Export-Learning and FDI with Heterogeneous Firms

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Abstract

Multinational production (MP) is driven by market access or offshoring to low-cost locations. When firms set up foreign affiliates in non-OECD countries (North-South MP), they may face issues such as weak intellectual property rights protection, poor investment climate, and high communication and monitoring costs. We develop a dynamic general equilibrium trade model with heterogeneous firms that addresses issues specific to North-South MP. We calibrate the model to match general trends in the global economy and explore the model's predictions for MP, exporting, innovation and consumer welfare through numerical exercises. We find that stronger intellectual property rights (TRIPS) lead to more innovation in the North, more MP, more technology transfer to the South and significantly higher long-run southern consumer welfare. In contrast, trade liberalization has the long-run effect of making consumers worse off in both regions.

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1 Introduction

Multinational production (MP), defined here as production done by affiliates outside of the country of origin of the parent firm, has become a central feature of world trade and economic globalization. In 1990, the foreign affiliate share of world GPD (value-added) was 4.6 percent but by 2005, this share had risen to 10 percent (UNCTAD, 2012). Within this measure, there are both foreign affiliates in OECD countries (North-North MP) and foreign affiliates in developing countries (North-South MP). During the time period from 1990 to 2005, there was a ten-fold increase in foreign direct investment (FDI) going to developing countries (UNCTAD *FDI Statistics*). Research and development (R&D) expenditure of US manufacturing firm foreign affiliates in Mexico, Latin America, non-OECD Asia, Africa and the Middle East increased seven-fold from 1995 to 2007. Only looking at US foreign affiliates located in non-OECD Asia, there was an eight-fold increase in R&D expenditure from 1994-1996 to 2004-2006 (*OECD.Stat*).[1]

In this paper, we take a closer look at the large increase in multinational firm activities in non-OECD countries. The Trade-Related Aspects of Intellectual Property Rights (TRIPS) agreement was signed as part of the Uruguay Round in 1994. This agreement formally introduced intellectual property rights into the World Trade Organization (WTO) and the world trading system. The TRIPS agreement covers copyrights and patents but also enforcement procedures and dispute mechanisms. Since most developed countries already had such systems in place, the implied changes in national regulation required by the TRIPS agreement mostly affects developing countries. They have been forced to increase their intellectual property rights (IPR) protection to remain inside the WTO.[2] McCalman (2001) estimates the value of transfers of income between countries implied by the TRIPS agreement. He finds that only a few countries gained from TRIPS (United States, Germany, France, Italy, Sweden, Switzerland) and that all other countries were made worse

[1] As the *OECD.Stat* data shows some yearly fluctuation in foreign affiliate R&D spending (for all regions), we compare an average of 1994-1996 R&D expenditure with an average of 2004-2006. For US firm foreign affiliates in non-OECD Asia, R&D expenditure is available for all years starting from 1994.

[2] The TRIPS agreement has been controversial. A *New York Times* op-ed provides an example of opposition to stronger IPRs. Krugman (2014) writes, "Basically, old-fashioned trade deals are victim of their own success: there just isn't much more protectionism to eliminate. Average U.S. tariff rates have fallen by two-thirds since 1960 . . . these days, "trade agreements" are mainly about other things. What they're really about, in particular, is property rights – things like the ability to enforce patents on drugs and copyrights on movies . . . Is this a good thing from a global point of view? Doubtful. The kind of property rights we're talking about here can alternatively be described as legal monopolies. True, temporary monopolies are, in fact, how we reward new ideas; but arguing that we need even more monopolization is very dubious . . . and has nothing at all to do with classical arguments for free trade."

off, including all developing countries. But it is just assumed in McCalman's cost-benefit analysis that there are no dynamic benefits from TRIPS. Recently, evidence has emerged indicating that, not only are there dynamic benefits from TRIPS, but these dynamic benefits take more forms than economists had previously realized. Branstetter, Fisman, Foley and Saggi (2011) study the response of host country industrial production to stronger IPR protection. They find that following patent reform, not only did US-based multinational firms expand their activities in reforming countries, but this lead to exports of new goods increasing in these reforming countries.

We develop a dynamic general equilibrium trade model with heterogeneous firms that incorporates issues that are specific to North-South MP. Examples of such issues are weak IPR protection, poor host-country investment climate, and high communication costs between parent firms and their foreign affiliates. We calibrate the model to match general trends in the global economy and explore the model's predictions for MP, exporting, innovation and consumer welfare through numerical exercises. Firm heterogeneity plays a central role in our analysis and we study how high productivity firms behave differently from low productivity firms. Consistent with the empirical literature, the model implies that only a small share of firms export and an even smaller share of firms are multinationals.[3] The model is calibrated to match the evidence of an almost seven-fold increase in R&D expenditure by foreign affiliates in non-OECD countries from 1995 to 2007. We show how stronger IPR protection and lower communication costs between parents and foreign affiliates can explain the large increase in MP activities by foreign affiliates in non-OECD countries.

In the model, firms in the North (developed countries) engage in innovative R&D to develop new product varieties. Upon successful innovation, a northern firm starts to produce in the North (serving the home market) and learns if it is a low or high productivity firm.[4] Firms in the North can engage in export-learning R&D to access the southern market and earn higher profits from

[3]For evidence, see for example Bernard, Eaton, Jensen and Kortum (2003). Even though only a small share of firms are multinationals, they account for a large share of world trade. For example, foreign affiliates of US-based multinational firms and foreign affiliates located in the US accounted for 2/3 of US goods exports and imports in 2010. For France, the figures for 2010 were 64 percent of goods exports and 62 percent of goods imports (UNCTAD 2013).

[4]This feature of the model is of course inspired by the seminal paper Melitz (2003) about trade with heterogeneous firms, where firms develop new product varieties and then learn their productivities. One problem with the Melitz model is that its implication for the effect of trade liberalization on industrial productivity is the exact opposite of what researchers like Trefler (2004) find empirically. This problem is discussed in Segerstrom and Sugita (2015) and a solution is suggested in Segerstrom and Sugita (2016). The proposed solution involves replacing constant returns to R&D with decreasing returns to R&D in an otherwise standard Melitz model. In this paper, we also assume decreasing returns to R&D and essentially present a dynamic version of the static Melitz model.

selling in both markets. The export-learning costs are of a similar nature to the fixed export costs in the static Arkolakis (2010) model, where firms need to pay a fixed cost for marketing or setting up a distribution network in each export market.[5] Northern exporting firms can then choose to engage in MP-learning R&D (or FDI) to learn how to produce their products in the lower-wage South (developing countries), and once successful, their foreign affiliates located in the South earn even higher global monopoly profits. Our assumption that MP follows exporting is motivated by the recent evidence in Conconi, Sapir and Zanardi (2016). Looking at all Belgian manufacturing firms that started to engage in FDI during 1998-2008, they find that 86 percent of these firms were already serving the foreign market via exports. This suggests to us that learning how to export is a stepping stone to MP.[6] For comparison, Gumpert, Moxnes, Ramondo and Tintelnot (2016) find that among French and Norwegian multinationals, 40 percent had tried out the host market first via exporting.[7] Once any foreign affiliate starts producing in the South, it faces the risk of imitation from southern firms. If imitation occurs, the product market becomes perfectly competitive and the foreign affiliate no longer earns any profits. Stronger IPR protection in the South (TRIPS) is modelled as a decrease in this imitation rate.[8]

We calibrate the model to fit two benchmark cases: a 1990-1995 or pre-TRIPS benchmark (the world prior to the implementation of the TRIPS agreement and substantial trade liberalization for developing countries) and a 2005-2007 or post-TRIPS benchmark (the world after the implementation of the TRIPS agreement and trade liberalization). In both benchmark equilibria, we find that the export-learning rate is higher for high productivity firms than for low productivity firms, and the MP-learning rate is higher for high productivity firms than for low productivity firms. Because of these differences, exporting firms are more productive on average than non-exporting firms and

[5]The fixed exporting cost in the static Arkolakis (2010) model conceptually involves advertising costs. In a dynamic setting, such costs would be per period fixed costs. In our framework, we do not have any per period fixed costs of exporting, only an entry cost paid in destination market labor.

[6]In Helpman, Melitz and Yeaple (2004), firms are heterogeneous in productivity and face fixed costs for selling domestically, for entering a foreign market via exports, and for entering a foreign market via FDI. The fixed costs of FDI are higher than the fixed costs for exporting and all firms with productivity above a threshold level engage in FDI. Firms with productivity below this threshold level but above another lower threshold level decide to export instead. The model is static and the decision to enter the foreign market via exports or FDI is a one-time decision.

[7]The Norwegian data spans all foreign affiliates of Norwegian firms in the manufacturing sector for the years 1996-2006. The French data spans the years 1999-2011 with 230,000 to 245,000 firm observations per year of which 0.3 percent are multinationals and 10.1 percent exporters (Gumpert et al, 2016).

[8]In Jakobsson and Segerstrom (2016), we study the impact of TRIPS using a model where imitation is costly and the imitation rate depends on the decisions of profit-maximizing firms. In that model, southern firms can choose between imitating a foreign affiliate's variety or reverse engineering an imported variety.

multinational firms are even more productive on average than exporting firms.[9] Our results suggest that MP benefits southern consumers. Lower communication costs between parents and foreign affiliates, and lower entry costs of MP lead to more innovation and higher long-run southern consumer welfare. Going from the pre-TRIPS to the post-TRIPS benchmark, we find that stronger southern IPR protection (TRIPS) lead to more FDI, more production taking place in foreign affiliates (more MP), more innovation and considerably higher long-run southern consumer welfare.[10] In contrast, we find that trade liberalization leads to more export-learning and actually lowers long-run southern consumer welfare by diverting northern resources away from innovative activities (to production for export).

