A Solution to the Melitz-Trefler Puzzle

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Preliminary

December 4, 2015

Abstract: An empirical finding by Trefler (2004) and others that industrial productivity rises more strongly in liberalized industries than in non-liberalized industries has been widely accepted as evidence for the Melitz (2003, Econometrica) model. However, our recent paper shows that under fairly general assumptions, a multi-industry version of the Melitz model predicts the exact opposite relationship. In this paper, we present a simple solution to this "Melitz-Trefler" puzzle: introducing decreasing returns to scale in entry costs into an otherwise standard Melitz model. The model predicts the Trefler finding and removes other counter-intuitive predictions.

JEL classification: F12, F13.

Keywords: Trade liberalization, firm heterogeneity, industrial productivity.

Acknowledgments: Financial supports from the Wallander Foundation and from the JSPS KAKENHI (Grant Number 80240761) are gratefully acknowledged.

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1 Introduction

In the last decade, the empirical trade literature have established a new mechanism of gains from trade by using firm-level data: trade liberalization improves industrial productivity by shifting resources from less productive to more productive firms within industries. For instance, by investigating the impact of the Canada-USA free trade agreement on Canadian manufacturing industries, Trefler (2004) found that industrial productivity increased more strongly in liberalized industries that experienced large Canadian tariff cuts than in non-liberalized industries, and that the rise in industrial productivity was mainly due to the shift of resources from less productive to more productive firms. Similar productivity gains through intra-industry reallocation are also observed in other large liberalization episodes (e.g. Pavcnik 2002, for Chile; Eslava, Haltiwanger, Kugler and Kugler, 2012, for Colombia; Nataraji, 2001, for India).

The empirical finding by Trefler (2004) and others that intra-industry reallocation improves productivity more strongly in liberalized industries than in non-liberalized industries has been widely accepted as evidence for the seminal model by Melitz (2003) on intra-industry reallocation due to trade liberalization. For instance, virtually all recently published survey papers cite Trefler (2004) as evidence for the Melitz model (Bernard, Jensen, Redding, and Schott, 2007, 2012; Helpman, 2011; Redding, 2011; Melitz and Trefler, 2012). However, in Segerstrom and Sugita (2015a), we show that the Trefler finding is actually evidence against the Melitz model. Under fairly standard assumptions, a multi-industry version of the Melitz model predicts that productivity rises more strongly in non-liberalized industries than in liberalized industries. This is the exact opposite of the Trefler finding.

This disconnect between theory and evidence we call the Melitz-Trefler puzzle. In this paper, we present a solution to the Melitz-Trefler puzzle. We develop a new model that can predict the Trefler finding as well as other major facts that the Melitz model explains. The model features decreasing returns to scale (DRS) in research and development (R&D) and nests the Melitz model as a special case of constant returns to scale (CRS) in R&D. A large empirical literature on patents and R&D has shown that R&D is subject to significant decreasing returns at the sector level (e.g. Kortum 2003). However, the Melitz model and most of its applications assume the CRS in R&D for convenience.

Following Segerstrom and Sugita (2015a, b), we consider that one country unilaterally reduces tariffs for one industry and not for other industries. Trade liberalization has two effects on industries in the liberalizing country, a competitiveness effect and a wage effect. In the Melitz model, the competitiveness effect contributes to lowering productivity in the liberalized industry, while the wage effect contributes to raising productivity in both liberalized and non-liberalized industries. The competitiveness effect makes the Melitz model to predict the opposite of the Trefler finding. The wage effect generates a counter-intuitive prediction that an exogenous rise in wage decreases the domestic productivity cutoff and industrial productivity.

In the new model with the DRS in R&D, trade liberalization still has the competitiveness and wage effects, but they both go in the opposite direction. The competitiveness effect of trade liberalization contributes to raising productivity in the liberalized industry and the wage effect of trade liberalization contributes to lowering productivity in both liberalized and non-liberalized industries. The competitiveness effect predicts the Trefler finding. The wage effect implies a more empirically plausible prediction that an exogenous rise in wage increases the domestic productivity cutoff and industrial productivity.

The new model predicts several other predictions of the Melitz model that have been confirmed in many empirical studies. For instance, Redding (2011) mentions two other facts as empirical motivations for the Melitz model: (1) exporters are larger and more productive than non-exporters; (2) entry and exit simultaneously occur within the same industry even without trade liberalization. The new model continues to predict these two facts. The Melitz model also predicts the Home Market effect, which receives empirical supports (e.g. Hanson and Xiang, 2004) and plays an important role in the New Economic Geography as well as in international trade. With a moderate degree of the DRS, the new model predicts both the Home Market effect and the Trefler finding.

The rest of the paper is organized as follows. In section 2, we present a model and our main results. In section 3, we present an intuitive explanation for our results in subsection 3.1, solve the model numerically to illustrate the intuition in section 3.2, and discuss other predictions of the model in section 3.3. In section 4, we offer some concluding comments and there is an Appendix where calculations that we did to solve the model are presented in more detail.

2 Model

2.1 Setting

Consider two countries, 1 and 2, with two differentiated goods sectors (or industries), A and B. Countries and sectors are initially symmetric (except the sector size) and become asymmetric after asymmetric trade liberalization. Though the model has infinitely many periods, there is no means for saving over periods. Following Melitz (2003), we focus on a stationary steady state equilibrium where aggregate variables do not change over time and omit notation for time periods. Throughout the paper, subscripts i and j denote countries ($i, j \in \{1, 2\}$) and subscript s denotes sectors ($s \in \{A, B\}$).

The representative consumer in country i has a two-tier (Cobb-Douglas plus CES) utility function:

$$U_i \equiv C_{iA}^{\alpha_A} C_{iB}^{\alpha_B} \qquad \text{where } \alpha_A + \alpha_B = 1 \text{ and } C_{is} \equiv \left[\int_{\omega \in \Omega_{is}} q_{is}(\omega)^{\rho} \, d\omega \right]^{1/\rho} \text{ for } s = A, B.$$

In the utility equation, $q_{is}(\omega)$ is country i's consumption of a product variety ω produced in sector s, Ω_{is} is the set of available varieties in sector s in country i and ρ measures the degree of product differentiation. We assume the within-sector elasticity of substitution $\sigma \equiv 1/(1-\rho)$ satisfies $\sigma > 1$. Given that $\alpha_A + \alpha_B = 1$, α_s represents the share of consumer expenditure on sector s products. Before trade liberalization, sectors differ only in α_s.

Countries are endowed with identical L units of labor as the only factor of production. Labor is inelastically supplied and workers in country i earn the competitive wage rate w_i. We measure all prices relative to the price of labor in country 2 by setting $w_2 = 1$.

Firms are risk neutral and maximize expected profits. In each time period, the measure M_{ise} of firms choose to enter in sector s in country i. Each firm uses f_{ise} units of labor to enter and incurs the fixed entry cost $w_i f_{ise}$. Each firm then independently draws its productivity φ from a Pareto distribution. The cumulative distribution function $G(\varphi)$ and the corresponding density function $g(\varphi) = G'(\varphi)$ are given by $G(\varphi) = 1 - (b/\varphi)^\theta$ and $g(\varphi) = \theta b^\theta / \varphi^{\theta+1}$ for $\varphi \in [b, \infty)$, where $\theta > 0$ and $b > 0$ are the shape and scale parameters of the distribution. We assume that $\theta > \sigma - 1$ to guarantee that expected profits are finite. In each period, there is an exogenous probability δ with which actively operating firms in country i and sector s die and exit.

A firm with productivity φ uses $1/\varphi$ units of labor to produce one unit of output and has constant marginal cost w_i/φ in country i. This firm must use f_{ij} units of domestic labor and incur the fixed "marketing" cost $w_i f_{ij}$ to sell in country j. Denoting $f_{ii} = f_d$ and $f_{ij} = f_x$ for $i \neq j$, we assume that exporting require higher fixed costs than local selling ($f_x > f_d$). There are also iceberg trade costs associated with shipping products across countries: a firm that exports from country i to country $j \neq i$ in sector s needs to ship $\tau_{ijs} > 1$ units of a product in order for one unit to arrive at the foreign destination (if $j = i$, then $\tau_{iis} = 1$).

Decreasing Returns to Scale in Entry Costs Individual firms take entry costs f_{ise} as given, but at the aggregate level, entry costs exhibit decreasing returns to scale (DRS). More specifically, entry costs increase in the mass of entrants:

$$f_{ise} = F \cdot M_{ise}^{\zeta}, \tag{1}$$

where $\zeta \geq 0$ expresses the extent of decreasing returns to scale. Notice that the model nests the original Melitz model as a special case of $\zeta = 0$.

Formulation (1) aims to introduce the DRS in research and development (R&D) in the simplest possible way.[1] Since M_{ise} is the number of firms that enter and FM_{ise}^{ζ} is the labor used per firm, the

[1] An alternative specification is that entry costs also depend on the mass of existing firms: $f_{ise} = FM_{ise}^{\zeta} M_{is}^{\xi}$ where $\zeta \geq 0$

total labor used for R&D in country i and sector s is $L_{ise} \equiv FM_{ise}^{1+\zeta}$. Solving this expression for M_{ise} yields $M_{ise} = (L_{ise}/F)^{1/(1+\zeta)}$, where M_{ise} can be thought of as the flow of new products developed by researchers and L_{ise} is the sector level of R&D labor. Thus the parameter ζ determines the degree of decreasing returns to R&D at the sector level. A large empirical literature on patents and R&D has shown that R&D is subject to significant decreasing returns at the sector level. According to Kortum (1993), point estimates of $1/(1+\zeta)$ lie between 0.1 and 0.6, which corresponds to ζ values between 0.66 and 9.

2.2 Equilibrium Conditions

A country i firm in sector s with productivity φ sets an optimal price $p_{ijs}(\varphi)$ for goods it sells to country j, earns revenue $r_{ijs}(\varphi)$ and gross profits $r_{ijs}(\varphi)/\sigma$ from selling to country j:

$$p_{ijs}(\varphi) = \frac{w_i \tau_{ijs}}{\rho\varphi} \text{ and } r_{ijs}(\varphi) = \frac{\alpha_s w_j L}{P_{js}^{1-\sigma_s}} \left(\frac{\tau_{ijs} w_i}{\rho\varphi}\right)^{1-\sigma_s}, \tag{2}$$

where P_{js} is the price index.

Because of the fixed marketing costs, there exist productivity cut-off levels φ^*_{ijs} such that only firms with $\varphi \geq \varphi^*_{ijs}$ sell products from country i to country j in sector s. We solve the model for an equilibrium where both countries produces both goods A and B, and the more productive firms export ($\varphi^*_{iis} < \varphi^*_{ijs}$). A firm with cut-off productivity φ^*_{ijs} just breaks even from selling to country j:

$$\frac{r_{ijs}\left(\varphi^*_{ijs}\right)}{\sigma} = \frac{\alpha_s w_j L}{\sigma} \left(\frac{p_{ijs}(\varphi)}{P_{js}}\right)^{1-\sigma_s} = w_i f_{ijs}, \tag{3}$$

where $P_{js} \equiv \left[\sum_{i=1,2} \int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma} M_{is} \mu_{is}(\varphi) d\varphi\right]^{1/(1-\sigma)}$ is the price index for sector s products in country j, M_{is} is the actively operating firms in country i, and $\mu_{is}(\varphi) = g(\varphi)/[1 - G(\varphi^*_{iis})]$ is the distribution of productivity. In a stationary steady state equilibrium, the mass of actively operating firms M_{is} and the mass of entrants M_{ise} in country i and sector s satisfy

$$[1 - G(\varphi^*_{iis})] M_{ise} = \delta M_{is}, \tag{4}$$

that is, firm entry in each time period is matched by firm exit.

From (2) and (3), the cut-off productivity levels of domestic and foreign firms in country j are related

and $\xi \geq 0$. This is in line with the specification of R&D costs in Jones (1995). Our main results continue to hold under this alternative specification but calculations become more complex.

by trade costs and labor costs as follows:

$$\varphi_{ijs}^{*} = T_{ijs}\left(\frac{w_i}{w_j}\right)^{1/\rho}\varphi_{jjs}^{*}, \tag{5}$$

where $T_{ijs} \equiv \tau_{ijs}\left(f_{ij}/f_{jj}\right)^{1/(\sigma-1)}$ captures both variable and fixed trade costs from country i to country j relative to the fixed trade cost within country j. Let ϕ_{ijs} denote the ratio of the expected profit of an entrant in country i from selling to country j in sector s to that captured by an entrant in country j from selling to country j. Using (2), (3), (4), and (5), the relative expected profit simplifies to:

$$\phi_{ijs} \equiv \frac{\delta^{-1}\int_{\varphi_{ijs}^{*}}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma} - w_i f_{ij}\right]dG(\varphi)}{\delta^{-1}\int_{\varphi_{jjs}^{*}}^{\infty}\left[\frac{r_{jjs}(\varphi)}{\sigma} - w_j f_{jj}\right]dG(\varphi)} = \frac{f_{ij}}{f_{jj}}T_{ijs}^{-\theta}\left(\frac{w_j}{w_i}\right)^{(\theta-\rho)/\rho}. \tag{6}$$

Variable ϕ_{ijs} summarizes the degree of country i's market access to country j. It decreases in variable trade costs T_{ijs}, relative marketing costs f_{ij}/f_{jj}, and the relative wage w_i/w_j.

Using the optimal price (2), the cutoff conditions (5) and the relative expected profit (6), we simplify the price index as

$$P_{is}^{1-\sigma} = \eta p\left(\varphi_{iis}^{*}\right)^{1-\sigma}\left(\frac{b}{\varphi_{iis}^{*}}\right)^{\theta}\left(\frac{M_{ise}}{\delta} + \phi_{jis}\frac{M_{jse}}{\delta}\right), \tag{7}$$

where $\eta = \left[\left(\theta-\sigma+1\right)/\theta\right]^{1/\sigma-1}$. To understand equation (7), consider first autarky with $\phi_{jis} = 0$. Then, from (4), it becomes that $P_{is}^{1-\sigma} = \eta p\left(\varphi_{iis}^{*}\right)^{1-\sigma}M_{is}$. The price index depends on the mass of varieties and the distribution of prices. Under the Pareto distribution, the latter is summarized by the highest price set by the least productive firms on the market. In the open economy with $\phi_{ijs} > 0$, the price index also depends on the mass of foreign varieties (M_{jse}/δ) and the degree of their market access (ϕ_{jis}).

Substituting the price index (7) into the cutoff condition (3), we obtain

$$\varphi_{11s}^{*\theta} = \frac{\theta b^{\theta}}{\left(\theta-\sigma+1\right)}\frac{\sigma f_d}{\alpha_s L_1}\left(M_{1se} + \phi_{21s}M_{2se}\right). \tag{8}$$

The domestic productivity cutoff φ_{11s}^{*} rises if and only if $\left(M_{1se} + \phi_{21s}M_{2se}\right)$ rises. In the following, we study how trade liberalization affects $\left(M_{1se} + \phi_{21s}M_{2se}\right)$.

A convenient property of the model with the Cobb-Douglas upper tier utility and the Pareto distribution is that we can solve for the mass of entrants M_{ise} as a function of the wage w_i and trade costs τ_{ijs}.

First, free entry implies that the expected profits from entry must equal the cost of entry:

$$\frac{1}{\delta}\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma}-w_i f_{ij}\right]dG(\varphi)=w_i f_{ise}. \tag{9}$$

Following Melitz (2003) and Demidova (2008), equation (9) can be rewritten as

$$\frac{1}{\delta}\left(\frac{\sigma-1}{\theta-\sigma+1}\right)\sum_{j=1,2} f_{ij}\left(\frac{b}{\varphi^*_{ijs}}\right)^{\theta}=f_{ise}. \tag{10}$$

Second, equation (10) implies that the fixed costs (the entry costs plus the marketing costs) are proportional to the mass of entrants in each country i and sector s:

$$w_i\left(M_{ise}f_{ise}+\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty} f_{ij}M_{is}\mu_{is}(\varphi)\,d\varphi\right)=w_i M_{ise}\left(\frac{\theta}{\sigma-1}\right)f_{ise}, \tag{11}$$

where $\mu_{is}(\varphi)$ is the density of productivity of active firms in sector s in country i such that $\mu_{is}(\varphi)=g(\varphi)/[1-G(\varphi_{iis})]$. Third, free entry (10) also implies that the fixed costs are equal to the gross profits in each country i and sector s, that is,

$$w_i M_{ise}\left(\frac{\theta}{\sigma-1}\right)f_{ise}=\frac{1}{\sigma}\sum_{j=1,2}R_{ijs} \tag{12}$$

where $R_{ijs}\equiv\int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi)M_{is}\mu_{is}(\varphi)d\varphi$ is the total revenue associated with shipments from country i to country j in sector s. Fourth, from (7), the total revenue R_{ijs} can be simplified as

$$R_{ijs}=\alpha_s w_j L_j\left(\frac{M_{ise}\phi_{ijs}}{\sum_{k=1,2}M_{kse}\phi_{kjs}}\right). \tag{13}$$

Thus, from (11) and (13), we obtain

$$\sum_{j=1,2}\alpha_s w_j L_j\left(\frac{\phi_{ijs}}{\sum_{k=1,2}M_{kse}\phi_{kjs}}\right)=w_i f_{ise}\left(\frac{\theta}{\rho}\right)\ \text{for } i=1,2. \tag{14}$$

Since f_{ise} is a function of M_{ise}, it is possible to express the mass of entrants $M_{ise}(T_{12s},T_{21s},w_1)$ as a function of variable trade costs and the wage. Then, from (5) and (8), we obtain the domestic and export productivity cutoffs as functions of variable trade costs and the wage.

