Export-Learning and FDI with Heterogeneous Firms

Amanda Jakobsson[*] Paul S. Segerstrom[†]

Singapore Management University Stockholm School of Economics

May 19, 2016

Abstract

This paper evaluates the impact of stronger intellectual property rights protection for multinational production (MP) and developing countries. We calibrate a North-South trade model with heterogeneous firms to match the world economy before and after the TRIPS agreement went into effect. The model can account for the ten-fold increase in foreign direct investment going to developing countries during the time period 1990-2005. We find that stronger intellectual property rights (TRIPS) lead to more innovation in the North, more MP, more technology transfer to the South and considerably higher southern consumer welfare. In contrast, the trade liberalization that occurred actually lowered long-run southern consumer welfare by diverting resources away from innovative activities.

Keywords: Multinational Firms, Heterogeneous Firms, North-South Trade, Intellectual Property Rights, Foreign Direct Investment, Product Cycles, Economic Growth.

JEL Classification: F12, F23, F43, O31, O34.

[*]School of Economics, Singapore Management University, 90 Stamford Road, Singapore 178903. Tel: +65 6808 5467. Email: ajakobsson@smu.edu.sg

[†]Department of Economics, Stockholm School of Economics, Box 6501, SE-11383 Stockholm, Sweden. Tel: +46 8 736 9203. Email: paul.segerstrom@hhs.se

1 Introduction

Multinational production (MP), defined here as production done by affiliates outside of the country of origin of the parent firm, has become a central feature of economic globalization. In the time period between 1990 and 2005, there was ten-fold increase in foreign direct investment (FDI) going to developing countries (UNCTAD, 2011). In 1990, foreign affiliates' share of world GPD (value-added) was 4.6 per cent but by 2005, this share had risen to 10 per cent of world GDP (UNCTAD, 2012).

Motivated by this huge increase in FDI going to developing countries, we turn to an aspect of MP that has been a topic of debate for many years: intellectual property rights (IPR) protection in developing countries. The Trade-Related Aspects of Intellectual Property Rights (TRIPS) agreement was signed as part of the Uruguay Round in 1994. This agreement formally introduced intellectual property rights into the World Trade Organization (WTO) and the world trading system. The TRIPS agreement covers copyrights and patents but also enforcement procedures and dispute mechanisms. Since most developed countries already had such systems in place, the implied changes in national regulation required by the TRIPS agreement mostly affects developing countries. They have been forced to increase their IPR protection to remain inside the WTO.

The TRIPS agreement has come in for intense criticism. As Irwin (2009, p.231) explains, "Many developing countries complain that, unlike mutually beneficial tariff reductions, the TRIPS agreement merely transfers income from developing to developed countries by strengthening the ability of multinational corporations to charge higher prices in poorer countries." In his book *In Defense of Globalization*, Bhagwati (2004, p.183) describes TRIPS as "like the introduction of cancer cells into a healthy body." For this influential economist, the otherwise healthy body is the World Trade Organization and TRIPS is killing it. Birdsall, Rodrik and Subramanian (2005) concur. They write "An international community that presides over TRIPS and similar agreements forfeits any claim to being development-friendly. This must change: the rich countries cannot just amend TRIPS; they must abolish it altogether."[1]

Turning to the economics literature, perhaps the best support for this critique is provided by McCalman (2001), who estimates the value of transfers of income between countries implied by the TRIPS agreement. He finds that only a few countries gained from TRIPS (United States, Germany, France, Italy, Sweden, Switzerland) and that all other countries were made worse off, including all developing countries. But it is just assumed in McCalman's cost-benefit analysis that there are no dynamic benefits from TRIPS. Recently, evidence has emerged indicating that, not only are there dynamic benefits from TRIPS, but these dynamic benefits take more forms than economists had previously realized. For example, Branstetter, Fisman, Foley and Saggi (2011) study the response of host country industrial production to stronger IPR protection. They find that following patent reform, not only did US-based multinational firms expand their activities in reforming countries, but this lead to exports of new goods increasing in these reforming countries.

This paper evaluates the impact of the TRIPS agreement for developing countries in light of MP. We calibrate a North-South trade model with heterogeneous firms to match the world economy before and after the TRIPS agreement went into effect. Firm heterogeneity plays a central role in our analysis and we study how high productivity firms behave differently from low productivity firms. Consistent with the empirical literature, the model implies that only a small share of firms export and an even smaller share of firms are multinationals.[2] Most importantly, the model can account for the ten-fold increase in foreign direct investment (FDI) going to developing countries during the time period 1990-2005 (UNCTAD, 2014). This is the first paper to study intellectual property rights within the context of a heterogeneous firms trade model that can account for the huge increase in FDI going to developing countries.

[1] A recent *New York Times* op-ed provides another example of opposition to stronger IPRs. Krugman (2014) writes, "Basically, old-fashioned trade deals are victim of their own success: there just isn't much more protectionism to eliminate. Average U.S. tariff rates have fallen by two-thirds since 1960 . . . these days, "trade agreements" are mainly about other things. What they're really about, in particular, is property rights – things like the ability to enforce patents on drugs and copyrights on movies . . . Is this a good thing from a global point of view? Doubtful. The kind of property rights we're talking about here can alternatively be described as legal monopolies. True, temporary monopolies are, in fact, how we reward new ideas; but arguing that we need even more monopolization is very dubious . . . and has nothing at all to do with classical arguments for free trade."

[2] For evidence, see Bernard, Eaton, Jensen and Kortum (2003). Even though only a small share of firms are multinationals, they account for 70 percent of world trade.

In the model, firms in the North (developed countries) engage in innovative research and development (R&D) to develop new product varieties. Upon successful innovation, a northern firm starts to produce in the North (serving the home market) and learns if it is a low or high productivity firm.[3] Firms in the North can engage in export-learning R&D to access the southern market and earn higher profits from selling in both markets. The export-learning costs are of a similar nature to the fixed export costs in Arkolakis (2010), where firms need to pay a fixed cost for marketing (or setting up a distribution network) to enter into each export market. Northern exporting firms can then choose to engage in adaptive R&D (FDI) to learn how to produce their products via MP in the lower-wage South (developing countries), and once successful, their foreign affiliates located in the South earn even higher global monopoly profits. Our assumption that MP follows exporting is motivated by the recent evidence in Conconi, Sapir and Zanardi (2016). Looking at all Belgian manufacturing firms that started to engage in FDI during 1998-2008, they find that almost 90 per cent of these firms were already serving the foreign market via exports. This suggests to us that learning how to export is a stepping stone to MP.[4] Once any foreign affiliate starts producing in the South, it faces the risk of imitation from southern firms. If imitation occurs, then the product market becomes perfectly competitive and the foreign affiliate no longer earns any profits. Stronger IPR protection in the South (TRIPS) is modelled as a decrease in this imitation rate.[5]

We calibrate the model to fit two benchmark cases: the 1990 benchmark (the world prior to the implementation of the TRIPS agreement) and the 2005 benchmark (the world after the implementation of the TRIPS agreement). In both benchmark equilibria, we find that the export-learning rate is higher for higher productivity firms than for low productivity firms,

[3]This feature of the model is of course inspired by the seminal paper by Melitz (2003) about trade with heterogeneous firms, where firms develop new product varieties and then learn their productivities. One problem with the Melitz model is that its implication for the effect of unilateral trade liberalization on industrial productivity is the exact opposite of what researchers like Trefler (2004) find empirically. This problem is discussed in Segerstrom and Sugita (2015) and a solution is suggested in Segerstrom and Sugita (2016).

[4]In Helpman, Melitz and Yeaple (2004), firms are heterogeneous in productivity and face fixed costs for selling domestically, for entering a foreign market via exports, and for entering a foreign market via FDI. The fixed costs of FDI are higher than the fixed costs for exporting and all firms with productivity above a threshold level engage in FDI. Firms with productivity below this threshold level but above another lower threshold level decide to export instead. The decision to enter the foreign market via exports or FDI is a one-time decision.

[5]In Jakobsson and Segerstrom (2016), we study the impact of TRIPS using a model where imitation is costly and the imitation rate depends on the decisions of profit-maximizing firms, but the results are similar.

and the FDI rate is higher for high productivity firms than for low productivity firms. Because of these differences, northern exporting firms are more productive on average than non-exporting firms and multinational firms are even more productive on average than northern exporting firms.[6] Going from the 1990 to the 2005 benchmark, we find that TRIPS lead to more FDI, more production taking place in foreign affiliates (more MP), more innovation and considerably higher long-run southern consumer welfare.[7] In contrast, the trade liberalization that occurred lead to more export-learning and actually lowered long-run southern consumer welfare by diverting northern resources away from innovative activities (to production for export).

This paper is related to the MP literature of trade models with heterogenous firms that study the interaction of trade and MP flows to quantify the gains from openness. In their quantitative application of Helpman, Melitz, Yeaple (2004), Irarrazabal, Moxnes and Opromolla (2013) study the role of geography and trade costs for intra-firm trade, but without innovation and with wages fixed by assumption. Tintelnot (2015) also has a monopolistic competition framework, but assumes that each firm produces a continuum of goods as in Eaton and Kortum (2002) such that the firm consists of a continuum of products with production-location-specific productivity shocks. His general equilibrium setting incorporates export-platform FDI but, as in Irarrazabal et al (2013), firm entry is exogenous so there is no innovation. Ramondo and Rodriguez-Clare (2013) introduce MP in an Eaton and Kortum (2002) Ricardian framework to study the substitutability and complimentarity between trade and MP. Also in their perfect competition framework it is not possible to study innovation. Arkolakis et al (2014) do model innovation as creating heterogenous firms selling differentiated goods in markets characterized by monopolistic competition à la Melitz (2003). In their model, comparative advantage and home market effects coming from increasing re-

[6]For evidence about the productivity differences between non-exporting, exporting and multinational firms, see Bernard, Eaton, Jensen and Kortum (2003), Bernard and Jensen (2004), Mayer and Ottaviano (2008), and Lileeva and Trefler (2010).

[7]Studying a slightly different time period 1999-2009, Arkolakis, Ramondo, Rodriguez-Claire and Yeaple (2014) report that R&D expenditures in the U.S. relative to local manufacturing value-added grew from 8.7% to 12.7%, and U.S. firms increased the share of their total global employment that is located in their foreign affiliates from 22% to 31%.

turns to innovation and geographical frictions determine the location of innovation activities and production activities across countries. Our paper departs from this literature in two ways. First, in our dynamic setting we can look at the dynamic gains from trade and MP arising from speeding up or slowing down the international product cycle. The dynamic setting also allows us to incorporate recent evidence that learning how to export is a stepping-stone to MP. Second, we are able to analyze the effect that IPR protection has for the location of MP and the resulting implications for home- and host-country welfare.

This paper is also related to the large literature on IPR protection in developing countries. Early models of North-South trade and IPR protection by Chin and Grossman (1990) and Deardorff (1992) do not have FDI and no international technology transfer takes place within multinational firms. Models with costless FDI have been developed by Helpman (1993), Lai (1998), Glass and Wu (2007), Branstetter and Saggi (2011), and He and Maskus (2012). Glass and Saggi (2002) present a North-South trade model with costly FDI but their results are not robust to allowing for decreasing returns to R&D. This is shown in Gustafsson and Segerstrom (2011), where a North-South trade model with costly FDI and decreasing returns to R&D is developed. A version of this model is calibrated in Jakobsson and Segerstrom (2016) to match the world economy before and after the TRIPS agreement went into effect. Unlike in previous papers where firms are homogeneous and all firms export, in this paper we take seriously the evidence that firm-level productivity differences are important and study the impact of the TRIPS agreement in a setting where firms differ in their productivities and most firms do not export.

The rest of the paper is organized as follows. In Section 2, we present the model and derive eight steady-state equilibrium conditions. In Section 3, we solve the model numerically for different parameter values and present the results. Then in Section 4 we offer some concluding remarks. There is an Appendix where we present calculations that we did to solve the model in more detail and present results from solving the model for alternative parameter values.

2 The Model

2.1 Overview

Consider a global economy with two regions, the North and the South. Labor is the only factor of production. It is used to manufacture product varieties, develop new product varieties (innovation), adapt existing product varieties for entry into the foreign market (export-learning) and adapt exported varieties for production in the South (FDI or MP-learning). Labor is perfectly mobile across activities within a region, but cannot move across regions. Since labor markets are perfectly competitive, there is one single wage rate paid to all northern workers w^N and one single wage rate paid to all southern workers w^S. Although labor cannot move across regions, goods can. International trade between the North and the South is subject to iceberg trade costs: $\tau > 1$ units of a good must be shipped for one unit to arrive at its destination.

Only firms in the North, *northern* firms, have the capacity to innovate. A northern firm can hire workers to engage in innovative R&D with the purpose of developing the blueprint for a new product variety. After successful innovation, the firm earns monopoly profits from selling to the domestic market (the North) and learns if it is a low or high productivity firm. When the northern firm makes the decision of how much labor to hire for innovation, the firm does not know its own productivity in manufacturing, and there is therefore uncertainty about its expected profit flow. With probability $q_L = q$, the northern firm will be a low productivity firm with unit labor requirement c_L and with probability $q_H = 1 - q$, the northern firm will be a high productivity firm with unit labor requirement c_H, where $c_H < c_L$. Even though firms are heterogeneous in their productivities, high and low productivity firms face the same labor requirement for R&D.

After learning its productivity, a northern firm can hire southern workers to engage in export-learning R&D to access the southern market. Such R&D costs can be thought of as marketing, setting up distribution networks and learning how to comply with regulations in the foreign market. Upon successful export-learning, the firm earns higher monopoly profits since it earns profits from selling in both markets (the North and the South). Such a firm is called an *exporter*.

A northern exporter can then choose to hire southern workers to engage in MP-learning R&D (or FDI) to learn how to produce in the South.[8] When successful in MP-learning R&D, a firm earns higher global monopoly profits because the cost of production is lower in the South. Such a firm is called a *foreign affiliate* since, even though all production takes place in the South, a fraction of its profits is repatriated back to its stockholders in the North in the form of royalty payments for the right to use the blueprint of the particular product variety. MP-learning R&D is the cost that firms incur when they learn how to do MP and can therefore be interpreted as an index of FDI.

R&D done in the South (export-learning R&D and MP-learning R&D) is financed by southern savings but northern firms control the amount of R&D in order to maximize their global expected discounted profits. Upon successfully adapting production to the South, a foreign affiliate sells to the southern market and also exports back to the North without incurring any additional export-learning costs. Foreign affiliates are exposed to a positive rate of imitation from *southern* firms. Once a product variety has been imitated, the blueprint becomes available to all southern firms, the product market becomes perfectly competitive and the foreign affiliate no longer earns any profits.

As illustrated in Figure 1, the model generates one-way product cycles à la Vernon (1966). The number of varieties in the economy grows at the rate g as a result of the innovative R&D activities of northern firms. Each product variety is initially produced by a northern firm that sells to its home market. It is at this point that the northern firm learns its own productivity. With probability q_z, the firm draws the productivity $z = H, L$. The firm can then engage in export-learning R&D with the aim of exporting to the southern market. Export-learning occurs at the rate χ_z. After the firm has become an exporter, it can engage in MP-learning R&D with the aim of producing in the lower-wage South. Such international technology transfer occurs at the FDI rate ϕ_z. Each foreign affiliate is then exposed to the positive rate

[8]We will only solve for equilibria where $w^N > w^S$, since lower production costs in the South creates the incentive for FDI in the model.

of imitation ι_S from southern firms, resulting in southern firms producing the product variety for the entire world market.

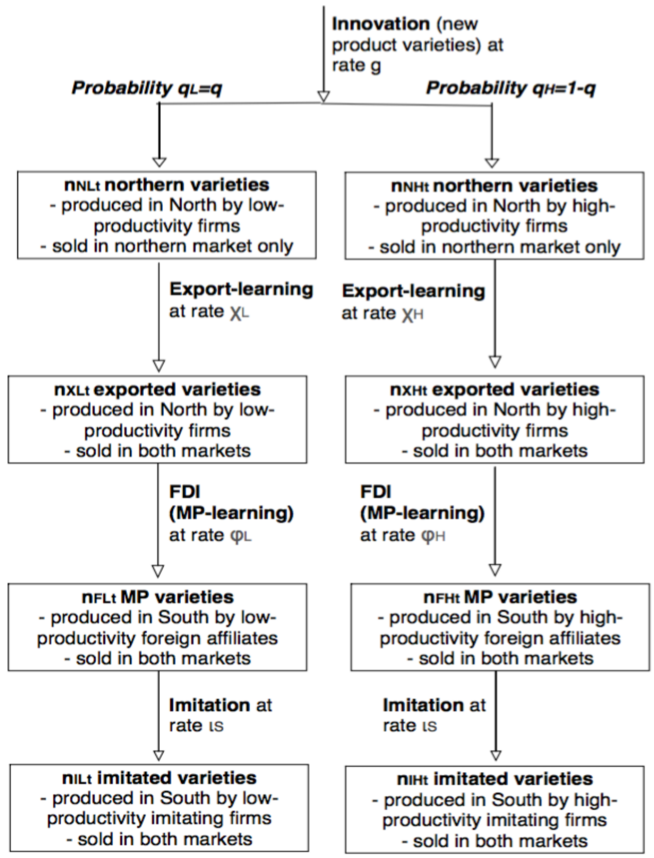

Figure 1: One-way Product Cycles

2.2 Households

In both the North and the South, there is a fixed measure of households that provide labor services in exchange for wage payments. Each individual member of a household lives forever and is endowed with one unit of labor, which is inelastically supplied. The size of each household, measured by the number of its members, grows exponentially at a fixed rate g_L, the population growth rate. Let $L_t^N = L_0^N e^{g_L t}$ denote the supply of labor in the North at time t, let $L_t^S = L_0^S e^{g_L t}$ denote the corresponding supply of labor in the South, and let

$L_t = L_t^N + L_t^S$ denote the world supply of labor. In addition to wage income, households also receive asset income from their ownership of firms.

Households in both the North and the South share identical preferences. Each household is modeled as a dynastic family that maximizes discounted lifetime utility

$$U = \int_0^\infty e^{-(\rho - g_L)t} \ln(u_t) dt \tag{1}$$

where $\rho > g_L$ is the subjective discount rate and u_t is the static utility of an individual at time t. The static constant elasticity of substitution (CES) utility function is

$$u_t = \left[\int_0^{n_t} x_t(\omega)^\alpha d\omega \right]^{\frac{1}{\alpha}}, \qquad 0 < \alpha < 1. \tag{2}$$

In (**??**), $x_t(\omega)$ is the per capita quantity demanded of the product variety ω at time t and n_t is the total number of invented varieties at time t. We assume that varieties are gross substitutes. Then with α measuring the degree of product differentiation, the elasticity of substitution between different product varieties is $\sigma \equiv 1/(1-\alpha) > 1$.

Solving the static consumer optimization problem yields the familiar demand function

$$x_t(\omega) = \frac{p_t(\omega)^{-\sigma} e_t}{P_t^{1-\sigma}} \tag{3}$$

where e_t is individual consumer expenditure at time t, $p_t(\omega)$ is the price of variety ω at time t, and $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ is an index of consumer prices. We will shortly define one such price index for each region. By substituting the demand function (**??**) into (**??**) and using the definition of the price index P_t, it can be shown that $u_t = e_t/P_t$. Then maximizing (**??**) subject to the relevant intertemporal budget constraint yields the intertemporal optimization condition

$$\frac{\dot{e}_t}{e_t} = r_t - \rho \tag{4}$$

implying that individual consumer expenditure only grows over time if the market interest rate r_t is larger than the subjective discount rate ρ.

The representative consumer in each region has different wage income ($w^N > w^S$) and hence different consumer expenditure. Let e_t^N and e_t^S denote the representative consumer's expenditure in the North and the South, respectively. We treat the southern wage rate as the numeraire price ($w^S = 1$) so all prices are measured relative to the price of southern labor. We solve the model for a steady-state equilibrium where wages w^N, w^S and consumer expenditure e^N, e^S are all constant over time. Then $\dot{e}_t/e_t = 0$ in (**??**) and $r_t = \rho$. The steady-state market interest rate is thus constant over time and equal in the two regions.

For each level of productivity $z = H, L$, there are four types of firms indexed by $j = N, X, F, I$. There are northern firms that only sell to the home market ("N" for "northern"), exporters who serve both markets ("X" for "export"), foreign affiliates that produce in the South ("F" for "FDI") and southern firms that have imitated foreign affiliates ("I" for "imitation"). Let n_{jzt} denote the number of product varieties produced by type j firms with productivity z at time t. Due to the positive trade costs, the prices of products will also differ between the two regions $r = N, S$. Let p_{jz}^r denote the price charged to consumers in region r by firms of type j with productivity z. In steady-state equilibrium, all product prices are constant over time.

2.3 Steady-State Dynamics

Let $g \equiv \dot{n}_t/n_t$ denote the steady-state growth rate of the number of varieties. From the variety condition $n_t = \sum_j \sum_z n_{jzt}$, it follows that the number of varieties produced by each type of firm must grow at the same rate $g = \dot{n}_{jzt}/n_{jzt}$. Therefore the variety shares $\gamma_{jz} \equiv n_{jzt}/n_t$ are necessarily constant over time in any steady-state equilibrium and satisfy $\sum_j \sum_z \gamma_{jz} = 1$.

Let $\chi_z \equiv (\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt})/n_{Nzt}$ denote the steady-state export-learning rate, which is constant over time since $\chi_z = (g/\gamma_{Nz})(\gamma_{Xz} + \gamma_{Fz} + \gamma_{Iz})$. In this definition, we take into account that some of the exported varieties are adapted for production by foreign affiliates, and in turn, some of these foreign affiliate varieties are imitated by southern firms. Let $\phi_z \equiv (\dot{n}_{Fzt} + \dot{n}_{Izt})/n_{Xzt}$ denote the steady-state FDI rate, which is constant over time since $\phi_z = (g/\gamma_{Xz})(\gamma_{Fz} + \gamma_{Iz})$. In the definition of the FDI rate, we take into account that moving production to a foreign affiliate exposes the firm to a positive rate of imitation by southern firms. Let $\iota_S \equiv \dot{n}_{Izt}/n_{Fzt}$ denote the imitation rate of foreign affiliate-produced varieties. It is constant over time in steady-state equilibrium since $\iota_S = g\left(\gamma_{Iz}/\gamma_{Fz}\right)$.