This paper is related to the recent MP literature that studies the interaction of trade and MP flows to quantify the gains from openness in trade models with heterogenous firms. In their quantitative application of Helpman, Melitz and Yeaple (2004) with intra-firm trade, Irarrazabal, Moxnes and Opromolla (2013) use a dataset of Norwegian multinational firms and their affiliates in OECD countries. Tintelnot (2016) uses a general equilibrium framework with export-platform FDI to estimate the unit input costs for German multinationals' foreign affiliates located in 11 OECD countries. These estimates are then used in calibration of a general equilibrium version of the model with MP data for 1996-2001 from Ramondo, Rodríguez-Clare and Tintelnot (2015) and bilateral trade flows for the same 12 OECD countries to analyze welfare effects of trade and MP, and to back out iceberg MP costs.[11] Ramondo and Rodríguez-Clare (2013) introduce MP in an Eaton and Kortum (2002) Ricardian framework to study the substitutability and complementarity between trade and MP for OECD countries. Arkolakis, Ramondo, Rodríguez-Clare and Yeaple (2014) develop a monopolistic competition framework to determine the location of innovation activities and production activities across OECD countries.[12] To understand the large increase in

[9]For evidence about the productivity differences between non-exporting, exporting and multinational firms, see for example Bernard, Eaton, Jensen and Kortum (2003), Bernard and Jensen (2004), Mayer and Ottaviano (2008), and Lileeva and Trefler (2010).

[10]Using Bureau of Economic Analysis (BEA) data to study a slightly different time period 1999-2009, Arkolakis, Ramondo, Rodríguez-Claire and Yeaple (2014) report that R&D expenditures in the US relative to local manufacturing value-added grew from 8.7% to 12.7%, and US firms increased the share of their total global employment that is located in their foreign affiliates from 22% to 31%. Their data include foreign affiliates in OECD locations. As discussed, in our paper we focus on MP with non-OECD production locations.

[11]The monopolistic competition framework in Tintelnot (2016) nests an Eaton and Kortum (2002) structure within each firm so that a firm produces a continuum of goods. Each firm faces productivity shocks that are specific to the production location of a particular good.

[12]In Arkolakis et al (2014), comparative advantage and home market effects coming from increasing returns to

FDI inflow going to developing countries, we instead focus on MP with foreign affiliates located in developing countries. Entry costs of exporting and MP, and communication costs between parents and foreign affiliates are common issues for all MP. When studying MP among non-OECD countries, weak IPR protection is of particular concern. The evidence in Branstetter et al (2011) suggests that patent reform in developing countries lead to more MP by US-based multinational firms. In our model, IPR protection in developing countries plays a crucial role in determining both MP flows and innovation. With the exception of Arkolakis et al (2014) who model innovation as creating heterogenous firms selling differentiated goods in markets characterized by monopolistic competition à la Melitz (2003), the other papers in this new literature do not address innovation. In Irarrazabal et al (2013) wages are fixed by assumption and there is no innovation, in Tintelnot (2014) firm entry is exogenous, and Ramondo and Rodríguez-Clare's perfect competition model does not allow for innovation. Importantly, while the previously-mentioned models within this new literature are static models, we are able to study the dynamic gains that arise from trade and North-South MP. One exception is Gumpert et al (2016), who develop a dynamic two-country version of Helpman et al (2004) by assuming that firm productivity follows a Markov process (and there are sunk entry costs for MP). They are able to capture the observed entry and exit rates of exporters and multinationals for France and Norway. Our focus is instead on the welfare effects of North-South MP where the primary incentive for MP is low-cost production as opposed to market access.

This paper is also related to the large literature on IPR protection in developing countries. Early models of North-South trade and IPR protection by Chin and Grossman (1990) and Deardorff (1992) do not have FDI and no international technology transfer takes place within multinational firms. Models with costless FDI have been developed by Helpman (1993), Lai (1998), Glass and Wu (2007), Branstetter and Saggi (2011), and He and Maskus (2012). Glass and Saggi (2002) present a North-South trade model with costly FDI but their results are not robust to allowing for decreasing returns to R&D. This is shown in Gustafsson and Segerstrom (2011), where a North-South trade model with costly FDI and decreasing returns to R&D is developed. A version of this model with endogenous imitation of both imported and MP varieties is calibrated in Jakobsson and Segerstrom (2016) to match the world economy before and after the TRIPS agreement went into

innovation and geographical frictions determine specialization in production or innovation across countries.

effect. In all of the previously-mentioned models of IPR protection in developing countries, firms are homogeneous and all firms export. In this paper we take seriously the evidence that firm-level productivity differences are important and study the impact of stronger IPR protection in a setting where firms differ in their productivities and most firms do not export.

The rest of the paper is organized as follows. In Section 2, we present the model and derive eight steady-state equilibrium conditions. In Section 3, we solve the model numerically for different parameter values and present the results. Then in Section 4 we offer some concluding remarks. There is an Appendix where we present results from solving the model for alternative parameter values and present the calculations that we did to solve the model in more detail.

2 The Model

2.1 Overview

Consider a global economy with two regions, the North and the South. Labor is the only factor of production. It is used to manufacture product varieties, develop new product varieties (innovation), adapt existing product varieties for entry into the foreign market (export-learning) and adapt exported varieties for production in the South (FDI or MP-learning). Labor is perfectly mobile across activities within a region, but cannot move across regions. Since labor markets are perfectly competitive, there is one single wage rate paid to all northern workers w^N and one single wage rate paid to all southern workers w^S. Although labor cannot move across regions, goods can. International trade between the North and the South is subject to iceberg trade costs: $\tau > 1$ units of a good must be shipped for one unit to arrive at its destination.

Only firms in the North, *northern* firms, have the capacity to innovate. A northern firm can hire workers to engage in innovative R&D with the purpose of developing the blueprint for a new product variety. After successful innovation, the firm earns monopoly profits from selling to the domestic market (the North) and learns if it is a low or high productivity firm. When the northern firm makes the decision of how much labor to hire for innovation, the firm does not know its own productivity in manufacturing, and there is therefore uncertainty about its expected profit flow. With probability $q_L = q$, the northern firm will be a low productivity firm with unit labor requirement c_L and with probability $q_H = 1 - q$, the northern firm will be a high productivity firm with unit labor requirement c_H, where $c_H < c_L$. Even though firms are heterogeneous in their productivities, high and low productivity firms face the same labor requirement for R&D.

After learning its productivity, a northern firm can hire southern workers to engage in export-learning R&D to access the southern market. Such R&D costs can be thought of as marketing, setting up distribution networks and learning how to comply with regulations in the foreign market. Upon successful export-learning, the firm earns higher monopoly profits since it earns profits from selling in both markets (the North and the South). Such a firm is called an *exporter*.

An exporter can then choose to hire southern workers to engage in MP-learning R&D to learn how to produce in the South. When successful in MP-learning R&D, a firm earns higher global monopoly profits because the cost of production is lower in the South.[13] Such a firm is called a *foreign affiliate* since, even though all production takes place in the South, its profits are repatriated back to its stockholders in the North. MP-learning R&D is the entry cost that firms incur when they learn how to do MP in the South and can therefore be interpreted as an index of FDI. Manufacturing by foreign workers involves an iceberg productivity loss that can be thought of as arising from information frictions or monitoring and communication costs between the parent firm and its foreign affiliate. To produce one unit of a good, a foreign affiliate requires ξc_z units of labor where $\xi > 1$, $(z = H, L)$.

R&D done in the South (export-learning R&D and MP-learning R&D) is financed by northern savings and northern firms control the amount of R&D in order to maximize their global expected discounted profits. Upon successfully adapting production to the South, a foreign affiliate sells to the southern market and also exports back to the North without incurring any additional export-learning costs. Foreign affiliates are exposed to a positive rate of imitation from *southern* firms. Once a product variety has been imitated, the blueprint becomes available to all southern firms, the product market becomes perfectly competitive and the foreign affiliate no longer earns any profits.

As illustrated in Figure 1, the model generates one-way product cycles à la Vernon (1966). The number of varieties in the economy grows at the rate g as a result of the innovative R&D activities of northern firms. Each product variety is initially produced by a northern firm that sells to its

[13] We will only solve for equilibria where $w^N > w^S$, since lower production costs in the South creates the incentive for MP in the model.

home market. It is at this point that the northern firm learns its own productivity. With probability q_z, the firm draws the productivity $z = H, L$. The firm can then engage in export-learning R&D with the aim of exporting to the southern market. Export-learning occurs at the rate χ_z. After the firm has become an exporter, it can engage in MP-learning R&D with the aim of producing in the lower-wage South. Such international technology transfer occurs at the FDI rate ϕ_z. Each foreign affiliate is exposed to the positive rate of imitation ι_S from southern firms, resulting in southern firms producing the product variety for the entire world market.

2.2 Households

In both the North and the South, there is a fixed measure of households that provide labor services in exchange for wage payments. Each individual member of a household lives forever and is endowed with one unit of labor, which is inelastically supplied. The size of each household, measured by the number of its members, grows exponentially at a fixed rate g_L, the population growth rate. Let $L_t^N = L_0^N e^{g_L t}$ denote the supply of labor in the North at time t, let $L_t^S = L_0^S e^{g_L t}$ denote the corresponding supply of labor in the South, and let $L_t = L_t^N + L_t^S$ denote the world supply of labor. In addition to wage income, northern households also receive asset income from their ownership of firms. We assume that R&D done by innovating, export-learning and MP-learning firms is financed by northern savings, which is roughly consistent with the Feldstein and Horioka (1980) finding that domestic savings finances domestic investments.[14]

Households in both the North and the South share identical preferences. Each household is modeled as a dynastic family that maximizes discounted lifetime utility

$$U = \int_0^\infty e^{-(\rho - g_L)t} \ln(u_t) dt \tag{1}$$

where $\rho > g_L$ is the subjective discount rate and u_t is the static utility of an individual at time t.

[14]French and Poterba (1991) document that around 94% of Americans investors held their equity wealth in the US stock market and Japanese investors held around 98% of their equity wealth in the Japanese stock market. Tesar and Werner (1995) also document this home bias in equity portfolios. More recent evidence of this home equity bias can be found in Cummings et al. (2010). They document that in 2007, US residents held 86 percent of the total market value of all US company equities either directly as individual investors or indirectly through pension funds, and retirement and insurance accounts.

