T_w}$. Then $\beta(1-\delta_K+q(G)f'(K))u'(c_0) \geq \mathbf{T_w}\omega(K,G)$. Since $(c_t,k_{t+1},g_{t+1})_{t=0}^{\infty} \in \Gamma(K,G)$ is arbitrary, $\omega(K,G) \geq \mathbf{T_w}\omega(K,G)$. ■

Proof of Lemma 3. The operator $\mathbf{T_w}$ is clearly monotone, so $\mathbf{T_w}\mathbf{w} \geq \mathbf{w}$ implies $\mathbf{T_w}^2\mathbf{w} \geq \mathbf{T_w}\mathbf{w}$ and then by induction $\mathbf{T_w}^{n+1}\mathbf{w} \geq \mathbf{T_w}^n\mathbf{w}$. Similarly, the assumption $\mathbf{w} \leq \omega$ implies $\mathbf{T_w}^n\mathbf{w} \leq \mathbf{T_w}^n\omega = \omega$, where we used Lemma F.2. Then for any (K,G), $(\mathbf{T_w}^n\mathbf{w})(K,G)$ is a monotone and bounded sequence of real numbers, hence it has a limit.

Next we show that $\lim_{n\to\infty}(\mathbf{T_w}^n\mathbf{w})(K,G) = \omega(K,G)$. Fix K and G. Let H_n be the policy correspondence that solves problem $\mathbf{T_w}$ given $\mathbf{T_w}^{n-1}\mathbf{w}$. Let $k_{0,n} = K, g_{0,n} = G$. Then for a fixed n, define $(c_{t,n}, k_{t+1,n}, g_{t+1,n})$ recursively by $(c_{t,n}, k_{t+1,n}, g_{t+1,n}) \in H_{n-t}(k_{t,n}, g_{t,n})$ if $t \leq n-1$ and $(c_{t,n}, k_{t+1,n}, g_{t+1,n}) = (0,0,0)$ if $t \geq n$.

Since $\mathbf{T_w}^n\mathbf{w} \leq \omega$, it follows that $c_{0,n} \geq \tilde{C}(K,G)$ for all n, where $\tilde{C}(K,G)$ is defined in the proof of Proposition 6. Then by the same reasoning as in the proof of Proposition 6 $c_{t,n} \geq \underline{c}_t(K,G)$ if $t \leq n-1$, where $\underline{c}_t(K,G)$ is defined in the proof of Proposition 6.

Let $y_n = (c_{t,n}, k_{t+1,n}, g_{t+1,n})_{t=0}^{\infty}$. Define

$$\Delta_{K,G} = \{\{J_t\}_{t=0}^{\infty}, J_t \in [0, \bar{C}(K,G)] \times [0, \bar{K}(K,G)] \times [\bar{G}(K,G)]\},$$

where $\bar{C}(K,G)$, $\bar{K}(K,G)$, and $\bar{G}(K,G)$ are defined in the proof of Proposition 6. Then $y_n \in \Delta_{K,G}$ for all n. Since $\Delta_{K,G}$ is compact in the metric we defined above, (y_n) has a convergent subsequence (y_{n_ℓ}) with a limit in $\Delta_{K,G}$. Denote $c_t = \lim_\ell c_{t,n_\ell}, k_{t+1} = \lim_\ell k_{t+1,n_\ell}, g_{t+1} = \lim_\ell g_{t+1,n_\ell}$.

By construction $c_{n,t}, k_{n,t}, g_{n,t}, c_{n,t+1}, k_{n,t+1}, g_{n,t+1}$ satisfy (24), (25), (26) and (27) evaluated at t if $n \geq t+1$. This fact and the continuity of the constraint functions, implies that $(c_t, k_{t+1}, g_{t+1})_{t=0}^{\infty}$ satisfy (24), (25), (26) and (27) for all t. Therefore, $(c_t, k_{t+1}, g_{t+1})_{t=0}^{\infty} \in$

$\Gamma(K,G)$, so $\beta(1-\delta_K q(G) f'(K))u'(c_0) \geq \omega(K,G)$.