The labor market clearing condition of country 1 determines the wage w_1. Free entry implies that wage payments to labor equal total revenue in each country i and sector s, that is, $w_i L_{is}=\sum_{j=1,2}R_{ijs}$, where L_{is} is the industrial labor demand. From (1) and (12), this immediately leads to:

$$L_{is} = \frac{1}{w_i} \sum_{j=1,2} R_{ijs} = M_{ise} \left(\frac{\sigma\theta}{\sigma - 1} \right) f_{ise} = M_{ise}^{1+\zeta} \left(\frac{\theta F}{\rho} \right). \tag{15}$$

Thus, the labor market clearing condition of country 1 determines the wage w_1 as follows:

$$L_1 = \left(\frac{\theta F}{\rho} \right) \sum_{s=A,B} M_{1se} \left(T_{12s}, T_{21s}, w_1 \right)^{1+\zeta}. \tag{16}$$

Following Segerstrom and Sugita (2015b), we consider two measures of industrial labor productivity $\Phi_{is}^L \equiv \left(\sum_{j=1,2} R_{1js} \right) / \left(\tilde{P}_{1s} L_{1s} \right)$ and $\Phi_{is}^W \equiv \left(\sum_{j=1,2} R_{1js} \right) / \left(P_{1s} L_{1s} \right)$. The price deflater $\tilde{P}_{1s} \equiv \int_{\varphi_{11s}^*}^{\infty} p_{11s}(\varphi) \mu_{1s}(\varphi) d\varphi$ in the first measure is the simple average of prices set by domestic firms at the factory gate and aims to resemble the industrial product price index, which is used for the calculation of the real industrial output.[2] The price deflator in the second measure is the exact consumer price index. This latter measure is motivated by thinking about consumer welfare. The welfare is expressed as a simple function of Φ_{is}^W: $U = \left(\alpha_A \Phi_{1A}^W \right)^{\alpha_A} \left(\alpha_B \Phi_{1B}^W \right)^{\alpha_B}$. From (12) and (15), they are simplified as

$$\Phi_{1s}^L = \left(\frac{\theta + 1}{\theta} \right) \rho \varphi_{11s}^* \text{ and } \Phi_{1s}^W = \left(\frac{\alpha_s L_1}{\sigma f_{11}} \right)^{1/(\sigma-1)} \rho \varphi_{11s}^*. \tag{17}$$

Thus, these two measures are increasing functions of the domestic cutoffs.

2.3 Log-Linearization

We analyze how trade liberalization in variable trade costs affects industrial productivity and domestic productivity cutoffs. Since countries and sectors are initially symmetric before liberalization, $M_{1se} = M_{2se}$ and $\phi_{ijs} = \phi$ hold.

First, we differentiate (8) and (17) to obtain the changes in industrial productivity and domestic productivity cutoff:

$$d \ln \Phi_{1s}^{k=L,W} = d \ln \varphi_{11s}^* = \frac{1}{\theta (1 + \phi)} \left[d \ln M_{1se} + \phi \left(d \ln M_{2se} + d \ln \phi_{21s} \right) \right]. \tag{18}$$

Therefore, it is sufficient to consider how the mass of entrants in both countries and the relative expected

[2] The term $\sum_{j=1,2} R_{1js}$ is the total revenue of firms in country 1 and sector s. Dividing by the price index $\tilde{P}_{1s}$ gives a measure of the real output of sector s. Then dividing by the number of workers L_{1s} gives a measure of real output per worker.

profit ϕ_{21s} change. Differentiating (6), we obtain

$$d\ln\phi_{21s} = -\theta d\ln T_{21s} + \left(\frac{\theta}{\rho} - 1\right) d\ln w_1. \tag{19}$$

Differentiating (14), we express the mass of entrants as:

$$\begin{aligned} d\ln M_{1se} &= \iota_T d\ln T_{21s} - \iota_T d\ln T_{12s} - \iota_w d\ln w_1 - \iota_1 d\ln f_{1se} + \iota_2 d\ln f_{2se} \\ d\ln M_{2se} &= -\iota_T d\ln T_{21s} + \iota_T d\ln T_{12s} + \iota_w d\ln w_1 + \iota_2 d\ln f_{1se} - \iota_1 d\ln f_{2se}, \end{aligned} \tag{20}$$

where

$$\iota_T \equiv \frac{\phi\theta}{(1-\phi)^2} > 0,\ \iota_w \equiv \frac{\phi}{1-\phi}\left(\frac{2\theta}{\rho(1-\phi)} - 1\right) > 0,\ \iota_1 \equiv \frac{1+\phi^2}{(1-\phi)^2} > 0 \text{ and } \iota_2 \equiv \frac{2\phi}{(1-\phi)^2} > 0.$$

Increases in the wage ($w_1 \uparrow$), export barriers ($T_{12s} \uparrow$) and domestic entry costs ($f_{1se} \uparrow$) discourage entry ($M_{1se} \downarrow$), while an increase in import barriers ($T_{21s} \uparrow$) and foreign entry costs ($f_{2se} \uparrow$) encourages entry ($M_{1se} \uparrow$). Substituting $d\ln f_{ise} = \zeta\, d\ln M_{ise}$ into (20), we obtain

$$\begin{aligned} d\ln M_{1se} &= \varepsilon_T d\ln T_{21s} - \varepsilon_T d\ln T_{12s} - \varepsilon_w d\ln w_1 \\ d\ln M_{2se} &= -\varepsilon_T d\ln T_{21s} + \varepsilon_T d\ln T_{12s} + \varepsilon_w d\ln w_1 \end{aligned} \tag{21}$$

where

$$\varepsilon_T \equiv \frac{\phi\theta}{(1-\phi)^2 + \zeta(1+\phi)^2} > 0 \text{ and } \varepsilon_w \equiv \frac{\phi\left[2\theta - \rho(1-\phi)\right]}{\rho\left[(1-\phi)^2 + \zeta(1+\phi)^2\right]} > 0.$$

Since both ε_T and ε_w are decreasing in ζ, we can see that the DRS in entry costs makes entry less responsive to changes in trade costs and the wage. Using $d\ln f_{ise} = \zeta\, d\ln M_{ise}$ and substituting (21) into (20), the above elasticities can be also expressed:

$$\varepsilon_T = \iota_T - \zeta(\iota_1 + \iota_2)\varepsilon_T \text{ and } \varepsilon_w = \iota_w - \zeta(\iota_1 + \iota_2)\varepsilon_w. \tag{22}$$

From (6), (18) and (21), we obtain our key equation:

$$d\ln\Phi_{1s}^{k=L,W} = d\ln\varphi_{11s}^* = \gamma_1 d\ln T_{21s} - \gamma_2 d\ln T_{12s} - \gamma_3 d\ln w_1 \tag{23}$$

where

$$\gamma_1 \equiv \frac{(1-\phi)\,\varepsilon_T - \phi\theta}{\theta\,(1+\phi)} = \frac{\phi^2}{1-\phi^2} - \frac{\zeta\phi\,(1+\phi)}{(1-\phi)\left[(1-\phi)^2 + \zeta\,(1+\phi)^2\right]},$$
$$\gamma_2 \equiv \frac{1-\phi}{\theta\,(1+\phi)}\varepsilon_T > 0,$$
$$\gamma_3 \equiv \frac{(1-\phi)\,\varepsilon_w - \phi\,(\theta/\rho - 1)}{\theta\,(1+\phi)} = \frac{\phi}{\rho\,(1-\phi)} - \frac{\zeta\phi\,(1+\phi)\,[2\theta - \rho\,(1-\phi)]}{\rho\theta(1-\phi)\left[(1-\phi)^2 + \zeta\,(1+\phi)^2\right]}. \tag{24}$$

Segerstrom and Sugita (2015a) derive a similar equation to (23) for the Melitz model with $\zeta = 0$ where γ_1, γ_2, and γ_3 are all positive. The sign of γ_2 is always positive, but the signs of γ_1 and γ_3 are ambiguous and depend on the size of ζ. With some manipulation, we establish the following lemma for the sign of γ_1:

Lemma 1. *There exists a positive threshold $\zeta_1 \equiv \frac{\phi(1-\phi)}{(1+\phi)^2} > 0$ such that $\gamma_1 < 0$ if and only if $\zeta > \zeta_1$ and that $\zeta_1 < 1/8$.*

Segerstrom and Sugita (2015b) decompose the effect of unilateral liberalization by country 1 ($d\ln T_{21s} < 0 = d\ln T_{12s}$) into two effects, the competitiveness effect and the wage effect. In their terminology, $\gamma_1 d\ln T_{21s}$ in (23) expresses the competitiveness effect, while $\gamma_3 d\ln w_1$ expresses the wage effect. Lemma 1 implies that as decreasing returns to scale (DRS) in R&D becomes stronger ($\zeta \uparrow$), the competitiveness effect becomes weaker ($\gamma_1 \downarrow$) and eventually takes the opposite sign ($\gamma_1 < 0$). Even a small degree of DRS ($\zeta > 1/8$) is sufficient for flipping the sign of the competitiveness effect. The intuition behind Lemma 1 will be discussed in section 3.1.

Lemma 1 offers a solution to the Melitz-Trefler puzzle. When country 1 opens up to trade in industry A but not industry B ($d\ln T_{21A} < d\ln T_{21B} = d\ln T_{12A} = d\ln T_{12B} = 0$), it follows that

$$\begin{aligned} d\ln\Phi^k_{1A} - d\ln\Phi^k_{1B} &= (\gamma_1\, d\ln T_{21A} - \gamma_3\, d\ln w_1) - (-\gamma_3\, d\ln w_1) \\ &= \gamma_1\, d\ln T_{21A}. \end{aligned}$$

That is, the competitiveness effect is equal to difference-in-difference changes of productivity between liberalized and non-liberalized industries in the liberalizing country. The Melitz model with $\zeta = 0$ predicts that $\gamma_1 > 0$, that is, productivity rises more strongly in non-liberalized industries than in liberalized industries ($d\ln T_{21A} < 0 \Rightarrow d\ln\Phi^k_{1A} < \ln\Phi^k_{1B}$). This is the exact opposite of the Trefler finding ($d\ln T_{21A} < 0 \Rightarrow d\ln\Phi^k_{1A} > \ln\Phi^k_{1B}$). On the other hand, when ζ is significantly greater than zero, the current model can predict $\gamma_1 < 0$, which is consistent with the Trefler finding.

Corollary 1. *Productivity rises more strongly in liberalized industries than in non-liberalized industries if and only if* $\zeta > \zeta_1 > 0$*.*

The DRS in R&D also affects the wage effect, which consists of $\gamma_3 d\ln w_1$. To determine the size of the wage effect, we need to solve for the wage change from the labor market clearing condition. Suppose that trade costs change in sector A but not in sector B. Totally differentiating (16) and substituting (21), we obtain that the wage changes as follows:

$$d\ln w_1 = \frac{\alpha_A \theta \rho}{[2\theta - \rho(1-\phi)]} \left(d\ln T_{21A} - d\ln T_{12A}\right).$$

The wage change does not depend on the size of ζ, so the size of γ_3 determines the size of the wage effect. The next lemma establishes

Lemma 2. *There exists a positive threshold* $\zeta_3 \equiv \frac{\theta(1-\phi)}{(\theta-\rho)(1+\phi)} > 0$ *such that* $\gamma_3 < 0$ *if and only if* $\zeta > \zeta_3$ *and that* $\zeta_3/\zeta_1 = \left(1 + \frac{1}{\phi}\right)\left(1 + \frac{\rho}{\theta-\rho}\right) > 1$*.*

As the DRS in entry costs becomes stronger from $\zeta = 0$, γ_3 is initially positive, decreases and eventually turns to be negative. The intuition behind Lemma 3 will be discussed in section 3.1. The case that $\gamma_3 < 0$ seems to be intuitive. When the domestic wage exogenously rises, one might expect the lowest productive firm to exit and the domestic cutoff to rise. However, the Melitz model with $\zeta = 0$ actually predicts the exact opposite: when the domestic wage increases, the domestic cutoff falls. On the other hand, the current model can predict the domestic cutoff rises if $\zeta > \zeta_3$. Again, introducing the DRS in R&D makes the model more intuitive.

Corollary 2. *When the domestic wage exogenously rises, the domestic productivity cutoffs and industrial productivity rise if and only if* $\zeta > \zeta_3 > 0$*.*

Finally, we analyze symmetric trade liberalization that Melitz (2003). Suppose country 1 and country 2 liberalize industry A by the same amount ($d\ln T_{21A} = d\ln T_{12A} = d\ln T_A < d\ln T_{21B} = d\ln T_{12B} = 0$). Since countries remain symmetric, the wage continues to satisfy $w_1 = 1$. Therefore, productivity does not change in non-liberalized industry B. Productivity changes in liberalized industry A as

$$\begin{aligned} d\ln \Phi_{1A}^k &= (\gamma_1 - \gamma_2)\, d\ln T_A \\ &= -\phi\theta d\ln T_A > 0. \end{aligned}$$

That is, productivity always rises regardless of the size of ζ.

3 Discussion

3.1 Intuition for Lemmas 1 and 2

This section explains intuition for why the DRS in entry costs changes the signs of the competitiveness effect and the wage effect. A key equation in the model (both when $\zeta = 0$ and when $\zeta > 0$) is

$$\varphi_{11s}^{*\theta} = \frac{\theta b^{\theta}}{(\theta - \sigma + 1)} \frac{\sigma f_d}{\alpha_s L_1} \left(M_{1se} + \phi_{21s} M_{2se} \right).$$

This equation implies that the domestic productivity cutoff φ_{11s}^{*} rises if and only if $M_{1se} + \phi_{21s} M_{2se}$ rises. Since industrial productivity

$$\Phi_{1s}^{L} = \left(\frac{\theta + 1}{\theta} \right) \rho \varphi_{11s}^{*}$$

is proportional to the domestic productivity cutoff φ_{11s}^{*}, industrial productivity Φ_{1s}^{L} rises as a result of trade liberalization if and only if $M_{1se} + \phi_{21s} M_{2se}$ rises.

The term $M_{1se} + \phi_{21s} M_{2se}$ can be interpreted as a mass of entrants index relevant for consumers in country 1 and sector s. M_{1se} is the mass of firms that directly enter in country 1 and sector s. But consumers also buy imported products, so the mass M_{2se} of firms that enter in country 2 and sector s is also relevant for country 1 consumers. Since not all country 2 firms export to country 1, we multiply M_{2se} by the relative expected profit term ϕ_{21s} and then add M_{1se} to obtain the total number of entering firms $M_{1se} + \phi_{21s} M_{2se}$ relevant for country 1 consumers in sector s. ϕ_{21s} is higher when more firms export from country 2 to country 1 in sector s.

Competitiveness Effect We first focus on the competitiveness effect, considering a unilaterally liberalizing industry s and fixing the wage ($d \ln T_{21s} < 0 = d \ln T_{12s} = d \ln w_1$).

If trade liberalization results in $M_{1se} + \phi_{21s} M_{2se}$ increasing, this means that more firms are entering and competition is becoming tougher in country 1 and sector s. With tougher competition, firms need to have a higher productivity level to profitably survive, so the domestic productivity cutoff φ_{11s}^{*} increases, and it follows that productivity Φ_{1s}^{L} rises. If trade liberalization results in $M_{1se} + \phi_{21s} M_{2se}$ decreasing, then fewer firms enter, competition becomes less tough, lower productivity firms can now survive and industrial productivity falls.

When country 1 unilaterally liberalizes industry s, country 2's market access ϕ_{21s} rises, the mass of entrants in country 2 M_{2se} increases, and that in country 1 M_{1se} decreases. The first two effects increase $M_{1se} + \phi_{21s} M_{2se}$, while the last effect decreases it. When $\zeta = 0$ (the Melitz model case), M_{1se} falls so much that it offsets the increase in $\phi_{21s} M_{2se}$ and $M_{1se} + \phi_{21s} M_{2se}$ falls.

As seen in (21), the DRS in entry costs weakens the adjustment of firm's entry. To understand why this is happening, it suffices to recall that for firms in country i and sector s, the cost of entry is $w_i M_{ise}^{\zeta} F$. When $\zeta = 0$ (the Melitz model case), the cost of entry does not depend on the mass of entering firms M_{ise} but when $\zeta > 0$, the cost of entry goes up when M_{ise} increases and the cost of entry goes down when M_{ise} decreases. So in a sector where trade liberalization encourages more entry, as more firms enter, the cost of entry goes up, which serves to discourage further entry. And in a sector where trade liberalization leads to less entry, as less firms enter, the cost of entry goes down, which serves to make entry more attractive. As ζ increases, we get less adjustment in the up direction because the cost of entry is going up and we get less adjustment in the down direction because the cost of entry is going down.

On the other hand, equation (19) with $d \ln w_1$ implies that the increase in ϕ_{12s} does not depend on the size of ζ. Therefore, as ζ increases and the adjustment of entry becomes smaller, the dominant change eventually becomes the increase in ϕ_{12s} so that $M_{1se} + \phi_{21s} M_{2se}$ rises.