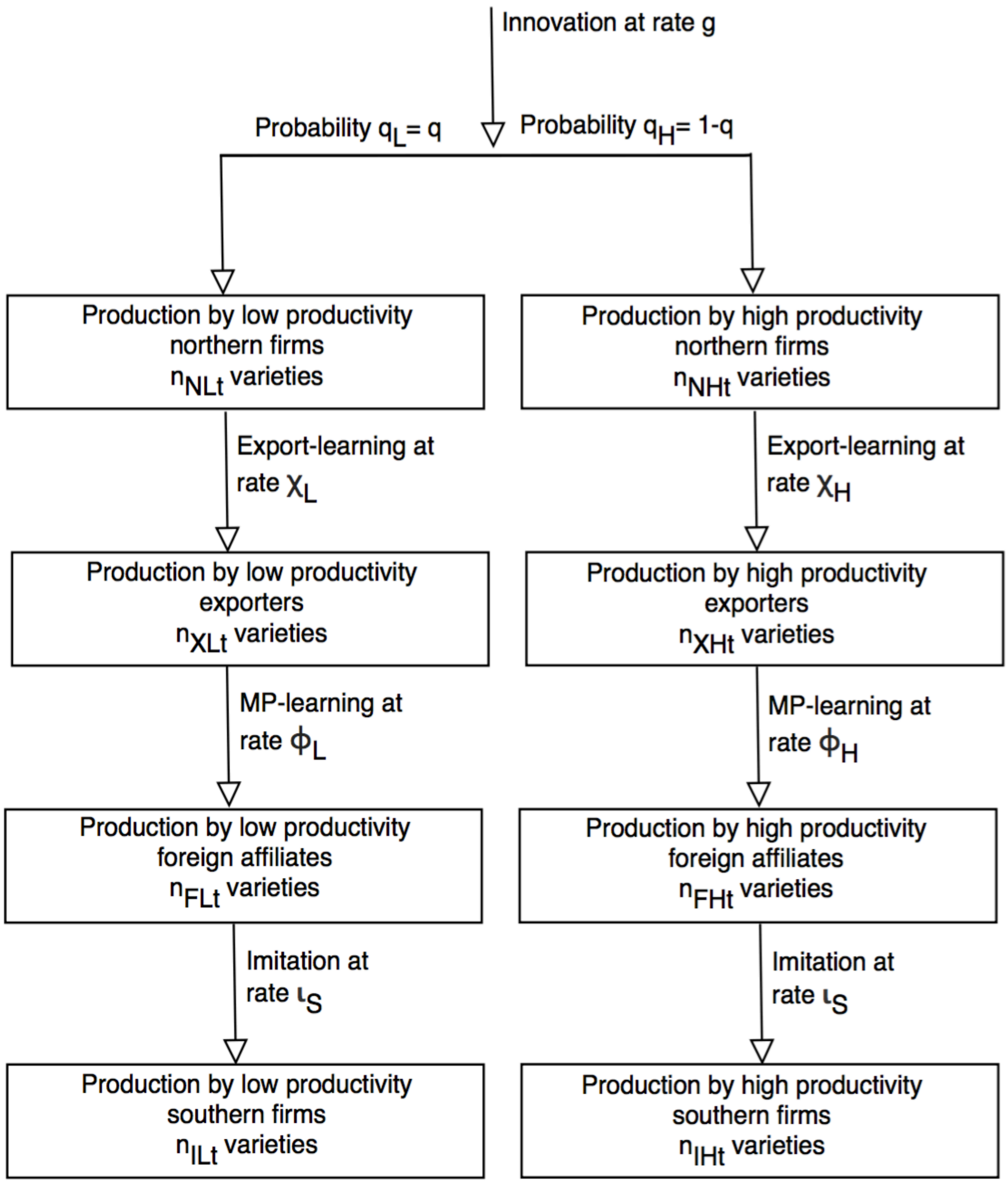

Figure 1: One-Way Product Cycles

The static constant elasticity of substitution (CES) utility function is

$$u_t = \left[\int_0^{n_t} x_t(\omega)^\alpha d\omega\right]^{\frac{1}{\alpha}}, \qquad 0 < \alpha < 1. \tag{2}$$

In (2), $x_t(\omega)$ is the per capita quantity demanded of the product variety ω at time t and n_t is the total number of invented varieties at time t. We assume that varieties are gross substitutes. Then with α measuring the degree of product differentiation, the elasticity of substitution between different product varieties is $\sigma \equiv 1/(1-\alpha) > 1$.

Solving the static consumer optimization problem yields the familiar demand function

$$x_t(\omega) = \frac{p_t(\omega)^{-\sigma} e_t}{P_t^{1-\sigma}} \tag{3}$$

where e_t is individual consumer expenditure at time t, $p_t(\omega)$ is the price of variety ω at time t, and $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ is an index of consumer prices. We will shortly define one such price index for each region. By substituting the demand function (3) into (2) and using the definition of the price index P_t, it can be shown that $u_t = e_t/P_t$. Then maximizing (1) subject to the relevant intertemporal budget constraint yields the intertemporal optimization condition

$$\frac{\dot{e}_t}{e_t} = r_t - \rho \tag{4}$$

implying that individual consumer expenditure only grows over time if the market interest rate r_t is larger than the subjective discount rate ρ.

The representative consumer in each region has different wage income ($w^N > w^S$) and different asset income and hence different consumer expenditure. Let e_t^N and e_t^S denote the representative consumer's expenditure in the North and the South, respectively. We treat the southern wage rate as the numeraire price ($w^S = 1$) so all prices are measured relative to the price of southern labor. We solve the model for a steady-state equilibrium where wages w^N, w^S and consumer expenditure e^N, e^S are all constant over time. Then $\dot{e}_t/e_t = 0$ in (4) and $r_t = \rho$. The steady-state market interest rate is thus constant over time and equal in the two regions.[15]

[15]The two regions typically have different interest rates along the transition path leading to a new steady-state equilibrium. But in a steady-state equilibrium, the two regions must have the same interest rate because consumers in both regions have the same subjective discount rate ρ.

For each level of productivity $z = H, L$, there are four types of firms indexed by $j = N, X, F, I$. There are northern firms that only sell to the home market ("N" for "northern"), exporters who serve both markets ("X" for "export"), foreign affiliates that produce in the South ("F" for "FDI") and southern firms that have imitated foreign affiliates ("I" for "imitation"). Let n_{jzt} denote the number of product varieties produced by type j firms with productivity z at time t. Due to positive trade costs, the prices of products will also differ between the two regions $r = N, S$. Let p^r_{jz} denote the price charged to consumers in region r by firms of type j with productivity z. In steady-state equilibrium, all product prices are constant over time. Even though consumer expenditure is constant over time, the steady-state equilibrium involves a positive rate of economic growth. As we will show, the number of product varieties that consumers can buy n_t gradually increases over time and this contributes to growth in living standards because consumers benefit from greater variety in consumption.

2.3 Steady-State Dynamics

Let $g \equiv \dot{n}_t/n_t$ denote the steady-state growth rate of the number of varieties. From the variety condition $n_t = \sum_j \sum_z n_{jzt}$, it follows that the number of varieties produced by each type of firm must grow at the same rate $g = \dot{n}_{jzt}/n_{jzt}$. Therefore the variety shares $\gamma_{jz} \equiv n_{jzt}/n_t$ are necessarily constant over time in any steady-state equilibrium and satisfy $\sum_j \sum_z \gamma_{jz} = 1$.

Let $\chi_z \equiv (\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt})/n_{Nzt}$ denote the steady-state export-learning rate, which is constant over time since $\chi_z = (g/\gamma_{Nz})(\gamma_{Xz} + \gamma_{Fz} + \gamma_{Iz})$. In this definition, we take into account that some of the exported varieties are adapted for production by foreign affiliates, and in turn, some of these foreign affiliate varieties are imitated by southern firms. Let $\phi_z \equiv (\dot{n}_{Fzt} + \dot{n}_{Izt})/n_{Xzt}$ denote the steady-state MP-learning rate, which is constant over time since $\phi_z = (g/\gamma_{Xz})(\gamma_{Fz} + \gamma_{Iz})$. In the definition of the MP-learning rate, we take into account that moving production to a foreign affiliate exposes the firm to a positive rate of imitation by southern firms. Let $\iota_S \equiv \dot{n}_{Izt}/n_{Fzt}$ denote the imitation rate of foreign affiliate-produced varieties. It is constant over time in steady-state equilibrium since $\iota_S = g(\gamma_{Iz}/\gamma_{Fz})$.

By the law of large numbers, $\sum_j \gamma_{jz} = q_z$. From the variety condition $n_t = \sum_j \sum_z n_{jzt}$, it follows that a share $q_L = q$ of total varieties are low productivity varieties and the remaining share

$q_H = 1 - q$ are high productivity varieties. Taking the time derivative of $q_z n_t = \sum_j n_{jzt}$, it is straightforward to show that the steady-state variety shares are

$$\gamma_{Nz} = q_z \frac{g}{g + \chi_z}, \tag{5}$$

$$\gamma_{Xz} = q_z \frac{\chi_z}{g + \chi_z} \frac{g}{g + \phi_z}, \tag{6}$$

$$\gamma_{Fz} = q_z \frac{\chi_z}{g + \chi_z} \frac{\phi_z}{g + \phi_z} \frac{g}{g + \iota_S} \tag{7}$$

and

$$\gamma_{Iz} = q_z \frac{\chi_z}{g + \chi_z} \frac{\phi_z}{g + \phi_z} \frac{\iota_S}{g + \iota_S}. \tag{8}$$

As expected, faster export-learning rates for northern firms correspond to larger shares of world production being done by northern exporters, more exporters learning how to become multinationals and more varieties being imitated ($\chi_z \uparrow \Longrightarrow \gamma_{Xz} \uparrow, \gamma_{Fz} \uparrow, \gamma_{Iz} \uparrow$). Faster MP-learning rates correspond to smaller shares of world production being done by northern exporters, larger shares being produced by foreign affiliates, and larger shares being produced by southern firms ($\phi_z \uparrow \Longrightarrow \gamma_{Xz} \downarrow, \gamma_{Fz} \uparrow, \gamma_{Iz} \uparrow$). And as expected, a faster imitation rate corresponds to smaller shares being produced by foreign affiliates and larger shares by southern firms ($\iota_S \uparrow \Longrightarrow \gamma_{Fz} \downarrow, \gamma_{Iz} \uparrow$).