By the law of large numbers, $\sum_j \gamma_{jz} = q_z$. From the variety condition $n_t = \sum_j \sum_z n_{jzt}$, it follows that a share $q_L = q$ of total varieties consists of low productivity varieties and the remaining share $q_H = 1 - q$ consists of high productivity varieties. Taking the time derivative of $q_z n_t = \sum_j n_{jzt}$, it is straightforward to show that the steady-state variety shares are

$$\gamma_{Nz} = q_z \frac{g}{g + \chi_z} \tag{5}$$

$$\gamma_{Xz} = q_z \frac{\chi_z}{g + \chi_z} \frac{g}{g + \phi_z} \tag{6}$$

$$\gamma_{Fz} = q_z \frac{\chi_z}{g + \chi_z} \frac{\phi_z}{g + \phi_z} \frac{g}{g + \iota_S} \tag{7}$$

and

$$\gamma_{Iz} = q_z \frac{\chi_z}{g + \chi_z} \frac{\phi_z}{g + \phi_z} \frac{\iota_S}{g + \iota_S}. \tag{8}$$

As expected, faster export-learning rates for northern firms correspond to larger shares of world production being done by northern exporters, more exporters learning how to become multinationals and more varieties being imitated ($\chi_z \uparrow \Longrightarrow \gamma_{Xz} \uparrow, \gamma_{Fz} \uparrow, \gamma_{Iz} \uparrow$). Faster MP-learning rates correspond to smaller shares of world production being done by northern exporters, larger shares being produced by foreign affiliates, and larger shares being produced by southern firms ($\phi_z \uparrow \Longrightarrow \gamma_{Xz} \downarrow, \gamma_{Fz} \uparrow, \gamma_{Iz} \uparrow$). And as expected, a faster imitation rate corresponds to larger shares being produced by southern firms and smaller shares by foreign affiliates ($\iota_S \uparrow \Longrightarrow \gamma_{Iz} \uparrow, \gamma_{Fz} \downarrow$).

The price index in the North will be different than the price index in the South for two reasons. First, products prices differ across regions because of trade costs τ. Second, the set of product varieties available in the northern market is larger than the set of product varieties available in the southern market, since some northern product varieties are only sold domestically. Let P_t^r denote the price index for rgion r. Given the definition of the price index $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ it follows that the northern price index satisfies $\left(P_t^N\right)^{1-\sigma} = \sum_j \sum_z \left[n_{jzt} \left(p_{jz}^N\right)^{1-\sigma}\right]$ and the southern price index satisfies $\left(P_t^S\right)^{1-\sigma} = \sum_{j \neq N} \sum_z \left[n_{jzt} \left(p_{jz}^S\right)^{1-\sigma}\right]$. Using the variety shares defined earlier, we can rewrite these expressions as

$$\left(P_t^N\right)^{1-\sigma} = \sum_{j=N,X,F,I} \sum_{z=H,L} \left[\gamma_{jz} \left(p_{jz}^N\right)^{1-\sigma}\right] n_t \tag{9}$$

$$\left(P_t^S\right)^{1-\sigma} = \sum_{j=X,F,I} \sum_{z=H,L} \left[\gamma_{jz} \left(p_{jz}^S\right)^{1-\sigma}\right] n_t \tag{10}$$

where the terms in brackets are constant over time. Thus, $\left(P_t^N\right)^{1-\sigma}$ and $\left(P_t^S\right)^{1-\sigma}$ both grow over time at the rate g in any steady-state equilibrium.

2.4 Product Markets

The firms producing different product varieties compete in prices and maximize profits. There are constant returns to scale in production. A northern firm that is not an exporter and only sells to its home market has the marginal cost $c_z w^N$, a northern exporting firm has the marginal cost $c_z w^N$ when selling to the home market and $\tau c_z w^N$ when selling to the export market. A foreign affiliate in the South and a local southern firms on the other hand have the marginal cost $c_z w^S$ when serving the home market (South) and $\tau c_z w^S$ when serving the export market (North).

A northern firm with earns the (domestic) profit flow $\pi_{Nzt} = \left(p_{Nz}^N - c_z w_N\right) x_{Nzt}^N L_t^N$, where x_{jz}^r is the quantity demanded by the typical consumer in region r of the type j firm's product, produced with productivity z. A northern firm chooses its price to maximize profits, and it is straightforward to verify that the profit-maximizing price is the monopoly price $p_{Nz}^N = c_z w^N / \alpha$. A low productivity northern firm has a higher marginal cost than a high productivity northern firm so the price of a low productivity firm's product variety will be higher. Using these prices, we can write the northern firm's profit flow as

$$\pi_{Nzt} = \left[\frac{c_z w^N X^N_{Nz}}{(\sigma - 1)\,\gamma_{Nz}} \right] \frac{L_t}{n_t} \qquad (z = H, L) \tag{11}$$

where $X^r_{jz} \equiv \frac{\left(p^r_{jz}\right)^{-\sigma} e^r L^r_t n_{jzt}}{\left(P^r_t\right)^{1-\sigma} L_t}$ is the population-adjusted aggregate demand term of type j firm's product sold in market r. X^r_{jz} is constant over time in steady-state equilibrium since prices and consumer expenditure are constant over time, L^r_t grows at the same rate g_L as the world population L_t, and $\left(P^r_t\right)^{1-\sigma}$ grow at the same rate g as n_{jzt}. In (**??**), the marginal cost terms c_z and the elasticity of substitution σ are parameters, while the wage rate w^N and the variety share γ_{Nz} are constant over time in steady-state equilibrium. Therefore, profits earned by a northern firm only change because L_t/n_t changes over time. L_t/n_t is a measure of the size the market relevant for each northern firm. Population growth increases the size of the market for firms but variety growth has the opposite effect because firms have to share consumer demand with more competing firms.

A northern firm that has learned how to export to the South earns the global profit flow $\pi_{Xzt} = \left(p^N_{Xz} - c_z w^N\right) x^N_{Xzt} L^N_t + \left(p^S_{Xz} - \tau c_z w^N\right) x^S_{Xzt} L^S_t$. The exporter's profit-maximizing price in the home market is $p^N_{Xz} = c_z w^N/\alpha$ and in the export market $p^S_{Xz} = \tau c_z w^N/\alpha$. Using these prices, the global profit flow of a northern exporter can be written as

$$\pi_{Xzt} = \frac{c_z w^N}{(\sigma - 1)} \frac{\left(X^N_{Xz} + \tau X^S_{Xz}\right)}{\gamma_{Xz}} \frac{L_t}{n_t}, \qquad (z = H, L). \tag{12}$$

The global profit flow for a foreign affiliate is $\pi_{Fzt} = \left(p^S_{Fz} - c_z w^S\right) x^S_{Fzt} L^S_t + \left(p^N_{Fz} - \tau c_z w^S\right) x^N_{Fzt} L^N_t$. Profit-maximizing monopoly prices are $p^S_{Fz} = c_z w^S/\alpha$ in the domestic market (the South) and $p^N_{Fz} = \tau c_z w^S/\alpha$ in the export market (the North). The incentive for an exporter to become a multinational firm and move production to the South is not primarily market access, but to earn higher profits by lowering production cost. Therefore we will solve for equilibria where the inequality condition $w^N > \tau w^S$ holds so each foreign affiliate exports back to the North and the parent firm in the North ceases to produce there.[9]

[9] In Helpman et al (2004), firms choose to enter into the foreign market either through exporting or through

Using these prices, the global profit flow for a foreign affiliate can be written as

$$\pi_{Fzt} = \frac{c_z w^S}{(\sigma - 1)} \frac{\left(X_{Fz}^S + \tau X_{Fz}^N\right)}{\gamma_{Fz}} \frac{L_t}{n_t}, \qquad (z = H, L). \tag{13}$$

Once imitation has occurred, the blueprint is freely available to all southern firms. Southern firms do not incur any imitation costs. A southern firm that imitates a firm of high productivity becomes a high productivity southern firm and vice versa. Imitation involves learning the production technology for the variety as well as the ability to sell the product variety in all markets. After successful imitation, southern imitators do not incur any export-learning costs to introduce their product to the northern market.[10] No southern firm can set its price higher than marginal cost, and all southern firms earn zero profits. The resulting prices are $p_{Iz}^S = c_z w^S$ and $p_{Iz}^N = \tau c_z w_S$.

The above analysis implies that as a product shifts from being produced by a northern firm (an exporter) to its foreign affiliate and then by a southern firm, the equilibrium price of the product declines in the North ($p_{Nz}^N = p_{Xz}^N = c_z w^N/\alpha > p_{Fz}^N = \tau c_z w_S/\alpha > p_{Iz}^N = \tau c_z w^S$) as well as in the South ($p_{Xz}^S = \tau c_z w^N/\alpha > p_{Fz}^S = c_z w^S/\alpha > p_{Iz}^S = c_z w^S$). This price pattern is consistent with Vernon's (1966) description of the product life cycle, in which multinational firms play a central role.

FDI. Market access is driving (horizontal) FDI in their model since a multinational firm continues to serve the parent firm's market via production at home. The assumption that exporters always keep serving the domestic market in our model is the same as in Helpman et al (2004). However, they assume that firms that engage in FDI serve the foreign market through the foreign affiliate but do not export back to the host country. This assumption is relaxed in the working paper version of their paper where they allow for export platform FDI. We assume that once a firm has successfully adapted production to a foreign affiliate, the parent firm no longer produces the variety in the domestic market and instead serves both markets via the foreign affiliate.

[10] Intuitively, the particular product variety has already been introduced to the northern market by the northern firm whose blueprint the imitator is using. It is possible to consider an alternative setting where the imitator can only sell the product in the South due to IPR protection in the northern market, or that only a small share of southern imitators export due to export-learning costs. If southern firms only take over the southern market and a multinational firm producing in the South only looses the southern market upon imitation, the loss from imitation would be less severe for the multinational. In the results section, we show how the relative purchasing power of the South increases when there is more MP. Over time, the loss from losing only the southern market would therefore be more noticeable. For most manufacturing products except perhaps pharmaceuticals, we consider the assumption of imitators producing at a lower cost and taking over the entire world market the most plausible.

2.5 Technology for Innovation, FDI and Export-Learning

There is free entry into innovative R&D activities in the North, with every northern firm having access to the same R&D technology. To innovate and develop a new product variety, a representative northern firm i must devote $a_N g^\beta / n_t^\theta$ units of labor to innovative R&D, where a_N is an innovative R&D productivity parameter, n_t is the disembodied stock of knowledge at time t and θ is an intertemporal knowledge spillover parameter.[11] The parameter $\beta > 0$ captures decreasing returns to R&D at the industry level. When there is more innovation in the economy ($g \equiv \dot{n}_t / n_t$ is higher), each individual northern firm must devote more resources to innovation in order to successfully develop one new product variety. A large empirical literature on patents and R&D has shown that R&D is subject to significant decreasing returns at the industry level [point estimates of $1/(1+\beta)$ lie between 0.1 and 0.6 according to Kortum(1993)].[12] Given this technology, the flow of new products developed by northern firm i is

$$\dot{n}_t^i = \frac{l_{Rt}^i}{a_N g^\beta / n_t^\theta} = \frac{n_t^\theta l_{Rt}^i}{a_N g^\beta}, \tag{14}$$

where l_{Rt}^i is the northern labor employed by firm i in innovative R&D. Aggregating over all northern firms, the aggregate flow of new products developed in the North is

$$\dot{n}_t = \frac{n_t^\theta L_{Rt}}{a_N g^\beta} = \left[\frac{n_t^{\theta+\beta} L_{rt}}{a_N} \right]^{\frac{1}{1+\beta}}, \tag{15}$$

where $L_{Rt} \equiv \sum_i l_{Rit}$ is the total amount of northern labor employed in innovative activities.

In any steady-state equilibrium, the share of labor employed in innovative R&D must be constant over time. Given that the northern supply of labor grows at the population growth rate g_L, northern R&D employment L_{Rt} must grow at this rate as well. Dividing both sides of (**??**) by n_t yields $g \equiv \frac{\dot{n}_t}{n_t} = \frac{n_t^{\theta-1} L_{Rt}}{a_N g^\beta}$. Since g is constant over time in any steady-state equilibrium, $n_t^{\theta-1}$ and L_{Rt} must grow at offsetting rates, that is, $(\theta - 1)\frac{\dot{n}_t}{n_t} + \frac{\dot{L}_{Rt}}{L_{Rt}} = (\theta - 1)\, g +$

[11] For $\theta > 0$, R&D labor becomes more productive as time passes and a northern firm needs to devote less labor to develop a new variety as the stock of knoweldge increases. For $\theta < 0$, R&D becomes more difficult over time.

[12] When we solve the model, we set $\beta = 1$ which yields $1/(1+\beta) = 0.5$.

$g_L = 0$. It immediately follows that

$$g \equiv \frac{\dot{n}_t}{n_t} = \frac{g_L}{1-\theta}. \tag{16}$$

Thus, the steady-state rate of innovation g is pinned down by parameter values and is proportional to the population growth rate g_L. As in Jones (1995), when there is positive population growth, the parameter restriction $\theta < 1$ is needed to guarantee that the steady-state rate of innovation is positive and finite.

We can now solve for the steady-state rate of economic growth. The representative consumer in region r has utility $u_t^r = e^r/P_t^r$. In steady-state equilibrium, individual consumer expenditure is constant over time but consumer utility nevertheless grows because the price indexes fall over time. Since $(P_t^r)^{1-\sigma}$ grows over time at the rate g, it follows that consumer utility growth is

$$g_u \equiv \frac{\dot{u}_{Nt}}{u_{Nt}} = \frac{\dot{u}_{St}}{u_{St}} = \frac{g}{\sigma-1} = \frac{g_L}{(1-\theta)(\sigma-1)}. \tag{17}$$

With consumer utility in both regions being proportional to consumer expenditure holding prices fixed, consumer utility growth equals real wage growth, which we use as a measure of economic growth. Equation (**??**) implies that public policy changes like trade liberalization (a decrease in τ) or stronger IPR protection (and increase in a_I) have no effect on the steady-state rate of economic growth. In this model, growth is "semi-endogenous". We view this as a virtue of the model because both total factor productivity and per capita GDP growth rates have been remarkably stable over time in spite of many public policy changes that one might think would be growth-promoting. For example, plotting data on per capita GDP (in logs) for the US from 1870 to 1995, Jones (2005, Table 1) shows that a simple linear trend fits the data extremely well. Further evidence for the R&D assumptions underlying semi-endogenous growth models is provided by Venturini (2012). Looking at US manufacturing industry data for the period 1975-1996, he finds that the exhaustion of technological opportunities, which leads to increasing R&D difficulty, is the mechanism best matching the real dynamics of business innovation.

In the unit labor requirement for innovation $a_N g^\beta/n_t^\theta$, the term $1/n_t^\theta$ is a measure of absolute R&D difficulty. It increases over time if $\theta < 0$ and decreases over time if $\theta \in (0, 1)$. By taking the ratio of R&D difficulty and the market size term L_t/n_t, we obtain a measure of relative R&D difficulty (or R&D difficulty relative to the size of the market):

$$\delta \equiv \frac{n_t^{-\theta}}{L_t/n_t} = \frac{n_t^{1-\theta}}{L_t}. \tag{18}$$

To see that δ is constant over time in steady-state equilibrium, note that $\frac{\dot{\delta}}{\delta} = (1-\theta)\frac{\dot{n}_t}{n_t} - \frac{\dot{L}_t}{L_t} = (1-\theta)\frac{g_L}{1-\theta} - g_L = 0$.[13]

To learn how to export one product variety to the South, a northern firm with productivity z must employ $a_X \chi_z^{\beta}/n_t^{\theta}$ units of southern labor to export-learning R&D.[14] The parameter a_X is an export-learning R&D productivity parameter. As with innovation, $\beta > 0$ captures the decreasing returns to export-learning R&D. When more firms learn how to become exporters (the rate of export-learning χ_z is higher), each individual northern firm must devote more resourcers to successfully enter into the southern market via exports. The flow of new (northern) products entering the southern market due to northern exporter i's export-learning activities is given by

$$\dot{n}^i_{Xzt} + \dot{n}^i_{Fzt} + \dot{n}^i_{Izt} = \frac{l^i_{Xzt}}{a_X \chi_z^{\beta}/n_t^{\theta}} = \frac{n_t^{\theta} l^i_{Xzt}}{a_X \chi_z^{\beta}}, \qquad (z = H, L) \tag{19}$$

where l^i_{Xzt} is the southern labor employed in export-learning R&D by firm i with productivity z. Aggregating over all northern exporters, the flow of new products sold in the South as a consequence of export-learning activities is

$$\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt} = \frac{n_t^{\theta} L_{Xzt}}{a_X \chi_z^{\beta}}, \qquad (z = H, L) \tag{20}$$

[13]The innovation rate g is constant in steady-state equilibrium, but a larger δ in one steady-state compared to an earlier steady-state means that there has been more innovation in the transition to the new steady-state, and that the stock of knowledge (number of varieties) has increased permanently. In the short run, the rate of innovation increases, but in the long run, the rate of innovation returns to its steady-state rate.

[14]Following Arkolakis (2010) and Arkolakis et al (2014), we assume that southern labor is employed for northern firms' export-learning activities. This can be thought of as hiring local labor for marketing and to set up distribution networks in the export market. The assumption also facilitates comparison between MP-learning activities and export-learning activities in the model.

where $L_{Xzt} \equiv \sum_i l^i_{Xzt}$ is the total amount of southern labor employed in export-learning activities by firms with productivity z. Some exporters then go on to become multinational firms after engaging in adaptive R&D to learn how to do MP, and some of these foreign affiliate-produced varieties become imitated by southern firms. Therefore, the flows $\dot{n}_{Fzt}$ and $\dot{n}_{Izt}$ must be taken into account in the exported product flow.

Adaptive R&D (or FDI) is undertaken by northern exporters. To learn how to produce an exported variety in the South via MP, the foreign affiliate of a northern exporting firm with productivity z must devote $a_F \phi_z^\beta / n_t^\theta$ units of southern labor to adaptive R&D. The parameter a_F is an adaptive R&D productivity parameter that is common to all firms and can be thought of as measuring the ease of doing FDI in the South. There are decreasing returns also to adaptive R&D. When northern exporters are doing more FDI (ϕ_z is higher), each individual exporting firm must devote more resources to adaptive R&D in order to be successful in transferring production to a foreign affiliate in the South. The flow of products for which production is transferred to the South due to firm i's adaptive R&D activities is

$$\dot{n}^i_{Fzt} + \dot{n}^i_{Izt} = \frac{l^i_{Fzt}}{a_F \phi_z^\beta / n_t^\theta} = \frac{n_t^\theta l^i_{Fzt}}{a_F \phi_z^\beta}, \qquad (z = H, L) \tag{21}$$

where l^i_{Fzt} is the southern labor employed by firm i with productivity z in adaptive R&D (learning to do MP). Aggregating over all foreign affiliates generates the product flow

$$\dot{n}_{Fzt} + \dot{n}_{Izt} = \frac{n_t^\theta L_{Fzt}}{a_F \phi_z^\beta}, \qquad (z = H, L) \tag{22}$$

where $L_{Fzt} \equiv \sum_i l^i_{Fzt}$ is the aggregate amount of southern labor employed in adaptive R&D by firms with productivity z.

Imitation targets foreign affiliates in the South. Let $\iota_S \equiv 1/a_I$ where a_I is a measure of the strength of southern IPR protection. With stronger southern IPR protection, the rate of imitation is lower ($a_I \uparrow \Longrightarrow \iota_S \downarrow$).

2.6 R&D Incentives

Denote the expected discounted profits associated with innovating in the North at time t for a firm with productivity z by v_{Nzt}. The R&D labor used to develop one new variety is $a_N g^\beta / n_t^\theta$ and the cost of developing this variety is $w_N a_N g^\beta / n_t^\theta$. Taking into account the probability of a high (low) productivity draw, free entry into innovative R&D activities in the North implies that the cost of innovating must be exactly balanced by the expected benefit from innovating in equilibrium:

$$qv_{NLt} + (1-q)\, v_{NHt} = \frac{w^N a_N g^\beta}{n_t^\theta}. \tag{23}$$

Let v_{Xzt} be the expected discounted profits that a northern exporter with productivity z earns. The benefit of becoming an exporter is $v_{Xzt} - v_{Nzt}$ since v_{Nzt} must be subtracted because the expected discounted profits earned in the domestic market are already included in v_{Xzt}.[15] A firm with productivity z will decide to become an exporter if $v_{Xzt} - v_{Nzt} \geq \frac{w^S a_X \chi_z^\beta}{n_t^\theta}$. If this holds with strict inequality, there will be infinite export-learning and if $v_{Xzt} - v_{Nzt} < \frac{w^S a_X \chi_z^\beta}{n_t^\theta}$, no northern firm will choose to become an exporter. Therefore, in steady-state equilibrium, it must hold that

$$v_{Xzt} - v_{Nzt} = \frac{w^S a_X \chi_z^\beta}{n_t^\theta}, \qquad (z = H, L). \tag{24}$$

Let v_{Fzt} denote the expected discounted profits that a foreign affiliate with productivity z earns from producing a product variety in the South at time t. The benefit of the transfer is not the expected discounted profits that a firm could earn from starting to do MP v_{Fzt} but the gain in expected profits $v_{Fzt} - v_{Xzt}$ since the exporter is already earning profits from producing in the North and serving both markets. Since the cost of transferring production to the South must be exactly balanced by the benefit in steady-state equilibrium, we obtain

$$v_{Fzt} - v_{Xzt} = \frac{w^S a_F \phi_z^\beta}{n_t^\theta}, \qquad (z = H, L). \tag{25}$$

When technology transfer occurs, each foreign affiliate pays its parent firm a royalty payment

[15] There are no "pure exporters" in the model. All exporting firms also serve their domestic market.

v_{Xzt} for the use of its technology in the South, since the adaptive R&D accounts for the increase in the firm's value $v_{Fzt} - v_{Xzt}$.

We assume that there is a stock market in each region that channels household savings to firms that engage in R&D in each region and helps households to diversify the risk of holding stocks issued by these firms. There is no aggregate risk, so it is possible for households to earn a safe return by holding the market portfolio in each region. Hence, ruling out any arbitrage opportunities implies that the total return on equity claims must equal the opportunity cost of invested capital, which is given by the risk-free market interest rate ρ.