Wage Effect Second, we consider the wage effect, by considering an exogenous increase in country 1's wage and fixing trade costs ($d \ln w_1 > 0 = d \ln T_{12s} = d \ln T_{21s}$). When country 1's wage increases, country 2's market access ϕ_{21s} rises, the mass of entrants in country 2 M_{2se} increases, and that in country 1 M_{1se} decreases. When $\zeta = 0$ (the Melitz model case), $M_{1se} + \phi_{21s} M_{2se}$ rise because the fall in M_{1se} dominates the increase in $\phi_{21s} M_{2se}$. On the other hand, when ζ increases from zero, the adjustment of entrants becomes smaller, while the increase in ϕ_{12s} remains the same. Therefore, the increase in ϕ_{12s} becomes the dominant change, so $M_{1se} + \phi_{21s} M_{2se}$ increases.

3.2 Numerical Results

As a check that our analytically derived results are correct, we also solve the model numerically. Looking at numerical examples is helpful for understanding the intuition behind the results.[3] We focus on what happens when country 1 unilaterally opens up to trade in industry A but not industry B (τ_{21A} decreases from 1.3 to 1.15). We study two cases.

The first case is where $\alpha_A = 0.1$, that is, where country 1 opens up to trade in a small industry that only attracts 10 percent of consumer expenditure. Then the wage effect of trade liberalization is small and this effect is dominated by the competitiveness effect in the Melitz model. Looking at the $\alpha_A = 0.1$ case, one mainly sees the competitiveness effect of trade liberalization on industrial productivity. The country 1 relative wage w_1/w_2 does decrease as a result of trade liberalization but this general equilibrium effect is small. The results when ζ equals 0 and 0.25 are reported in Table 1 and the results when ζ equals 1.5

[3]The MATLAB files used to solve the model can be obtained from the authors upon request.

and 5 are reported in Table 2. The value $\zeta = 0.25$ is large enough so that the condition $\zeta > \zeta_1$ is satisfied and the value $\zeta = 1.5$ is large enough so that the stronger condition $\zeta > \zeta_3$ is satisfied. By increasing ζ from 0 to 0.25 to 1.5 to 5, we are able to see clearly the implications of stronger decreasing returns to R&D.

	$\zeta = 0$ Case			$\zeta = .25$ Case		
	$\tau_{21A} = 1.30$	$\tau_{21A} = 1.15$	% Change	$\tau_{21A} = 1.30$	$\tau_{21A} = 1.15$	% Change
Φ^L_{1A}	.2011	.1997	-0.7%	.2483	.2509	+1.0%
Φ^L_{1B}	.2011	.2019	+0.4%	.2256	.2258	+0.1%
Φ^L_{2A}	.2011	.2100	+4.4%	.2483	.2539	+2.3%
Φ^L_{2B}	.2011	.2005	-0.3%	.2256	.2253	-0.1%
U_1	.2028	.2033	+0.2%	.2297	.2301	+0.2%
U_2	.2028	.2031	+0.1%	.2297	.2300	+0.1%
w_1/w_2	1.0000	.9923	-0.8%	1.0000	.9935	-0.6%
M_{1Ae}	.0080	.0052	-35.0%	.0211	.0172	-18.0%
M_{1Be}	.0724	.0752	+3.9%	.1224	.1248	+2.0%
M_{2Ae}	.0080	.0109	+36.2%	.0211	.0248	+17.6%
M_{2Be}	.0724	.0695	-4.0%	.1224	.1199	-2.0%
ϕ_{21A}	.2457	.4138	+68.4%	.2457	.4164	+69.5%
ϕ_{21B}	.2457	.2360	-3.9%	.2457	.2375	-3.3%
φ^*_{11A}	.2241	.2224	-0.8%	.2766	.2795	+1.0%
φ^*_{12A}	.3261	.3369	+3.3%	.4026	.4081	+1.4%
φ^*_{11B}	.2241	.2249	+0.4%	.2513	.2516	+0.1%
φ^*_{12B}	.3261	.3216	-1.4%	.3657	.3621	-1.0%
φ^*_{22A}	.2241	.2339	+4.4%	.2766	.2829	+2.3%
φ^*_{21A}	.3261	.2894	-11.3%	.4026	.3630	-9.8%
φ^*_{22B}	.2241	.2233	-0.4%	.2513	.2510	-0.1%
φ^*_{21B}	.3261	.3308	+1.4%	.3657	.3695	+1.0%

Table 1: Effects of Trade Liberalization when $\alpha_A = 0.1$

The second case is where $\alpha_A = 0.5$, that is, where country 1 opens up to trade in a large industry that attracts 50 percent of consumer expenditure. Then the wage effect of trade liberalization is large and dominates the competitiveness effect in the Melitz model. The results when ζ equals 0 and 0.25 are reported in Table 3 and the results when ζ equals 1.5 and 5 are reported in Table 4.

For the numerical results reported in Tables 1-4, we assume that countries and industries are symmetric before trade liberalization. Then there are only nine remaining parameters that need to be chosen.

	$\zeta = 1.5$ Case			$\zeta = 5$ Case		
	$\tau_{21A} = 1.30$	$\tau_{21A} = 1.15$	% Change	$\tau_{21A} = 1.30$	$\tau_{21A} = 1.15$	% Change
Φ_{1A}^{L}	.3783	.3871	+2.3%	.4836	.4966	+2.7%
Φ_{1B}^{L}	.2837	.2835	-0.1%	.3243	.3240	-0.1%
Φ_{2A}^{L}	.3783	.3811	+0.7%	.4836	.4852	+0.3%
Φ_{2B}^{L}	.2837	.2839	+0.1%	.3243	.3246	+0.1%
U_1	.2944	.2950	+0.2%	.3403	.3409	+0.2%
U_2	.2944	.2948	+0.1%	.3403	.3408	+0.1%
w_1/w_2	1.0000	.9942	-0.6%	1.0000	.9944	-0.6%
M_{1Ae}	.1452	.1374	-5.4%	.4476	.4394	-1.8%
M_{1Be}	.3498	.3518	+0.6%	.6455	.6468	+0.2%
M_{2Ae}	.1452	.1525	+5.0%	.4476	.4551	+1.7%
M_{2Be}	.3498	.3478	-0.6%	.6455	.6443	-0.2%
ϕ_{21A}	.2457	.4181	+70.2%	.2457	.4185	+70.3%
ϕ_{21B}	.2457	.2384	-3.0%	.2457	.2386	-2.9%
φ_{11A}^{*}	.4214	.4312	+2.3%	.5387	.5532	+2.7%
φ_{12A}^{*}	.6133	.6131	-0.0%	.7841	.7808	-0.4%
φ_{11B}^{*}	.3160	.3159	-0.0%	.3613	.3609	-0.1%
φ_{12B}^{*}	.4600	.4566	-0.7%	.5258	.5223	-0.7%
φ_{22A}^{*}	.4214	.4246	+0.8%	.5387	.5405	+0.3%
φ_{21A}^{*}	.6133	.5596	-8.8%	.7841	.7178	-8.4%
φ_{22B}^{*}	.3160	.3162	+0.1%	.3613	.3616	+0.1%
φ_{21B}^{*}	.4600	.4634	+0.7%	.5258	.5293	+0.7%

Table 2: Effects of Trade Liberalization when $\alpha_A = 0.1$ and ζ is large

We use the following benchmark parameter values: $\sigma = 3.8$, $\delta = .025$, $b = .2$, $\theta = 4.582$, $F = 2$, $f_{ii} = .043$, $L_i = 1$, $\tau_{ijs} = 1.3$ and $f_{ij} = .0588$. The first six parameter values come from Balistreri, Hillbery and Rutherford (2011), where a version of the Melitz model is calibrated to fit trade data. $L_i = 1$ is a convenient normalization given that an increase in country size L_i has no effect on the key endogenous variables that we are solving for (the relative wage w_1/w_2, productivity cutoff levels φ^*_{ijs} and industry productivity levels Φ^L_{is}). $\tau_{ijs} = 1.3$ corresponds to a 30 percent tax on all traded goods. Finally, we chose $f_{ij} = .0588$ to guarantee that 18 percent of firms export in the initial equilibrium, consistent with evidence for the United States (Bernard et al., 2007).

The first column of numbers in Table 1 shows the benchmark equilibrium for the Melitz model (when $\zeta = 0$ and $\tau_{21A} = 1.30$). The second column shows what happens when country 1 unilaterally opens up to trade in industry A (τ_{21A} is decreased from 1.30 to 1.15 holding $\tau_{21B} = \tau_{12A} = \tau_{12B} = 1.30$ fixed) and the third column shows the percentage change. The fourth and fifth columns of numbers show the effects of the same trade liberalization when there is slightly decreasing returns to scale in R&D ($\zeta = .25$, all other parameter values unchanged).

Looking at Tables 1 and 2, we see the most important result in this paper: it is possible to write down a model of international trade that has exact opposite properties compared to the Melitz model. In the Melitz model, trade liberalization results in productivity falling in the liberalized industry and rising in the non-liberalized industry (Φ^L_{1A} decreases by 0.7% and Φ^L_{1B} increases by 0.4% when $\zeta = 0$ in Table 1). But if we increase the degree of decreasing returns to R&D enough by increasing ζ, then we obtain opposite effects: trade liberalization results in productivity rising in the liberalized industry and falling in the non-liberalized industry (Φ^L_{1A} increases by 1.0% when $\zeta = .25$ in Table 1 and Φ^L_{1B} decreases by 0.1% when $\zeta = 1.5$ in Table 2). Furthermore, as we increase ζ, these opposite effects become quantitatively stronger. For the highest value of ζ (when $\zeta = 5$ in Table 2), productivity rises by 2.7% in the liberalized industry.

To see the intuition behind these results, we begin by considering the Melitz model case ($\zeta = 0$ in Table 1) and focus on what happens in industry A. When country 1 opens up to trade in industry A, country 2 firms earn higher profits from exporting. These higher export profits lead to more entry and greater industrial employment (the mass of entrants M_{2Ae} increases by 36.2%). As the industry becomes more populated with firms, the country 2 demand for each individual firm's product decreases, so the least productive firms are forced to exit (φ^*_{22A} increases by 4.4%). Even though the increase in labor demand bids up the wage rate in country 2 (w_1/w_2 decreases by 0.8%), the wage increase is not large enough to completely offset the tariff reduction by country 1 and more country 2 firms become exporters (φ^*_{21A} decreases by 11.3%). Since expanding exporters are more productive than exiting non-exporters, productivity rises for country 2 in industry A (Φ_{2A}^{L} increases by 4.4%). For firms in country 1, the picture is very different. Now they are competing against more productive firms in their export market, they earn lower profits from exporting and this sets into motion the opposite effects. Fewer country 1 firms become exporters (φ_{12A}^{*} increases by 3.3%), entry is discouraged and the mass of firms in the industry falls (M_{1Ae} decreases by 35.0%) until the expected profits from domestic sales increase to offset the loss of expected profits from exporting. The increase in domestic profits allows less productive firms to survive in the domestic market (φ_{11A}^{*} decreases by 0.8%). Thus, we get a reallocation of resources from more productive to less productive firms in country 1, lowering industry productivity (Φ_{1A}^{L} decreases by 0.7%).

Next focus on what happens in industry B when country 1 opens up to trade in industry A. Whereas we observe both a partial equilibrium competitiveness effect and a general equilibrium wage effect of trade liberalization in the liberalized industry A, there is only the general equilibrium wage effect in the non-liberalized industry B. Because wages rise in country 2 (w_1/w_2 decreases by 0.8%), it becomes less profitable for country 2 firms to export in industry B, fewer firms choose to export (φ_{21B}^{*} increases by 1.4%) and there is a reallocation of resources from more productive to less productive firms, lowering productivity (Φ_{2B}^{L} decreases by 0.3%). This general equilibrium wage effect is small simply because we are studying a case where only a small industry is opened up to trade in country 1 ($\alpha_A = 0.1$). Because wages fall in country 1 (w_1/w_2 decreases by 0.8%), there it becomes more profitable for firms to export in industry B, more firms choose to export (φ_{12B}^{*} decreases by 1.4%) and there is a reallocation of resources from less productive to more productive firms, raising productivity (Φ_{1B}^{L} increases by 0.4%).

The properties of the Melitz model change somewhat when the industry that opens up to trade is sufficiently large. In the case where $\alpha_A = .5$ and $\zeta = 0$ shown in Table 3, we obtain the same qualitative effects of trade liberalization in the non-liberalized industry B. Because wages rise in country 2 (w_1/w_2 decreases by 2.9%), productivity falls (Φ_{2B}^{L} decreases by 1.2%) and because wages fall in country 1 (w_1/w_2 decreases by 2.9%), productivity rises (Φ_{1B}^{L} increases by 1.5%). But the qualitative effects are different for the industry A that opens up to trade because there is a larger fall in the country 1 wage rate. Even though trade liberalization raises productivity in country 2 (Φ_{2A}^{L} increases by 2.4%), which by itself makes exporting less attractive for country 1 firms, the larger fall in the country 1 wage rate now dominates and country 1 productivity in industry A actually rises (Φ_{1A}^{L} increases by 0.4%).

Regardless of whether productivity falls or rises in the liberalized industry A, the Melitz model has the property that consumer welfare rises as a result of trade liberalization. In the tables, U_1 and U_2 denote the steady-state utility levels of the representative consumer in countries 1 and 2, respectively. In the $\alpha_A = .1$ case, trade liberalization by country 1 raises consumer welfare in country 2 and raises even

	$\zeta = 0$ Case			$\zeta = .25$ Case		
	$\tau_{21A} = 1.30$	$\tau_{21A} = 1.15$	% Change	$\tau_{21A} = 1.30$	$\tau_{21A} = 1.15$	% Change
Φ_{1A}^{L}	.2012	.2020	+0.4%	.2315	.2343	+1.2%
Φ_{1B}^{L}	.2012	.2042	+1.5%	.2315	.2328	+0.6%
Φ_{2A}^{L}	.2012	.2061	+2.4%	.2315	.2353	+1.6%
Φ_{2B}^{L}	.2012	.1988	-1.2%	.2315	.2306	-0.4%
U_1	.1231	.1243	+1.0%	.1416	.1429	+0.9%
U_2	.1231	.1239	+0.6%	.1416	.1425	+0.6%
w_1/w_2	1.0000	.9707	-2.9%	1.0000	.9724	-2.8%
M_{1Ae}	.0402	.0339	-15.7%	.0765	.0698	-8.8%
M_{1Be}	.0402	.0465	+15.7%	.0765	.0830	+8.5%
M_{2Ae}	.0402	.0463	+15.2%	.0765	.0828	+8.2%
M_{2Be}	.0402	.0341	-15.2%	.0765	.0700	-8.5%
ϕ_{21A}	.2463	.3698	+50.1%	.2463	.3732	+51.5%
ϕ_{21B}	.2463	.2109	-14.4%	.2463	.2128	-13.6%
φ_{11A}^{*}	.2241	.2250	+0.4%	.2578	.2610	+1.2%
φ_{12A}^{*}	.3258	.3206	-1.6%	.3748	.3668	-2.1%
φ_{11B}^{*}	.2241	.2275	+1.5%	.2578	.2593	+0.6%
φ_{12B}^{*}	.3258	.3092	-5.1%	.3748	.3595	-4.1%
φ_{22A}^{*}	.2241	.2296	+2.5%	.2578	.2621	+1.7%
φ_{21A}^{*}	.3258	.3013	-7.5%	.3748	.3486	-7.0%
φ_{22B}^{*}	.2241	.2215	-1.2%	.2578	.2569	-0.3%
φ_{21B}^{*}	.3258	.3443	+5.7%	.3748	.3916	+4.5%

Table 3: Effects of Trade Liberalization when $\alpha_A = 0.5$

more consumer welfare in country 1 (U_2 increases by 0.1% and U_1 increases by 0.2% in Table 1). Thus country 2 benefits when country 1 opens up to trade and country 1 benefits even more by unilaterally opening up to trade. For country 1, even though productivity falls by 0.7% in industry A, this only represents one-tenth of the economy. Productivity rises by 0.4% in industry B and this is the dominant effect for consumer welfare because industry B represents nine-tenths of the economy. Looking at the $\alpha_A = .5$ case in Table 3, we obtain qualitatively similar welfare effects.

