Reputation and Information Aggregation*

Emiliano Catonini[†] and Sergey Stepanov[‡]

October 2, 2016

Abstract

We analyze how reputation concerns of a partially informed decision-maker affect her ability to extract information from reputation-concerned advisors. Contrary to most of the literature, we show that the decision-maker's concerns for her reputation as an expert can improve information aggregation. When the decision-maker's reputation concerns are very low, she is tempted to ask for advice regardless of her private information, which undermines advisors' truth-telling incentives. Very high reputation concerns destroy the incentives to seek advice. The optimal strength of the decision-maker's reputation concerns maximizes advice-asking without undermining advisors' incentives. Prior uncertainty about the state of nature calls for a more reputation-concerned decision-maker, unless the uncertainty becomes too high, in which case the reputation concerns become (almost) irrelevant. Finally, higher prior competence of advisors may worsen the quality of decisions when the decision-maker's reputation concerns are not sufficiently strong.

JEL classification: D82, D83

Keywords: reputation concerns, information aggregation, advice

1 Introduction

It is well documented that people can be reluctant to ask for advice or help from other people, even when such advice/help can improve the quality of their decisions (e.g., Lee (2002), Brooks et al. (2015)). One frequently cited reason for such behavior in

*This study was funded within the framework of the Basic Research Program at the National Research University Higher School of Economics (HSE) and by the Russian Academic Excellence Project '5-100'. We thank Elena Panova, Marco Ottaviani, Sergei Severinov, Pierpaolo Battigalli, and Thomas Tröger for helpful discussions and comments.

†ICEF, National Research University Higher School of Economics, Russian Federation. Postal address: Office 3431, ul. Shabolovka 26, 119049 Moscow, Russia. Email: ecatonini@hse.ru

‡ICEF and Faculty of Economics, National Research University Higher School of Economics, Russian Federation. Postal address: Office 4318, ul. Shabolovka 26, 119049 Moscow, Russia. Email: sstepanov@hse.ru

the management and psychology literature is the fear to appear incompetent, inferior, or dependent (e.g., DePaulo and Fisher (1980), Lee (1997), Lee (2002), Brooks et al. (2015)). In the economic theory literature, Levy (2004) provides a model in which a decision maker excessively ignores/neglects the opportunity to ask advice in order to be perceived competent.

Overall, the existing studies suggest that reputation concerns of a decision maker may be detrimental to her ability to collect information from potential advisors, such as her colleagues or subordinates. We show that the opposite can actually be true: even if asking for advice damages the decision maker's reputation in equilibrium, her reputation concerns may actually *help* her to aggregate information possessed by other agents. The key feature of our story, which distinguishes it from the previous literature, is that the decision maker's advice-seeking behavior affects advisors' incentives to provide truthful information. We argue that without reputation concerns the decision maker may ask for advice *excessively*, that is, so often that it adversely affects the advisors' incentives to provide truthful information. The positive role for reputation concerns then is to ensure that the decision maker asks for advice more often when it is needed more, that is, when her available information leaves high uncertainty about the state of the world. This advice-asking behavior improves the advisors' information provision incentives and, therefore, results in better aggregation of information.

We consider a model in which a decision maker needs to take a decision/action from a binary set. Which action is optimal depends on the unknown state of nature, which is also binary. Prior to taking an action, the decision maker receives an informative binary signal about the state. In addition, she can solicit advice from other agents ("advisors"), each of whom has also received an informative binary signal. Our setup fits a variety of real-life settings. For example, the decision maker can be a CEO or a prime-minister, and the advisors can be her colleagues, subordinates, designated advisors or any kind of experts in the domain of the decision maker's responsibilities. The crucial feature of the model is that both the decision maker and the advisors have reputation concerns – they want to appear competent, i.e., able to receive precise signals. The decision maker can be one of two types: good and bad, the difference being that the good type receives more informative signals. Similarly, each of the advisors can also be one of two types: high and low. Neither the decision maker nor any of the advisors knows his or her own type, but the prior probabilities of good and high types are common knowledge. All advisors are ex-ante identical. Whereas the decision maker cares both about taking the right action and appearing competent (i.e., being of a good type), the advisors only have reputation concerns (for simplicity).

In this setup, similarly to Ottaviani and Sørensen (2001), an advisor's reporting incentives can be described as simply "guessing the state", given all available information. Thus, an advisor will report his signal truthfully if and only if his posterior beliefs about the state *before* accounting for his own signal (i.e., based only on the prior and decision maker's decision to ask for advice[1]) are sufficiently close to 1/2, so that different signals result in different states appearing more likely for the advisor. Otherwise, no informative advice happens ("babbling" or "herding" by the advisors).

Now, if the decision maker cares *only* about the quality of decisions, she will care only about receiving advice, and, thus, will *always* want to ask for it. This means that, in equilibrium, no information can be inferred by advisors from the decision maker's behavior. This has a beneficial effect when the prior belief about the state is close to 1/2: truthful reporting is ensured. However, if the prior is sufficiently far from 1/2, the advisors will herd on the prior, and no informative advice will be provided. This is what we call the situation of "excessive advice-seeking": the decision maker's "unrestrained" advice-seeking behavior destroys provision of advice.

Now suppose the decision maker could *commit* to ask for advice only when she receives a signal contradicting the prior. When "unrestrained" advice-seeking leads to herding by the advisors, such commitment could improve the situation, provided that the combination of the prior and the decision maker's signal contradicting the prior results in a belief sufficiently close to 1/2 so that truthful reporting by advisors occurs. As a result, the decision maker would manage to receive decision-relevant information precisely when it is most needed (when her signal confirms the prior, extra information is of much lower value for her).

We show that decision maker's reputation concerns (unless they are too extreme) can help implement such commitment as a separating equilibrium. The key intuition can be explained through a kind of "single-crossing" argument. The decision maker who received the signal confirming the prior has a strong reputational motive to convey this news to the advisors. At the same time, her need for extra information is low, because she is already quite confident about the state. In contrast, the decision maker who received the signal contradicting the prior has either a weaker reputational incentive to lie (when the signal is weaker than the prior) or a reputational incentive to actually reveal her true signal (when the signal is stronger than the prior). At the same time, such decision maker cares a lot about information aggregation, because the signal contradicting the prior makes her more uncertain about the state, compared to the signal confirming the prior. Thus, whenever the weight on reputation in the decision maker's utility is big enough, the separation of the signal-types of the decision maker becomes possible in equilibrium.

[1]We assume that all advisors speak simultaneously. Sequential advice would not alter our results qualitatively.

In other words, in this equilibrium, the decision maker's reputation concerns make the decision maker ask for advice only when she is very uncertain about the state of the world, and, realizing this decision maker's behavior, the advisors have incentives to report their information truthfully when asked.

We further show that, for a range of weights on reputation, there exists an equilibrium with even more information aggregation. In this equilibrium, the decision maker always asks for advice when her signal contradicts the prior and *mixes* between asking and not asking when her signal confirms the prior, and the advisors report truthfully when asked.[2] We call this equilibrium partially separating. The optimal weight on reputation then is the one that results in the partially separating equilibrium maximizing the frequency of advice-asking without destroying the advisors' truthtelling incentives.

A further rise in the reputation concerns damages information aggregation, as it becomes too tempting for the decision maker who got the signal confirming the prior to reveal her type by refusing to ask for advice. As a result, the partially separating equilibrium becomes unsustainable. In fact, when the weight on reputation is very high, even the purely separating equilibrium may disappear, because even the decision maker with the signal contradicting the prior may find it profitable to pretend having received the other signal and refuse to ask for advice.

We further study the interaction between the prior uncertainty about the state of the world and reputation concerns. We show that greater uncertainty leads to a higher optimal weight on reputation. The intuition is that higher prior uncertainty increases the decision maker's incentives to ask for advice even when decision maker's signal confirms the prior. A higher weight on reputation is then needed to restrain this temptation. However, when the prior uncertainty becomes so high that truthtelling by the advisors arises even when the decision maker *always* asks for advice, restraining advice-asking is not needed anymore, and any weight on reputation from 0 up to a certain value becomes optimal.

The implication of this result is that at times of low uncertainty, i.e., when the organization is performing well and seems to be on the right track, it needs a leader concerned with her reputation, but not too strongly. When the uncertainty about the right strategy is large but not extremely large, the best leader is the one with sufficiently strong reputation concerns. Finally, in the situations of a very high uncertainty (i.e., when the organization faces important and non-obvious strategic choices), the reputation concerns of the leader are actually irrelevant (unless they are too extreme) for two reasons: (1) the advice-seeking incentives are so strong that reputation concerns cannot undermine them,

[2] This equilibrium co-exists with the purely separating equilibrium for a range of parameters. However, we argue that the partially separating equilibrium is more plausible than the purely separating one because it is preferred ex-ante by the decision maker.

(2) the advisors are willing to tell the truth if their beliefs stay at the prior.

Another interesting result of our model is that an increase in the prior competence of advisors may undermine information aggregation and, hence, worsen the quality of decisions. This effect is due to a greater temptation of the decision maker to ask for advice even when her signal confirms the prior, which may destroy advisors' truthtelling. It is more likely to arise when the prior is strong and the reputation concerns are rather weak.

There are a number of papers arguing that reputation concerns can be detrimental for efficiency, because they distort behavior of agents (e.g., Scharfstein and Stein (1990), Trueman (1994), Prendergast and Stole (1996), Effinger and Polborn (2001), Morris (2001), Levy (2004), Prat (2005), Ottaviani and Sørensen (2001, 2006a, 2006b), Ely and Välimäki (2003)).[3] In these papers, like in our work, reputation concerns are "career concerns for expertise" which arise due to the future gains from being perceived smart (except for Morris (2001) and Ely and Valimaki (2003), in which the agent have concerns for being perceived as having certain preferences).

Of these papers, Levy (2004) and Ottaviani and Sørensen (2001, 2006a, 2006b) most closely relate to our work. Ottaviani and Sørensen consider aggregation of information from agents possessing private signals about the state of nature. Due to their reputation concerns, agents have incentives to misreport their signals, which may result in herd behavior in reporting and, ultimately, in the failure to aggregate information.

Levy (2004) presents a model in which a decision maker who knows her type needs to take a decision. Like in our setup, the decision maker cares both about the outcome of her action and the public perception of his ability. Levy shows that the decision maker excessively contradicts prior public information or may abstain from asking for valuable advice in order to raise her perceived competence.

Our model shares certain features of Levy (2004) and Ottaviani and Sørensen (2001, 2006a, 2006b): we have a reputation-concerned decision maker who decides whether to ask for advice or not (Levy, 2004), and reputation-concerned advisors who are tempted to herd on the public belief in their reporting behavior (the papers by Ottaviani and Sørensen). Yet, the crucial distinction of our model from these papers is strategic interaction between reputation-concerned agents. In our model, the strategy of the DM (to ask for advice or not depending on her signal) impacts on the advisors' behavior. Absent such influence, the decision maker's reputation concerns could only harm. Indeed,

[3] A few papers provide a positive view of reputation concerns. Suurmond et al. (2004) present a model in which reputation concerns help to implement better decisions through their effect on information acquisition by the agent. Klein and Mylovanov (2014) show that reputation concerns may provide incentives for truthful reporting in a model of long-term dynamic interaction between the agent and the principal. Also, in Morris (2001), reputation concerns of an advisor may actually make the reporting behavior of a misaligned advisor less biased.

assume, that the advisors would report their signals truthfully with the same (positive) probability whenever asked, regardless of the decision maker's strategy. Then the abstention from advice-asking after receiving the signal confirming the prior would be of no help in inducing truthful reporting. Then, like in Levy (2004), the decision maker would excessively avoid advice-seeking in order to signal her smartness, and zero reputation concerns would thus be optimal.

Finally, we would like to note that our main results would arguably hold in an alternative setup in which advisors' reputation concerns are replaced with concerns about right decisions but acquisition and/or transmission of information is costly. Such a setup generates the same problem of "excessive asking" by the decision maker with a signal confirming the prior, for if the advisors believe that they face such a decision maker, they will lose incentives to acquire/transmit information. We elaborate on this more in the Conclusion section.

The rest of the paper is organized as follows. In Section 2 we set up the model. Section 3 analyzes the equilibria of the model. In Section 4 we examine the effects of the prior uncertainty about the state and their prior competence. Section 5 provides a numerical example. Section 6 concludes the paper.

2 The model

2.1 Players and information

There is a state of the world $\omega \in \{0,1\}$. A decision maker has to take a decision $d \in \{0,1\}$. The instrumental utility for the decision maker from the decision is 1 if the decision matches the state of the world and 0 otherwise. The decision maker receives a private signal $\sigma \in \{0,1\}$ about the state. Before taking her decision, she can, at no cost, consult N advisors, each of whom has also received a private signal $s_i \in \{0,1\}$, $i \in \{1,...,N\}$. Conditional on the state, all signals are independent.

The decision maker can be of two types, $\theta \in \{G,B\}$, which influence the precision of her signal. Specifically,

$$g := \Pr(\sigma = \omega|\theta = G) > b := \Pr(\sigma = \omega|\theta = B) \geq 1/2.$$

That is, the Good type of the decision maker receives a more informative signal than the Bad type.

Analogously, each advisor $i = 1,...,N$ can be of type $t_i \in \{H,L\}$, with the High type receiving a more informative signal than the Low type.

$$h := \Pr(s_i = \omega | t_i = H) > l := \Pr(s_i = \omega | t_i = L) \geq 1/2.$$

The types of all agents are independent of each other and of the state of the world. No agent knows his/her own type and types of others. There are common priors about the state of the world, the type of the decision maker, and the type of each advisor, namely:

$$p := \Pr(\omega = 0);\ \ q := \Pr(\theta = G);\ r := \Pr(t_i = H), \forall i = 1, ..., N.$$

Without loss of generality, we assume that $p \geq 1/2$.

We will call the decision maker "signal-type 0" when she has received signal $\sigma = 0$ and "signal-type 1" otherwise (not to confuse the private information of the decision maker with her unknown type θ.)

