Why so much consensus? Reciprocal aggregation and the duality between persons and groups

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Abstract

Why do shareholders so often vote like one man in general assembly meetings? Unless markets are perfect, this challenges the conventional wisdom. We address this puzzle by proposing a general equilibrium notion of *reciprocal aggregation*: Firms aggregate the preferences of shareholders, and reciprocally shareholders aggregate the of perceived preferences (or beliefs) of firms. We show how a mere Pareto condition for both aggregation mechanisms results in full alignment of shareholders, and restores the first welfare theorem even in case of severe market failures.

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1 Introduction

The problem

Why so much stability? This puzzle was raised by Tullock (1981) in the context of public choice. Indeed in theory, absent very restrictive conditions on individual preferences, endless cycling is expected in the collective decision process. But as Tullock puts it: "If we look at the real world, however, we observe not only there is no endless cycling, but acts are passed with reasonable dispatch and then remain unchanged for very long periods of time. Thus, theory and reality seem to be not only out of contact, but actually in sharp conflict".

Tullock's puzzle is not limited to the public sphere. A quick look at the corporate world raises a similar puzzle: Why so much consensus?

We looked at the 794 decisions[1] taken in the 2014 general assemblies meetings of the forty largest French corporations (CAC40). On average each decision received a support of 94.9% of the voters. The *consensus index* is 87.6%, meaning that 87.6% of the decisions were approved by majorities higher than 87.6%. These decisions can be split into three categories: 1/ Collective decisions, such as the approval of the company's financial statements, the setting of dividends, the delegation of authority granted to the board members, the amendments to the company's charter etc; 2/ Decisions on individual compensation of senior managers and board members; 3/ Decisions on appointment or renewal of individual board members. The following table shows, for each category, the total number of decisions, and in percentage the lowest, highest and average support for all considered decisions, and the consensus index.

Decisions/rates	*Number*	*Min%*	*Max%*	*Average%*	*Index%*
Collective	519	17.1	100	96.2	89.4
Individual	99	53.7	99.7	89.2	78.8
Appointment	175	50.3	100	94.3	87.5
Total	**794**	**17.1**	**100**	**94.9**	**87.6**

Even decisions on individual compensation (e.g., say on pay) obtain overwhelming support, close to 90% on average; and roughly 80% of these decisions are supported by roughly 80% or more of the voters. For anybody familiar with the social choice literature, these numbers are striking.

Is it because markets are perfect and complete? Then indeed at equilibrium all individual vectors of marginal utilities are collinear, and coincide with the vector of market prices; hence everybody agrees that profit should be maximized (there are no externalities - neither direct, nor pecuniary); and everybody agrees on how to discount future income flows (markets are complete); end of the story.

[1] There were 800 in total. We discard 6 decisions that were carried to the vote against the approval of the board – and which obtained very low support (from 2 to 16%).

This solution is insightful, but limited. First, there certainly are some externalities between those firms (e.g., in the energy sector, between EDF, Total and Engie). The French State is a major shareholder in some of these 40 firms, and holds a specific portfolio. It should internalize externalities through the prism of this portfolio,[2] and this should lead it to not systematically seek profit maximization. Second, these firms are so big that they certainly do not all behave competitively. Third, there is growing evidence that markets are incomplete.

In the presence of externalities, or imperfect competition, or market incompleteness, at equilibrium shareholders typically disagree on how to run the firms − as they have different present value vectors. Hence a genuine problem of social choice appears in each and every firm. How are production plans endogenized then? The social choice literature is quite powerless at providing a compelling notion of (political) equilibrium which would generate the hope that, even if efficiency is a dream, at least some stability is reachable.

This difficulty has been discarded in the classical literature, either by assuming that the firm is a monolithic decision maker endowed with an ad hoc objective function, or by assuming that the space of political heterogeneity is one-dimensional, and then resorting to the median voter theorem. The attempts to address that question in a general equilibrium framework have been severely exposed to the lack of structure at the aggregate level. There is an extensive literature on the subject, where groups of decision makers bargain (Britz, Herings & Predtetchinski, 2013), or vote (Gevers, 1974; Drèze 1989; DeMarzo, 1993; Tvede & Crès, 2005), or agree to maximize some appropriate objective function (Drèze, 1974; Grossman & Hart, 1979; Kelsey & Milne, 1996; Dierker, Dierker & Grodal, 1999). Alas! There seems to be no aggregation mechanism, as simple and parsimonious as the market mechanism, that yields the same remarkable, coincidental combination of stability and efficiency as the latter offers. This seems inescapable, at least when one sticks to the strict view of a monolithic and immutable individual decision maker.

The argument

Our strategy to approach the problem is to open the possibility of *aggregation simultaneously at the collective and individual levels.* It is naturally accepted that an aggregation mechanism takes place in every collective entity (boards or assemblies) − either through deliberation, bargaining, log-rolling, side-paying, voting... or all at once − at the end of which some decision is taken. It is less frequently assumed that aggregation takes place at the individual level,[3] although this seems quite intuitive. Indeed, an individual agent has to

[2] The French State's portfolio is managed par the Agence des Participations de l'Etat (APE), de facto a bureau of the Ministry of Economics and Finance.

[3] That individual preferences are formed by aggregation is well accepted when taking households as the individual decision making unit (see, e.g. Chiappori & Ekeland, 2006, and the survey therein). It is also a major avenue in the theory of choice rationalization (see the survey in Ambrus & Rozen, 2008). The literature on aggregation of experts' judgements also assumes that the preferences of the individual decision maker are

take actions, make decisions, or express opinions in different capacities, and various arenas. As an individual consumer, she trades goods and rebalances her portfolio according to her insurance needs; and as a shareholder, she weighs in the firms' decision processes in general assembly meetings. In the latter capacity, the individual is exposed to the decision dynamics of many groups − in our setup: of all boards and assemblies of firms in which she holds shares.

We argue that what happens at the collective level backward loops to the individual level, providing some new parameters for individual decision making. These parameters can convey positive, informational content, and feedback the beliefs of the individual; they can also convey some normative content and feedback the preferences of the individual.

We formalize the feedback mechanism on the graph of portfolio holdings: Each agent is a node; an individual node (a consumer) is linked to a collective node (a firm) according to whether the considered consumer has shares in the considered firm. This graph, denoted $\mathcal{G}$ offers a *dual* perspective (see Simmel, 1955; Breiger, 1974): The individual level is the dual of the collective level. Collective nodes aggregate the preferences of their neighboring individual nodes; and *reciprocally*, individuals nodes aggregate the 'preferences' of their neighboring collective nodes. In broad strokes, individuals shape the collectives they join, *and thereafter they are shaped by them*. In this paper, we stick closely to this duality between nodes, in order to grasp the essence of what we call *reciprocal aggregation*.

