Information Acquisition under Persuasive Precedent versus Binding Precedent (Preliminary and Incomplete)

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Abstract

We analyze a dynamic model of judicial decision making. A court regulates a set of activities by permitting or banning them. In each period a new case arises and the appointed judge has to decide whether the case should be permitted or banned. The judge is uncertain about the correct ruling until she conducts a costly investigation.

We compare two institutions: persuasive precedent and binding precedent. Under persuasive precedent, the judge is not required to follow previous rulings but can use the information acquired in an investigation made in a previous period. Under binding precedent, however, the judge must follow previous rulings when they apply. In both a three-period model and an infinite-horizon model, we find that the incentive to investigate for the judge is stronger in earlier periods when there are few precedents under binding precedent than under persuasive precedent, but as more precedents are established over time, the incentive to investigate becomes weaker under binding precedent. Even though the judge's dynamic payoff is always higher under persuasive precedent, social welfare can be higher under binding precedent because of the more intensive investigation conducted early on.

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1 Introduction

We analyze a dynamic model of judicial decision making under uncertainty. A court regulates a set of activities by permitting or banning them. In each period a new case arises which must be decided by a judge. The judge is uncertain about the correct ruling until a costly investigation is made. Following Baker and Mezzetti [2012], we assume that the judge can either investigate the case before making a ruling or summarily decide without a careful investigation.

We compare two institutions: persuasive precedent and binding precedent. Under persuasive precedent, the judge is not required to follow previous rulings but can use the information acquired by the investigation of a previous case. Under binding precedent, however, the judge must follow previous rulings when they apply.

We study the judge's incentives first in a simple three-period model and then in an infinite-horizon model. In both models, we find that in early periods, when few precedents have been established, the incentives to acquire information for the judge is stronger under binding precedent than under persuasive precedent. But as more precedents are established over time, the incentive to acquire information for the judge becomes weaker under binding precedent than under persuasive precedent.

To see why, note that the cost of making a wrong summary decision is higher under binding precedent than under persuasive precedent since in the future, the judge has to follow precedents when they are binding even if previous rulings turn out to be erroneous. Because of the long-run repercussions of an early erroneous ruling when precedents are binding, a judge who faces few precedents is more inclined to investigate to avoid making mistakes. As more precedents are established over time, however, the value of information acquired through investigation becomes lower under binding precedent since the judge may not be able to use the information to make rulings, and this discourages the judge from acquiring information under binding precedent.

Since binding precedent places constraints on what the judge's rulings can be once certain precedents are established, the judge's dynamic payoff is always higher under persuasive precedent than under binding precedent. However, since the court's rulings have broad implications that affect the society at large, it would be misleading to use just the judge's payoff to measure social welfare. Although it is not obvious what the social welfare criterion should be since the judge is the only agent explicitly modeled in our paper, if the investigation cost that the judge privately bears is small relative to the social implications of her rulings, then a reasonable measure of social welfare may be simply the payoffs coming from the ruling decisions. When we use this as our welfare measure, we find that the social welfare can be higher under binding precedent than under persuasive precedent. This happens when the social benefit coming from the more intensive investigation that the court conducts early on outweighs the loss coming from the persistent mistakes in ruling that can potentially arise under binding precedent.

One way to view judicial decision-making is that it involves a principal-agent problem in which the society delegates important decisions to courts. As is typical in principal-agent relationships, the agent may have her own self-interest that conflicts with the task given by the principal. The conflict we focus on is the private cost that the judge has to bear in order to make socially sound decisions, and our result shows that binding precedent may be an effective way to increase the judge's incentive to gather information and improve the quality of the rulings.

Related Literature

Landes and Posner [1976], Schwartz [1992], Rasmusen [1994], Daughety and Reinganum [1999], Talley [1999], Bueno De Mesquita and Stephenson [2002], Gennaioli and Shleifer [2007], Baker and Mezzetti [2012], Ellison and Holden [2014], Anderlini, Felli, and Riboni [2014], Callander and Hummel [2014], Callander and Clark [forthcoming], Li [2001], Szalay [2005].

2 Model

A court regulates a set of activities by permitting or banning them. In each period, a new case arises which must be decided by the appointed judge. The judge prefers to permit activities that she regards as beneficial and ban activities which she regards as harmful. Specifically, denote a case by $x \in [0, 1]$. The judge has a threshold value $\theta \in [0, 1]$ such that she regards case x as beneficial and would like it to be permitted if and if $x \leq \theta$. The preference parameter θ is unknown initially, and we assume that θ is distributed according to a continuous cumulative distribution function F. The support of θ is $[\underline{\theta}, \overline{\theta}]$ with $\underline{\theta} < \overline{\theta}$.

In period $t \in \{1, ...\infty\}$, a case x_t randomly arises according to a continuous cumulative distribution function G on $[0, 1]$. We assume that the cases are independent across periods. The precedent at time t is captured by two numbers L_t and R_t where L_t is the highest case that was ever permitted and R_t is the lowest case that was ever banned

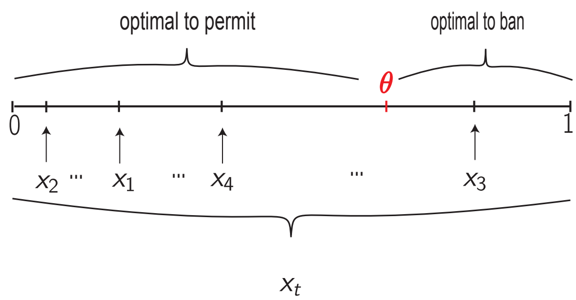

Figure 1: Random arrival of cases.

by time t. Assume that $(L_1, R_1) = (0, 1)$, that is, the precedent at the beginning of the first period is consistent with the judge's preferences and does not impose any mistake in ruling.

The timing of the events is as follows. In period t, after case x_t is brought to the court, the judge chooses whether to investigate the case or not before deciding whether to permit it or ban it.[1] For tractability, suppose that an investigation allows the judge to learn the value of θ at a fixed cost $z > 0$. If the case is decided without an investigation, we say the judge made a summary decision.[2]

Let $s = ((L, R), x)$. In what follows, for expositional convenience, we refer to s as the state even though it does not include the information about θ. Let S denote the set of possible states, i.e., $S = [0, 1]^3$.

Denote the ruling at time t by $r_t \in \{0, 1\}$, where $r_t = 0$ if the case is banned and $r_t = 1$ if the case is permitted. After the judge makes her ruling, the precedent changes to L_{t+1} and R_{t+1}. If x_t was permitted, then $L_{t+1} = \max\{L_t, x_t\}$ and $R_{t+1} = R_t$; if x_t was banned, then $L_{t+1} = L_t$ and $R_{t+1} = \min\{R_t, x_t\}$. Formally, the transition of the precedent is captured by the function $\pi : S \times \{0, 1\} \to [0, 1]^2$ where

$$\pi(s_t, r_t) = \begin{cases} (L_t, \min\{R_t, x_t\}) & \text{if } r_t = 0 \\ (\max\{L_t, x_t\}, R_t) & \text{if } r_t = 1. \end{cases} \tag{1}$$

[1]We assume that the judge learns about her preference parameter θ through investigation. Alternatively, we can assume that the judge learns about her preferences in terms of the consequences of cases, but does not know the consequence of a particular case unless she investigates. To illustrate, let $c(x)$ denote the consequence of a case x and assume that $c(x) = x + \gamma$. The judge would like to permit case x if $c(x)$ is below some threshold $\bar{c}$ and would like to ban it otherwise. Suppose that the judge knows $\bar{c}$ and observe x, but γ is unknown until the judge investigates. This alternative model is equivalent to ours.

[2]For expositional simplicity, we assume that the judge investigates the case when indifferent.

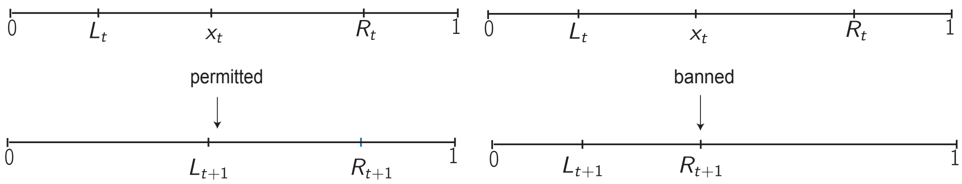

Figure 2: Evolution of precedents.

We consider two institutions: *persuasive precedent* and *binding precedent*. Under persuasive precedent, the judge is free to make any ruling, and the role of the precedent is potentially to provide information regarding whether the case is beneficial or not. Under binding precedent, the judge must permit x_t in period t if $x_t \leq \min\{L_t, R_t\}$ and must ban x_t if $x_t \geq \max\{L_t, R_t\}$. To understand this assumption, note that if $x_t \leq \min\{L_t, R_t\}$, then there must be some case higher than x_t that was permitted in the past and there is no case lower than x_t that was banned in the past. When the precedent is binding, the only ruling that is consistent with precedent in this case is to permit x_t. Similarly, if $x_t \geq \max\{L_t, R_t\}$, then there must be some case lower than x_t that was banned in the past and there is no case higher than x_t that was permitted in the past. Therefore, binding precedent requires that the judge bans x_t. More generally, we say that a ruling regarding x *violates* precedent (L, R) if $x \leq \min\{L, R\}$ and the judge bans x or if $x \geq \max\{L, R\}$ and the judge permits x. We can think of the cost of violating precedent to be infinite when it is binding and zero when it is persuasive. We focus on these two extremes to highlight the difference in the incentives that the judge faces. In Section 5, we discuss an extension in which the cost of violating precedent is positive but not infinite, reflecting different degrees of "bindingness" of precedent in different situations.

Note that under binding precedent, we always have $L_t < R_t$ on the equilibrium path. But off the equilibrium path, we may have $R_t < L_t$; in this case, for $x_t \in (R_t, L_t)$, the judge can either permit or ban x_t even under binding precedent. This is because if the judge permits x_t, the ruling is still supported by precedent since there is a higher case that has been permitted before, and if the judge bans x_t, the ruling is also supported by precedent since there is a lower case that has been banned before. Let S^p denote the set of possible precedents that can arise on the equilibrium path under binding precedent, $S^p = \{(L, R) \in [0, 1]^2 : L < R\}$.[3]

[3]Another way to formalize how binding precedent affects the decision problem is to assume that

The payoff of the judge from the ruling r_t on case x_t in period t is given by

$$u(x_t, \theta, r_t) = \begin{cases} 0 & \text{if } x_t \leq \theta \text{ and } r_t = 1, \text{ or } x_t \geq \theta \text{ and } r_t = 0, \\ -\ell(x_t, \theta) & \text{otherwise,} \end{cases}$$

where $\ell(x_t, \theta) > 0$ for $x_t \neq \theta$ is the cost of making a mistake, that is, permitting a case when it is above θ or banning a case when it is below θ. Assume that $\ell(x, \theta)$ is continuous in x and θ for $x \neq \theta$,[4] strictly increasing in x and strictly decreasing in θ if $x > \theta$ and strictly decreasing in x and strictly increasing in θ if $x < \theta$. For example, if $\ell(x, \theta) = f(|x - \theta|)$ where $f(y) : \mathbb{R}^+ \to \mathbb{R}^+$ is continuous for $y > 0$, strictly increasing, and $f(0) = 0$, then these assumptions are satisfied.

The dynamic payoff of the judge is the sum of her discounted payoffs from the rulings made in each period net of the cost of violating a precedent and net of the investigation cost if the judge carries out one, appropriately discounted. The discount factor is denoted by $\delta \in (0, 1)$.

Persuasive precedent

In the model with persuasive precedent, the payoff-relevant state in any period is the realized case $x \in [0, 1]$ and the information about θ.

If θ is known at the time when the relevant decisions are made, then it is optimal not to investigate the case for any $x \in [0, 1]$ and it is optimal to permit x if $x < \theta$ and to ban x if $x > \theta$.

If θ is unknown at the time when the relevant decisions are made, a policy for the judge is a pair of functions $\sigma_P = (\mu_P, \rho_P)$, where $\mu_P : [0, 1] \to \{0, 1\}$ is an investigation policy and $\rho_P : [0, 1] \to \{0, 1\}$ is an uninformed ruling policy, with $\mu_P(x) = 1$ if and only if an investigation is made when the case is x and $\rho_P(x) = 1$ if and only if case x is permitted.

For each policy $\sigma_P = (\mu_P, \rho_p)$, let $V_P(\cdot; \sigma_P)$ be the associated value function, that is, $V_P(x; \sigma_P)$ represents the dynamic payoff of the judge when she is uninformed, faces case x in the current period, and follows the policy σ_P. In what follows, we suppress the dependence of the dynamic payoffs on σ_P for notational convenience. For notational

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the set of feasible actions depends on the precedent. Specifically, under binding precedent (L_t, R_t), if $x \leq L_t$, then the only feasible ruling r_t is 1, and if $x \geq R_t$, then the only feasible ruling r_t is 0. Under these assumptions on feasible actions, $L_t < R_t$ always holds. One advantage of modeling binding precedent as imposing infinite cost of violating precedent is that it allows us to easily extend the framework to allow for positive but bounded cost of violating precedent, as we discuss in Section 5.

[4] We allow there to be a discontinuity at $x = \theta$ to reflect a fixed cost of making a mistake in ruling.

convenience, let $EV_P = \int_0^1 V_P(x')dG(x')$.