	$\zeta = 1.5$ Case			$\zeta = 5$ Case		
	$\tau_{21A} = 1.30$	$\tau_{21A} = 1.15$	% Change	$\tau_{21A} = 1.30$	$\tau_{21A} = 1.15$	% Change
Φ_{1A}^L	.3064	.3122	+1.9%	.3609	.3684	+2.1%
Φ_{1B}^L	.3064	.3058	-0.2%	.3609	.3593	-0.4%
Φ_{2A}^L	.3064	.3092	+0.9%	.3609	.3635	+0.7%
Φ_{2B}^L	.3064	.3073	+0.3%	.3609	.3627	+0.5%
U_1	.1875	.1891	+0.9%	.2208	.2227	+0.9%
U_2	.1875	.1886	+0.6%	.2208	.2222	+0.6%
w_1/w_2	1.0000	.9738	-2.6%	1.0000	.9743	-2.6%
M_{1Ae}	.2765	.2690	-2.7%	.5853	.5798	-0.9%
M_{1Be}	.2765	.2837	+2.6%	.5853	.5905	+0.9%
M_{2Ae}	.2765	.2835	+2.5%	.5853	.5904	+0.9%
M_{2Be}	.2765	.2692	-2.6%	.5853	.5800	-0.9%
ϕ_{21A}	.2457	.3753	+52.7%	.2457	.3762	+53.1%
ϕ_{21B}	.2457	.2140	-12.9%	.2457	.2145	-12.7%
φ_{11A}^*	.3413	.3478	+1.9%	.4020	.4104	+2.1%
φ_{12A}^*	.4968	.4837	-2.6%	.5851	.5689	-2.8%
φ_{11B}^*	.3413	.3406	-0.2%	.4020	.4003	-0.4%
φ_{12B}^*	.4968	.4807	-3.2%	.5851	.5677	-3.0%
φ_{22A}^*	.3413	.3445	+0.9%	.4020	.4049	+0.7%
φ_{21A}^*	.4968	.4642	-6.6%	.5851	.5475	-6.4%
φ_{22B}^*	.3413	.3423	+0.3%	.4020	.4040	+0.5%
φ_{21B}^*	.4968	.5140	+3.5%	.5851	.6036	+3.2%

Table 4: Effects of Trade Liberalization when $\alpha_A = 0.5$ and ζ is large

For the Melitz model, the effects of trade liberalization on industrial productivity are summarized in Table 5. The wage effect tends to increase productivity in both industries symmetrically, while the competitiveness effect tends to decrease productivity in the liberalized industry. As a consequence, industrial productivity unambiguously rises in the non-liberalized industry B but it can rise or fall in the liberalized industry A, depending on the relative size of the wage effect and the competitiveness effect. Productivity falls in the liberalized industry when the wage effect is small (Table 1) and productivity rises in the liberalized industry when the wage effect is large (Table 3). Regardless of the relative size of the two effects, we always get that productivity rises more in the non-liberalized industry ($\Delta\tau_{21A} < 0 \Rightarrow \Delta \ln \Phi_{1A}^L - \Delta \ln \Phi_{1B}^L < 0$). This result is derived under general assumptions in Segerstrom and Sugita (2015).

	Impact on Industrial Productivity		
	Liberalized (A) $\Delta \ln \Phi_{1A}^L$	Non-liberalized (B) $\Delta \ln \Phi_{1B}^L$	Difference-in-Differences $\Delta \ln \Phi_{1A}^L - \Delta \ln \Phi_{1B}^L$
Competitiveness Effect	$-$	0	$-$
Wage Effect	$+$	$+$	0
Total Effect	$+$ or $-$	$+$	$-$

Table 5: The effects of trade liberalization in the Melitz model ($\zeta = 0$)

For the Melitz model, trade liberalization has its biggest effects on the mass of entering firms in different countries and industries (M_{ise}). Looking at Table 1, the reduction in τ_{21A} from 1.3 to 1.15 results in a 36.2% increase in M_{2Ae} and a 35.0% decrease in M_{1Ae}. When the degree of decreasing returns to R&D becomes stronger (ζ increases above zero), the mass of entering firms M_{ise} does not change as much due to trade liberalization. The change in M_{2Ae} goes from +36.2% to +17.6% to +5.0% to +1.7% and the change in M_{1Ae} goes from -35.0% to -18.0% to -5.4% to -1.8% as ζ increases from 0 to .25 to 1.5 to 5. Increasing ζ makes firm entry sluggish. There is less adjustment both in the up direction and in the down direction. We see the same pattern when we look at Tables 3 and 4. The change in M_{2Ae} goes from +15.2% to +8.2% to +2.5% to +0.9% and the change in M_{1Ae} goes from -15.7% to -8.8% to -2.7% to -0.9% as ζ increases from 0 to .25 to 1.5 to 5.

To understand why this is happening, it suffices to recall that for firms in country i and sector s, the cost of entry is $w_i M_{ise}^{\zeta} F$. When $\zeta = 0$ (the Melitz model case), the cost of entry does not depend on the mass of entering firms M_{ise} but when $\zeta > 0$, the cost of entry goes up when M_{ise} increases and the cost of entry goes down when M_{ise} decreases. So in a sector where trade liberalization encourages more entry, as more firms enter, the cost of entry goes up, which serves to discourage further entry. And in a sector where trade liberalization leads to less entry, as less firms enter, the cost of entry goes down, which serves to make entry more attractive. As ζ increases, we get less adjustment in the up direction because the cost of entry is going up and we get less adjustment in the down direction because the cost of entry is going down.

A key equation in the model (both when $\zeta = 0$ and when $\zeta > 0$) is

$$\varphi_{11s}^{*\theta} = \frac{\theta b^{\theta}}{(\theta - \sigma + 1)} \frac{\sigma f_d}{\alpha_s L_1} \left(M_{1se} + \phi_{21s} M_{2se} \right).$$

This equation implies that the domestic productivity cutoff φ_{11s}^{*} rises if and only if $M_{1se} + \phi_{21s} M_{2se}$ rises. Since industrial productivity

$$\Phi_{1s}^{L} = \left(\frac{\theta + 1}{\theta} \right) \rho \varphi_{11s}^{*}$$

is proportional to the domestic productivity cutoff φ_{11s}^{*}, industrial productivity Φ_{1s}^{L} rises as a result of trade liberalization if and only if $M_{1se} + \phi_{21s} M_{2se}$ rises.

The term $M_{1se} + \phi_{21s} M_{2se}$ can be interpreted as a mass of entrants index relevant for consumers in country 1 and sector s. M_{1se} is the mass of firms that directly enter in country 1 and sector s. But consumers also buy imported products, so the mass M_{2se} of firms that enter in country 2 and sector s is also relevant for country 1 consumers. Since not all country 2 firms export to country 1, we multiply M_{2se} by the relative expected profit term ϕ_{21s} and then add M_{1se} to obtain the total number of entering firms $M_{1se} + \phi_{21s} M_{2se}$ relevant for country 1 consumers in sector s. ϕ_{21s} is higher when more firms export from country 2 to country 1 in sector s.

If trade liberalization results in $M_{1se} + \phi_{21s} M_{2se}$ increasing, this means that more firms are entering and competition is becoming tougher in country 1 and sector s. With tougher competition, firms need to have a higher productivity level to profitably survive, so the domestic productivity cutoff φ_{11s}^{*} increases, and it follows that productivity Φ_{1s}^{L} rises. If trade liberalization results in $M_{1se} + \phi_{21s} M_{2se}$ decreasing, then fewer firms enter, competition because less tough, lower productivity firms can now survive and industrial productivity falls.

Returning to Table 1 and the $\zeta = 0$ case, trade liberalization results in $M_{1Ae} + \phi_{21A} M_{2Ae}$ decreasing from $.0080 + (.2457)(.0080) = .0100$ to $.0052 + (.4138)(.0109) = .0097$. Although ϕ_{21A} increases by 68.4% (from .2457 to .4138) and M_{2Ae} increases by 36.2%, the dominant change in the expression $M_{1Ae} + \phi_{21A} M_{2Ae}$ is the 35.0% decrease in M_{1Ae}. Because trade liberalization results in significantly fewer firms entering in country 1 and sector A, the overall level of competition in this sector drops, lower productivity firms can now survive (φ_{11A}^{*} decreases by 0.8%) and industrial productivity falls (Φ_{1A}^{L} decreases by 0.7%). In the Melitz model, trade liberalization results in productivity falling in the liberalized industry.

The properties of the model, however, are fundamentally different when $\zeta = .25$. Then trade liberalization results in $M_{1Ae} + \phi_{21A} M_{2Ae}$ increasing from $.0211 + (.2457)(.0211) = .0263$ to $.0172 + (.4164)(.0248) = .0275$. Given the decreasing returns to R&D, the changes in M_{1Ae} and

M_{2Ae} are now much smaller. With M_{1Ae} decreasing by 18.0% and M_{2Ae} increasing by 17.6%, the dominant change in the expression $M_{1Ae} + \phi_{21A} M_{2Ae}$ is the 69.5% increase in ϕ_{21A} (from .2457 to .4164). Because trade liberalization results in significantly more firms exporting from country 2 to country 1 in sector A, the overall level of competition in this sector rises, firms need to have higher productivity to survive (φ^*_{11A} increases by 1.0%) and industrial productivity rises (Φ^L_{1A} increases by 1.0%).

To summarize, we see that when $\zeta = 0$, the dominant change in $M_{1Ae} + \phi_{21A} M_{2Ae}$ is the decrease in M_{1Ae} and when $\zeta = .25$, the dominant change in $M_{1Ae} + \phi_{21A} M_{2Ae}$ is the increase in ϕ_{21A}. In the Melitz model, the main effect of trade liberalization is to reduce the number of firms entering the liberalized sector. Because competition becomes less tough as a result of trade liberalization, lower productivity firms can survive and the overall level of productivity in the liberalized sector falls. With slightly decreasing returns to R&D, the properties of the model fundamentally change. Then the main effect of trade liberalization is to increase the number of firms that export to the liberalized sector. Because competition becomes more tough as a result of trade liberalization, firms need higher productivity to survive and the overall level of productivity in the liberalized sector rises.

Turning now to the wage effect of trade liberalization, we focus on what happens in the non-liberalized sector B in Tables 3 and 4. In the $\zeta = 0$ Melitz case, trade liberalization results in $M_{1Be} + \phi_{21B} M_{2Be}$ increasing from $.0402 + (.2463)(.0402) = .0501$ to $.0465 + (.2109)(.0341) = .0537$. Although ϕ_{21B} decreases by 14.4% (from .2463 to .2109) and M_{2Be} decreases by 15.2%, the dominant change in the expression $M_{1Be} + \phi_{21B} M_{2Be}$ is the 15.7% increase in M_{1Be}. Because trade liberalization results in significantly more firms entering in country 1 and sector B, the overall level of competition in this sector rises, lower productivity firms can no longer survive (φ^*_{11B} increases by 1.5%) and industrial productivity rises (Φ^L_{1B} increases by 1.5%). In the Melitz model, trade liberalization results in productivity rising in the non-liberalized industry. The falling wage rate (w_1/w_2 decreases by 2.9%) contributes to rising productivity.

The wage effect properties of the model, however, are fundamentally different when $\zeta = 1.5$. Then trade liberalization results in $M_{1Be} + \phi_{21B} M_{2Be}$ decreasing from $.2765 + (.2457)(.2765) = .3444$ to $.2837 + (.2140)(.2692) = .3413$. Given the decreasing returns to R&D, the changes in M_{1Be} and M_{2Be} are now much smaller. With M_{1Be} increasing by 2.6% and M_{2Be} decreasing by 2.6%, the dominant change in the expression $M_{1Be} + \phi_{21B} M_{2Be}$ is the 12.9% decrease in ϕ_{21B} (from .2457 to .2140). Because trade liberalization in sector A results in significantly fewer firms exporting from country 2 to country 1 in sector B, the overall level of competition in this sector falls, firms with lower productivity can now survive (φ^*_{11B} decreases by 0.2%) and industrial productivity falls (Φ^L_{1B} decreases by 0.2%).

To summarize, we see that when $\zeta = 0$, the dominant change in $M_{1Be} + \phi_{21B} M_{2Be}$ is the increase in M_{1Be} and when $\zeta = 1.5$, the dominant change in $M_{1Be} + \phi_{21B} M_{2Be}$ is the decrease in ϕ_{21B}. In the Melitz model, the main effect of trade liberalization in the non-liberalized sector is to raise the number of firms entering the non-liberalized sector. Because competition becomes more tough as a result of trade liberalization, lower productivity firms can no longer survive and the overall level of productivity in the non-liberalized sector rises (this is the wage effect of trade liberalization). A falling wage rate is associated with rising productivity in the Melitz model. However, with decreasing returns to R&D, the properties of the model fundamentally change. Then the main effect of trade liberalization on the non-liberalized sector is to decrease the number of firms that export to the non-liberalized sector. Because competition becomes less tough as a result of trade liberalization, lower productivity firms can now survive and the overall level of productivity in the non-liberalized sector falls. A falling wage rate is associated with falling productivity (when ζ is sufficiently large).

3.3 Other "Melitz" Predictions

The Melitz model has several other predictions that have been confirmed in many empirical studies. For instance, Redding (2011) mentions two other facts as empirical motivations for the Melitz model: (1) exporters are larger and more productive than non-exporters; (2) entry and exit simultaneously occur within the same industry even without trade liberalization. This section shows that the new model continues to predict other central facts that the Melitz model predicts.

Selection into Exporting A large number of empirical studies on firm-level data shows that within industries, firm's productivity is positively correlated with the probability that the firm exports (e.g. Bernard and Jensen, 1995) and the number of markets to which a firm exports (e.g. Eaton, Kortum, and Kramarz, 2011). Eaton, Kortum, and Kramarz (2011) show that the Melitz model (with idiosyncratic trade costs and fixed entry) successfully predicts these cross-sectional facts. The new model also predicts these facts since firm's behaviors after entry is exactly the same as those in the Melitz model.

Simultaneous Entry and Exit Another fact emphasized by Redding (2011) is that firm's entry and exit simultaneously occur within industries even without trade liberalization. This fact is robustly found in the industrial organization literature and motivates a seminal model by Hopenhayn (1992) with random productivity draws at free entry and probabilistic exit. Similar to the Melitz model, the current model features random productivity draws at free entry and probabilistic exit, so it can predict simultaneous entry and exit.

Home Market Effect As an extension of the Krugman (1980) model, the Melitz model predicts the Home Market effect: a larger demand for an industry creates an export base of the industry. The Home Market effect receives empirical supports (e.g. Hanson and Xiang, 2004) and plays an important role in the New Economic Geography as well as in the trade literature. Therefore, the new model would be empirically appealing if it can predict both the Home Market effect and the Trefler finding.

To answer this question, we consider the model with fixed wages, following Helpman and Krugman (1985). Industry B produces a homogenous numeraire good with constant returns to scale technology. The good is also traded under free trade and perfectly competition. Thus, industry B fixes the wage. Suppose that the two countries are initially symmetric and that population of country 1 increases ($d\ln L_1 > 0 = d\ln L_2$). Then, we analyze whether the net export of country 1 in industry A, $R_{12A} - R_{21A}$, becomes positive or negative. If it becomes positive, we conclude that the model predicts the Home Market effect.

Totally differentiating (14), we obtain

$$d\ln M_{1se} = \varepsilon_L d\ln L_1 \text{ and } d\ln M_{2se} = -\phi\varepsilon_L d\ln L_1,$$
$$\text{where } \varepsilon_L \equiv \frac{[1-\phi+\zeta(1+\phi)]}{(1+\zeta)\left[(1-\phi)^2+\zeta(1+\phi)^2\right]} > 0.$$

Using this and differentiating (14), we obtain

$$\frac{d\ln(R_{12A}/R_{21A})}{d\ln L_1} = 2\varepsilon_L - 1.$$

Since $R_{12A} = R_{21A}$ initially holds, the net export of country 1 in industry A, $R_{12A} - R_{21A}$, becomes positive if and only if $d\ln(R_{12A}/R_{21A})/d\ln L_1 > 0$, that is, $\varepsilon_L > 1/2$. Since ε_L is a function of ζ, we can find a range of ζ for which $2\varepsilon_L > 1$ holds. Thus, we establish the following lemma.

Lemma 3. *There exists a positive threshold* $\zeta_H \equiv \zeta_1 + \sqrt{\zeta_1^2 + (1-\phi)/(1+\phi)} > \zeta_1 > 0$ *such that (1) the model predicts the Home Market effect if and only if* $\zeta < \zeta_H$*; that (2) if* $\zeta \in (\zeta_1, \zeta_H)$*, then the model predicts both the Home Market effect and the Trefler finding.*

4 Conclusion

To be added.