Also by continuity $\beta(1-\delta_K q(G) f'(K))u'(c_0) = \lim_n \mathbf{T_w}^n \mathbf{w}(K,G) \leq \omega(K,G)$. Then $\lim_n \mathbf{T_w}^n \mathbf{w}(K,G) = \omega(K,G)$. ■

Lemma F.3 $\mathbf{T_v} v^* = v^*$.

Proof. Standard. ■

Proof of Proposition 7. Denote $\mathbf{T_v}^n \hat{v} \equiv v_n$ and $\hat{v} = v_0$. The operator $\mathbf{T_v}$ is monotone, so by induction $v_1 = \mathbf{T_v}\hat{v} \leq \hat{v} = v_0$ implies that $v_{n+1} = \mathbf{T_v}^{n+1}\hat{v} \leq \mathbf{T_v}^n \hat{v} = v_n$ for all n. Similarly, $\hat{v} \geq v^*$ implies that $v_n = \mathbf{T_v}^n \hat{v} \geq \mathbf{T_v}^n v^* = v^*$, where we use Lemma F.3. This implies that for all (K,G,λ), $\lambda \in \Omega(K,G)$, $\{v_n(K,G,\lambda)\}_{n=0}^\infty$ is a decreasing sequence of real numbers, hence it has a limit. Denote the pointwise limit v. Then $v \geq v^*$.

Next we show that $v \leq v^*$. Fix (K,G,λ) such that $\lambda \in \Omega(K,G)$. We will construct a sequence $\{c_t, k_{t+1}, g_{t+1}\}_{t=0}^\infty$ that is feasible for (SP1) given K, G, λ and

$$v(K,G,\lambda) \leq \sum_{t=0}^{\infty} \beta^t u(c_t).$$

Then from the definition of v^*, the claim follows.

Let n be an arbitrary integer. Set $k_{0,n} = K, g_{0,n} = G, \lambda_{0,n} = \lambda$. We construct the n-th sequence of choice variables $\{c_{t,n}, k_{t+1,n}, g_{t+1,n}, \lambda_{t+1,n}\}_{t=0}^\infty$ as follows. For $t < n$, choose $(c_{t,n}, k_{t+1,n}, g_{t+1,n}, \lambda_{t+1,n})$ recursively such that

$$u(c_{t,n}) + \beta v_{n-t-1}(k_{t+1,n}, g_{t+1,n}, \lambda_{t+1,n}) = v_{n-t}(k_{t,n}, g_{t,n}, \lambda_{t,n})$$

and $(c_{t,n}, k_{t+1,n}, g_{t+1,n}, \lambda_{t+1,n})$ are feasible given $k_{t,n}, g_{t,n}, \lambda_{t,n}$.

Set $(c_{t,n}, k_{t+1,n}, g_{t+1,n}, \lambda_{t+1,n}) = (0,0,0,0)$ if $t \geq n$.

By construction,

$$v_n(K_0, G_0, \lambda_0) = \sum_{t=0}^{n-1} \beta^t u(c_{t,n}) + \beta^n v_0(k_{n,n}, g_{n,n}, \lambda_{n,n}).$$

Furthermore, by construction, the sequence is physically feasible, so for all t and n, $c_{t,n} \leq \bar{C}(K_0, G_0)$, $k_{t,n} \leq \bar{K}(K_0, G_0)$ and $g_{t,n} \leq \bar{G}(K_0, G_0)$. Since $\hat{v} = v_0$ is strictly increasing in

K and constant in G and λ, it follows that $v_0(k_{n,n}, g_{n,n}, \lambda_{n,n}) \le v_0(\bar{K}(K_0, G_0), 1, 1) \equiv A$. Therefore

$$v_n(K_0, G_0, \lambda_0) \le \sum_{t=0}^{n-1} \beta^t u(c_{t,n}) + \beta^n A.$$

Next, we show that this sequence of sequences has a converging subsequence with a limit that has the desired properties.