The price index in the North will be different from the price index in the South for two reasons. First, product prices differ across regions because of trade costs τ. Second, the set of product varieties available in the northern market is larger than the set of product varieties available in the southern market, since some northern product varieties are only sold domestically. Let P_t^r denote the price index for region r. Given the definition of the price index $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ it follows that the northern price index satisfies $\left(P_t^N\right)^{1-\sigma} = \sum_j \sum_z \left[n_{jzt} \left(p_{jz}^N\right)^{1-\sigma}\right]$ and the southern price index satisfies $\left(P_t^S\right)^{1-\sigma} = \sum_{j \neq N} \sum_z \left[n_{jzt} \left(p_{jz}^S\right)^{1-\sigma}\right]$. Using the variety shares definition $\gamma_{jz} \equiv n_{jzt}/n_t$, we can rewrite these expressions as

$$\left(P_t^N\right)^{1-\sigma} = \sum_j \sum_z \left[\gamma_{jz} \left(p_{jz}^N\right)^{1-\sigma}\right] n_t \tag{9}$$

$$\left(P_t^S\right)^{1-\sigma} = \sum_{j \neq N} \sum_z \left[\gamma_{jz} \left(p_{jz}^S\right)^{1-\sigma}\right] n_t \tag{10}$$

where the terms in brackets are constant over time. Thus, $\left(P_t^N\right)^{1-\sigma}$ and $\left(P_t^S\right)^{1-\sigma}$ both grow over time at the rate g in any steady-state equilibrium.[16]

2.4 Product Markets

The firms producing different product varieties compete in prices and maximize profits. There are constant returns to scale in production. For each firm operating in the North and for each of the firms that have imitated a foreign affiliate, c_z units of labor produces one unit of output. However, following Arkolakis et al (2014), Ramondo and Rodríguez-Clare (2013) and Tintelnot (2016), we assume that foreign affiliates face a productivity loss due to monitoring and communication costs.[17] We model this variable cost of MP as an iceberg cost: to produce one unit of a good, a foreign affiliate requires ξc_z units of labor where $\xi > 1$. There are added costs of doing business in another country and as is explained in Arkolakis et al (2014), these added costs reflect "various impediments that multinationals face when operating in a different economic, legal or social environment."

A northern firm that is not an exporter and only sells to its home market has the marginal cost $c_z w^N$. An exporter has the marginal cost $c_z w^N$ when selling to the home market and $\tau c_z w^N$ when selling to the export market. A foreign affiliate in the South has the marginal cost $\xi c_z w^S$ when serving its home market (the South) and $\tau \xi c_z w^S$ when serving its export market (the North). A southern firm has lower marginal costs than a foreign affiliate: $c_z w^S$ when serving the southern market and $\tau c_z w^S$ when serving the northern market.

A northern firm earns the (domestic) profit flow $\pi_{Nzt} = \left(p_{Nz}^N - c_z w^N\right) x_{Nzt}^N L_t^N$, where x_{jzt}^r is the quantity demanded by the typical consumer in region r of the product produced by a type j firm with productivity z. A northern firm chooses its price to maximize profits, and it is straightforward to verify that the profit-maximizing price is the monopoly price $p_{Nz}^N = c_z w^N/\alpha$. A low productivity northern firm has a higher marginal cost than a high productivity northern firm so the price

[16]The changes in the price indexes over time do not reflect price inflation but rather technological change. The prices that firms charge are constant over time, so the steady-state equilibrium involves a zero rate of price inflation. The only reason why the price indexes P_t^r change over time is that n_t increases. Thus, the nominal interest rate in steady-state equilibrium $r_t = \rho$ is also the real interest rate. There is no need to distinguish between nominal and real values of variables.

[17]Similarly, Glass and Saggi (2002), Parello (2008) as well as Gustafsson and Segerstrom (2011) in their extended model with endogenous imitation assume that southern (local) firms have a productivity advantage over foreign affiliates. Markusen (1995) provides further motivation for the assumption used in these models.

charged by a low productivity firm will be higher. Using these prices, we can write the northern firm's profit flow as

$$\pi_{Nzt} = \left[\frac{c_z w^N X^N_{Nz}}{(\sigma - 1)\gamma_{Nz}}\right]\frac{L_t}{n_t} \qquad (z = H, L) \tag{11}$$

where $X^r_{jz} \equiv \left(p^r_{jz}\right)^{-\sigma} e^r L^r_t n_{jzt} / \left(P^r_t\right)^{1-\sigma} L_t$ is the population-adjusted aggregate demand for the products of type j firms in market r. X^r_{jz} is constant over time in steady-state equilibrium since L^r_t grows at the same rate g_L as the world population L_t, and $\left(P^r_t\right)^{1-\sigma}$ grow at the same rate g as n_{jzt}. In (11), the marginal cost terms c_z and the elasticity of substitution σ are parameters, while the wage rate w^N and the variety share γ_{Nz} are constant over time in steady-state equilibrium. Therefore, profits earned by a northern firm only change because L_t/n_t changes over time. L_t/n_t is a measure of the size of the market relevant for each northern firm. Population growth increases the size of the market for firms but variety growth has the opposite effect because firms have to share consumer demand with more competing firms.

A northern firm that has learned how to export to the South earns the global profit flow $\pi_{Xzt} = \left(p^N_{Xz} - c_z w^N\right) x^N_{Xzt} L^N_t + \left(p^S_{Xz} - \tau c_z w^N\right) x^S_{Xzt} L^S_t$. The exporter's profit-maximizing price in the home market is $p^N_{Xz} = c_z w^N/\alpha$ and in the export market is $p^S_{Xz} = \tau c_z w^N/\alpha$. Using these prices, the global profit flow of a northern exporter can be written as

$$\pi_{Xzt} = \left[\frac{c_z w^N \left(X^N_{Xz} + \tau X^S_{Xz}\right)}{(\sigma - 1)\gamma_{Xz}}\right]\frac{L_t}{n_t}, \qquad (z = H, L). \tag{12}$$

The global profit flow for a foreign affiliate is $\pi_{Fzt} = \left(p^S_{Fz} - \xi c_z w^S\right) x^S_{Fzt} L^S_t + \left(p^N_{Fz} - \tau \xi c_z w^S\right) \cdot x^N_{Fzt} L^N_t$. Profit-maximizing monopoly prices are $p^S_{Fz} = \xi c_z w^S/\alpha$ in the domestic market (the South) and $p^N_{Fz} = \tau \xi c_z w^S/\alpha$ in the export market (the North). The incentive for an exporter to become a multinational firm and move production to the South is not primarily market access, but to earn higher profits by lowering production cost. Therefore we will solve for equilibria where the inequality condition $w^N > \tau \xi w^S$ holds so each foreign affiliate exports back to the North and the parent firm in the North ceases to produce there.[18] Using these prices, the global profit flow for

[18] In Helpman et al (2004), firms choose to enter into the foreign market either through exporting or through FDI. Market access is driving (horizontal) FDI in their model since a multinational firm continues to serve the parent firm's market via production at home. The assumption that exporters always keep serving the domestic market in our model is the same as in Helpman et al (2004). However, they assume that firms that engage in FDI serve the foreign market through the foreign affiliate but do not export back to the host country. This assumption is relaxed in the working paper version of their paper where they allow for export platform FDI. We assume that once a firm has successfully

a foreign affiliate can be written as

$$\pi_{Fzt} = \left[\frac{\xi c_z w^S \left(X_{Fz}^S + \tau X_{Fz}^N \right)}{(\sigma - 1)\, \gamma_{Fz}} \right] \frac{L_t}{n_t}, \qquad (z = H, L). \tag{13}$$

Once imitation has occurred, the blueprint is freely available to all southern firms. Southern firms do not incur any imitation costs. A southern firm that imitates a firm of high productivity becomes a high productivity southern firm and vice versa. Imitation involves learning the production technology for the variety as well as the ability to sell the product variety in all markets. After successful imitation, southern imitators do not incur any export-learning costs to introduce their product to the northern market.[19] No southern firm can set its price higher than marginal cost, and all southern firms earn zero profits. The resulting prices are $p_{Iz}^S = c_z w^S$ and $p_{Iz}^N = \tau c_z w_S$.

The above analysis implies that as a product shifts from being produced by a northern firm to its foreign affiliate and then by a southern firm, the equilibrium price of the product declines in the North ($p_{Nz}^N = p_{Xz}^N = c_z w^N/\alpha > p_{Fz}^N = \tau \xi c_z w_S/\alpha > p_{Iz}^N = \tau c_z w^S$) as well as in the South ($p_{Xz}^S = \tau c_z w^N/\alpha > p_{Fz}^S = c_z \xi w^S/\alpha > p_{Iz}^S = c_z w^S$). This price pattern is consistent with Vernon's (1966) description of the product life cycle, in which multinational firms play a central role.

2.5 Technology for Innovation, Export-Learning and MP-Learning

There is free entry into innovative R&D activities in the North, with every northern firm having access to the same R&D technology. To innovate and develop a new product variety, a representative northern firm i must devote $a_R g^\beta / n_t^\theta$ units of labor to innovative R&D, where a_R is an innovative R&D productivity parameter, n_t is the disembodied stock of knowledge at time t and θ is an intertemporal knowledge spillover parameter.[20] The parameter $\beta > 0$ captures decreasing returns to R&D at the industry level. When there is more innovation in the economy ($g \equiv \dot{n}_t/n_t$ is higher),

adapted production to a foreign affiliate, the parent firm no longer produces the variety in the domestic market and instead serves both markets via the foreign affiliate.

[19]The rationale is that the particular product variety has already been introduced to the northern market by the northern firm whose blueprint the imitator is using. It is possible to consider an alternative setting where the imitator can only sell the product in the South due to IPR protection in the northern market, or that only a small share of southern imitators export due to export-learning costs.