2.2 Sequence of the events and payoffs

The sequence of the events is as follows:

1. The nature draws the state ω and the competences of all players.
2. All players receive their private signals.
3. The decision maker decides whether to ask for advice or not. This is a binary choice $m \in \{m^0, m^1\}$, where m^0 and m^1 denote "not asking" and "asking" respectively. It is impossible to ask a subgroup of advisors: either all advisors are invited to provide advice or none. If the decision maker does not ask, the game proceeds to stage 5. If she asks, the game proceeds to the next stage.[4]
4. If asked, the advisors provide their advice publicly to the decision maker. Specifically, all advisors *simultaneously*[5] and *publicly* send binary *cheap-talk* messages $a_i \in \{0, 1\}$, $i \in \{1, ..., N\}$.
5. The decision maker takes a decision $d \in \{0, 1\}$.
6. The state is revealed and the players receive their payoffs.

The decision maker cares about matching her action with the state (instrumental objective). However, she would also like to appear informed (reputation concerns). We model the decision maker's reputational payoff as the posterior belief of an "external

[4]In principle, after asking the decision maker could also make a non-verifiable statement about her signal. At the end of Section 3.3.1, we will argue that such an option would not affect our results qualitatively.

In some real cases, it may be impossible to shut down advice-giving by simply not asking. Then, m^0 and m^1 can be interpreted as two non-verifiable statements about the signal before receiving advice. At the end of Section 3.3.1, we will argue that all our results would survive under this modification.

[5]The model can be extended to sequential advice, the qualitative results would remain the same.

observer", who observes the realized state, the decision, the whole profile of the advisors's messages (if they were asked for advice), and the decision maker's decision whether to ask for advice or not: $\Pr(G|m,a,d,\omega)$, where $a = (a_i)_{i=1}^N$ (to be omitted if the decision maker did not ask for advice).[6]

The decision maker's aggregate payoff is a convex combination of the instrumental and reputational objectives with weight $\rho \in [0,1]$ attached to reputation:

$$u_D(m,a,d,\omega) = (1-\rho)I(d,\omega) + \rho \Pr(G|m,a,d,\omega), \text{ where}$$

$$I(d,\omega) = \begin{cases} 1 \text{ if } d = \omega; \\ 0, \text{ if } d = 1-\omega. \end{cases}$$

For simplicity, we assume that advisors only have reputation concerns, i.e., an advisor's payoff is

$$u_i(m,a,d,\omega) = \Pr(H|m,a_i,\omega), \ \forall i = 1,...,N,$$

provided that the decision maker asked for advice.[7]

The values of reputation at different terminal nodes are computed in the Appendix.

Note that, in any equilibrium of the game, the *ex-ante* expected reputation of any player is equal to the prior belief about her/him, i.e., does not depend on a particular equilibrium. Thus, since the agents' payoffs are linear in reputation, the *ex-ante* welfare comparisons boil down to comparing the likelihoods of taking a correct decision.

2.3 Assumptions

We make the following *equilibrium selection* assumptions.

- A1 An advisor always reports the state that he considers more likely. When he considers both states equally likely, he reports his true signal.

- A2 The decision maker always takes the decision that corresponds to the state that she considers more likely. When she considers two states equally likely, she takes the decision that corresponds to her signal.

[6] The modeling assumption that the observer learns the advisors' suggestions is not a simplifying assumption, quite the contrary: it entails that the two signal-types of the decision maker can separate also through the decision. If the observer would not learn the suggestions, his opinion about the signal-type of the decision maker would be an average of the opinions he has under the different suggestions that induce, at least for one of the two signal-types, the observed decision. All our results would go through.

[7] If the decision maker did not ask for advice, an advisor's payoff is simply the prior belief r, but this will not play any role in the model.

A3 After observing a sequence of events that has probability 0 in equilibrium, the observer puts probability 1 on the signal-type that corresponds to the observed decision.

We will show in Section 3.1 that there always exists an equilibrium of the decision stage which is compatible with A2. Analogously, we will show in Section 3.2 that the advisors have no incentive to deviate from the behavior prescribed by A1.

A3 implies that the observer *strongly believes* (Battigalli and Siniscalchi, 2002) in A2. That is, if the unexpected decision corresponds to a state that only one of the two signal-types, at that point of the game, considers more likely, the observer believes that the decision has been taken by that signal type, *even if the observed asking or not asking move was supposed to be chosen only by the other signal-type.* A3 may seem rather restrictive, but we make it for simplicity. Weaker assumptions on off-the-path beliefs would not alter our qualitative results, but the exposition would get more complicated[8].

Finally, to avoid uninteresting cases, we introduce the following restrictions on the parameters.

A4 Signal-type 1, after the truthful report of only 0's, considers state 0 more likely, and after the truthful report of only 1's, considers state 1 more likely.

A5 Upon inferring that the decision maker has received signal 0, each advisor believes that state 0 is more likely *regardless of the own signal*; upon inferring that the decision maker has received signal 1, an advisor who received signal 1 believes that state 1 is more likely.

A4 allows to focus on the case where at least signal-type 1 can change her mind after the advices. A5 eliminates the trivial cases in which the advisors' opinions about which state is more likely are independent from what they infer about the decision maker's signal. The first part of A5 is true if

$$\Pr(\omega = 0|s_i = 1, \sigma = 0) = \frac{\Pr(s_i = 1|\omega = 0)\Pr(\omega = 0|\sigma = 0)}{num. + \Pr(s_i = 1|\omega = 1)\Pr(\omega = 1|\sigma = 0)} > 1/2 \Leftrightarrow$$

$$\Pr(s_i = 1|\omega = 0)\Pr(\omega = 0|\sigma = 0) > \Pr(s_i = 1|\omega = 1)\Pr(\omega = 1|\sigma = 0) \Leftrightarrow$$

$$\Pr(\omega = 0|\sigma = 0)\,(r(1-h) + (1-r)(1-l)) > (1 - \Pr(\omega = 0|\sigma = 0))\,(rh + (1-r)l) \Leftrightarrow$$

$$\Pr(\omega = 0|\sigma = 0) > rh + (1-r)l$$

[8]For example we could instead assume that after observing an out-of-equilibrium history of events ending with decision i, the observer puts probability 1 on signal-type i if the other signal-type considers state $j \neq i$ weakly more likely given the pre-decision history.

that is, the average advisors' signal precision cannot overturn the initial bias of the prior *plus* a signal 0 to the decision maker. Analogously, the second part of A5 is true if

$$\Pr(\omega = 0|\sigma = 1) < rh + (1-r)l$$

The probabilities of the states conditional on the decision maker's signal are computed in the Appendix.

3 Equilibrium analysis

3.1 The decision stage

Proceeding by backward induction, we start the equilibrium analysis from the final decision stage. As anticipated in Section 2.3, for any history of preceding events, A2 pins down one equilibrium of the decision stage.

If the two signal-types consider different states more likely, the prescribed decisions are clearly optimal in terms of expected instrumental utility, and in terms of expected reputation each signal-type prefers to be recognized as such rather than as the other one. If the two signal-types consider the same state more likely, pooling on the corresponding decision is sustained by the off-the-path beliefs pinned down by A3: after the opposite decision, each signal-type, beside a lower instrumental utility, expects also a lower reputation, since she would be recognized as the signal-type that corresponds to the less likely state.

Formally, the following lemma is true:

Lemma 1 *Consider an arbitrary history of events ψ prior to the decision stage (that is, ψ is either m^0 or (m^1, a)). Then, for any beliefs about the signal-types after history ψ, the behavior prescribed by A2 is a Bayesian equilibrium of the game that starts after ψ.*[9]
Proof. *See the Appendix.* ■

Apart from the considered equilibrium, there may exist other equilibria at the decision stage. However, assuming different equilibrium behavior at the decision stage would not change our qualitative results.

[9]The decision stage is a one-player game, where the player can be of two types. Still, the payoff of one type depends on the action that the other type would choose because the game has belief-dependent payoffs. For an in-depth analysis of games with belief-dependent payoffs, see Battigalli and Dufwenberg (2009).

3.2 The advising stage and its welfare consequences

Ex-ante, the advisors are indifferent among all kinds of behaviors: their expected reputation always coincides with the prior. As anticipated in Section 2.3, Assumption A1, we select the behavior under which the advisors always report the state that they consider more likely[10], and, when they consider the two states equally likely, report their signal. Using the arguments from the proof of Lemma 1, one can easily see that this is equilibrium behavior: the problem of an advisor is analogous to the decision stage problem of the decision maker who cares only about her reputation.

Thus, when the two signal-types of advisor consider different states more likely, we will say that the advisors *report truthfully.* When the two signal-types consider the same state more likely, we will say that the advisors *herd*[11] (on the corresponding message).

Given A5, upon inferring that the decision maker has received signal 1, truthful reporting occurs if and only if an advisor who received signal 0 believes that state 0 is more likely: $\Pr(\omega = 0|s_i = 0, \sigma = 1) > 1/2$. Similarly to the derivations following the statement of A5, one can easily show that this inequality is equivalent to

$$\Pr(\omega = 1|\sigma = 1) \leq rh + (1 - r)l. \tag{TR1}$$

Note that this condition is always satisfied if $\Pr(\omega = 1|\sigma = 1) \leq 1/2$. In general, calling $\overline{\omega}$ the more likely state conditional on asking, an advisor with the opposite signal still believes that the state corresponding to his signal is more likely if:

$$\Pr(\overline{\omega}|m^1) \leq rh + (1 - r)l. \tag{TR2}$$

Now, suppose that signal-type 1 always asks and signal-type 0 asks with the highest probability such that (TR2) is satisfied, so that the advisors report truthfully. If such probability is 1, the *first best* is realized: all information is always aggregated by the decision maker, who then takes the decision that corresponds to the state that emerges as more likely. If such probability is less than 1, we say that the *second best* is realized: the decision maker aggregates all information with the highest possible frequency under the incentive compatibility constraint of the advisors to report truthfully.

It is easy to see that signal-type 1 must indeed ask with probability 1 in the second

[10] When the two signal-types consider different states more likely, there is also a partially informative communication equilibrium, in which one of the signal-types randomizes between reporting his signal and lying (Ottaviani and Sørensen, 2001). Our qualitative results would remain intact if we assumed that the advisors play in this way.

[11] Equivalently, we could assume that they "babble" instead of herding. In either case, what matters is that their communication is totally uninformative. In general, when both signal-types of an advisor consider the same state more likely, equilibrium communication is necessarily uninformative (Ottaviani and Sørensen, 2001, Lemma 1).

best. If she did not, then efficiency could be improved in either of the two following ways without violating (TR2). If signal-type 0 was asking with probability below 1, then, obviously, the probabilities of asking by both signal-types could be increased in such a way that (TR2) remains satisfied. If signal-type 0 was already always asking, then $\overline{\omega} = 0$, and $\Pr(\overline{\omega}|m^1)$ can only be reduced by increasing the probability of asking by signal-type 1; thus, efficiency will improve and (TR2) will remain satisfied.

3.3 The choice between asking and not asking and overall equilibrium behavior

Before presenting our main propositions we formulate two auxiliary lemmas. The first one concerns the behavior of expected reputation for signal-type 0.

Lemma 2 *The expected reputation of signal-type* 0 *conditional on* $m = m^0, m^1$ *(i.e. conditional on not asking or asking) is:*

i) *for a fixed* $m = m^0, m^1$*, strictly increasing in* $\mu := \Pr(m|\sigma = 0)/\Pr(m|\sigma = 1)$ *when* $\mu \leq 1$*, and also when* $\mu > 1$ *if* $p > gq + b(1-q)$*.*

ii) *higher for* $m = m^0$ *than for* $m = m^1$ *when* $\Pr(m^1|\sigma = 1) = 1$*.*

Proof. *See the Appendix.* ■

The following lemma establishes an important "single crossing" result.

Lemma 3 *Consider a strategy of the decision maker such that:*

1. *given the asking/not asking behavior prescribed by this strategy, truthful reporting occurs after asking, i.e., (TR2) holds;*
2. *signal-type* 1 *always asks;*
3. *signal-type* 0 *weakly prefers to ask.*

Then signal-type 1 *strictly prefers to ask.*

Proof. *See the Appendix.* ■

Condition 2 of the lemma cannot be dispensed with. Consider the following situation. Both signal-types ask with probability 1/2; the only advisor reports truthfully; signal type 1 considers state 1 just slightly more likely when the advice is 1, so she always follows the advice, whereas signal-type 0 considers 0 more likely regardless of the advice, so she always decides 0. Signal-type 0 prefers to ask, because it allows her to distinguish herself through the decision when $a = 1$ instead of pooling on non-asking (in terms of the instrumental payoff, asking is irrelevant for her, as it cannot affect her decision). In contrast, signal-type 1 prefers to pool with signal-type 0 by not asking. This is because asking has almost no effect her instrumental payoff (as she is instrumentally almost indifferent between the two decision when $a = 1$), while reputationally she would like to pool rather than risk being distinguished after advice.[12]

3.3.1 Equilibria with information aggregation

First, we partition the space of parameters according to the following driver: which state does signal-type 1 consider more likely? From Equation (P) in the Appendix we get:

$$\Pr(\omega = 1|\sigma = 1) = \frac{g(1-p)q + b(1-p)(1-q)}{g(1-p)q + b(1-p)(1-q) + (1-g)pq + (1-b)p(1-q)}.$$

It is straightforward to show that $\Pr(\omega = 1|\sigma = 1) \geq 1/2$ if and only if:

$$gq + b(1-q) \geq p,$$

that is, the average signal precision has to be stronger than the bias.

If $\Pr(\omega = 1|\sigma = 1) < 1/2$, clearly the advisors will report truthfully a 0 even if they recognize signal-type 1 Then, truthful reporting of a 1 is guaranteed by A5. If $\Pr(\omega = 1|\sigma = 1) \geq 1/2$, we partition the space of parameters according to the following driver: do advisors report truthfully if they learn that the decision maker has received signal 1? This is true if Condition (TR1) is satisfied.

So, we have three cases.

Case 1. $gq + b(1-q) < p$ (implying $\Pr(\omega = 1|\sigma = 1) \leq hr + l(1-r)$);

Case 2. $gq + b(1-q) \geq p$ and $\Pr(\omega = 1|\sigma = 1) \leq hr + l(1-r)$;

Case 3. $\Pr(\omega = 1|\sigma = 1) > hr + l(1-r)$ (implying $gq + b(1-q) \geq p$).