A diagram can help fix ideas. Nodes, whether individual or collective, are equipped with a device to take decisions − represented in our setup by (shadow) present value vectors. Let us denote ∇ a present value vector (pv. for short) for a person i, or a group j. Classically, the individual pv. ∇_i is 'socialized' into a vector ∇_{ij} translating how individual i's self-interest is expressed in group j. Then an aggregation mechanism $\mathcal{E}$ generates a collective pv. ∇_j from the collection $(\nabla_{ij})_i$.

Reciprocally, the group pv. ∇_j is 'individualized' into a new subjective pv. ∇_{ji} translating how group j's decision resonates in individual i's decision environment.[4] Finally an aggregation mechanism $\mathcal{S}$ generates new individual present values ∇_i from the collection

shaped by aggregation (e.g., Crès & al., 2012, and the survey therein). Finally, preference incompleteness is often axiomatized, if not by sheer aggregation, at least by multi-utility representation (see the survey in Evren & Ok, 2011).

[4]This is reminiscent of the Arrow-Lindhal model for internalization of external effects, where $\mathcal{S}$ is a price mechanism. Every 'collective' consumption x_j is individualized, in the sense that each individual i expresses a demand for collective consumption j, x_{ji}, given a price p_{ji}. The latter price level is fixed through a market clearing process: at equilibrium, all individualized demand for collective consumption have to coincide, $x_{ji} = x_j$, for all i

$(\nabla_{ji})_j$. Hence the diagram:

$$\begin{array}{ccc} (\nabla_i)_i & \overset{\text{socialization}}{\rightarrow} & (\nabla_{ij})_{ij} \\ \mathcal{S}\ \uparrow & & \downarrow\ \mathcal{E} \\ (\nabla_{ji})_{ij} & \overset{\text{individualization}}{\leftarrow} & (\nabla_j)_j \end{array} \tag{1}$$

Our notion of reciprocal aggregation identifies fixed points along this loop.

The results

We remain as little specific as possible on the nature of the aggregation mechanisms taking place at nodes. We only assume that they respect the Pareto principle: Consider node n; the Pareto principle establishes that if all neighbors nodes of n agree that plan a is at least as good as plan b, and one believes a is better than b, then so should node n. So the Pareto principle prescribes that node n surrenders to the view of its neighbors in case the latter are unanimous, *and only in that case*. Otherwise, it does not prescribe anything.

Our finding is that *with sufficient interconnectedness in the graph $\mathcal{G}$, the Pareto principle yields a total alignment of present value vectors*: The only stable outcomes are those where all present value vectors are identical. I.e., all agents (both individual and collective) agree on how to run the firms, as seems to be the case in the casual evidence of the CAC40.

The Pareto principle is probably one of the mildest assumption to make on an aggregation mechanism. It is a founding assumption of the literature on aggregation of individual preferences, starting with Arrow (1951). It is compatible with all types of aggregation processes that naturally come to mind, including voting mechanisms of all sorts. The Pareto principle is also a founding assumption of the logical aggregation theory (Kornhauser & Sager, 1986). It is finally a founding assumption of the literature on aggregation of judgements and beliefs, starting with Harsanyi (1955).

We build on the latter literature to assume that the Pareto principle also holds at the individual level. We argue indeed that individuals grant some wisdom, or authority, to the boards: they consider collective decisions as compelling *judgements*. Indeed a firm is a collective property. Its directors and shareholders decide collectively how to run it. At the end of the day some decision is taken. This decision is made by boards and assemblies, according to some protocol, usually very formal. Therefore, the underlying ‘preference’ of the firm is not as subjective as the individual ones. It acquires objectivity in the sense that its links with individual experiences, desires, perceptions or emotions becomes more distant. It acquires objectivity not in the sense that it is related to actual, external facts, but because it is less distorted by feelings and personal bias. In short: Individuals have beliefs, collectives make judgments.

The paper is organized as follow. The model is introduced in Section 2. Section 3 defines the governance of firms. It introduces the Pareto principle (at the collective level) and links it to value maximization of the firm's output (Lemma 2). Section 4 introduces the notion of reciprocal aggregation in a network of shareholders and firms. It provides the main insight of the paper, i.e., the total alignment of present value vectors when there is enough interlock in the shareholdings (Theorem 1), and explores the link between stability and optimality. It also shows how the Pareto principle holds, at the individual level, through a mere updating of beliefs. Section 5 develops the analysis of Section 4 in a general equilibrium setup to understand how trading on the financial markets endogenously shapes, through portfolio choices, the network of affiliation of shareholders to firms. Finally Section 6 offers a discussion of the related literature.

2 The economy

Consider an economy with two dates $t \in \{0, 1\}$ and uncertainty at date 1,with S contingencies $s \in \mathcal{S} =\{1, \dots, S\}$. To keep the model to the simplest, discard the question of consumption and production at date 0, and assume there is one commodity at every contingency at date 1.

Firms

There are J firms in the economy with $\mathcal{J} = \{1, \dots, J\}$. Every firm j is characterized by its production set $Y_j \subset \mathbb{R}^S_+$.

Production sets are supposed to be compact, convex and smooth manifolds: Y_j is non-empty and compact and there is a smooth and strictly convex map $g_j : \mathbb{R}^S_+ \to \mathbb{R}$ such that $Y_j = \{ y_j \in \mathbb{R}^S_+ \mid g_j(y_j) \leq 0 \}$.. This property ensures us that there is a unique value-maximizing production plan for all non-trivial present value vector (see Lemma 1).

Let $y = (y_1, \dots, y_J) \in Y = \prod_j Y_j$ be a list of production plans.

Consumers

Consumer preferences are represented by a utility function $U_i : \mathbb{R}^S \to \mathbb{R}$, assumed to be differentiable, increasing and strictly quasi-concave. Every consumer is endowed with an initial endowment $\bar{x}_i = (\bar{x}_i^1, \dots, \bar{x}_i^S) \in \mathbb{R}^S$ and an initial portfolio of shares in firms $\bar{\theta}_i = (\bar{\theta}_{i1}, \dots, \bar{\theta}_{iJ}) \in \mathbb{R}^J$ where $\sum_i \bar{\theta}_{ij} = 1$ for all j. Hence an economy is described by

$$((U_i, \bar{x}_i, \bar{\theta}_i)_{i \in \mathcal{I}}, (g_j)_{j \in \mathcal{J}}).$$

Let $x = (x_1, \dots, x_I)$ with $x_i \in \mathbb{R}^S$ for every i be a list of consumption bundles and $\theta = (\theta_1, \dots, \theta_I)$ with $\theta_i \in \mathbb{R}^J$ for every j a list of portfolios.

Allocations and states

For a list of production plans y and a portfolio θ_i for consumer i, the actual consumption of consumer i in state s is the sum of her initial endowment $\bar{x}_i^s$ and return on her portfolio:

$$x_i^s \;=\; \bar{x}_i^s + \sum_{j\in\mathcal{J}} \theta_{ij} y_j^s$$

Let $A(y) = (y_1 \dots y_J)$ be the $S{\times}J$-matrix of production plans. Then $x_i = \bar{x}_i{+}A(y)\theta_i$.