The policy σ_P^* is optimal if σ_P^* and the associated value function V_P^* satisfy the following conditions:

(P1) The uninformed ruling policy satisfies $\rho_P^*(x) = 1$ if

$$\int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) \geq \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta)$$

and $\rho_P^*(x) = 0$ if

$$\int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) < \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta)$$

for any case x.[5]

(P2) Given V_P^* and the uninformed ruling policy ρ_P^*, the investigation policy for the uninformed judge satisfies $\mu_P^*(x) = 1$ if and only if

$$-z \geq \rho_P^*(x) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) + (1-\rho_P^*(x)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_P^*.$$

(P3) Given σ^*, for any state s, the dynamic payoff satisfies

$$\begin{aligned} V_P^*(x) &= -z\mu_P^*(x) + (1-\mu_P^*(x)) \left[\rho_P^*(x) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) \right. \\ &+ \left. (1-\rho_P^*(x)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_P^* \right]. \end{aligned}$$

Condition (P1) says that the the ruling decision depends on only the current period payoff, and in particular, the judge chooses the ruling that minimizes the expected cost of making a mistake in the current period. This is because under persuasive precedent, the ruling does not affect the judge's continuation payoff. Condition (P2) says that when uninformed, the judge chooses to investigate a case if and only if her dynamic payoff from investigating is higher than her expected dynamic payoff from not investigating. If a judge investigates case x, then θ becomes known and no mistake

[5]For expositional convenience, we assume that when the judge is indifferent between permitting and banning a case, she permits it.

in ruling will be made in the current period as well as in the future. In this case, the dynamic payoff of the judge is negative of the cost of investigation. If a judge does not investigate case x, then her dynamic payoff is the sum of the expected cost of making a mistake in the current period and the continuation payoff. This payoff is the value function given in condition (P3).

Binding precedent

In the model with binding precedent, the payoff-relevant state in any period is the precedent pair (L, R), the realized case x, and the information about θ.[6]

If θ is known at the time when the relevant decisions are made, then it is optimal not to investigate; moreover, since the precedent is binding, it is optimal to permit x if $x < \max\{L, \theta\}$ and to ban x if $x > \min\{R, \theta\}$. Let $C(L, R)$ denote the expected dynamic payoff of the judge when the precedent is (L, R), conditional on θ being known when decisions regarding the cases are made where the expectation is taken over θ before it is revealed and over all future cases x. Formally

$$C(L,R) = \frac{1}{1-\delta}\left[\int_{\mathcal{L}}\int_{\theta}^{L} -\ell(x,\theta)dG(x)dF(\theta) + \int_{\mathcal{R}}\int_{R}^{\theta} -\ell(x,\theta)dG(x)dF(\theta)\right], \quad (2)$$

where $\mathcal{L}$ is the (possibly degenerate) interval $[\underline{\theta}, \max\{L, \underline{\theta}\}]$ and $\mathcal{R}$ is the (possibly degenerate) interval $[\min\{R, \bar{\theta}\}, \bar{\theta}]$. Equivalently,

$$C(L,R) = \begin{cases} 0 & \text{if } L \leq \underline{\theta} \text{ and } R \geq \bar{\theta} \\ \frac{1}{1-\delta}\left[\int_{\underline{\theta}}^{L}\int_{\theta}^{L} -\ell(x,\theta)dG(x)dF(\theta) + \int_{R}^{\bar{\theta}}\int_{R}^{\theta} -\ell(x,\theta)dG(x)dF(\theta)\right] & \text{if } L > \underline{\theta} \text{ and } R < \bar{\theta} \\ \frac{1}{1-\delta}\left[\int_{\underline{\theta}}^{L}\int_{\theta}^{L} -\ell(x,\theta)dG(x)dF(\theta)\right] & \text{if } L > \underline{\theta} \text{ and } R \geq \bar{\theta} \\ \frac{1}{1-\delta}\left[\int_{R}^{\bar{\theta}}\int_{R}^{\theta} -\ell(x,\theta)dG(x)dF(\theta)\right] & \text{if } L \leq \underline{\theta} \text{ and } R < \bar{\theta} \end{cases}$$

To see how we derive $C(L, R)$, note that if $\theta < L$ and $x \in (\theta, L]$, then the judge incurs a cost of $-\ell(x, \theta)$ since she has to permit x; similarly, if $\theta > R$ and $x \in [R, \theta)$, then the judge incurs a cost of $-\ell(x, \theta)$ since she has to ban x. It follows that the expected per-period payoff of a judge conditional on θ being known is $\int_{\mathcal{L}}\int_{\theta}^{L} -\ell(x,\theta)dG(x)dF(\theta) + \int_{\mathcal{R}}\int_{R}^{\theta} -\ell(x,\theta)dG(x)dF(\theta)$, and her dynamic payoff in the infinite horizon model is $1/(1-\delta)$ times the per-period payoff. Note that $\max\{L, \underline{\theta}\}$ is

[6] For expositional simplicity, we consider precedents with $L < R$ in our analysis of binding precedent. Under binding precedent, this must happen on the equilibrium path and the analysis is without loss of generality.

increasing in L and $\min\{R, \bar{\theta}\}$ is decreasing in R, and therefore $C(L, R)$ is decreasing in L and increasing in R.

If θ is unknown at the time when the decisions regarding the cases are made, a policy for the judge is a pair of functions $\sigma_B = (\mu_B, \rho_B)$, where $\mu_B : S \to \{0, 1\}$ is an investigation policy and $\rho_B : S \to \{0, 1\}$ is an uninformed ruling policy, where $\mu_B(s) = 1$ if and only if an investigation is made when the state is s, and $\rho_B(s) = 1$ if and only if case x is permitted when the state is s.

Let $A(s)$ denote the judge's dynamic payoff if she investigates in state $s = ((L, R), x)$, not including the investigation cost. Formally,

$$A(s) = \mathbf{1}_{\mathcal{L}}(x) \int_{\underline{\theta}}^{x} -\ell(x, \theta) dF(\theta) + \mathbf{1}_{\mathcal{R}}(x) \int_{x}^{\bar{\theta}} -\ell(x, \theta) dF(\theta) + \delta C(L, R).$$

For each policy $\sigma_B = (\mu_B, \rho_B)$, let $V_B(\cdot; \sigma_B)$ denote the associated value function, that is, $V_B(s; \sigma_B)$ represents the dynamic payoff of the judge when the state is s, θ is unknown, and she follows the policy σ_B. In what follows, we suppress the dependence V_B on σ_B for notational convenience. For notational convenience, let $EV_B(L, R) = \int_0^1 V_B(L, R, x') dG(x')$.

Recall that the transition of precedent is captured by the function π, defined in (1). The policy σ_B^* is optimal if σ_B^* and the associated value function V_B^* satisfy the following conditions:

(B1) Given V_B^*, the uninformed ruling policy satisfies $\rho_B^*(s) = 1$ if either $x \leq L$ or $x \in (L, R)$ and

$$\begin{aligned} &\int_{\underline{\theta}}^{\max\{x, \underline{\theta}\}} -\ell(x, \theta) dF(\theta) + \delta EV_B^*(\pi(s, 1)) \\ \geq &\int_{\min\{x, \bar{\theta}\}}^{\bar{\theta}} -\ell(x, \theta) dF(\theta) + \delta EV_B^*(\pi(s, 0)), \end{aligned}$$

and $\rho_B^*(s) = 0$ if either $x \geq R$ or $x \in (L, R)$ and

$$\begin{aligned} &\int_{\underline{\theta}}^{\max\{x, \underline{\theta}\}} -\ell(x, \theta) dF(\theta) + \delta EV_B^*(\pi(s, 1)) \\ < &\int_{\min\{x, \bar{\theta}\}}^{\bar{\theta}} -\ell(x, \theta) dF(\theta) + \delta EV_B^*(\pi(s, 0)), \end{aligned}$$

for any state s.

(B2) Given V_B^* and the uninformed ruling policy ρ_B^*, for any state s, the investigation policy for the uninformed judge satisfies $\mu_B^*(s) = 1$ if and only if

$$-z + A(s) \geq \rho_B^*(s) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta)$$
$$+(1-\rho_B^*(s)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV^*(\pi(s,\rho^*(s)))$$

(B3) Given σ^*, for any state s, the dynamic payoff satisfies

$$\begin{aligned} V_B^*(s) &= \mu_B^*(s)\left[-z + A(s)\right] \\ &+ (1-\mu_B^*(s)) \left[\rho_B^*(s) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta)\right. \\ &+ \left.(1-\rho_B^*(s)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_B^*(\pi(s,\rho^*(s)))\right]. \end{aligned}$$

Under binding precedent, the ruling decision may change the precedent, which in turn may affect the continuation payoff. As such, condition (B1) says the ruling decision depends on both the current period payoff and the continuation payoff. In particular, the judge chooses the ruling that maximizes the sum of the current period payoff and the continuation payoff, taking into consideration how her ruling affects the precedent in the next period. Condition (B2) says that the judge chooses to investigate a case if and only if her dynamic payoff from investigating is higher than her expected dynamic payoff from not investigating.

If a judge investigates case x, then θ becomes known. When the precedents are binding, however, mistakes in ruling can still happen if $\theta < L$ or if $\theta > R$. In this case, the dynamic payoff $V_B^*(s)$ is the expected cost of making mistakes in the ruling, both in the current period and in future periods, minus the cost of investigation. If a judge does not investigate case x, then her dynamic payoff is the sum of the expected cost of making a mistake in the current period and the continuation payoff. Condition (B3) formalizes this.

3 A three-period model

Before we analyze the infinite-horizon model, we discuss a three-period model to illustrate some of the intuition.

3.1 Persuasive Precedent

Consider the judge in period t. If the judge has investigated in a previous period, then θ is known and the judge permits or bans case x_t according to θ. If the judge has not investigated in a previous period, then her belief about θ is the same as the prior. If she decides to investigate in period t, her payoff is $-z$ in period t and 0 in future periods. The following result says that for an uninformed judge, there exists a threshold in $(\underline{\theta}, \bar{\theta})$ such that she permits x_t if it is below this threshold and bans x_t if it is above this threshold.

Lemma 1. *Under persuasive precedent, there exists $\hat{x} \in (\underline{\theta}, \bar{\theta})$ such that if the judge is uninformed, she permits x_t if $x_t \leq \hat{x}$ and bans x_t if $x_t > \hat{x}$.*

Now we analyze the judge's investigation decisions. The following lemma says that when the investigation cost is sufficiently low, the uninformed judge investigates with positive probability in each period, the cases that she investigates in period t forms an interval, and the interval of investigation is larger in an earlier period. Intuitively, for the cases that fall in the middle, it is less clear to a judge whether she should permit it or ban it and the expected cost of making a mistake is higher. Hence, the value of investigation for these cases is higher. Moreover, since the judge can use the information she acquires in an earlier period for later periods, the value of investigation is higher in an earlier period, resulting in more cases being investigated in an earlier period.

Lemma 2. *In the three-period model under persuasive precedent, there exists $z^* > 0$ such that if $z < z^*$, the uninformed judge investigates x_t ($t = 1, 2, 3$) with positive probability in equilibrium. Specifically, there exist x_t^L and $x_t^H > x_t^L$ such that the uninformed judge investigates x_t in period t if and only if $x_t \in [x_t^L, x_t^H]$. Moreover, $x_1^L < x_2^L < x_3^L$ and $x_3^H < x_2^H < x_1^H$.*

3.2 Binding precedent

We first show that in each period t, the cases that the judge optimally investigates form a (possibly degenerate) interval under binding precedent as well.

Lemma 3. *Under binding precedent, the set of cases that the uninformed judge investigates in period t is convex for any precedent (L_t, R_t); if $x_t \notin (L_t, R_t)$, then the judge does not investigate x_t in period t.*

In the next proposition, we show that the judge investigates more under binding precedent than under persuasive precedent in period 1, but she investigate less under binding precedent than under persuasive precedent in periods 2 and 3.

Proposition 1. *The judge investigates less under binding precedent than under persuasive precedent in period 3. Specifically, for any precedent (L_3, R_3), the uninformed judge investigates x_3 if and only if $x_3 \in (L_3, R_3) \cap [x_3^L, x_3^H]$.*

The judge also investigates less under binding precedent than under persuasive precedent in period 2. Specifically, if $[\underline{\theta}, \bar{\theta}] \subseteq [L_2, R_2]$, then the set of cases that the uninformed judge investigates in period 2 under binding precedent is the same as $[x_2^L, x_2^H]$; otherwise the set of cases she investigates under binding precedent is a subset of $[x_2^L, x_2^H]$.

The judge investigates more under binding precedent than under persuasive precedent in period 1, that is, $[x_1^L, x_1^H]$ is a subset of the set of cases that the uninformed judge investigates under binding precedent in period 1.

The reason for the judge to investigate less in period 3 under binding precedent is that investigation has no value if $x_3 \leq L_3$ or if $x_3 \geq R_3$ since the judge must permit any $x_3 \leq L_3$ and must ban any $x_3 \geq R_3$ no matter what the investigation outcome is; moreover, since period 3 is the last period, the information about θ has no value for the future either. For $x_3 \in (L_3, R_3)$, the judge faces the same incentives under binding and persuasive precedent and therefore investigates the same set of cases.