We are interested in the existence of equilibria with at least some information aggregation, meaning that the decision maker sometimes asks for advice, and the advisors report truthfully. Three types of equilibria will be of primary importance for us:

- Pooling on asking: both signal-types always ask for advice;

- Separating: signal-type 0 never asks for advice, signal-type 1 always asks;

[12] Here we rely on our equilibrium selection at the decision stage. Of course, if, at the decision stage, signal-type 1 pooled with signal-type 0 by always taking $d = 0$, then she would be indifferent between asking and not asking.

- "Good"[13] partially separating: signal-type 0 randomizes between asking and not asking, signal-type 1 always asks.

In the "pooling on asking" equilibrium the first best is attained. However, it cannot exist when "pooling on asking" fails to induce truthful reporting (which will be the case when $p > hr + l(1-r)$). In such a case, our focus will be on the separating and, especially, the "good" partially separating equilibrium, because the latter implements the second best. Importantly, these two equilibria will generally exist only for an intermediate range of ρ; whereas equilibria arising for too high or too low values of ρ will result in poorer or no information aggregation (we will discuss them at the end of this subsection). Thus, generally, the ex-ante efficiency will be non-monotonic in ρ.

We start with the existence conditions for the separating and the "good" partially separating equilibria. The following result provides the main insight of the paper.

Proposition 1 *Suppose* $\Pr(\omega = 1|\sigma = 1) \leq hr + l(1-r)$ *(Cases 1 and 2). Then, a separating equilibrium in which signal-type* 0 *never asks for advice and signal-type* 1 *always asks for advice exists if and only if* $\rho \in [\underline{\rho}, \overline{\rho}]$*, with* $\underline{\rho} \in [0, 1)$*, and* $\overline{\rho} \in (\underline{\rho}, 1)$ *when* $gq + b(1-q) < p$*,* $\overline{\rho} = 1$ *when* $gq + b(1-q) \geq p$*. Moreover, a partially separating equilibrium in which signal-type* 0 *asks for advice with probability* $\mu > 0$ *and signal-type* 1 *always asks exists if and only if* $\rho \in [\underline{\rho}, \widehat{\rho}]$*, where* $\widehat{\rho} \in [\underline{\rho}, 1)$*. In both equilibria the advisors report truthfully, and in the partially separating equilibrium at* $\widehat{\rho}$*, if* $p > hr + l(1-r)$ *the second best is attained (else the first best is attained, i.e.,* $\mu = 1$*).*

Proof. Take a candidate separating equilibrium in which signal-type 1 always asks and signal-type 0 never asks.

It is easy to observe that the difference in expected reputation between asking and not asking is negative for signal-type 0 and, in Case 1, signal-type 1,[14] whereas it is zero for signal-type 1 in Case 2.[15] By truthful reporting after asking, the difference in the expected instrumental payoff between asking and not asking is non negative for signal-type 0 and, by A4, positive for signal-type 1.[16]

Hence, the difference in the expected utility between not asking and asking is strictly increasing in ρ for both signal-types. For $\rho = 0$, signal-type 0 prefers to ask and signal-type 1 strictly prefers to ask. For $\rho = 1$, signal-type 0 strictly prefers not to ask and

[13] We call it "good" because other partially separating equilibria that may exist result in lower information aggregation (we will call them "bad" thereafter).

[14] A signal-type prefers to be recognized as the signal-type that corresponds to the state that she considers more likely rather than as the opposite signal-type. For the formalization of this argument, see the proof of Lemma 1 in the Appendix.

[15] In Case 2, after not asking signal-type 1 decides 1, so by A3 she is recognized as signal-type 1, just like after asking.

[16] This is because, by A4, advisors' information is decision-relevant for signal-type 1 with a positive probability. See the proof of Lemma 3 for the formal argument.

signal-type 1 strictly prefers not to ask in Case 1 and is indifferent in Case 2. Thus, each signal-type is indifferent in the candidate separating equilibrium only for one value of ρ. Let $\underline{\rho}$ be the value at which signal-type 0 is indifferent and let $\overline{\rho}$ be the value at which signal-type 1 is indifferent. By Lemma 3, at $\underline{\rho}$ signal-type 1 strictly prefers to ask. Thus $\overline{\rho} > \underline{\rho}$ ($\overline{\rho} = 1$ in Case 2) and at $\overline{\rho}$ signal-type 0 strictly prefers not to ask.

Thus the separating equilibrium exists if and only if $\rho \in [\underline{\rho}, \overline{\rho}]$

Consider now the partially separating equilibrium of the lemma. For $\rho < \underline{\rho}$, no such equilibrium can exist: since signal-type 0 prefers to ask with $\Pr(m^1|\sigma = 0) = 0$, by Lemma 2 (part (i)) a fortiori she strictly prefers to ask when $\Pr(m^1|\sigma = 0) > 0$.

For $\rho = 1$, by Lemma 2 (part (ii)), signal-type 0 strictly prefers not to ask for any value of $\Pr(m^1|\sigma = 0)$. Thus, for any given $\Pr(m^1|\sigma = 0)$ there must be a value of ρ between $\underline{\rho}$ and 1 such that signal-type 0 is indifferent between asking and not asking. Let $\widehat{\rho} < 1$ be such value for the *maximum* $\Pr(m^1|\sigma = 0)$ such that the advisors report truthfully. By Lemma 2 (part (i)), for every $\rho \in [\underline{\rho}, \widehat{\rho})$ there exists a lower value of $\Pr(m^1|\sigma = 0)$ such that signal-type 0 is indifferent between asking and not asking, and the advisors report truthfully. In contrast, any $\rho > \widehat{\rho}$ (if $\widehat{\rho} < 1$) would require a higher value of $\Pr(m^1|\sigma = 0)$ for signal-type 0 to be indifferent, but this would be incompatible with the advisors' truthtelling. By Lemma 3, signal-type 1 strictly prefers to ask, thus she will not deviate. ■

Note that $\widehat{\rho}$ can be smaller or larger than $\overline{\rho}$. Note also that when signal-type 0 cannot change her mind after advices, she has no instrumental gain from asking and, thus, $\underline{\rho} = \widehat{\rho} = 0$.

In Case 3, for separating and partially separating equilibrium in which signal-type 1 always asks the following holds.

Proposition 2 *Suppose* $\Pr(\omega = 1|\sigma = 1) > hr + l(1-r)$ *(Case 3). There exists a separating equilibrium for every value of* ρ *but it does not trigger truthful reporting. There exists a partially separating equilibrium in which signal-type* 0 *asks for advice with probability* $\mu > 0$ *and signal-type* 1 *always asks if and only if* $\rho \in [\underline{\widehat{\rho}}, \widehat{\rho}]$*, where* $\underline{\widehat{\rho}}, \widehat{\rho} \in (0, 1)$*. In the partially separating equilibrium the advisors report truthfully and at* $\widehat{\rho}$*, if* $p > hr + l(1-r)$ *the second best is attained (else the first best is attained, i.e.,* $\mu = 1$*).*

Proof. It is straightforward to observe that the candidate separating equilibrium is always an equilibrium: since $\Pr(\omega = 1|\sigma = 1) > hr + l(1 - r)$, the advisors herd, hence there is no difference in expected instrumental utility between asking and not asking, and in terms of expected reputation, since $\Pr(\omega = 1|\sigma = 1) > 1/2$, both signal-types prefer to be recognized as such rather than as the other one.

For the partially separating equilibrium, let $\underline{\mu}$ be the minimum value of $\Pr(m^1|\sigma = 0)$ such that the advisors report truthfully when $\Pr(m^1|\sigma = 1) = 1$. It exists because by $\Pr(\omega = 0|\sigma = 1) < 1/2$ and $p \geq 1/2$, $\Pr(\omega = 0|m^1) = 1/2 = \Pr(\omega = 1|m^1)$ for some $\Pr(m^1|\sigma = 0) < 1$ when $\Pr(m^1|\sigma = 1) = 1$. The partially separating equilibrium kicks in at $\underline{\widehat{\rho}}$ such that signal-type 0 is indifferent between not asking and asking given that $\Pr(m^1|\sigma = 0) = \underline{\mu}$. It exists up to $\widehat{\rho}$ defined like in Case 1-2 and optimal in the same sense. ■

The intuition behind Propositions 1 and 2 is as follows. The decision maker who received the signal confirming the prior (signal-type 0) has a strong reputational incentive to convey this news to the advisors. At the same time, her need for extra information is low, because she is already quite confident about the state. In contrast, the decision maker who received the signal contradicting the prior (signal-type 1) has either a weaker reputational incentive to lie (when the signal is weaker than the prior – Case 1) or a weaker reputational incentive to actually reveal her true signal (when the signal is stronger than the prior – Cases 2 and 3). At the same time, such decision maker cares more about information aggregation, because the signal contradicting the prior makes her more uncertain about the state, compared to the signal confirming the prior.

Thus, whenever the weight on reputation in the decision maker's utility is big enough, the separation (either full or partial) of the signal-types of the decision maker becomes possible in equilibrium.

When pooling on asking triggers truthful reporting, the first best can be implemented in a pooling equilibrium up to precisely $\widehat{\rho}$. Note indeed that if pooling triggers truthful reporting, the partially separating equilibrium at $\widehat{\rho}$ coincides with the pooling one with weak incentive to ask for signal-type 0.

Proposition 3 *If $p \leq hr + l(1 - r)$ a pooling equilibrium in which both signal-types always ask for advice and the advisors report truthfully exists if and only if $\rho \in [0, \widehat{\rho}]$. If $p > hr + l(1 - r)$ such an equilibrium does not exist.*

Proof. For $\rho = \widehat{\rho}$, if $p \leq hr + l(1 - r)$, by Proposition 1 for Case 1-2 and by Proposition 2 for Case 3, there exists a partially separating equilibrium with $\Pr(m^1|\sigma = 0) = 1$ in which signal-type 0 is indifferent between asking and not asking. Thus, the partially separating equilibrium is a pooling equilibrium, in which by Lemma 2 (part (ii)) the expected reputation of signal-type 0 after not asking, under A3, is higher than after asking. Thus, for $\rho > \widehat{\rho}$ signal-type 0 strictly prefers not to ask and for $\rho < \widehat{\rho}$ she strictly prefers to ask, and by Lemma 3 signal-type 1 too. ■

Apart from the three described equilibria, there may exist other equilibria with information aggregation. To begin with, an equilibrium in which only signal-type 0 asks with a positive probability and the advisors report truthfully does not exist due to A5. There may exist, however, the following equilibria:

- "Bad" partially separating I: signal-type 0 never asks for advice, signal-type 1 randomizes between asking and not asking;
- "Bad" partially separating II: signal-type 0 always asks for advice, signal-type 1 randomizes between asking and not asking;
- "Fully mixed" equilibrium: both signal-types randomize between asking and not asking.

We will prove in Proposition 4 that none of these equilibria exist for $\rho < \underline{\rho}$ in Cases 1 and 2, and for $\rho < \underline{\widehat{\rho}}$ in Case 3. Then, it is easy to observe that each of these equilibria, (i) if it exists for $\rho \leq \widehat{\rho}$, is ex-ante strictly worse than the pooling equilibrium on asking or the "good" partially separating one for the same ρ, and (ii) if it exists for $\rho > \widehat{\rho}$, is ex-ante strictly worse than the pooling equilibrium on asking or the "good" partially separating one arising at $\rho = \widehat{\rho}$.

To see (ii), just notice that any profile of strategies in which signal-type 1 asks with probability less than one is strictly worse than the second best (see Section 3.2).

To see (i), let us consider the three "bad" equilibria one by one.

First, take a "bad" partially separating equilibrium of type I. With respect to this equilibrium, both signal-types ask with a non lower probability in the "good" partially separating equilibrium for any $\rho \in [\underline{\rho}, \widehat{\rho}]$ in Cases 1 and 2 and for any $\rho \in [\underline{\widehat{\rho}}, \widehat{\rho}]$ in Case 3 (as well as in the separating equilibrium, existing for $\rho \in [\underline{\rho}, \overline{\rho}]$ in Cases 1 and 2).

Second, take a "bad" partially separating equilibrium of type II. Signal-type 0 asks with higher probability than signal-type 1. If this still triggers truthful reporting by the advisors, then pooling on asking triggers truthful reporting by the advisors too ((TR2) is a fortiori satisfied), and it is an equilibrium for any $\rho \leq \widehat{\rho}$.

Finally, take a "fully mixed" equilibrium. If signal-type 0 asks more frequently than signal-type 1, then pooling on asking must trigger truthful reporting too, and it is an equilibrium for any $\rho \leq \widehat{\rho}$. If signal-type 0 asks less frequently than signal-type 1, for any $\rho \in [\underline{\rho}, \widehat{\rho}]$ in Cases 1 and 2 and for any $\rho \in [\underline{\widehat{\rho}}, \widehat{\rho}]$ in Case 3, the "fully mixed" equilibrium is worse than the "good" partially separating equilibrium, for the following reason. In order not to be inferior to the "good" partially separating equilibrium, the "fully mixed" equilibrium must yield a higher probability of asking by signal-type 0. This, coupled with a lower than 1 probability of asking by signal-type 1, implies by Lemma 2 (part (i)) that the expected reputation of signal-type 0 after asking is higher than in the "good" partially separating equilibrium. After not asking, if signal-type 1 considers state 1 more likely and hence decides 1, the expected reputation of signal-type 0 is the same in the two equilibria. Else, we have $p \geq gq + b(1-q)$, so by Lemma 2 (part (i)) the expected reputation of signal-type 0 after not asking is higher in the "good" partially separating equilibrium. Hence, in both cases, in the "fully mixed equilibrium" signal-type 0 would strictly prefer to ask, a contradiction.

Remark on declarations after asking. Given the selected behavior of the advisors, we can argue that modifying the game by allowing the decision maker to make a declaration about her signal after asking would change substantially nothing in the model.

Consider an equilibrium of the modified game in which both signal-types ask with positive probability, make different and informative declarations δ and δ', and, to make the case interesting, at least one declaration, say δ, triggers truthful reporting. Call α the relative probability that signal-type 0 asks and makes declaration δ, i.e. $\alpha := \Pr(m^1, \delta|\sigma = 0)/\Pr(m^1, \delta|\sigma = 1)$.

First, suppose that $\alpha \leq 1$ and δ' triggers herding. If δ' and not asking are played with the same relative probability by the two signal-types, *or* the two signal-types consider different states more likely (so that after δ they separate with the decision), δ' can obviously be eliminated and substituted with not asking. Else, by Lemma 2 (part (i)), signal-type 0, between δ' and not asking, will strictly prefer and play only the one that she plays relatively more often, say δ'. Clearly, signal-type 1 will imitate signal-type 0. Thus, δ' can be eliminated and substituted with not asking.