Definition 1 *An **allocation** is a list of consumption bundles, portfolios and production plans (x, θ, y) such that*

- *Consumption is obtainable: $x_i = \bar{x}_i{+}A(y)\theta_i$ for every i.*
- *Ownership of firms is balanced: $\sum_i \theta_{ij} = 1$ for every j.*

  *A **state** is a list of utility functions and an allocation (U, x, θ, y) with $U = (U_1, \dots, U_I)$.*

Given an allocation, the aggregate demand for commodities $\sum_i x_i$ and the aggregate supply of commodities $\sum_i \bar{x}_i{+}\sum_j y_j$ are equal because $\sum_i \theta_{ij} = 1$ implies $\sum_i x_i = \sum_i \bar{x}_i{+}A(y)\theta_i = \sum_i \bar{x}_i + \sum_j y_j$.

Clusters

We consider shareholder governances where the decision makers in firm j are the shareholders with strictly positive shares. Let $\mathcal{I}_j \subset \mathcal{I}$ be the set of shareholders in firm j, and let $\mathcal{J}_i \subset \mathcal{J}$ be the set of firms in which consumer i has shares. The web of affiliation of shareholders and firms is defined by the list of portfolios θ: $i \in \mathcal{I}_j$ and $j \in \mathcal{J}_i$ if and only if $\theta_{ij} > 0$.

There is a graph representation of a portfolio, $\mathcal{G}(\theta)$, defined by nodes i and j being connected if and only if $\theta_{ij} > 0$. It has some number of maximal strongly connected components or simply clusters.

Definition 2 *For θ a **cluster** is a nonempty subset of consumers and firms $\mathcal{C} \subset \mathcal{I}{\cup}\mathcal{J}$ with $\cup_{i\in\mathcal{C}}\mathcal{J}_i \cup \cup_{j\in\mathcal{C}}\, \mathcal{I}_j = \mathcal{C}$ such that $\mathcal{D} \subset \mathcal{C}$ and $\cup_{i\in\mathcal{D}}\mathcal{J}_i \cup \cup_{j\in\mathcal{D}}\, \mathcal{I}_j = \mathcal{D}$ imply $\mathcal{D} = \mathcal{C}$ or $\mathcal{D} = \emptyset$.*

Present value vectors

Agents use vectors of present values to take decisions. These vectors are normalized such that their coordinates sum to one. Let $\triangle_+^S$ be the unit simplex in $\mathbb{R}^S$

$$\triangle_+^S \;=\; \left\{ \nabla \in \mathbb{R}_+^S \mid \sum_{s\in\mathcal{S}} \nabla^s = 1 \right\}.$$

Production plans are optimal provided they maximize value for some present value vectors.

Definition 3 *A production plan $y_j \in Y_j$ is **optimal** for the present value vector $\nabla_j \in \mathbb{R}^S$ provided $\nabla_j \cdot y_j \geq \nabla_j \cdot y'_j$ for all $y'_j \in Y_j$.*

The following observation is immediate, but important. It shows a one to one relationship between value-maximizing production plans and non-trivial present value vectors.

Lemma 1 *Suppose Y_j is not a singleton. Then*

(i) $y_j \in \partial Y_j$ if and only if $g_j(y_j) = 0$.

(ii) $y_j \in Y_j$ is optimal with respect to $\nabla_j \in \mathbb{R}^S \backslash \{0\}$ if and only if $y_j \in \partial Y_j$ and $Dg_j(y_j)$ and ∇_j are collinear with $Dg_j(y_j) \cdot \nabla_j > 0$.

(iii) For every $y_j \in \partial Y_j$ there is a unique present value vector $\nabla_j \in \mathbb{R}^S \backslash \{0\}$ up to normalization such that y_j is optimal with respect to ∇_j.

(iv) For every $\nabla_j \in \mathbb{R}^S \backslash \{0\}$ there is a unique production plan $y_j \in \partial Y_j$ such that y_j is optimal with respect to ∇_j.

Proof: See Appendix. □

We now turn toward the governance of firms.

3 The shareholder governance

This section reviews how individual and collective decisions are taken in the economy.

Individual decisions

For a utility function U_i and a consumption bundle x_i, let $\nabla_i(U_i, x_i) \in \mathbb{R}^S_+$ be the normalized gradient of U_i with respect to x_i

$$\nabla_i(U_i, x_i) \;=\; \frac{1}{\sum_s D_{x_i^s} U_i(x_i)} DU_i(x_i).$$

Shareholder i uses her present value vector ∇_i to choose production plans, as stated in the following observation, which comes as an immediate consequence of the strict quasi-concavity of U_i.

Proposition 1 *For a consumer i and a firm j with $\theta_{ij} > 0$ and an alternative production $y'_j \in Y_j$ with $y'_j \neq y_j$.*

- *If $U_i(x_i + \theta_{ij}(y'_j - y_j)) > U_i(x_i)$, then $\nabla_i(U_i, x_i) \cdot (y'_j - y_j) > 0$.*
- *If $\nabla_i(U_i, x_i) \cdot (y'_j - y_j) > 0$, then there is $\tau > 0$ such that $U_i(x_i + \tau\theta_{ij}(y'_j - y_j)) > U_i(x_i)$.*

Proof: See Appendix. □

In the standard neoclassical model with perfectly competitive and complete markets, all individual agents use the equilibrium price vector to value production plans, and therefore there is unanimity within the assembly of shareholders. But in case of market incompleteness, no general equilibrium condition guarantees equality of individual present value vectors. Hence there is a potential conflict in the assembly that has to be solved through some aggregation mechanism.

This is what we turn toward now.

Collective decisions

In assessing the value of a plan $y_j \in Y_j$, the present value vectors $(\nabla_i)_{i \in \mathcal{I}_j}$ of shareholders in firm j are aggregated into a present value vector $\nabla_j \in \triangle^S$ for firm j. The aggregation mechanism is supposed to respect the Pareto principle: if all shareholders value y'_j at least as much as y_j and at least one shareholder values y'_j more than y_j, then the firm values y'_j more than y_j.

Definition 4 *For present value vectors $((\nabla_i)_{i \in \mathcal{I}}, (\nabla_j)_{j \in \mathcal{J}})$:*

- *∇_j respects the* ***Pareto principle*** *(across shareholders) provided that for every change $\Delta y \in \mathbb{R}^S$, $\nabla_i \cdot \Delta y \geq 0$ for every $i \in \mathcal{I}_j$ with $>$ for some $i \in \mathcal{I}_j$ implies $\nabla_j \cdot \Delta y > 0$.*
- *$(\nabla_j)_{j \in \mathcal{J}}$ is* ***Pareto stable*** *provided that for every j, ∇_j respects the Pareto principle.*

The Pareto principle is admittedly a mild, low-demanding requirement. It is a founding assumption of the literature on aggregation of individual preferences, starting with Arrow (1951), followed by Sen (1970). It is also a building block of welfare economics, especially of the literature on the joint aggregation of beliefs and tastes, starting with Harsanyi (1955)'s social aggregation theorem, extended to Savage's framework[5] by Hylland & Zeckhauser (1979) and Mongin (1995), among others, and more recently to the case of multiple priors by Crès, Gilboa & Vieille (2011).