If the precedent in period 2 satisfies $[\underline{\theta}, \bar{\theta}] \subseteq [L_2, R_2]$, then investigation avoids mistakes in ruling in the current period as well as the future period even under binding precedent. In this case, the judge faces the same incentives under binding and persuasive precedent and therefore investigates the same set of cases. However, if the precedent in period 2 does not satisfy $[\underline{\theta}, \bar{\theta}] \subseteq [L_2, R_2]$, then even if $x_2 \in (L_2, R_2)$ and the judge investigates, mistakes in ruling can still happen in period 3 under binding precedent if $\theta \notin [L_2, R_2]$ since the judge is bound to follow the precedent. In this case, the value of investigation is lower under binding precedent than under persuasive precedent and therefore the judge investigates less under binding precedent.

Since the precedent in period 1 satisfies $[\underline{\theta}, \bar{\theta}] \subseteq [L_1, R_1]$, investigation avoids mistakes in ruling in the current period as well as in future periods even under binding precedent. However, for $x_1 \in (\underline{\theta}, \bar{\theta})$, if the judge does not investigate x_1 and makes a summary ruling, then she changes the precedent in a way that $[\underline{\theta}, \bar{\theta}] \not\subseteq [L_2, R_2]$. As discussed in the previous paragraph, the binding precedent arising from a summary ruling potentially results in mistakes in the future and diminishes the judge's incentive to investigate future periods, which in turn lowers the judge's dynamic payoff. Hence, the judge's payoff from not investigating in period 1 is lower under binding precedent than under persuasive precedent, and therefore she has a stronger incentive to investigate under binding precedent.

4 Infinite-horizon model

We now consider the infinite-horizon model, that is, $T = \infty$. We first show that the judge's value functions and optimal policies as defined in (P1-P3) and (B1-B3) are unique.

Proposition 2. *Under either persuasive precedent or binding precedent, the judge's optimal policy is unique.*

To prove this, we first apply the Contraction Mapping Theorem to show that the value functions V_P^* and V_B^* are unique. The optimality conditions (P1-P2) and (B1-B2) then uniquely determine the optimal policies σ_P^* and σ_B^*. We next turn to the characterization of the value function and optimal policy, first under persuasive precedent and then under binding precedent.

4.1 Persuasive precedent

If the judge already investigated in a previous period, then she knows the value of θ and would permit or ban a case according to θ. We next show that the set of cases that the uninformed judge investigates in period t is convex.

Proposition 3. *Under persuasive precedent, if it is optimal for the judge to investigate x_1 and $x_2 > x_1$ in period t, then it is optimal for her to investigate any $x \in [x_1, x_2]$.*

Let $M_P = \{x : \mu_P^*(x) = 1\}$, that is, M_P is the set of cases that the uninformed judge investigates under persuasive precedent. Let $\hat{x}$ be such that $\int_{\underline{\theta}}^{\hat{x}} \ell(x,\theta)dF(\theta) = \int_{\hat{x}}^{\bar{\theta}} \ell(x,\theta)dF(\theta)$, and let $\hat{z} = \int_{\underline{\theta}}^{\hat{x}} \ell(x,\theta)dF(\theta)$. If $M_P \neq \varnothing$, let $a_P = \inf\{x : \mu_P^*(x) = 1\}$ and $b_P = \sup\{x : \mu_P^*(x) = 1\}$. We next show that if the judge faces a case such that there is no uncertainty about what the correct ruling is for that case (that is, if $x \leq \underline{\theta}$ or if $x \geq \bar{\theta}$), then the judge does not investigate the case, even though the information from investigation is valuable for future rulings. The uninformed judge investigates with positive probability if the investigation cost is below the threshold $\hat{z}$. In that case, the uninformed judge permits any case below a_P and bans any case above b_P.

Proposition 4. *Under persuasive precedent, the judge does not investigate any case $x \notin (\underline{\theta}, \bar{\theta})$. If $z > \hat{z}$, then $M_P = \varnothing$, and the uninformed judge permits x if $x \leq \hat{x}$ and bans x otherwise. If $z \leq \hat{z}$, then $M_P = [a_P, b_P] \neq \varnothing$ and the uninformed judge permits x if $x < a_P$ and bans x if $x > b_P$.*

Suppose $M_P \neq \varnothing$. Recall that $EV_P^* = \int_0^1 V_P^*(x')dG(x')$. To characterize the optimal policy and the value function, note that

$$V_P^*(x) = \begin{cases} \delta EV_P^* & \text{if } x \leq \underline{\theta}, \text{ or if } x \geq \bar{\theta}, \\ \int_{\underline{\theta}}^{x} -\ell(x,\theta)dF(\theta) + \delta EV_P^* & \text{if } \underline{\theta} < x < a_P, \\ -z & \text{if } x \in [a_P, b_P], \\ \int_{x}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_P^* & \text{if } b_P < x < \bar{\theta}. \end{cases} \tag{3}$$

To see how we derive this, note that if $x \leq \underline{\theta}$ or if $x \geq \bar{\theta}$, then the judge does not investigate and makes no mistake in her ruling in the current period. In this case, her current-period payoff is 0 and her continuation payoff is δEV_P^*. If $\underline{\theta} < x < a_P$ or if $b_P < x < \bar{\theta}$, the judge does not investigate in the current period and incurs some cost of making a mistake in expectation. Since θ remains unknown, her continuation payoff is δEV_P^*. If $x \in [a_P, b_P]$, then the judge investigates. Since she makes no mistake in her ruling both in the current period and in all future periods, her current period payoff is $-z$ and her continuation payoff is 0.

Hence, we have

$$\begin{aligned} EV_P^* = & -z[G(b_P) - G(a_P)] + \delta EV_P^* [G(a_P) + 1 - G(b_P)] \\ & + \int_{\underline{\theta}}^{a_P} \int_{\underline{\theta}}^{x} -\ell(x,\theta)dF(\theta)dG(x) + \int_{b_P}^{\bar{\theta}} \int_{x}^{\bar{\theta}} -\ell(x,\theta)dF(\theta)dG(x). \end{aligned}$$

For any a, b such that $\underline{\theta} < a \leq b < \bar{\theta}$, let $h(a,b) = \int_{\underline{\theta}}^{a} \int_{\underline{\theta}}^{x} -\ell(x,\theta)dF(\theta)dG(x) + \int_{b}^{\bar{\theta}} \int_{x}^{\bar{\theta}} -\ell(x,\theta)dF(\theta)dG(x)$. Intuitively, $h(a,b)$ is the expected cost of making a mistake before the realization of the case when the judge's investigation interval is $[a,b]$. Then

$$EV_P^* = \frac{h(a_P, b_P) - z[G(b_P) - G(a_P)]}{1 - \delta[G(a_P) + 1 - G(b_P)]} \tag{4}$$

Since the judge is indifferent between investigating and not investigating when $x = a_P$, we have

$$-z = \int_{\underline{\theta}}^{a_P} -\ell(a_P, \theta)dF(\theta) + \delta EV_P^*. \tag{5}$$

Similarly, since the judge is indifferent between investigating and not investigating when $x = b_P$, we have

$$-z = \int_{b_P}^{\bar{\theta}} -\ell(b_P, \theta)dF(\theta) + \delta EV_P^*. \tag{6}$$

We can solve for EV_P^*, a_P, b_P from equations (4), (5), and (6). Plugging these in (3), we can solve for $V_P^*(x)$.

4.2 Binding precedent

We now consider binding precedent. We first establish that the value function V_B^* is decreasing in L and increasing in R and the optimal investigation policy μ^* is also decreasing in L and increasing in R. This result says that as the precedent gets tighter, the judge investigates less and her payoff also becomes lower. Recall that $EV_B^*(L,R) = \int_0^1 V_B^*(L,R,x')dG(x')$. The proposition also establishes the continuity of EV_B^*.

Proposition 5. *Suppose the precedent $(\hat{L}, \hat{R})$ is tighter than (L,R), that is, $L \leq \hat{L} < \hat{R} \leq R$. Under binding precedent, for any case $x \in [0,1]$, if the judge investigates x under precedent $(\hat{L}, \hat{R})$, then she also investigates x under precedent (L,R), that is, $\mu_B^*(L,R,x)$ is decreasing in L and increasing in R. Moreover, the value function $V_B^*(L,R,x)$ is decreasing in L and increasing in R, and $EV_B^*(L,R)$ is continuous in L and R for any $(L,R) \in S^p$.*

We next show that the set of cases that the judge investigates is convex, and the judge does not investigate any case for which she must follow the precedent in her ruling of the case.

Proposition 6. *Under binding precedent, for any precedent $(L, R) \in S^p$, (i) if the judge investigates case x_1 and $x_2 > x_1$ in period t, then she also investigates any $x \in [x_1, x_2]$ in period t, and (ii) the judge does not investigate x if $x \leq L$ or if $x \geq R$.*

For any $(L, R) \in S^p$ such that $\{x : \mu_B^*(L, R, x) = 1\} \neq \varnothing$, let $a(L, R) = \inf\{x : \mu_B^*(L, R, x) = 1\}$ and $b(L, R) = \sup\{x : \mu_B^*(L, R, x) = 1\}$. Since the set of cases that the judge investigates is convex, under the precedent (L, R), the judge investigates any case $x \in (a, b)$ and does not investigate any case $x < a$ or any case $x > b$. We refer to $\{x : \mu_B^*(L, R, x) = 1\}$ as the investigation interval under (L, R). If $a(L, R) = L$ and $b(L, R) = R$, then it follows from Proposition 6 that the investigation interval under precedent (L, R) is open. We next show that if $L < a(L, R) < b(L, R) < R$, then the investigation interval under precedent (L, R) is closed.

Suppose $L < a(L, R) < b(L, R) < R$ and consider $x \in (L, R)$. Since $x \in (L, R)$, the judge does not have to follow any binding precedent in her ruling of x. It follows that her expected current-period payoff in making a summary ruling regarding x is continuous in x for $x \in (L, R)$. Since EV_B^* is also continuous by Proposition 5, the judge's dynamic payoff if she makes a summary ruling regarding x is continuous in x for $x \in (L, R)$. Note also that the judge's dynamic payoff if she investigates x is constant in x. It follows the judge is indifferent between investigating and not investigating when $x = a(L, R)$ and when $x = b(L, R)$. Recall that we assume that when the judge is indifferent between investigating and not investigating, she investigates. Hence, if $L < a(L, R) < b(L, R) < R$, then the set of cases that the uninformed judge investigates is the closed interval $[a(L, R), b(L, R)]$.

Suppose that given the initial precedent (L_1, R_1), the set of cases that the uninformed judge investigates is nonempty (if it is empty, then no investigation will be carried out in any period). For notational simplicity, let $a_1 = a(L_1, R_1)$ and $b_1 = b(L_1, R_1)$. Recall that the initial precedent is consistent with the judge's preference, that is, $L_1 < \underline{\theta}$ and $R_1 > \bar{\theta}$. Hence, we have $L_1 < a_1 < b_1 < R_1$, and the uninformed judge investigates x_1 if and only if $x_1 \in [a(L_1, R_1), b(L_1, R_1)]$.

If $x_1 \in [a_1, b_1]$, the judge investigates x_1. In this case, since θ becomes known, no more investigation will be carried out. If $x_1 \notin [a_1, b_1]$, then the judge makes a summary ruling without any investigation and changes the precedent to $(L_2, R_2) = (x_1, R_1)$ if she permits the case and to $(L_2, R_2) = (L_1, x_1)$ if she bans the case. Note that when the judge makes a summary ruling, the resulting new precedent satisfies $L_2 < a_1$ and

$b_1 < R_2$. Monotonicity of μ_B^* in L and R as established in Proposition 5 implies that the investigation interval in period 2, if nonempty, satisfies $a(L_2, R_2) \geq a_1$ and $b(L_2, R_2) \leq b_1$. Therefore we have $L_2 < a(L_2, R_2) \leq b(L_2, R_2) < R_2$ and the judge investigates x_2 if and only if $x \in [a(L_2, R_2), b(L_2, R_2)]$. An iteration of this argument shows that on any realized equilibrium path, the investigation interval is a strict subset of the precedent in any period and closed (possibly empty). Denote a nonempty investigation interval on an equilibrium path by $[a(L^e, R^e), b(L^e, R^e)]$. By Propositions 5 and 6, we have $L^e < a(L^e, R^e) \leq b(L^e, R^e) < R^e$ and given the precedent (L^e, R^e), the judge is indifferent between investigating x and making a summary ruling if $x = a(L^e, R^e)$ or if $x = b(L^e, R^e)$. The investigation intervals either converge to $\varnothing$ or to some nonempty set $[\hat{a}, \hat{b}]$ such that if the precedent is $(L, R) = (\hat{a}, \hat{b})$, then $a(L, R) = \hat{a}$ and $b(L, R) = \hat{b}$.

More formally, we define a *limit investigation interval under binding precedent*, denoted by M_B, as follows. If $\{x : \mu_B^*(L_1, R_1, x) = 1\} = \varnothing$, then $M_B = \varnothing$. If $\{x : \mu_B^*(L_1, R_1, x) = 1\} \neq \varnothing$, then construct a sequence $\{a_n, b_n, L_n, R_n\}$ as follows. Given L_n and R_n, if $\{x : \mu_B^*(L_n, R_n, x) = 1\} \neq \varnothing$, then let $a_n = a(L_n, R_n)$, $b_n = b(L_n, R_n)$ and pick L_{n+1} and R_{n+1} such that $L_n < L_{n+1} < a(L_n, R_n)$, $b(L_n, R_n) < R_{n+1} < R_n$, if then let $a_n = b_n = \frac{L_n + R_n}{2}$, $a_{n+1} = a_n$, $b_{n+1} = b_n$, $L_{n+1} = L_n$, $R_{n+1} = R_n$ and $M_B = \varnothing$. Note that a_n is increasing and b_n is decreasing. Since a monotone and bounded sequence converges, $\lim a_n$ and $\lim b_n$ are well defined. If $\{x : \mu_B^*(L_n, R_n, x) = 1\} \neq \varnothing$ for all n, then let $M_B = (\lim a_n, \lim b_n)$.