[20]For $\theta > 0$, R&D labor becomes more productive as time passes and a northern firm needs to devote less labor to develop a new variety as the stock of knoweldge increases. For $\theta < 0$, R&D becomes more difficult over time.

each individual northern firm must devote more resources to innovation in order to successfully develop one new product variety. Given this technology, the flow of new products developed by northern firm i is

$$\dot{n}_t^i = \frac{l_{Rt}^i}{a_R g^\beta / n_t^\theta} = \frac{n_t^\theta l_{Rt}^i}{a_R g^\beta}, \tag{14}$$

where l_{Rt}^i is the northern labor employed by firm i in innovative R&D. Aggregating over all northern firms, the aggregate flow of new products developed in the North is

$$\dot{n}_t = \frac{n_t^\theta L_{Rt}}{a_R g^\beta} = \left[\frac{n_t^{\theta+\beta} L_{Rt}}{a_R}\right]^{\frac{1}{1+\beta}}, \tag{15}$$

where $L_{Rt} \equiv \sum_i l_{Rit}$ is the total amount of northern labor employed in innovative activities. A large empirical literature on patents and R&D has shown that R&D is subject to significant decreasing returns at the industry level [point estimates of $1/(1+\beta)$ lie between 0.1 and 0.6 according to Kortum (1993), which corresponds to β values between .66 and 9]. Blundell, Griffith and Windmeijer (2002) find a long-run elasticity of patents to R&D of 0.5, which in our notation corresponds to $1/(1+\beta) = 0.5$ or $\beta = 1$.

In any steady-state equilibrium, the share of labor employed in innovative R&D must be constant over time. Given that the northern supply of labor grows at the population growth rate g_L, northern R&D employment L_{Rt} must grow at this rate as well. Dividing both sides of (15) by n_t yields $g \equiv \dot{n}_t/n_t = n_t^{\theta-1} L_{Rt}/a_R g^\beta$. Since g is constant over time in any steady-state equilibrium, $n_t^{\theta-1}$ and L_{Rt} must grow at offsetting rates, that is, $(\theta-1)\,\dot{n}_t/n_t + \dot{L}_{Rt}/L_{Rt} = (\theta-1)\,g + g_L = 0$. It immediately follows that

$$g \equiv \frac{\dot{n}_t}{n_t} = \frac{g_L}{1-\theta}. \tag{16}$$

Thus, the steady-state rate of innovation g is pinned down by parameter values and is proportional to the population growth rate g_L. As in Jones (1995), when there is positive population growth, the parameter restriction $\theta < 1$ is needed to guarantee that the steady-state rate of innovation is positive and finite.

We can now solve for the steady-state rate of economic growth. The representative consumer in region r has utility $u_t^r = e^r/P_t^r$. In steady-state equilibrium, individual consumer expenditure is constant over time but consumer utility nevertheless grows because the price indexes fall over time. Since $(P_t^r)^{1-\sigma}$ grows over time at the rate g, it follows that consumer utility growth is

$$g_u \equiv \frac{\dot{u}_{Nt}}{u_{Nt}} = \frac{\dot{u}_{St}}{u_{St}} = \frac{g}{\sigma - 1} = \frac{g_L}{(1-\theta)(\sigma - 1)}. \tag{17}$$

With consumer utility in both regions being proportional to consumer expenditure holding prices fixed, consumer utility growth equals real wage growth, which we use as a measure of economic growth. Equation (17) implies that public policy changes like trade liberalization (a decrease in τ) or stronger IPR protection (a decrease in ι_S) have no effect on the steady-state rate of economic growth. In this model, growth is "semi-endogenous". We view this as a virtue of the model because both total factor productivity and per capita GDP growth rates have been remarkably stable over time in spite of many public policy changes that one might think would be growth-promoting. For example, plotting data on per capita GDP (in logs) for the US from 1870 to 1995, Jones (2005, Table 1) shows that a simple linear trend fits the data extremely well. Further evidence for the R&D assumptions underlying semi-endogenous growth models is provided by Venturini (2012). Looking at US manufacturing industry data for the period 1975-1996, he finds that the exhaustion of technological opportunities, which leads to increasing R&D difficulty, is the mechanism best matching the real dynamics of business innovation.

In the unit labor requirement for innovation $a_R g^\beta / n_t^\theta$, the term $1/n_t^\theta$ is a measure of absolute R&D difficulty. It increases over time if $\theta < 0$ and decreases over time if $\theta \in (0,1)$. By taking the ratio of R&D difficulty and the market size term L_t/n_t, we obtain a measure of relative R&D difficulty (or R&D difficulty relative to the size of the market):

$$\delta \equiv \frac{n_t^{-\theta}}{L_t/n_t} = \frac{n_t^{1-\theta}}{L_t}. \tag{18}$$

To see that δ is constant over time in steady-state equilibrium, note that $\dot{\delta}/\delta = (1-\theta)\,\dot{n}_t/n_t - \dot{L}_t/L_t = (1-\theta)\,g_L/\,(1-\theta) - g_L = 0$.[21]

To learn how to export one product variety to the South, a northern firm with productivity z must employ $a_X \chi_z^\beta / n_t^\theta$ units of southern labor to export-learning R&D.[22] The parameter a_X is an

[21] The innovation rate g is constant in steady-state equilibrium. However, if a public policy change like stronger IPR protection leads to a permanently higher value of δ, then there will be more innovation on the transition path to the new steady-state equilibrium.

[22] Following Arkolakis (2010) and Arkolakis et al (2014), we assume that southern labor is employed for northern

export-learning R&D productivity parameter. As with innovation, $\beta > 0$ captures the decreasing returns to export-learning R&D. The flow of new products entering the southern market due to northern firm i's export-learning activities is given by

$$\dot{n}^i_{Xzt} + \dot{n}^i_{Fzt} + \dot{n}^i_{Izt} = \frac{l^i_{Xzt}}{a_X \chi^\beta_z / n^\theta_t} = \frac{n^\theta_t l^i_{Xzt}}{a_X \chi^\beta_z}, \qquad (z = H, L) \tag{19}$$

where l^i_{Xzt} is the southern labor employed in export-learning R&D by firm i with productivity z. Aggregating over all northern firms, the flow of new products sold in the South as a consequence of export-learning activities is

$$\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt} = \frac{n^\theta_t L_{Xzt}}{a_X \chi^\beta_z}, \qquad (z = H, L) \tag{20}$$

where $L_{Xzt} \equiv \sum_i l^i_{Xzt}$ is the total amount of southern labor employed in export-learning activities by firms with productivity z.

MP-learning R&D (or FDI) is undertaken by exporters. To learn how to produce an exported variety in the South via MP, the foreign affiliate of an exporter must devote $a_F \phi^\beta_z / n^\theta_t$ units of southern labor to MP-learning R&D. The parameter a_F is an R&D productivity parameter that can be thought of as measuring the ease of doing FDI in the South. There are decreasing returns also to MP-learning R&D. The flow of products for which production is transferred to the South due to firm i's R&D activities is

$$\dot{n}^i_{Fzt} + \dot{n}^i_{Izt} = \frac{l^i_{Fzt}}{a_F \phi^\beta_z / n^\theta_t} = \frac{n^\theta_t l^i_{Fzt}}{a_F \phi^\beta_z}, \qquad (z = H, L) \tag{21}$$

where l^i_{Fzt} is the southern labor employed by firm i with productivity z in MP-learning R&D. Aggregating over all foreign affiliates generates the product flow

$$\dot{n}_{Fzt} + \dot{n}_{Izt} = \frac{n^\theta_t L_{Fzt}}{a_F \phi^\beta_z}, \qquad (z = H, L) \tag{22}$$

where $L_{Fzt} \equiv \sum_i l^i_{Fzt}$ is the aggregate amount of southern labor employed in MP-learning R&D

firms' export-learning activities. This can be thought of as hiring local labor for marketing and to set up distribution networks in the export market.

by firms with productivity z.

Imitation targets foreign affiliates in the South. Let $\iota_S \equiv 1/a_I$ where a_I is a measure of the strength of southern IPR protection. With stronger southern IPR protection, the rate of imitation is lower ($a_I \uparrow \Longrightarrow \iota_S \downarrow$).

2.6 R&D Incentives

Denote the expected discounted profits associated with innovating in the North at time t for a firm with productivity z by v_{Nzt}. The R&D labor used to develop one new variety is $a_R g^\beta / n_t^\theta$ and the cost of developing this variety is $w_N a_R g^\beta / n_t^\theta$. Taking into account the probability of a high (low) productivity draw, free entry into innovative R&D activities in the North implies that the cost of innovating must be exactly balanced by the expected benefit from innovating in equilibrium:

$$q v_{NLt} + (1-q)\, v_{NHt} = \frac{w^N a_R g^\beta}{n_t^\theta}. \tag{23}$$

Let v_{Xzt} be the expected discounted profits that an exporter with productivity z earns. The benefit of becoming an exporter is not the expected discounted profits that an exporter earns v_{Xzt} but the gain in expected discounted profits $v_{Xzt} - v_{Nzt}$ since the firm is already earning profits from selling in the North. Since the cost of becoming an exporter must be exactly balanced by the benefit in steady-state equilibrium, we obtain

$$v_{Xzt} - v_{Nzt} = \frac{w^S a_X \chi_z^\beta}{n_t^\theta}, \qquad (z = H, L). \tag{24}$$

Let v_{Fzt} denote the expected discounted profits that a foreign affiliate with productivity z earns from producing a product variety in the South at time t. The benefit of becoming a multinational firm is not the expected discounted profits that a foreign affiliate earns v_{Fzt} but the gain in expected discounted profits $v_{Fzt} - v_{Xzt}$. Since the cost of transferring production to the South must be exactly balanced by the benefit of MP-learning in steady-state equilibrium, we obtain

$$v_{Fzt} - v_{Xzt} = \frac{w^S a_F \phi_z^\beta}{n_t^\theta}, \qquad (z = H, L). \tag{25}$$

When technology transfer occurs, each foreign affiliate pays its parent firm a royalty payment v_{Xzt}

for the use of its technology in the South, since the MP-learning R&D accounts for the increase in the firm's value $v_{Fzt} - v_{Xzt}$.

We assume that there is a stock market in the North that channels household savings to firms that engage in R&D and helps households to diversify the risk of holding stocks issued by these firms. There is no aggregate risk, so it is possible for northern households to earn a safe return by holding the market portfolio in the region. Hence, ruling out any arbitrage opportunities implies that the total return on equity claims must equal the opportunity cost of invested capital, which is given by the risk-free market interest rate ρ.