The Pareto principle is an influential principle of normative economics and moral philosophy. In particular, it appears as a founding assumption of the literature on aggregation of judgements and logical aggregation theory, rendered popular by the doctrinal paradox of Kornhauser & Sager (1986), and the discursive dilemma of List & Pettit (2002) − see Mongin (2012) for a panoramic and authoritative survey of these questions.

We now characterize stability in terms of value maximization in firms.

[5] Note that it has been fingered that the unanimity of individual preferences can be 'spurious', i.e., falsely driven by the fact that disagreement over tastes and disagreement over beliefs neutralize each other − see, e.g., Mongin (1997) and Gilboa, Samet and Schmeidler (2004).

The Pareto principle and value maximization

The following result is well-known and widely used throughout this paper. The Pareto principle yields that production plans are maximizing value for some present value vector in the positive cone of the present value vectors of the shareholders.

Lemma 2 *For present value vectors* $((\nabla_i)_{i\in\mathcal{I}}, (\nabla_j)_{j\in\mathcal{J}})$, ∇_j *respects the Pareto principle across shareholders if and only if there is* $(\mu_{ij})_{i\in\mathcal{I}_j}$ *with* $\mu_{ij} > 0$ *for every* i *such that* $\nabla_j = \sum_{i\in\mathcal{I}_j}\mu_{ij}\nabla_i$.

Proof: According to Theorem 22.2 in Rockafellar (1970) either, corresponding to ∇_j does not respect the Pareto principle, there is $\Delta y \in \mathbb{R}^S$ such that

$$\begin{aligned} \nabla_i\cdot\Delta y &\geq 0 \text{ for every } (i,m)\in\mathcal{M}_i\times\mathcal{I}_j \\ \textstyle\sum_{i\in\mathcal{I}_j}\nabla_i\cdot\Delta y &> 0 \\ \nabla_j\cdot\Delta y &\leq 0 \end{aligned}$$

or, corresponding to ∇_j does respect the Pareto principle, there are $(\alpha_i)_{i\in\mathcal{I}_j}$ with $\alpha_i \geq 0$ for every i, $\beta > 0$ and $\gamma \geq 0$ such that

$$\textstyle\sum_{i\in\mathcal{I}_j}\alpha_i\nabla_i + \beta\sum_{i\in\mathcal{I}_j}\nabla_i - \gamma\nabla_j \;=\; 0.$$

Clearly $\alpha_i \geq 0$ and $\nabla_i \in \triangle_+^S$ for every i and $\beta > 0$ implies $\gamma > 0$. Therefore

$$\nabla_j \;=\; \sum_{i\in\mathcal{I}_j}\frac{\alpha_i+\beta}{\gamma}\nabla_i$$

hence the result with $\mu_{ij} \;=\; \dfrac{\alpha_i+\beta}{\gamma} > 0$. □

Hence ∇_j is averaging of the present values of shareholders. If we consider that the latter are 'biased' by idiosyncrasies, then these biases are somewhat purged in the aggregate by averaging. An interpretation is that each ∇_j proposes an 'objective' collective judgement[6]. How these various collective judgements interact through the network of shareholding is the object of the next section.

4 Reciprocal aggregation

We assume that, in forming their present values, shareholders cannot ignore the (collective) present values of the firms in which they invest. They must take them into account, one way or another, and when appropriate use them to update their own present values. In short: they aggregate them.

[6]Following Gilboa & alii (2010), a more precise, although more cumbersome term would be 'inter-subjectivity' instead of sheer objectivity, that would refer to some external, validated 'truth'.

Hence individuals also act as collectives. The assertion that there is a duality between persons and groups has a long tradition in sociology. It dates back to the early XXth century and the work of Georg Simmel. The question of the interpenetration of group-affiliation and individual personality is central in his theory. A first quote of Simmel (1955) will help catch the essence of our argument.

> "[...] as individuals, we form the personality out of particular elements of life, each of which has arisen from, or is interwoven with, society. This personality is subjectivity par excellence in the sense that it combines the elements of culture in an individual manner. There is here a reciprocal relation between the subjective and the objective. As the person becomes affiliated with a social group, he surrenders himself to it. A synthesis of such subjective affiliations creates a group in an objective sense. But the person also regains his individuality because his pattern of participation is unique: hence the fact of multiple group-participation creates in turn a new subjective element. Causal determination of, and purposive action by, the individual appear as two sides of the same coin." (p. 141)

In broad strokes, individuals shape the collectives they join, and thereafter they are shaped by them. Individuals are *subjective*, but the groups they shape are *objective*.

The Pareto principle across firms

In assessing the opportunity of a change in production plan $\Delta y \in \mathbb{R}^S$, shareholder i faces $|\mathcal{J}_i|$ potentially different evaluations represented by the present value vectors $(\nabla_j)_{j\in\mathcal{J}_i}$.

Definition 5 *For present value vectors* $((\nabla_i)_{i\in\mathcal{I}}, (\nabla_j)_{j\in\mathcal{J}})$*:*

- ∇_i *respects the Pareto principle (across firms) provided that for every change* $\Delta y \in \mathbb{R}^S$, $\nabla_j \cdot \Delta y \geq 0$ *for every* $j \in \mathcal{J}_i$ *with* $>$ *for some* $j \in \mathcal{J}_i$ *implies* $\nabla_i \cdot \Delta y > 0$.
- $(\nabla_i)_{i\in\mathcal{I}}$ *is Pareto stable provided that for every* i, ∇_i *respects the Pareto principle.*

A useful image to illustrate the Pareto principle (across groups) is that of juries in criminal matters. As an individual, in the absence of hard evidence, one might lean toward the belief that a suspect is guilty. Suppose that several collective juries *all* come to the conclusion that the suspect is not guilty, shouldn't it push one to revise one's beliefs?

A second force behind the Pareto principle is the character of local public good of the firm. If a consumer takes shares in some firm, it is because its production plan provides an insurance service that fits this investor's needs. Hence all investors in a given firm share this feature in common. We argue that this commonality of needs, and will to invest in common (the *affectio societatis* of the French civil law), might foster a tendency to surrender to the views of the group, especially when these views have *unanimous* support from all groups to which one has *freely* adhered.[7] Following Hirshman's rhetoric, a shareholder has chosen not to exit from the firm (or vote with her feet), and is thus embedded within a shared loyalty.