We next show that in the first period when the precedent is (L_1, R_1), the judge investigates more under binding precedent than under persuasive precedent. But as more precedents are established over time and the judge has less freedom in making her ruling under binding precedent, eventually the uninformed judge investigates less than under persuasive precedent. Recall that M_P is the set of cases that the uninformed judge investigates under persuasive precedent.

Proposition 7. *We have* $M_B \subseteq M_P \subseteq [a(L_1, R_1), b(L_1, R_1)]$.

Proposition 7 is analogous to Proposition 1 in the three-period model, in that they both say that the judge investigates more under binding precedent than under persuasive precedent early on but investigates less under binding precedent in later periods. The example below illustrates Proposition 7.

Example 1. *Suppose that* θ *is uniformly distributed on* $[0.2, 0.8]$, x *is uniformly distributed on* $[0, 1]$, $\delta = 0.95$, $z = 0.1$ *and* $\ell(x, \theta) = |x - \theta|$.

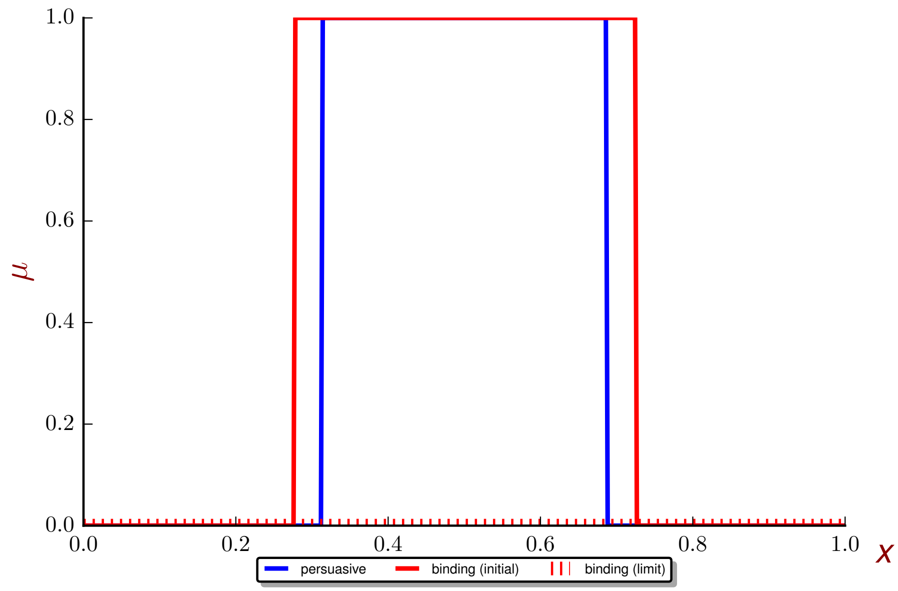

Figure 3: Investigation policies

In Figure 3, the solid blue curve illustrates the uninformed judge's optimal investigation policy under persuasive precedent, the solid red curve illustrates her optimal investigation policy under binding precedent in the initial period with the precedent being $(L_1, R_1) = (0, 1)$, and the dashed red curve illustrates what the uninformed judge's investigation policy converges to under binding precedent.

The figure shows that the judge investigates more under binding precedent in the first period than under persuasive precedent, but as the precedents are established over time, the set of cases that the uninformed judge investigates eventually becomes empty under binding precedent.

4.3 Welfare comparison

Since binding precedent places constraints on what the judge can do in terms of her rulings, clearly the judge's total payoff is higher under persuasive precedent than under binding precedent. However, since the rulings affect the society at large, the judge's payoff does not capture the social welfare completely in the presence of this externality. If the investigation cost that the judge privately bears is small relative to the social implications of her rulings, then a reasonable measure of social welfare may simply be the payoffs coming from the ruling decisions. Formally, we define a social welfare function $V_P^S(x)$ under persuasive precedent and a social welfare function $V_B^S(x, L, R)$ as follows.

Under persuasive precedent, the optimal policy that the judge chooses is given by (μ_P^*, ρ_p^*). If $\mu_P^*(x) = 1$, then the current ruling as well as all future rulings are correct, and therefore $V_P^S(x) = 0$. If $\mu_P^*(x) = 0$, then the social welfare consists of the expected social cost from the potential mistake in ruling today as well the discounted continuation payoff EV_p^S. That is,

$$V_P^S(x) \;=\; \rho^*(x) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) + (1-\rho^*(x)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_P^S.$$

Similarly, under binding precedent, if $\mu_B^*(x) = 1$, then $V_B^S(s) = A(s)$, and if $\mu_B^*(x) = 0$, then

$$\begin{aligned} V_B^S(s) \;&=\; \rho^*(s) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) \\ &+\; (1-\rho^*(s)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_B^S(\pi(s, \rho^*(s))). \end{aligned}$$

Let the expected social welfare under persuasive precedent be $EV_P^S = \int_0^1 V_P^S(x')dG(x')$ and the expected social welfare under binding precedent be $EV_B^S = \int_0^1, V_B^S(L_1, R_1, x')dG(x')$.

Proposition 8. *The expected social welfare under binding precedent can be higher than that under persuasive precedent.*

The next example is an illustration of Proposition 8.

Example 1 (continued). *Figure 4 shows that the judge's dynamic payoff is higher under persuasive precedent than under binding precedent. As to the comparison of social welfare under the two institutions, recall that the judge investigates more intensively in early periods under binding precedent, as illustrated in Figure 3 before. The social benefit from the early intensive investigation is so high that the social welfare is higher under binding precedent, as can be seen in Figure 5.*

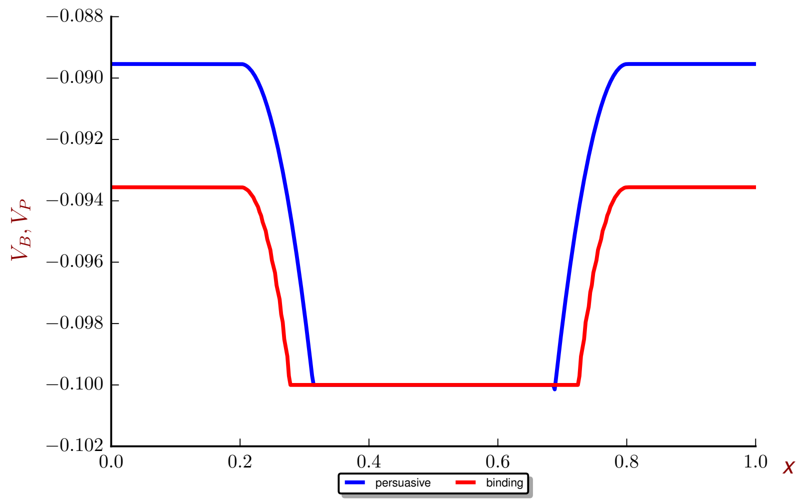

Figure 4: Comparison of the judge's dynamic payoffs

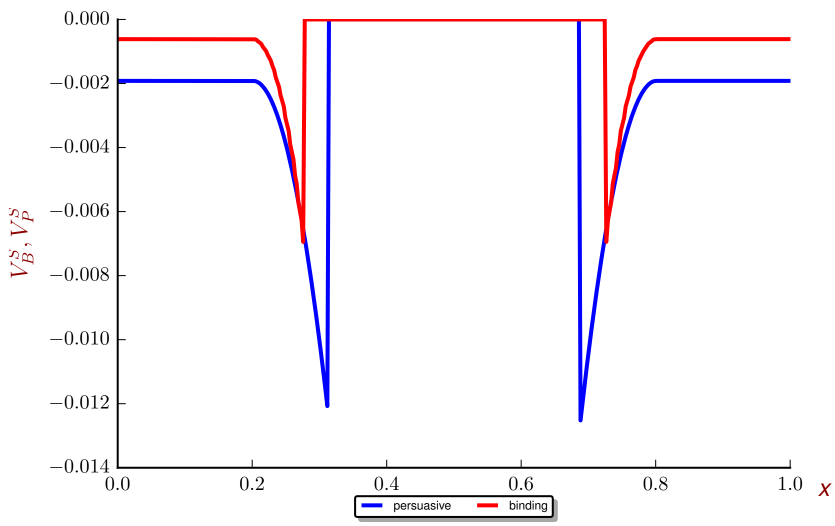

Figure 5: Comparison of social welfare

5 Extension: varying costs of violating precedent (to be completed)

So far we have compared two opposite cases: either it is costless to violate a precedent (persuasive case), or it is infinitely costly to do so (binding case). In practice, it may be possible to violate precedent by incurring some cost.[7] This cost may not be

[7]For example, Posner [1995], page 122 discusses the reputational cost that a judge may incur for flouting precedent.

so large as to completely deter the judge from going against a previous ruling when investigation makes it clear that it is the socially beneficial thing to do. To capture these intermediate cases, we extend our model to allow a richer cost structure of violating precedent, reflecting the varying degrees of bindingness that exist in different institutions.

Suppose the cost of violating precedent (L, R) in the ruling of x is given by $\beta d(s, r)$ where $\beta \geq 0$ parameterizes the degree to which (L, R) is binding for future rulings. When $\beta = 0$, the precedent is persuasive; when $\beta = \infty$, the precedent is binding. From now on, we refer to the case with $\beta = \infty$ as *strictly binding* for clarity. For the intermediate cases with $\beta \in (0, \infty)$, we say that the precedent is *somewhat binding*.

When there is no violation of precedent, that is, if $x \in [\min\{L, R\}, \max\{L, R\}]$, or if $x < \min\{L, R\}$ and $r = 1$, or if $x > \max\{L, R\}$ and $r = 0$, then $d(s, r) = 0$; otherwise $d(s, r) > 0$. Assume that the cost of violating the precedent is higher when the distance between the case and the precedent it is violating is higher. Specifically, if $x < \min\{L, R\}$, then $d(s, 0)$ is decreasing in x and increasing in $\min\{L, R\}$; if $x > \max\{L, R\}$, then $d(s, 1)$ is increasing in x and decreasing in $\max\{L, R\}$.

Fix $\beta \geq 0$. If θ is known at the time when the relevant decisions are made, then it is optimal not to investigate any case. Let $\lambda : S \times \Theta \to \{0, 1\}$ denote the informed ruling policy where $\lambda(s, \theta) = 1$ if and only if x is permitted when the precedent is (L, R) given the threshold θ. As before, ρ is the judge's uninformed ruling policy and μ is the judge's investigation policy. Hence, the judge's policy is $\sigma = (\mu, \rho, \lambda)$.

For each policy $\sigma = (\mu, \rho, \lambda)$, let $H(\cdot; \sigma)$ be the associated value function for the informed judge and $V(\cdot; \sigma)$ denote the associated value function for the uninformed judge. In what follows, we suppress the dependence of H and V on σ for notational convenience. Also, let $EH^*(L, R, \theta) = \int_0^1 H^*(L, R, x', \theta) dG(x')$.

The policies σ^* is optimal if σ^* and the associated value functions H^* and V^* satisfy the following conditions:

(C1) Given H^*, the informed ruling policy satisfies $\lambda^*(s, \theta) = 1$ if and only if

$$u(x, \theta, 1) - \beta d(s, 1) + \delta EH^*(\pi(s, 1), \theta) \geq u(x, \theta, 0) - \beta d(s, 0) + \delta EH^*(\pi(s, 0), \theta),$$

for any s and θ.

(C2) Given λ^*, for any s and θ, the dynamic payoff for the informed judge satisfies

$$H^*(s, \theta) = \max\{u(x, \theta, 1) - \beta d(s, 1) + \delta EH^*(\pi(s, 1), \theta), u(x, \theta, 0) - \beta d(s, 0) + \delta EH^*(\pi(s, 0), \theta)\}.$$

(C3) Given V^*, the uninformed ruling policy satisfies $\rho^*(s) = 1$ if and only if

$$\int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) - \beta d(s,1) + \delta EV^*(\pi(s,1))$$
$$> \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) - \beta d(s,1) + \delta EV^*(\pi(s,0)),$$

for any state s.