Let $y_n = \{c_{t,n}, k_{t+1,n}, g_{t+1,n}\}_{t=0}^{\infty}$. Define

$$\Delta_{K,G} = \{\{x_t\}_{t=0}^{\infty}, x_t \in [0, \bar{C}(K,G)] \times [0, \bar{K}(K,G)] \times [\bar{G}(K,G)]\},$$

where $\bar{C}(K,G)$, $\bar{K}(K,G)$, and $\bar{G}(K,G)$ are defined in the proof of Proposition 6. Then $y_n \in \Delta_{K,G}$ for all n. Since $\Delta_{K,G}$ is compact in the metric we defined above, (y_n) has a convergent subsequence (y_{n_ℓ}) with a limit in $\Delta_{K,G}$. Denote $c_t = \lim_\ell c_{t,n_\ell}$, $k_{t+1} = \lim_\ell k_{t+1,n_\ell}$, and $g_{t+1} = \lim_\ell g_{t+1,n_\ell}$. Next, we show that the limit sequence satisfies all the constraints in (SP1).

By construction, $u'(c_{0,n}) = \lambda_0/[\beta(1 - \delta_q + q(G_0)f'(K_0))]$. As in the proof of Proposition 6 (Claim 3), we can derive that if $n \ge t+1$

$$u'(c_{t,n}) \le \frac{u'(c_{0,n})}{N(K_0)^t} = \frac{\lambda_0}{\beta(1 - \delta_q + q(G_0)f'(K_0))N(K_0, G_0)^t}.$$

Then if we define

$$\underline{c}_t = u'^{-1}\left(\frac{\lambda_0}{[\beta(1 - \delta_q + q(G_0)f'(K_0))]N(K_0, G_0)^t}\right),$$

$c_{t,n} \ge \underline{c}_t$ for all t, n such that $n \ge t+1$.

Since the Euler equation must hold if $n > t$,

$$\beta(1 - \delta_k + q(0)f'(k_{t,n})) \le \beta(1 - \delta_k + q(g_{t,n})f'(k_{t,n})) = \frac{u'(c_{t-1,n})}{u'(c_{t,n})} \le \frac{u'(\underline{c}_t)}{u'(\bar{C}(K_0, G_0))}.$$

Then,

$$f'(k_{t,n}) \le \frac{1}{\beta q(0)}\left[\frac{u'(\underline{c}_t)}{u'(\bar{C}(K_0, G_0))} - (1 - \delta_k)\right],$$

which defines a lower bound for $k_{t,n}$. Denote it $\underline{k}_t$.

By construction, $c_{n,t}, k_{n,t}, g_{n,t}, c_{n,t+1}, k_{n,t+1}, g_{n,t+1}$ satisfy (24), (25), (26) and (27) evaluated at t if $n \geq t+1$. This fact and the continuity of the constraint functions imply that $(c_t, k_{t+1}, g_{t+1})_{t=0}^{\infty}$ satisfy (24), (25), (26) and (27) for all t; moreover, by construction $u'(c_0)\beta(1-\delta_k+q(G_0)f'(K_0)) = \lambda_0$. Therefore, $\{c_0, k_{t+1}, g_{t+1}\}_{t=0}^{\infty}$ satisfies all the constraints in the problem (SP1) given K_0, G_0, λ_0. Also $u(c_t) \leq u(\bar{C}(K_0, G_0))$ for all t, therefore $\sum_{t=0}^{\infty} \beta^t u(c_t)$ is well-defined (though it may be $-\infty$). Therefore,

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \leq v^*(K_0, G_0, \lambda_0).$$

Finally, using this inequality, we have that

$$\begin{aligned} v(K_0, G_0, \lambda_0) &= \lim_n v_n(K_0, G_0, \lambda_0) \\ &= \lim_{\ell} v_{n_\ell}(K_0, G_0, \lambda_0) \\ &\leq \lim_{\ell} \left[\sum_{t=0}^{n_\ell - 1} \beta^t u(c_{t,n_\ell}) + \beta^{n_\ell} A \right] \\ &= \sum_{t=0}^{\infty} \beta^t u(c_t) \\ &\leq v^*(K_0, G_0, \lambda_0). \end{aligned}$$