For a northern firm i, the relevant no-arbitrage condition is $\left(\pi_{Nzt} - w^S l^i_{Xzt}\right) dt + \dot{v}_{Nzt} dt + \left(\dot{n}^i_{Xzt} + \dot{n}^i_{Fzt} + \dot{n}^i_{Izt}\right) dt \left(v_{Xzt} - v_{Nzt}\right) = \rho v_{Nzt} dt$. The northern firm earns the profit flow $\pi_{Nzt} dt$ during the time interval dt but also incurs the export-learning cost $w^S l^i_{Xzt} dt$ during this time interval. In addition, the firm experiences the gradual capital gain $\dot{v}_{Nzt} dt$ during the time interval dt and its market value jumps up by $v_{Xzt} - v_{Nzt}$ for each product that it succeeds in introducing to the southern market. The firm succeeds in introducing $\left(\dot{n}^i_{Xzt} + \dot{n}^i_{Fzt} + \dot{n}^i_{Izt}\right) dt$ varieties to the southern market during the time interval dt. To rule out any arbitrage opportunities for investors, the rate of return for a northern firm must be the same as the return on an equal sized investment in a risk-free bond $\rho v_{Nzt} dt$. From (19) and (24), it follows that $\left(\dot{n}^i_{Xzt} + \dot{n}^i_{Fzt} + \dot{n}^i_{Izt}\right) \left(v_{Xzt} - v_{Nzt}\right) = w^S l^i_{Xzt}$. Equation (23) implies that v_{Nzt} must grow at the rate $-\theta g$. Thus, after dividing by $v_{Nzt} dt$, the no-arbitrage condition for the z-productivity northern firm simplifies to $\pi_{Nzt}/v_{Nzt} - \theta g = \rho$ or $v_{Nzt} = \pi_{Nzt}/\left(\rho + \theta g\right)$. Combining this expression with (23), the northern no-arbitrage condition can be written as $\left(q\pi_{NLt} + (1-q)\,\pi_{NHt}\right)/\left(\rho + \theta g\right) = w^N a_R g^\beta / n^\theta_t$. In this equation, the left-hand side is the expected discounted profit from innovating and the right-hand side is the cost of innovation. The northern firm's expected discounted profits or market value is equal the expected profit flow $q\pi_{NLt} + (1-q)\,\pi_{NHt}$ appropriately discounted by the market interest rate ρ and the capital loss term θg. Substituting for expected profit flow using (11), dividing both sides by w^N and then by the market size term L_t/n_t yields the northern steady-state no-arbitrage condition

$$\frac{1}{\sigma - 1}\left(\frac{q\, c_L X^N_{NL}}{\gamma_{NL}\left(\rho + \theta g\right)} + \frac{(1-q)\, c_H X^N_{NH}}{\gamma_{NH}\left(\rho + \theta g\right)}\right) = a_R g^\beta \delta. \tag{26}$$

The left-hand side of (26) is the market size-adjusted expected benefit from innovating and the right-hand side is the market size-adjusted cost of innovating. In steady-state calculations, we need to adjust for market size L_t/n_t because market size changes over time if $g_L \neq g$ or $\theta \neq 0$. The market size-adjusted benefit from innovating is higher when the average consumer buys more of non-exported northern varieties ($X_{Nz}^N \uparrow$), future profits are less heavily discounted ($\rho \downarrow$), and northern firms experience larger capital gains over time ($\theta g \downarrow$). The market size-adjusted cost of innovating is higher when northern researchers employed in innovative R&D are less productive ($a_R \uparrow$), and when innovating is relatively more difficult ($\delta \uparrow$).

For an exporter i, the relevant no-arbitrage condition is $\left(\pi_{Xzt} - w^S l_{Fzt}^i\right) dt + \dot{v}_{Xzt} dt + (\dot{n}_{Fzt}^i + \dot{n}_{Izt}^i) dt \left(v_{Fzt} - v_{Xzt}\right) = \rho v_{Xzt} dt$. Following the same procedure as for northern firms, we obtain the steady-state exporter no-arbitrage condition

$$\frac{c_z w}{\sigma - 1}\left[\frac{X_{Xz}^N + \tau X_{Xz}^S}{\gamma_{Xz}\left(\rho + \theta g\right)} - \frac{X_{Nz}^N}{\gamma_{Nz}\left(\rho + \theta g\right)}\right] = a_X \chi_z^\beta \delta, \qquad (z = H, L) \tag{27}$$

where $w = w^N/w^S$ is the northern relative wage or the North-South wage ratio.

A foreign affiliate i faces the no-arbitrage condition $\pi_{Fzt} dt + \dot{v}_{Fzt} dt - \left(\iota_S dt\right) v_{Fzt} = \rho v_{Fzt} dt$. It is exposed to a positive rate of imitation by southern firms and experiences a total capital loss if it is imitated, which occurs with the probability $\iota_S dt$ during the time interval dt. Following the same procedure as for northern firms, we obtain the foreign affiliate steady-state no-arbitrage condition

$$\frac{c_z}{\sigma - 1}\left[\frac{\xi X_{Fz}^S + \tau \xi X_{Fz}^N}{\gamma_{Fz}\left(\rho + \theta g + \iota_S\right)} - \frac{w\left(X_{Xz}^N + \tau X_{Xz}^S\right)}{\gamma_{Xz}\left(\rho + \theta g\right)}\right] = a_F \phi_z^\beta \delta, \qquad (z = H, L). \tag{28}$$

2.7 Labor Markets

Each labor market is perfectly competitive and wages adjust instantaneously to equate labor demand and labor supply. Northern labor is employed in innovative R&D, in production by northern firms selling only to the home market and in exporting firms serving both markets. Each innovation requires $a_R g^\beta / n_t^\theta$ units of labor, so total employment in innovative R&D is $\left(a_R g^\beta / n_t^\theta\right) \dot{n}_t = a_R g^\beta \left(n_t^{1-\theta}/L_t\right)\left(\dot{n}_t/n_t\right) L_t = a_R g^{1+\beta} \delta L_t$. Northern firms use $c_z \left(p_{Nz}^N\right)^{-\sigma} e^N L_t^N / \left(P_t^N\right)^{1-\sigma}$ units of labor for each variety produced and there are n_{Nzt} such varieties produced. Exporters use $c_z \left(p_{Nz}^N\right)^{-\sigma} e^N L_t^N / \left(P_t^N\right)^{1-\sigma} + \tau c_z \left(p_{Nz}^S\right)^{-\sigma} e^S L_t^S / \left(P_t^S\right)^{1-\sigma}$ units of labor for each variety produced and there are n_{Xzt} such varieties produced, so total employment in production activities in the North is $\sum_z c_z X^N_{Nz} L_t + c_z \left(X^N_{Xz} + \tau X^S_{Xz} \right) L_t$. As L^N_t denotes labor supply in the North, full employment requires that $L^N_t = a_R g^{1+\beta} \delta L_t + \sum_z c_z X^N_{Nz} L_t + c_z \left(X^N_{Xz} + \tau X^S_{Xz} \right) L_t$. Evaluating at time $t = 0$ yields the steady-state full employment of labor condition for the North:

$$L^N_0 = L_0 \left[a_R g^{1+\beta} \delta + \sum_{z=H,L} c_z X^N_{Nz} + c_z \left(X^N_{Xz} + \tau X^S_{Xz} \right) \right]. \tag{29}$$

Southern labor is employed in export-learning R&D, MP-learning R&D, production by foreign affiliates and production by southern firms that have imitated foreign affiliates. Following the same procedure as for the northern labor market, full employment in the South requires that $L^S_t = \sum_z \left(a_X \chi^\beta_z / n^\theta_t \right) \left(\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt} \right) + \left(a_F \phi^\beta_z / n^\theta_t \right) \left(\dot{n}_{Fzt} + \dot{n}_{Izt} \right) + c_z \left[\xi X^S_{Fz} + \tau \xi X^N_{Fz} \right] L_t + c_z \left[X^S_{Iz} + \tau X^N_{Iz} \right] L_t$. Using the definitions of χ_z, ϕ_z and δ and evaluating at time $t = 0$, we obtain the steady-state full employment of labor condition for the South:

$$L^S_0 = L_0 \left[\sum_{z=H,L} a_X \delta \chi^{1+\beta}_z \gamma_{Nz} + a_F \delta \phi^{1+\beta}_z \gamma_{Xz} + c_z \left(\xi X^S_{Fz} + \tau \xi X^N_{Fz} + X^S_{Iz} + \tau X^N_{Iz} \right) \right]. \tag{30}$$

2.8 Aggregate Demand

To solve the model, we need steady-state values for the aggregate demand terms X^N_{Nz}, X^N_{Xz}, X^S_{Xz}, X^S_{Fz}, X^N_{Fz}, X^S_{Iz} and X^N_{Iz}. Solving for the ratio X^N_{Nz}/X^N_{Fz} yields

$$\begin{aligned} \frac{X^N_{Nz}}{X^N_{Fz}} &= \frac{\left[\left(p^N_{Nz} \right)^{-\sigma} e^N L^N_t n_{Nzt} \right] / \left[\left(P^N_t \right)^{1-\sigma} L_t \right]}{\left[\left(p^N_{Fz} \right)^{-\sigma} e^N L^N_t n_{Fzt} \right] / \left[\left(P^N_t \right)^{1-\sigma} L_t \right]} = \left(\frac{p^N_{Nz}}{p^N_{Fz}} \right)^{-\sigma} \frac{n_{Nzt}/n_t}{n_{Fzt}/n_t} \\ &= \left(\frac{c_z w^N / \alpha}{\tau \xi c_z w^S / \alpha} \right)^{-\sigma} \frac{\gamma_{Nz}}{\gamma_{Fz}} = \left(\frac{w}{\tau \xi} \right)^{-\sigma} \frac{q_z g / \left(g + \chi_z \right)}{q_z \chi_z \phi_z g / \left[\left(g + \chi_z \right) \left(g + \phi_z \right) \left(g + \iota_S \right) \right]}, \end{aligned}$$

and by doing similar calculations looking at other ratios, we obtain that $X^N_{Nz} = X^N_{Fz} \left(\tau \xi / w \right)^\sigma \left(g + \phi_z \right) \left(g + \iota_S \right) / \left(\chi_z \phi_z \right)$, $X^N_{Xz} = X^N_{Fz} \left(\tau \xi / w \right)^\sigma \left(g + \iota_S \right) / \phi_z$, $X^S_{Xz} = X^S_{Fz} \left(\xi / w \tau \right)^\sigma \left(g + \iota_S \right) / \phi_z$, $X^S_{Iz} = X^S_{Fz} \left(\xi / \alpha \right)^\sigma \iota_S / g$ and $X^N_{Iz} = X^N_{Fz} \left(\xi / \alpha \right)^\sigma \iota_S / g$.