(C4) Given H^*, V^* and the uninformed ruling policy ρ^*, for any state s, the investigation policy μ^* satisfies $\mu^*(s) = 1$ if and only if

$$-z + \int_{\underline{\theta}}^{\bar{\theta}} H^*(s,\theta)dF(\theta) \geq \rho^*(s) \left[\int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) - \beta d(s,1)\right]$$
$$+(1-\rho^*(s)) \left[\int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) - \beta d(s,0)\right] + \delta EV^*(\pi(s,\rho^*(s))).$$

(C5) Given σ^*, for any state s, the dynamic payoff satisfies

$$\begin{aligned} V^*(s) &= \mu^*(s)\left[-z + \int_{\underline{\theta}}^{\bar{\theta}} H^*(s,\theta)dF(\theta)\right] \\ &+ (1-\mu^*(s))\left[\rho^*(s)\int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta)\right. \\ &+ \left.(1-\rho^*(s))\int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV^*(\pi(s,\rho^*(s)))\right]. \end{aligned}$$

6 Appendix

Proof of Lemma 1: First note that

$$E_\theta[u(x_t,\theta,0)-u(x_t,\theta,1)] = \left[\int_{\min\{x_t,\bar{\theta}\}}^{\bar{\theta}} -\ell(x_t,\theta)dF(\theta) - \int_{\underline{\theta}}^{\max\{\underline{\theta},x_t\}} -\ell(x_t,\theta)dF(\theta)\right],$$

where $\int_{\min\{x_t,\bar{\theta}\}}^{\bar{\theta}} -\ell(x_t,\theta)dF(\theta)$ is the judge's expected payoff if she bans x_t since she incurs a cost if and only if $\theta > x_t$, and $\int_{\underline{\theta}}^{\max\{\underline{\theta},x_t\}} -\ell(x_t,\theta)dF(\theta)$ is the judge's expected payoff if she permits x_t since she incurs a cost if and only if $x_t > \theta$. If $x_t \geq \bar{\theta}$, then clearly $E_\theta[u(x_t,\theta,0)-u(x_t,\theta,1)] > 0$; and if $x_t \leq \underline{\theta}$, then clearly $E_\theta[u(x_t,\theta,0)-u(x_t,\theta,1)] < 0$.

We next consider $x_t \in (\underline{\theta},\bar{\theta})$. Since $-\ell(x,\theta)$ is increasing in x for $x < \theta$ and $-\ell(x,\theta) < 0$ for $x \neq \theta$, it follows that $\int_{x_t}^{\bar{\theta}} -\ell(x_t,\theta)dF(\theta)$ is increasing in x_t. Also, since $-\ell(x,\theta)$ is decreasing in x for $x > \theta$ and $-\ell(x,\theta) < 0$ for $x \neq \theta$, it follows that $\int_{\underline{\theta}}^{x_t} -\ell(x_t,\theta)dF(\theta)$ is decreasing in x_t. Hence,

$$E_\theta[u(x_t,\theta,0)-u(x_t,\theta,1)] = \int_{x_t}^{\bar{\theta}} -\ell(x_t,\theta)dF(\theta) - \int_{\underline{\theta}}^{x_t} -\ell(x_t,\theta)dF(\theta)$$

is increasing in x_t. Since $E_\theta[u(x_t,\theta,0)-u(x_t,\theta,1)] < 0$ if $x_t = \underline{\theta}$ and $E_\theta[u(x_t,\theta,0)-u(x_t,\theta,1)] > 0$ if $x_t = \bar{\theta}$, and $E_\theta[u(x_t,\theta,0)-u(x_t,\theta,1)]$ is continuous, it follows that there exists $\hat{x} \in (\underline{\theta},\bar{\theta})$ such that $E_\theta[u(x_t,\theta,0)-u(x_t,\theta,1)] < 0$ for $x_t < \hat{x}$ and $E_\theta[u(x_t,\theta,0)-u(x_t,\theta,1)] > 0$ for $x_t > \hat{x}$. ∎

Proof of Lemma 2: Consider period 3 first. If $x_3 < \underline{\theta}$, then the judge knows that $\theta > x_3$ and therefore permits the case without investigation. If $x_3 > \bar{\theta}$, then the judge knows that $\theta > x_3$ and therefore bans the case without investigation. For $x_3 \in [\underline{\theta},\hat{x})$, if the judge does not investigate, she permits the case and her expected payoff is $\int_{\underline{\theta}}^{x_3} -\ell(x_3,\theta)dF(\theta)$. For $x_3 \in (\hat{x},\bar{\theta}]$, if the judge does not investigate, she bans the case and her expected payoff is $\int_{x_3}^{\bar{\theta}} -\ell(x_3,\theta)dF(\theta)$. It follows that the judge's expected payoff if she does not investigate is the highest when $x_3 = \hat{x}$ and it is equal to $\int_{\underline{\theta}}^{\hat{x}} -\ell(\hat{x},\theta)dF(\theta) = \int_{\hat{x}}^{\bar{\theta}} -\ell(\hat{x},\theta)dF(\theta)$. Let $z^* = -\int_{\underline{\theta}}^{\hat{x}} -\ell(\hat{x},\theta)dF(\theta) > 0$. If $z < z^*$, then the judge investigates some cases in period 3. Specifically, suppose $z < z^*$ and let $x_3^L < \hat{x}$ and $x_3^H > \hat{x}$ be such that $\int_{\underline{\theta}}^{x_3^L} -\ell(x_3^L,\theta)dF(\theta) = -z$ and $\int_{x_3^H}^{\bar{\theta}} -\ell(x_3^H,\theta)dF(\theta) = -z$. If $x_3 \in [x_3^L, x_3^H]$, then the judge investigates case x_3 if she is uninformed. Let EV_t^P be the expected continuation payoff of of the judge in period t if no investigation was carried out in any previous period. Then, we have

$$EV_3^P = \int_{\underline{\theta}}^{x_3^L} \int_{\underline{\theta}}^{x} -\ell(x,\theta)dF(\theta)dG(x) + \int_{x_3^H}^{\bar{\theta}} \int_{x}^{\bar{\theta}} -\ell(x,\theta)dF(\theta)dG(x) - [G(x_3^H) - G(x_3^L)]z > -z.$$

Now consider period 2. Suppose the judge did not investigate in period 1. If the judge chooses to investigate in period 2, then her payoff in period 2 is $-z$ and her expected payoff in period 3 is 0. If the judge chooses not to investigate in period 2, then by Lemma 1, she permits any case $x_2 < \hat{x}$ and bans any case $x_2 > \hat{x}$. Note that if $x_2 \leq \underline{\theta}$ or if $x_2 \geq \overline{\theta}$, then her payoff is 0 if she does not investigate since she makes the correct decision.

Consider $\underline{\theta} < x_2 < \hat{x}$ and suppose the judge does not investigate the case. Since she permits such a case, her expected payoff in period 2 is $\int_{\underline{\theta}}^{x_2} -\ell(x_2,\theta)dF(\theta)$. Similarly, for $\hat{x} < x_2 < \overline{\theta}$, if the judge does not investigate the case, she bans it and in this case, her expected payoff in period 2 is $\int_{x_2}^{\bar{\theta}} -\ell(x_2,\theta)dF(\theta)$.

Now consider the judge's optimal investigation policy in period 2. For $x_2 \notin [\underline{\theta}, \overline{\theta}]$, since the judge's expected payoff is $\delta EV_3^P > -\delta z$ if she does not investigate the case and $-z$ if she investigates, it is optimal for her not to investigate x_2.

For $\underline{\theta} < x_2 < \hat{x}$, if

$$-z \geq \int_{\underline{\theta}}^{x_2} -\ell(x_2,\theta)dF(\theta) + \delta EV_3^P$$

then it is optimal for the judge to investigate x_2 in period 2. Similarly, for $\hat{x} < x_2 < \bar{x}$, if

$$-z \geq \int_{x_2}^{\bar{\theta}} -\ell(x_2,\theta)dF(\theta) + \delta EV_3^P,$$

then it is optimal for the judge to investigate x_2 in period 2. For $z < z^*$, since $EV_3^P < 0$, there exist $x_2^L \in (\underline{\theta}, x_3^L)$ and $x_2^H \in (x_3^H, \bar{\theta})$ such that

$$-z = \int_{\underline{\theta}}^{x_2^L} -\ell(x_2^L,\theta)dF(\theta) + \delta EV_3^P = \int_{x_2^H}^{\bar{\theta}} -\ell(x_2^H,\theta)dF(\theta) + \delta EV_3^P.$$

For $x_2 \in [x_2^L, x_2^H]$, it is optimal for the judge to investigate x_2. Thus, we have

$$EV_2^P = \int_{\underline{\theta}}^{x_2^L} \left[\int_{\underline{\theta}}^{x} -\ell(x,\theta)dF(\theta) + \delta EV_3^P\right] dG(x) + \int_{x_2^H}^{\bar{\theta}} \left[\int_{x}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_3^P\right] dG(x) - [G(x_2^H) -$$

Note that

$$EV_3^P = \max_{\{a,b\in[\underline{\theta},\bar{\theta}],b>a\}} \int_{\underline{\theta}}^{a}\int_{\underline{\theta}}^{x} -\ell(x,\theta)dF(\theta)dG(x) + \int_{b}^{\bar{\theta}}\int_{x}^{\bar{\theta}} -\ell(x,\theta)dF(\theta)dG(x) - [G(b)-G(a)]z,$$

and $EV_3^P < 0$. It follows that $EV_2^P < EV_3^P$.

Now consider period 1. If $-z \geq \delta EV_2^P$, then the judge investigates all cases in period 1. Suppose $z < z^*$ and $z > -\delta EV_2^P$, then by a similar argument as in period 2, there exist $x_1^L \in (\underline{\theta}, x_2^L)$ and $x_1^H \in (x_2^H, \bar{\theta})$ such that

$$-z = \int_{\underline{\theta}}^{x_1^L} -\ell(x_1^L,\theta)dF(\theta) + \delta EV_2^P = \int_{x_1^H}^{\bar{\theta}} -\ell(x_1^H,\theta)dF(\theta) + \delta EV_2^P.$$

For $x_1 \in [x_1^L, x_1^H]$, it is optimal for the judge to investigate x_1 in period 1. ■

Proof of Lemma 3: Consider period 3 first. Suppose the judge has not investigated in a previous period. Recall that under persuasive precedent, the judge investigates x_3 if and only if $x_3 \in [x_3^L, x_3^H]$. Since under binding precedent, investigation has no value if $x_3 \leq L_3$ or if $x_3 \geq R_3$, the judge investigates x_3 if $x_3 \in [x_3^L, x_3^H] \cap (L_3, R_3)$. Hence, the set of cases that the judge investigates in period 3 is either empty or convex and the judge does not investigate x_3 if $x_3 \notin (L_3, R_3)$.

Let $k(L,R)$ denote the judge's expected payoff in period t under binding precedent when the precedents are (L,R) in period t conditional on θ being known where the expectation is taken over θ before it is revealed and over all possible cases x. Formally

$$k(L,R) = \left[\int_{\mathcal{L}}\int_{\theta}^{L} -\ell(x,\theta)dG(x)dF(\theta) + \int_{\mathcal{R}}\int_{R}^{\theta} -\ell(x,\theta)dG(x)dF(\theta)\right]$$

where $\mathcal{L}$ is the (possibly degenerate) interval $[\underline{\theta}, \max\{L, \underline{\theta}\}]$ and $\mathcal{R}$ is the (possibly degenerate) interval $[\min\{R, \bar{\theta}\}, \bar{\theta}]$. Note that $k(L,R) = (1-\delta)C(L,R)$, $k(L,R) \leq 0$ and $k(L,R) < 0$ if $L > \underline{\theta}$ or if $R < \bar{\theta}$. Note also that $k(L,R)$ is decreasing in L and increasing in R.

To prove the lemma for periods 2 and 3, we first establish Claim 1 below. Let $EV_t^B(L,R)$ denote the judge's expected equilibrium continuation payoff in period t under binding precedent given that the precedent in period t is (L,R) and no investigation has been made in a previous period.

Claim 1. *If $EV_t^B(L,R)$ is decreasing in L and increasing in R, then the set of cases* *that the judge investigates in period* $t-1$ *is either empty or convex for any precedent in period* $t-1$.

Proof: Suppose that $EV_t^B(L,R)$ is decreasing in L and increasing in R. Fix the precedent in period $t-1$ and denote it by (L_{t-1},R_{t-1}). Suppose that the judge investigates cases x' and $x''>x'$ in period $t-1$. We next show that the judge also investigates case $\hat{x}\in[x',x'']$.

Let $g^p(L,R,x)$ be the judge's current-period payoff if she permits the case without investigation in state $s=(L,R,x)$ and $g^b(s)$ be her current-period payoff if she bans the case without investigation in state s. Note that for any (L,R), $g^p(L,R,x)$ is decreasing in x and $g^b(L,R,x)$ is increasing in x.

Suppose $\hat{x}\in(L_{t-1},R_{t-1})$. If the judge investigates $\hat{x}$, then her continuation payoff is $-z+\delta k(L_{t-1},R_{t-1})$. Suppose the judge does not investigate $\hat{x}$ and without loss of generality, suppose it is optimal for her to permit $\hat{x}$ if she does not investigate it. Since the judge investigates x' under precedent (L_{t-1},R_{t-1}), we have $-z+\delta k(L_{t-1},R_{t-1})\geq g^p(L_{t-1},R_{t-1},x')+\delta EV_t^B(\max\{x',L_{t-1}\},R_{t-1})$. Since g^p is decreasing in x, we have $g^p(L_{t-1},R_{t-1},x')>g^p(L_{t-1},R_{t-1},\hat{x})$. Moreover, since $\hat{x}>\max\{x',L_{t-1}\}$ and EV_t^B is decreasing in L, we have $EV_t^b(\max\{x',L_{t-1}\},R_{t-1})>EV_t^B(\hat{x},R_{t-1})$. Hence, we have $-z+\delta k(L_{t-1},R_{t-1})\geq g^p(L_{t-1},R_{t-1},\hat{x})+\delta EV_t^B(\hat{x},R_{t-1})$, which implies that it is optimal for the judge to investigate case $\hat{x}$.