Second, suppose that $\alpha \leq 1$, δ' triggers truthful reporting, and $\Pr(m^1, \delta'|\sigma = 0)/\Pr(m^1, \delta'|\sigma = 1) \leq 1$ too. Hence, signal-type 0 does not always ask. By Lemma 2 (part (i)), signal-type 0 strictly prefers and makes only one of the two declarations, say δ. Then, by analogous argument, signal-type 1 would strictly prefer δ to δ' if she would consider state 0 more likely. Since sometimes she declares δ', it must be that she considers state 1 more likely. Hence, signal-type 0 is indifferent between δ and not asking when after asking she is recognized. But then, since expected reputation depends only on relative probabilities, there also exists (and aggregates more information) our "good" partially separating equilibrium, where signal-type 0 asks with frequency α.

Third, suppose that $\alpha > 1$. Then, also pooling on asking triggers truthful reporting and can be implemented in equilibrium without declarations for all $\rho \leq \widehat{\rho}$. The original equilibrium using δ and δ' can exist also above $\widehat{\rho}$, but not up to $\rho = 1$, by the incentives of signal-type 0 as formalized by Lemma 2 (part (i)). So it is true that under some restrictive conditions on the parameters, the declarations extend the implementation of the first best above $\widehat{\rho}$. However, as shown, the introduction of the declarations would not affect at all our results for the intermediate values of ρ we are interested in, and it would only confirm the message that intermediate values ρ are generally optimal, while too high or too low values of ρ harm information aggregation.

Remark on substituting "asking" and "not asking" with declarations. In some real-life contexts, it could be impossible to prevent an advisor from expressing his opinion by not asking. In such cases, "not asking" essentially becomes unfeasible, and m^0 and m^1 should be interpreted as two non-verifiable statements about the signal prior to receiving advice. Such a modification would not affect our results. First, all the "good" equilibria of our model would survive. To see this, simply notice that in any of these equilibria non-asking is played only by signal-type 0. Then, we can substitute non-asking with message m^0 (and asking – with message m^1) without any effect, because, due to A5, the advisors will herd after m^0. Second, any novel equilibrium that could appear would have exactly the same features as pooling on asking with subsequent declarations δ and δ' in the game with declarations after asking – simply replace δ and δ' with m^0 and m^1. So, the argument and the conclusions of the remark on declarations after asking apply here as well.

3.3.2 General picture and the effect of reputation concerns

Consider first $p \leq hr + l(1-r)$. The "pooling on asking" exists and, thus, the first best can be implemented in equilibrium, if and only if $\rho \in [0, \widehat{\rho}]$. Any equilibrium existing for $\rho > \widehat{\rho}$ is obviously inferior. Thus, for $p \leq hr + l(1-r)$, we reach the familiar from the literature conclusion that too high reputation concerns hamper efficient decision making.

Consider now $p > hr + l(1-r)$. For $\rho > \widehat{\rho}$ the second best cannot be implemented anymore; thus the conclusion is qualitatively the same as in the case when $p \leq hr + l(1-r)$: too high reputation concerns are harmful. However, for low ρ the picture changes drastically. Specifically, the following proposition is true:

Proposition 4 *Assume $p > hr + l(1-r)$. Then, for $\rho < \underline{\rho}$ in Cases 1 and 2, and for $\rho < \underline{\widehat{\rho}}$ in Case 3, there exists no equilibrium with any information aggregation.*

Proof. Consider first Cases 1 and 2 and assume there is such an equilibrium for some $\rho < \underline{\rho}$. Then, it must be that signal-type 0 always asks, because, by Proposition 1, when $\rho < \underline{\rho}$, signal-type 0 prefers asking even when the reputational loss from asking, by Lemma 2, is the largest (i.e., when the observer believes that the two signal-types always separate). But then $\Pr(\overline{\omega}|m^1) \geq p > rh + (1-r)l$, implying no truthful reporting by the advisors.

Consider now Case 3 and assume there is an equilibrium with some information aggregation for some $\rho < \underline{\widehat{\rho}}$. From the proof of Proposition 2, it is clear that the ratio $\Pr(m^1|\sigma = 0)/\Pr(m^1|\sigma = 1)$ must be at least $\underline{\mu}$ in order to induce truthtelling by the advisors. At the same time, for $\widehat{\underline{\rho}}$, signal-type 0 is indifferent between asking and not asking when this ratio is exactly $\underline{\mu}$ and, by Lemma 2 (part (i)), would strictly prefer asking if $\Pr(m^1|\sigma=0)/\Pr(m^1|\sigma=1) > \underline{\mu}$. Since the difference in expected intrumental utility between asking and not asking is positive, for $\rho < \widehat{\underline{\rho}}$ and $\Pr(m^1|\sigma=0)/\Pr(m^1|\sigma=1) > \underline{\mu}$ she would strictly prefer asking as well. ■

Thus, *when the prior is sufficiently strong ($p > hr + l(1-r)$), too low reputation concerns are unambiguously bad* as they result in a complete failure of information aggregation. The intuition is simple: when the decision makers cares little about her reputation, she is tempted to ask for advice regardless of her signal. However, the advisors then have no incentives to report truthfully, as they keep believing strongly in the state suggested by the prior.

Given the negative effect of crossing $\widehat{\rho}$, our overall analysis suggests that the effect of the decision maker's reputation concerns on information aggregation is generally non-monotonic. Both too high and too low reputation concerns are detrimental for information aggregation. Too low reputation concerns provoke excessive advice-seeking, which undermines the advisors' reporting incentives. Too high reputation concerns result in excessive advice avoidance.

4 Comparative statics

In this section we perform the analysis of the impact of the prior uncertainty about the state of nature and the prior competence of the advisors on the effects of reputation concerns. We start from the effect of the uncertainty on $\underline{\rho}$.

Proposition 5 *$\underline{\rho}$ is decreasing in p, that is, it is increasing in the prior uncertainty.*
Proof. *See the Appendix.* ■

The intuition behind this result is as follows. Recall that $\underline{\rho}$ is determined by the incentive compatibility constraint of signal-type 0 under full separation. As p goes down, advisors' information becomes more valuable for signal-type 0, while her expected reputation payoff from revealing her signal relative to pooling with signal-type 1 diminishes, because she is less sure that the state is 0. Thus, a higher minimum weight of reputation, $\underline{\rho}$, is needed to make signal-type 0 abstain from asking for advice.

Now let us move to the effects of p on $\widehat{\rho}$ and $\widehat{\underline{\rho}}$.

Proposition 6 *Both $\widehat{\underline{\rho}}$ and $\widehat{\rho}$ are decreasing in p, that is, they are increasing in the prior uncertainty.*
Proof. *See the Appendix.* ■

Like at $\underline{\rho}$, at both $\underline{\widehat{\rho}}$ and $\widehat{\rho}$ signal-type 0 is indifferent between asking and not asking. Thus, for given μ, the intuition is the same as for Proposition 5: as the uncertainty rises, the weight of reputation needs to be increased in order to keep signal-type 0 indifferent. In addition, when p moves towards $1/2$, the posterior conditional on asking also decreases for given μ. Consequently, the maximum and the minimum μ under which the advisors report truthfully, corresponding to $\widehat{\rho}$ and $\underline{\widehat{\rho}}$ respectively, go up. In order to support a higher μ in equilibrium, ρ needs to be raised even further, because an increase in μ decreases the relative reputational benefit from not asking.

Notice also that by lowering p we move from Cases 1 and 2 to Case 3 at some point, meaning that the lower bound on ρ switches from $\underline{\rho}$ to $\underline{\widehat{\rho}}$. However, it is easy to observe that $\underline{\widehat{\rho}} > \underline{\rho}$, because a higher weight of reputation is needed to keep signal-type 0 indifferent when μ is positive rather than 0. Thus, the switch does not break the monotonicity of the change in the lower bound on ρ.

It should be noted that considering the lower bounds on the reputation concerns, $\underline{\rho}$ and $\underline{\widehat{\rho}}$, is relevant only when $p > hr + l(1-r)$. When $p \leq hr + l(1-r)$, by proposition 3, the equilibrium with pooling on asking exists for any $\rho \in [0, \widehat{\rho}]$, so ρ below $\underline{\rho}$ or $\underline{\widehat{\rho}}$ is not detrimental.

The implications of the above analysis can be summarized as follows:

Corollary 1 *When the prior uncertainty is sufficiently low, $p > hr + l(1-r)$, greater prior uncertainty calls for higher reputation concerns, as both $\underline{\rho}$ (or $\underline{\widehat{\rho}}$) and $\widehat{\rho}$ rise. When the prior uncertainty is high enough, $p \leq hr + l(1-r)$, reputation concerns do not matter unless they are too high (above $\widehat{\rho}$), with the upper bound increasing in the prior uncertainty.*

Higher prior uncertainty increases the decision maker's incentives to ask for advice even when decision maker's signal confirms the prior. A higher weight on reputation is then needed to restrain this temptation. However, when the prior uncertainty becomes so high that truthtelling by the advisors arises even when the decision maker *always* asks for advice, restraining advice-asking is not needed anymore, and any weight on reputation from 0 up to a certain value becomes optimal.

The implication of this result is that at times of low uncertainty, i.e., when the organization is performing well and seems to be on the right track, it needs a leader concerned with her reputation, but not too strongly. When the uncertainty about the right strategy is large but not extremely large, the best leader is the one with rather strong reputation concerns. Finally, in the situations of a very high uncertainty (i.e., when the organization faces important and non-obvious strategic choices), the reputation concerns of the leader are actually irrelevant (unless they are too extreme) for two reasons: (1) the advice-seeking incentives are so strong that reputation concerns cannot undermine them, (2) there is no problem of "excessive asking", as the advisors are willing to tell the truth when they believe that the decision maker always asks for advice.

Let us consider the effect of the prior competence of advisors, i.e., $hr+l(1-r)$. *Prima facie*, it seems that the arguments we applied for the analysis of the prior uncertainty work here as well. Indeed, higher advisors' competence raises the instrumental payoff from asking, which makes asking more attractive for signal-type 0 and, thus, should push the thresholds on ρ upwards. In addition, it allows for higher μ to be compatible with truthelling by advisors, which should also work towards increasing $\widehat{\rho}$. Yet, there are complications.

First, in contrast to decreasing p, an increase in the advisors' competence improves their truthtelling incentives for *any* beliefs formed after being asked: not just when signal-type 0 asks "too often", but also when signal-type 0 asks "too rarely" (given that signal-type 1 always asks). This means, that, in Case 3, the lowest μ compatible with the advisors' truthelling *decreases* rather than increases, which works towards decreasing $\underline{\widehat{\rho}}$.

More troublesome, the expected reputational payoff of signal-type 0 from asking is likely to *decrease*. This is because, with higher advisors' competence, there is a lower chance for signal-type 0 to separate and reveal her signal *after* asking when the profile of advises coupled with the prior still favors 0. For instance, consider a profile of advises with more 0s than 1s and such that signal-type 1 still considers state 1 more likely after receiving this advice. Then, the two signal-types separate with the decisions after observing such a profile. Reputation-wise this is good for signal-type 0, because she prefers revealing her signal to pooling with signal-type 1. However, when the competence of advisors rises, the same profile of advices eventually makes signal-type 1 believe that state 0 is more likely and switch to decision 0. This kills the possibility for signal-type 0 to reveal her signal and leads to a discrete drop in her reputation.

The ultimate effect on the thresholds is unclear then (for given μ). For each specific change in the advisors' competence, the ultimate answer will depend on what change, caused by switching from not asking to asking, is larger: an increase in the instrumental utility of signal-type 0 or a fall in her expected reputational payoff.

Nevertheless, it is rather clear that an increase in the prior competence of advisors *may worsen* the quality of decisions for given ρ. To show this, it is sufficient to consider Case 1 under the assumption that $p > hr + l(1-r)$ (so that pooling on asking with subsequent truthful reporting is impossible) and show that $\underline{\rho}$ can increase, which would mean widening the zone with no information aggregation (see Proposition 4). As an example, consider the following setup: (1) there are two advisors, (2) signal-type 1 considers state 0 more likely prior to advices, (3) the profile of advices $(1,1)$ makes signal-type 0 believe that state 1 is more likely. Then the two signal-types pool with their decisions after asking for any profile of advices: after $(0,0)$ or $(0,1)$, both signal-types take decision 0, whereas after $(1,1)$ they both take decision 1. Now consider $\rho = \underline{\rho}$ and raise the competence of advisors. The expected instrumental utility of signal-type 0 after asking clearly increases (this can be formally derived looking at ΔIU_0 from the proof of Lemma 3 in the Appendix). At the same time, her expected reputation after asking does not change, because the two signal-types still pool on the same decision all the time. Hence, $\underline{\rho}$ should go up in order to keep signal-type 0 indifferent between asking and not asking. If, for given ρ, this results in $\underline{\rho}$ crossing ρ from below, efficiency drops as no information aggregation is possible below $\underline{\rho}$.

Thus, higher advisors' competence may be detrimental to efficiency because it may produce "excessive asking", thereby killing advisors' truthtelling altogether. The following proposition can thus be formulated:

Proposition 7 *For given reputation concerns, when $p > hr + l(1-r)$ and the reputation concerns are not sufficiently strong, greater prior competence of advisors (i.e., higher $hr + l(1-r)$) can* ***worsen*** *the quality of decisions.*

5 Numerical example

Fix the following values of the parameters:

$$q = r = \frac{1}{2};\ g = h = \frac{7}{9};\ b = l = \frac{5}{9};\ n = 3.$$

We leave the prior uncertainty p free to study how it influences the effect of reputation concerns on information aggregation. Note that the average signal precision, i.e. the ex-ante probability that a state generates the corresponding signal, is the same for the decision maker and for the advisors (2/3). This has two implications. First, if both signal-types of the decision maker always ask, each signal-type of the advisor has the same posterior over the state of the world as the decision maker of the same signal-type. Second, the posterior over the state of the world of the decision maker depends only on the total number of signals of each kind that she learns, *including her own.* Note that this is not a "limit case", in the sense that whether the decision maker is better informed than the advisors or not does not determine per se any qualitative difference in the results.