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Appendix: Derivations

Equilibrium Conditions

From firm's pricing (2) and the cutoff condition (5), we obtain

$$
\begin{aligned}
\frac{p_{ijs}(\varphi^*_{ijs})}{p_{ijs}(\varphi^*_{jjs})} &= \frac{w_i \tau_{ijs}}{w_j} \left(\frac{\varphi^*_{jjs}}{\varphi^*_{ijs}} \right) \\
&= \frac{w_i \tau_{ijs}}{w_j} \left(\frac{1}{T_{ijs}} \left(\frac{w_i}{w_j} \right)^{-1/\rho} \right) \\
&= \frac{w_i \tau_{ijs}}{w_j} \left(\frac{1}{\tau_{ijs}} \left(\frac{f_{ij}}{f_{jj}} \right)^{1/(1-\sigma)} \left(\frac{w_i}{w_j} \right)^{-1/\rho} \right) \\
&= \left(\frac{w_i f_{ij}}{w_j f_{jj}} \right)^{1/(1-\sigma)}.
\end{aligned}
$$

Since

$$
\begin{aligned}
\int_x^\infty \left(\frac{\varphi}{x} \right)^{\sigma-1} dG(\varphi) &= \int_x^\infty \left(\frac{\varphi}{x} \right)^{\sigma-1} b^\theta \theta \varphi^{-\theta-1} \, d\varphi \\
&= b^\theta \theta x^{1-\sigma} \int_x^\infty \varphi^{\sigma-1-\theta-1} \, d\varphi \\
&= b^\theta \theta x^{1-\sigma} \frac{x^{\sigma-1-\theta}}{\theta - \sigma + 1} \\
&= \eta \left(\frac{b}{x} \right)^\theta, \text{ where } \eta \equiv \frac{\theta}{\theta - \sigma + 1},
\end{aligned}
$$

it holds that

$$\begin{aligned}
\int_{\varphi_{ijs}^*}^{\infty} p_{ijs}(\varphi)^{1-\sigma}\, dG(\varphi) &= \int_{\varphi_{ijs}^*}^{\infty} p_{ijs}(\varphi_{ijs}^*)^{1-\sigma} \left(\frac{\varphi}{\varphi_{ijs}^*}\right)^{\sigma-1} dG(\varphi) \\
&= p_{ijs}(\varphi_{ijs}^*)^{1-\sigma} \int_{\varphi_{ijs}^*}^{\infty} \left(\frac{\varphi}{\varphi_{ijs}^*}\right)^{\sigma-1} dG(\varphi) \\
&= p\left(\varphi_{jjs}^*\right)^{1-\sigma} \left(\frac{w_i f_{ij}}{w_j f_{jj}}\right) \eta \left(\frac{b}{\varphi_{ijs}^*}\right)^{\theta} \\
&= \eta p\left(\varphi_{jjs}^*\right)^{1-\sigma} \left(\frac{w_i f_{ij}}{w_j f_{jj}}\right) \left(\frac{b}{T_{ijs}\left(w_i/w_j\right)^{1/\rho} \varphi_{jjs}^*}\right)^{\theta} \\
&= \eta p\left(\varphi_{jjs}^*\right)^{1-\sigma} \left(\frac{f_{ij}}{f_{jj}} T_{ijs}^{-\theta} \left(\frac{w_i}{w_j}\right)^{1-\theta/\rho}\right) \left(\frac{b}{\varphi_{jjs}^*}\right)^{\theta} \\
&= \eta p\left(\varphi_{jjs}^*\right)^{1-\sigma} \left(\frac{b}{\varphi_{jjs}^*}\right)^{\theta} \phi_{ijs} \qquad (25)
\end{aligned}$$

Substituting this into the price index we obtain equation (7) for the price index

$$\begin{aligned}
P_{js}^{1-\sigma} &= \sum_{i=1,2} \int_{\varphi_{ijs}^*}^{\infty} p_{ijs}(\varphi)^{1-\sigma}\, M_{is}\mu_{is}(\varphi) d\varphi \\
&= \sum_{i=1,2} \frac{M_{ise}}{\delta} \int_{\varphi_{ijs}^*}^{\infty} p_{ijs}(\varphi)^{1-\sigma}\, dG(\varphi) \\
&= \eta p\left(\varphi_{jjs}^*\right)^{1-\sigma} \left(\frac{b}{\varphi_{jjs}^*}\right)^{\theta} \sum_{i=1,2} \frac{M_{ise}}{\delta} \phi_{ijs}. \qquad (26)
\end{aligned}$$

Using these results, the cutoff condition (3) for country 1 can be written as

$$\begin{aligned}
\frac{r_{11s}(\varphi_{11s}^*)}{\sigma} &= w_1 f_d \\
\frac{\alpha_s w_1 L_1}{\sigma} \left(\frac{p_{11s}(\varphi_{11s}^*)}{P_{1s}}\right)^{1-\sigma} &= w_1 f_d \\
\frac{\alpha_s L_1}{\sigma} \eta^{\sigma-1} \left[\left(b/\varphi_{11s}^*\right)^{\theta} \left(M_{1se} + \phi_{21s} M_{2se}\right)\right]^{-1} &= f_d.
\end{aligned}$$

Rearranging terms then yields

$$\varphi_{11s}^{*\theta} = \frac{\theta b^{\theta}}{\delta\left(\theta - \sigma + 1\right)} \frac{\sigma f_d}{\alpha_s L_1} \left(M_{1se} + \phi_{21s} M_{2se}\right).$$

This is equation (8).

Labor Demand

Let L_{is} denote labor demand by all firms in country i and sector s. We use a three step argument to solve for labor demand.

First, we rewrite the free entry condition. Using the following relationship

$$\begin{aligned}
\int_{\varphi_{ijs}^{*}}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma_s}-w_i f_{ij}\right]dG(\varphi) &= \int_{\varphi_{ijs}^{*}}^{\infty}\left[w_i f_{ij}\left(\frac{\varphi}{\varphi_{ijs}^{*}}\right)^{\sigma_s-1}-w_i f_{ij}\right]dG(\varphi)\\
&= w_i f_{ij}\left[\int_{\varphi_{ijs}^{*}}^{\infty}\left(\frac{\varphi}{\varphi_{ijs}^{*}}\right)^{\sigma-1}dG(\varphi)-\left[1-G(\varphi_{ijs}^{*})\right]\right]\\
&= w_i f_{ij}\,(\eta-1)\left(\frac{b}{\varphi_{ijs}^{*}}\right)^{\theta}\\
&= \left(\frac{\sigma-1}{\theta-\sigma+1}\right)w_i f_{ij}\left(\frac{b}{\varphi_{ijs}^{*}}\right)^{\theta}, \qquad (27)
\end{aligned}$$

we simplify the free entry condition as

$$\begin{aligned}
\frac{1}{\delta}\sum_{j=1,2}\int_{\varphi_{ijs}^{*}}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma_s}-w_i f_{ij}\right]dG(\varphi) &= w_i f_{ise}\\
\frac{w_i}{\delta}\left(\frac{\sigma-1}{\theta-\sigma+1}\right)\sum_{j=1,2}f_{ijs}\left(\frac{b}{\varphi_{ijs}^{*}}\right)^{\theta} &= w_i f_{ise}\\
\frac{1}{\delta}\left(\frac{\sigma-1}{\theta-\sigma+1}\right)\sum_{j=1,2}f_{ijs}\left(\frac{b}{\varphi_{ijs}^{*}}\right)^{\theta} &= f_{ise}.
\end{aligned}$$

This is equation (10).

Second, we show that the fixed costs (the entry costs plus the marketing costs) are proportional to the mass of entrants in each country i and sector s.

$$\begin{aligned}
w_i\left(M_{ise}f_{ise}+\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty}f_{ijs}M_{is}\mu_{is}(\varphi)\,d\varphi\right) &= w_i\left(M_{ise}f_{ise}+\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty}f_{ij}\frac{M_{ise}}{\delta}g(\varphi)\,d\varphi\right) \text{ from (4)}\\
&= w_i\left(M_{ise}f_{ise}+\frac{M_{ise}}{\delta}\sum_{j=1,2}f_{ij}[1-G(\varphi^*_{ijs})]\right)\\
&= w_i\left(M_{ise}f_{ise}+\frac{M_{ise}}{\delta}\sum_{j=1,2}f_{ij}\left(\frac{b}{\varphi^*_{ijs}}\right)^{\theta}\right)\\
&= w_i\left(M_{ise}f_{ise}+\frac{M_{ise}}{\delta}\delta f_{ise}\left(\frac{\theta-\sigma+1}{\sigma-1}\right)\right) \text{ from (10)}\\
&= w_iM_{ise}f_{ise}\left(\frac{\sigma-1+\theta-\sigma+1}{\sigma-1}\right)
\end{aligned}$$

from which it follows that

$$w_i\left(M_{ise}f_{ise}+\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty}f_{ij}M_{is}\mu_{is}(\varphi)\,d\varphi\right)=w_iM_{ise}\left(\frac{\theta f_{ise}}{\sigma-1}\right).$$

Second, we show that the fixed costs are equal to the gross profits in each country i and sector s. From the free entry condition (9), we obtain

$$\begin{aligned}
\delta w_i f_{ise} &= \sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma}-w_if_{ij}\right]dG(\varphi)\\
w_i\left(\delta f_{ise}+\sum_{j=1,2}f_{ij}[1-G(\varphi^*_{ijs})]\right) &= \sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty}\frac{r_{ijs}(\varphi)}{\sigma}dG(\varphi)\\
w_i\left(M_{ise}f_{ise}+\frac{M_{ise}}{\delta}\sum_{j=1,2}f_{ij}[1-G(\varphi^*_{ijs})]\right) &= \frac{M_{ise}}{\delta}\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty}\frac{r_{ijs}(\varphi)}{\sigma}dG(\varphi)\\
w_iM_{ise}\left(\frac{\theta f_{ise}}{\sigma-1}\right) &= \frac{M_{is}}{1-G(\varphi^*_{iis})}\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty}\frac{r_{ijs}(\varphi)}{\sigma}dG(\varphi) \text{ from (12)}\\
&= \frac{1}{\sigma}\sum_{j=1,2}\int_{\varphi^*_{ijs}}^{\infty}r_{ijs}(\varphi)M_{is}\mu_{is}(\varphi)d\varphi \text{ from (A.1)}\\
&= \frac{1}{\sigma}\sum_{j=1,2}R_{ijs}
\end{aligned}$$

where $R_{ijs}\equiv\int_{\varphi^*_{ijs}}^{\infty}r_{ijs}(\varphi)M_{is}\mu_{is}(\varphi)d\varphi$ is the total revenue associated with shipments from country i to country j in sector s.

Third, we show that the wage payments to labor equals the total revenue in each country i and sector s. Firms use labor for market entry, for the production of goods sold to domestic consumers and for the production of goods sold to foreign consumers. Taking into account both the marginal and fixed costs of production, we obtain

$$\begin{aligned}
w_i L_{is} &= w_i M_{ise} f_{ise} + w_i \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} \left[f_{ij} + q_{ijs}(\varphi) \frac{\tau_{ijs}}{\varphi} \right] M_{is} \mu_{is}(\varphi)\, d\varphi \\
&= w_i \left(M_{ise} f_{ise} + \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} f_{ij} M_{is} \mu_{is}(\varphi)\, d\varphi \right) + \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} q_{ijs}(\varphi) \frac{w_i \tau_{ijs}}{\rho \varphi} \rho M_{is} \mu_{is}(\varphi)\, d\varphi \\
&= w_i M_{ise} \left(\frac{\theta f_{ise}}{\sigma - 1} \right) + \rho \sum_{j=1,2} \int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi) M_{is} \mu_{is}(\varphi)\, d\varphi \text{ from (\textbf{??}), (\textbf{??}) and (11)} \\
&= \frac{1}{\sigma} \sum_{j=1,2} R_{ijs} + \rho \sum_{j=1,2} R_{ijs} \\
&= (1 - \rho + \rho) \sum_{j=1,2} R_{ijs} \\
&= \sum_{j=1,2} R_{ijs}.
\end{aligned}$$

Thus

$$L_{is} = \frac{1}{w_i} \sum_{j=1,2} R_{ijs} = \frac{1}{w_i} w_i M_{ise} \left(\frac{\theta f_{ise}}{\sigma - 1} \right) \sigma = M_{ise} \left(\frac{\theta}{\rho} \right) f_{ise}.$$

and it immediately follows that

$$L_{ise} = M_{ise}^{1+\zeta} \left(\frac{\theta F}{\rho} \right).$$

Relative Expected Profit

The expected profit of an entrant in country i from selling to country j in sector s (after the entrant has paid the entry cost $w_i F_{is}$) is

$$\frac{[1 - G(\varphi^*_{iis})]}{\delta} \int_{\varphi^*_{ijs}}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma_s} - w_i f_{ij} \right] \frac{g(\varphi)}{1 - G(\varphi^*_{iis})}\, d\varphi = \delta^{-1} \int_{\varphi^*_{ijs}}^{\infty} \left[\frac{r_{ijs}(\varphi)}{\sigma} - w_i f_{ij} \right] dG(\varphi).$$

The expected profit of an entrant in country j from selling to country j in sector s (after the entrant has paid the entry cost $w_i F_{is}$) is

$$\frac{\left[1 - G(\varphi^*_{jjs})\right]}{\delta} \int_{\varphi^*_{jjs}}^{\infty} \left[\frac{r_{jjs}(\varphi)}{\sigma} - w_j f_{jj} \right] \frac{g(\varphi)}{1 - G(\varphi^*_{jjs})}\, d\varphi = \delta^{-1} \int_{\varphi^*_{jjs}}^{\infty} \left[\frac{r_{jjs}(\varphi)}{\sigma} - w_j f_{jj} \right] dG(\varphi).$$

Thus the expected profit of an entrant in country i from selling to country j in sector s relative to that captured by an entrant in country j from selling to country j (or the relative expected profit) is given by

$$
\begin{aligned}
\phi_{ijs} &\equiv \frac{\delta^{-1}\int_{\varphi^*_{ijs}}^{\infty}\left[\frac{r_{ijs}(\varphi)}{\sigma}-w_i f_{ij}\right]\,dG(\varphi)}{\delta^{-1}\int_{\varphi^*_{jjs}}^{\infty}\left[\frac{r_{jjs}(\varphi)}{\sigma}-w_j f_{jj}\right]\,dG(\varphi)} \\
&= \frac{\delta^{-1}w_i f_{ij}\frac{\sigma-1}{\theta-\sigma+1}\left(\frac{b}{\varphi^*_{ijs}}\right)^{\theta}}{\delta^{-1}w_j f_{jj}\frac{\sigma-1}{\theta-\sigma+1}\left(\frac{b}{\varphi^*_{jjs}}\right)^{\theta}} \text{ from (27)} \\
&= \frac{w_i f_{ij}}{w_j f_{jj}}\left(\frac{\varphi^*_{jjs}}{\varphi^*_{ijs}}\right)^{\theta} \\
&= \frac{w_i f_{ij}}{w_j f_{jj}}\left[T_{ijs}^{-1}\left(\frac{w_i}{w_j}\right)^{-1/\rho}\right]^{\theta} \text{ from (5)}
\end{aligned}
$$

or

$$
\phi_{ijs} = \frac{f_{ij}}{f_{jj}}T_{ijs}^{-\theta}\left(\frac{w_i}{w_j}\right)^{1-\theta/\rho}. \tag{14}
$$

Total Revenue

$$
\begin{aligned}
R_{ijs} &\equiv \int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi)M_{is}\mu_{is}(\varphi)\,d\varphi \\
&= \frac{M_{is}}{1-G(\varphi^*_{iis})}\int_{\varphi^*_{ijs}}^{\infty} r_{ijs}(\varphi)\,dG(\varphi) \text{ from (A.1)} \\
&= \frac{[1-G(\varphi^*_{iis})]M_{ise}}{\delta[1-G(\varphi^*_{iis})]}\int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)q_{ijs}(\varphi)\,dG(\varphi) \\
&= \frac{M_{ise}}{\delta}\int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)\frac{p_{ijs}(\varphi)^{-\sigma}\alpha_s w_j L_j}{P_{js}^{1-\sigma}}\,dG(\varphi) \\
&= \frac{\alpha_s w_j L_j}{P_{js}^{1-\sigma}}\frac{M_{ise}}{\delta}\int_{\varphi^*_{ijs}}^{\infty} p_{ijs}(\varphi)^{1-\sigma}\,dG(\varphi) \\
&= \alpha_s w_j L_j \frac{\frac{M_{ise}}{\delta}\eta p\left(\varphi^*_{jjs}\right)^{1-\sigma}\left(\frac{b}{\varphi^*_{jjs}}\right)^{\theta}\phi_{ijs}}{\eta p\left(\varphi^*_{jjs}\right)^{1-\sigma}\left(\frac{b}{\varphi^*_{jjs}}\right)^{\theta}\sum_{i=1,2}\frac{M_{ise}}{\delta}\phi_{ijs}} \text{ from (26)} \\
&= \alpha_s w_j L_j \frac{M_{ise}\phi_{ijs}}{\sum_{i=1,2}M_{ise}\phi_{ijs}}.
\end{aligned}
$$

Log-Linearization

Since $\phi_{ijs} = \frac{f_x}{f_d} T_{ijs}^{-\theta} \left(\frac{w_i}{w_j}\right)^{1-\theta/\rho}$ and $T_{ijs} \equiv \tau_{ijs} \left(\frac{f_x}{f_d}\right)^{1/(\sigma-1)}$ imply that $T_{iis} = 1$ and $\phi_{iis} = 1$, equations (14) can be written out as

$$\begin{aligned}\frac{\alpha_s w_1 L_1}{M_{1se} + M_{2se}\phi_{21s}} + \frac{\alpha_s L_2}{M_{1se}\phi_{12s} + M_{2se}}\phi_{12s} &= \left(\frac{\theta F}{\rho}\right) w_1 f_{1se} \\ \frac{\alpha_s w_1 L_1}{M_{1se} + M_{2se}\phi_{21s}}\phi_{21s} + \frac{\alpha_s L_2}{M_{1se}\phi_{12s} + M_{2se}} &= \left(\frac{\theta F}{\rho}\right) f_{2se}.\end{aligned}$$

Written in matrix form, these systems of linear equations become

$$\begin{pmatrix} 1 & \phi_{12s} \\ \phi_{21s} & 1 \end{pmatrix} \begin{pmatrix} \alpha_s w_1 L_1 / \left(M_{1se} + M_{2se}\phi_{21s}\right) \\ \alpha_s L_2 / \left(M_{1se}\phi_{12s} + M_{2se}\right) \end{pmatrix} = \left(\frac{\theta F}{\rho}\right) \begin{pmatrix} w_1 f_{1se} \\ f_{2se} \end{pmatrix}.$$