Suppose $\hat{x}\leq L_{t-1}$. Then the judge has to permit $\hat{x}$ regardless of whether she investigates it or not. Hence, the judge investigates $\hat{x}$ if $-z+\delta k(L_{t-1},R_{t-1})\geq\delta EV_t^B(L_{t-1},R_{t-1})$. Since the judge investigates $x<\hat{x}$, we have $-z+\delta k(L_{t-1},R_{t-1})\geq\delta EV_t^B(L_{t-1},R_{t-1})$, implying that the judge investigates $\hat{x}$. A similar argument shows that the judge investigates $\hat{x}$ if $\hat{x}\geq R_{t-1}$ as well. Hence, the set of cases that the judge investigates in period $t-1$ is convex for any precedent in period $t-1$. ∎

Now consider period 2 and suppose the judge did not investigate in period 1. Consider precedents (L_3,R_3) and $(\hat{L}_3,\hat{R}_3)$ such that $\hat{L}_3\leq L_3$ and $\hat{R}_3\geq R_3$. As shown before, under the precedent (L_3,R_3), the judge's optimal policy is to investigate x_3 if $x_3\in(L_3,R_3)\cap(x_3^L,x_3^H)$ and otherwise to make a summary ruling. By following the same policy under precedent $(\hat{L}_3,\hat{R}_3)$, the judge receives the same payoff as under precedent (L_3,R_3). Hence, $EV_3^B(L_3,R_3)\leq EV_3^B(\hat{L}_3,\hat{R}_3)$. By Claim 1, the set of cases that the judge investigates in period 2 is either empty or convex. Consider $x_2\notin(L_2,R_2)$. The difference in the judge's continuation payoff in period 2 if she investigates the case and if she does not is given by $-z+\delta k(L_2,R_2)-\delta EV_3^B(L_2,R_2)$.

Since $EV_3^B(L_2, R_2) > -z + k(L_2, R_2)$, it follows that the judge does not investigate $x_2 \notin (L_2, R_2)$ in period 2.

Now consider period 1. Consider precedents (L_2, R_2) and $(\hat{L}_2, \hat{R}_2)$ such that $\hat{L}_2 \leq L_2$ and $\hat{R}_2 \geq R_2$. We next show that if judge 2 follows the same policy under precedent $(\hat{L}_2, \hat{R}_2)$ as the optimal policy under (L_2, R_2), then the judge's continuation payoff is higher under precedent $(\hat{L}_2, \hat{R}_2)$ than under (L_2, R_2). First consider x_2 such that the judge investigates x_2 under precedent (L_2, R_2). Note that $x_2 \in (L_2, R_2)$. In this case, the judge's continuation payoff is $-z$ under either (L_2, R_2) or $(\hat{L}_2, \hat{R}_2)$. Next consider x_2 such that the judge makes a summary ruling and permits x_2 under precedent (L_2, R_2). In this case, the precedent in period 3 becomes $(\max\{x_2, L_2\}, R_2)$. If the judge follows the same policy under precedent $(\hat{L}_2, \hat{R}_2)$, then the precedent in period 3 becomes $(\max\{x_2, \hat{L}_2\}, \hat{R}_2)$. Since $EV_3^B(L, R)$ is decreasing in L and increasing in R, we have $EV_3^B(\max\{x_2, L_2\}, R_2) \leq EV_3^B(\max\{x_2, \hat{L}_2\}, \hat{R}_2)$. Since the judge's period 2 payoff is the same under either (L_2, R_2) or $(\hat{L}_2, \hat{R}_2)$, it follows that her continuation payoff in period 2 is higher under precedent $(\hat{L}_2, \hat{R}_2)$ than under (L_2, R_2). A similar argument shows that the result hold for x_2 such that the judge makes a summary ruling and bans x_2 under precedent (L_2, R_2). Hence, $EV_2^B(L, R)$ is decreasing in L and increasing in R. And by Claim 1, the set of cases that the judge investigates in period 1 is either empty or convex. ∎

Proof of Proposition 1: Consider period 3 first. Suppose the judge did not investigate in a previous period. As shown in the proof of Lemma 3, under binding precedent, the judge investigates x_3 if $x_3 \in [x_3^L, x_3^H] \cap (L_3, R_3)$.

Now consider period 2 and suppose that the judge did not investigate in period 1. Recall that under persuasive precedent, the judge investigates x_2 if and only if $x_2 \in [x_2^L, x_2^H]$ where $[x_2^L, x_2^H] \supset [x_3^L, x_3^H]$. First suppose $[\underline{\theta}, \bar{\theta}] \subseteq [L_2, R_2]$. Then the incentive of the judge in period 2 is the same under binding precedent as under persuasive precedent. In this case, under binding precedent, the judge investigates x_2 if and only if $x_2 \in [x_2^L, x_2^H]$. Next suppose $[\underline{\theta}, \bar{\theta}] \not\subseteq [L_2, R_2]$. We show below that under binding precedent, the judge does not investigate case x_2^L.

Recall that the judge is indifferent between investigating and not investigating x_2^L in period 2 under persuasive precedent. That is, we have

$$-z = \int_{\underline{\theta}}^{x_2^L} -\ell(x_2^L, \theta) dF(\theta) + \delta k(x_3^L, x_3^H) - \delta z[G(x_3^H) - G(x_3^L)] \tag{7}$$

Consider binding precedent. If $x_2^L \notin (L_2, R_2)$, then the judge does not investigate x_2^L in period 2, as shown in Lemma 3. Suppose $x_2^L \in (L_2, R_2)$. The difference in the judge's continuation payoff between investigating and not investigating x_2^L is

$$\begin{aligned}&-z + \delta k(\max\{L_2, x_3^L\}, \min\{R_2, x_3^H\})\\&-\left[\int_{\underline{\theta}}^{x_2^L} -\ell(x_2^L, \theta)dF(\theta) + \delta k(\max\{x_2^L, x_3^L\}, \min\{R_2, x_3^H\}) - \delta z[G(\min\{R_2, x_3^H\}) - G(\max\{x_2^L, x_3^L\})]\right]\end{aligned}$$

Since $\max\{L_2, x_3^L\} = \max\{x_2^L, x_3^L\} = x_3^L$, this is equal to

$$-z - \int_{\underline{\theta}}^{x_2^L} -\ell(x_2^L, \theta)dF(\theta) + \delta z[G(\min\{R_2, x_3^H\}) - G(x_3^L)].$$

Substituting for $-z$ from (7), the difference in the judge's continuation payoff between investigating and not investigating x_2^L is

$$\delta k(x_3^L, x_3^H) - \delta z[G(x_3^H) - G(x_3^L)] + \delta z\left[G(\min\{R_2, x_3^H\}) - G(x_3^L)\right] < \delta k(x_3^L, x_3^H) < 0$$

Hence, the judge does not investigate x_2^L under binding precedent.

A similar argument establishes that under binding precedent, the judge does not investigate x_2^H in period 2. Given the convexity of the set of cases that the judge investigates under either binding or persuasive precedent, this implies that the set of cases that the judge investigates in period 2 under binding precedent is contained in the set of cases he investigates under persuasive precedent.

Now consider period 1. We show below that the judge investigates x_1^L under binding precedent. Recall that the judge is indifferent between investigating and not investigating x_1^L under persuasive precedent. That is,

$$\begin{aligned}-z &= \int_{\underline{\theta}}^{x_1^L} -\ell(x_1^L, \theta)dF(\theta) + \delta V_2^P\\&= \int_{\underline{\theta}}^{x_1^L} -\ell(x_1^L, \theta)dF(\theta) + \delta k(x_2^L, x_2^H) + \delta^2[1 - G(x_2^H) + G(x_2^L)]k(x_3^L, x_3^H)\\&- \delta z[G(x_2^H) - G(x_2^L) + \delta(G(x_3^H) - G(x_3^L))]\end{aligned}$$

Under binding precedent, if the judge investigates x_1^L in period 1, her continuation payoff is $-z$; if the judge does not investigate x_1^L, her continuation payoff is $\int_{\underline{\theta}}^{x_1^L} -\ell(x_1^L,\theta)dF(\theta) + \delta EV_2^B(x_1^L, R_1)$. Note that $EV_2^B(x_1^L, R_1) < V_2^p$ since the judge can follow the same policy under persuasive precedent as the optimal policy under binding precedent and receive a higher payoff. Hence, the judge investigates x_1^L in period 1 under binding precedent.

A similar argument establishes that under binding precedent, the judge investigates x_1^H. Given the convexity of the set of cases that the judge investigates under either binding or persuasive precedent, this implies that the set of cases that the judge investigates under binding precedent in period 1 contains the set of cases he investigates under persuasive precedent in period 1. ∎

Proof of Proposition 2: Let $\mathcal{F}$ denote the set of bounded measurable functions on S taking values in $\mathbb{R}$. For $f \in \mathcal{F}$, let $||f|| = \sup\{|f(s)| : s \in S\}$. An operator $Q : \mathcal{F} \to \mathcal{F}$ satisfies the contraction property for $||\cdot||$ if there is a $\beta \in (0,1)$ such that for $f^1, f^2 \in \mathcal{F}$, we have $||Q(f^1) - Q(f^2)|| \leq \beta||f^1 - f^2||$. For any operator Q that satisfies the contraction property, there is a unique $f \in V$ such that $Q(f) = f$.

We prove the proposition for binding precedent. A similar and less involved argument shows uniqueness under persuasive precedent as well.

Let $g^p(s)$ be the judge's current-period payoff if she permits the case without investigation in state s and $g^b(s)$ be her current period payoff if she bans the case without investigation in state s. Formally,

$$g^p(s) = \begin{cases} \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) & \text{if } x < R, \\ -\infty & \text{if } x \geq R, \end{cases}$$

$$g^b(s) = \begin{cases} \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) & \text{if } x > L, \\ -\infty & \text{if } x \leq L. \end{cases}$$

For any $V \in \mathcal{F}$ and $(L,R) \in S^p$, let $EV(L,R) = \int_0^1 V(L,R,x')dG(x')$. Note that for any $s \in S$, $\mu^*(s)$ as defined in (B2) satisfies

$$\mu_B^*(s) \in \arg\max_{\mu \in \{0,1\}} \mu[-z + A(s)] \\ + (1-\mu)\max\{g^p(s) + \delta EV_B^*(\max\{x,L\},R), g^b(s) + \delta EV_B^*(L,\min\{x,R\})\}.$$

For $V \in \mathcal{F}$ and any $s \in S$, define

$$TV(s) = \max\{-z + A(s), g^p(s) + \delta EV(\max\{x, L\}, R), g^b(s) + \delta EV(L, \min\{x, R\})\}.$$

Note that V^* as defined in (B3) satisfies $V_B^* = TV_B^*$.

Note that $\mu(s) = 1$ if and only if $TV(s) = -z + A(s)$.

Suppose that $V^1, V^2 \in \mathcal{F}$ and consider any $s \in S^p \times [0, 1]$. Without loss of generality, suppose that $TV^1(s) \geq TV^2(s)$. For notational convenience, define μ^1 and μ^2 relative to V^1 and V^2. There are three cases to consider.

(i) Suppose that $TV^1(s) = -z + A(s)$. Since $TV^1(s) \geq TV^2(s)$, we have $TV^2(s) = -z + A(s)$. We also have that $\mu^1(s) = 1$ and $\mu^2(s) = 1$. It follows that $TV^1(s) - TV^2(s) = 0$.

(ii) Suppose that $TV^1(s) = g^p(s) + \delta EV^1(\max\{L, x\}, R)$. Then $\mu^1(s) = 0$. We have

$$\begin{aligned}
|TV^1(s) - TV^2(s)| &\leq g^p(s) + \delta EV^1(\max\{L, x\}, R) - g^p(s) - \delta EV^2(\max\{L, x\}, R) \\
&\leq \delta \int_0^1 \left[V^1(\max\{L, x\}, R, x') - V^2(\max\{L, x\}, R, x')\right] dG(x') \\
&\leq \delta \int_0^1 \left[|V^1(\max\{L, x\}, R, x') - V^2(\max\{L, x\}, R, x')|\right] dG(x')] \\
&\leq \delta ||V^1 - V^2||.
\end{aligned}$$

(iii) Suppose that $TV^1(s) = g^b(s) + \delta EV^1(L, \min\{x, R\})$. Then a similar argument as in case (ii) shows that $|TV^1(s) - TV^2(s)| \leq \delta ||V^1 - V^2||$.

Since either $|TV^1(s) - TV^2(s)| = 0$ or $|TV^1(s) - TV^2(s)| \leq \delta ||V^1 - V^2||$ for any $s \in S$ in all three cases, we have $||TV^1 - TV^2|| \leq \delta ||V^1 - V^2||$ and therefore T is a contraction. Since T is a contraction, there is a unique V_B^* that satisfies $TV_B^* = V_B^*$ and therefore a unique V^* that satisfies (B3). Once we solve for V_B^*, (B1) and (B2) determine the optimal policies ρ_B^* and μ_B^* uniquely. ∎

Proof of Proposition 3: Since the judge investigates x_1 and x_2, by (P1) and (P2), we have

$$-z \geq \max\left\{\int_{\underline{\theta}}^{\max\{x, \underline{\theta}\}} -\ell(x, \theta) dF(\theta), \int_{\min\{x, \bar{\theta}\}}^{\bar{\theta}} -\ell(x, \theta) dF(\theta)\right\} + \delta EV_P^*.$$

for $x \in \{x_1, x_2\}$.