■

Proof of Proposition 5. If the policy functions are differentiable, then the value function v is also differentiable. Let $R_1 \equiv u_1(C_1 - \tilde{M}_1)$, $R_2 \equiv u_1(C_2 - \tilde{M}_2)$, and $R_3 \equiv u_1(C_4 - \tilde{M}_3)$. The government's first-order condition is:

$$R_3 + \beta[v_1' \tilde{M}_3 + v_2'] = 0, \tag{F.40}$$

where v_i is the partial derivative with respect to the ith term, and $'$ represents the next period. By definition,

$$v(K,G) = u(C(K,G,\tilde{M}(K,G,\nu(K,G)),\nu(K,G))) + \beta v(\tilde{M}(K,G,\nu(K,G)),\nu(K,G)).$$

Differentiating the value function we get:

$$v_1 = R_1 + \beta v_1' \tilde{M}_1 \tag{F.41}$$

$$v_2 = R_2 + \beta v_1' \tilde{M}_2, \tag{F.42}$$

where we used the FOC to eliminate all terms involving derivatives of ν.

Updating equations (F.40), (F.41) and (F.42) one period, we obtain a system of six linear equations and six unknowns: v_1, v_2, v_1', v_2', v_1'', v_2'', which we can express in matrix notation as:

$$\begin{bmatrix} 1 & 0 & -\beta\tilde{M}_1 & 0 & 0 & 0 \\ 0 & 1 & -\beta\tilde{M}_2 & 0 & 0 & 0 \\ 0 & 0 & \beta\tilde{M}_3 & \beta & 0 & 0 \\ 0 & 0 & 1 & 0 & -\beta\tilde{M}_1' & 0 \\ 0 & 0 & 0 & 1 & -\beta\tilde{M}_2' & 0 \\ 0 & 0 & 0 & 0 & \beta\tilde{M}_3' & \beta \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_1' \\ v_2' \\ v_1'' \\ v_2'' \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ -R_3 \\ R_1' \\ R_2' \\ -R_3' \end{bmatrix}$$

The system can be solved analytically with standard tools of linear algebra. Using Gaussian elimination, we obtain:

$$v_1 = R_1 + \beta\tilde{M}_1 R_1' - \frac{\beta\tilde{M}_1\tilde{M}_1'}{\tilde{M}_2' + \tilde{M}_1'\tilde{M}_3}[R_2' + R_3/\beta + R_1'\tilde{M}_3]$$

$$v_2 = R_2 + \beta\tilde{M}_2 R_1' - \frac{\beta\tilde{M}_2\tilde{M}_1'}{\tilde{M}_2' + \tilde{M}_1'\tilde{M}_3}[R_2' + R_3/\beta + R_1'\tilde{M}_3].$$

Then updating the expressions for v_1 and v_2 one period and plugging into the first-order condition, we get the following expression:

$$\begin{aligned} R_3 \quad & +\beta\tilde{M}_3 R_1' + \beta^2\tilde{M}_3\tilde{M}_1' R_1'' - \tfrac{\beta^2\tilde{M}_3\tilde{M}_1'\tilde{M}_1''}{\tilde{M}_2''+\tilde{M}_1''\tilde{M}_3'}[R_2'' + R_3'/\beta + R_1''\tilde{M}_3'] \\ & +\beta R_2' + \beta^2\tilde{M}_2' R_1'' - \tfrac{\beta^2\tilde{M}_2'\tilde{M}_1''}{\tilde{M}_2''+\tilde{M}_1''\tilde{M}_3'}[R_2'' + R_3'/\beta + R_1''\tilde{M}_3'] = 0. \end{aligned}$$

Finally, substituting the expression for R_i and grouping, we get the expression in the text.