A dual of Lemma 2 holds for the Pareto principle at the individual level.

Lemma 3 *For present value vectors* $((\nabla_i)_{i\in\mathcal{I}}, (\nabla_j)_{j\in\mathcal{J}})$, ∇_i *respects the Pareto principle if and only if there is* $(\nu_{ij})_{j\in\mathcal{J}_i}$ *with* $\nu_{ij} > 0$ *such that* $\nabla_i = \sum_{j\in\mathcal{J}_i}\nu_{ij}\nabla_j$.

Proof: See proof of Lemma 2. □

Informally, an interpretation of $\nabla_i = \sum_j \nu_{ij}\nabla_j$ is that shareholders do not think 'out of the box'. The box is delimited by the collective judgements of the firms. If shareholders give in to these collective judgements, as they stem themselves from the shareholders' present values, a self-referential dynamics is at work. If the network of shareholdings is sufficiently interlocked, at a stable state all persons and groups have the same present values. This is what we formalize next.

Stable states and alignment of present values

Consider a production plan y_j for firm j. By Lemma 1, for any $y_j \in \partial Y_j$ there is a unique present value vector $\nabla_j \in \mathbb{R}^S\setminus\{0\}$ up to normalization with respect to which y_j is optimal. Let us denote it $\nabla_j(y_j)$.

Definition 6 *A state* (U, x, θ, y) *is* ***stable*** *provided:*

- $(\nabla_j(y_j))_{j\in\mathcal{J}}$ *is Pareto stable across shareholders;*
- $(\nabla_i(U_i, x_i))_{i\in\mathcal{I}}$ *is Pareto stable across firms.*

Utility functions U play no role in the definition of a state. However they play a central role for the stability of states.

If the graph of affiliations $\mathcal{G}(\theta)$ is connected, at a stable state, all firms and shareholders agree on how to discount future income flows. This provides an interpretation of why one observes such a wide consensus in general assembly meetings, even in case of severe market failures.

Theorem 1 *Consider a stable state* (U, x, θ, y) *such that the graph* $\mathcal{G}(\theta)$ *is connected. Then there is* $\nabla \in \triangle^S_{++}$ *such that* $\nabla_i(U_i, x_i) = \nabla_j(y_j) = \nabla$ *for all* i *and* j.

Proof : Let $P_{\mathcal{I}}$ be the convex hull of the present value vectors of the shareholders $(\nabla_i)_{i\in\mathcal{I}}$. Suppose p is an extreme point of $P_{\mathcal{I}}$ and let $\mathcal{I}(p) = \{\, i \in \mathcal{I} \mid \nabla_i = p \,\}$. Then $\mathcal{I}(p) \neq \emptyset$ by

[7] We insist that both the unanimity of 'jurys', and the freedom of affiliation, make a difference. Section 6 discusses why this is not conformism, or imitation. In particular, these last words apply generally to direct inter-individual influences; here these influences are mediated by groups through institutional designs.

construction. If $\nabla_i = p$, then $\nabla_k = p$ for every $k \in \mathcal{I}_i$ and so on. Therefore $\mathcal{I}(p) = \mathcal{I}$ because every pair of shareholders is connected. According to Lemma 3, $P_{\mathcal{J}} \subset P_{\mathcal{I}}$, so $\nabla_j(y_j) = p$ for every j. $\square$

This alignment of present value vectors remains true in general within each cluster.

Corollary 1 *Suppose (U, x, θ, y) is a stable state. Then for every cluster $\mathcal{C}$ there is $\nabla_{\mathcal{C}} \in \triangle^S_{++}$ such that $\nabla_i(U_i, x_i) = \nabla_j(y_j) = \nabla_{\mathcal{C}}$ for every $i, j \in \mathcal{C}$.*

Stability and optimality

Definition 7 *Consider a state (U, x, θ, y). The allocation (x, θ, y) is* ***Pareto optimal*** *if there is no other allocation (x', θ', y') such that $U_i(x'_i) \geq U_i(x_i)$ for all i (with strict inequality for at least one i).*

We immediately get the following property for stable states with a unique cluster.

Proposition 2 *Consider a stable state (U, x, θ, y). Suppose the underlying graph $\mathcal{G}(\theta)$ is connected then it is Pareto optimal.*

Proof: A sufficient condition for Pareto optimality is that all consumer gradients be collinear and that production be optimized with respect to this common present value vector. We know from Theorem 1 that both conditions hold. $\square$

The Pareto principle across firms somewhat challenges a rigorous obedience to methodological individualism,[8] but we argue that it does so in ways that are already widely accepted in the realm of mainstream economic thought. It does not necessarily require a revision of deep, primitive characteristics of the individual. When consumers are expected utility maximizers, the Pareto principle can be vehicled by a mere updating of beliefs, i.e. subjective probabilities over investment prospects.

The expected-utility framework

The equations of Lemma 3 involve a change in preferences. But they do not necessarily involve a change in *tastes.* To account for fixed tastes, if *de gustibus non est disputandum*, let us introduce the special case where consumers are expected utility maximizers as in the Savage framework. Consumer i is characterized by a subjective belief $\pi_i \in \triangle^S_+$ about the likelihood of the different contingencies, and an elementary utility function $u_i : \mathbb{R} \to \mathbb{R}$,

[8] List & Spiekermann (2013) argue that 'supervenience individualism' (i.e., the view according to which the individual-level facts fully determine the social facts) is compatible with 'causal-expalanatory holism' (i.e., the view according to which some causal relations are distinct from any individual-level causal relations). They illustrates this compatibility with, among others, the example of social-network theory.

accounting for her tastes, such that $U_i = \pi_i \diamond u_i$:

$$U_i(x_i) = \pi_i \diamond u_i(x_i) = \sum_{s\in\mathcal{S}} \pi_i^s u_i(x_i^s). \tag{2}$$

The elementary utility function u_i is assumed to be differentiable, increasing and concave. In this section, u_i is assumed to be fixed, but beliefs π_i are updated.

The Savage framework is particularly well adapted to the type of uncertainty we have in mind. Firms invest over contingencies, or prospects (e.g., whether subprime assets are sustainable or not; whether the oil price in 5 years from now is going to be above, or below, 60$ a barrel...) for the distribution of which it is extremely difficult to come with an objective probabilistic assessment. It is widely accepted that beliefs can be updated with acquisition of new information. Here we assume that, absent any informational content, in the course of economic transactions, agents infer beliefs about other agents' beliefs and choose to update their priors accordingly, as axiomatized by the Pareto principle.

Updating beliefs

At any state, shareholders of firm j observe the present value vector $\nabla_j(y_j)$ used by firm j to maximize value. They infer from this present value vector the underlying probability distribution over contingencies.