First, we compute the decision maker and the advisors beliefs as a function of p. We can use the average signals precision (2/3) as a deterministic signal precision (see, for instance, Equation (P) in the Appendix). Denote by $o(s)$ the number of 0's in a profile of advices s. Then we have:

$$\Pr(\omega = 0|\sigma = 0) = \frac{2p}{p+1} = \Pr(\omega = 0|s_i = 0);$$
$$\Pr(\omega = 0|\sigma = 1) = \frac{p}{2-p} = \Pr(\omega = 0|s_i = 1);$$
$$\Pr(\omega = 0|\sigma = 0, s) = \frac{(\frac{2}{3})^{o(s)+1}(\frac{1}{3})^{3-o(s)}p}{(\frac{2}{3})^{o(s)+1}(\frac{1}{3})^{3-o(s)}p + (\frac{1}{3})^{o(s)+1}(\frac{2}{3})^{3-o(s)}(1-p)} = \begin{cases} \frac{16p}{1+15p} & \text{if } o(s) = 3 \\ \frac{4p}{1+3p} & \text{if } o(s) = 2 \\ p & \text{if } o(s) = 1 \\ \frac{p}{4-3p} & \text{if } o(s) = 0 \end{cases}$$
$$\Pr(\omega = 0|\sigma = 1, s) = \begin{cases} \frac{4p}{1+3p} & \text{if } o(s) = 3 \\ p & \text{if } o(s) = 2 \\ \frac{p}{4-3p} & \text{if } o(s) = 1 \\ \frac{p}{16-15p} & \text{if } o(s) = 0 \end{cases}$$
$$\Pr(\omega = 0|m^1) = \frac{(2\Pr(m^1|\sigma = 0) + \Pr(m^1|\sigma = 1))p}{(2\Pr(m^1|\sigma = 0) + \Pr(m^1|\sigma = 1))p + (2\Pr(m^1|\sigma = 1) + \Pr(m^1|\sigma = 0))(1-p)}.$$

As p changes, we have the following situations.

- $p > \frac{16}{17}$. Then $\Pr(\omega = 0|\sigma = 1, s) > \frac{1}{2}$ for $o(s) = 0$. This case contradicts A4.(second part) and thus it is not analyzed.

- $\frac{4}{5} < p \leq \frac{16}{17}$. Then, for $\Pr(m^1|\sigma = 0) = 0$ and $\Pr(m^1|\sigma = 1) > 0$, $\Pr(\omega = 0|m^1) > hr + l(1-r)$, thus the advisors never report truthfully. This case contradicts A5 (second part) and thus it is not analyzed.

- $\frac{2}{3} < p \leq \frac{4}{5}$. Then $\Pr(\omega = 0|\sigma = 1) = \Pr(\omega = 0|s_i = 1) > \frac{1}{2}$. This is Case 1; moreover the advisors herd in case of pooling on asking. Note also that $\Pr(\omega = 0|\sigma = 0, s) \leq 1/2$ if $o(s) = 0$. Hence, unless $p = \frac{4}{5}$, signal-type 0 changes her mind if all the advisors suggest 1. Signal-type 1, instead, follows the majority of the advisors.

- $\frac{1}{2} < p \leq \frac{2}{3}$. Then $\Pr(\omega = 0|\sigma = 1) = \Pr(\omega = 0|s_i = 1) \leq \frac{1}{2}$. This is Case 2; moreover the advisors report truthfully in case of pooling on asking. The reactions of the decision maker to the advices are the same as in the previous case.

For no value of p we fall in Case 3, for which it is necessary (but not sufficient) that the advisors' signals have worse average precision than the decision maker's one.

So, we consider only Case 1 ($\frac{2}{3} < p \leq \frac{4}{5}$) and Case 2 ($\frac{1}{2} < p \leq \frac{2}{3}$).

Both signal-types of the decision maker react to the advisors' suggestions in the same way in the two cases. Moreover, signal-type 0 always decides 0 after not asking. Thus, we can compute all values of instrumental utility and reputation in the same way for both cases, except for signal-type 1 when she does not ask.

The expected instrumental utility for signal-type 0 after not asking is $\Pr(\omega = 0|\sigma = 0) = \frac{2p}{p+1}$ and for signal-type 1 it is $\Pr(\omega = 0|\sigma = 1) = \frac{p}{2-p}$ in Case 1 and $\Pr(\omega = 1|\sigma = 1) = \frac{2-2p}{2-p}$ in Case 2. After asking, the expected instrumental utility for signal-type 0 is

$$\begin{aligned}
&\textstyle\sum_{s:o(s)\geq 1} \Pr(\omega = 0, s|\sigma = 0) + \Pr(\omega = 1, s = (1,1,1)|\sigma = 0) = \\
&= \textstyle\sum_{s:o(s)\geq 1} \Pr(s|\omega = 0)\Pr(\omega = 0|\sigma = 0) + \Pr(s = (1,1,1)|\omega = 1)\Pr(\omega = 1|\sigma = 0) = \\
&= (1 - \frac{1}{3^3})\frac{2p}{p+1} + \frac{2^3}{3^3}(1 - \frac{2p}{p+1}) = \frac{2}{3}\frac{2p}{p+1} + \frac{8}{27} = \frac{44p+8}{27p+27};
\end{aligned}$$

and for signal-type 1 it is

$$\begin{aligned}
&\textstyle\sum_{s:o(s)\geq 2} \Pr(\omega = 0, s|\sigma = 1) + \sum_{s:o(s)<2} \Pr(\omega = 1, s|\sigma = 1) = \\
&= \textstyle\sum_{s:o(s)\geq 2} \Pr(s|\omega = 0)\Pr(\omega = 0|\sigma = 1) + \sum_{s:o(s)<2} \Pr(s|\omega = 1)\Pr(\omega = 1|\sigma = 1) = \\
&= (\frac{2^3}{3^3} + 3\cdot\frac{1}{3}\cdot\frac{2^2}{3^2})\frac{p}{2-p} + (\frac{2^3}{3^3} + 3\cdot\frac{1}{3}\cdot\frac{2^2}{3^2})(1 - \frac{p}{2-p}) = (\frac{2^3}{3^3} + 3\cdot\frac{1}{3}\cdot\frac{2^2}{3^2}) = \frac{20}{27}.
\end{aligned}$$

Suppose now that signal-type 1 always asks and signal-type 0 asks with probability μ. Then, after not asking, the advisors believe that the decision maker has received signal 0 after decision 0 (by equilibrium strategy or A3) and signal 1 after decision 1 (by A3). Using the same notation as in the Appendix ($x := \Pr(G|\sigma = \omega)$, $y := \Pr(G|\sigma \neq \omega)$), the expected reputation for signal-type 0 after not asking is:

$$\begin{aligned}
&\Pr(\omega = 0|\sigma = 0)x + \Pr(\omega = 1|\sigma = 0)y = \\
&\quad = \frac{2p}{p+1}\cdot\frac{7}{12} + (1 - \frac{2p}{p+1})\cdot\frac{1}{3} = \frac{1}{4}\frac{2p}{p+1} + \frac{1}{3} = \frac{5p+2}{6p+6};
\end{aligned}$$

and for signal-type 1, in Case 1 (where she optimally decides 0), it is:

$$\begin{aligned}
&\Pr(\omega = 0|\sigma = 1)x + \Pr(\omega = 1|\sigma = 1)y = \\
&\quad = \frac{p}{2-p}\cdot\frac{7}{12} + (1 - \frac{p}{2-p})\cdot\frac{1}{3} = \frac{1}{4}\frac{p}{2-p} + \frac{1}{3} = \frac{8-p}{24-12p}.
\end{aligned}$$

and in Case 2 (where she optimally decides 1), it is:

$$\begin{aligned}
&\Pr(\omega = 0|\sigma = 1)y + \Pr(\omega = 1|\sigma = 1)x = \\
&\quad = \frac{p}{2-p}\cdot\frac{1}{3} + (1 - \frac{p}{2-p})\cdot\frac{7}{12} = \frac{7}{12} - \frac{1}{4}\frac{p}{2-p} = \frac{7-5p}{12-6p}.
\end{aligned}$$

After asking, the expected reputation of the two signal-types is different since they decide differently if $o(s) = 1$. Using v and w defined in the Appendix (at the end of subsection Preliminaries), for signal-type 0 it is:

$$\begin{aligned}
&\Pr(\omega{=}1, o(s) \neq 1|0)v + \Pr(\omega{=}0, o(s) \neq 1|0)w + \Pr(\omega{=}1, o(s){=}1|0)y + \Pr(\omega{=}0, o(s){=}1|0)x = \\
&\quad = (1 - 3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{2p}{p+1})v + (1 - 3\frac{2}{3}\frac{1}{3^2})\frac{2p}{p+1}w + (3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{2p}{p+1})y + (3\frac{2}{3}\frac{1}{3^2})\frac{2p}{p+1}x = \\
&\quad = \frac{5}{9}(1 - \frac{2p}{p+1})\frac{7+2\mu}{12+6\mu} + \frac{7}{9}\frac{2p}{p+1}\frac{7\mu+2}{12\mu+6} + \frac{4}{9}(1 - \frac{2p}{p+1})\frac{1}{3} + \frac{2}{9}\frac{2p}{p+1}\frac{7}{12} = \\
&\quad = \frac{35+10\mu}{108+54\mu} + \frac{2p}{p+1}(\frac{49\mu+14}{108\mu+54} - \frac{35+10\mu}{108+54\mu} - \frac{2}{54}) + \frac{4}{27};
\end{aligned}$$

and for signal-type 1:

$$\begin{aligned}
&\Pr(\omega{=}1, o(s) \neq 1|1)v + \Pr(\omega{=}0, o(s) \neq 1|1)w + \Pr(\omega{=}1, o(s){=}1|1)x + \Pr(\omega{=}0, o(s){=}1|1)y = \\
&\quad = (1 - 3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{p}{2-p})v + (1 - 3\frac{2}{3}\frac{1}{3^2})\frac{p}{2-p}w + (3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{p}{2-p})x + (3\frac{2}{3}\frac{1}{3^2})\frac{p}{2-p}y = \\
&\quad = \frac{5}{9}(1 - \frac{p}{2-p})\frac{7+2\mu}{12+6\mu} + \frac{7}{9}\frac{p}{2-p}\frac{7\mu+2}{12\mu+6} + \frac{4}{9}(1 - \frac{p}{2-p})\frac{7}{12} + \frac{2}{9}\frac{p}{2-p}\frac{1}{3} = \\
&\quad = \frac{35+10\mu}{108+54\mu} + \frac{p}{2-p}(\frac{49\mu+14}{108\mu+54} - \frac{35+10\mu}{108+54\mu} - \frac{5}{27}) + \frac{7}{27}.
\end{aligned}$$

To look for $\underline{\rho}$ and $\overline{\rho}$ we need to compute the values of reputation for $\mu = 0$. Then, for signal-type 0 the expected reputation after asking is:

$$\frac{35}{108} + \frac{2p}{p+1}(\frac{28}{108} - \frac{35}{108} - \frac{4}{108}) + \frac{16}{108} = \frac{51}{108} - \frac{11}{108}\frac{2p}{p+1} = \frac{29p+51}{108p+108}$$

Recalling that for $\mu = 0$, $w = y$ and $v = x$ (see the Appendix, Preliminaries), for signal-type 1 it simply is:

$$\begin{aligned}
&\Pr(\omega{=}1, o(s) \neq 1|1)x + \Pr(\omega{=}0, o(s) \neq 1|1)y + \Pr(\omega{=}1, o(s){=}1|1)x + \Pr(\omega{=}0, o(s){=}1|1)y = \\
&\quad = \Pr(\omega{=}0|1)y + \Pr(\omega{=}1|1)x = \frac{p}{2-p}\frac{1}{3} + (1 - \frac{p}{2-p})\frac{7}{12} = \frac{7}{12} - \frac{1}{4}\frac{p}{2-p} = \frac{7-5p}{12-6p}.
\end{aligned}$$

The difference in expected utility between asking and not asking for signal-type 0 is zero for $\underline{\rho}$ such that:

$$\begin{gathered}
(1-\underline{\rho})(\frac{44p+8}{27p+27} - \frac{2p}{p+1}) + \underline{\rho}(\frac{29p+51}{108p+108} - \frac{5p+2}{6p+6}) = 0 \\
(1-\underline{\rho})(32-40p) + \underline{\rho}(15-61p) = 0 \\
\underline{\rho} = \frac{32-40p}{17+21p}.
\end{gathered}$$

As expected, for $p = 4/5$, $\underline{\rho} = 0$: signal-type 0 has no strict incentive to follow three 1 suggestions, so there is no gain from asking. For $p = 2/3$, $\underline{\rho} = \frac{16}{93}$. So, in Case 1 $\underline{\rho} \in \left[0, \frac{16}{93}\right)$. Recall that in Case 1 pooling on asking does not trigger truthful reporting. Thus, there is no information aggregation up to $\underline{\rho}$, i.e. there is no information aggregation for too low reputation concerns. As uncertainty increases, i.e. as p decreases, $\underline{\rho}$ increases. That is, there higher reputation concerns are needed to obtain some degree of information aggregation. For $p = 1/2$, $\underline{\rho} = \frac{24}{55}$, so in Case 2, $\underline{\rho} \in \left[\frac{16}{93}, \frac{24}{55}\right)$. However in Case 2, pooling on asking triggers truthful reporting and can be implemented from $\rho = 0$.

The difference in expected utility between asking and not asking for signal-type 1 in Case 1 is zero for $\overline{\rho}$ such that:

$$(1-\overline{\rho})(\frac{20}{27} - \frac{p}{2-p}) + \overline{\rho}(\frac{7-5p}{12-6p} - \frac{8-p}{24-12p}) = 0$$
$$(1-\overline{\rho})(\frac{160-134p}{108(2-p)}) + \overline{\rho}(\frac{54-81p}{108(2-p)}) = 0$$
$$\overline{\rho} = \frac{160-134p}{106-53p}.$$

As expected, for $p = 2/3$, $\overline{\rho} = 1$. Note indeed that in Case 2, the expected reputation after asking and not asking is the same, so no value of ρ makes signal-type 1 indifferent between asking and not asking. For $p = 4/5$, $\overline{\rho} = \frac{132}{209}$. Thus, also $\overline{\rho}$ increases as p decreases.