Finally, we solve for the ratio $X^r_{FH}/X^r_{FL} = \left(c_H / c_L \right)^{-\sigma} \gamma_{FH} / \gamma_{FL}$. Inserting steady-state variety share expressions, we obtain

$$X_{FH}^{r}=X_{FL}^{r}\left(\frac{c_H}{c_L}\right)^{-\sigma}\left(\frac{1-q}{q}\right)\left(\frac{g+\chi_L}{g+\chi_H}\right)\left(\frac{\chi_H}{\chi_L}\right)\left(\frac{g+\phi_L}{g+\phi_H}\right)\left(\frac{\phi_H}{\phi_L}\right).$$

2.9 Asset Ownership and Consumer Expenditure

To determine consumer expenditures e^N and e^S, we need to specify who owns the firms and how wealth is distributed between the North and the South. We assume that R&D done by innovating, export-learning and MP-learning firms is financed by northern savings. Then in equilibrium, northern firms, exporters and foreign affiliates end up being owned by northern consumers.

Let A_t^N denote the aggregate value of northern financial assets and A_t^S denote the aggregate value of southern financial assets. There is perfect competition among southern firms, so $A_t^S=0$ and the aggregate value of all financial assets is $A_t=A_t^N=\sum_z\sum_{j\neq I}n_{jzt}v_{jzt}$. Substituting into this expression firm values from the no-arbitrage conditions $v_{Nzt}=\pi_{Nzt}/\left(\rho+\theta g\right)$, $v_{Xzt}=\pi_{Xzt}/\left(\rho+\theta g\right)$ and $v_{Fzt}=\pi_{Nzt}/\left(\rho+\theta g+\iota_S\right)$ along with profit expressions (11), (12) and (13) yields

$$A_t^N=\sum_z\frac{c_zL_t}{\sigma-1}\left[\frac{w^NX_{Nz}^N}{\rho+\theta g}+\frac{w^N\left(X_{Xz}^N+\tau X_{Xz}^S\right)}{\rho+\theta g}+\frac{w^S\xi\left(X_{Fz}^S+\tau X_{Fz}^N\right)}{\rho+\theta g+\iota_S}\right].$$

Let $\tilde{a}_t^r$ denote the financial asset holdings of the typical consumer in region r. The intertemporal budget constraint of a typical consumer in region r is $\dot{\tilde{a}}_t^r=w^r+\rho\tilde{a}_t^r-e^r-g_L\tilde{a}_t^r$. In any steady-state equilibrium where the wage rates w^r are constant over time, we must have that $\dot{\tilde{a}}_t^r=0$ and it follows that $e^r=w^r+\left(\rho-g_L\right)\tilde{a}_t^r$. For the typical consumer in region r, $\tilde{a}_t^r=A_t^r/L_t^r$. It follows that typical northern and southern consumer expenditure levels are given by

$$e^S=w^S \tag{31}$$

and

$$e^N=w^N+\sum_z\frac{\left(\rho-g_L\right)c_z}{\sigma-1}\frac{L_0}{L_{N0}}\left[\frac{w^NX_{Nz}^N}{\rho+\theta g}+\frac{w^N\left(X_{Xz}^N+\tau X_{Xz}^S\right)}{\rho+\theta g}+\frac{w^S\xi\left(X_{Fz}^S+\tau X_{Fz}^N\right)}{\rho+\theta g+\iota_S}\right]. \tag{32}$$

Having solved for consumer expenditures e^N and e^S, we can determine the ratio X^N_{FL}/X^S_{FL} and obtain the steady-state asset condition

$$\frac{X^N_{FL}}{X^S_{FL}} = \left(\frac{1}{\tau}\right)^{\sigma} \frac{e^N L^N_0}{e^S L^S_0} \frac{\left(P^S_t\right)^{1-\sigma}}{\left(P^N_t\right)^{1-\sigma}} \tag{33}$$

where $\left(P^S_t\right)^{1-\sigma} / \left(P^N_t\right)^{1-\sigma} = \sum_{j \neq N} \sum_z \left[\gamma_{jz} \left(p^S_{jz}\right)^{1-\sigma}\right] / \sum_j \sum_z \left[\gamma_{jz} \left(p^N_{jz}\right)^{1-\sigma}\right]$ is constant over time.

Thus, solving the model for a steady-state equilibrium reduces to solving a system of eight equations [(26), (27) and (28) for $z = H, L$, (29), (30) and (33)] in 8 unknowns (w, δ, χ_L, χ_H, ϕ_L, ϕ_H, X^S_{FL} and X^N_{FL}), where the eight equations are: five R&D conditions (innovation, two export-learning, two MP-learning), two labor market conditions (North and South) and one asset condition.

3 Numerical Results

3.1 Parameters

The subjective discount rate ρ is set at 0.07 to reflect a real interest rate of 7 percent, consistent with the average real return on the US stock market over the 20th century (Mehra and Prescott, 1985). The measure of product differentiation α determines the markup of price over marginal cost $1/\alpha$. It is set at 0.714 to generate a northern markup of 40 percent, which is within the range of estimates from Basu (1996) and Norrbin (1993). The parameter g_L is set at 0.014 to reflect a 1.4 percent population growth rate. This was the average annual world population growth rate during the 1990s according to the World Development Indicators (World Bank, 2016). The steady-state economic growth rate is calculated from $g_u = g_L / \left((\sigma - 1)(1 - \theta)\right)$. In order to generate a steady-state economic growth rate of 2 percent, consistent with the average US GDP per capita growth rate from 1950 to 1994 (Jones, 1995), the R&D spillover parameter θ is set at 0.72. Since only the ratio L^N_0/L^S_0 matters, we set $L^N_0 = 1$ and $L^S_0 = 2$ so L^N_0/L^S_0 equals the ratio of working-age population in high-income countries to that in upper middle-income countries during the 1990s (World Bank, 2016). Only the relative productivity advantage of high productivity firms over low productivity firms matters, so we normalize $c_L = 1$. Helpman et al (2004) find that, for US firms, the productivity advantage of exporters over domestic firms is 0.388 (and the productivity advantage of multinationals over domestic firms is 0.537). Consistent with this evidence, we set $c_H = 1 - 0.388 = 0.612$. Empirical studies on patents and R&D suggests that there are significant decreasing returns to R&D at the industry-level. Blundell et al (2002) find a long-run elasticity of patents to R&D of 0.5. This corresponds to $1/(1+\beta) = 0.5$, so we set $\beta = 1$. The iceberg MP cost parameter ξ is set at $\xi = 1.211$. This is the iceberg MP cost backed out from the calibration of the general equilibrium model in Tintelnot (2016). In his framework, there are firm-country-specific fixed costs of setting up a foreign affiliate and foreign affiliates also face a production efficiency loss, i.e. a variable MP cost.[23]

During the time period 1990-2005, trade costs were falling. We use the micro-founded measure of bilateral trade costs developed by Novy (2013) that indirectly infers trade frictions from observable trade data. By linear extrapolation of the bilateral trade cost estimates between the US and Mexico in 1970 and 2000, we obtain a tariff-equivalent of 54 percent for 1990 ($\tau = 1.54$) and 33 percent in 2005 ($\tau = 1.33$).[24]

The remaining parameters are the R&D productivity parameters a_R (innovation), a_X (export-learning), a_F (MP-learning), a_I (imitation), and the probability q for a low productivity draw. Since only the relative difference between the R&D productivity parameters matters, we normalize $a_R = 1$.

We set the export-learning R&D productivity parameter a_X and the probability of a low productivity draw q to match the following two facts: (i) Bernard et al (2003) find that 79 percent of US plants do not export any of their output; and (ii) the share of high-tech exports out of all manufacturing exports for the US in 1990 was 0.325 (World Bank, 2016). By setting $a_X = 4.8$ and $q = 0.957$

[23]Tintelnot (2016) first estimates the unit input costs for German foreign affiliates located in 11 OECD countries with input costs in Germany normalized to 1. He then uses these estimates for German foreign affiliates in OECD countries along with the model's predicted trade and MP shares to calibrate the general equilibrium and thereby obtain a value for the iceberg MP cost. The MP cost has a gravity pattern: it decreases from 1.211 with common language and common border between host and parent company, and increases with distance (with a coefficient .004). We set $\xi = 1.211$ but for robustness solve our model for smaller and larger iceberg MP costs. For comparison, Gumpert et al (2016) use the ratio of of affiliate sales to domestic sales for Norwegian multinationals to obtain an iceberg MP cost of 1.22.

[24]By using this data, we do not aim to directly measure welfare changes for Mexico and the US. Instead, in our numerical exercise, we are using Mexico's trade costs and consumer expenditure as an example of a typical middle-income country in the relevant time period, and the US as an example of a typical high-income country.

we obtain a 0.79 share of non-exporting northern firms ($f_N^N \equiv \sum_z \gamma_{Nz} / \left(\sum_z \gamma_{Nz} + \gamma_{Xz} + \gamma_{Fz}\right) = 0.7906$) and a high productivity share of northern exports of 0.326 ($f_{XH}^S \equiv X_{XH}^S / \left(X_{XH}^S + X_{XL}^S\right) = 0.326$).

We set the MP-learning productivity parameter a_F and the parameter a_I that is our measure of IPR protection in the South to (i) generate a foreign affiliate share in "world" GDP of 2.0 percent in the early 1990s benchmark; [25] and (ii) match the ratio of consumption share-adjusted real GDP per employed for U.S. and Mexico of 2.59 in 1990 (Feenstra, Inklaar and Timmer, 2015). By setting $a_F = 26.6$ and $a_I = 3.5$, we obtain that $Y_F \equiv \sum_r \sum_z X_{Fz}^r / \left[\sum_r \sum_j \sum_z X_{jz}^r\right] = 0.020$ and $e^N / e^S = 2.59$ in our pre-TRIPS benchmark.