Suppose $x' \in [x_1, x_2]$. Since $\int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta)$ is decreasing in x, we have

$$\int_{\underline{\theta}}^{\max\{x',\underline{\theta}\}} -\ell(x',\theta)dF(\theta) \leq \int_{\underline{\theta}}^{\max\{x_1,\underline{\theta}\}} -\ell(x_1,\theta)dF(\theta).$$

Since $\int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta)$ is increasing in x, we have

$$\int_{\min\{x',\bar{\theta}\}}^{\bar{\theta}} -\ell(x',\theta)dF(\theta) \leq \int_{\min\{x_2,\bar{\theta}\}}^{\bar{\theta}} -\ell(x_2,\theta)dF(\theta).$$

It follows that

$$-z \geq \max\left\{\int_{\underline{\theta}}^{\max\{x',\underline{\theta}\}} -\ell(x',\theta)dF(\theta), \int_{\min\{x',\bar{\theta}\}}^{\bar{\theta}} -\ell(x',\theta)dF(\theta)\right\} + \delta EV_P^*$$

and therefore the judge investigates x'. ∎

Proof of Proposition 4: If $M_P = \varnothing$, then clearly $M_P \subset (\underline{\theta}, \bar{\theta})$. We next show by contradiction that if $M_P \neq \varnothing$, then $\underline{\theta} < a_P < b_P < \bar{\theta}$. Suppose $a_P < \underline{\theta}$. Consider $x = a_P$. Since $a_P < \underline{\theta}$, the judge's dynamic payoff equals $-z$ if she investigates x, and equals δEV_P^* if she does not investigate x. Since $a_P = \inf\{x : \mu^*(x) = 1\}$, it follows that $-z \geq \delta EV^*$. Note that for any $x > \bar{\theta}$, the judge's dynamic payoff is $-z$ if she investigates, and δEV_P^* if she does not investigate. Hence, it must be the case that the judge investigates any case $x > \bar{\theta}$. It follows that $b_P = 1$. Moreover, since $a_P < \underline{\theta}$, the judge makes the correct decision for any case $x < a_P$. It follows that

$$EV_P^* = \int_0^{a_P} \delta EV_P^* dG(x) - z(1 - G(a_P)) = \delta G(a_P)EV_P^* - z(1 - G(a_P)).$$

Since $-z \geq \delta EV_P^*$, this implies that $EV_P^* > \delta EV_P^*$, but this is impossible since $EV_P^* < 0$.

We next prove that the uninformed judge permits $x < a_P$ and bans $x > b_P$. Suppose not, and assume that there exists $\hat{x} < a_P$ such that the uninformed judge bans $\hat{x}$. So we have

$$\int_{\hat{x}}^{\bar{\theta}} -\ell(\hat{x},\theta)dF(\theta) + EV_P^* > -z.$$

Since the uninformed judge's payoff from banning x is increasing in x, it follows that for $x \in [a_P, b_P]$, we have

$$\int_x^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + EV_P^* > -z,$$

which is a contradiction. Hence, the uninformed judge permits $x < a_P$. A similar argument shows that the uninformed judges bans $x > b_P$. ∎

Proof of Proposition 5: Recall that

$$TV(s) = \max\{-z + A(s), g^p(s) + \delta EV(\max\{x, L\}, R), g^b(s) + \delta EV(L, \min\{x, R\})\}.$$

Let $KV(s) = 1$ if $TV(s) = -z + A(s)$ and $KV(s) = 0$ otherwise. To prove the proposition, we first establish the following lemma.

Lemma 4. *For $(L, R) \in S^p$, if $V \in \mathcal{F}$ satisfies the following properties: (i) V is decreasing in L and increasing in R, (ii) $EV(L, R) - EV(\hat{L}, \hat{R}) \leq C(L, R) - C(\hat{L}, \hat{R})$, and (iii) KV is decreasing in L and increasing in R, then TV also satisfies these properties, that is, (i) TV is decreasing in L and increasing in R, (ii) $ETV(L, R) - ETV(\hat{L}, \hat{R}) \leq C(L, R) - C(\hat{L}, \hat{R})$, and (iii) KTV is decreasing in L and increasing in R.*

Proof: We first show that if $V \in \mathcal{F}$ is decreasing in L and increasing in R, then TV is also increasing in L and decreasing in R. Fix $x \in [0, 1]$. If V is decreasing in L and increasing in R, then $EV(\max\{x, L\}, R)$ and $EV(L, \min\{x, R\})$ are decreasing in L and increasing in R. Note that $A(s)$ is decreasing in L and increasing in R, $g^p(s)$ is constant in L and increasing in R, $g^b(s)$ is constant in R and decreasing in L. Hence, $TV(s)$ is decreasing in L and increasing in R.

Let $\hat{s} = (\hat{L}, \hat{R}, x)$. We next show that if $V \in \mathcal{F}$ satisfies properties $(i), (ii)$, and (iii), then $ETV(L, R) - ETV(\hat{L}, \hat{R}) \leq C(L, R) - C(\hat{L}, \hat{R})$. Consider the following cases.

(a) Suppose $TV(\hat{s}) = -z + A(\hat{s})$. Then $KV(\hat{s}) = 1$. Since KV is decreasing in L and increasing in R, we have $KV(s) = 1$, which implies that $TV(s) = -z + A(s)$. Hence $TV(s) - TV(\hat{s}) = A(s) - A(\hat{s})$.

(b) Suppose $TV(\hat{s}) > -z + A(\hat{s})$. Without loss of generality, suppose that $TV(\hat{s}) = g^p(\hat{s}) + \delta EV(\max\{x, \hat{L}\}, \hat{R})$. Note that $KV(\hat{s}) = 0$ and $x < \hat{R}$. Suppose $KV(s) = 1$. Then $TV(s) = -z + A(s)$ and $TV(s) - TV(\hat{s}) < A(s) - A(\hat{s})$. Suppose $KV(s) = 0$

and $TV(s) = g^p(s) + \delta EV(\max\{x, L\}, R)$. Then

$$TV(s) - TV(\hat{s}) = \delta \left[EV(\max\{x, L\}, R) - EV(\max\{x, \hat{L}\}, \hat{R}) \right]$$
$$\leq \delta[EV(L, R) - EV(\hat{L}, \hat{R})] \leq \delta[C(L, R) - C(\hat{L}, \hat{R})].$$

Suppose $TV(s) = g^b(s) + \delta EV(L, \min\{x, R\})$. There are two cases to consider, either $x > \hat{L}$ or $x \leq \hat{L}$. First suppose $x > \hat{L}$. Then $g^b(\hat{s}) = g^b(s)$. Since $TV(\hat{s}) \geq g^b(\hat{s}) + \delta EV(\hat{L}, \min\{x, \hat{R}\})$, it follows that $TV(s) - TV(\hat{s}) \leq \delta EV(L, \min\{x, R\}) - \delta EV(\hat{L}, \min\{x, \hat{R}\}) \leq \delta[C(L, R) - C(\hat{L}, \hat{R})]$. Next suppose $x \leq \hat{L}$. Note that $TV(\hat{s}) = g^p(\hat{s}) + \delta EV(\max\{x, \hat{L}\}, \hat{R}) = g^p(\hat{s}) + \delta EV(\hat{L}, \hat{R})$ and $A(\hat{s}) = g^p(s) + \delta C(\hat{L}, \hat{R})$. Hence, $A(\hat{s}) - TV(\hat{s}) = \delta C(\hat{L}, \hat{R}) - \delta EV(\hat{L}, \hat{R})$. Note also that $TV(s) = g^b(s) + \delta EV(L, x) \leq g^b(s) + \delta EV(L, R)$ and $A(s) \geq g^b(s) + \delta C(L, R)$. Hence $A(s) - TV(s) > \delta C(L, R) - \delta EV(L, R)$. It follows that $A(s) - TV(s) - A(\hat{s}) + TV(\hat{s}) > \delta C(L, R) - \delta EV(L, R) - \delta C(\hat{L}, \hat{R}) + \delta EV(\hat{L}, \hat{R}) \geq 0$. Therefore $TV(s) - TV(\hat{s}) \leq A(s) - A(\hat{s})$. It follows that for all $x \in [0, 1]$, we have $TV(s) - TV(\hat{s}) \leq A(s) - A(\hat{s})$, and therefore $ETV(L, R) - ETV(\hat{L}, \hat{R}) \leq E[A(s) - A(\hat{s})] = C(L, R) - C(\hat{L}, \hat{R})$.

Lastly we show that if $V \in \mathcal{F}$ satisfies properties $(i), (ii)$, and (iii), then KTV is decreasing in L and increasing in R. Since $KTV(s) \in \{0, 1\}$ for any $s \in S$, it is sufficient to show that if $KTV(\hat{s}) = 1$, then $KTV(s) = 1$.

Suppose $KTV(\hat{s}) = 1$. Consider $x \in (\hat{L}, \hat{R})$ first. Then we have

$$-z + A(\hat{s}) \geq \max\{g^p(\hat{s}) + \delta ETV(x, \hat{R}), g^b(\hat{s}) + \delta ETV(\hat{L}, x)\}. \tag{8}$$

Note that in this case, $A(\hat{s}) = \delta C(\hat{L}, \hat{R})$, $A(s) = \delta C(L, R)$, $g^p(\hat{s}) = g^p(s), g^b(\hat{s}) = g^b(s)$. As established earlier, if $V \in \mathcal{F}$ satisfies properties $(i), (ii)$, and (iii), then TV is decreasing in L and increasing in R and $C(L, R) - C(\hat{L}, \hat{R}) \geq ETV(L, R) - ETV(\hat{L}, \hat{R})$. Since $L < \hat{L} < x < \hat{R} < R$, we have $\max\{L, x\} = \max\{\hat{L}, x\} = x$ and $\min\{x, R\} = \min\{x, \hat{R}\} = x$. It follows that $ETV(\max\{L, x\}, R) - ETV(\max\{\hat{L}, x\}, \hat{R}) = ETV(x, R) - ETV(x, \hat{R})$ and $ETV(L, \min\{x, R\}) - ETV(\hat{L}, \min\{x, \hat{R}\}) = ETV(L, x) - ETV(\hat{L}, x)$. Since $C(x, R) - C(x, \hat{R}) \geq ETV(x, R) - ETV(x, \hat{R})$ and $C(L, R) - C(\hat{L}, \hat{R}) \geq C(x, R) - C(x, \hat{R})$, it follows that $C(L, R) - C(\hat{L}, \hat{R}) \geq ETV(\max\{L, x\}, R) - ETV(\max\{\hat{L}, x\}, \hat{R})$. Similarly, since $C(L, x) - C(\hat{L}, x) \geq ETV(L, x) - ETV(\hat{L}, x)$ and $C(L, R) - C(\hat{L}, \hat{R}) > C(L, x) - C(\hat{L}, x)$, it follows that and $C(L, R) - C(\hat{L}, \hat{R}) \geq ETV(L, \min\{x, R\}) -$

$ETV(\hat{L}, \min\{x, \hat{R}\})$. It then follows from (8) that

$$-z + A(s) \geq \max\{g^p(s) + \delta ETV(\max\{L, x\}, R), g^b(s) + \delta ETV(L, \min\{x, R\})\}$$

and therefore $KTV(s) = 1$.

Next consider $x \notin (\hat{L}, \hat{R})$, and without loss of generality, suppose that $x \leq \hat{L}$. In this case, $A(\hat{s}) - \delta C(\hat{L}, \hat{R}) = g^p(\hat{s})$ and $A(s) - \delta C(L, R) \geq g^p(s)$. Specifically, $A(s) - \delta C(L, R) = g^p(s)$ if $x \leq L$ and $A(s) - \delta C(L, R) = 0$ if $L < x \leq \hat{L}$. Hence,

$$\begin{aligned}
& - z + A(s) - g^p(s) - \delta ETV(\max\{L, x\}, R) - [-z + A(\hat{s}) - g^p(\hat{s}) - \delta ETV(\max\{\hat{L}, x\}, \hat{R})] \\
& \geq \delta[C(L, R) - C(\hat{L}, \hat{R})] - \delta[ETV(x, R) - ETV(x, \hat{R})] \geq 0.
\end{aligned}$$

It follows that $-z + A(s) \geq g^p(s) + \delta ETV(\max\{L, x\}, R)$. We next show that $-z + A(s) \geq g^b(s) + \delta ETV(L, \min\{x, R\})$. If $x \leq L$, then clearly $-z + A(s) \geq g^b(s) + \delta ETV(L, \min\{x, R\})$. Suppose $L < x \leq \hat{L}$. Note that $-z + A(\hat{s}) \geq g^p(\hat{s}) + \delta ETV(\hat{L}, \hat{R})$ implies that $-z + \delta C(\hat{L}, \hat{R}) \geq \delta ETV(\hat{L}, \hat{R})$. Since $A(s) = \delta C(L, R) \geq g^b(s) + \delta C(L, R)$ and $C(L, R) - C(\hat{L}, \hat{R}) \geq ETV(L, R) - ETV(\hat{L}, \hat{R})$, it follows that $-z + A(s) \geq g^b(s) + \delta ETV(L, R) \geq g^b(s) + ETV(L, \min\{x, R\})$. Hence $KTV(s) = 1$. ∎

Since $V_B^* = TV_B^*$ and Lemma 4 shows that the contraction mapping T preserves properties (i), (ii) and (iii), it follows that V_B^* is decreasing in L and increasing in R. It also follows that $EV_B^*(L, R) - EV_B^*(\hat{L}, \hat{R}) \leq C(L, R) - C(\hat{L}, \hat{R})$. Since $C(L, R)$ is continuous in L and R, we have $EV_B^*(L, R)$ is continuous in L and R. Since the optimal policy satisfies $\mu_B^*(s) = KV_B^*(s)$, it also follows from Lemma 4 that μ_B^* is decreasing in L and increasing in R. ∎

Proof of Proposition 6: Fix $(L, R) \in S^p$. We first prove part (i). Note that $g^p(s)$ is decreasing in x and $g^b(s)$ is increasing in x. Since V_B^* is decreasing in L by Proposition 5, $V_B^*(\pi(s, P), x') = V_B^*((\max\{L, x\}, R), x')$ is decreasing in x. Similarly, since V_B^* is increasing in R, $V_B^*(\pi(s, B), x') = V^*((L, \min\{R, x\}), x')$ is increasing in x.