To compute $\widehat{\rho}$, we need to compute the highest value of μ such that the advisors report truthfully. It solves:

$$\Pr(\omega = 0|m^1) = \frac{2\widehat{\mu}p + p}{\widehat{\mu}p + 2 + \widehat{\mu} - p} = \frac{2}{3}.$$
$$\widehat{\mu} = \frac{4-5p}{4p-2}.$$

For $p = 4/5$, $\widehat{\mu} = 0$, so the "good" partially separating equilibrium where signal-type 1 always asks boils down to the separating equilibrium with weak incentive for signal-type 0. In this sense, $\widehat{\rho} = \underline{\rho}$. As anticipated, for $p = 2/3$, $\widehat{\mu} = 1$ so for every $p > 2/3$ there is no pooling equilibrium on asking with truthful reporting. Now we look for the value of ρ such that signal-type 0 is indifferent between asking and not asking for $p = 2/3$ and $\mu = 1$: this is the upper bound for $\widehat{\rho}$ in Case 1. Substituting $\mu = 1$ in the reputation after asking, the difference in expected utility for signal-type 0 between asking and not asking is zero for $\widehat{\rho}$ such that:

$$(1-\widehat{\rho})(\frac{44p+8}{27p+27}-\frac{2p}{p+1})+\widehat{\rho}(\frac{45}{162}+\frac{2p}{p+1}(\frac{12}{162})+\frac{4}{27}-\frac{5p+2}{6p+6})=0$$
$$(1-\widehat{\rho})(48-60p)+\widehat{\rho}(59-136p)=0$$
$$\widehat{\rho}=\frac{48-60p}{76p-11}.$$

For $p=2/3$, it is $\frac{144-120}{152-33}=\frac{24}{119}$. So, in Case 1, $\widehat{\rho}\in\left[0,\frac{24}{119}\right)$. Note that, as expected, $\widehat{\rho}>\underline{\rho}$: at $p=2/3$, $\frac{24}{119}>\frac{16}{93}$.

In case 2, pooling on asking triggers truthful reporting. So, we are interested in $\widehat{\rho}$ as the maximum weight on reputation such that the pooling equilibrium on asking exists under A3. For the limit case $p=1/2$, we obtain $\widehat{\rho}=\frac{18}{27}=2/3$. Thus, in Case 2, the pooling equilibrium on asking exists up to $\widehat{\rho}\in\left[\frac{24}{119},\frac{2}{3}\right)$.

Also $\widehat{\rho}$ increases as p decreases. That is, more uncertainty requires (in Case 1) or allows (in Case 2) higher reputation concerns to achieve the best feasible level of information aggregation.

6 Conclusion

In this paper we have studied how reputation concerns of a decision maker affect her ability to extract decision-relevant information from potential advisors. Too high reputation concerns provoke excessive advice-avoidance due to the decision-maker's desire to appear well informed. Too low reputation concerns result in excessive advice-seeking, which destroys advisors' incentives to provide truthful information. In general, some intermediate reputation concerns are optimal, as they create a credible commitment (in equilibrium) to abstain from asking for advice too frequently and, at the same time, do not trigger too much advice-avoidance.

A rise in the prior uncertainty about the state of nature increases the temptation to ask for advice. This may disrupt aggregation of information when the prior uncertainty is relatively low, i.e., when the problem of excessive advise-seeking is relevant. In such a case, higher optimal reputation concerns are needed in order to restrain excessive advice-seeking. For the same reason, when the prior uncertainty is low and the reputation concerns are not strong enough, higher prior competence of advisors may destroy information aggregation and worsen the quality of decisions.

A key ingredient of our story is that advisors are willing to provide information only when they feel uncertain about the state of nature. Although in our model this behavior stems from their reputation concerns, there may also be other reasons that generate a similar incentive. For example, assume that advisors have no reputation concerns but need to invest in acquiring or transmitting information to the decision maker. Assume in addition that they care about the quality of decisions. Then their incentives to invest will be stronger (and hence the quality of information received by the decision maker will be higher) the more undecided they think the decision maker is. Consequently, like in our model, it will be crucial to avoid "excessive asking" by a decision maker with the signal confirming the prior. At the same time, the temptation to ask for advice should increase in the prior uncertainty and the competence of advisors. Thus, we conjecture that such a framework will generate the same main results as the current one.[17] A formal analysis of this alternative setup can be a subject of future work.

Appendix

Preliminaries

Fix a signal-type σ and a state ω. Let $\overline{\omega}$ be the other state. The probability of ω conditional on σ is

$$\Pr(\omega|\sigma) = \frac{\Pr(\sigma|\omega, G)\Pr(\omega)\Pr(G) + \Pr(\sigma|\omega, B)\Pr(\omega)\Pr(B)}{numerator + \Pr(\sigma|\overline{\omega}, G)\Pr(\overline{\omega})\Pr(G) + \Pr(\sigma|\overline{\omega}, B)\Pr(\overline{\omega})\Pr(B)} \tag{P}$$

For the theoretical analysis, we do not need to compute the probability of a state conditional on the advices. For the numerical example of Section 5, such probabilities are computed for the specific case.

For any profile of advisors' truthfully reported signals s, let $o(s)$ denote the number of 0's in s. By A2 the decision after s is 1 if and only if $o(s) < j$ for some $0 \leq j \leq n$ in case $\sigma = 0$ and $o(s) < j'$ for some $j' \geq j$ in case $\sigma = 1$. Denote by S the set of all possible s. Let $\overline{S}$ be the set of s such that $j \leq o(s) < j'$ and $\widehat{S}$ its complement. In other words, $\overline{S}$ is the subset of S such that, for any $s \in \overline{S}$, either signal-type ignores advisors' information and takes the decision corresponding to her own signals. In contrast, for any $s \in \widehat{S}$, both signal-types ignore their signals and take the same decision, suggested by s.

While $\overline{S}$ is empty when $j' = j$, $\widehat{S}$ is always non-empty because by A4 $j' \leq n$. For a profile s to belong to $\widehat{S}$, it must contain either enough 0's to make signal-type 1 believe that state 0 is more likely, or sufficiently many 1's (definitely more than $n/2$) to make

[17] One difference of such a setting from the current one is that it is not the uncertainty about the state per se that would matter for the advisors' incentives, but whether they believe that they face a *decision maker* who is undecided. This would matter when the decision maker after receiving signal 1 is rather confident that the state is 1. In the current model, pooling on asking triggers truthful reporting in such a case. Yet, in the alternative setup, the advisors will have weak incentives, for they know that the decision maker is not undecided.

signal-type 0 believe that state 1 is more likely. However, since $\omega = 0$ is weakly more likely a priori, the minimum number of 1's needed to "change the mind" of signal-type 0 is weakly higher than the minimum number of 0's needed to "change the mind" of signal-type 1. Therefore, the likelihood that s falls into $\widehat{S}$ should be weakly higher when $\omega = 0$.

Formally, consider first all profiles s belonging to $\widehat{S}$ such that $o(s) \leq n/2$. It must be that either $\Pr(\omega = 1|\sigma = 0, s) > 1/2$ (s contains so many 1's that signal-type 0 considers $\omega = 1$ more likely) or $\Pr(\omega = 0|\sigma = 1, s) > 1/2$ (despite $o(s) \leq n/2$, s contains enough 0's to make signal-type 1 believe that $\omega = 0$ is more likely). Then, *any* profile s' such that $o(s') = n - o(s)$ also belongs to $\widehat{S}$, because: (1) if $\Pr(\omega = 1|\sigma = 0, s) > 1/2$, then $\Pr(\omega = 0|\sigma = 1, s') > 1/2$ as well (s' contains as many 0's as s contains 1's, and $p \geq 1/2$), (2) if $\Pr(\omega = 0|\sigma = 1, s) > 1/2$, then $\Pr(\omega = 0|\sigma = 1, s') > 1/2$ (s' contains more 0's than s does).

Since all advisors are identical and, for every i, $\Pr(s_i = \omega|\omega)$ does not depend on ω, $\Pr(s|\omega = 1) = \Pr(s'|\omega = 0)$.

If there are any remaining profiles s'' belonging to $\widehat{S}$, they must have $o(s'') \geq n/2$, implying $\Pr(s''|\omega = 0) \geq \Pr(s''|\omega = 1)$. Thus, we conclude that

$$\Pr(\widehat{S}|\omega = 0) \geq \Pr(\widehat{S}|\omega = 1). \tag{S}$$

This formula will be used in the proof of Lemma 3.

It will be convenient to label and compare the reputations at some specific terminal nodes under A2 and A3. Fix a terminal history ξ.

Suppose first that, after observing ξ, the observer infers that decision maker has definitely received a specific signal σ: $\Pr(\sigma|\xi) = 1$.[18] Then the reputation depends only on whether $\sigma = \omega$ or $\sigma \neq \omega$, i.e., one of the two values of reputation is realized:

$$\begin{aligned} \Pr(G|\xi, \omega) &= \Pr(G|\sigma = \omega) = \frac{\Pr(\sigma = \omega|G)\Pr(G)}{\Pr(\sigma = \omega)} = \frac{gq}{gq + b(1-q)} =: x; \\ \Pr(G|\xi, \omega) &= \Pr(G|\sigma \neq \omega) = \frac{\Pr(\sigma \neq \omega|G)\Pr(G)}{\Pr(\sigma \neq \omega)} = \frac{(1-g)q}{(1-g)q + (1-b)(1-q)} := y. \end{aligned}$$

It is straightforward to show that, since $1/2 \leq b < g$, we have $x > y$.

Suppose now that ξ does not necessarily reveal the signal-type perfectly. Specifically, suppose that either (i) $\xi = (m^1, a, d)$ and both signal-types after a consider state $\omega = d$ strictly more likely (for instance, $a = s \in \widehat{S}$), or (ii) $\xi = (m^0, d = 0)$ with $\Pr(m^0, d =$

[18] This is the case when (i) $\xi = (m^1, a, d)$ and the two signal-types after a consider different states more likely, (ii) $\xi = (m^1, s, d)$ and $s \in \overline{S}$, (iii) $\xi = (m^0, d = 1)$, (iv) $\xi = (m^0, d = 0)$ and signal-type 1 always asks or considers state 1 more likely, (v) ξ has probability 0 in equilibrium (by A3).

$0|\sigma) = \Pr(m^0|\sigma) \neq 0$ for both σ. In case (i), note that

$$\Pr(\xi|\omega,\sigma) = \Pr(m^1|\sigma) \cdot \Pr(a|\omega, m^1) \cdot \Pr(d|\sigma, a, m^1) = \Pr(m^1|\sigma) \cdot \Pr(a|\omega, m^1),$$

where $\Pr(d|\sigma, a, m^1) = 1$ by A2. The reputation of the decision maker at ξ when state ω is observed is then

$$\begin{aligned}\Pr(G|\xi,\omega) &= \frac{\Pr(\xi|\omega,G)\Pr(G|\omega)}{\Pr(\xi|\omega,G)\Pr(G|\omega)+\Pr(\xi|\omega,B)\Pr(B|\omega)} = \\ &= \frac{[\Pr(m|\omega,\sigma=\omega)\Pr(\sigma=\omega|\omega,G)+\Pr(m|\omega,\sigma\neq\omega)\Pr(\sigma\neq\omega|\omega,G)]\Pr(G)+}{numerator+[\Pr(m|\omega,\sigma=\omega)\Pr(\sigma=\omega|\omega,B)+\Pr(m|\omega,\sigma\neq\omega)\Pr(\sigma\neq\omega|\omega,B)]\Pr(B)} = \\ &= \frac{\Pr(m|\omega,\sigma=\omega)gq+\Pr(m|\omega,\sigma\neq\omega)(1-g)q}{numerator+\Pr(m|\omega,\sigma=\omega)b(1-q)+\Pr(m|\omega,\sigma\neq\omega)(1-b)(1-q)} = \Pr(G|m,\omega),\end{aligned} \tag{R}$$

where in case (i), $\Pr(a|\omega, m^1)$ has been simplified in the second line. Let $\mu = \Pr(m^1|\sigma = 0)/\Pr(m^1|\sigma = 1)$ or $\mu = \Pr(m^0, d = 0|\sigma = 0)/\Pr(m^0, d = 0|\sigma = 1)$. We have:

$$\Pr(G|\xi,\omega=1) = \frac{gq+\mu(1-g)q}{gq+\mu(1-g)q+b(1-q)+\mu(1-b)(1-q)} =: v;$$
$$\Pr(G|\xi,\omega=0) = \frac{\mu gq+(1-g)q}{\mu gq+(1-g)q+\mu b(1-q)+(1-b)(1-q)} =: w.$$

It is easy to observe that:

$$\begin{aligned} x &= v > w = y && \text{if } \mu = 0; \\ x &> v > w > y && \text{if } \mu \in (0,1); \\ x &> v = w > y && \text{if } \mu = 1; \\ x &> w > v > y && \text{if } \mu > 1; \end{aligned}$$

and that for $\mu > 0$,

$$v + w > x + y.$$

Proofs

Proof of Lemma 1

Consider an arbitrary history of events ψ prior to the decision stage (that is, ψ is either m^0 or (m^1, a)). Fix any signal-type $\overline{\sigma}$, and without loss of generality suppose that she considers state 0 (weakly) more likely, that is $\Pr(\omega = 0|\overline{\sigma}, \psi) \geq 1/2$. Suppose that if she takes $d = 1$, she is perceived as signal-type 1. This would be the equilibrium belief if signal-type 1 considers state 1 more likely or an off-the-path belief pinned down by A3 when signal-type 1 considers state 0 more likely.