Solving using Cramer's Rule yields

$$\begin{aligned}\frac{\alpha_s w_1 L_1}{M_{1se} + M_{2se}\phi_{21s}} &= \frac{\theta F}{\rho}\left(\frac{w_1 f_{1se} - \phi_{12s} f_{2se}}{1 - \phi_{12s}\phi_{21s}}\right) \\ \frac{\alpha_s L_2}{M_{1se}\phi_{12s} + M_{2se}} &= \frac{\theta F}{\rho}\left(\frac{f_{2se} - \phi_{21s} w_1 f_{1se}}{1 - \phi_{12s}\phi_{21s}}\right).\end{aligned}$$

where

$$\begin{aligned}1 - \phi_{12s}\phi_{21s} &= 1 - \left(\frac{f_x}{f_d}\right)^2 \left(T_{12s}T_{21s}\right)^{-\theta} \\ &= 1 - \left(\tau_{12s}\tau_{21s}\right)^{-\theta} \left(\frac{f_x}{f_d}\right)^{-2(\theta-\sigma+1)/(\sigma-1)} > 0\end{aligned}$$

since $\tau_{12s}\tau_{21s} > 1$, $f_x > f_d$, and $\theta - \sigma + 1 > 0$. For these equations to make sense, we need

$$\frac{1}{\phi_{12s}} > \frac{f_{2se}}{w_1 f_{1se}} > \phi_{21s},$$

which is satisfied in the current case of symmetric countries and industries. The above equations can be written as

$$\begin{aligned}\left(f_{1se} - \frac{\phi_{12s}}{w_1} f_{2se}\right)\left(M_{1se} + M_{2se}\phi_{21s}\right) &= \frac{\rho \alpha_s L_1}{\theta F}\left(1 - \phi_{12s}\phi_{21s}\right) \\ \left(f_{2se} - \phi_{21s} w_1 f_{1se}\right)\left(M_{1se}\phi_{12s} + M_{2se}\right) &= \frac{\rho \alpha_s L_2}{\theta F}\left(1 - \phi_{12s}\phi_{21s}\right).\end{aligned} \tag{28}$$

Taking logs and differentiating these lead to

$$d\ln\left(f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}\right)+d\ln\left(M_{1se}+M_{2se}\phi_{21s}\right)=d\ln\left(1-\phi_{12s}\phi_{21s}\right)$$
$$d\ln\left(f_{2se}-\phi_{21s}w_1f_{1se}\right)+d\ln\left(M_{1se}\phi_{12s}+M_{2se}\right)=d\ln\left(1-\phi_{12s}\phi_{21s}\right). \tag{29}$$

The definition of $\phi_{ijs}=\frac{f_x}{f_d}T_{ijs}^{-\theta}\left(\frac{w_i}{w_j}\right)^{1-\theta/\rho}$ implies that

$$d\ln\phi_{12s}=-\theta\,d\ln T_{12s}-\left(\frac{\theta}{\rho}-1\right)d\ln w_1$$
$$d\ln\phi_{21s}=-\theta\,d\ln T_{21s}+\left(\frac{\theta}{\rho}-1\right)d\ln w_1. \tag{30}$$

Since countries and industries are symmetric before liberalization, it follows that $\phi_{ijs}=\phi$, $w_1=1$, $M_{1se}=M_{2se}$, and $f_{1se}=f_{2se}=f_e$. Using this symmetry and (30), the changes in terms in (29) are obtained as follows:

$$\begin{aligned}
d\ln\left(1-\phi_{12s}\phi_{21s}\right)&=\frac{1}{1-\phi_{12s}\phi_{21s}}\left(-\phi_{12s}d\phi_{21s}-\phi_{21s}d\phi_{12s}\right)\\
&=-\frac{\phi_{12s}\phi_{21s}}{1-\phi_{12s}\phi_{21s}}\left(d\ln\phi_{12s}+d\ln\phi_{21s}\right)\\
&=\frac{\phi^2\theta}{1-\phi^2}\left(d\ln T_{12s}+d\ln T_{21s}\right),
\end{aligned} \tag{31}$$

$$\begin{aligned}
d\ln\left(f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}\right)&=\frac{f_{1se}}{f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}}d\ln f_{1se}-\frac{\frac{\phi_{12s}}{w_1}f_{2se}}{f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}}\left(d\ln f_{2se}+d\ln\phi_{12s}-d\ln w_1\right)\\
&=\frac{1}{1-\phi}d\ln f_{1se}-\frac{\phi}{1-\phi}\left(d\ln f_{2se}+d\ln\phi_{12s}-d\ln w_1\right)\\
&=\frac{1}{1-\phi}d\ln f_{1se}-\frac{\phi}{1-\phi}\left(d\ln f_{2se}-\theta d\ln T_{12s}-\frac{\theta}{\rho}d\ln w_1\right)\\
&=\frac{1}{1-\phi}d\ln f_{1se}-\frac{\phi}{1-\phi}d\ln f_{2se}+\frac{\phi\theta}{1-\phi}d\ln T_{12s}+\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)d\ln w_1,
\end{aligned} \tag{32}$$

$$
\begin{aligned}
d\ln\left(M_{1se}+M_{2se}\phi_{21s}\right) &= \frac{M_{1se}}{M_{1se}+M_{2se}\phi_{21s}}d\ln M_{1se} + \frac{M_{2se}\phi_{21s}}{M_{1se}+M_{2se}\phi_{21s}}\left(d\ln M_{2se}+d\ln\phi_{21s}\right) \\
&= \frac{1}{1+\phi}d\ln M_{1se} + \frac{\phi}{1+\phi}\left(d\ln M_{2se}+d\ln\phi_{21s}\right), \\
&= \frac{1}{1+\phi}d\ln M_{1se} + \frac{\phi}{1+\phi}\left(d\ln M_{2se} - \theta d\ln T_{21s} + \left(\frac{\theta}{\rho}-1\right)d\ln w_1\right) \\
&= \frac{1}{1+\phi}d\ln M_{1se} + \frac{\phi}{1+\phi}d\ln M_{2se} - \frac{\phi\theta}{1+\phi}d\ln T_{21s} + \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)d\ln w_1.
\end{aligned}
$$

From the symmetry of the two countries, we obtain the following corresponding relationships for Foreign:

$$
\begin{aligned}
d\ln\left(f_{2se}-\phi_{21s}w_1f_{1se}\right) &= \frac{f_{2se}}{f_{2se}-\phi_{21s}w_1f_{1se}}d\ln f_{2se} - \frac{\phi_{21s}w_1f_{1se}}{f_{2se}-\phi_{21s}w_1f_{1se}}\left(d\ln\phi_{21s}+d\ln w_1+d\ln f_{1se}\right) \\
&= \frac{1}{1-\phi}d\ln f_{2se} - \frac{\phi}{1-\phi}\left(-\theta\, d\ln T_{21s} + \left(\frac{\theta}{\rho}-1\right)d\ln w_1 + d\ln w_1 + d\ln f_{1se}\right) \\
&= \frac{1}{1-\phi}d\ln f_{2se} - \frac{\phi}{1-\phi}d\ln f_{1se} + \frac{\phi\theta}{1-\phi}d\ln T_{21s} - \frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)d\ln w_1
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
d\ln\left(M_{1se}\phi_{12s}+M_{2se}\right) &= \frac{M_{1se}\phi_{12s}}{M_{1se}\phi_{12s}+M_{2se}}\left(d\ln M_{1se}+d\ln\phi_{12s}\right) + \frac{M_{2se}}{M_{1se}\phi_{12s}+M_{2se}}d\ln M_{2se} \\
&= \frac{\phi}{1+\phi}\left(d\ln M_{1se} - \theta\, d\ln T_{12s} - \left(\frac{\theta}{\rho}-1\right)d\ln w_1\right) + \frac{1}{1+\phi}d\ln M_{2se} \\
&= \frac{1}{1+\phi}d\ln M_{2se} + \frac{\phi}{1+\phi}d\ln M_{1se} - \frac{\phi\theta}{1+\phi}d\ln T_{12s} - \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)d\ln w_1.
\end{aligned}
\tag{34}
$$

Now substituting into the equation

$$
d\ln\left(f_{1se}-\frac{\phi_{12s}}{w_1}f_{2se}\right) + d\ln\left(M_{1se}+M_{2se}\phi_{21s}\right) = d\ln\left(1-\phi_{12s}\phi_{21s}\right),
$$

we obtain

$$
\begin{aligned}
&\frac{1}{1-\phi}d\ln f_{1se} - \frac{\phi}{1-\phi}d\ln f_{2se} + \frac{\phi\theta}{1-\phi}d\ln T_{12s} + \frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)d\ln w_1 \\
&+\frac{1}{1+\phi}d\ln M_{1se} + \frac{\phi}{1+\phi}d\ln M_{2se} - \frac{\phi\theta}{1+\phi}d\ln T_{21s} + \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)d\ln w_1 \\
&= \frac{\phi^2\theta}{1-\phi^2}\left(d\ln T_{12s}+d\ln T_{21s}\right)
\end{aligned}
$$

and rearranging terms yields

$$\begin{aligned}\frac{1}{1+\phi}d\ln M_{1se}+\frac{\phi}{1+\phi}d\ln M_{2se} &= -\left(\frac{\phi\theta}{1-\phi}-\frac{\phi^2\theta}{1-\phi^2}\right)d\ln T_{12s}+\left(\frac{\phi\theta}{1+\phi}+\frac{\phi^2\theta}{1-\phi^2}\right)d\ln T_{21s}\\ &-\left[\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)+\frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)\right]d\ln w_1\\ &-\frac{1}{1-\phi}d\ln f_{1se}+\frac{\phi}{1-\phi}d\ln f_{2se}.\end{aligned}$$

This equation can be written more compactly as

$$\lambda_d d\ln M_{1se}+\lambda_f d\ln M_{2se}=-\nu_T d\ln T_{12s}+\nu_T d\ln T_{21s}-\nu_w d\ln w_1-\nu_d d\ln f_{1se}+\nu_f d\ln f_{2se}$$

where $\lambda_d\equiv 1/(1+\phi)$, $\lambda_f\equiv\phi/(1+\phi)$, $\nu_d=1/(1-\phi)$, $\nu_f=\phi/(1-\phi)$

$$\nu_T\equiv\frac{\phi\theta}{1-\phi}-\frac{\phi^2\theta}{1-\phi^2}=\frac{\phi\theta(1+\phi)-\phi^2\theta}{(1-\phi)(1+\phi)}=\frac{\phi\theta}{1-\phi^2}=\frac{\phi\theta(1-\phi)+\phi^2\theta}{(1-\phi)(1+\phi)}=\frac{\phi\theta}{1+\phi}+\frac{\phi^2\theta}{1-\phi^2}$$

and

$$\nu_w\equiv\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)+\frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)=\frac{\phi(1+\phi)+\phi(1-\phi)}{(1-\phi)(1+\phi)}\frac{\theta}{\rho}-\frac{\phi}{1+\phi}=\frac{\phi}{1+\phi}\left[\frac{2\theta}{\rho\,(1-\phi)}-1\right].$$

Next, substituting into the equation

$$d\ln\left(f_{2se}-\phi_{21s}w_1f_{1se}\right)+d\ln\left(M_{1se}\phi_{12s}+M_{2se}\right)=d\ln\left(1-\phi_{12s}\phi_{21s}\right),$$

we obtain

$$\begin{aligned}&\frac{1}{1-\phi}d\ln f_{2se}-\frac{\phi}{1-\phi}d\ln f_{1se}+\frac{\phi\theta}{1-\phi}d\ln T_{21s}-\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)d\ln w_1\\ +&\frac{1}{1+\phi}d\ln M_{2se}+\frac{\phi}{1+\phi}d\ln M_{1se}-\frac{\phi\theta}{1+\phi}d\ln T_{12s}-\frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)d\ln w_1\\ &=\frac{\phi^2\theta}{1-\phi^2}\left(d\ln T_{12s}+d\ln T_{21s}\right)\end{aligned}$$

and rearranging terms yields

$$\begin{aligned}\frac{\phi}{1+\phi}d\ln M_{1se}+\frac{1}{1+\phi}d\ln M_{2se} &= \left(\frac{\phi\theta}{1+\phi}+\frac{\phi^2\theta}{1-\phi^2}\right)d\ln T_{12s}-\left(\frac{\phi\theta}{1-\phi}-\frac{\phi^2\theta}{1-\phi^2}\right)d\ln T_{21s}\\ &+\left[\frac{\phi}{1-\phi}\left(\frac{\theta}{\rho}\right)+\frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)\right]d\ln w_1\\ &+\frac{\phi}{1-\phi}d\ln f_{1se}-\frac{1}{1-\phi}d\ln f_{2se}.\end{aligned}$$

This equation can be written more compactly as

$$\lambda_f d\ln M_{1se}+\lambda_d d\ln M_{2se}=\nu_T d\ln T_{12s}-\nu_T d\ln T_{21s}+\nu_w d\ln w_1+\nu_f d\ln f_{1se}-\nu_d d\ln f_{2se}.$$

The two equations

$$\begin{aligned}\lambda_d d\ln M_{1se}+\lambda_f d\ln M_{2se} &= -\nu_T d\ln T_{12s}+\nu_T d\ln T_{21s}-\nu_w d\ln w_1-\nu_d d\ln f_{1se}+\nu_f d\ln f_{2se}\\ \lambda_f d\ln M_{1se}+\lambda_d d\ln M_{2se} &= \nu_T d\ln T_{12s}-\nu_T d\ln T_{21s}+\nu_w d\ln w_1+\nu_f d\ln f_{1se}-\nu_d d\ln f_{2se}\end{aligned}$$

can be written in matrix form as:

$$\begin{aligned}\frac{1}{1+\phi}\begin{pmatrix}1 & \phi\\ \phi & 1\end{pmatrix}\begin{pmatrix}d\ln M_{1se}\\ d\ln M_{2se}\end{pmatrix} &= -\frac{\phi\theta}{1-\phi^2}\begin{pmatrix}1\\ -1\end{pmatrix}d\ln T_{12s}+\frac{\phi\theta}{1-\phi^2}\begin{pmatrix}1\\ -1\end{pmatrix}d\ln T_{21s}\\ &-\frac{\phi}{1+\phi}\left(\frac{2\theta}{\rho(1-\phi)}-1\right)\begin{pmatrix}1\\ -1\end{pmatrix}d\ln w_1\\ &-\frac{1}{1-\phi}\begin{pmatrix}1\\ -\phi\end{pmatrix}d\ln f_{1se}+\frac{1}{1-\phi}\begin{pmatrix}\phi\\ -1\end{pmatrix}d\ln f_{2se}.\end{aligned}$$

Since

$$(1+\phi)\begin{pmatrix}1 & \phi\\ \phi & 1\end{pmatrix}^{-1}=\frac{1+\phi}{1-\phi^2}\begin{pmatrix}1 & -\phi\\ -\phi & 1\end{pmatrix}=\frac{1}{1-\phi}\begin{pmatrix}1 & -\phi\\ -\phi & 1\end{pmatrix},$$

$$(1+\phi)\begin{pmatrix}1 & \phi\\ \phi & 1\end{pmatrix}^{-1}\begin{pmatrix}1\\ -1\end{pmatrix}=\frac{1}{1-\phi}\begin{pmatrix}1 & -\phi\\ -\phi & 1\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix}=\frac{1+\phi}{1-\phi}\begin{pmatrix}1\\ -1\end{pmatrix},$$

$$(1+\phi)\begin{pmatrix}1 & \phi\\ \phi & 1\end{pmatrix}^{-1}\begin{pmatrix}1\\ -\phi\end{pmatrix}=\frac{1}{1-\phi}\begin{pmatrix}1 & -\phi\\ -\phi & 1\end{pmatrix}\begin{pmatrix}1\\ -\phi\end{pmatrix}=\frac{1}{1-\phi}\begin{pmatrix}1+\phi^2\\ -2\phi\end{pmatrix},$$

and

$$(1+\phi)\begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix}^{-1}\begin{pmatrix} \phi \\ -1 \end{pmatrix} = \frac{1}{1-\phi}\begin{pmatrix} 1 & -\phi \\ -\phi & 1 \end{pmatrix}\begin{pmatrix} \phi \\ -1 \end{pmatrix} = \frac{1}{1-\phi}\begin{pmatrix} 2\phi \\ -(1+\phi^2) \end{pmatrix},$$

we obtain

$$\begin{aligned}
\begin{pmatrix} d\ln M_{1se} \\ d\ln M_{2se} \end{pmatrix} = &-\frac{\phi\theta}{(1-\phi)^2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln T_{12s} + \frac{\phi\theta}{(1-\phi)^2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln T_{21s} \\
&- \frac{\phi}{1-\phi}\left(\frac{2\theta}{\rho\,(1-\phi)} - 1\right)\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln w_1 \\
&- \frac{1}{(1-\phi)^2}\begin{pmatrix} 1+\phi^2 \\ -2\phi \end{pmatrix} d\ln f_{1se} + \frac{1}{(1-\phi)^2}\begin{pmatrix} 2\phi \\ -(1+\phi^2) \end{pmatrix} d\ln f_{2se}.
\end{aligned}$$