For a northern firm i, the relevant no-arbitrage condition is $\left(\pi_{Nzt} - w^S l^i_{Xzt}\right) dt + \dot{v}_{Nzt} dt + \left(\dot{n}^i_{Xzt} + \dot{n}^i_{Fzt} + \dot{n}^i_{Izt}\right) dt \left(v_{Xzt} - v_{Nzt}\right) = \rho v_{Nzt} dt$. The northern firm earns the profit flow $\pi_{Nzt} dt$ during the time interval dt but also incurs the export-learning cost $w^S l^i_{Xzt} dt$ during this time interval. In addition, the firm experiences the gradual capital gain $\dot{v}_{Nzt} dt$ during the time interval dt and its market value jumps up by $v_{Xzt} - v_{Nzt}$ for each product that it succeeds in introducing to the southern market. The firm succeeds in introducing $\left(\dot{n}^i_{Xzt} + \dot{n}^i_{Fzt} + \dot{n}^i_{Izt}\right) dt$ varieties to the southern market during the time interval dt. To rule out any arbitrage opportunities for investors, the rate of return for a northern firm must be the same as the return on an equal sized investment in a risk-free bond $\rho v_{Nzt} dt$. From (**??**) and (**??**), it follows that $\left(\dot{n}^i_{Xzt} + \dot{n}^i_{Fzt} + \dot{n}^i_{Izt}\right) \left(v_{Xzt} - v_{Nzt}\right) = w^S l^i_{Xzt}$. Equation (**??**) implies that v_{Nzt} must grow at the rate $-\theta g$. Thus, after dividing by $v_{Nzt} dt$, the no-arbitrage condition for the z-productivity northern firm simplifies to $v_{Nzt} = \frac{\pi_{Nzt}}{\rho + \theta g}$. Combining this expression with (**??**), the northern no-arbitrage condition can be written as $\frac{q\pi_{NHt} + (1-q)\pi_{NLt}}{\rho + \theta g} = \frac{w^N a_N g^\beta}{n_t^\theta}$. In this equation, the left-hand side is the expected discounted profit from innovating and the right-hand side is the cost of innovation. The northern firm's expected discounted profits or market value is equal the expected profit flow $q\pi_{NHt} + (1-q)\pi_{NLt}$ appropriately discounted by the market interest rate ρ and the capital loss term θg. Substituting for expected profit flow using (**??**), dividing both sides by w^N and then by the market size term L_t / n_t yields the northern steady-state no-arbitrage condition

$$\frac{\frac{1}{\sigma-1}\left(\frac{qc_L X_{NL}^N}{\gamma_{NL}}+\frac{(1-q)c_H X_{NH}^N}{\gamma_{NH}}\right)}{\rho+\theta g}=a_N g^{\beta}\delta. \tag{26}$$

The left-hand side of (**??**) is the market size-adjusted expected benefit from innovating and the right-hand side is the market size-adjusted cost of innovating. In steady-state calculations, we need to adjust for market size L_t/n_t because market size changes over time if $g_L \neq g$ or $\theta \neq 0$. The market size-adjusted benefit from innovating is higher when the average consumer buys more of non-exported northern varieties ($X_{Nz} \uparrow$), future profits are less heavily discounted ($\rho \downarrow$), and northern firms experience smaller capital losses over time ($\theta g \downarrow$). The market size-adjusted cost of innovating is higher when northern researchers employed in innovative R&D are less productive ($a_N \uparrow$), and when innovating is relatively more difficult ($\delta \uparrow$).

For a northern exporter i, the relevant no-arbitrage condition is $\left(\pi_{Xzt}-w^S l_{Fzt}^i\right)dt+\dot{v}_{Xzt}dt+\left(\dot{n}_{Fzt}^i+\dot{n}_{Izt}^i\right)dt\left(v_{Fzt}-v_{Xzt}\right)=\rho v_{Xzt}dt$. There is free entry into non-production (R&D) activities also in the South. Following the same procedure as for northern firms, the no-arbitrage condition for the northern exporter becomes $\frac{\pi_{Xzt}}{\rho+\theta g}-\frac{\pi_{Nzt}}{\rho+\theta g}=\frac{w_S a_X \chi_z^{\beta}}{n_t^{\theta}}$. After inserting the profit expressions (**??**) and (**??**), we obtain the steady-state northern exporter no-arbitrage condition

$$\frac{c_z w}{\sigma-1}\left[\frac{\frac{X_{Xz}^N+\tau X_{Xz}^S}{\gamma_{Xz}}-\frac{X_{Nz}^N}{\gamma_{Nz}}}{\rho+\theta g}\right]=a_X\chi_z^{\beta}\delta, \qquad (z=H,L) \tag{27}$$

where $w=w^N/w^S$ is the northern relative wage or the North-South wage ratio.

A foreign affiliate i faces the no-arbitrage condition $\pi_{Fzt}dt+\dot{v}_{Fzt}dt-(\iota_S dt)\,v_{Fzt}=\rho v_{Fzt}dt$. It is exposed to a positive rate of imitation by southern firms and experiences a total capital loss if it is imitated, which occurs with the probability $\iota_S dt$ during the time interval dt. In equilibrium, the benefit from adapting the product variety to MP must equal the cost of adaption so that $\frac{\pi_{Fzt}}{\rho+\theta g+\iota_S}-\frac{\pi_{Xzt}}{\rho+\theta g}=\frac{w^S a_F \phi_z^{\beta}}{n_t^{\theta}}$. Following the same procedure as for firms with production in the North, we obtain the foreign affiliate steady-state no-arbitrage condition

$$\frac{c_z}{\sigma - 1}\left[\frac{\frac{X^S_{Fz}+\tau X^N_{Fz}}{\gamma_{Fz}}}{\rho+\theta g+\iota_S} - \frac{\frac{w\left(X^N_{Xz}+\tau X^S_{Xz}\right)}{\gamma_{Xz}}}{\rho+\theta g}\right] = a_F \phi_z^\beta \delta, \qquad (z = H, L). \tag{28}$$

2.7 Labor Markets

Each labor market is perfectly competitive and wages adjust instantaneously to equate labor demand and labor supply. Northern labor is employed in innovative R&D, in production by northern firms selling only to the home market and in exporting firms serving both markets. Each innovation requires $a_N g^\beta / n_t^\theta$ units of labor, so total employment in innovative R&D is $\frac{a_N g^\beta}{n_t^\theta}\dot{n}_t = a_N g^\beta \frac{n_t^{1-\theta}}{L_t}\frac{\dot{n}_t}{n_t}L_t = a_N g^{1+\beta}\delta L_t$. Northern firms that have not learned to export use $\frac{c_z\left(p^N_{Nz}\right)^{-\sigma} e^N L^N_t}{\left(P^N_t\right)^{1-\sigma}}$ units of labor for each variety produced and there are n_{Nzt} such varieties produced. Northern exporters use $\frac{c_z\left(p^N_{Nz}\right)^{-\sigma} e^N L^N_t}{\left(P^N_t\right)^{1-\sigma}} + \frac{\tau c_z\left(p^S_{Nz}\right)^{-\sigma} e^S L^S_t}{\left(P^S_t\right)^{1-\sigma}}$ units of labor for each variety produced and there are n_{Xzt} such varieties produced, so total employment in production activities in the North is $\sum_z c_z X^N_{Nz} L_t + \left(X^N_{Xz} + \tau X^S_{Xz}\right) c_z L_t$. As L^N_t denotes labor supply in the North, full employment requires that $L^N_t = a_N g^{1+\beta}\delta L_t + \sum_z c_z X^N_{Nz} L_t + \left(X^N_{Xz} + \tau X^S_{Xz}\right) c_z L_t$. Evaluating at time $t = 0$ yields the steady-state full employment of labor condition for the North:

$$L^N_0 = L_0\left[a_N g^{1+\beta}\delta + \sum_{z=H,L} c_z X^N_{Nz} + c_z\left(X^N_{Xz} + \tau X^S_{Xz}\right)\right]. \tag{29}$$

Southern labor is employed in adaptive R&D, export-learning R&D, production by foreign affiliates and production by southern firms that have imitated foreign affiliates. Following the same procedure as for the northern labor market, full employment in the South requires that $L^S_t = \sum_z \frac{a_X \chi_z^\beta}{n_t^\theta}\left(\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}\right) + \frac{a_F \phi_z^\beta}{n_t^\theta}\left(\dot{n}_{Fzt} + \dot{n}_{Izt}\right) + \left[X^S_{Fz} + \tau X^N_{Fz}\right] c_z L_t + \left[X^S_{Iz} + \tau X^N_{Iz}\right] c_z L_t$. Using the definitions of χ_z, ϕ_z and δ and evaluating at time $t = 0$, we obtain the steady-state full employment of labor condition for the South:

$$L_0^S = L_0 \left[\sum_{z=H,L} a_X \delta \chi_z^{1+\beta} \gamma_{Nz} + a_F \delta \phi_z^{1+\beta} \gamma_{Xz} + \left(X_{Fz}^S + \tau X_{Fz}^N + X_{Iz}^S + \tau X_{Iz}^N \right) c_z \right]. \quad (30)$$

2.8 Aggregate Demand

To solve the model, we need steady-state values for the aggregate demand terms X_{Nz}^N, X_{Xz}^N, X_{Xz}^S, X_{Fz}^S, X_{Fz}^N, X_{Iz}^S and X_{Iz}^N. Solving for the ratio X_{Nz}^N / X_{Fz}^N yields

$$\frac{X_{Nz}^N}{X_{Fz}^N} = \frac{\frac{\left(p_{Nz}^N\right)^{-\sigma} e^N L_t^N n_{Nzt}}{\left(P_t^N\right)^{1-\sigma} L_t}}{\frac{\left(p_{Fz}^N\right)^{-\sigma} e^N L_t^N n_{Fzt}}{\left(P_t^N\right)^{1-\sigma} L_t}} = \left(\frac{p_{Nz}^N}{p_{Fz}^N} \right)^{-\sigma} \frac{n_{Nzt}/n_t}{n_{Fzt}/n_t} = \left(\frac{\frac{c_z w^N}{\alpha}}{\frac{\tau c_z w^S}{\alpha}} \right)^{-\sigma} \left[\frac{q_z \frac{g}{g+\chi_z}}{q_z \frac{\chi_z}{g+\chi_z} \frac{\phi_z}{g+\phi_z} \frac{g}{g+\iota_S}} \right],$$

and by doing similar calculations looking at other ratios, we obtain that $X_{Nz}^N = X_{Fz}^N \left(\frac{\tau}{w}\right)^\sigma \frac{(g+\phi_z)(g+\iota_S)}{\chi_z \phi_z}$, $X_{Xz}^N = X_{Fz}^N \left(\frac{\tau}{w}\right)^\sigma \frac{g+\iota_S}{\phi_z}$, $X_{Xz}^S = X_{Fz}^S \left(\frac{1}{w\tau}\right)^\sigma \frac{g+\iota_S}{\phi_z}$, $X_{Iz}^S = X_{Fz}^S \left(\frac{1}{\alpha}\right)^\sigma \frac{\iota_S}{g}$ and $X_{Iz}^N = X_{Fz}^N \left(\frac{1}{\alpha}\right)^\sigma \frac{\iota_S}{g}$.

Finally, we solve for the ratios $\frac{X_{FH}^S}{X_{FL}^S} = \left(\frac{c_H}{c_L}\right)^{-\sigma} \frac{\gamma_{FH}}{\gamma_{FL}}$ and $\frac{X_{FH}^N}{X_{FL}^N} = \left(\frac{c_H}{c_L}\right)^{-\sigma} \frac{\gamma_{FH}}{\gamma_{FL}}$. Inserting steady-state variety share expressions, we obtain

$$X_{FH}^r = X_{FL}^r \left(\frac{c_H}{c_L} \right)^{-\sigma} \left(\frac{q}{1-q} \right) \left(\frac{g+\chi_L}{g+\chi_H} \right) \left(\frac{\chi_H}{\chi_L} \right) \left(\frac{g+\phi_L}{g+\phi_H} \right) \left(\frac{\phi_H}{\phi_L} \right).$$

2.9 Asset Ownership and Consumer Expenditure

To determine consumer expenditures e^N and e^S, we need to specify who owns the firms and how wealth is distributed between the North and the South. We assume that R&D done in the North is financed by northern savings and R&D done in the South is financed by southern savings.[16] Then in equilibrium, northern firms that are only active in the domestic market will be fully owned by northern consumers while exporting firms and foreign affiliates will be owned jointly by northern and southern consumers.

[16] French and Poterba (1991) document that around 94% of Americans held their equity wealth in the U.S. stock market and Japanese help around 98% of their equity wealth in the Japanese stock market. Also Tesar and Werner (1995) document this home bias in equity portfolios.

Let A_t^N denote the aggregate value of northern financial assets and A_t^S denote the aggregate value of southern financial assets. The aggregate value of all financial assets is $A_t = A_t^N + A_t^S = \sum_z n_{Nzt}v_{Nzt} + n_{Xzt}v_{Xzt} + n_{Fzt}v_{Fzt}$. Since consumer savings within the South finance R&D investments in the South, $A_t^S = \sum_z (n_{Xzt} + n_{Fzt})(v_{Xzt} - v_{Nzt}) + n_{Fzt}(v_{Fzt} - v_{Xzt}) = \sum_z n_{Xzt}(v_{Xzt} - v_{Nzt}) + n_{Fzt}(v_{Fzt} - v_{Nzt})$. Substituting into this expression using the firm values (**??**) and (**??**), we obtain $A_t^S = w^S L_t \delta \left[\sum_z \gamma_{Xz} a_X \chi_z^\beta + \gamma_{Fz}\left(a_F \phi_z^\beta + a_X \chi_z^\beta\right)\right]$. Since $A_t^N = \sum_z (n_{Nzt} + n_{Xzt} + n_{Fzt}) v_{Nzt}$, substituting into this expression using northern firm value $v_{Nzt} = \pi_{Nzt}/(\rho + g)$ and profits from (**??**) yields $A_t^N = \frac{w^N L_t}{(\sigma-1)(\rho+\theta g)}\left[\sum_{z=H,L} c_z X_{Nz}^N \frac{\gamma_{Nz}+\gamma_{Xz}+\gamma_{Fz}}{\gamma_{Nz}}\right]$.

Let $\tilde{a}_t^r$ denote the financial asset holdings of the typical consumer in region r. The intertemporal budget constraint of a typical consumer in region r is $\dot{a}_t^r = w^r + \rho a_t^r - e^r - g_L a_t^r$. In any steady-state equilibrium where the wage rates w^r are constant over time, we must have that $\dot{\tilde{a}}_t^r = 0$ and it follows that $e^r = w^r + (\rho - g_L)\tilde{a}_t^r$. For the typical consumer in region r, $\tilde{a}_t^r = A_t^r / L_t^r$. Setting $w^S = 1$ and $w^N = w^N/w^S \equiv w$, it follows that typical northern and southern consumer expenditure levels are given by

$$e^S = w^S + (\rho - g_L) w^S \delta \frac{L_0}{L_0^S}\left[\sum_{z=H,L} \gamma_{Xz} a_X \chi_z^\beta + \gamma_{Fz}\left(a_F \phi_z^\beta + a_X \chi_z^\beta\right)\right] \tag{31}$$

and

$$e^N = w^N + \frac{(\rho - g_L) w^N}{(\sigma - 1)(\rho + \theta g)} \frac{L_0}{L_0^N}\left[\sum_{z=H,L} c_z X_{Nz}^N \frac{\gamma_{Nz} + \gamma_{Xz} + \gamma_{Fz}}{\gamma_{Nz}}\right]. \tag{32}$$

Having solved for consumer expenditures e^N and e^S, we can determine the ratio X_{FL}^N / X_{FL}^S and obtain the steady-state asset condition

$$\frac{X_{FL}^N}{X_{FL}^S} = \left(\frac{1}{\tau}\right)^\sigma \frac{e^N L_0^N}{e^S L_0^S} \frac{\left(P_t^S\right)^{1-\sigma}}{\left(P_t^N\right)^{1-\sigma}} \tag{33}$$

where $\frac{\left(P_t^S\right)^{1-\sigma}}{\left(P_t^N\right)^{1-\sigma}} = \frac{\sum_{j \neq N}\sum_z\left[\gamma_{jz}\left(p_{jz}^S\right)^{1-\sigma}\right]}{\sum_j \sum_z\left[\gamma_{jz}\left(p_{jz}^N\right)^{1-\sigma}\right]}$ is constant over time.

Thus, solving the model for a steady-state equilibrium reduces to solving a system of eight equations [(**??**), (**??**) and(**??**) for $z = H, L$, (**??**), (**??**) and (**??**)] in 8 unknowns (w, δ, χ_L, χ_H,

ϕ_L, ϕ_H, X^S_{FL} and X^N_{FL}), where the eight equations are: five R&D conditions (innovation, two export-learning, two FDI (MP-learning)), two labor market conditions (North and South) and one asset condition.

3 Numerical Results

3.1 Parameters

The subjective discount rate ρ is set at 0.07 to reflect a real interest rate of 7 percent, consistent with the average real return on the US stock market over the 20th century (Mehra and Prescott, 1985). The measure of product differentiation α determines the markup of price over marginal cost $1/\alpha$. It is set at 0.714 to generate a northern markup of 40 percent, which is within the range of estimates from Basu (1996) and Norrbin (1993). The parameter g_L is set at 0.014 to reflect a 1.4 percent population growth rate. This was the average annual world population growth rate during the 1990s according to the World Development Indicators (World Bank, 2016). The steady-state economic growth rate is calculated from $g_u = g_L/\left((\sigma-1)(1-\theta)\right)$. In order to generate a steady-state economic growth rate of 2 percent, consistent with the average US GDP per capita growth rate from 1950 to 1994 (Jones, 1995), the R&D spillover parameter θ is set at 0.72.[17] Since only the ratio L_{N0}/L_{S0} matters, we set $L_{N0} = 1$ and $L_{S0} = 2$ so L_{N0}/L_{S0} equals the ratio of working-age population in high-income countries to that in middle-income countries (World Bank, 2003). Only the relative productivity advantage of high productivity firms over low productivity firms matter, so we normalize $c_L = 1$. Helpman et al (2004) find that, for US firms, the productivity advantage of exporters over domestic firms is 0.388 (and the productivity advantage of multinationals over domestic firms is 0.537). Consistent with this evidence, we set $c_H = 1-0.388 = 0.612$. Empirical studies on patents and R&D suggests that there are significant decreasing returns to R&D at the industry level. Given that point estimates of $1/(1+\beta)$ lie between 0.1 and 0.6 (Kortum, 1993), we set $\beta = 1$ which yields the intermediate value $1/(1+\beta) = 0.5$.

[17] Recall that when $0 < \theta < 1$ knowledge spillovers are positive but weak.

During the time period 1990-2005 when the TRIPS agreement was being implemented, North-South trade costs were falling. We use the micro-founded measure of bilateral trade costs developed by Novy (2013) that indirectly infers trade frictions from observable trade data. By linear extrapolation of the bilateral trade cost estimates between the US and Mexico in 1970 and 2000, we obtain a tariff-equivalent of 54 percent for 1990 ($\tau = 1.54$) and 33 percent in 2005 ($\tau = 1.33$).

The remaining parameters are the R&D productivity parameters a_N (innovation), a_X (export-learning) and a_F (adaption or MP-learning), a_I (imitation), and the probability q for a low productivity draw. Since only the relative difference between the R&D productivity parameters matters, we normalize $a_N = 1$.

We set the export-learning R&D productivity parameter a_X and the probability of a low productivity draw q to match the following two facts: (i) Bernard et al (2003) find that 79 percent of US plants do not export any of their output; (ii) the share of high-tech exports out of all manufacturing exports for the US in 1990 was 0.325 (World Bank, 2016). By setting $a_X = 5.56$ and $q = 0.959$ we obtain a 79 percent share of non-exporting northern firms ($\sum_z \gamma_{Nz}/\left(\sum_z \gamma_{Nz} + \gamma_{Xz} + \gamma_{Fz}\right) = 0.79$) and a high productivity share of northern exports of 0.322 ($X^S_{XH}/\left(X^S_{XH} + X^S_{XL}\right) = 0.322$).

We set the FDI (MP-adaption) productivity parameter a_F and the parameter a_I that is our measure of IPR protection in the South to match that (i) the foreign affiliate share in world GDP was 4.6 per cent in 1990[18] (UNCTAD, 2012); (ii) the ratio of consumption share-adjusted real GDP per employed for U.S. and Mexico was 2.56 in 1990 (Feenstra, Inklaar and Timmer, 2015). By setting $a_F = 148.5$ and $a_I = 4.4$ we obtain $\sum_z X_{Fz}/\sum_z \sum_j \sum_r X^r_{jz} = 0.04630$ and $e^N/e^S = 2.5606$ in our 1990 benchmark.

Stronger IPR protection corresponds to a lower imitation rate $\iota_S \equiv 1/a_I$. By setting $a_I = 4.4$ we also capture weak IPR protection in the South prior to the TRIPS agreement with a high imitation rate ι_S in the 1990 benchmark (between one out of four and one out

[18]Foreign affiliate share in world GDP is $\left(\sum_z X^S_{Fz} + X^N_{Fz}\right)/\left[\sum_{r=N,S}\sum_{z=H,L}\sum_j X^r_{jz}\right]$. Value added (product) of foreign affiliates in 1990 was 1,018 billion US dollars and world GDP was 22,206 billion US dollar, measured in 2012 US dollar. By 2005, the value added (product) of foreign affiliates had risen to 4,949 billion US dollars and world GDP to 50,411 billion US dollars, i.e. 9.8 percent (UNCTAD, 2012).

of five products produced by foreign affiliate varieties is copied each year). We set a higher value for a_I in 2005 to capture stronger IPR protecion after the implementation of the TRIPS agreement. In particular, we set $a_I = 81.21$ so the model is consistent with the evidence of a ten-fold increase in the FDI inflow to developing countries between 1990 and 2005 (UNCTAD, 2011).