Since the judge investigates x_i, $i = 1, 2$, we have

$$-z + A(L, R, x_i) \geq g^p(L, R, x_i) + \delta EV_B^*(\max\{L, x_i\}, R) \tag{9}$$
$$-z + A(L, R, x_i) \geq g^b(L, R, x_i) + \delta EV_B^*(L, \min\{R, x_i\}). \tag{10}$$

Recall that $\mathcal{L} = [\underline{\theta}, \max\{L, \underline{\theta}\}]$ and $\mathcal{R} = [\min\{R, \bar{\theta}\}, \bar{\theta}]$. Suppose $\hat{x} \in [x_1, x_2]$. There are three cases to consider.

(a) Suppose $\mathbf{1}_{\mathcal{L}}(\hat{x}) = \mathbf{1}_{\mathcal{R}}(\hat{x}) = 0$. Then $A(L, R, \hat{x}) \geq A(L, R, x_i)$ for $i = 1, 2$. Since $\hat{x} > x_1$, $g^p(s)$ is decreasing in x and $EV_B^*(\max\{L, x\}, R)$ is decreasing in x, it follows from (9) that

$$-z + A(L, R, \hat{x}) \geq g^p(L, R, \hat{x}) + \delta EV_B^*(\max\{L, \hat{x}\}, R).$$

Since $\hat{x} < x_2$, $g^b(s)$ is increasing in x and $EV_B^*(L, \min\{R, x\})$ is increasing in x, it follows from (10) that

$$-z + A(L, R, \hat{x}) \geq g^p(L, R, \hat{x}) + \delta EV_B^*(L, \min\{R, \hat{x}\}).$$

Hence, it is optimal for the judge to investigate $\hat{x}$.

(b) Suppose $\mathbf{1}_{\mathcal{L}}(\hat{x}) = 1$. Then $\mathbf{1}_{\mathcal{R}}(\hat{x}) = 0$ and $\mathbf{1}_{\mathcal{R}}(x_1) = 0$. Moreover, we have $A(L, R, \hat{x}) = g^p(L, R, \hat{x}) + \delta C(L, R)$, $A(L, R, x_1) \geq g^p(L, R, x_1) + \delta C(L, R)$, $g^b(L, R, \hat{x}) = -\infty$, and $g^b(L, R, x_1) = -\infty$. From (9), we have

$$-z + \delta C(L, R) \geq \delta EV_B^*(\max\{L, x_1\}, R).$$

Since $EV_B^*(\max\{L, x\}, R)$ is decreasing in x and $\hat{x} > x_1$, it follows that

$$-z + \delta C(L, R) \geq \delta EV_B^*(\max\{L, \hat{x}\}, R)$$

and therefore

$$-z + A(L, R, \hat{x}) \geq g^p(L, R, \hat{x}) + \delta EV_B^*(\max\{L, \hat{x}\}, R).$$

Since $g^b(L, R, \hat{x}) = -\infty$, we also have

$$-z + A(L, R, \hat{x}) > g^b(L, R, \hat{x}) + \delta EV_B^*(L, \min\{R, \hat{x}\}).$$

Hence, it is optimal for the judge to investigate $\hat{x}$.

(c) Suppose $\mathbf{1}_{\mathcal{R}}(\hat{x}) = 1$. Then $\mathbf{1}_{\mathcal{L}}(\hat{x}) = \mathbf{1}_{\mathcal{L}}(x_2) = 0$. Moreover, we have $A(L, R, \hat{x}) = g^b(L, R, \hat{x}) + \delta C(L, R)$, $A(L, R, x_2) \geq g^b(L, R, x_2) + \delta C(L, R)$, $g^p(L, R, \hat{x}) = -\infty$, and $g^p(L, R, x_2) = -\infty$. A similar argument as in case (ii) shows that it is optimal for the judge to investigate $\hat{x}$.

We next prove part (ii). Consider a case $x \notin (L, R)$ and suppose that the judge investigates x. Since the judge has to follow the precedent in the current period regarding x, the difference in her current period payoff between investigating and not investigating is $-z$. If she investigates x, her continuation payoff is $\delta C(L, R)$; if she does not investigate x, she does not change the precedent since $x \notin (L, R)$ and therefore her continuation payoff is $\delta EV_B^*(L, R)$.

Since the judge investigates x, we have $-z + \delta C(L, R) \geq \delta EV_B^*(L, R)$. Note that neither side of the inequality depends on x, which implies that it is optimal for the judge to investigate any $x \notin (L, R)$ given precedent (L, R). It follows from part (i) of the proposition that the judge investigates all $x \in [0, 1]$ given precedent (L, R). Hence $EV_B^*(L, R) = -z + C(L, R)$, which contradicts $-z + \delta C(L, R) \geq \delta EV_B^*(L, R)$ since $-z < 0$ and $\delta < 1$. Hence, the judge does not investigate $x \notin (L, R)$. ■

Proof of Proposition 7: Let $a_0 = a(L_1, R_1)$ and $b_0 = b(L_1, R_1)$. We first show that the initial investigation interval under binding precedent is larger than the investigation interval under persuasive precedent. That is, $a_0 < a_P < b_P < b_0$. We show that $a_0 < a$. A similar argument shows that $b < b_0$.

For $0 \leq a < b \leq 1$, let $h(a, b) = \int_{\underline{\theta}}^{a} \int_{\theta}^{a} -\ell(x, \theta) dG(x) dF(\theta) + \int_{b}^{\bar{\theta}} \int_{b}^{\theta} -\ell(x, \theta) dG(x) dF(\theta)$.

Note that for any L, R, we have

$$EV_B^*(L, R) \leq h(a(L, R), b(L, R)) + [G(b(L, R)) - G(a(L, R))] [-z + \delta C(L, R)] \\ + \delta EV_B^*(L, R) [G(a(L, R)) + 1 - G(b(L, R))],$$

where the inequality comes from the property that $EV_B^*(L, R)$ is decreasing in L and increasing in R and when the appointed judge makes a summary decision, the precedent either stays the same or gets tighter.

It follows that

$$EV_B^*(L, R) \leq \frac{h(a(L, R), b(L, R)) + [G(b(L, R)) - G(a(L, R))] [-z + \delta C(L, R)]}{1 - \delta [G(a(L, R)) + 1 - G(b(L, R))]}.$$

Since $C(a(L, R), b(L, R)) \leq 0$, we have $EV_B^*(L, R) \leq \frac{h(a(L,R),b(L,R)) - z[G(b(L,R)) - G(a(L,R))]}{1 - \delta[G(a(L,R)) + 1 - G(b(L,R))]}$.

Consider any case $y \in (L_1, a_0)$. Since the judge does not investigate case y given precedent (L_1, R_1), we have $\int_{\underline{\theta}}^{y} -\ell(y,\theta)dF(\theta) + \delta EV_B^*(y, R_1) > -z$. It follows that

$$\int_{\underline{\theta}}^{y} -\ell(y,\theta)dF(\theta) > -z - \delta EV_B^*(y, R_1).$$

Recall that under persuasive precedent, a_P satisfies

$$\int_{\underline{\theta}}^{a_P} -\ell(a_P,\theta)dF(\theta) = -z - \delta EV_P^*.$$

Since $EV_B^*(y, R_1) \leq EV_P^*$, it follows that $\int_{\underline{\theta}}^{y} -\ell(y,\theta)dF(\theta) > \int_{\underline{\theta}}^{a_P} -\ell(a_P,\theta)dF(\theta)$ and therefore $y < a_P$. Since this is true for any $y < a_0$ we have $a_0 \leq a_P$. A similar argument shows that $b_P \leq b_0$.

We next show that $a_P < \hat{a} < \hat{b} < b_P$. Note that if the precedent is $(\hat{a}, \hat{b})$, then the judge investigates case x if and only if $x \in (\hat{a}, \hat{b})$. Hence, we have

$$EV_B^*(\hat{a},\hat{b}) = h(\hat{a},\hat{b}) + \left(-z + \delta C(\hat{a},\hat{b})\right)\left[G(\hat{b}) - G(\hat{a})\right] + \delta EV_B^*(\hat{a},\hat{b})\left[G(\hat{a}) + 1 - G(\hat{b})\right],$$

which implies that

$$EV_B^*(\hat{a},\hat{b}) = \frac{h(\hat{a},\hat{b})}{1-\delta[G(\hat{a}) + 1 - G(\hat{b})]} + \left(-z + \delta C(\hat{a},\hat{b})\right)\left(\frac{G(\hat{b}) - G(\hat{a})}{1-\delta[G(\hat{a}) + 1 - G(\hat{b})]}\right). \tag{11}$$

Moreover, we have

$$-z + \delta C(\hat{a},\hat{b}) = \int_{\underline{\theta}}^{\hat{a}} -\ell(\hat{a},\theta)dF(\theta) + \delta EV_B^*(\hat{a},\hat{b})$$

and

$$-z + \delta C(\hat{a},\hat{b}) = \int_{\hat{b}}^{\bar{\theta}} -\ell(\hat{b},\theta)dF(\theta) + \delta EV_B^*(\hat{a},\hat{b}).$$

From the indifference condition and (11), we have

$$\begin{aligned}\int_{\underline{\theta}}^{\hat{a}} -\ell(\hat{a},\theta)dF(\theta) &= -z + \delta C(\hat{a},\hat{b}) - \delta EV_B^*(\hat{a},\hat{b}) \\ &= \frac{-(1-\delta)z + (1-\delta)\delta C(\hat{a},\hat{b}) - \delta h(\hat{a},\hat{b})}{1-\delta[G(\hat{a}) + 1 - G(\hat{b})]}\end{aligned}$$

Since $(1-\delta)C(\hat{a},\hat{b}) = h(\hat{a},\hat{b})$, it follows that

$$\int_{\underline{\theta}}^{\hat{a}} -\ell(\hat{a},\theta)dF(\theta) = \frac{-(1-\delta)z}{1-\delta[G(\hat{a})+1-G(\hat{b})]}.$$

Similarly,

$$\int_{\hat{b}}^{\bar{\theta}} -\ell(\hat{b},\theta)dF(\theta) = \frac{-(1-\delta)z}{1-\delta[G(\hat{a})+1-G(\hat{b})]}.$$

Recall that under persuasive precedent, a_P and b_P satisfy

$$\int_{\underline{\theta}}^{a_P} -\ell(a_P,\theta)dF(\theta) = \int_{b_P}^{\bar{\theta}} -\ell(b_P,\theta)dF(\theta) = \frac{-(1-\delta)z}{1-\delta[G(a_P)+1-G(b_P)]} - \frac{\delta h(a_P,b_P)}{1-\delta[G(a_P)+1-G(b_P)]}.$$

For $a \in [\underline{\theta},\bar{\theta}]$, let $\beta(a)$ be defined by $\int_{\underline{\theta}}^{a} -\ell(a_P,\theta)dF(\theta) = \int_{\beta(a)}^{\bar{\theta}} -\ell(b_P,\theta)dF(\theta)$. Also, let A equal the constant $-\frac{\delta h(a_P,b_P)}{1-\delta[G(a_P)+1-G(b_P)]} > 0$.

Note that a_P is the solution to

$$\int_{\underline{\theta}}^{a} -\ell(a,\theta)dF(\theta) = \frac{-(1-\delta)z}{1-\delta[G(a)+1-G(\beta(a))]} + A$$

and $\hat{a}$ is the solution to

$$\int_{\underline{\theta}}^{a} -\ell(a,\theta)dF(\theta) = \frac{-(1-\delta)z}{1-\delta[G(a)+1-G(\beta(a))]}.$$

If $a = \underline{\theta}$, then $\int_{\underline{\theta}}^{a} -\ell(a,\theta)dF(\theta) = 0$. Moreover, since $-(1-\delta)z - \delta h(a_P,b_P) < 0$ and $\frac{-(1-\delta)z}{1-\delta[G(\underline{\theta})+1-G(\bar{\theta})]} + A < \frac{-(1-\delta)z-\delta h(a_P,b_P)}{1-\delta[G(\underline{\theta})+1-G(\bar{\theta})]}$, it follows that $\frac{-(1-\delta)z}{1-\delta[G(a)+1-G(\beta(a))]} + A < 0$ for $a = \underline{\theta}$. Hence, for any $a \in [\underline{\theta}, a_P)$, we have

$$\int_{\underline{\theta}}^{a} -\ell(a,\theta)dF(\theta) > \frac{-(1-\delta)z}{1-\delta[G(a)+1-G(\beta(a))]} + A.$$

Since $A > 0$, this implies that for any $a \in [\underline{\theta}, a_P)$, we have

$$\int_{\underline{\theta}}^{a} -\ell(a,\theta)dF(\theta) > \frac{-(1-\delta)z}{1-\delta[G(a)+1-G(\beta(a))]}.$$

It follows that $\hat{a} > a_P$. Since $\beta(a)$ is decreasing in a, it follows that $\hat{b} < b_P$. ∎

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