To fix ideas, consider risk-neutral agents. Then $\nabla_j(y_j)$ can be interpreted as the implicit probability distribution π_j over contingencies used by firm j to maximize value. The convex-hull condition of Lemma 3 reads as:

$$\pi_i = \sum_{j\in\mathcal{J}_i} \nu_{ij}\pi_j. \tag{3}$$

This corresponds to the DeGroot (1974) model of beliefs formation on a graph, where shareholder i gives 'weight' ν_{ij} to the belief of firm j. The weights might differ according to the authority that firm j carries in the eyes of shareholder i, or according to the degree of loyalty that shareholder i feels toward firm j.

This interpretation can be carried to risk-averse agents. Consumer i's present value vector is the normalized gradient $D_{x_i}U_i(x_i) = \left(\pi_i^1 u_i'(x_i^1), \cdots, \pi_i^S u_i'(x_i^S)\right)$. The marginal utility $u_i'(x_i^s)$ can be interpreted as the *shadow spot price* of individual i in contingency s. It measures the value for consumer i of one unit of the good in contingency s independently of its probability.

In the spirit of the competitive analysis, we assume that a consumer does not have access to any information about the marginal utilities of other consumers. Moreover firms are not endowed with preferences. Therefore, when facing the present value vector $\nabla_j(y_j)$, consumer i uses her own shadow spot prices to assess the implicit beliefs underlying $\nabla_j(y_j)$

When facing a present value vector ∇, consumer i can easily compute what belief it would take, given her current consumption, for her to display the same present value vector.

Given x_i, the S vectors $e_i^s(x_i) = (0, \cdots, 0, u_i'(x_i^s), 0, \cdots, 0)$, $1 \leq s \leq S$, form a basis of $\mathbb{R}^S$ (since $u_i'(x_i^s) \neq 0$). The same as, in this basis, $D_{x_i} U_i(\pi_i, x_i)$ has coordinates π_i, there exists a unique $\pi \in \triangle_+^S$ such that, up to normalization, ∇ has coordinates π. Denote $\pi(u_i, x_i, \nabla)$ the latter.

Assumption (CBP) - Competitive Beliefs Perceptions: Consumer i associates beliefs $\pi(u_i, x_i, \nabla)$ to a present value vector ∇, where $\pi(u_i, x_i, \nabla)$ is such that ∇ is collinear to $\left(\pi_i^1(u_i, x_i, \nabla) u_i'(x_i^1), \cdots, \pi_i^S(u_i, x_i, \nabla) u_i'(x_i^S)\right)$.

Assuming (CBP), the à la DeGroot equations (3) hold.

5 General equilibrium

We saw in Proposition 2 that a stable state is Pareto optimal if the web of affiliation $\mathcal{G}(\theta)$ of shareholders to firms is connected. In this section we endogenize the latter by modeling how the list of portfolios θ comes out of trading on financial markets. We explore whether stable states can result from a general economic (and social) equilibrium, framing Proposition 2 and Corollary 1 as variants of the first welfare theorem.

Stock market equilibrium

Consumers can trade assets at date t=0. The asset prices are $q = (q_1, \ldots, q_J)$ where q_j is the price of asset j. Asset prices are normalized to be in $\triangle^J$.

At date t=0 given asset prices q and production plans y consumers trade assets in order to maximize their utilities. The budget constraint for consumer i is $q \cdot \theta_i \leq q \cdot \bar{\theta}_i$. Given a portfolio θ_i and production plans y the consumption of consumer i is $x_i = \bar{x}_i + A(y)\theta_i$.

Definition 8 *Consider a list of utility functions and production plans (U, y). A* ***stock market equilibrium*** *is a price vector and a list of portfolios (q, θ) such that*

- $\theta_i \in \arg\max\{\, U_i(\bar{x}_i + A(y)\theta_i') \mid q \cdot \theta_i' \leq q \cdot \bar{\theta}_i \,\}$ *for every i.*
- $\sum_i \theta_{ij} = 1$ *for every j.*

If financial markets are complete, so $A(y)$ has rank $S-1$, then at stock market equilibria the present value vectors of consumers are identical $\nabla_i(U_i, x_i) = \nabla_{i'}(U_{i'}, x_{i'})$ for every pair of consumers i and i'. However if financial markets are incomplete, so $A(y)$ has rank $S-2$ or less, then at stock market equilibria the present value vectors of consumers do not need to be identical. Indeed as shown in Theorem 11.6 in Magill & Quinzii (1996) for almost all collections of initial endowments $\bar{x}$, at equilibrium no pair of consumers has identical present value vectors.

Reciprocal aggregation equilibrium

A notion of general equilibrium can be introduced which rests on three pillars: individual optimization, market clearing and stability of the reciprocal aggregation of shareholders' and firms' present values. It combines the notion of stock market equilibrium (Definition 8) and stable state (Definition 6). A first notion, purely self-referential, comes first to mind.

Definition 9 *A* ***reciprocal aggregation equilibrium*** *is defined by prices and a state* $(q^*, U^*, x^*, \theta^*, y^*)$ *such that:*

- (q^*, θ^*) *is a stock market equilibrium for* (U^*, y^*)*.*
- $(U^*, x^*, \theta^*, y^*)$ *is a stable state.*

Let us underline the fact that the initial preferences $\bar{U}$ play no role in the definition of a reciprocal aggregation equilibrium. Hence this notion does not account for memory. We introduce another, stronger concept of equilibrium.

Definition 10 *A* ***reciprocal aggregation equilibrium with memory*** *is defined by prices and a state* $(q^*, U^*, x^*, \theta^*, y^*)$ *such that:*

- (q^*, θ^*) *is a stock market equilibrium for* (U, y^*)*.*
- (q^*, θ^*) *is a stock market equilibrium for* (U^*, y^*)*.*
- $(U^*, x^*, \theta^*, y^*)$ *is a stable state.*

Existence

Proving existence of reciprocal aggregation equilibria is straightforward, even in the expected-utility framework.

Observation 1 *Consider a vector of primitive characteristics* $(\bar{\pi} \diamond \bar{u}, \bar{x}, \bar{\theta})$*. There always exists a reciprocal aggregation equilibrium.*

Proof: Fix a present value vector $\nabla \in \triangle^S_{++}$. Define $y_j^* = \arg\max \{\nabla \cdot y_j \mid y_j \in Y_j\}$ for all j, and consequently A^*. Define $x_i^* = \bar{x}_i + A^*\bar{\theta}_i$ for all i, and consequently $\pi_i^* = \pi(\bar{u}_i, x_i^*, \nabla)$ though competitive beliefs perceptions. Define finally q^* as the normalized vector of asset prices collinear to $A^{*T}\nabla$. Then $(\pi^* \diamond \bar{u}, x^*, y^*, \bar{\theta}, q^*)$ is a (no-trade, stable-taste) reciprocal aggregation equilibrium. □

Observe also that when markets are complete, given any $(\bar{\pi}, y)$, a stock market equilibrium (θ^*, q^*) is associated to the (fixed-belief, fixed-taste) reciprocal aggregation equilibrium $(\bar{\pi} \diamond \bar{u}, x^*, \theta^*, y, q^*)$ (with $x_i^* = \bar{x}_i + A\theta_i^*$), hence existence of the latter is also straightforward.