In the model, the FDI inflow to developing countries is captured by $\sum_z L_{Fzt}$ (the total amount of southern labor devoted to adaptive R&D activities by foreign affiliates multiplied by the southern wage rate $w_S = 1$). Rewriting (**??**) using the definitions for the FDI rate ϕ_z, the relative R&D difficulty δ and the variety share of northern exporters γ_{Xz}, the FDI inflow measure can be written as $L_{Ft} = \sum_z L_{Fzt} = \sum_z \phi_z^{1+\beta}\gamma_{Xz}\delta a_F L_t$. The ratio L_{Fzt}/L_t is constant over time in any steady-state equilibrium so we obtain $L_{F0} = \sum_z L_{Fz0} = \sum_z \phi_z^{1+\beta}\gamma_{Xz}\delta a_F L_0$. In 1990 the FDI inflow to developing countries (including transition economies) was 34.9 billion US dollars and in 2005 that FDI inflow was 363.4 billion US dollars (UNCTAD, 2011). This represents a roughly ten-fold increase in the FDI inflow to developing countries measured in current prices. Adjusting the FDI inflow in 1990 for population growth and inflation from 1990 to 2005 generates an expected FDI inflow of 59.7 billion US dollars for 2005.[19] The ratio of the observed FDI inflow to this expected FDI inflow yields a six-fold increase in FDI inflow to developing countries during the time period 1990-2005 that can be attributed to policy changes (the decrease in τ and the increase in a_I). So we set $a_I = 4.4$ in 1990 and $a_I = 81.21$ in 2005 to assure a small FDI inflow L_{F0} in 1990 and a six-fold increase in L_{F0} by 2005.[20]

3.2 Main results

The model is solved numerically using the parameter values discussed in Section **??**. The pre-TRIPS 1990 benchmark and the post-TRIPS 2005 benchmark are presented in Columns

[19] From 1990 to 2005, the US GDP implicit price deflator increased by 38.4 percent (Federal Reserve Bank of St Louis, 2011). During the same time period, the world population grew by 23.4 percent using the 1.4 percent annual population growth rate. Multiplying the observed FDI inflow in 1990 by the population growth and inflation over the period generates the expected FDI inflow in 2005 in the absence of any policy changes.

[20] In the 1990 benchmark with $a_I = 4.4$, $L_{FH0} + L_{FL0} = .028995$, such that $6 * L_{F0} = .173972$. Setting $a_I = 81.21$ in the 2005 benchmark with $\tau = 1.33$ generates $L_{FH0} + L_{FL0} = .173972$.

1 and 2 of Table 1. The stylized facts that emerge from Bernard et al (2003) and Bernard et al (2007), among others, are that multinationals are on average more productive than exporters and that exporters are on average more productive than non-exporters. The model generates a pattern that is consistent with this. The export-learning rate of northern firms is higher for high productivity firms than for low productivity firms ($\chi_H > \chi_L$ in both 1990 and 2005). Also, the rate of FDI is higher for high productivity firms than for low productivity firms ($\phi_H > \phi_L$ in both 1990 and 2005). Therefore, the share of high productivity firms is higher for exporting northern firms than for non-exporting northern firms, and the share of high productivity firms is higher for multinational firms than for northern exporters. In particular, in 1990, $\gamma_{NH}/(\gamma_{NH}+\gamma_{NL}) = .0277$, $\gamma_{XH}/(\gamma_{XH}+\gamma_{XL}) = .0786$ and $\gamma_{FH}/(\gamma_{FH}+\gamma_{FL}) = .2252$.

Going from the 1990 to the 2005 benchmark (with trade liberalization and stronger southern IPR protection to comply with TRIPS), the speed of learning how to export increases ($\chi_H \uparrow$ and $\chi_L \uparrow$) and the speed of adapting to MP increases ($\phi_H \uparrow$ and $\phi_L \uparrow$). There is a geographical redistribution of production from the North to the South ($\sum_z \gamma_{Nz} + \gamma_{Xz}$ decreases from .9853 to .9603 and $\sum_z \gamma_{Fz} + \gamma_{Iz}$ increases from .0147 to .0397). In the post-TRIPS scenario, the share of non-exporting firms in the North is smaller (the share of non-exporters decrease from .790 to .736). Also, MP has increased and foreign affiliates are more important in the world economy. The share of varieties that are produced in foreign affiliates have increased ($\sum_z \gamma_{Fz}$ increases from .0027 to .0318) and there is a large increase in foreign affiliate value-added as share of world GDP from .0463 to .3923. The sales from MP in the northern market increases from .005 to .058 and in the southern market from .046 to .335.[21] The share of MP sales (foreign affiliate-produced varieties) out of total sales in the northern market increases from 1.4 percent to 16.7 percent, while in the southern market this share increases from 6.1 percent to 51.2 percent.

Going from the 1990 to the 2005 benchmark, southern consumer welfare is improved (u_0^S increases from 76.14 to 94.55) but northern consumer welfare is worsened (u_0^N decreases from 288.13 to 272.31). To understand these welfare changes, we solve the model for two

[21] MP sales in market r is aggregate demand for foreign affiliate produced varieties $\sum_z X^r_{Fz}$.

counterfactual scenarios. In the first counterfactual, presented in Column 3 of Table 1, trade costs are assumed to be at their 1990 level, but IPR protection is set at its post-TRIPS 2005 level. This would have been the case if TRIPS had not been accompanied by any trade liberalization. Stronger IPR protection leads to a faster rate of adaption to MP for both high and low productivity firms in the North (ϕ_H increases from .01028 to .02750 and ϕ_L increases from .00302 to .00807). Consumer welfare is measured by $u_0^r = e^r/P_0^r$, $r = N, S$. With stronger southern IPR protection, consumer welfare is improved in both regions (u_0^N increases from 288.13 to 295.77 and u_0^S increases from 76.14 to 95.03). Southern consumer expenditure is higher (e^S increases from 1.032 to 1.113) and the southern price index is lower (P_0^S decreases from .01355 to .01171), which result in higher long-run consumer welfare. For northern consumers, there is a drop in consumer expenditure but this is out-weighted by a lower price index. In essence, with stronger IPR protection in the South, there is a substantial geographical redistribution of production from the North to the South. Less production is done by northern exporters ($\sum_z \gamma_{Nz} \downarrow$), and more production is done by foreign affiliates in the South ($\sum_z \gamma_{Fz} \uparrow$). This has two effects on consumer welfare. First, more production taking place in the lower-wage South translates to lower product prices in both regions. Second, labor resources are freed up from production by exporting firms and there is downward pressure on the northern wage rate (w_N/w_S decreases from 2.2833 to 2.0643), lowering the cost of innovation. Therefore, there is more innovation (δ increases from 19.0631 to 19.4528) and the resulting increase in invented varieties benefits consumers in both regions (n_0 increases from 1,850,760 to 1,989,254).[22]

In the second counterfactual presented in Column 4 of Table 1, trade costs are set at their 2005 level but IPR protection is the same as in the 1990 benchmark. This would have been the case if trade liberalization had occurred between 1990 and 2005 without being accompanied by any stronger southern IPR protection. Lower trade costs leads to faster rates of export-learning (χ_H increases from .04481 to .05807 and χ_L increases from .01315 to .01704). There is a redistribution across firms producing different varieties along the product

[22]The observed share of high-technology exports (out of manufactured exports) for the US in 2005 was .299 (down from .325 in 1990). The model generates $X_{XH}^S / \left(X_{XH}^S + X_{XL}^S\right) = 0.276$ for the 2005 benchmark.

	(1) 1990	(2) 2005	(3) $a_I \uparrow$	(4) $\tau \downarrow$
	$\tau = 1.54$ $a_I = 4.4$	$\tau = 1.33$ $a_I = 81.21$	$\tau = 1.54$ $a_I = 81.21$	$\tau = 1.33$ $a_I = 4.4$
w_N/w_S	2.2833	2.0437	2.0643	2.2792
δ	19.0631	18.1788	19.4528	17.8823
χ_H	.04481	.05933	.04543	.05807
χ_L	.01315	.01741	.01334	.01704
ϕ_H	.01028	.02538	.02750	.00936
ϕ_L	.00302	.00745	.00807	.00275
γ_{NH}	.02161	.01874	.02147	.01896
γ_{NL}	.75906	.71103	.75685	.71495
γ_{XH}	.01608	.01476	.01260	.01856
γ_{XL}	.18855	.21578	.17401	.23132
γ_{FH}	.00060	.00602	.00557	.00063
γ_{FL}	.00205	.02582	.02257	.00229
γ_{IH}	.00271	.00148	.00137	.00285
γ_{IL}	.00934	.00637	.00557	.01044
ι_S	.227	.012	.012	.227
L_{FH0}	.01446	.07700	.08258	.01297
L_{FL0}	.01457	.09697	.09828	.01392
Non-exporting firms' share	.790	.736	.784	.744
Foreign Affiliate share in VA	.04630	.39233	.39583	.04588
High prod. share of exports (sales)	.322	.276	.287	.309
MP Sales North market	.005	.058	.035	.008
MP Sales South market	.046	.335	.367	.041
MP Sales World	.051	.393	.401	.049
MP Sales share of North market	.0143	.1672	.1074	.0217
MP Sales share of South market	.0609	.5120	.5317	.0589
e^N	2.642	2.352	2.398	2.615
e^S	1.032	1.124	1.113	1.045
e^N/e^S	2.561	2.092	2.155	2.502
P_0^N	.00917	.00864	.00811	.00981
P_0^S	.01355	.01189	.01171	.01377
P_0^N/P_0^S	.677	.726	.692	.712
u_0^N	288.13	272.31	295.77	266.55
u_0^S	76.14	94.55	95.03	75.90
n_0	1,850,760	1,562,334	1,989,254	1,473,336

Table 1: Pre- and post-TRIPS benchmarks and two counterfactual scenarios.

cycle as the shares of northern firms that have not learned how to export decreases (γ_{NH} decreases from .02161 to .01896 and γ_{NL} decreases from .75906 to .71495) while the exported, MP-produced and imitator-produced varieties increase their shares.[23] Surprisingly, consumer welfare in both regions is worsened by trade liberalization (u_0^N decreases from 288.13 to 266.55 and u_0^S decreases from 76.14 to 75.90). Trade liberalization directly decreases prices of traded varieties in both regions. As exporting firms and multinational firms are owned jointly by southern and northern consumers, southern consumers benefit from the increase of market value of exporting firms (and multinationals). However, as northern exporters expand production in response to trade liberalization, resources are drawn from innovation into production (δ decreases from 19.0631 to 17.8823). Less innovation results in less product variety (n_0 decreases from 1,850,760 to 1,473,336) which puts upward pressure on the price index in both regions ($P_0^r \uparrow$). However, more exported varieties from the North also means that more product varieties are available to southern consumers. Therefore, the welfare-worsening effects of less innovation is less severe for the South.

3.2.1 Aggregate Sales and Aggregate Labor Demand

To understand the effects of the different policy changes, it is useful to look at labor demand by activity and across high productivity and low productivity firms. From expanding the left-hand side of (**??**) by $n_t L_t / n_t L_t$ and evaluating at time $t = 0$, it follows that $L_{R0} = g^{1+\beta}\delta a_N L_0$. It was seen earlier that aggregate labor demand from adaptive R&D by firms with productivity z is $L_{Fz0} = \phi_z^{1+\beta}\gamma_{Xz}\delta a_F L_0$. Similarly, using (**??**), we derive the aggregate demand from export-learning R&D activities, $L_{Xz0} = \chi_z^{1+\beta}\gamma_{Nz}\delta a_X L_0$. Aggregate labor demand from production in northern firms who have not yet learned how to export is $L_{Nz0}^{NProd} = c_z X_{Nz}^N L_0$. For northern exporters, aggregate labor demand from production for the home market is $L_{Xz0}^{NProd} = c_z X_{Xz} L_0$ and from production for the export market $L_{Xz0}^{SProd} = \tau c_z X_{Xz}^S L_0$. Foreign affiliates in the South and the local firms imitating them have

[23] γ_{XH} increases from .01608 to .01856, γ_{XL} increases from .18855 to .23132, γ_{FH} increases from .00060 to .00063, γ_{FL} increases from .00205 to .00229 and γ_{IH} and γ_{IL} increases from .00271 to .00285 and from .00934 to .01044, respectively.

aggregate labor demand $L_{jz0}^{SProd} = c_z X_{jz}^S L_0$ $(j = F, I)$ for production for the domestic market (South) and $L_{jz0}^{NProd} = \tau c_z X_{jz}^N L_0$ for production for the export market (North).

	(1) 1990	(2) 2005	(3) $a_I \uparrow$	(4) $\tau \downarrow$
	$\tau = 1.54$ $a_I = 4.4$	$\tau = 1.33$ $a_I = 81.21$	$\tau = 1.54$ $a_I = 81.21$	$\tau = 1.33$ $a_I = 4.4$
North $L_{N0} = 1$				
Non-production labor				
L_{R0}	.143	.136	.145	.134
Production labor				
L_{NH0}^{NProd}	.051	.042	.051	.042
L_{NL0}^{NProd}	.522	.466	.531	.461
L_{XH0}^{NProd}	.038	.033	.030	.041
L_{XL0}^{NProd}	.130	.141	.122	.149
L_{XH0}^{SProd}	.027	.034	.024	.037
L_{XL0}^{SProd}	.091	.147	.096	.136
South $L_{S0} = 2$				
Non-production labor				
L_{XH0}	.014	.020	.014	.019
L_{XL0}	.042	.065	.044	.062
L_{FH0}	.014	.077	.083	.013
L_{FL0}	.015	.097	.098	.014
Production labor				
L_{FH0}^{SProd}	.052	.348	.389	.046
L_{FL0}^{SProd}	.052	.438	.463	.049
L_{FH0}^{NProd}	.009	.080	.057	.012
L_{FL0}^{NProd}	.009	.101	.068	.013
L_{IH0}^{SProd}	.767	.278	.312	.677
L_{IL0}^{SProd}	.774	.351	.371	.726
L_{IH0}^{NProd}	.126	.064	.046	.178
L_{IL0}^{NProd}	.127	.081	.054	.191

Table 2: Pre- and post-TRIPS benchmarks and two counterfactual scenarios.

We calculate labor demand by activity and productivity type for the two benchmarks and for each of the counterfactual scenarios. The results are presented in Table 2. The top panel represents labor demand by activity and by productivity type for firms in the North, and the lower panel represents labor demand by activity and productivity type for firms in the South.[24] Going from the 1990 benchmark in Column 1 of Table 2 to the counterfactual with stronger IPR protection in Column 3, northern labor is redistributed mainly from exporters' production for the home market towards innovative activities and production by newly invented products that are not yet exported. (L_{R0} increases from .143 to .145 and L_{NL0}^{NProd} increases from .522 to .531 while L_{XH0}^{NProd} and L_{XL0}^{NProd} decrease from .038 to .030 and from .130 to .122 respectively). Comparing the benchmark in Column1 with the counterfactual with trade liberalization in Column 4, trade liberalization redistributes northern labor from innovative activities (L_{R0} decreases from .143 to .134) and production of newly invented (not yet exported varieties) towards exporting firms' production activities (L_{NH0}^{NProd} and L_{NL0}^{NProd} decrease while L_{Xz0}^{rProd} increase).

In the 1990 benchmark, 77.0 per cent of southern labor is employed in production by imitating firms for the home market and 12.7 per cent in imitating firms' production for the export market.[25] With stronger IPR protection (Column 3 of Table 2), these shares have dropped to 34.2 per cent and 5.0 per cent for the home and export market, respectively. At the same time, non-production employment in adaption to MP increases from 1.4 per cent to 9.0 per cent, while employment in foreign affiliate production for the southern market increases from 5.2 per cent to 42.6 per cent and employment in foreign affiliate production for the northern market increases from 0.9 per cent to 6.2 per cent. With stronger IPR protection, sales in the southern market becomes relatively more important for multinationals. With trade liberalization, the opposite occurs: sales in the southern market becomes relatively less important for multinationals and exports back to the northern market more relevant. In the counterfactual with only trade liberalization (Column 4 of Table 2), production activities in the South are directed more towards the northern market relative to the southern market. Employment in foreign affiliate production for the southern market decreases from 5.2 per cent to 4.7 per cent while employment in foreign affiliate production for the northern market increases from 0.9 per cent to 0.12 per cent. Employment in imitating southern firms selling

[24] When calibrating the model, recall that we set $L_{N0} = 1$ and $L_{S0} = 2$.

[25] $\sum_z L_{Iz}^{rProd}/2$ generates the percentage of labor employed in imitating firms' production for market r.

to the southern market decreases from 77.0 per cent to 70.1 per cent and for the northern market, the employment share increases from 12.7 per cent to 18.5 per cent.

Trade liberalization increases the share of MP sales in the northern market but decreases the share of MP sales in the southern market (in Table 1, comparing Column 1 and Column 4). In response to trade liberalization, southern firms devote more resources to producing for the export market (North) and less resources for production for the home market (South): L_{Iz0}^{SProd} decreases going from the 1990 benchmark in Column 1 of Table 2 to the counterfactual in Column 4 while L_{Iz0}^{NProd} increases and, similarly, L_{Fz0}^{SProd} decreases and L_{Fz0}^{NProd} increases. Looking at aggregate demand terms presented in Table 3 confirms that trade liberalization makes the northern market relatively more lucrative for firms with production in South while it makes the southern market relatively less attractive. Comparing Column 1 and Column 4 in Table 3, X_{Fz}^{S} and X_{Iz}^{S} decreases while X_{Fz}^{N} and X_{Iz}^{N} increases with trade liberalization.

	(1) 1990	(2) 2005	(3) $a_I \uparrow$	(4) $\tau \downarrow$
	$\tau = 1.54$ $a_I = 4.4$	$\tau = 1.33$ $a_I = 81.21$	$\tau = 1.54$ $a_I = 81.21$	$\tau = 1.33$ $a_I = 4.4$
Northern market				
X_{NH}^{N}	.0276	.0228	.0279	.0227
X_{NL}^{N}	.1739	.1553	.1769	.1537
X_{XH}^{N}	.0205	.0180	.0164	.0222
X_{XL}^{N}	.0432	.0471	.0407	.0497
X_{FH}^{N}	.0030	.0329	.0202	.0049
X_{FL}^{N}	.0019	.0253	.0147	.0032
X_{IH}^{N}	.0446	.0263	.0162	.0729
X_{IL}^{N}	.0275	.0203	.0118	.0479
Southern market				
X_{XH}^{S}	.0094	.0141	.0084	.0153
X_{XL}^{S}	.0198	.0369	.0209	.0342
X_{FH}^{S}	.0282	.1893	.2121	.0249
X_{FL}^{S}	.0175	.1459	.1545	.0164
X_{IH}^{S}	.4175	.1516	.1699	.3685
X_{IL}^{S}	.2581	.1168	.1237	.2421

Table 3: Aggregate demand (sales) by firm type and market.

3.3 Solving the model with a R&D subsidy to innovation

The result that trade liberalization is welfare-worsening is surprising. We saw in the previous section that trade liberalization draws labor resources from innovation into production. To understand this result we revisit the theory of second best. If there are market failures, for example, if firms do not internalize the social benefit of innovation via knowledge spillovers to other firms, there will be too little innovation in equilibrium. Similarly, if there are labor market frictions, trade protection can insure against too little labor being allocated to innovative activities. The first best solution would of course be to address these market failures and frictions directly, but trade protection offers a second best policy option.

To analyze this issue, we solve the model with a R&D-subsidy to northern firms. Let s_R denote the fraction of the firm's cost of innovative R&D that is subsidized by the government. As in Segerstrom (1998) we assume that the government finances the subsidy s_R by means of lump-sum taxation. Free entry in the North implies that (**??**) becomes

$$qv_{NLt} + (1-q)\,v_{NHt} = \frac{w^N a_N g^\beta}{n_t^\theta}\,(1-s_R)\,.$$

Consequently, the northern steady-state no-arbitrage condition is $\frac{q\pi_{NHt}+(1-q)\pi_{NLt}}{\rho+\theta g} = \frac{w^N a_N g^\beta}{n_t^\theta}\,(1-s_R)$ where the right-hand side now reflects the lower cost of innovation due to the subsidy. The northern steady-state no-arbitrage condition is

$$\frac{\frac{1}{\sigma-1}\left(\frac{qc_L X_{NL}^N}{\gamma_{NL}} + \frac{(1-q)c_H X_{NH}^N}{\gamma_{NH}}\right)}{\rho+\theta g} = a_N g^\beta \delta\,(1-s_R)\,.$$

In Table 4 we present the results from this exercise. Columns 1 and 2 reproduce the 1990 benchmark and the counterfactual with trade liberalization (from $\tau = 1.54$ to $\tau = 1.33$). Columns 3 and 4 present the results from this trade liberalization example but with an innovation subsidy $s_R = 0.2$, and Columns 5 and 6 the results with an innovation subsidy $s_R = 0.85$. Without any R&D subsidy to innovation, trade liberalization (that is not accompanied by stronger IPR protection in the South) worsens consumer welfare in both regions.

With a subsidy of 0.2 or higher, trade liberalization leads to higher consumer welfare in the South but northern consumer welfare is still worsened by trade liberalization. With a subsidy of 0.85 or higher, trade liberalization is welfare-improving for consumers in both regions. On the one hand, lower trade costs encourages international trade and consumers in both regions benefit from lower prices on imported varieties, and there is also an increase in the share of varieties on the world market that are produced in the lower-wage South. On the other hand, without the subsidy to innovation in the North, so much resources are allocated to production in the North that innovation suffers and consumers experience less product variety. The R&D subsidy can correct for this by preventing product variety from falling so much that the negative welfare effect from less innovation dominates the positive welfare effect of lower prices from trade liberalization.

4 Concluding Comments

This paper evaluates the impact of stronger intellectual property rights protection for multinational production (MP) and developing countries. We calibrate a North-South trade model with heterogeneous firms to match the world economy before and after the TRIPS agreement went into effect. Firms in the North engage in innovative R&D to develop new product varieties and then learn their productivities. Firms in the North can engage in export-learning R&D to access the southern market and can then engage in adaptive R&D (FDI) to learn how to produce their products in the lower-wage South via MP. Once any foreign affiliate of a northern firm starts producing in the South, it faces the risk of imitation from southern firms. Stronger IPR protection in the South (TRIPS) is modelled as a decrease in this imitation rate.