Defining

$$\iota_T \equiv \frac{\phi\theta}{(1-\phi)^2},\ \iota_w \equiv \frac{\phi}{1-\phi}\left(\frac{2\theta}{\rho\,(1-\phi)} - 1\right),\ \iota_1 \equiv \frac{1+\phi^2}{(1-\phi)^2} \text{ and } \iota_2 \equiv \frac{2\phi}{(1-\phi)^2}, \tag{35}$$

the system of equations can be written out as

$$\begin{aligned}
d\ln M_{1se} &= \iota_T d\ln T_{21s} - \iota_T d\ln T_{12s} - \iota_w d\ln w_1 - \iota_1 d\ln f_{1se} + \iota_2 d\ln f_{2se} \\
d\ln M_{2se} &= -\iota_T d\ln T_{21s} + \iota_T d\ln T_{12s} + \iota_w d\ln w_1 + \iota_2 d\ln f_{1se} - \iota_1 d\ln f_{2se}.
\end{aligned} \tag{36}$$

This system of equations can be further simplified by using $f_{ise} \equiv M_{ise}^{\zeta}$. From $d\ln f_{ise} = \zeta d\ln M_{ise}$,

$$\begin{pmatrix} 1+\zeta\iota_1 & -\zeta\iota_2 \\ -\zeta\iota_2 & 1+\zeta\iota_1 \end{pmatrix}\begin{pmatrix} d\ln M_{1se} \\ d\ln M_{2se} \end{pmatrix} = \iota_T\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln T_{21s} - \iota_T\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln T_{12s} - \iota_w\begin{pmatrix} 1 \\ -1 \end{pmatrix} d\ln w_1.$$

Since

$$\begin{pmatrix} 1+\zeta\iota_1 & -\zeta\iota_2 \\ -\zeta\,\iota_2 & 1+\zeta\iota_1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{(1+\zeta\iota_1)^2-(\zeta\iota_2)^2} \begin{pmatrix} 1+\zeta\iota_1 & \zeta\iota_2 \\ \zeta\iota_2 & 1+\zeta\iota_1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$= \frac{1+\zeta(\iota_1-\iota_2)}{[1+\zeta(\iota_1-\iota_2)]\,[1+\zeta(\iota_1+\iota_2)]} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$= \frac{1}{1+\zeta(\iota_1+\iota_2)} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$= \frac{1}{1+\zeta(1+2\phi+\phi^2)/(1-\phi)^2} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$= \frac{(1-\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2} \begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

we obtain

$$\begin{aligned} d\ln M_{1se} &= \varepsilon_T d\ln T_{21s} - \varepsilon_T d\ln T_{12s} - \varepsilon_w d\ln w_1 \\ d\ln M_{2se} &= -\varepsilon_T d\ln T_{21s} + \varepsilon_T d\ln T_{12s} + \varepsilon_w d\ln w_1, \end{aligned} \tag{37}$$

where

$$\varepsilon_T \equiv \frac{(1-\phi)^2\iota_T}{(1-\phi)^2+\zeta\,(1+\phi)^2} = \frac{(1-\phi)^2\phi\theta/(1-\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2} = \frac{\phi\theta}{(1-\phi)^2+\zeta\,(1+\phi)^2}$$

and

$$\varepsilon_w \equiv \frac{(1-\phi)^2\iota_w}{(1-\phi)^2+\zeta\,(1+\phi)^2} = \frac{(1-\phi)^2\frac{\phi}{1-\phi}\left(\frac{2\theta}{\rho(1-\phi)}-1\right)}{\left[(1-\phi)^2+\zeta\,(1+\phi)^2\right]} = \frac{\phi\,[2\theta-\rho\,(1-\phi)]}{\rho\left[(1-\phi)^2+\zeta\,(1+\phi)^2\right]}.$$

Using $d\ln f_{ise} = \zeta d\ln M_{ise}$ and substituting (37) into (36), we obtain

$$\begin{aligned} d\ln M_{1se} &= \iota_T d\ln T_{21s} - \iota_T d\ln T_{12s} - \iota_w d\ln w_1 - \iota_1 d\ln f_{1se} + \iota_2 d\ln f_{2se} \\ &= \iota_T d\ln T_{21s} - \iota_T d\ln T_{12s} - \iota_w d\ln w_1 - \iota_1\zeta d\ln M_{1se} + \iota_2\zeta d\ln M_{2se} \\ &= \iota_T d\ln T_{21s} - \iota_T d\ln T_{12s} - \iota_w d\ln w_1 - \iota_1\zeta\left[\varepsilon_T d\ln T_{21s} - \varepsilon_T d\ln T_{12s} - \varepsilon_w d\ln w_1\right] \\ &\qquad + \iota_2\zeta\left[-\varepsilon_T d\ln T_{21s} + \varepsilon_T d\ln T_{12s} + \varepsilon_w d\ln w_1\right] \\ &= \left[\iota_T - \zeta\,(\iota_1+\iota_2)\,\varepsilon_T\right] d\ln T_{21s} - \left[\iota_T - \zeta\,(\iota_1+\iota_2)\,\varepsilon_T\right] d\ln T_{12s} - \left[\iota_w - \zeta\,(\iota_1+\iota_2)\,\varepsilon_w\right] d\ln w_1. \end{aligned}$$

Comparing the last expression with (37), we obtain alternative expressions of ε_T and ε_w

$$\varepsilon_T = \iota_T - \zeta\left(\iota_1 + \iota_2\right)\varepsilon_T \text{ and } \varepsilon_w = \iota_w - \zeta\left(\iota_1 + \iota_2\right)\varepsilon_w. \tag{38}$$

The two measures of industrial labor productivity

$$\begin{aligned}
\Phi_{1s}^{L} &\equiv \frac{\sum_{j=1,2} R_{1js}}{\tilde{P}_{1s} L_{1s}} = \left(\frac{\theta+1}{\theta}\right)\rho\varphi_{11s}^{*} \\
\Phi_{1s}^{W} &\equiv \frac{\sum_{j=1,2} R_{1js}}{P_{1s} L_{1s}} = \left(\frac{\alpha_s L_1}{\sigma f_{11}}\right)^{1/(\sigma-1)}\rho\varphi_{11s}^{*}
\end{aligned}$$

imply that

$$d\ln\Phi_{1s}^{k=L,W} = d\ln\varphi_{11s}^{*}.$$

Taking logs and then differentiating

$$\varphi_{11s}^{*\theta} = \frac{\theta b^{\theta}}{\delta\left(\theta-\sigma+1\right)}\frac{\sigma f_d}{\alpha_s L_1}\left(M_{1se} + \phi_{21s} M_{2se}\right),$$

yields

$$\begin{aligned}
\theta\, d\ln\varphi_{11s}^{*} &= d\ln\left(M_{1se} + \phi_{21s} M_{2se}\right) \\
&= \frac{1}{1+\phi} d\ln M_{1se} + \frac{\phi}{1+\phi} d\ln M_{2se} - \frac{\phi\theta}{1+\phi} d\ln T_{21s} + \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right) d\ln w_1 \\
&= \frac{1}{1+\phi}\left[\varepsilon_T d\ln T_{21s} - \varepsilon_T d\ln T_{12s} - \varepsilon_w d\ln w_1\right] \\
&\quad + \frac{\phi}{1+\phi}\left[-\varepsilon_T d\ln T_{21s} + \varepsilon_T d\ln T_{12s} + \varepsilon_w d\ln w_1\right] \\
&\quad - \frac{\phi\theta}{1+\phi} d\ln T_{21s} + \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right) d\ln w_1 \\
&= \left[\frac{1-\phi}{1+\phi}\varepsilon_T - \frac{\phi\theta}{1+\phi}\right] d\ln T_{21s} - \left[\frac{1-\phi}{1+\phi}\varepsilon_T\right] d\ln T_{12s} \\
&\quad - \left[\frac{1-\phi}{1+\phi}\varepsilon_w - \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)\right] d\ln w_1
\end{aligned}$$

Substituting (35) and (38), the above equation becomes

$$d\ln\varphi_{11s}^{*} = \gamma_1 d\ln T_{21s} - \gamma_2 d\ln T_{12s} - \gamma_3 d\ln w_1$$

where

$$
\begin{aligned}
\gamma_1 &\equiv \frac{1}{\theta}\left[\frac{1-\phi}{1+\phi}\varepsilon_T - \frac{\phi\theta}{1+\phi}\right] \\
&= \frac{1}{\theta}\left[\frac{1-\phi}{1+\phi}\left(\iota_T - \zeta\left(\iota_1+\iota_2\right)\varepsilon_T\right) - \frac{\phi\theta}{1+\phi}\right] \\
&= \frac{1}{\theta}\left[\frac{1-\phi}{1+\phi}\left(\frac{\phi\theta}{(1-\phi)^2} - \zeta\left(\frac{1+\phi^2+2\phi}{(1-\phi)^2}\right)\left(\frac{\phi\theta}{(1-\phi)^2+\zeta\left(1+\phi\right)^2}\right)\right) - \frac{\phi\theta}{1+\phi}\right] \\
&= \frac{\phi}{1-\phi^2} - \zeta\left(\frac{1+\phi}{1-\phi}\right)\frac{\phi}{(1-\phi)^2+\zeta\left(1+\phi\right)^2} - \frac{\phi(1-\phi)}{(1+\phi)(1-\phi)} \\
&= \frac{\phi^2}{1-\phi^2} - \frac{\zeta\phi\left(1+\phi\right)}{\left(1-\phi\right)\left[\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2\right]},
\end{aligned}
$$

$$
\begin{aligned}
\gamma_2 &\equiv \frac{1-\phi}{\theta\left(1+\phi\right)}\varepsilon_T > 0 \\
&= \frac{1-\phi}{\theta\left(1+\phi\right)}\left(\iota_T - \zeta\left(\iota_1+\iota_2\right)\varepsilon_T\right) \\
&= \frac{1-\phi}{\theta\left(1+\phi\right)}\left(\frac{\phi\theta}{(1-\phi)^2} - \zeta\left(\frac{1+\phi^2+2\phi}{(1-\phi)^2}\right)\left(\frac{\phi\theta}{(1-\phi)^2+\zeta\left(1+\phi\right)^2}\right)\right) \\
&= \frac{\phi}{(1-\phi^2)} - \frac{\zeta\phi\left(1+\phi\right)}{\left(1-\phi\right)\left[\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2\right]}
\end{aligned}
$$

and

$$
\begin{aligned}
\gamma_3 &\equiv \frac{1}{\theta}\left[\frac{1-\phi}{1+\phi}\varepsilon_w - \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)\right] \\
&= \frac{1}{\theta}\left[\frac{1-\phi}{1+\phi}\left(\iota_w - \zeta\left(\iota_1+\iota_2\right)\varepsilon_w\right) - \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)\right] \\
&= \frac{1}{\theta}\left[\frac{1-\phi}{1+\phi}\left(\frac{\phi}{1-\phi}\left(\frac{2\theta}{\rho\left(1-\phi\right)}-1\right) - \zeta\left(\frac{1+\phi^2+2\phi}{(1-\phi)^2}\right)\varepsilon_w\right) - \frac{\phi}{1+\phi}\left(\frac{\theta}{\rho}-1\right)\right] \\
&= \frac{2\phi}{\rho(1+\phi)\left(1-\phi\right)} - \frac{\phi}{\theta\left(1+\phi\right)} - \frac{\zeta(1+\phi)}{\theta(1-\phi)}\varepsilon_w - \frac{\phi(1-\phi)}{\rho(1+\phi)(1-\phi)} + \frac{\phi}{\theta(1+\phi)} \\
&= \frac{\phi(1+\phi)}{\rho(1+\phi)\left(1-\phi\right)} - \frac{\zeta(1+\phi)}{\theta(1-\phi)}\frac{\phi\left[2\theta-\rho\left(1-\phi\right)\right]}{\rho\left[\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2\right]} \\
&= \frac{\phi}{\rho\left(1-\phi\right)} - \frac{\zeta\phi\left(1+\phi\right)\left[2\theta-\rho\left(1-\phi\right)\right]}{\rho\theta(1-\phi)\left[\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2\right]}.
\end{aligned}
$$

Proof for Lemma 1 and Lemma 2 We are ready to determine the sign of γ_1,

$$\begin{aligned}
\gamma_1 &\equiv \frac{[(1-\phi)\,\varepsilon_T - \phi\theta]}{\theta\,(1+\phi)} \\
&= \frac{1}{\theta\,(1+\phi)}\left[(1-\phi)\,\frac{\phi\theta}{(1-\phi)^2+\zeta\,(1+\phi)^2} - \phi\theta\right] \\
&= \frac{\phi}{1+\phi}\left(\frac{1-\phi}{(1-\phi)^2+\zeta\,(1+\phi)^2} - 1\right) \\
&= \frac{\phi}{1+\phi}\left(\frac{(1-\phi)-(1-\phi)^2-\zeta\,(1+\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2}\right) \\
&= \frac{\phi}{1+\phi}\left(\frac{(1-\phi)\,(1-[1-\phi])-\zeta\,(1+\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2}\right) \\
&= \frac{\phi}{1+\phi}\left(\frac{\phi\,(1-\phi)-\zeta\,(1+\phi)^2}{(1-\phi)^2+\zeta\,(1+\phi)^2}\right) < 0 \\
&\text{if and only if } \zeta > \zeta_1 \equiv \frac{\phi\,(1-\phi)}{(1+\phi)^2},
\end{aligned}$$

and the sign of γ_3,

$$\begin{aligned}
\gamma_3 &= \frac{1}{\theta\,(1+\phi)}\left[(1-\phi)\,\varepsilon_w - \phi\left(\frac{\theta}{\rho}-1\right)\right] \\
&= \frac{1}{\theta\,(1+\phi)}\left[(1-\phi)\,\frac{\phi\,(2\theta-\rho\,(1-\phi))}{\rho\left[(1-\phi)^2+(1+\phi)^2\,\zeta\right]} - \phi\left(\frac{\theta-\rho}{\rho}\right)\right] \\
&= \frac{\phi}{\rho\theta\,(1+\phi)}\left[\frac{(1-\phi)\,(2\theta-\rho\,(1-\phi))}{(1-\phi)^2+(1+\phi)^2\,\zeta} - (\theta-\rho)\right] \\
&= \frac{\phi}{\rho\theta\,(1+\phi)}\left[\frac{(1-\phi)\,(2\theta-\rho\,(1-\phi))-(\theta-\rho)\left((1-\phi)^2+(1+\phi)^2\,\zeta\right)}{(1-\phi)^2+(1+\phi)^2\,\zeta}\right] \\
&= \frac{\phi}{\rho\theta\,(1+\phi)}\left[\frac{(1-\phi)\,[(2\theta-\rho\,(1-\phi))-(\theta-\rho)\,(1-\phi)]-(\theta-\rho)\,(1+\phi)^2\,\zeta}{(1-\phi)^2+(1+\phi)^2\,\zeta}\right] \\
&= \frac{\phi}{\rho\theta\,(1+\phi)}\left[\frac{(1-\phi)\,[2\theta-\theta\,(1-\phi)]-(\theta-\rho)\,(1+\phi)^2\,\zeta}{(1-\phi)^2+(1+\phi)^2\,\zeta}\right] \\
&= \frac{\phi}{\rho\theta\,(1+\phi)}\left[\frac{\theta\,(1-\phi)\,(1+\phi)-(\theta-\rho)\,(1+\phi)^2\,\zeta}{(1-\phi)^2+(1+\phi)^2\,\zeta}\right] \\
&= \frac{\phi}{\rho\theta}\left[\frac{\theta\,(1-\phi)-(\theta-\rho)\,(1+\phi)\,\zeta}{(1-\phi)^2+(1+\phi)^2\,\zeta}\right] < 0 \\
&\text{if and only if } \zeta > \zeta_3 \equiv \frac{\theta\,(1-\phi)}{(\theta-\rho)\,(1+\phi)}.
\end{aligned}$$

A comparison of ζ_1 and ζ_3 leads to

$$\begin{aligned}\frac{\zeta_3}{\zeta_1} &= \left[\frac{\theta\,(1-\phi)}{(\theta-\rho)\,(1+\phi)}\right]\left[\frac{(1+\phi)^2}{\phi\,(1-\phi)}\right] \\ &= \left(\frac{1+\phi}{\phi}\right)\left(\frac{\theta}{\theta-\rho}\right) \\ &= \left(1+\frac{1}{\phi}\right)\left(1+\frac{\rho}{\theta-\rho}\right) > 1.\end{aligned}$$

To determine the maximum value of $\zeta_1 \equiv \phi\,(1-\phi)\,/\,(1+\phi)^2$, we take the derivative of $\ln\zeta_1 = \ln\phi + \ln(1-\phi) - 2\ln(1+\phi)$:

$$\begin{aligned}\frac{d\ln\zeta_1(\phi)}{d\phi} &= \frac{1}{\phi} - \frac{1}{1-\phi} - \frac{2}{1+\phi} \\ &= \frac{(1-\phi)\,(1+\phi) - \phi\,(1+\phi) - 2\phi(1-\phi)}{\phi(1-\phi)(1+\phi)} \\ &= \frac{1-\phi^2-\phi-\phi^2-2\phi+2\phi^2}{\phi(1-\phi^2)} \\ &= \frac{1-3\phi}{\phi(1-\phi^2)}.\end{aligned}$$

Note that the derivative is positive for $\phi < 1/3$ and negative for $1/3 < \phi < 1$, so the second order condition is satisfied and the maximum value of ζ_1 occurs when $\phi = 1/3$. Since $\zeta_1(\phi) \equiv \phi\,(1-\phi)\,/\,(1+\phi)^2$,

$$\zeta_1(1/3) = \frac{\frac{1}{3}\left(1-\frac{1}{3}\right)}{\left(1+\frac{1}{3}\right)^2} = \frac{\frac{1}{3}\left(\frac{2}{3}\right)}{\left(\frac{4}{3}\right)^2} = \frac{2}{16} = \frac{1}{8}.$$

Therefore, ζ_1 takes the maximum value $1/8$ at $\phi = 1/3$.