■

G Computational algorithm

G.1 Economy with evolving institutions (Section 5)

We solve the model using the Coleman (1990) method. The consumer's optimization yields the Euler equation

$$\frac{1}{C_t} = \beta(1 + r_{t+1} - \delta_K)\frac{1}{C_{t+1}}.$$

In the recursive competitive equilibrium (with state variables K and G), this can be rewritten as (using the firm's optimization $r_{t+1} = q(G_{t+1})f'(K_{t+1})$)

$$\frac{1}{C(K, G, K', G')} = \beta(1 + q(G')f'(K') - \delta_K)\frac{1}{C'(K', G', K'', G'')},$$

where

$$C(K, G, K', G') = \Theta(G)f(K) + (1 - \delta_K)K - K' + (1 - \delta_G)G - G' - \tau(G, G').$$

The computation is done with the following steps.

1. Set the grids over K and G.

2. Guess the initial decision rule for K' as $\mathbf{K}'_0(K, G)$. Note that G' (and G'') can be represented as a function of G:

$$G' = e^{-\zeta}(G - \bar{G}) + \bar{G}.$$

3. The Euler equation

$$\frac{1}{C(K, G, K', G')} = \beta(1 + q(G')f'(K') - \delta_K)\frac{1}{C'(K', G', \mathbf{K}'_n(K', G'), G'')}$$

can be solved for one unknown, K'. (Off-the-grid points are linearly interpolated.) Then we can obtain the new decision rule $\mathbf{K}'_{n+1}(K, G)$ as this solution.

4. $||\mathbf{K}_n - \mathbf{K}_{n+1}||_\infty < tol$, we end. Otherwise repeat from step 3.

G.2 Coercive government (Section 6.1)

The value function v is approximated by linear interpolation on a grid of points for physical capital and institutional capital, wherever necessary.

1. Set a grid for K and G.

   We define implicitly $\underline{K}$ and $\bar{K}$ by $\beta(1-\delta_K+q(0)f'(\underline{K})) = 1$ and $\beta(1-\delta_K+\theta f'(\bar{K})) = 1$. Then we set $K_{min} = 0.5 \times \underline{K}$ and $K_{max} = 2.55 \times \bar{K}$. We construct an equally spaced grid for physical capital between K_{min} and K_{max} with 100 points. We set $G_{min} = 10^{-5}$ and $G_{max} = 2.0$ and again construct an equally spaced grid for institutional level with 80 points.

2. Initialize v.

   We set $v(K, G)$ to be the value in the competitive equilibrium without distortions.

3. For all points on the grid (K, G), we set

$$Tv(K,G) = \max_{C,K',G'} u(C) + \beta v(K', G'),$$

$$C + K' + G' \leq \Theta(G)f(K) + (1-\delta_K)K + (1-\delta_G)G - \tau(G, G').$$

4. If $||v - Tv||_\infty < tol$, we end. If not, we update $v = Tv$ and return to step 3.

G.3 Market economy: commitment (Section 6.2)

The value function v and the admissible set Ω are approximated by linear approximation off the grid points, whenever necessary. Proposition 6 states that $\Omega(K,G) = [\omega(K,G), \infty)$, where ω is a continuous, real-valued function that is strictly decreasing in k.

The algorithm for solving the problem is as follows.

1. We set the grid for K and G as described in G.2.

2. Initialize the function ω.

We set the initial guess for the function to be

$$\omega(K,G) = \beta(1-\delta_K + q(G)f'(K))u'(\Theta(G)f(K) + (1-\delta_K)K + (1-\delta_G)G)$$

on all the points of the grid.

3. Find $T_{\mathbf{w}}\omega$.

   For all points (K,G) on the grid, we set

$$\tilde{C}(K,G) = \max_{G'} \Theta(G)f(K) + (1-\delta_k)K + (1-\delta_g)G - \tau(G,G') - \tilde{K}(K,G,G'),$$

   where $\tilde{K}(K,G,G')$ is the unique solution K' to the equation:

$$u'(C(K,G,K',G')) = \omega(K',G').$$

   These functions are independent of the value function. Then we set

$$T_{\mathbf{w}}\omega(K,G) = \beta(1-\delta_K + q(G)f'(K))u'(\tilde{C}(K,G)).$$

   This is the minimum value of feasible $\beta(1-\delta_K + q(G)f'(K))u'(C)$ for given K and G.