We find that stronger IPR protection in the South (TRIPS) induced foreign affiliates of northern firms to increase their R&D expenditures and resulted in a faster rate of technology transfer within these multinational firms, consistent with the empirical evidence in Branstetter, Fisman and Foley (2006). As a result of TRIPS, more product varieties ended up being produced in the South and exports of new products increased, consistent with the empirical evidence in Branstetter, Fisman, Foley and Saggi (2011). TRIPS encouraged firms to adapt to

	(1) 1990 $s_R = 0$ $\tau = 1.54$	(2) $\tau \downarrow$ $s_R = 0$ $\tau = 1.33$	(3) 1990 w. $s_R = 0.2$ $\tau = 1.54$	(4) $\tau \downarrow$ $s_R = 0.2$ $\tau = 1.33$	(5) 1990 w. $s_R = 0.85$ $\tau = 1.54$	(6) $\tau \downarrow$ $s_R = 0.85$ $\tau = 1.33$
w_N/w_S	2.2833	2.2792	2.4007	2.3764	5.1792	4.3350
δ	19.0631	17.8823	23.3358	21.9887	74.4183	73.8278
χ_H	.04481	.05807	.03719	.04874	.00509	.00881
χ_L	.01315	.01704	.01092	.01430	.00149	.00259
ϕ_H	.01028	.00936	.01012	.00914	.01512	.01143
ϕ_L	.00302	.00275	.00297	.00268	.00444	.00335
γ_{NH}	.02161	.01896	.02350	.02075	.03721	.03485
γ_{NL}	.75906	.71495	.78695	.74543	.93115	.91178
γ_{XH}	.01608	.01856	.01455	.01712	.00291	.00500
γ_{XL}	.18855	.23132	.16239	.20268	.02558	.04424
γ_{FH}	.00060	.00063	.00053	.00056	.00016	.00021
γ_{FL}	.00205	.00229	.00174	.00196	.00041	.00054
γ_{IH}	.00271	.00285	.00242	.00257	.00072	.00094
γ_{IL}	.00934	.01044	.00792	.00893	.00186	.00244
ι_S	.227	.227	.227	.227	.227	.227
L_{FH0}	.01446	.01297	.01548	.01400	.02207	.02149
L_{FL0}	.01457	.01392	.01488	.01428	.01670	.01637
Non-exporting firms' share	.790	.744	.819	.775	.971	.950
Foreign Affiliate share in VA	.04630	.04588	.04678	.04638	.05268	.05292
High prod./ exports sales	.322	.309	.333	.320	.388	.386
MP Sales North market	.005	.008	.005	.008	.012	.014
MP Sales South market	.046	.041	.046	.0042	.038	.037
MP Sales World	.051	.049	.051	.0050	.050	.051
MP Sales/North market	.0143	.0217	.0147	.0219	.0346	.0370
MP Sales/South market	.0609	.0589	.0612	.0594	.0633	.0630
e^N	2.642	2.615	2.771	2.721	5.662	4.736
e^S	1.032	1.045	1.029	1.042	1.006	1.009
e^N/e^S	2.561	2.502	2.694	2.612	5.630	4.692
P_0^N	.00917	.00981	.00722	.00762	.00286	.00240
P_0^S	.01355	.01377	.01083	.01091	.00374	.00335
P_0^N/P_0^S	.677	.712	.667	.698	.766	.713
u_0^N	288.13	266.55	383.81	357.23	1977.80	1980.21
u_0^S	76.14	75.90	94.98	95.50	269.11	301.00
n_0	1,850,760	1,473,336	3,807,001	3,079,571	238,145,619	231,474,984

Table 4: 1990 benchmark with trade liberalization in presence of innovative R&D subsidy

MP and led to an increase in the share of world GDP produced via MP, consistent with data from UNCTAD (2012). TRIPS led to more employment in production in foreign affiliates in the South and more employment in innovative R&D in the North, consistent with the empirical evidence in Arkolakis et al (2014) for the time period 1999-2009. TRIPS also stimulated innovative R&D spending by northern firms and resulted in faster economic growth in the South, consistent with the empirical evidence in Gould and Gruben (1996). When we solve the model numerically for plausible parameter values, we find that TRIPS led to significantly higher long-run southern consumer welfare. In contrast, the trade liberalization that occurred led to more export-learning and actually lowered long-run southern consumer welfare by diverting northern resources away from innovative activities (to production for export).

References

[1] Antras, Pol and Elhanan Helpman (2008), "Contractual Frictions and Global Sourcing", In *The Organization of Firms in a Global Economy* eds. Elhanan Helpman, Dalia Marin and Thierry Verdier, 9-54. Cambridge, MA: Harvard University Press.

[2] Arkolakis, Costas (2010), "Market Penetration Costs and New Consumers Margin in International Trade", *Journal of Political Economy*, 118(6), 1151-1199.

[3] Arkolakis, C., N. Ramondo, A. Rodríguez-Clare, and S. Yeaple (2014), "Innovation and Production in the Global Economy," NBER Working Paper No. 18972, National Bureau of Economic Research, Inc.

[4] Basu, Susanto (1996), "Procyclical Productivity: Increasing Returns or Cyclical Utilization," *Quarterly Journal of Economics*, 111(3), 709-751.

[5] Bernard, Andrew B., Jonathan Eaton, Bradford Jensen and Samuel S. Kortum (2003), "Plants and Productivity in International Trade", *American Economic Review*, 93(4), 1268-1290.

[6] Bernard, Andrew B., J. Bradford Jensen, Stephen J Redding and Peter K. Schott (2007), "Firms in International Trade", *Journal of Economic Perspectives*, 21(3), 105-130.

[7] Bernard, Andrew B. and J. Bradford Jensen (2004), "Why Some Firms Export", *Review of Economics and Statistics*, 86(2), 561-569.

[8] Branstetter, Lee and Kamal Saggi (2011), "Intellectual Property Rights, Foreign Direct Investment and Industrial Development," *Economic Journal*, 121(555), 1161-1191.

[9] Branstetter, Lee, Raymond Fisman, Fritz Foley and Kamal Saggi (2011), "Does Intellectual Property Rights Reform Spur Industrial Development?", *Journal of International Economics*, 83(1), 27-36.

[10] Branstetter, Lee, Raymond Fisman and Fritz Foley (2006), "Do Stronger Intellectual Property Rights Increase International Technology Transfer? Empirical Evidence from U.S. Firm-Level Panel Data," *Quarterly Journal of Economics*, 121(1), 321-349.

[11] Castro, Luis, Ben Li, Keith E. Maskus and Yiqing Xie (2013), "Fixed Export Costs and Firm-Level Export Behavior", University of Colorado at Boulder, Mimeo.

[12] Conconi, Paola, Andre Zapir and Maurizio Zanardi (2016), "The Internationalization Process of Firms: From Exports to FDI", *Journal of International Economics*, 99, 16-30.

[13] Eaton, Jonathan and Samuel Kortum, "Technology, Geography, and Trade", *Econometrica*, 70(5), 1741-1779.

[14] Federal Reserve Bank of St. Louis (2011), *Federal Reserve Economic Data*.

[15] Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer (2015), "The Next Generation of the Penn World Table" forthcoming *American Economic Review*, available for download at www.ggdc.net/pwt

[16] French, Kenneth R. and James M. Poterba (1991), "Investor Diversification and International Equity Markets", *American Economic Review*, 81(2), 222-226.

[17] Glass, Amy and Kamal Saggi (2002), "Intellectual Property Rights and Foreign Direct Investment", *Journal of International Economics*, 56(2), 387-410.

[18] Glass, Amy and Xiaodong Wu (2007), "Intellectual Property Rights and Quality Improvement," *Journal of Development Economics*, 82(2), 393-415.

[19] Gould, David M. and William C. Gruben (1996), "The Role of Intellectual Property Rights in Economic Growth", *Journal of Development Economics*, 48(2), 323-350.

[20] Helpman, Elhanan, Marc J. Melitz and Stephen R. Yeaple (2004), "Export Versus FDI with Heterogeneous Firms", *American Economic Review*, 94(1), 300-316.

[21] Irarrazabal, Alfonso, Andreas Moxnes and Luca David Opromolla (2013), "The Margins of Multinational Production and the Role of Intra-Firm Trade", *Journal of Political Economy*, 121(1), 74-126.

[22] Jakobsson, Amanda (2013), "A Simple Model of TRIPS", Stockholm School of Economics, mimeo.

[23] Jakobsson, Amanda and Paul S. Segerstrom (2016), "In Support of the TRIPs Agreement," Stockholm School of Economics and Singapore Management University, mimeo.

[24] Jones, Charles I. (1995), "R&D-based Models of Economic Growth," *Journal of Political Economy*, 103(4), 759-784.

[25] Kortum, Samuel (1993), "Equilibrium R&D and the Patent-R&D Ratio: U.S. Evidence," *American Economic Review*, 83(2), 450-457.

[26] Lileeva, Alla and Daniel Trefler (2010), "Improved Access to Foreign Markets Raises Plant-Level Productivity... for Some Plants," *Quarterly Journal of Economics*, 125(3), 1051-1099.

[27] Mayer, Thierry and Gianmarco Ottaviano (2008), "The Happy Few: The Internationalisation of European Firms," *Intereconomics: Review of European Economic Policy*, 43(3), 135-148.

[28] McCaig, Brian and Pavcnik, Nina (2013), “Export Markets and labor reallocation,” NBER Working Paper No. 19616.

[29] McMillan, Margaret S. and Dani Rodrik (2011), “Globalization, Structural Change and Productivity Growth,” NBER Working Paper No. 17143.

[30] Mehra, Rajnish and Edward Prescott (1985), “The Equity Premium: A Puzzle,” *Journal of Monetary Economics*, 15(2), 145-161.

[31] Melitz, Marc J. (2003), “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity,” *Econometrica*, 71(6), 1695-1725.

[32] Norrbin, S.C. (1993), “The Relationship between Price and Marginal Cost in US Industry: A Contradiction,” *Journal of Political Economy*, 101(6), 1149-1164.

[33] Novy, Dennis (2013), “Gravity Redux: Measuring International Trade Costs with Panel Data,” *Economic Inquiry*, 51(1), 101-121.

[34] Ramondo, N. and A. Rodríguez-Clare (2013), “Trade, Multinational Production, and the Gains from Openness,” *Journal of Political Economy*, 121(2), 273 - 322.

[35] Segerstrom, Paul S. (1998), ”Endogenous Growth Without Scale Effects,” *American Economic Review*, 88(5), 1290-1310.

[36] Tesar, Linda L. and Ingrid M. Werner (1995), “Home bias and high turnover”, *Journal of International Money and Finance*, 14(4), 467-492.

[37] Tintelnot, Felix (2015), “Global Production with Export Platforms”, University of Chicago, mimeo.

[38] UNCTAD (2011), *FDI Statistics*, United Nations Conference on Trade and Development, at http://www.unctad.org/fdistatistics.

[39] UNCTAD (2012), *World Investment Report 2012*, United Nations Conference on Trade and Development.

[40] Venturini, F. (2012),“Looking into the black box of Schumpeterian growth theories: An empirical assessment of R&D races”, *European Economic Review*, 56(8), 1530-1545.

[41] World Bank (2016), *World Development Indicators*, Washington, D.C.

[42] World Bank (2003), *World Development Indicators*, Washington, D.C.

Appendix: Solving The Model

In this appendix, calculations done to solve the model are spelled out in more detail.

Households

The static consumer optimization problem is

$$\max_{x_t(\cdot)} \int_0^{n_t} x_t(\omega)^\alpha d\omega \qquad \text{s.t.} \qquad \dot{y}(\omega) = p_t(\omega)x_t(\omega),\, y(0) = 0,\, y(n_t) = c_t.$$

where $y(\omega)$ is a new state variable and $\dot{y}(\omega)$ is the derivative of y with respect to ω. The Hamiltonian function for this optimal control problem is

$$H = x_t(\omega)^\alpha + \gamma(\omega)p_t(\omega)x_t(\omega)$$

where $\gamma(\omega)$ is the costate variable. The costate equation $\frac{\partial H}{\partial y} = 0 = -\dot{\gamma}(\omega)$ implies that $\gamma(\omega)$ is constant across ω. $\frac{\partial H}{\partial x} = \alpha x_t(\omega)^{\alpha-1} + \gamma \cdot p_t(\omega) = 0$ implies that

$$x_t(\omega) = \left(\frac{\alpha}{-\gamma \cdot p_t(\omega)}\right)^{1/(1-\alpha)}.$$

Substituting this back into the budget constraint yields

$$\begin{aligned} e_t &= \int_0^{n_t} p_t(\omega)x_t(\omega)d\omega = \int_0^{n_t} p_t(\omega)\left(\frac{\alpha}{-\gamma \cdot p_t(\omega)}\right)^{1/(1-\alpha)} d\omega \\ &= \left(\frac{\alpha}{-\gamma}\right)^{1/(1-\alpha)} \int_0^{n_t} p_t(\omega)^{\frac{1-\alpha-1}{1-\alpha}} d\omega. \end{aligned}$$

Now $\sigma \equiv \frac{1}{1-\alpha}$ implies that $1 - \sigma = \frac{1-\alpha-1}{1-\alpha} = \frac{-\alpha}{1-\alpha}$, so

$$\frac{e_t}{\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega} = \left(\frac{\alpha}{-\gamma}\right)^{1/(1-\alpha)}.$$

It immediately follows that the consumer demand function is

$$x_t(\omega) = \frac{p_t(\omega)^{-\sigma} e_t}{P_t^{1-\sigma}} \qquad \textbf{(??)}$$

where $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ is an index of consumer prices.

Substituting this consumer demand function back into the consumer utility function yields

$$u_t = \left[\int_0^{n_t} x_t(\omega)^\alpha \, d\omega \right]^{\frac{1}{\alpha}} = \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha} e_t^\alpha}{P_t^{(1-\sigma)\alpha}} d\omega \right]^{\frac{1}{\alpha}} = e_t \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha}}{P_t^{(1-\sigma)\alpha}} d\omega \right]^{\frac{1}{\alpha}}.$$

Taking into account that $-\sigma\alpha = \frac{-\alpha}{1-\alpha} = 1 - \sigma$, consumer utility can be simplified further to

$$u_t = \frac{e_t}{P_t^{1-\sigma}} \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} \, d\omega \right]^{\frac{1}{\alpha}} = \frac{e_t}{P_t^{1-\sigma}} \left[P_t^{1-\sigma} \right]^{\frac{1}{\alpha}} = \frac{e_t}{P_t^{1-\sigma}} P_t^{-\sigma} = \frac{e_t}{P_t}$$

or

$$\ln u_t = \ln e_t - \ln P_t.$$

The individual household takes the prices of all products as given, as well as how prices change over time, so the $\ln P_t$ term can be ignored in solving the household's dynamic optimization problem. This problem simplifies to:

$$\max_{c_t} \int_0^\infty e^{-(\rho - g_L)t} \ln e_t \, dt \qquad \text{s.t.} \qquad \dot{a}_t = w_t + r_t a_t - g_L a_t - e_t,$$

where a_t represents the asset holding of the representative consumer, w_t is the wage rate and r_t is the interest rate.

The Hamiltonian function for this optimal control problem is

$$H = e^{-(\rho - g_L)t} \ln e_t + \lambda_t \left[w_t + r_t a_t - g_L a_t - e_t \right]$$

where λ_t is the relevant costate variable. The costate equation $-\dot{\lambda}_t = \frac{\partial H}{\partial \tilde{a}} = \lambda_t \left[r_t - g_L \right]$ implies that

$$\frac{\dot{\lambda}_t}{\lambda_t} = g_L - r_t.$$

$\partial H / \partial d_t = e^{-(\rho - g_L)t} \frac{1}{e_t} - \lambda_t = 0$ implies that $e^{-(\rho - g_L)t} \frac{1}{e_t} = \lambda_t$. Taking logs of both sides yields $-(\rho - g_L)t - \ln e_t = \ln \lambda_t$ and then differentiating with respect to time yields

$$-(\rho - g_L) - \frac{\dot{e}_t}{e_t} = \frac{\dot{\lambda}_t}{\lambda_t} = g_L - r_t.$$

It immediately follows that

$$\frac{\dot{e}_t}{e_t} = r_t - \rho. \qquad \textbf{(??)}$$

Steady-State Dynamics

We will now derive some steady-state equilibrium implications of the model.

The export-learning rate is $\chi_z \equiv (\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt})/n_{Nzt}$. It is constant over time in any steady-state equilibrium since

$$\begin{aligned}\chi_z &\equiv \frac{\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Nzt}} = \frac{\dot{n}_{Xzt}}{n_{Xzt}}\frac{n_{Xzt}/n_t}{n_{Nzt}/n_t} + \frac{\dot{n}_{Fzt}}{n_{Fzt}}\frac{n_{Fzt}/n_t}{n_{Nzt}/n_t} + \frac{\dot{n}_{Izt}}{n_{Izt}}\frac{n_{Izt}/n_t}{n^z_{Nt}/n_t} \\ &= g\frac{\gamma^z_X}{\gamma^z_N} + g\frac{\gamma^z_F}{\gamma_{Nz}} + g\frac{\gamma_{Iz}}{\gamma_{Nz}}.\end{aligned}$$

The FDI rate is $\phi_z \equiv (\dot{n}_{Fzt} + \dot{n}_{Izt})/n_{Xzt}$. It is constant over time in any steady-state equilibrium since

$$\phi_z \equiv \frac{\dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Xzt}} = \frac{\dot{n}_{Fzt}}{n_{Fzt}}\frac{n_{Fzt}/n_t}{n_{Xzt}/n_t} + \frac{\dot{n}_{Izt}}{n_{Izt}}\frac{n_{Izt}/n_t}{n_{Xzt}/n_t} = g\frac{\gamma_{Fz}}{\gamma_{Xz}} + g\frac{\gamma_{Iz}}{\gamma_{Xz}}.$$

The imitation rate of foreign affiliates is $\iota_S \equiv \dot{n}_{Izt}/n_{Fzt}$. It is constant over time in steady-state equilibrium since

$$\iota_S \equiv \frac{\dot{n}_{Izt}}{n_{Fzt}} = \frac{\dot{n}_{Izt}}{n_{Izt}}\frac{n_{Izt}/n_t}{n_{Fzt}/n_t} = g\frac{\gamma_{Iz}}{\gamma_{Fz}}.$$

We can now solve for γ_{Xz}. By differentiating the variety condition for z-productivity firms $q_z n_t = n_{Nzt} + n_{Xzt} + n_{Fzt} + n_{Izt}$, we obtain that

$$\begin{aligned} q_z\dot{n}_t &= \dot{n}_{Nzt} + \dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt} \\ q_z\frac{\dot{n}_t}{n_t} &= \frac{\dot{n}_{Nzt} + \dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}}{n_t} \\ q_z g &= \frac{\dot{n}_{Nzt}}{n_{Nzt}}\frac{n_{Nzt}}{n_t} + \frac{\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Nzt}}\frac{n_{Nzt}}{n_t} \\ q_z g &= g\gamma_{Nz} + \chi_z\gamma_{Nz}\end{aligned}$$

and solving for γ_{Nz} yields

$$\gamma_{Nz} = q^z\frac{g}{g+\chi_z}, \qquad (z = H, L). \qquad \textbf{(??)}$$

To solve for γ_{Xz}, note that

$$\begin{aligned}\chi_z &= \frac{\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Nzt}} \\ &= \frac{\dot{n}_{Xzt}}{n_{Xzt}}\frac{n_{Xzt}/n_t}{n_{Nzt}/n_t} + \frac{\dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Xzt}}\frac{n_{Xzt}/n_t}{n_{Nzt}/n_t} \\ &= (g+\phi_z)\frac{\gamma_{Xz}}{\gamma_{Nz}}\end{aligned}$$

from which it follows that $\gamma_{Xz} = \gamma_{Nz}\left(\frac{\chi_z}{g+\phi_z}\right)$. Inserting the steady-state expression for γ_{Nz} (**??**) yields

$$\gamma_{Xz} = q_z \frac{\chi_z}{g+\chi_z} \frac{g}{g+\phi_z}, \qquad (z = H, L). \qquad \textbf{(??)}$$

To solve for γ_{Fz}, note that

$$\begin{aligned} \phi_z &= \frac{\dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Xzt}} \\ &= \frac{\dot{n}_{Fzt}}{n_{Fzt}} \frac{n_{Fzt}/n_t}{n_{Xzt}/n_t} + \frac{\dot{n}_{Izt}}{n_{Fzt}} \frac{n_{Fzt}/n_t}{n_{Xzt}/n_t} \\ &= (g + \iota_S) \frac{\gamma_{Fz}}{\gamma_{Xz}} \end{aligned}$$

from which it follows that $\gamma_{Fz} = \gamma_{Xz}\phi_z/\left(g+\iota_S\right)$. Inserting the steady-state expressions for γ_{Xz} from (**??**) yields

$$\gamma_{Fz} = q_z \frac{\chi_z}{g+\chi_z} \frac{\phi_z}{g+\phi_z} \frac{g}{g+\iota_S}, \qquad (z = H, L). \qquad \textbf{(??)}$$

To solve for γ_{Iz}, note that

$$\begin{aligned} \iota_S &\equiv \frac{\dot{n}_{Izt}}{n_{Fzt}} \\ &= \frac{\dot{n}_{Izt}}{n_{Izt}} \frac{n_{Izt}/n_t}{n_{Fzt}/n_t} \\ &= g \frac{\gamma_{Iz}}{\gamma_{Fz}}. \end{aligned}$$

from which it follows that $\gamma_{Iz} = \left(\iota_S/g\right)\gamma_{Fz}$. Inserting the steady-state expressions for γ_{Fz} from (**??**) yields

$$\gamma_{Iz} = q_z \frac{\chi_z}{g+\chi_z} \frac{\phi_z}{g+\phi_z} \frac{\iota_S}{g+\iota_S}, \qquad (z = H, L). \qquad \textbf{(??)}$$

Because prices differ between the North and the South due to trade costs, and because the set of varieties available to consumers in the South is a subset of the set of varieties available to consumers in the North, we need to define a different price index for each region. Let P_t^N be the price index for the North and P_t^S be the price index for the South. Given the definition of the price index $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$, it follows that the northern price index satisfies

$$\begin{aligned}\left(P_t^N\right)^{1-\sigma} &= \int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \\ &= \sum_{z=H,L} \left[n_{Nzt}\left(p_{Nz}^N\right)^{1-\sigma} + n_{Xzt}\left(p_{Nz}^N\right)^{1-\sigma} + n_{Fzt}\left(p_{Fz}^N\right)^{1-\sigma} + n_{Izt}\left(p_{Iz}^N\right)^{1-\sigma}\right] \\ &= \sum_{z=H,L} \left[\gamma_{Nz} n_t\left(p_{Nz}^N\right)^{1-\sigma} + \gamma_{Xz} n_t\left(p_{Nz}^N\right)^{1-\sigma} + \gamma_{Fz} n_t\left(p_{Fz}^N\right)^{1-\sigma} + \gamma_{Iz} n_t\left(p_{Iz}^N\right)^{1-\sigma}\right] \\ &= \sum_{z=H,L} \left[\gamma_{Nz}\left(p_{Nz}^N\right)^{1-\sigma} + \gamma_{Xz}\left(p_{Nz}^N\right)^{1-\sigma} + \gamma_{Fz}\left(p_{Fz}^N\right)^{1-\sigma} + \gamma_{Iz}\left(p_{Iz}^N\right)^{1-\sigma}\right] n_t\end{aligned}$$

where the term in brackets is constant over time. Likewise, the southern price index satisfies

$$\begin{aligned}\left(P_t^S\right)^{1-\sigma} &= \int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \\ &= \sum_{z=H,L} \left[n_{Xzt}\left(p_{Nz}^S\right)^{1-\sigma} + n_{Fzt}\left(p_{Fz}^S\right)^{1-\sigma} + n_{Izt}\left(p_{Iz}^S\right)^{1-\sigma}\right] \\ &= \sum_{z=H,L} \left[\gamma_{Xz} n_t\left(p_{Nz}^S\right)^{1-\sigma} + \gamma_{Fz} n_t\left(p_{Fz}^S\right)^{1-\sigma} + \gamma_{Iz} n_t\left(p_{Iz}^S\right)^{1-\sigma}\right] \\ &= \sum_{z=H,L} \left[\gamma_{Xz}\left(p_{Nz}^S\right)^{1-\sigma} + \gamma_{Fz}\left(p_{Fz}^S\right)^{1-\sigma} + \gamma_{Iz}\left(p_{Iz}^S\right)^{1-\sigma}\right] n_t\end{aligned}$$

where the term in brackets is constant over time. Thus, we obtain

$$\left(P_t^N\right)^{1-\sigma} = \sum_j \sum_z \left[\gamma_{jz}\left(p_{jz}^N\right)^{1-\sigma}\right] n_t \qquad \textbf{(??)}$$

$$\left(P_t^S\right)^{1-\sigma} = \sum_{j\neq N} \sum_z \left[\gamma_{jz}\left(p_{jz}^S\right)^{1-\sigma}\right] n_t. \qquad \textbf{(??)}$$

The representative northern consumer's static utility is $u_t^N = e_t^N/P_t^N$ and the representative southern consumer's static utility is $u_{St} = c_{St}/P_{St}$. In any steady-state equilibrium, consumer expenditure is constant but the price indexes P_t^N and P_t^S fall over time, and therefore consumer utility grows over time in steady-state equilibrium. Define $g_u \equiv \dot{u}_t^N/u_t^N = \dot{u}_t^S/u_t^S$. It is straightforward to see that $\dot{u}_t^N/u_t^N = -\dot{P}_t^N/P_t^N = g/(\sigma-1)$.