Optimality

As a corollary of Proposition 2, if the endogenous graph of shareholding $\mathcal{G}(\theta^*)$ is connected, then the corresponding equilibrium allocation is Pareto optimal. Hence the first welfare theorem holds.

6 Discussion and related literature

Our results are based on one hypothesis and one assumption. The hypothesis is that some aggregation takes place at the individual level. The assumption is that both the collective and individual aggregation mechanisms satisfy the Pareto principle.

The assumption that aggregation mechanisms satisfy the Pareto principle is quite conventional. Although it is mild, it has strong consequences in our model. The strength is triggered by the dual perspective between individuals and collectives, and the concept of reciprocal aggregation.

As we have abundantly argued, the hypothesis that some aggregation takes place at the individual level is not unconventional in the realm of economic thinking. There is nothing revolutionary in interpreting Equations (3) as a mere updating of beliefs à la DeGroot. But some argue that one cannot disentangle beliefs from tastes, even for expected utility maximizers (e.g., Duffie, 2014). Then the equations of Lemma 3 become more problematic, as they involve a change in tastes, a phenomenon that economists have long been reluctant to explore but on which there is now a rich literature.

We start this section by reviewing various aspects of this literature, then appeal to the concept of adaptive preferences in support of the descriptive force of our model, and finally discuss the normative consequences of our results.

Do individuals change tastes?

It has long been recognized that individuals change taste over time based on past behavior and experiences (Becker, 1976, 1996). Recent evolutionary studies predict how cardinal properties of hedonic utility adapt to the decision environment (see, e.g., Netzer, 2009). Also, the field of industrial organization has for long identified the phenomenon of 'experience goods', and consumer theory the phenomenon of 'habit formation' (axiomatized by Rozen, 2010), two instances of endogenous change of tastes (see von Weizsäcker, 1971, Hammond, 1976, Pollack, 1978). In this early literature, a change of tastes is triggered by past consumption, but the subsequent literature on 'reference-dependent preferences' (Kahneman & Tversky, 1979) has extended possible causes of change to rational expectations about future variables (see, e.g., Köszegi & Rabin, 2006). The changes in tastes induced by the Pareto principle are clearly forward looking, as we argue in the next section.

The assumption of stability of tastes, although a parsimonious one to describe and explain economic phenomena, seems to be contradicted also by some evidence from behavioral economics. An example is the apparent tendency of decision makers to choose the default option. In the context of the present paper, the 'default option bias' (Kahneman et al., 1991) is supporting the Pareto principle: If some production policy is bound to be chosen because of unanimous support by firms, then it acquires de facto the status of default option. In addition, once an option is bound to be imposed on us by the force of unanimity of the groups to which *one chooses to affiliate*, surrendering to this option is the most direct way to avoid 'cognitive dissonance' in the future.

More directly relevant to the present model is the theory of deliberation-induced preference change (e.g., Miller, 1992, Knight & Johnson, 1994, Dryzek and List, 2003). For many scholars following Habermas (1984), the central concern of politics is to change preferences, rather than aggregate them. And the goal of political interaction is to forge a unanimous, rational consensus rather than optimally compromise between diverging, inflexible opinions or interests.

Our model is compatible with both views. The Pareto principle across shareholders is compatible with pure voting without deliberation, as well as with deliberation converging toward a unique, common present value vectors, consensual with the shareholders' initial present values, i.e. in their convex hull. As for the Pareto principle across firms, it is compatible with both views too: it can be seen as an optimal compromise between multiple selves,[9] as well as an intimate deliberation weighing various considerations for a better judgement; both mechanisms occurring in the heart of the considered individual. In any case, we believe that aggregation and deliberation are complementary, and probably difficult to disentangle in the real world. The more entanglement, the richer the notion of reciprocal aggregation.

Updating beliefs or changing tastes: adaptive preferences

In support of the Pareto principle at the individual level, there are good reasons (if not a full-fledged 'rationality') to 'adapt' to (if not adopt) the unanimous views expressed by groups to which one chooses to adhere. Given unanimity, most probably these views will be imperative. They will generate outcomes that everyone will have to put up with. Adapting to these ineluctable circumstances is reasonable − like a trader has to follow the mainstream expectations, or she loses money. If all firms in which one invests unanimously value a type of investment policy, then most probably this type will be widely adopted, and therefore the concerned shareholder would be better off resolving to surrender to such policies, or vote with her feet, short-sell the firm's stock and cut links in the graph. Updating one's beliefs for the sake of welfare is rational. Changing one's tastes for the same sake sounds reasonable, in which case it shows the strength, and freedom, to embrace the ineluctable: it is *amor fati*, the 'love of fate', or 'faire de nécessité vertu' (Elster, 1983). When not driven by updating of

[9] Bismarck said: "Faust complained that he had two souls in his breast. I have a whole squabbling crowd. It goes on as in a republic." (cited in Steedman & Krause, 1985).

belief, the Pareto principle applied at the individual level rests on *forward-looking adaptive preferences.*[10]

In support of the Pareto principle, we follow Elster (1983) and build a formal parallel between beliefs and tastes. In his search for a broad theory of individual and collective rationality, Elster appeals to the notion of *autonomy.* Autonomy is for *desires* what judgement is for belief: "[...] autonomous desires are desires that have been deliberately chosen, acquired, or modified – either by an act of will or by a process of character planning". Both for beliefs and desires, a crucial condition for substantive rationality is the absence of distortions and illusions.

We consider that judgements are substantively rational beliefs, not in the sense that they are grounded in available evidence, but in the sense that they purged from idiosyncrasies by the operation of the Pareto principle (Proposition 1). Boards and assemblies have their own rules and regulations, so are typically *autonomous*, in the etymological sense. To paraphrase Elster, they typically are in control of the processes whereby their tastes are formed, or at least not in the grip of the individual preference-formation processes. Would there remain scoria of idiosyncratic distortions and illusions in the aggregate, these scoria would be washed away in the backward loop toward the individual level by the operation of the Pareto principle across firms.

According to Dworkin (1988): "Autonomy is conceived of as a second-order capacity of persons to reflect critically upon their first-order preferences, desires, wishes, and so forth and the capacity to accept or attempt to change these in light of higher-order preferences and values." The backward looping individual aggregation mechanism $\mathcal{S}$ is typically a second-order capacity to reflect critically upon one's first-order preferences. And when an investor joins an assembly, there might be a will, based on self-interest, to invest in common, akin to Hirshman's loyalty, which creates a higher-order value.