Then if signal-type $\overline{\sigma}$ takes $d = 1$, her expected reputation is

$$\begin{aligned}\Pr(\omega &= 0|\overline{\sigma}, \psi) \cdot \Pr(G|\sigma \neq \omega) + [1 - \Pr(\omega = 0|\overline{\sigma}, \psi)] \cdot \Pr(G|\sigma = \omega) = \\ &= \Pr(\omega = 0|\overline{\sigma}, \psi) \cdot y + [1 - \Pr(\omega = 0|\overline{\sigma}, \psi)] \cdot x.\end{aligned}$$

If signal-type $\overline{\sigma}$ takes $d = 0$ and the other signal-type, at ψ, considers state 1 more likely (which implies $\overline{\sigma} = 0$), the expected reputation of signal-type $\overline{\sigma}$ is:

$$\begin{aligned}\Pr(\omega &= 0|\overline{\sigma}, \psi) \cdot \Pr(G|\sigma = \omega) + [1 - \Pr(\omega = 0|\overline{\sigma}, \psi)] \cdot \Pr(G|\sigma \neq \omega) = \\ &= \Pr(\omega = 0|\overline{\sigma}, \psi) \cdot x + [1 - \Pr(\omega = 0|\overline{\sigma}, \psi)] \cdot y.\end{aligned}$$

Since $\Pr(\omega = 0|\overline{\sigma}, \psi) \geq 1/2$ and $x > y$, $d = 0$ yields non lower reputation than $d = 1$ to signal-type $\overline{\sigma}$.

If signal-type $\overline{\sigma}$ takes $d = 0$ and also the other signal-type, at ψ, considers state 0 more likely, the expected reputation of signal-type $\overline{\sigma}$ is:

$$\begin{aligned}\Pr(\omega &= 0|\overline{\sigma}, \psi) \cdot \Pr(G|\psi, d = 0, \omega = 0) + [1 - \Pr(\omega = 0|\overline{\sigma}, \psi)] \cdot \Pr(G|\psi, d = 0, \omega = 1) = \\ &= \Pr(\omega = 0|\overline{\sigma}, \psi) \cdot w + [1 - \Pr(\omega = 0|\overline{\sigma}, \psi)] \cdot v.\end{aligned}$$

Since $\Pr(\omega = 0|\overline{\sigma}, \psi) \geq 1/2$, $w > y$, and $w + v \geq x + y$, $d = 0$ yields non lower reputation than $d = 1$ to signal-type $\overline{\sigma}$.

Obviously, instrumental utility only reinforces the no-deviation incentives.

Proof of Lemma 2.

For brevity, let $\overline{q} := 1 - q$, $\overline{p} := 1 - p$, $\overline{g} := 1 - g$, $\overline{b} := 1 - b$. For $m = m^0, m^1$ and $\mu = \Pr(m|\sigma = 0)/\Pr(m|\sigma = 1)$, from Equations (P) and (R) we get:

$$\begin{aligned}\Pr(\omega = 0|\sigma = 0) &= \frac{gpq + bp\overline{q}}{gpq + bp\overline{q} + \overline{g}\overline{p}q + \overline{b}\overline{p}\overline{q}};\\ \Pr(G|m, \omega = 0) &= \frac{\mu gq + \overline{g}q}{\mu gq + \overline{g}q + \mu b\overline{q} + \overline{b}\overline{q}};\\ \Pr(\omega = 1|\sigma = 0) &= \frac{\overline{g}\overline{p}q + \overline{b}\overline{p}\overline{q}}{\overline{g}\overline{p}q + \overline{b}\overline{p}\overline{q} + gpq + bp\overline{q}};\\ \Pr(G|m, \omega = 1) &= \frac{\mu\overline{g}q + gq}{\mu\overline{g}q + gq + \mu\overline{b}\overline{q} + b\overline{q}}.\end{aligned}$$

For brevity, let

$$\begin{aligned}
\alpha &:= \frac{1}{gpq + bp\overline{q} + \overline{g}\overline{p}q + \overline{b}\overline{p}\overline{q}} \\
C(\mu) &:= \Pr(\omega = 0|\sigma = 0)\Pr(G|m, \omega = 0) + \Pr(\omega = 1|\sigma = 0)\Pr(G|m, \omega = 1) = \\
&= \alpha\left(\frac{(gpq + bp\overline{q})(\mu gq + \overline{g}q)}{\mu gq + \overline{g}q + \mu b\overline{q} + \overline{b}\overline{q}} + \frac{(\overline{g}\overline{p}q + \overline{b}\overline{p}\overline{q})(\mu\overline{g}q + gq)}{\mu\overline{g}q + gq + \mu\overline{b}\overline{q} + b\overline{q}}\right).
\end{aligned}$$

We have

$$\begin{aligned}
D(\mu) &:= \Pr(\omega = 0|\sigma = 0)\frac{\partial \Pr(G|m, \omega = 0)}{\partial \mu} = \\
&= \alpha\frac{(gpq + bp\overline{q})(gq(\mu gq + \overline{g}q + \mu b\overline{q} + \overline{b}\overline{q}) - (gq + b\overline{q})(\mu gq + \overline{g}q))}{(\mu gq + \overline{g}q + \mu b\overline{q} + \overline{b}\overline{q})^2} = \\
&= \alpha\frac{(gpq + bp\overline{q})(gq\overline{b}\overline{q} - b\overline{q}\overline{g}q)}{(\mu gq + \overline{g}q + \mu b\overline{q} + \overline{b}\overline{q})^2} = \alpha pq\overline{q}(g\overline{b} - b\overline{g})\frac{(gq + b\overline{q})}{(\mu gq + \overline{g}q + \mu b\overline{q} + \overline{b}\overline{q})^2} > 0;
\end{aligned}$$

and

$$\begin{aligned}
E(\mu) &:= \Pr(\omega = 1|\sigma = 0)\frac{\partial \Pr(G|m, \omega = 1)}{\partial \mu} = \\
&= \alpha\frac{(\overline{g}\overline{p}q + \overline{b}\overline{p}\overline{q})(\overline{g}q(\mu\overline{g}q + gq + \mu\overline{b}\overline{q} + b\overline{q}) - (\overline{g}q + \overline{b}\overline{q})(\mu\overline{g}q + gq))}{(\mu\overline{g}q + gq + \mu\overline{b}\overline{q} + b\overline{q})^2} = \\
&= \alpha\frac{(\overline{g}\overline{p}q + \overline{b}\overline{p}\overline{q})(\overline{g}qb\overline{q} - gq\overline{b}\overline{q})}{(\mu\overline{g}q + gq + \mu\overline{b}\overline{q} + b\overline{q})^2} = \alpha\overline{p}q\overline{q}(b\overline{g} - g\overline{b})\frac{(\overline{g}q + \overline{b}\overline{q})}{(\mu\overline{g}q + gq + \mu\overline{b}\overline{q} + b\overline{q})^2} < 0.
\end{aligned}$$

Denote:

$$\begin{aligned}
A(s) &:= \Pr(s|\omega{=}0)\Pr(\omega{=}0|\sigma{=}0)\Pr(G|m^1, \omega{=}0) + \Pr(s|\omega{=}1)\Pr(\omega{=}1|\sigma{=}0)\Pr(G|m^1, \omega{=}1); \\
B(s) &:= \Pr(s|\omega{=}0)\Pr(\omega{=}0|\sigma{=}0)\Pr(G|\sigma{=}0, \omega{=}0) + \Pr(s|\omega{=}1)\Pr(\omega{=}1|\sigma{=}0)\Pr(G|\sigma{=}0, \omega{=}1).
\end{aligned}$$

First, we show that Part (i) holds for $m = m^1$. The expected reputation of signal-type 0 after asking is:

$$\sum_{s\in\widehat{S}} A(s) + \sum_{s\in\overline{S}} B(s).$$

Since $\sum_{s\in\overline{S}} B(s)$ does not depend on μ, we can focus on $\sum_{s\in\widehat{S}} A(s)$. Fix any s, s' with $o(s) \geq n/2$ and $o(s') = n - o(s)$. As already observed in the Preliminaries, $\widehat{S}$ can be partitioned into pairs s, s' with $o(s') = n - o(s)$ and unpaired s with $o(s) \geq n/2$. Thus, $\sum_{\widehat{s}\in\widehat{S}} A(\widehat{s})$ is increasing in μ when $A(s) + A(s')$ and $A(s)$ are increasing in μ. This is what we show next.

Since $\Pr(s_i = \omega|\omega)$ depends neither on ω, nor on i, we have $\Pr(s'|\omega = 1) = \Pr(s|\omega = 0)$

and $\Pr(s|\omega=1)=\Pr(s'|\omega=0)$. Thus,

$$A(s)+A(s')=[\Pr(s|\omega=0)+\Pr(s'|\omega=0)]\cdot C(\mu).$$

Since the first factor does not depend on μ, the sign of the derivative is determined by:

$$\frac{\partial C(\mu)}{\partial\mu}=D(\mu)+E(\mu)=\alpha q\overline{q}(g\overline{b}-b\overline{g})\left(\frac{p(gq+b\overline{q})}{(\mu gq+\overline{g}q+\mu b\overline{q}+\overline{b}\overline{q})^2}-\frac{\overline{p}(\overline{g}q+\overline{b}\overline{q})}{(\mu\overline{g}q+gq+\mu\overline{b}\overline{q}+b\overline{q})^2}\right);$$

$$\frac{\partial C(\mu)}{\partial\mu}>0\Leftrightarrow\left(\frac{\overline{g}q+\frac{1}{\mu}gq+\overline{b}\overline{q}+\frac{1}{\mu}b\overline{q}}{gq+\frac{1}{\mu}\overline{g}q+b\overline{q}+\frac{1}{\mu}\overline{b}\overline{q}}\frac{\mu}{\mu}\right)^2>\frac{\overline{g}q+\overline{b}\overline{q}}{gq+b\overline{q}}\frac{\overline{p}}{p}.$$

The latter inequality is always verified when $\mu\leq 1$, because then the left hand side is bigger than 1, whereas the right hand side is smaller than 1. Since the term in brackets is always bigger than $(\overline{g}q+\overline{b}\overline{q})/gq+b\overline{q}$, a sufficient condition for the inequality to hold also when $\mu>1$ is:

$$\begin{aligned}\frac{\overline{g}q+\overline{b}\overline{q}}{gq+b\overline{q}} &> \frac{\overline{p}}{p}\\ \frac{(1-g)q+(1-b)(1-q)}{gq+b(1-q)} &> \frac{(1-p)}{p}\\ p &> gq+b(1-q),\end{aligned}$$

as desired. Moreover,

$$\frac{\partial A(s)}{\partial\mu}=\Pr(s|\omega=0)D(\mu)+\Pr(s|\omega=1)E(\mu)$$

is positive too whenever $\partial C(\mu)/\partial\mu=D(\mu)+E(\mu)>0$, because $\Pr(s|\omega=0)\geq\Pr(s|\omega=1)$ and $D(\mu)>0>E(\mu)$.

Now we want to show that Part (i) holds also for $m=m^0$. Note that $C(\mu)$ represents precisely the expected reputation of signal-type 0 after not asking. Hence Part (i) holds also for $m=m^0$.

For Part (ii), write the expected reputation of signal-type 0 after not asking when signal-type 1 always asks as $\sum_{s\in\overline{S}\cup\widehat{S}}B(s)$. Then, the difference with the expected reputation of signal-type 0 after asking reads:

$$\textstyle\sum_{s\in\widehat{S}}B(s)-\sum_{s\in\widehat{S}}A(s).$$

Fix s, s' with $o(s) \geq n/2$ and $o(s') = n - o(s)$. As before, it is enough to show that:

$$\begin{aligned} B(s) + B(s') &> (A(s) + A(s'))|_{\mu=1}; \\ B(s) &> A(s)|_{\mu=1}. \end{aligned}$$

Since $\Pr(G|\sigma = 0, \omega) = \lim_{\mu\to\infty} \Pr(G|m^1, \omega)$, $B(s) + B(s') = (A(s) + A(s'))|_{\mu\to\infty}$. Note that $\lim_{\mu\to\infty} C(\mu) > C(1) = q$. Then, $B(s) + B(s') > (A(s) + A(s'))|_{\mu=1}$ and

$$\Pr(\omega = 0|\sigma = 0) \cdot x + \Pr(\omega = 1|\sigma = 0) \cdot y > \Pr(\omega = 0|\sigma = 0) \cdot q + \Pr(\omega = 1|\sigma = 0) \cdot q.$$

Thus, by $\Pr(s|\omega = 0) \geq \Pr(s|\omega = 1)$ and $x > q > y$,

$$\begin{aligned} B(s) &= \Pr(s|\omega = 0) \Pr(\omega = 0|\sigma = 0) \cdot x + \Pr(s|\omega = 1) \Pr(\omega = 1|\sigma = 0) \cdot y > \\ &> \Pr(s|\omega = 0) \Pr(\omega = 0|\sigma = 0) \cdot q + \Pr(s|\omega = 1) \Pr(\omega = 1|\sigma = 0) \cdot q = A(s)|_{\mu=1}, \end{aligned}$$

where the last equality comes from $\Pr(G|m, \omega = 1) = \Pr(G|m, \omega = 0) = q$ when $\mu = 1$. ■

Proof of Lemma 3.

Throughout the proof, the 0 and 1 after the conditioning bar means $\sigma = 0$ and $\sigma = 1$. The difference in expected instrumental utility between asking and not asking for signal-type 0 is:

$$\begin{aligned} \Delta IU_0 \; : &= \sum_{s:o(s)<j} [\Pr(\omega = 1, s|0) - \Pr(\omega = 0, s|0)] = \\ &= \Pr(\omega = 1|0) \sum_{s:o(s)<j} \Pr(s|\omega = 1) - \Pr(\omega = 0|0) \sum_{s:o(s)<j} \Pr(s|\omega = 0). \end{aligned}$$

For signal-type 1, if $\Pr(\omega = 0|\sigma = 1) > 1/2$ it is:

$$\begin{aligned} \Delta IU_1 \; : &= \sum_{s:o(s)<j'} [\Pr(\omega = 1, s|1) - \Pr(\omega = 0, s|1)] \geq \\ &\geq \Pr(\omega = 1|1) \sum_{s:o(s)<j} \Pr(s|\omega = 1) - \Pr(\omega = 0|1) \sum_{s:o(s)<j} \Pr(s|\omega = 0), \end{aligned}$$

where the inequality holds because for every s with $o(s) < j'$,

$$\Pr(\omega = 1, s|1) - \Pr(\omega = 0, s|1) = [\Pr(\omega = 1|s, 1) - \Pr(\omega = 0|s, 1)] \cdot \Pr(s|1) > 0. \quad \text{(F)}$$

If $\Pr(\omega = 1|\sigma = 1) \geq 1/2$, it is:

$$\begin{aligned}\Delta IU_1' \quad : \quad &= \sum_{s:o(s)\geq j'} [\Pr(\omega = 0, s|1) - \Pr(\omega = 1, s|1)] \geq \\ &\geq \Pr(\omega = 0|1) \sum_{s:o(s)>n-j} \Pr(s|\omega = 0) - \Pr(\omega = 1|1) \sum_{s:o(s)>n-j} \Pr(s|\omega = 1),\end{aligned}$$

where the inequality holds because $j' \leq n - j + 1$ and for every s with $o(s) \geq j'$,

$$\Pr(\omega{=}0, s|1) - \Pr(\omega{=}1, s|1) = (\Pr(\omega{=}0|s, 1) - \Pr(\omega{=}1|s, 1)) \Pr(s|1) > 0.$$

It follows immediately from $\Pr(\omega = 0|\sigma = 0) > \Pr(\omega = 0|\sigma = 1)$ that ΔIU_1 is higher than ΔIU_0. Note furthermore that since $\Pr(s_i = \omega|\omega)$ depends neither on ω, nor on i, we have:

$$\sum_{s:o(s)<j} \Pr(s|\omega = 1) = \sum_{s:o(s)>n-j} \Pr(s|\omega = 0).$$

Then it follows immediately from $\Pr(\omega = 0|\sigma = 0) > \Pr(\omega = 1|\sigma = 1)$ that $\Delta IU_1'$ is higher than ΔIU_0.