4. Update ω.

   If $||\omega - T_{\mathbf{w}}\omega||_\infty < tol$, we stop updating ω and move to step 5. Otherwise, set $\omega = T_{\mathbf{w}}\omega$ and return to step 2.

5. Construct a grid for λ. We set $\lambda_{min} = \min_{K,G}\omega(K,G)$, where the minimum is over the grid. We set $\lambda_{max} = 6$ and verify that this bound is not binding. Then we construct an equally spaced grid for λ with 110 points.

   In all cases when $\lambda > \lambda_{max}$, we set[26]

$$v(K,G,\lambda) = v(K,G,\lambda_{max}) + \frac{1}{1-\beta}(\log(\lambda_{max}) - \log(\lambda)).$$

[26]This approximation follows from the log preferences assumption. Similar approximations exist for CRRA and CARA preferences.

6. We construct functions $\mathcal{C}(K,G,\lambda)$, $\mathcal{U}(K,G,\lambda)$ and $\lambda'(K,G,\lambda)$ to satisfy the following:

$$\beta(1-\delta_K+q(G)f'(K))u'(\mathcal{C}(K,G,\lambda))=\lambda,$$

$$\mathcal{U}(K,G,\lambda)=u(\mathcal{C}(K,G,\lambda)),$$

$$\lambda'(K,G,\lambda)=u'(\mathcal{C}(K,G,\lambda)).$$

7. Initialize v as described in G.2.

8. Find Tv

   For all points on the grid (K,G,λ) such that $\lambda \geq \omega(K,G)$, we set

$$Tv(K,G,\lambda)=\max_{K',G'}\mathcal{U}(K,G,\lambda)+\beta v(K',G',\lambda'(K,G,\lambda)),$$

   subject to

$$\lambda'(K,G,\lambda)\geq\omega(K',G')$$

   and

$$\mathcal{C}(K,G,\lambda)+K'+G'\leq\Theta(G)f(K)+(1-\delta_K)K+(1-\delta_G)G-\tau(G,G').$$

9. If $||v-Tv||_\infty < tol$, we end. If not, we update $v=Tv$ and return to step 8.

G.4 Market economy: non-commitment (Section 6.3)

In this case, we need to find three objects: the government policy function ν, the private saving function M and the government's value function v. We use the notation from Section 6.3. As in G.2, for all off-grid value of K and G, we approximate v, ν and M by linear interpolation.

The algorithm is then as follows:

1. Set the grids for K and G as in Section G.2.

2. Initialize the government's policy function $\nu(K,G)=G_{min}$.

3. Initialize the private saving function $M(K,G) = K_{min}$.

4. Set the government's value function to be the fixed point of:

$$v(K,G) = u(C(K,G,M(K,G),\nu(K,G))) + \beta v(M(K,G),\nu(K,G)).$$

5. Find updates for ν and M.

   For all (K,G) on the grid:

   (a) Set the new ν function as

   $$\nu'(K,G) \in \arg\max_{G'} u(C(K,G,\tilde{M}(K,G,G'),G') + \beta v(\tilde{M}(K,G,G'),G'),$$

   where $\tilde{M}(K,G,G')$ is computed as the solution K' of the problem:

   $$u'(C(K,G,K',G')) = \beta(1-\delta_K + q(G')f'(K'))u'(C(K',G',M(K',G'),\nu(K',G'))).$$

   Note that given the (linearly interpolated) ν and M, $\tilde{M}$ is always computed exactly for any (K,G,G'), so the condition above always holds exactly.

   (b) Set the new M function as $M'(K,G) = \tilde{M}(K,G,\nu'(K,G))$.

6. If $||\nu - \nu'||_\infty + ||M - M'||_\infty < tol$, end. Otherwise: update $\nu = \nu'$ and $M = M'$ and go to step 4.

Additional References for Appendix

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