Product Markets

A northern firm with productivity z earns the flow of domestic profits

$$\pi_{Nzt} = \left(p_{Nz}^N - c_z w^N\right) x_{Nzt} L_t^N$$

where x_{Nzt} is the quantity demanded by the typical northern consumer of the northern firm's product. From the earlier demand function, it follows that $x_{Nzt} = \left(p_{Nz}^{N}\right)^{-\sigma} e^{N} / \left(P_{t}^{N}\right)^{1-\sigma}$. Hence, we can write a northern firm's profit flow as:

$$\pi_{Nzt} = \left(p_{Nz}^{N} - c_z w^{N}\right) \frac{\left(p_{Nz}^{N}\right)^{-\sigma} e^{N} L_{t}^{N}}{\left(P_{t}^{N}\right)^{1-\sigma}}.$$

Maximizing π_{Nzt} with respect to p_{Nz}^{N} yields the first-order condition

$$\frac{\partial \pi_{Nzt}}{\partial p_{Nz}} = \left[(1-\sigma)\left(p_{Nz}^{N}\right)^{-\sigma} + \sigma c_z w^{N}\left(p_{Nz}^{N}\right)^{-\sigma-1}\right] \frac{e^{N} L_{t}^{N}}{\left(P_{t}^{N}\right)^{1-\sigma}} = 0,$$

which implies that $(1-\sigma)\left(p_{Nz}^{N}\right)^{-\sigma} + \sigma c_z w^{N}\left(p_{Nz}^{N}\right)^{-\sigma-1} = 0$ since $\frac{e^{N} L_{t}^{N}}{\left(P_{t}^{N}\right)^{1-\sigma}} \neq 0$. Dividing by $\left(p_{Nz}^{N}\right)^{-\sigma}$ yields $\frac{\sigma c_z w^{N}}{p_{Nz}^{N}} = \sigma - 1$ or

$$p_{Nz}^{N} = \frac{\sigma c_z w^{N}}{\sigma - 1} = \frac{c_z w^{N}}{\alpha}.$$

To demonstrate the second equality, first note that $\sigma \equiv \frac{1}{1-\alpha}$ implies that $\sigma - 1 = \frac{1-(1-\alpha)}{1-\alpha} = \frac{\alpha}{1-\alpha}$. It follows that $\frac{\sigma}{\sigma-1} = (\frac{1}{1-\alpha})/(\frac{\alpha}{1-\alpha}) = \frac{1}{\alpha}$. Plugging the prices back into the profit expression, we obtain

$$\begin{aligned}
\pi_{Nzt} &= \left(p_{Nz}^{N} - c_z w^{N}\right) \frac{\left(p_{Nz}^{N}\right)^{-\sigma} e^{N} L_{t}^{N}}{\left(P_{t}^{N}\right)^{1-\sigma}} \\
&= \left(\frac{c_z w^{N}}{\alpha} - c_z w^{N}\right) \frac{\left(p_{Nz}^{N}\right)^{-\sigma} e^{N} L_{t}^{N}}{\left(P_{t}^{N}\right)^{1-\sigma}} \\
&= \frac{c_z w^{N}}{\sigma - 1}\left[\frac{\left(p_{Nz}^{N}\right)^{-\sigma} e^{N} L_{t}^{N}}{\left(P_{t}^{N}\right)^{1-\sigma}}\right]
\end{aligned}$$

where we have used that $\frac{1}{\alpha} - 1 = \frac{\sigma}{\sigma-1} - \frac{\sigma-1}{\sigma-1} = \frac{1}{\sigma-1}$. It turns out to be convenient to rewrite profits by multiplying the RHS by $\frac{L_t}{L_t}\frac{n_{Nzt}}{n_{Nzt}}\frac{n_t}{n_t}$:

$$\pi_{Nzt} = \frac{c_z w^{N}}{\sigma - 1}\left[\frac{\left(p_{Nz}^{N}\right)^{-\sigma} e^{N} L_{t}^{N} n_{Nzt}}{\left(P_{t}^{N}\right)^{1-\sigma} L_t}\right] \frac{L_t}{n_t \frac{n_{Nzt}}{n_t}}.$$

Now $\gamma_{Nz} \equiv \frac{n_{Nzt}}{n_t}$ is constant over time, $X_{Nz}^{N} \equiv \frac{\left(p_{Nz}^{N}\right)^{-\sigma} e^{N} L_{t}^{N} n_{Nzt}}{\left(P_{t}^{N}\right)^{1-\sigma} L_t}$ is constant over time since

$\left(P_t^N\right)^{1-\sigma}$ grows at the same rate g as n_{Nzt}. Thus we can write π_{Nzt} more simply as:

$$\pi_{Nzt} = \frac{c_z w^N X_{Nz}^N}{(\sigma - 1)\,\gamma_{Nz}} \frac{L_t}{n_t}. \qquad \textbf{(??)}$$

A northern exporting firm earns the flow of global profits

$$\pi_{Xzt} = \left(p_{Xz}^N - c_z w^N\right) x_{Xzt}^N L_t^N + \left(p_{Xz}^S - \tau c_z w^N\right) x_{Xzt}^S L_t^S$$

where $x_{Xzt}^N = \left(p_{Xz}^N\right)^{-\sigma} e^N / \left(P_t^N\right)^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the northern exporting firm's product and $x_{Xzt}^S = \left(p_{Xz}^S\right)^{-\sigma} e^S / \left(P_t^S\right)^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the northern exporting firm's product. Hence, we can write a northern exporting firm's global profit flow as:

$$\pi_{Xzt} = \left(p_{Xz}^N - c_z w^N\right) \frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t\right)^{1-\sigma}} + \left(p_{Xz}^S - \tau c_z w^N\right) \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}}.$$

Maximizing π_{Xzt} with respect to p_{Xz}^N yields the first-order condition

$$\frac{\partial \pi_{Xzt}}{\partial p_{Xz}^N} = \left[(1-\sigma)\left(p_{Xz}^N\right)^{-\sigma} + \sigma c_z w^N \left(p_{Xz}^N\right)^{-\sigma-1}\right] \frac{e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} = 0,$$

which implies that $(1-\sigma)\left(p_{Xz}^N\right)^{-\sigma} + \sigma c_z w^N \left(p_{Xz}^N\right)^{-\sigma-1} = 0$ since $\frac{e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} \neq 0$. Dividing by $\left(p_{Xz}^N\right)^{-\sigma}$ yields $\frac{\sigma c_z w^N}{p_{Xz}^N} = \sigma - 1$ or

$$p_{Xz}^N = \frac{\sigma c_z w^N}{\sigma - 1} = \frac{c_z w^N}{\alpha}.$$

Similarly, maximizing π_{Xzt} with respect to p_{Xz}^S yields the first-order condition

$$\frac{\partial \pi_{Xzt}}{\partial p_{Xz}^S} = \left[(1-\sigma)\left(p_{Xz}^S\right)^{-\sigma} + \sigma \tau c_z w^N \left(p_{Xz}^S\right)^{-\sigma-1}\right] \frac{e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} = 0,$$

which implies that $(1-\sigma)\left(p_{Xz}^S\right)^{-\sigma} + \sigma \tau c_z w^N \left(p_{Xz}^S\right)^{-\sigma-1} = 0$. Dividing by $\left(p_{Xz}^S\right)^{-\sigma}$ yields $\frac{\sigma \tau c_z w^N}{p_{Xz}^S} = \sigma - 1$ or

$$p_{Xz}^S = \frac{\sigma \tau c_z w^N}{\sigma - 1} = \frac{\tau c_z w^N}{\alpha}.$$

Plugging the prices back into the profit expression, we obtain

$$\begin{aligned}\pi_{Xzt} &= \left(p_{Xz}^N - c_z w^N\right) \frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} + \left(p_{Xz}^S - \tau c_z w^N\right) \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} \\ &= \left(\frac{c_z w^N}{\alpha} - c_z w^N\right) \frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} + \left(\frac{\tau c_z w^N}{\alpha} - \tau c_z w_N\right) \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} \\ &= \frac{c_z w^N}{\sigma - 1}\left[\frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} + \tau \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}}\right]\end{aligned}$$

where we have used that $\frac{1}{\alpha} - 1 = \frac{\sigma}{\sigma-1} - \frac{\sigma-1}{\sigma-1} = \frac{1}{\sigma-1}$. It turns out to be convenient to rewrite profits by multiplying the RHS by $\frac{L_t}{L_t}\frac{n_{Xzt}}{n_{Xzt}}\frac{n_t}{n_t}$:

$$\pi_{Xzt} = \frac{c_z w^N}{\sigma - 1}\left[\frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N n_{Xzt}}{\left(P_t^N\right)^{1-\sigma} L_t} + \tau \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S n_{Xzt}}{\left(P_t^S\right)^{1-\sigma} L_t}\right] \frac{L_t}{n_t \frac{n_{Xzt}}{n_t}}.$$

Now $\gamma_{Xz} \equiv \frac{n_{Xzt}}{n_t}$ is constant over time, $X_{Xz}^N \equiv \frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N n_{Xzt}}{\left(P_t^N\right)^{1-\sigma} L_t}$ is constant over time since $\left(P_t^N\right)^{1-\sigma}$ grows at the same rate g as n_{Xzt}, and $X_{Xz}^S \equiv \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S n_{Xzt}}{\left(P_t^S\right)^{1-\sigma} L_t}$ is constant over time since $\left(P_t^S\right)^{1-\sigma}$ grows at the same rate g as n_{Xzt}. Thus we can write π_{Xzt} more simply as:

$$\pi_{Xzt} = \left[\frac{c_z w^N \left(X_{Xz}^N + \tau X_{Xz}^S\right)}{(\sigma - 1)\gamma_{Xz}}\right] \frac{L_t}{n_t}. \qquad \textbf{(??)}$$

A foreign affiliate earns the flow of global profits:

$$\pi_{Fzt} = \left(p_{Fz}^S - c_z w^S\right) x_{Fzt}^S L_t^S + \left(p_{Fz}^N - \tau c_z w^S\right) x_{Fzt}^N L_t^N$$

where $x_{Fzt}^S = \left(p_{Fz}^S\right)^{-\sigma} e^S / \left(P_t^S\right)^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the foreign affiliate's product and $x_{Fzt}^N = \left(p_{Fz}^N\right)^{-\sigma} e^N / \left(P_t^N\right)^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the foreign affiliate's product. Hence, we can write a foreign affiliate's profit flow as

$$\pi_{Fzt} = \left(p_{zF}^S - c_z w^S\right) \frac{\left(p_{Fz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} + \left(p_{Fz}^N - \tau c_z w^S\right) \frac{\left(p_{Fz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}}.$$

Maximizing π_{Fzt} with respect to p_{Fz}^S yields the first-order condition

$$\frac{\partial \pi_{Fzt}}{\partial p_{Fz}^S} = \left[(1-\sigma)\left(p_{Fz}^S\right)^{-\sigma} + \sigma c_z w^S \left(p_{Fz}^S\right)^{-\sigma-1}\right] \frac{e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} = 0$$

which implies that $(1-\sigma)\left(p_{Fz}^S\right)^{-\sigma}+\sigma c_z w^S\left(p_{Fz}^S\right)^{-\sigma-1}=0$. Dividing by $\left(p_{Fz}^S\right)^{-\sigma}$ yields $\frac{\sigma c_z w^S}{p_{Fz}^S}=\sigma-1$ or

$$p_{Fz}^S=\frac{\sigma c_z w^S}{\sigma-1}=\frac{c_z w^S}{\alpha}.$$

Similarly, maximizing π_{Fzt} with respect to p_{Fz}^N yields the first-order condition

$$\frac{\partial \pi_{Fzt}}{\partial p_{Fz}^N}=\left[(1-\sigma)\left(p_{Fz}^N\right)^{-\sigma}+\sigma\tau c_z w^S\left(p_{Fz}^N\right)^{-\sigma-1}\right]\frac{e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}}=0,$$

which implies that $(1-\sigma)\left(p_{Fz}^N\right)^{-\sigma}+\sigma\tau c_z w^S\left(p_{Fz}^N\right)^{-\sigma-1}=0$. Dividing by $\left(p_{Fz}^N\right)^{-\sigma}$ yields $\frac{\sigma\tau c_z w^S}{p_{Fz}^N}=\sigma-1$ or

$$p_{Fz}^N=\frac{\sigma\tau c_z w^S}{\sigma-1}=\frac{\tau c_z w^S}{\alpha}.$$

When the inequality $\tau w_S<w_N$ holds, each foreign affiliate exports to the northern market. The trade costs parameter τ cannot be too high. Plugging the prices back into the profit expression, we obtain

$$\begin{aligned}\pi_{Fzt} &= \left(\frac{c_z w^S}{\alpha}-c_z w^S\right)\frac{\left(p_{Fz}^S\right)^{-\sigma}e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}}+\left(\frac{\tau c_z w^S}{\alpha}-\tau c_z w^S\right)\frac{(p_{Fz}^N)^{-\sigma}e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}}\\ &= \frac{c_z w^S}{\sigma-1}\left[\frac{\left(p_{Fz}^S\right)^{-\sigma}e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}}+\tau\frac{\left(p_{Fz}^N\right)^{-\sigma}e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}}\right].\end{aligned}$$

We rewrite profits by multiplying the RHS by $\frac{L_t}{L_t}\frac{n_{Fzt}}{n_{Fzt}}\frac{n_t}{n_t}$:

$$\pi_{Fzt}=\frac{c_z w^S}{\sigma-1}\left[\frac{\left(p_{Fz}^S\right)^{-\sigma}e^S L_t^S n_{Fzt}}{\left(P_t^S\right)^{1-\sigma}L_t}+\tau\frac{\left(p_{Fz}^N\right)^{-\sigma}e^N L_t^N n_{Fzt}}{\left(P_t^N\right)^{1-\sigma}L_t}\right]\frac{L_t}{n_t\frac{n_{Fzt}}{n_t}}.$$

Now $\gamma_{Fz}\equiv\frac{n_{Fzt}}{n_t}$ is constant over time, $X_{Fz}^S\equiv\frac{\left(p_{Fz}^S\right)^{-\sigma}e^S L_t^S n_{Fzt}}{\left(P_t^S\right)^{1-\sigma}L_t}$ is constant over time since $\left(P_t^S\right)^{1-\sigma}$ grows at the same rate g as n_{Fzt}, and $X_{Fz}^N\equiv\frac{\left(p_{Fz}^N\right)^{-\sigma}e^N L_t^N n_{Fzt}}{\left(P_t^N\right)^{1-\sigma}L_t}$ is constant over time since $\left(P_t^N\right)^{1-\sigma}$ grows at the same rate g as n_{Fzt}. Thus, we can write π_{Fzt} more simply as:

$$\pi_{Fzt}=\left[\frac{c_z w^S\left(X_{Fz}^S+\tau X_{Fz}^N\right)}{(\sigma-1)\gamma_{Fz}}\right]\frac{L_t}{n_t}. \qquad \textbf{(??)}$$

A foreign affiliate's variety is imitated by southern firms at the exogenously given rate ι_S. Once the imitation technology is available to southern firms, competition drives down price to marginal cost and southern firms therefore earn zero profits. The quantity demanded by the typical southern consumer of southern firm products is $x_{Izt}^S=\left(p_{Iz}^S\right)^{-\sigma}e^S/\left(P_t^S\right)^{1-\sigma}$ and

$x_{Izt}^{N}=\left(p_{Iz}^{N}\right)^{-\sigma}e^{N}/\left(P_{t}^{N}\right)^{1-\sigma}$ is the quantity demanded by the typical northern consumer of southern firm products. Since southern firms set price equal to marginal cost, we must have $p_{Iz}^{S}=c_{z}w^{S}$ and $p_{Iz}^{N}=\tau c_{z}w^{S}$.

R&D Incentives

For a non-exporting northern firm, the relevant no-arbitrage condition is

$$v_{Nt}=\frac{q\pi_{NHt}+(1-q)\,\pi_{NLt}}{\rho+\theta g}=\frac{w^{N}a_{N}g^{\beta}}{n_{t}^{\theta}}.$$

Substituting for π_{NHt} and π_{NLt} yields

$$\begin{aligned}\frac{\frac{qc_{H}w^{N}X_{NH}^{N}}{(\sigma-1)\gamma_{NH}}\frac{L_{t}}{n_{t}}+\frac{(1-q)c_{L}w^{N}X_{NL}^{N}}{(\sigma-1)\gamma_{NL}}\frac{L_{t}}{n_{t}}}{\rho+\theta g} &= \frac{w^{N}a_{N}g^{\beta}}{n_{t}^{\theta}}\\ \frac{\frac{qc_{H}X_{NH}^{N}}{(\sigma-1)\gamma_{NH}}+\frac{(1-q)c_{L}X_{NL}^{N}}{(\sigma-1)\gamma_{NL}}}{\rho+\theta g} &= a_{N}g^{beta}\frac{n_{t}^{1-\theta}}{L_{t}}.\end{aligned}$$

Thus the st4eady-state northern no-aribitrage condition is

$$\frac{\frac{1}{\sigma-1}\left(\frac{qc_{H}X_{NH}^{N}}{\gamma_{NH}}+\frac{(1-q)c_{L}X_{NL}^{N}}{\gamma_{NL}}\right)}{\rho+\theta g}=a_{N}g^{\beta}\delta. \tag{??}$$

For a northern exporting firm, the relevant no-arbitrage condition is

$$v_{Xzt}-v_{Nzt}=\frac{\pi_{Xzt}}{\rho+\theta g}-\frac{\pi_{Nzt}}{\rho+\theta g}=\frac{w^{S}a_{X}\chi_{z}^{\beta}}{n_{t}^{\theta}}.$$

Using the profits for northern exporters and non-exporters from earlier, we can write this as:

$$\begin{aligned}\frac{\frac{c_{z}w^{N}}{(\sigma-1)}\frac{X_{Xz}^{N}+\tau X_{Xz}^{S}}{\gamma_{Xz}}\frac{L_{t}}{n_{t}}}{\rho+\theta g}-\frac{\frac{c_{z}w^{N}X_{Nz}^{N}}{(\sigma-1)\gamma_{Nz}}\frac{L_{t}}{n_{t}}}{\rho+\theta g} &= \frac{w^{S}a_{X}\chi_{z}^{\beta}}{n_{t}^{\theta}}\\ \frac{\frac{c_{z}w}{(\sigma-1)}\frac{X_{Xz}^{N}+\tau X_{Xz}^{S}}{\gamma_{Xz}}}{\rho+\theta g}-\frac{\frac{c_{z}wX_{Nz}^{N}}{(\sigma-1)\gamma_{Nz}}}{\rho+\theta g} &= a_{X}\chi_{z}^{\beta}\frac{n_{t}^{1-\theta}}{L_{t}}.\end{aligned}$$

It follows that the steady-state exporter no-arbitrage condition is

$$\frac{c_{z}w}{\sigma-1}\left[\frac{\frac{X_{Xz}^{N}+\tau X_{Xz}^{S}}{\gamma_{Xz}}-\frac{X_{Nz}^{N}}{\gamma_{Nz}}}{\rho+\theta g}\right]=a_{X}\chi_{z}^{\beta}\delta \tag{??}$$

where $w\equiv w_{N}/w_{S}$ is the northern relative wage.