Wage change

Suppose that trade costs change in sector A but not in sector B ($d\ln T_{12B} = d\ln T_{21B} = 0$). Starting with the labor market clearing condition

$$L_1 = L_{1A} + L_{1B} = \left(\frac{\theta F}{\rho}\right)\left(M_{1Ae}^{1+\zeta} + M_{1Be}^{1+\zeta}\right),$$

first taking logs of both sides

$$\ln L_1 = \ln\left(\frac{\theta F}{\rho}\right) + \ln\left(M_{1Ae}^{1+\zeta} + M_{1Be}^{1+\zeta}\right)$$

and then differentiating yields

$$
\begin{aligned}
0 &= \frac{1}{M_{1Ae}^{1+\zeta}+M_{1Be}^{1+\zeta}}\left[(1+\zeta)M_{1Ae}^{\zeta}dM_{1Ae}+(1+\zeta)M_{1Be}^{\zeta}dM_{1Be}\right]\\
&=(1+\zeta)\left[\frac{M_{1Ae}^{1+\zeta}}{M_{1Ae}^{1+\zeta}+M_{1Be}^{1+\zeta}}d\ln M_{1Ae}+\frac{M_{1Be}^{1+\zeta}}{M_{1Ae}^{1+\zeta}+M_{1Be}^{1+\zeta}}d\ln M_{1Be}\right]\\
&=(1+\zeta)\left[\frac{L_{1A}}{L_1}d\ln M_{1Ae}+\frac{L_{1B}}{L_1}d\ln M_{1Be}\right].
\end{aligned}
$$

It follows that

$$
0=\frac{L_{1A}}{L_1}\left[\varepsilon_T d\ln T_{21A}-\varepsilon_T d\ln T_{12A}-\varepsilon_w d\ln w_1\right]-\frac{L_{1B}}{L_1}\varepsilon_w d\ln w_1
$$

and rearranging terms yields

$$
\frac{L_{1A}}{L_1}\varepsilon_T\left[d\ln T_{21A}-d\ln T_{12A}\right]=\frac{L_{1A}}{L_1}\varepsilon_w d\ln w_1+\frac{L_{1B}}{L_1}\varepsilon_w d\ln w_1=\varepsilon_w d\ln w_1 \tag{39}
$$

Since countries are initially symmetric, it holds that before liberalization $f_{1se}=f_{2se}=f_{se}$, $M_{1se}=M_{2se}=M_{se}$, $\phi_{12s}=\phi_{21s}=\phi$, $L_1=L_2=L$ and $w_1=1$. Thus, two equations in (28) becomes one equation

$$
\left(f_{se}-\phi_s f_{se}\right)\left(M_{se}+M_{se}\phi_s\right)=\frac{\rho\alpha_s L}{\theta F}(1-\phi_s^2).
$$

This equation implies

$$
\begin{aligned}
\left(f_{se}-\phi_s f_{se}\right)\left(M_{se}+M_{se}\phi_s\right)&=\frac{\rho\alpha_s L}{\theta F}(1-\phi_s^2)\\
M_{se}f_{se}\left(1-\phi_s\right)\left(1+\phi_s\right)&=\frac{\rho\alpha_s L}{\theta F}(1-\phi_s^2)\\
M_{se}f_{se}&=\frac{\rho\alpha_s L}{\theta F}\\
\frac{\theta}{\rho}FM_{se}^{1+\zeta}&=\alpha_s L
\end{aligned}
$$

Since $L_{is}=\frac{\theta}{\rho}FM_{ise}^{1+\zeta}$ from (15), $\alpha_s=L_{is}/L$ holds. Using this and rearranging equation (39), we obtain

$$\begin{aligned}
d\ln w_1 &= \frac{L_{1A}}{L_1}\frac{\varepsilon_T}{\varepsilon_w}\left(d\ln T_{21A} - d\ln T_{12A}\right) \\
&= \alpha_A \frac{\frac{\phi\theta}{(1-\phi)^2+\zeta(1+\phi)^2}}{\frac{\phi[2\theta-\rho(1-\phi)]}{\rho\left[(1-\phi)^2+\zeta(1+\phi)^2\right]}}\left(d\ln T_{21A} - d\ln T_{12A}\right) \\
&= \frac{\alpha_A\phi\theta\rho}{\phi\left[2\theta-\rho\left(1-\phi\right)\right]}\left(d\ln T_{21A} - d\ln T_{12A}\right) \\
&= \frac{\alpha_A\theta\rho}{\left[2\theta-\rho\left(1-\phi\right)\right]}\left(d\ln T_{21A} - d\ln T_{12A}\right).
\end{aligned}$$

Thus, the wage change does not depend on the size of ζ.

Home Market Effect

From $f_{ise} = M_{ise}^{\zeta}$ and $\phi_{12s} = \phi_{21s} = \phi$, system of equations (28) becomes

$$\begin{aligned}
\left(M_{1se}^{\zeta} - \phi M_{2se}^{\zeta}\right)\left(M_{1se} + M_{2se}\phi\right) &= \frac{\rho\alpha_s L_1}{\theta F}(1-\phi^2) \\
\left(M_{2se}^{\zeta} - \phi M_{1se}^{\zeta}\right)\left(M_{1se}\phi + M_{2se}\right) &= \frac{\rho\alpha_s L_2}{\theta F}(1-\phi^2).
\end{aligned}$$

Taking logs and total differentiation lead to

$$\begin{aligned}
d\ln\left(M_{1se}^{\zeta} - \phi M_{2se}^{\zeta}\right) + d\ln\left(M_{1se} + M_{2se}\phi\right) &= d\ln L_1 \\
d\ln\left(M_{2se}^{\zeta} - \phi M_{1se}^{\zeta}\right) + d\ln\left(M_{1se}\phi + M_{2se}\right) &= 0.
\end{aligned}$$

Since

$$
\begin{aligned}
d\ln\left(M_{1se}^{\zeta}-\phi M_{2se}^{\zeta}\right) &= \frac{M_{1se}^{\zeta}}{M_{1se}^{\zeta}-\phi M_{2se}^{\zeta}}\zeta d\ln M_{1se}-\frac{\phi M_{2se}^{\zeta}}{M_{1se}^{\zeta}-\phi M_{2se}^{\zeta}}\zeta d\ln M_{2se} \\
&= \frac{\zeta}{1-\phi}d\ln M_{1se}-\frac{\zeta\phi}{1-\phi}d\ln M_{2se} \\
d\ln\left(M_{1se}+M_{2se}\phi\right) &= \frac{M_{1se}}{M_{1se}+M_{2se}\phi}d\ln M_{1se}+\frac{\phi M_{2se}}{M_{1se}+M_{2se}\phi}d\ln M_{2se} \\
&= \frac{1}{1+\phi}d\ln M_{1se}+\frac{\phi}{1+\phi}d\ln M_{2se} \\
d\ln\left(M_{2se}^{\zeta}-\phi M_{1se}^{\zeta}\right) &= \frac{M_{2se}^{\zeta}}{M_{2se}^{\zeta}-\phi M_{1se}^{\zeta}}\zeta d\ln M_{2se}-\frac{\phi M_{1se}^{\zeta}}{M_{2se}^{\zeta}-\phi M_{1se}^{\zeta}}\zeta d\ln M_{1se} \\
&= \frac{\zeta}{1-\phi}d\ln M_{2se}-\frac{\zeta\phi}{1-\phi}d\ln M_{1se} \\
d\ln\left(M_{1se}\phi+M_{2se}\right) &= \frac{M_{1se}\phi}{M_{1se}\phi+M_{2se}}d\ln M_{1se}+\frac{M_{2se}}{M_{1se}\phi+M_{2se}}d\ln M_{2se} \\
&= \frac{\phi}{1+\phi}d\ln M_{1se}+\frac{1}{1+\phi}d\ln M_{2se},
\end{aligned}
$$

we have

$$
\begin{aligned}
\left(\frac{1}{1+\phi}+\frac{\zeta}{1-\phi}\right)d\ln M_{1se}+\left(\frac{\phi}{1+\phi}-\frac{\zeta\phi}{1-\phi}\right)d\ln M_{2se} &= d\ln L_1 \\
\left(\frac{\phi}{1+\phi}-\frac{\zeta\phi}{1-\phi}\right)d\ln M_{1se}+\left(\frac{1}{1+\phi}+\frac{\zeta}{1-\phi}\right)d\ln M_{2se} &= 0.
\end{aligned}
$$

In a matrix form, this is written as

$$
\begin{pmatrix} (1-\phi)+\zeta(1+\phi) & \phi\{(1-\phi)-\zeta(1+\phi)\} \\ \phi\{(1-\phi)-\zeta(1+\phi)\} & (1-\phi)+\zeta(1+\phi) \end{pmatrix}\begin{pmatrix} d\ln M_{1se} \\ d\ln M_{2se} \end{pmatrix}=\begin{pmatrix} 1-\phi^2 \\ 0 \end{pmatrix}d\ln L_1.
$$

The determinant of the matrix in the left hand side becomes

$$
\begin{aligned}
&\{(1-\phi)-\zeta(1+\phi)\}^2-\phi^2\{(1-\phi)-\zeta(1+\phi)\}^2 \\
=&\left[(1-\phi)-\zeta(1+\phi)-\phi\{(1-\phi)-\zeta(1+\phi)\}\right]\left[(1-\phi)-\zeta(1+\phi)+\phi\{(1-\phi)-\zeta(1+\phi)\}\right] \\
=&\left[(1-\phi)(1+\phi)+\zeta(1+\phi)(1-\phi)\right]\left[(1-\phi)^2+\zeta(1+\phi)^2\right] \\
=&(1-\phi^2)(1+\zeta)\left\{(1-\phi)^2+\zeta(1+\phi)^2\right\}
\end{aligned}
$$

Using Cremer's law, we obtain

$$\begin{aligned}\frac{d\ln M_{1se}}{d\ln L_1} &= \frac{\left(1-\phi^2\right)\left\{(1-\phi)+\zeta\left(1+\phi\right)\right\}}{\left(1-\phi^2\right)\left(1+\zeta\right)\left\{\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2\right\}} \\ &= \frac{\left\{(1-\phi)+\zeta\left(1+\phi\right)\right\}}{\left(1+\zeta\right)\left\{\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2\right\}} \equiv \varepsilon_L\end{aligned}$$

and

$$\begin{aligned}\frac{d\ln M_{2se}}{d\ln L_1} &= -\frac{\left(1-\phi^2\right)\phi\left\{(1-\phi)+\zeta\left(1+\phi\right)\right\}}{\left(1-\phi^2\right)\left(1+\zeta\right)\left\{\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2\right\}} \\ &= -\frac{\phi\left\{(1-\phi)+\zeta\left(1+\phi\right)\right\}}{\left(1+\zeta\right)\left\{\left(1-\phi\right)^2+\zeta\left(1+\phi\right)^2\right\}} = -\phi\varepsilon_L.\end{aligned}$$

From (13), it holds

$$\frac{R_{12A}}{R_{21A}} = \frac{L_2}{L_1}\left(\frac{M_{1Ae}}{M_{2Ae}}\right)\left(\frac{M_{1Ae}+M_{2Ae}\phi}{M_{1Ae}\phi+M_{2Ae}}\right).$$

Taking logs and differentiating this, we obtain

$$d\ln\frac{R_{12A}}{R_{21A}} = -d\ln L_1 + d\ln\left(\frac{M_{1Ae}}{M_{2Ae}}\right) + d\ln\left(M_{1Ae}+M_{2Ae}\phi\right) + d\ln\left(\phi M_{1Ae}+M_{2Ae}\right).$$

Using the following relationships

$$
\begin{aligned}
d\ln\left(\frac{M_{1Ae}}{M_{2Ae}}\right) &= d\ln M_{1Ae} - d\ln M_{2Ae} = \varepsilon_L\left(1+\phi\right)d\ln L_1, \\
d\ln\left(M_{1Ae}+M_{2Ae}\phi\right) &= \frac{M_{1Ae}}{M_{1Ae}+M_{2Ae}\phi}d\ln M_{1Ae} + \frac{M_{2Ae}\phi}{M_{1Ae}+M_{2Ae}\phi}d\ln M_{2Ae} \\
&= \frac{1}{1+\phi}d\ln M_{1Ae} + \frac{\phi}{1+\phi}d\ln M_{2Ae} \\
&= \frac{\varepsilon_L}{1+\phi}d\ln L_1 - \frac{\phi^2\varepsilon_L}{1+\phi}d\ln L_1 \\
&= \frac{\varepsilon_L\left(1-\phi^2\right)}{1+\phi}d\ln L_1 \\
&= \varepsilon_L\left(1-\phi\right)d\ln L_1, \text{ and} \\
d\ln\left(\phi M_{1Ae}+M_{2Ae}\right) &= \frac{\phi M_{1Ae}}{\phi M_{1Ae}+M_{2Ae}}d\ln M_{1Ae} + \frac{M_{2Ae}}{\phi M_{1Ae}+M_{2Ae}}d\ln M_{2Ae} \\
&= \frac{\phi}{1+\phi}d\ln M_{1Ae} + \frac{1}{1+\phi}d\ln M_{2Ae} \\
&= \frac{\phi}{1+\phi}\varepsilon_L d\ln L_1 - \frac{\phi}{1+\phi}\varepsilon_L d\ln L_1 \\
&= 0,
\end{aligned}
$$

we obtain

$$
\begin{aligned}
\frac{d\ln\left(R_{12A}/R_{21A}\right)}{d\ln L_1} &= -1 + \varepsilon_L\left(1+\phi\right) + \varepsilon_L\left(1-\phi\right) \\
&= -1 + 2\varepsilon_L.
\end{aligned}
$$

Since $R_{12A}/R_{21A} = 1$ initially holds, the model predicts the Home market effect if and only if $\frac{d\ln(R_{12A}/R_{21A})}{d\ln L_1} > 0$.

From the definition of ε_L, $2\varepsilon_L - 1 > 0$ holds if and only if

$$
\begin{aligned}
2\left\{(1-\phi)+\zeta(1+\phi)\right\} &> (1+\zeta)\left\{(1-\phi)^2+\zeta(1+\phi)^2\right\} \\
(1-\phi)\left\{2-(1+\zeta)(1-\phi)\right\}+\zeta(1+\phi)\left\{2-(1+\zeta)(1+\phi)\right\} &> 0 \\
(1-\phi)\left\{2-(1-\phi)-\zeta(1-\phi)\right\}+\zeta(1+\phi)\left\{2-(1+\phi)-\zeta(1+\phi)\right\} &> 0 \\
(1-\phi)(1+\phi)-\zeta(1-\phi)^2+\zeta(1+\phi)(1-\phi)-\zeta^2(1+\phi)^2 &> 0 \\
(1-\phi)(1+\phi)-\zeta(1-\phi)(1-\phi-1-\phi)-\zeta^2(1+\phi)^2 &> 0 \\
(1-\phi)(1+\phi)+2\phi(1-\phi)\zeta-\zeta^2(1+\phi)^2 &> 0 \\
(1+\phi)^2\zeta^2-2\phi(1-\phi)\zeta-(1-\phi)(1+\phi) &< 0.
\end{aligned}
$$

Define $\Gamma(\zeta) \equiv (1+\phi)^2\zeta^2 - 2\phi(1-\phi)\zeta - (1-\phi)(1+\phi)$. Then, $\frac{d\ln(R_{12A}/R_{21A})}{d\ln L_1} > 0$ if and only if $\Gamma(\zeta) < 0$. Notice that $\Gamma(\zeta) = 0$ has the following solutions

$$
\begin{aligned}
\zeta &= \frac{\phi(1-\phi)}{(1+\phi)^2} \pm \sqrt{\frac{\phi^2(1-\phi)^2+(1-\phi)(1+\phi)^3}{(1+\phi)^4}} \\
&= \frac{\phi(1-\phi)}{(1+\phi)^2} \pm \sqrt{\left(\frac{\phi(1-\phi)}{(1+\phi)^2}\right)^2+\frac{1-\phi}{1+\phi}} \\
&= \zeta_1 \pm \sqrt{\zeta_1^2+\frac{1-\phi}{1+\phi}}.
\end{aligned}
$$

Since $\zeta \geq 0$, $\Gamma(\zeta) < 0$ if and only if $\zeta < \zeta_H \equiv \zeta_1 + \sqrt{\zeta_1^2 + \frac{1-\phi}{1+\phi}}$. Therefore, the model predicts the Home Market effect if and only if $\zeta < \zeta_H$. Since $\zeta_H > \zeta_1$, the model predicts the Home Market effect and the Trefler finding if $\zeta_1 < \zeta < \zeta_H$.