The difference in expected reputation for signal type 0 is:

$$\Delta R_0 := \sum_{s\in\widehat{S}} [\Pr(\omega{=}0, s|0)(w{-}x){+}\Pr(\omega{=}1, s|0)(v{-}y)] {+} \sum_{s\in\overline{S}} [\Pr(\omega{=}0, s|0)(x{-}x){+}(\Pr(\omega{=}1, s|0)(y{-}y)].$$

For signal-type 1, if $\Pr(\omega = 0|\sigma = 1) > 1/2$ it is:

$$\Delta R_1 := \sum_{s\in\widehat{S}} [\Pr(\omega{=}0, s|1)(w{-}x){+}\Pr(\omega{=}1, s|1)(v{-}y)] {+} \sum_{s\in\overline{S}} [\Pr(\omega{=}0, s|1)(y{-}x){+}\Pr(\omega{=}1, s|1)(x{-}y)];$$

and if $\Pr(\omega = 1|\sigma = 1) \geq 1/2$ it is:

$$\Delta R_1' := \sum_{s\in\widehat{S}} [\Pr(\omega{=}0, s|1)(w{-}y){+}\Pr(\omega{=}1, s|1)(v{-}x)] {+} \sum_{s\in\overline{S}} [\Pr(\omega{=}0, s|1)(y{-}y){+}\Pr(\omega{=}1, s|1)(x{-}x)].$$

The second term of ΔR_0 and $\Delta R_1'$ is zero, whereas the second term of ΔR_1 is non negative because for every $s \in \overline{S}$, Equation (F) holds. The first term of ΔR_1 is strictly bigger than the first term of ΔR_0 because $w - x < 0$, $v - y > 0$, and, by $\Pr(\omega = 0|0) > \Pr(\omega = 0|1)$,

$$\begin{aligned}\sum_{s\in\widehat{S}} \Pr(\omega &= 0, s|1) = \Pr(\widehat{S}|\omega = 0) \Pr(\omega = 0|1) < \Pr(\widehat{S}|\omega = 0) \Pr(\omega = 0|0) = \sum_{s\in\widehat{S}} \Pr(\omega = 0, s|0); \\ \sum_{s\in\widehat{S}} \Pr(\omega &= 1, s|1) = \Pr(\widehat{S}|\omega = 1) \Pr(\omega = 1|1) > \Pr(\widehat{S}|\omega = 1) \Pr(\omega = 1|0) = \sum_{s\in\widehat{S}} \Pr(\omega = 1, s|0).\end{aligned}$$

So, if $\Pr(\omega = 0|\sigma = 1) > 1/2$, signal-type 1 strictly prefers to ask. If $\Pr(\omega = 1|\sigma = 1) \geq$

$1/2$, suppose by contraposition that signal-type 1 prefers not to ask. Then, since $\Delta IU_1'$ is positive, $\Delta R_1'$ must be negative. So, by $w - y > x - v$, we have $\Pr(\omega = 0, \widehat{S}|1) < \Pr(\omega = 1, \widehat{S}|1)$. Then, rewriting the first term of $\Delta R_1'$ as:

$$\begin{array}{c}(w-x)\sum_{s\in\widehat{S}}\Pr(\omega=1,s|\sigma=1)+(v-y)\sum_{s\in\widehat{S}}\Pr(\omega=0,s|\sigma=1)+\\(v-w)\sum_{s\in\widehat{S}}\Pr(\omega=1,s|\sigma=1)+(w-v)\sum_{s\in\widehat{S}}\Pr(\omega=0,s|\sigma=1),\end{array}$$

the second line is positive, and then the first line is negative. The first line is bigger than ΔR_0, because $w - x < 0$, $v - y > 0$, and, by Equation (S) and $\Pr(\omega = 0|0) > \Pr(\omega = 1|1)$,

$$\begin{aligned}\sum_{s\in\widehat{S}}\Pr(\omega &= 1,s|1)=\Pr(\widehat{S}|\omega=1)\Pr(\omega=1|1)<\Pr(\widehat{S}|\omega=0)\Pr(\omega=0|0)=\sum_{s\in\widehat{S}}\Pr(\omega=0,s|0);\\\sum_{s\in\widehat{S}}\Pr(\omega &= 0,s|1)=\Pr(\widehat{S}|\omega=0)\Pr(\omega=0|1)>\Pr(\widehat{S}|\omega=1)\Pr(\omega=1|0)=\sum_{s\in\widehat{S}}\Pr(\omega=1,s|0).\end{aligned}$$

Hence ΔR_0 is smaller than $\Delta R_1'$, and since also ΔIU_0 is smaller than $\Delta IU_1'$, signal-type 0 strictly prefers not to ask, contradicting Condition 3 of the Lemma. ■

The technical reason why Condition 2 of the Lemma cannot be dispensed with is that the proof fails in the comparisons of reputations for $s \in \overline{S}$.

Proof of Proposition 5

By inspection of ΔIU_0 in the proof of Lemma 3, it is easy to observe that the difference in expected instrumental utility between asking and not asking increases when p decreases. Since we are interested in $\underline{\rho}$, we fix the beliefs of the observer that she has when signal-type 1 always asks, signal-type 0 never asks, and A3 holds. Then, after asking and each vector of advices, it is optimal for signal-type 0 to take the decision that corresponds to the state that she considers more likely.

Suppose first that, as p changes, $\widehat{S}$ does not change. Since $\mu = 0$, it follows from section Preliminaries of the Appendix that $v = x$ and $w = y$. Then the difference in expected reputation between asking and not asking for signal-type 0, ΔR_0, reads:

$$\Pr(\widehat{S}|\omega=0)\Pr(\omega=0|\sigma=0)(y-x)+\Pr(\widehat{S}|\omega=1)\Pr(\omega=1|\sigma=0)(x-y).$$

As only $\Pr(\omega|\sigma = 0)$ depends on p, $\Pr(\omega = 0|\sigma = 0)$ decreases as p decreases, and $y < x$, the difference in expected reputation is increasing as p decreases.

Thus, the difference in the overall expected payoff of signal-type 0 between asking and not asking goes up. Then, since the difference in expected instrumental utility is positive,[19] for signal-type 0 to remain indifferent between asking and not asking as p

[19] Hence, when signal-type 0 is indifferent between asking and not asking, the difference in expected

decreases, $\underline{\rho}$ must increase.

Consider now a change in $\widehat{S}$ as p marginally decreases. Namely, suppose that for some $k \leq n$ and each vector of advices s with $o(s) = k$, one of the two signal-types switches from considering state 0 to considering state 1 more likely. When the switching signal-type is 0, if she were to still decide 0, the reasoning for the case in which $\widehat{S}$ does not change holds. By switching to decision 1, she improves her expected utility of asking. When the switching signal-type is 1, as she switches to decision 1, the expected utility of signal-type 0 improves too, because she considers state 0 more likely and hence prefers to be recognized as signal-type 0 rather than as signal-type 1 as it happened before. Thus, a change in $\widehat{S}$ may only increase the difference in the expected payoff of signal-type 0 between asking and not asking, and this makes $\underline{\rho}$ increase even further. ■

Proof of Proposition 6

By inspection of ΔIU_0 in the proof of Lemma 3, it is easy to observe that the difference in expected instrumental utility between asking and not asking increases when p decreases. Since we are interested in $\widehat{\rho}$ and $\underline{\widehat{\rho}}$, we must focus on the case in which signal-type 1 always asks and signal-type 0 asks with probability $\mu > 0$.

Suppose first that, as p changes, $\widehat{S}$ does not change. The difference in expected reputation between asking and not asking for signal-type 0, ΔR_0, reads:

$$\Pr(\widehat{S}|\omega = 0) \Pr(\omega = 0|\sigma = 0)(w - x) + \Pr(\widehat{S}|\omega = 1) \Pr(\omega = 1|\sigma = 0)(v - y)].$$

As only $\Pr(\omega|\sigma = 0)$ depends on p, $\Pr(\omega = 0|\sigma = 0)$ decreases as p decreases, and $w - x < v - y$, the difference in expected reputation is increasing as p decreases. Moreover, the maximum and, in Case 3, the minimum μ under which the advisors report truthfully weakly increase as p decreases. This observation follows from the fact that the probability of state 0 conditional on asking decreases as p decreases; thus, to restore the maximum or the minimum probability of state 0 conditional on asking under which the advisors report truthfully, the probability that signal-type 0 asks must increase (see Condition (TR2)). By Lemma 2, part (i), an increase in μ when signal-type 1 always asks induces an increase in expected reputation of signal-type 0 after asking.

Thus, the difference in the overall expected payoff of signal-type 0 between asking and not asking goes up. Then, since the difference in expected instrumental utility is positive, for signal-type 0 to remain indifferent between asking and not asking as p decreases, $\widehat{\rho}$ and, in Case 3, $\underline{\widehat{\rho}}$ must increase.

reputation is negative. Note that the difference in expected instrumental utility could also be zero, but in this case the difference in expected reputation would be negative, hence signal-type 0 would ask only for $\underline{\rho} = \widehat{\rho} = 0$.

Consider now a change in $\widehat{S}$ as p marginally decreases. Namely, suppose that for some $k \leq n$ and each vector of advices s with $o(s) = k$, one of the two signal-types switches from considering state 0 to considering state 1 more likely. When the switching signal-type is 0, if she were to still decide 0, the reasoning for the case in which $\widehat{S}$ does not change holds. In the new equilibrium where she takes decision 1, she improves her expected utility, since in case of deviation to decision 0 she would obtain exactly the same expected utility as if she were still expected to decide 0 (by A3). When the switching signal-type is 1, this means that, after s, signal-type 1 considers state 0 and state 1 equally likely. Then, conditional on s only, state 0 is more likely than 1. Thus, given s, signal-type 0 prefers to be recognized as signal-type 0 rather than pooling with signal-type 1 on the decision. This observation is equivalent to Lemma 2, part (ii), as the probability of state 0 conditional on s is higher than 1/2 like the prior p. Hence, the switch of signal-type 1 to decision 1 makes the expected utility of signal-type 0 after s increase. Thus, a change in $\widehat{S}$ may only increase the difference in the expected payoff of signal-type 0 between asking and not asking, and this makes $\widehat{\rho}$ and $\underline{\widehat{\rho}}$ increase even further. ■

References

[1] Battigalli, P., and M. Dufwenberg, 2009, "Dynamic psychological games," *Journal of Economic Theory*, 144(1), 1-35.

[2] Battigalli, P., and M. Siniscalchi, 2002, "Strong belief and forward induction reasoning," *Journal of Economic Theory*, 106(2), 356–391.

[3] Brooks, A.W., F. Gino, and M.E. Schweitzer, 2015, "Smart people ask for (my) advice: Seeking advice boosts perceptions of competence," *Management Science*, 61(6), 1421–1435.

[4] DePaulo, B., and J. Fisher, 1980, "The cost of asking for help," *Basic and Applied Social Psychology*, 1(1), 23–35.

[5] Effinger, M.R., and M.K. Polborn, 2001, "Herding and anti-herding: A model of reputational differentiation," *European Economic Review*, 45(3), 385–403.

[6] Ely, J.C., and J. Välimäki, 2003, "Bad Reputation," *The Quarterly Journal of Economics*, 118(3), 785–814.

[7] Klein, N., and T. Mylovanov, 2014, "Will truth out?—An advisor's quest to appear competent," unpublished paper.

[8] Lee, F., 1997, "When the going gets tough, do the tough ask for help? Help seeking and power motivation in organizations," *Organizational Behavior and Human Decision Processes*, 72(3), 336–363.

[9] Lee, F., 2002, "The social costs of seeking help," *Journal of Applied Behavioral Science*, 38(1), 17–35.

[10] Levy, G., 2004, "Anti-herding and strategic consultation," *European Economic Review*, 48(3), 503–525.

[11] Morris, S., 2001, "Political Correctness," *Journal of Political Economy*, 109(2), 231–265.

[12] Ottaviani, M., and P.N. Sørensen, 2001, "Information Aggregation in Debate: Who Should Speak First?" *Journal of Public Economics*, 81(3), 393–421.

[13] Ottaviani, M., and P.N. Sørensen, 2006a, "Professional advice," *Journal of Economic Theory*, 126(1), 120–142.

[14] Ottaviani, M., and P.N. Sørensen, 2006b, "Reputational Cheap Talk," *RAND Journal of Economics*, 37(1), 155–175.

[15] Prat, A., 2005, "The Wrong Kind of Transparency," *American Economic Review*, 95(3): 862–877.

[16] Prendergast, C., and L. Stole, 1996, "Impetuous Youngsters and Jaded Old-Timers: Acquiring a Reputation for Learning," *Journal of Political Economy*, 104(6), 1105–1134.

[17] Scharfstein, D.S., and J.C. Stein, 1990, "Herd Behavior and Investment," *American Economic Review*, 80(3), 465–479.

[18] Suurmond, G., O.H. Swank, and B. Visser, 2004, "On the bad reputation of reputational concerns," *Journal of Public Economics*, 88(12), 2817–2838.

[19] Trueman, B., 1994, "Analyst Forecasts and Herding Behavior," *Review of Financial Studies*, 7(1), 97–124.