For a foreign affiliate, the relevant no-arbitrage condition is

$$\frac{\pi_{Fzt}}{\rho+\theta g+\iota_S}-\frac{\pi_{Xzt}}{\rho+\theta g} \quad = \quad \frac{w^S a_F \phi_z^\beta}{n_t^\theta}.$$

Using the foreign affiliate profits from earlier, we can write this as

$$\begin{aligned}
\frac{\frac{c_z w^S}{\sigma-1}\frac{X_{Fz}^S+\tau X_{Fz}^N}{\gamma_{Fz}}\frac{L_t}{n_t}}{\rho+\theta g+\iota_S}-\frac{\frac{c_z w^N}{\sigma-1}\frac{X_{Xz}^N+\tau X_{Xz}^S}{\gamma_{Xz}}\frac{L_t}{n_t}}{\rho+\theta g} &= \frac{w_S a_F \phi_z^\beta}{n_t^\theta} \\
\frac{\frac{c_z}{\sigma-1}\frac{X_{Fz}^S+\tau X_{Fz}^N}{\gamma_{Fz}}}{\rho+\theta g+\iota_S}-\frac{\frac{c_z w}{\sigma-1}\frac{X_{Xz}^N+\tau X_{Xz}^S}{\gamma_{Xz}}}{\rho+\theta g} &= a_F \phi_z^\beta \frac{n_t^{1-\theta}}{L_t}.
\end{aligned}$$

It follows that the steady-state foreign affiliate no-arbitrage condition is

$$\frac{c_z}{\sigma-1}\left[\frac{\frac{X_{Fz}^S+\tau X_{Fz}^N}{\gamma_{Fz}}}{\rho+\theta g+\iota_S}-\frac{\frac{w\left(X_{Xz}^N+\tau X_{Xz}^S\right)}{\gamma_{Xz}}}{\rho+\theta g}\right]=a_F\phi_z^\beta\delta. \qquad \text{(??)}$$

Labor Markets

In the South, labor is employed in adaptive R&D, export-learning R&D, production by foreign affiliates and production by southern firms that have imitated foreign affiliates. Each northern product variety introduced to the southern market via exports requires $a_X\chi_z^\beta/n_t^\theta$ units of labor, so total employment in export-learning R&D by firms is $\sum_z\left(a_X\chi_z^\beta/n_t^\theta\right)(\dot{n}_{Xzt}+\dot{n}_{Fzt}+\dot{n}_{Izt})$. Each variety transferred to the South by a foreign affiliate requires $a_F\phi_z^\beta/n_t^\theta$ units of labor, so total employment in adaptive R&D is $\sum_z\left(a_F\phi_z^\beta/n_t^\theta\right)(\dot{n}_{Fzt}+\dot{n}_{Izt})$. Turning to production in the South, a foreign affiliate with productivity z uses $\frac{c_z\left(p_{Fz}^S\right)^{-\sigma}e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}}+\frac{\tau c_z\left(p_{Fz}^N\right)^{-\sigma}e^N L_t^N}{\left(P_t^N\right)1-\sigma} = X_{Fz}^S\frac{c_zL_t}{n_{Fzt}}+\tau X_{Fz}^N\frac{c_zL_t}{n_{Fzt}}$ units of labor for each variety produced, and there are n_{Fzt} such varieties produced, so total employment in foreign affiliate production is $\sum_z\left(X_{Fz}^S\frac{c_zL_t}{n_{Fzt}}+\tau X_{Fz}^N\frac{c_zL_t}{n_{Fzt}}\right)n_{Fzt}=\sum_z\left[X_{Fz}^S+\tau X_{Fz}^N\right]c_zL_t$. A southern firm that has imitated a foreign affiliate with productivity z uses $\frac{c_z\left(p_{Iz}^S\right)^{-\sigma}e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}}+\frac{\tau c_z\left(p_{Iz}^N\right)^{-\sigma}e^N L_t^N}{\left(P_t^N\right)1-\sigma} = X_{Iz}^S\frac{c_zL_t}{n_{Izt}}+\tau X_{Iz}^N\frac{c_zL_t}{n_{Izt}}$ units of labor for each variety produced, and there are n_{Izt} such varieties produced, so total employment in southern (local) production is $\sum_z\left(X_{Iz}^S\frac{c_zL_t}{n_{Fzt}}+\tau X_{Iz}^N\frac{c_zL_t}{n_{Izt}}\right)n_{Izt}=\sum_z\left[X_{Iz}^S+\tau X_{Iz}^N\right]c_zL_t$. As L_t^S denotes the labor supply in the South, full employment requires that

$$\begin{aligned}
L_t^S \quad = \quad & \sum_{z=H,L}\frac{a_X\chi_z^\beta}{n_t^\theta}(\dot{n}_{Xzt}+\dot{n}_{Fzt}+\dot{n}_{Izt})+\frac{a_F\phi_z^\beta}{n_t^\theta}(\dot{n}_{Fzt}+\dot{n}_{Izt}) \\
& +\left[X_{Fz}^S+\tau X_{Fz}^N\right]c_zL_t+\left[X_{Iz}^S+\tau X_{Iz}^N\right]c_zL_t.
\end{aligned}$$

Now using $\delta \equiv \frac{n_t^{1-\theta}}{L_t}$, $\chi_z \equiv \frac{\dot{n}_{Xzt}+\dot{n}_{Fzt}+\dot{n}_{Izt}}{n_{Nzt}}$, $\phi_z \equiv \frac{\dot{n}_{Fzt}+\dot{n}_{Izt}}{n_{Xzt}}$ and $\iota_S = \frac{\dot{n}_{Izt}}{n_{Fzt}}$, southern R&D employment can be written as

$$\begin{aligned}
& \sum_{z=H,L} \left[\frac{a_X \chi_z^\beta}{n_t^\theta} \left(\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt} \right) + \frac{a_F \phi_z^\beta}{n_t^\theta} \left(\dot{n}_{Fzt} + \dot{n}_{Izt} \right) \right] \\
= & \sum_{z=H,L} \left[\frac{a_X \chi_z^\beta \left(\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt} \right)}{n_{Nzt}} \frac{n_{Nzt}}{n_t} \frac{n_t^{1-\theta}}{L_t} L_t + \frac{a_F \phi_z^\beta \left(\dot{n}_{Fzt} + \dot{n}_{Izt} \right)}{n_{Xzt}} \frac{n_{Xzt}}{n_t} \frac{n_t^{1-\theta}}{L_t} L_t \right] \\
= & \sum_{z=H,L} \left[a_X \chi_z^{1+\beta} \gamma_{Nz} \delta L_t + a_F \phi_z^{1+\beta} \gamma_{Xz} \delta L_t \right].
\end{aligned}$$

It follows that

$$L_t^S = L_t \left[\sum_{z=H,L} a_X \delta \left(\chi_z \right)^{1+\beta} \gamma_{Nz} + a_F \delta \left(\phi_z \right)^{1+\beta} \gamma_{Xz} + \left(X_{Fz}^S + \tau X_{Fz}^N + X_{Iz}^S + \tau X_{Iz}^N \right) c_z \right].$$

and evaluating at time $t = 0$ yields the steady-state full employment of labor condition for the South:

$$L_0^S = L_0 \left[\sum_{z=H,L} a_X \delta \left(\chi_z \right)^{1+\beta} \gamma_{Nz} + a_F \delta \left(\phi_z \right)^{1+\beta} \gamma_{Xz} + \left(X_{Fz}^S + \tau X_{Fz}^N + X_{Iz}^S + \tau X_{Iz}^N \right) c_z \right]. \quad \textbf{(??)}$$

Aggregate Demand

We need to solve for steady-state values of the aggregate demand expressions X_{Nz}^N, X_{Xz}^N, X_{Xz}^S, X_{Fz}^S, X_{Fz}^N, X_{Iz}^S and X_{Iz}^N. The calculations

$$\begin{aligned}
\frac{X_{Nz}^N}{X_{Fz}^N} &= \frac{\frac{\left(p_{Nz}^N\right)^{-\sigma} e^N L_t^N n_{Nzt}}{\left(P_t^N\right)^{1-\sigma} L_t}}{\frac{\left(p_{Fz}^N\right)^{-\sigma} e^N L_t^N n_{Fzt}}{\left(P_t^N\right)^{1-\sigma} L_t}} \\
&= \left(\frac{p_{Nz}^N}{p_{Fz}^N} \right)^{-\sigma} \frac{n_{Nzt}/n_t}{n_{Fzt}/n_t} \\
&= \left(\frac{\frac{c_z w^N}{\alpha}}{\frac{\tau c_z w^S}{\alpha}} \right)^{-\sigma} \frac{\gamma_{Nz}}{\gamma_{Fz}} \\
&= \left(\frac{w}{\tau} \right)^{-\sigma} \frac{q_z \frac{g}{g+\chi_z}}{q_z \frac{\chi_z}{g+\chi_z} \frac{\phi_z}{g+\phi_z} \frac{g}{g+\iota_S}} \\
&= \left(\frac{w}{\tau} \right)^{-\sigma} \frac{\left(g+\phi_z\right)\left(g+\iota_S\right)}{\chi_z \phi_z},
\end{aligned}$$

$$
\begin{aligned}
\frac{X_{Xz}^N}{X_{Fz}^N} &= \frac{\frac{\left(p_{Nz}^N\right)^{-\sigma} e^N L_t^N n_{Xzt}}{\left(P_t^N\right)^{1-\sigma} L_t}}{\frac{\left(p_{Fz}^N\right)^{-\sigma} e^N L_t^N n_{Fzt}}{\left(P_t^N\right)^{1-\sigma} L_t}} \\
&= \left(\frac{p_{Nz}^N}{p_{Fz}^N}\right)^{-\sigma} \frac{n_{Xzt}/n_t}{n_{Fzt}/n_t} \\
&= \left(\frac{\frac{c_z w^N}{\alpha}}{\frac{\tau c_z w^S}{\alpha}}\right)^{-\sigma} \frac{\gamma_{Xz}}{\gamma_{Fz}} \\
&= \left(\frac{w}{\tau}\right)^{-\sigma} \frac{q_z \frac{\chi_z}{g+\chi_z} \frac{g}{g+\phi_z}}{q_z \frac{\chi_z}{g+\chi_z} \frac{\phi_z}{g+\phi_z} \frac{g}{g+\iota_S}} \\
&= \left(\frac{w}{\tau}\right)^{-\sigma} \frac{g+\iota_S}{\phi_z},
\end{aligned}
$$

$$
\begin{aligned}
\frac{X_{Xz}^S}{X_{Fz}^S} &= \frac{\frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S n_{Xzt}}{\left(P_t^S\right)^{1-\sigma} L_t}}{\frac{\left(p_{Fz}^S\right)^{-\sigma} e^S L_t^S n_{Fzt}}{\left(P_t^S\right)^{1-\sigma} L_t}} \\
&= \left(\frac{p_{Xz}^S}{p_{Fz}^S}\right)^{-\sigma} \frac{n_{Xzt}/n_t}{n_{Fzt}/n_t} \\
&= \left(\frac{\frac{\tau c_z w^N}{\alpha}}{\frac{c_z wS}{\alpha}}\right)^{-\sigma} \frac{\gamma_{Xz}}{\gamma_{Fz}} \\
&= (\tau w)^{-\sigma} \frac{q_z \frac{\chi_z}{g+\chi_z} \frac{g}{g+\phi_z}}{q_z \frac{\chi_z}{g+\chi_z} \frac{\phi_z}{g+\phi_z} \frac{g}{g+\iota_S}} \\
&= (\tau w)^{-\sigma} \frac{g+\iota_S}{\phi_z},
\end{aligned}
$$

$$
\begin{aligned}
\frac{X_{Iz}^S}{X_{Fz}^S} &= \frac{\frac{\left(p_{Iz}^S\right)^{-\sigma} e^S L_t^S n_{Izt}}{\left(P_t^S\right)^{1-\sigma} L_t}}{\frac{\left(p_{Fz}^S\right)^{-\sigma} e^S L_t^S n_{Fzt}}{\left(P_t^S\right)^{1-\sigma} L_t}} \\
&= \left(\frac{p_{Iz}^S}{p_{Fz}^S}\right)^{-\sigma} \frac{n_{Izt}/n_t}{n_{Fzt}/n_t} \\
&= \left(\frac{c_z w^S}{\frac{c_z w_S}{\alpha}}\right)^{-\sigma} \frac{\gamma_{Iz}}{\gamma_{Fz}} \\
&= \left(\frac{1}{\alpha}\right)^{\sigma} \frac{q_z \frac{\chi_z}{g+\chi_z} \frac{\phi_z}{g+\phi_z} \frac{\iota_S}{g+\iota_S}}{q_z \frac{\chi_z}{g+\chi_z} \frac{\phi_z}{g+\phi_z} \frac{g}{g+\iota_S}} \\
&= \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota_S}{g},
\end{aligned}
$$

$$
\begin{aligned}
\frac{X_{Iz}^N}{X_{Fz}^N} &= \frac{\frac{\left(p_{Iz}^N\right)^{-\sigma} e^N L_t^N n_{Izt}}{\left(P_t^N\right)^{1-\sigma} L_t}}{\frac{\left(p_{Fz}^N\right)^{-\sigma} e^N L_t^N n_{Izt}}{\left(P_t^N\right)^{1-\sigma} L_t}} \\
&= \left(\frac{p_{Iz}^N}{p_{Fz}^N}\right)^{-\sigma} \frac{n_{Izt}/n_t}{n_{Fzt}/n_t} \\
&= \left(\frac{\tau c_z w^S}{\frac{\tau c_z w^S}{\alpha}}\right)^{-\sigma} \frac{\gamma_{Iz}}{\gamma_{Fz}} \\
&= \left(\frac{1}{\alpha}\right)^{\sigma} \frac{q_z \frac{\chi_z}{g+\chi_z} \frac{\phi_z}{g+\phi_z} \frac{\iota_S}{g+\iota_S}}{q_z \frac{\chi_z}{g+\chi_z} \frac{\phi_z}{g+\phi_z} \frac{g}{g+\iota_S}} \\
&= \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota_S}{g}
\end{aligned}
$$

imply that

$$
X_{Nz}^N = X_{Fz}^N \left(\frac{w}{\tau}\right)^{-\sigma} \frac{\left(g+\phi_z\right)\left(g+\iota_S\right)}{\chi_z \phi_z},
$$

$$
X_{Xz}^N = X_{Fz}^N \left(\frac{w}{\tau}\right)^{-\sigma} \frac{g+\iota_S}{\phi_z},
$$

$$
X_{Xz}^S = X_{Fz}^S \left(w\tau\right)^{-\sigma} \frac{g+\iota_S}{\phi_z},
$$

$$
X_{Iz}^S = X_{Fz}^S \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota_S}{g},
$$

and

$$X_{Iz}^N = X_{Fz}^N \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota_S}{g}.$$

Finally, we need to express X_{FH}^S in terms of X_{FL}^S and X_{FH}^N in terms of X_{FL}^N. The calculations

$$\begin{aligned}
\frac{X_{FH}^S}{X_{FL}^S} &= \frac{\frac{\left(p_{FH}^S\right)^{-\sigma} e^S L_t^S n_{FHt}}{\left(P_t^S\right)^{1-\sigma} L_t}}{\frac{\left(p_{FL}^S\right)^{-\sigma} e^S L_t^S n_{FLt}}{\left(P_t^S\right)^{1-\sigma} L_t}} \\
&= \left(\frac{p_{FH}^S}{p_{FL}^S}\right)^{-\sigma} \frac{n_{FHt}/n_t}{n_{FLt}/n_t} \\
&= \left(\frac{\frac{c_H w^S}{\alpha}}{\frac{c_L w^S}{\alpha}}\right)^{-\sigma} \frac{\gamma_{FH}}{\gamma_{FL}} \\
&= \left(\frac{c_H}{c_L}\right)^{-\sigma} \frac{q\left(\frac{\chi_H}{g+\chi_H}\frac{\phi_H}{g+\phi_H}\frac{g}{g+\iota_S}\right)}{(1-q)\left(\frac{\chi_L}{g+\chi^L}\frac{\phi_L}{g+\phi_L}\frac{g}{g+\iota_S}\right)}
\end{aligned}$$

yields

$$X_{FH}^S = X_{FL}^S \left(\frac{c_H}{c_L}\right)^{-\sigma} \left(\frac{q}{1-q}\right) \left(\frac{g+\chi_L}{g+\chi_H}\right) \left(\frac{\chi_H}{\chi_L}\right) \left(\frac{g+\phi_L}{g+\phi_H}\right) \left(\frac{\phi_H}{\phi_L}\right)$$

where we have used that $q_H = q$ and $q_L = 1-q$.

Similarly, the calculations

$$\begin{aligned}
\frac{X_{FH}^N}{X_{FL}^N} &= \frac{\frac{\left(p_{FH}^N\right)^{-\sigma} e^N L_t^N n_{FHt}}{\left(P_t^N\right)^{1-\sigma} L_t}}{\frac{\left(p_{FL}^N\right)^{-\sigma} e^N L_t^N n_{FLt}}{\left(P_t^N\right)^{1-\sigma} L_t}} \\
&= \left(\frac{p_{FH}^N}{p_{FL}^N}\right)^{-\sigma} \frac{n_{FHt}/n_t}{n_{FLt}/n_t} \\
&= \left(\frac{\frac{\tau c_H w^S}{\alpha}}{\frac{\tau c_L w^S}{\alpha}}\right)^{-\sigma} \frac{\gamma_{FH}}{\gamma_{FL}} \\
&= \left(\frac{c_H}{c_L}\right)^{-\sigma} \frac{q\left(\frac{\chi_H}{g+\chi_H}\frac{\phi_H}{g+\phi_H}\frac{g}{g+\iota_S}\right)}{(1-q)\left(\frac{\chi_L}{g+\chi_L}\frac{\phi_L}{g+\phi_L}\frac{g}{g+\iota_S}\right)}
\end{aligned}$$

yields

$$X^N_{FH} = X^N_{FL}\left(\frac{c_H}{c_L}\right)^{-\sigma}\left(\frac{q}{1-q}\right)\left(\frac{g+\chi_L}{g+\chi_H}\right)\left(\frac{\chi_H}{\chi_L}\right)\left(\frac{g+\phi_L}{g+\phi_H}\right)\left(\frac{\phi_H}{\phi_L}\right).$$

Asset Ownership and Consumer Expenditure

The aggregate value of all financial assets is the total value of firms:

$$A_t = A^N_t + A^S_t = \sum_{z=H,L} n_{Nzt}v_{Nzt} + n_{Xzt}v_{Xzt} + n_{Fzt}v_{Fzt}.$$

The aggregate value of northern financial assets A^N_t is given by

$$A^N_t = \sum_{z=H,L}\left(n_{Nzt} + n_{Xzt} + n_{Fzt}\right)v_{Nzt}$$

while the aggregate value of southern financial assets A^S_t is given by

$$A^S_t = A_t - A^S_t = \sum_{z=H,L} n_{Xzt}\left(v_{Xzt} - v_{Nzt}\right) + n_{Fzt}\left(v_{Fzt} - v_{Nzt}\right)$$

. From $v_{Xzt} - v_{Nzt} = \frac{w^S a_X \chi_z^\beta}{n_t^\theta}$ and $v_{Fzt} - v_{Xzt} = \frac{w^S a_F \phi_z^\beta}{n_t^\theta}$, it follows that

$$\begin{aligned}
v_{Fzt} - v_{Xzt} &= \frac{w^S a_F \phi_z^\beta}{n_t^\theta} \\
v_{Fzt} - \left(\frac{w^S a_X \chi_z^\beta}{n_t^\theta} + v_{Nzt}\right) &= \frac{w^S a_F \phi z^\beta}{n_t^\theta} \\
v_{Fzt} - v_{Nzt} &= \frac{w^S a_F \phi_z}{n_t^\theta} + \frac{w^S a_X \chi_z^\beta}{n_t^\theta}.
\end{aligned}$$

Substituting for $v_{Fzt} - v_{Nzt}$ in the expression for aggregate southern assets

$$\begin{aligned}
A^S_t &= \sum_{z=H,L} n_{Xzt}\left(v_{Xzt} - v_{Nzt}\right) + n_{Fzt}\left(v_{Fzt} - v_{Nzt}\right) \\
&= \sum_{z=H,L} n_{Xzt}\frac{w^S a_X \chi_z^\beta}{n_t^\theta} + n_{Fzt}\left(\frac{w^S a_F \phi_z^\beta}{n_t^\theta} + \frac{w^S a_X \chi_z}{n_t^\theta}\right) \\
&= w^S L_t \frac{n_t^{1-\theta}}{L_t}\left[\sum_{z=H,L}\left(\frac{n_{Xzt}}{n_t}\right)a_X\chi_z^\beta + \left(\frac{n_{Fzt}}{n_t}\right)\left(a_F\phi_z^\beta + a_X\chi_z^\beta\right)\right]
\end{aligned}$$

yields

$$A_t^S = w^S L_t \delta \left[\sum_{z=H,L} \gamma_{Xz} a_X \chi_z^\beta + \gamma_{Fz} \left(a_F \phi_z^\beta + a_X \chi_z^\beta \right) \right].$$

Using $v_{Nzt} = \frac{\pi_{Nzt}}{\rho+\theta g}$ and the steady-state profit expressions $\pi_{Nzt} = \frac{c_z w^N X_{Nz}^N}{(\sigma-1)\gamma_{Nz}} \frac{L_t}{n_t}$, northern aggregate assets can be written as

$$\begin{aligned} A_t^N &= \sum_{z=H,L} \left(n_{Nzt} + n_{Xzt} + n_{Fzt} \right) v_{Nzt} \\ &= \sum_{z=H,L} \left(\frac{n_{Nzt}}{n_t} + \frac{n_{Xzt}}{n_t} + \frac{n_{Fzt}}{n_t} \right) \frac{\pi_{Nzt} n_t}{\rho + \theta g} \\ &= \sum_{z=H,L} \left(\gamma_{Nz} + \gamma_{Xz} + \gamma_{Fz} \right) \frac{c_z w^N X_{Nz}^N n_t}{(\sigma - 1) \gamma_{Nz} (\rho + \theta g)} \frac{L_t}{n_t} \\ &= \frac{w^N L_t}{(\sigma - 1)(\rho + \theta g)} \left[\sum_{z=H,L} c_z X_{Nz}^N \frac{\gamma_{Nz} + \gamma_{Xz} + \gamma_{Fz}}{\gamma_{Nz}} \right]. \end{aligned}$$