The question remains to which point adapting one's taste is not a token of conformism, or imitation, however autonomous. Beyond recalling the traditional argument of sociology,[11] we want here to raise two points. First, individual i does not take, at least directly, into

[10] The concept of adaptive preferences has a long history (Elster, 1983, Sen, 1985, von Weizsäcker, 2013). It is often used to describe how people living in great deprivation tend to get used to it. Some even end up being happy about their lot. This is why tenants of the capability approach (Sen, 1985, 1999, Nussbaum, 2003, 2004) discard utility as a tool for measuring welfare.

[11] A quote of Simmel (1955) discards the idea that through the operation of a dual aggregation mechanism $\mathcal{S}$ an individual agent is bound to lose its core identity: "It is true that external and internal conflicts arise through the multiplicity of group-affiliations, which threaten the individual with psychological tensions or even a schizophrenic break. But it is also true that multiple group-affiliations can strengthen the individual and reenforce the integration of his personality. Conflicting and integrating tendencies are mutually reenforcing. Conflicting tendencies can arise just because the individual has a core of inner unity. The ego can become more clearly conscious of this unity, the more he is confronted with the task of reconciling within himself a diversity of group-interests. [...] These conflicts may induce the individual to make external adjustments, but also to assert himself energetically" (p. 141-142).

account the preference parameters of other individuals, but that of the groups she affiliates to. It is not imitation of individual peers. At worst it is 'conformity to the party line', and we insist: a party that one has *freely chosen* to join, not conformity to norms imposed upon the individual by the accident of birth. Second, and to carry the metaphor further, one is affiliated to potentially many different parties, and synthesizes all their respective lines; the Pareto principle requires adapting tastes *only if* there is a massive alignment in the form of *unanimous agreement* of all the parties one joins.

We now turn to the discussion of the normative consequences of reciprocal aggregation based on the Pareto principle, in particular the validity of the first welfare theorem despite potential market incompleteness.

Market incompleteness and Pareto optimality

Incomplete markets fail to promote allocative efficiency because at equilibrium consumers typically use different present value vectors to discount future income flows. Even though we know since Hart (1975) that adding assets to reduce the incompleteness of the financial structure might turn harmful (Cass & Citanna, 1998), the conventional wisdom is unequivocal: only complete markets guarantee the first welfare theorem to hold, and Pareto optimality is desirable.

This conventional wisdom has recently been challenged, at least in the case of expected utility maximizing agents. The main objection is that Pareto optimality is not an appropriate welfare criterion when traders have heterogeneous beliefs over contingencies. Because the unanimity behind a Pareto improvement might be 'spurious', i.e., based on differences in beliefs *and tastes* that offset each other. Blume & al. (2014) introduce a welfare criterion according to which various restrictions on financial markets are desirable. Gilboa, Samuelson & Schmeidler (2014) point at the growing complexity of financial assets, and the fact that financial innovation creates its own, endogenous uncertainty; they suggest that this complexity creates mostly opportunities to bet, and argue that the welfare implications of betting are dubious. Building on an early argument by Stiglitz (1989), this literature (see also Brunnermeier, Simsek & Xiong, 2014) aims at separating the (dubious) trades triggered by differences in beliefs from the (unquestionable) trades based on differences in tastes. The bottom line is: With incomplete markets the first welfare theorem does not hold, but why should we care if the notion of Pareto optimality ceases to be compelling?

We do also challenge the conventional wisdom, but from a different perspective. We claim that equilibria might be Pareto optimal, even though markets are incomplete. But Pareto optimality is weakened by the fact that it obtains through an endogenous change of beliefs. So in accordance with the above-mentioned literature, we agree that Pareto optimality is less compelling in the presence of heterogenous priors. It is however not so much because some agents must have mistaken beliefs, but rather because subjective beliefs are plastic, subject to lots of influence, and are determined endogenously.

Conclusion

Great accomplishments are achieved by markets when they work perfectly. Existence and optimality of economic equilibrium are the epitome of such great achievements. And the general equilibrium theory provides profound, sharp and elegant arguments to advocate for the power of the invisible hand. But anyone doubting that markets are perfect will find it difficult to explain the high level of consensus in general assembly meetings illustrated by CAC40 firms in 2014.

We provide a rationale for this puzzle. We show that when the general equilibrium perspective is extended to include reciprocal aggregation mechanisms to *supplement* the market mechanism, then the mere Pareto principle yields that markets potentially never fail.

7 Appendix.

Proof of Lemma 1: (The subscript j is dropped for lightness of natation.) (i) There is y such that $g(y) < 0$ because g is strictly convex. There is y' such that $g(y') > 0$ because Y is compact and $Y = \{\, y \mid g(y) \leq 0 \,\}$. Therefore there is y'' such that $g(y'') = 0$. For all y and y', $g(y') \geq g(y){+}Dg(y){\cdot}(y'{-}y)$ because g is convex. Assume $g(y) = 0$ and $g(y') < 0$, then $Dg(y){\cdot}(y'{-}y) < 0$, so $Dg(y) \neq 0$. Hence $g(y) = 0$ implies $y \in \partial Y$. Clearly $g(y) \neq 0$ implies $y_j \notin \partial Y$.

(ii) Suppose $y \in \partial Y$ and $Dg(y)$ and ∇ are collinear. For all y' with $g(y') \leq 0$ and $y' \neq y$, $Dg(y){\cdot}(y'{-}y) < 0$ because g is strictly convex. Therefore y is optimal with respect to ∇. Suppose $y \in Y$ with $y \notin \partial Y$. Then $g(y') < 0$ so there is $\varepsilon > 0$ such that $g(y{+}\varepsilon\nabla) \leq 0$. Hence y is not optimal with respect to ∇. Suppose $Dg(y)$ and ∇ are not collinear. Then there is $v \in \mathbb{R}^S$ such that $Dg(y){\cdot}v < 0 < \nabla{\cdot}v$ so there is $\varepsilon > 0$ such that $g(y{+}\varepsilon v) \leq 0$. Hence y is not optimal with respect to ∇.

(iii) It follows from the proof of (ii) that $y \in \partial Y$ is optimal with respect to $Dg(y)$ and not with respect to any ∇ that is not collinear with $Dg(y)$.

(iv) For every $\nabla \in \mathbb{R}^S \backslash \{0\}$ there is y such that y is optimal with respect to ∇ because Y is compact. Suppose y and y' are both optimal with respect to ∇, then $g((1{-}\tau)y{+}\tau y') < 0$ for all $\tau \in]0,1[$ because g is strictly convex. Therefore for every $\tau \in]0,1[$ there is $\varepsilon > 0$ such that $g((1{-}\tau)y{+}\tau y'{+}\varepsilon\nabla) \leq 0$. However $\nabla{\cdot}((1{-}\tau)y{+}\tau y'{+}\varepsilon\nabla) > \nabla{\cdot}y = \nabla{\cdot}y'$ contradicting that y and y' are optimal with respect to ∇. □

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