Consumer expenditure for the typical southern consumer is

$$\begin{aligned} e^S &= w^S + (\rho - g_L) a_t^S \\ &= w^S + (\rho - g_L) \frac{A_t^S}{L_t^S} \\ &= w^S + (\rho - g_L) w^S \frac{L_t}{L_t^S} \delta \left[\sum_{z=H,L} \gamma_{Xz} a_X \chi_z^\beta + \gamma_{Fz} \left(a_F \phi_z^\beta + a_X \chi_z^\beta \right) \right]. \end{aligned}$$

Evaluating at time 0 yields steady-state southern consumer expenditure

$$e^S = e^S + (\rho - g_L) w^S \frac{L_0}{L_0^S} \delta \left[\sum_{z=H,L} \gamma_{Xz} a_X \chi_z^\beta + \gamma_{Fz} \left(a_F \phi_z^\beta + a_X \chi_z^\beta \right) \right]. \qquad \textbf{(??)}$$

Consumer expenditure for the typical northern consumer is

$$\begin{aligned} e^N &= w^N + (\rho - g_L) a_t^N \\ &= w^N + (\rho - g_L) \frac{A_t^N}{L_t^N} \\ &= w^N + (\rho - g_L) \frac{w^N}{(\sigma - 1)(\rho + \theta g)} \frac{L_t}{L_t^N} \left[\sum_{z=H,L} c_z X_{Nz}^N \frac{\gamma_{Nz} + \gamma_{Xz} + \gamma_{Fz}}{\gamma_{Nz}} \right]. \end{aligned}$$

Evaluating at time 0 yields steady-state northern consumer expenditure

$$e^N = w^N + \frac{(\rho - g_L)\, w^N}{(\sigma - 1)\,(\rho + \theta g)} \frac{L_0}{L_0^N} \left[\sum_{z=H,L} c_z X_{Nz}^N \frac{\gamma_{Nz} + \gamma_{Xz} + \gamma_{Fz}}{\gamma_{Nz}} \right]. \qquad \textbf{(??)}$$

Having solved for steady-state consumer expenditure e^N and e^S, we can take the ratio

$$\begin{aligned} \frac{X_{FL}^N}{X_{FL}^S} &= \frac{\frac{\left(p_{FL}^N\right)^{-\sigma} e^N L_t^N n_{FLt}}{\left(P_t^N\right)^{1-\sigma} L_t}}{\frac{\left(p_{FL}^S\right)^{-\sigma} e^S L_t^S n_{FLt}}{\left(P_t^S\right)^{1-\sigma} L_t}} = \left(\frac{p_{FL}^N}{p_{FL}^S}\right)^{-\sigma} \frac{e^N L_t^N}{e^S L_t^S} \frac{\left(P_t^S\right)^{1-\sigma}}{\left(P_t^N\right)^{1-\sigma}} \\ &= \left(\frac{\frac{\tau c_L w^S}{\alpha}}{\frac{c_L w^S}{\alpha}}\right)^{-\sigma} \frac{e^N L_t^N}{e^S L_t^S} \frac{\left(P_t^S\right)^{1-\sigma}}{\left(P_t^N\right)^{1-\sigma}} = \left(\frac{1}{\tau}\right)^{\sigma} \frac{e^N L_t^N}{e^S L_t^S} \frac{\left(P_t^S\right)^{1-\sigma}}{\left(P_t^N\right)^{1-\sigma}}. \end{aligned}$$

Evaluating at time 0 yields the steady-state asset condition

$$\frac{X_{FL}^N}{X_{FL}^S} = \left(\frac{1}{\tau}\right)^{\sigma} \frac{e^N L_0^N}{e^S L_0^S} \frac{\left(P_t^S\right)^{1-\sigma}}{\left(P_t^N\right)^{1-\sigma}}. \qquad \textbf{(??)}$$

Aggregate Labor Demand

Total employment in innovative R&D L_{Rt} is derived from the flow of new products developed in the North. From (**??**) it follows that

$$\begin{aligned} \dot{n}_t &= \frac{n_t^{\theta} L_{Rt}}{a_N g^{\beta}} \\ \frac{\dot{n}_t}{n_t} n_t \frac{L_t}{L_t} &= \frac{n_t^{\theta} L_{Rt}}{a_N g^{\beta}} \\ g^{1+\beta} a_N \frac{n_t^{1-\theta}}{L_t} L_t &= L_{Rt}. \end{aligned}$$

Evaluating at time $t = 0$ yields steady-state employment in innovative R&D

$$L_{R0} = g^{1+\beta} a_N \delta L_0.$$

Total employment in export-learning R&D by firms with productivity z is denoted by L_{Xzt}. It is derived from the flow of new products sold in the South as a consequence of export-learning activities. From (**??**) it follows that

$$\begin{aligned}
\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt} &= \frac{n_t^{\theta} L_{Xzt}}{a_X \chi_z^{\beta}} \\
\frac{\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Nzt}} \frac{n_{Nzt}}{n_t} n_t \frac{L_t}{L_t} &= \frac{n_t^{\theta} L_{Xzt}}{a_X \chi_z^{\beta}} \\
\chi_z^{1+\beta} a_X \gamma_{Nz} \frac{n_t^{1-\theta}}{L_t} L_t &= L_{Xzt}
\end{aligned}$$

Evaluating at time $t = 0$ yields steady-state employment in export-learning R&D

$$L_{Xz0} = \chi_z^{1+\beta} a_X \gamma_{Nz} \delta L_0, \qquad (z = H, L).$$

Total employment in adaptive R&D by firms with productivity z is denoted by L_{Fzt}. It is derived from the flow of products that are adapted for production in the South as a result of firms' FDI activities. From (**??**), we obtain

$$\begin{aligned}
\dot{n}_{Fzt} + \dot{n}_{Izt} &= \frac{n_t^{\theta} L_{Fzt}}{a_F \phi_z^{\beta}} \\
\frac{\dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Xzt}} \frac{n_{Xzt}}{n_t} n_t \frac{L_t}{L_t} &= \frac{n_t^{\theta} L_{Fzt}}{a_F \phi_z^{\beta}} \\
\phi_z^{\beta} a_F \gamma_{Xz} \frac{n_t^{1-\theta}}{L_t} L_t &= L_{Fzt}.
\end{aligned}$$

Evaluating at time $t = 0$ yields steady-state employment in adaptive R&D

$$L_{Fz0} = \phi_z^{1+\beta} \gamma_{Xz} \delta a_F L_0, \qquad (z = H, L).$$

Appendix: Solving the Model with Alternative Parameter Values

Fixed costs of export-learning and MP-adaption

So far, we have presented two counterfactuals: trade liberalization ($\tau \downarrow$) and stronger southern IPR protection ($a_I \uparrow$). In Tables 4 and 5, we reproduce the results from Tables 1 and 2, respectively, along with the results from solving the model for two additional counterfactuals. In Column 3 of both tables, we present the results using the 1990 benchmark as our starting point and then lower the cost of export-learning ($a_X \downarrow$). In the counterfactual presented in Column 4, we lower the cost of adapting production to MP, or the fixed cost of FDI ($a_F \downarrow$).

In the 1990 benchmark, $a_X = 5.56$. In the counterfactual with lower fixed cost of export-learning we set $a_X = 2$. With lower fixed costs of exporting, a larger share of northern

	(1) 1990	(2) 2005	(3) $a_X \downarrow$	(4) $a_F \downarrow$
	$\tau = 1.54$	$\tau = 1.33$	$\tau = 1.54$	$\tau = 1.54$
	$a_I = 4.4$	$a_I = 81.21$	$a_I = 4.4$	$a_I = 4.4$
	$a_X = 5.56$	$a_X = 5.56$	$a_X = 2$	$a_X = 5.56$
	$a_F = 148.5$	$a_F = 148.5$	$a_F = 148.5$	$a_F = 50$
w_N/w_S	2.2833	2.0437	2.3029	1.9705
δ	19.0631	18.1788	18.5300	19.1355
χ_H	.04481	.05933	.09559	.04332
χ_L	.01315	.01741	.02806	.01271
ϕ_H	.01028	.02538	.00805	.01714
ϕ_L	.00302	.00745	.00236	.00503
γ_{NH}	.02161	.01874	.01407	.02195
γ_{NL}	.75906	.71103	.61399	.76437
γ_{XH}	.01608	.01476	.02319	.01418
γ_{XL}	.18855	.21578	.32943	.17682
γ_{FH}	.00060	.00602	.00067	.00088
γ_{FL}	.00205	.02582	.00281	.00321
γ_{IH}	.00271	.00148	.00307	.00399
γ_{IL}	.00934	.00637	.01278	.01461
ι_S	.227	.012	.227	.227
L_{FH0}	.01446	.07700	.01240	.01196
L_{FL0}	.01457	.09697	.01517	.01284
Non-exporting firms' share	.790	.736	.638	.801
Foreign Affiliate share in VA	.04630	.39233	.04589	.04637
High prod. share of exports (sales)	.322	.276	.282	.309
MP Sales North market	.005	.058	.006	.004
MP Sales South market	.046	.335	.043	.046
MP Sales World	.051	.393	.048	.051
MP Sales share of North market	.0143	.1672	.0170	.0133
MP Sales share of South market	.0609	.5120	.0602	.0609
e^N	2.642	2.352	2.653	2.279
e^S	1.032	1.124	1.039	1.029
e^N/e^S	2.561	2.092	2.554	2.215
P_0^N	.00917	.00864	.00954	.00789
P_0^S	.01355	.01189	.01262	.01196
P_0^N/P_0^S	.677	.726	.756	.685
u_0^N	288.13	272.31	278.14	288.97
u_0^S	76.14	94.55	82.30	89.41
n_0	1,850,760	1,562,334	1,672,677	1,875,944

Table 5: Pre- and post-TRIPS benchmarks and two alternative counterfactual scenarios.

	(1) 1990	(2) 2005	(3) $a_X \downarrow$	(4) $a_F \downarrow$
Key parameters	$\tau = 1.54$	$\tau = 1.33$	$\tau = 1.54$	$\tau = 1.54$
	$a_I = 4.4$	$a_I = 81.21$	$a_I = 4.4$	$a_I = 4.4$
	$a_X = 5.56$	$a_X = 5.56$	$a_X = 2$	$a_X = 5.56$
	$a_F = 148.5$	$a_F = 148.5$	$a_F = 148.5$	$a_F = 50$
North $L_{N0} = 1$				
Non-production labor				
L_{R0}	.143	.136	.139	.143
Production labor				
L_{NH0}^{NProd}	.051	.042	.032	.052
L_{NL0}^{NProd}	.522	.466	.410	.527
L_{XH0}^{NProd}	.038	.033	.053	.033
L_{XL0}^{NProd}	.130	.141	.220	.122
L_{XH0}^{SProd}	.027	.034	.028	.026
L_{XL0}^{SProd}	.091	.147	.118	.096
South $L_{S0} = 2$				
Non-production labor				
L_{XH0}	.014	.020	.014	.013
L_{XL0}	.042	.065	.054	.039
L_{FH0}	.014	.077	.012	.012
L_{FL0}	.015	.097	.015	.013
Production labor				
L_{FH0}^{SProd}	.052	.348	.045	.051
L_{FL0}^{SProd}	.052	.438	.055	.055
L_{FH0}^{NProd}	.009	.080	.010	.008
L_{FL0}^{NProd}	.009	.101	.012	.008
L_{IH0}^{SProd}	.767	.278	.660	.757
L_{IL0}^{SProd}	.774	.351	.807	.813
L_{IH0}^{NProd}	.126	.064	.142	.111
L_{IL0}^{NProd}	.127	.081	.174	.119

Table 6: Pre- and post-TRIPS benchmarks and two counterfactual scenarios.

firms go on to become exporters and southern consumers thereby get access to larger product variety resulting in higher southern consumer welfare. At the same time, northern resources are drawn into production resulting in less innovation. Therefore, northern consumer utility decreases.

In the 1990 benchmark, $a_F = 148.5$. In the counterfactual with lower fixed cost of export-learning we set $a_F = 50$ (Column 4 of Tables 5 and 6). Lowering the fixed cost of MP lowers the North-South relative wage and increases innovation ($\delta \uparrow$ and $n_0 \uparrow$) and therefore also consumer welfare increase in both regions. However, as will be discussed in the following section, only by lowering the fixed MP costs it is not possible to replicate the ten-fold increase in FDI inflow going to developing countries between 1990 and 2005.

Less costly FDI (MP)

In our model, the cost of MP is a fixed cost (all produced varieties face an iceberg trade cost when shipped across countries. (This is in contrast to for example Arkolakis et al (2014) who model MP costs as an iceberg cost levied on each unit produced via MP). In this exercise we start from our 1990 benchmark and gradually lower a_F (the cost of adapting to MP).

In Table 7, the results from the 1990 benchmark along with counterfactuals with $a_F = 50$, $a_F = 20$ and $a_F = 6$ are presented. For $a_F = 5$ and lower, the assumption $p^N_{Nz} = p^N_{Xz} = c_z w^S/\alpha > \tau c_z w^S/\alpha = p^N_{Fz}$ is violated so there would be no exports of foreign affiliate-produced varieties back to the North.

Both northern and southern consumer welfare increases with lower cost of adapting to MP. However, lower cost of learning to do MP means a smaller FDI inflow to developing countries ($w_S * L_{F0} = w_S * \sum_z L_{Fz}$ becomes very small). In particular, it is not possible to replicate the observed ten-fold increase in FDI going to developing countries only by lowering the costs of MP-adaption (the fixed costs of FDI).

Additional R&D subsidy results

In Table 8, we present additional results for the counterfactual with trade liberalization in the presence of a R&D subsidy (s_R of 0.5, 0.75 and 0.85).

Alternative 2005 benchmark

For the calibration of the model we changed trade costs (τ) using Novy's (2013) estimated bilateral trade costs for US and Mexico for 1990 and 2005 along with IPR protection in the South, measured by $1/a_I$, to account for the ten-fold increase in FDI inflow between 1990 and 2005 (UNCTAD 2011). Even though we are able to replicate the ten-fold increase in FDI inflow, this benchmark resulted in a foreign affiliate share (value added) of world GDP of 39 per cent. In 2005, foreign affiliates actually accounted for 9.8 percent of world GDP (UNCTAD 2012).

In this section, we present an alternative 2005 benchmark that replicates the observed 9.8 percent share of foreign affiliates in world GDP while also generating $e^N/e^S = 3.1125$, which

	(1) 1990	(2)	(3)	(4)
	$\tau = 1.54$ $a_I = 4.4$ $a_X = 5.56$ $a_F = 148.5$	$\tau = 1.54$ $a_I = 4.4$ $a_X = 5.56$ $a_F = 50$	$\tau = 1.54$ $a_I = 4.4$ $a_X = 5.56$ $a_F = 20$	$\tau = 1.54$ $a_I = 4.4$ $a_X = 5.56$ $a_F = 6$
w_N/w_S	2.2833	1.9705	1.7595	1.5220
δ	19.0631	19.1355	19.2680	19.5781
χ_H	.04481	.04332	.04239	.04159
χ_L	.01315	.01271	.01244	.01221
ϕ_H	.01028	.01714	.02576	.04446
ϕ_L	.00302	.00503	.00756	.01305
γ_{NH}	.02161	.02195	.02217	.02237
γ_{NL}	.75906	.76437	.76771	.77061
γ_{XH}	.01608	.01418	.01242	.00986
γ_{XL}	.18855	.17682	.16614	.14935
γ_{FH}	.00060	.00088	.00115	.00158
γ_{FL}	.00205	.00321	.00453	.00703
γ_{IH}	.00271	.00399	.00525	.00720
γ_{IL}	.00934	.01461	.02062	.03200
ι_S	.227	.227	.227	.227
L_{FH0}	.01446	.01196	.00952	.00572
L_{FL0}	.01457	.01284	.01098	.00747
Non-exporting firms' share	.790	.801	.811	.825
Foreign Affiliate share in VA	.04630	.04637	.04641	.04642
High prod. share of exports (sales)	.322	.309	.294	.269
MP Sales North market	.005	.004	.004	.004
MP Sales South market	.046	.046	.047	.047
MP Sales World	.051	.051	.051	.050
MP Sales share of North market	.0143	.0133	.0125	.0113
MP Sales share of South market	.0609	.0609	.0608	.0608
e^N	2.642	2.279	2.034	1.759
e^S	1.032	1.029	1.026	1.023
e^N/e^S	2.561	2.215	1.982	1.719
P_0^N	.00917	.00789	.00699	.00595
P_0^S	.01355	.01196	.01012	00856
P_0^N/P_0^S	.677	.685	.691	.695
u_0^N	288.13	288.97	290.87	295.62
u_0^S	76.14	89.41	101.38	119.53
n_0	1,850,760	1,875,944	1,922,674	2,035,341

Table 7: Pre-TRIPS benchmarks and gradually lower a_F (lower fixed cost of adapting to MP).

	(1) 1990 $s_R = 0.5$ $\tau = 1.54$	(2) $\tau \downarrow$ $s_R = 0.5$ $\tau = 1.33$	(3) 1990 w. $s_R = 0.75$ $\tau = 1.54$	(4) $\tau \downarrow$ $s_R = 0.75$ $\tau = 1.33$	(5) 1990 w. $s_R = 0.85$ $\tau = 1.54$	(6) $\tau \downarrow$ $s_R = 0.85$ $\tau = 1.33$
w_N/w_S	2.7539	2.6467	3.7954	3.3587	5.1792	4.3350
δ	34.8191	33.1776	56.9294	55.5776	74.4183	73.8278
χ_H	.02405	.03297	.01054	.01658	.00509	.00881
χ_L	.00706	.00968	.00309	.00487	.00149	.00259
ϕ_H	.01016	.00891	.01190	.00960	.01512	.01143
ϕ_L	.00298	.00262	.00349	.00282	.00444	.00335
γ_{NH}	.02767	.02469	.03385	.03078	.03721	.03485
γ_{NL}	.84023	.80330	.90306	.87381	.93115	.91178
γ_{XH}	.01107	.01384	.00577	.00857	.00291	.00500
γ_{XL}	.11207	.14795	.05228	.08064	.02558	.04424
γ_{FH}	.00041	.00044	.00025	.00030	.00016	.00021
γ_{FL}	.00121	.00140	.00066	.00082	.00041	.00054
γ_{IH}	.00185	.00203	.00113	.00135	.00072	.00094
γ_{IL}	.00549	.00636	.00300	.00373	.00186	.00244
ι_S	.227	.227	.227	.227	.227	.227
L_{FH0}	.01773	.01625	.02073	.01955	.02207	.02149
L_{FL0}	.01546	.01497	.01618	.01584	.01670	.01637
Non-exporting firms' share	.874	.835	.941	.909	.971	.950
Foreign Affiliate share in VA	.04808	.04775	.05060	.05054	.05268	.05292
High prod. % of exports sales	.355	.342	.381	.372	.388	.386
MP Sales North market	.006	.009	.008	.011	.012	.014
MP Sales South market	.046	.042	.043	.041	.038	.037
MP Sales World	.052	.051	.051	.052	.050	.051
MP Sales % of North market	.0167	.0233	.0248	.0291	.0346	.0370
MP Sales % of South market	.0621	.0606	.0630	.0623	.0633	.0630
e^N	3.152	3.010	4.246	3.747	5.662	4.736
e^S	1.021	1.033	1.010	1.018	1.006	1.009
e^N/e^S	3.087	2.915	4.205	3.682	5.630	4.692
P_0^N	.00466	.00471	.00314	.00283	.00286	.00240
P_0^S	.00709	.00697	.00450	.00419	.00374	.00335
P_0^N/P_0^S	.658	.676	.697	.676	.766	.713
u_0^N	675.99	638.94	1352.64	1322.77	1977.80	1980.21
u_0^S	144.04	148.20	224.18	242.74	269.11	301.00
n_0	15,863,975	13,354,086	91,603,600	84,079,434	238,145,619	231,474,984

Table 8: 1990 benchmark with trade liberalization in presence of innovative R&D subsidy

is the household consumption share adjusted real GDP per worker ratio for US-Mexico in 2005 (Feenstra, Inklaar and Timmer, 2015). By setting $a_I = 10.3$ and $a_F = 1212.8$ we match these two facts. All other parameter values are the same as in the 1990 benchmark, except trade costs $\tau = 1.33$. In this alternative 2005 benchmark, the model generates a share of high productivity export sales of .317 (the observed high-tech sales out of total manufacturing export sales for 2005 is 30 percent). Innovation is similar to the chosen benchmark: $\delta = 18.0465$ and $n_0 = 1,522,159$. However, the North-South relative wage is higher than in our chosen benchmark ($w^N/w^S = 2.8724$ compared to 2.0437 in Table 1, Column 2). The rates of FDI are even lower than in the 1990 benchmark ($\phi_H = .00484$ and $\phi_L = .00142$). MP sales in the northern market is .020 and in the southern market .084 – much lower than in the chosen 2005 benchmark. Northern consumer welfare is similar to the chosen 2005 benchmark ($u_0^N = 270.01$) but southern consumer welfare is substantially lower ($u_0^S = 60.69$, even lower than in the 1990 benchmark). The share of foreign affiliate-produced varieties out of total varieties on the world market is very low ($\gamma_{FH} = .00068$ and $\gamma_{FL} = .00239$, only slightly higher than in the 1990 benchmark).

In conclusion, even though this benchmark matches the observed 2005 consumption-share adjusted real GDP per worker ratio for US-Mexico and the 9.8 percent share of foreign affiliates in world GDP, it fails to match the ten-fold increase in FDI inflow to developing countries. Most importantly, setting such a high a_F results in very little FDI (L_{F0} only increases by a factor of two), the share of foreign affiliate-produced varieties on the world market remains close to its 1990 share. Going from the 1990 benchmark to this alternative 2005 benchmark, MP sales in the northern market increases from .005 to .084 (0.084 is 4.97 percent of the total sales in northern market) while MP sales in the southern market decreases from .051 to .020 (.020 is 12.73 percent of the total sales in the southern market). It is not plausible to assume that FDI has become so much more difficult during 1990-2005 (a_F increasing from the 1990 benchmark value of 148.5 to the alternative 2005 benchmark value of 1212.8.). The chosen 2005 benchmark overestimates the share of foreign affiliates in world GDP, but it generates a substantial increase in MP while replicating the observed increase in FDI inflow going to developing countries – consistent with empirical evidence.