Stronger IPR protection corresponds to a lower imitation rate $\iota_S \equiv 1/a_I$. By setting $a_I = 3.5$, we capture weak IPR protection in the South prior to the TRIPS agreement (one out of 3.5 products produced by foreign affiliates is copied each year). We set a higher value for a_I in the post-TRIPS benchmark to capture stronger IPR protection after the implementation of the TRIPS agreement. In particular, we set $a_I = 13.43$ so the model is consistent with the evidence of a seven-fold increase in R&D expenditure by non-OECD foreign affiliates (including Mexico) of US manufacturing firms from 1995 to 2007.

In the model, R&D expenditure by foreign affiliates is captured by L_{Ft} (the total amount of southern labor devoted to adaptive R&D activities by foreign affiliates multiplied by the southern wage rate $w^S = 1$). Rewriting (22) using the definitions for the FDI rate ϕ_z, the relative R&D difficulty δ and the variety share of northern exporters γ_{Xz}, the FDI inflow measure can be written as $L_{Ft} = \sum_z L_{Fzt} = \sum_z \phi_z^{1+\beta} \gamma_{Xz} \delta a_F L_t$. The ratio L_{Fzt}/L_t is constant over time in any steady-state equilibrium so we obtain $L_{F0} = \sum_z L_{Fz0} = \sum_z \phi_z^{1+\beta} a_F \gamma_{Xz} \delta L_0$. Pre-TRIPS, the only available OECD data starts in 1994. For all regions, data is available for 1995 pre-TRIPS and 2007 post-TRIPS. In 1995 the R&D expenditure by foreign affiliates of US manufacturing firms in non-OECD Asia, Latin America, Mexico, Middle East and Africa was 609 billion US dollars, and in

[25] Looking at the same Mexican and non-OECD foreign affiliates of US manufacturing firms from *OECD.Stats* as described below, the share of foreign affiliate value added in total US value added for 1994 (pre-TRIPS) was 0.6 percent (*UNCTAD FDI Statistics*). However, the share of foreign affiliate value added in US *manufacturing* value added in 1994 is 3.4 percent (Bureau of Economic Analysis, 2016). For our calibration we choose the intermediate value 2.0 percent. For comparison, in 1990 the global foreign affiliate share of world GPD (value-added) was 4.6 percent and by 2005, this share had risen to 10 percent (UNCTAD, 2012). Importantly, this 4.6 percent share includes foreign affiliates in OECD countries (North-North MP), which is not the type of MP considered in our North-South model.

2007 their R&D expenditure was 4007 billion US dollars (*OECD.Stats*, 2016).[26] This represents a 6.6-fold increase in the R&D expenditure of US manufacturing firm foreign affiliates in non-OECD countries plus Mexico from 1995 to 2007. Adjusting the R&D expenditure of these foreign affiliates in 1995 for population growth and inflation from 1995 to 2007 generates an expected foreign affiliate R&D expenditure of 930.9 billion US dollars for 2007.[27] The ratio of the observed R&D expenditure to this expected R&D expenditure yields a 4.3-fold increase during the time period 1995-2007 that can be attributed to policy changes.[28] Such policy changes could be trade liberalization, stronger IPR protection, a decrease in monitoring and communication costs ($\xi \downarrow$) and a decrease in the entry cost of MP (captured by $a_F \downarrow$) due to for example FDI-promoting policies or financial development. For our numerical exercise, we focus mainly on stronger IPR protection ($a_I \uparrow$) and trade liberalization ($\tau \downarrow$). In particular, we set $a_I = 3.5$ in the pre-TRIPS benchmark and $a_I = 13.43$ in the post-TRIPS benchmark so that the model generates small foreign affiliate R&D expenditure before TRIPS and a four-fold increase in $\sum_z L_{Fz0}$ (approximately a seven-fold increase in foreign affiliate R&D expenditure after TRIPS).[29] In an online appendix, to assess the robustness of our main results, we explore the effect of stronger IPR protection and trade liberalization for different levels of ξ, a_X and a_F.

3.2 Main results

The model is solved numerically using the parameter values discussed in Section 3.1. The pre-TRIPS benchmark and the post-TRIPS benchmark are presented in Columns 1 and 2 of Table 1. The stylized facts that emerge from Bernard et al (2003) and Bernard et al (2007), among

[26] *OECD.Stat* records foreign affiliate data in millions of national currency for monetary variables. The monetary variables for US foreign affiliates are in current USD.

[27] From 1995 to 2007, the US GDP implicit price deflator increased by 29.2 percent (Federal Reserve Bank of St Louis, 2016). During the same time period, the world population grew by 18.3 percent using the 1.4 percent annual population growth rate. Multiplying the observed foreign affiliate R&D expenditure in 1995 by the population growth and inflation over the period generates the expected foreign affiliate R&D expenditure in 2007 in the absence of any policy changes.

[28] Looking at the 1994-1996 average instead of 1995 (despite Middle East data missing for 1994), we obtain a 5.4-fold increase in R&D expenditure of southern foreign affiliates of US firms until 2007. This corresponds to a 3.9-fold increase in L_{F0} that can be attributed to policy changes. For comparison, using R&D expenditure for non-OECD Asia foreign affiliates of US manufacturing firms from 1994-1996 to 2004-2006, there is an 8.2-fold increase in R&D expenditures (*OECD. Stats*, 2016). This corresponds to a 5.1-fold increase that can be attributed to policy changes.

[29] In the pre-TRIPS benchmark with $a_I = 3.5$, $L_{F0} = .008176$, such that $4 * L_{F0} = .03270$. Setting $a_I = 13.43$ in the post-TRIPS benchmark with $\tau = 1.33$ generates $L_{F0} = .03269$.

others, are that multinationals are on average more productive than exporters and that exporters are on average more productive than non-exporters. The model generates a pattern that is consistent with this. The export-learning rate of northern firms is higher for high productivity firms than for low productivity firms ($\chi_H > \chi_L$ in Columns 1 and 2). Also, the rate of MP-learning is higher for high productivity firms than for low productivity firms ($\phi_H > \phi_L$ in Columns 1 and 2). Therefore, the share of high productivity firms is higher for exporting northern firms than for non-exporting northern firms, and the share of high productivity firms is higher for multinational firms than for northern exporters. In particular, in our pre-TRIPS benchmark, $\gamma_{NH}/(\gamma_{NH}+\gamma_{NL}) = .029$, $\gamma_{XH}/(\gamma_{XH}+\gamma_{XL}) = .080$ and $\gamma_{FH}/(\gamma_{FH}+\gamma_{FL}) = .229$.

Going from the pre-TRIPS to the post-TRIPS benchmark (with trade liberalization and stronger southern IPR protection), the speed of learning how to export increases ($\chi_H \uparrow$ and $\chi_L \uparrow$) and the speed of learning to do MP increases ($\phi_H \uparrow$ and $\phi_L \uparrow$). There is a geographical redistribution of world production from the North to the South ($\sum_z \gamma_{Nz} + \gamma_{Xz}$ decreases from .982 to .961 and $\sum_z \gamma_{Fz} + \gamma_{Iz}$ increases from .018 to .039). The share of non-exporting firms in the North decreases from .791 to .757. Also, MP increases and foreign affiliates become more important in the world economy. The share of varieties that are produced in foreign affiliates $\sum_z \gamma_{Fz}$ increases from .003 to .016 and there is an increase in foreign affiliate value-added as share of world GDP (Y_F increases from .020 to .069). The share of total sales in the northern market that is coming from sales by foreign affiliates (Y_F^N) increases from .006 to .032 and in the southern market (Y_F^S) from .026 to .089.[30] Consumer welfare is measured by $u_0^r = e^r/P_0^r$, $r = N, S$. Going from the pre-TRIPS to the post-TRIPS benchmark, southern consumers are made better off (u_0^S increases from 81.45 to 96.09) but northern consumers are made worse off (u_0^N decreases from 302.2 to 295.8). To understand these long-run welfare effects, we solve the model for two counterfactual scenarios.

In the first counterfactual, presented in Column 3 of Table 1, trade costs are kept at the same level as in the pre-TRIPS benchmark ($\tau = 1.54$), but southern IPR protection is at the post-TRIPS level ($a_I = 13.43$). This would be the case if the TRIPS agreement had been implemented but not accompanied by any trade liberalization. Stronger IPR protection leads to faster MP-learning for both high and low productivity firms in the North (ϕ_H increases from .0128 to .0294 and

[30]The share of foreign affiliate sales in market r is captured by the ratio of market r's aggregate demand for foreign affiliate-produced varieties to market r's aggregate demand for all the varieties they consume. In the northern market, this is $Y_F^N \equiv \sum_z X_{Fz}^N / \sum_j \sum_z X_{jz}^N$, and in the southern market, it is $Y_F^S \equiv \sum_z X_{Fz}^S / \sum_{j \neq N} \sum_z X_{jz}^S$.

ϕ_L increases from .0038 to .0086) and a larger share of varieties being produced via MP (γ_{FH} increases from .0006 to .0030 and γ_{FL} increases from .0021 to .0115). With stronger southern IPR protection, consumers are made better off in both regions (u_0^N increases from 302.2 to 314.2 and u_0^S increases from 81.5 to 98.1). Southern consumers do not hold any assets so their consumer expenditure is only wage income (normalized to 1). However, the southern price index is lower (P_0^S decreases from .0123 to .0102), which results in higher long-run southern consumer welfare. For northern consumers, there is a drop in consumer expenditure but this is out-weighed by a lower price index (P_0^N decreases from .0086 to .0072). In essence, with stronger IPR protection in the South, there is a substantial geographical redistribution of production from the North to the South. Less production is done by northern exporters ($\sum_z \gamma_{Xz} \downarrow$), and more production is done by foreign affiliates in the South ($\sum_z \gamma_{Fz} \uparrow$). This has two effects on consumer welfare. First, more production taking place in the lower-wage South translates to lower product prices in both regions. Second, labor resources are freed up from production by exporting firms and there is downward pressure on the northern wage rate (w_N/w_S decreases from 2.19 to 1.90), lowering the cost of innovation. Therefore, there is more innovation (δ increases from 19.35 to 19.82) and the resulting increase in invented varieties benefits consumers in both regions ($n_0^{1/(\sigma-1)}$ increases from 330.8 to 342.6).

In the second counterfactual presented in Column 4 of Table 1, trade costs are set at their post-TRIPS level ($\tau = 1.33$) but IPR protection is the same as in the pre-TRIPS benchmark ($a_I = 3.5$). This would be the case if trade liberalization had occurred, but the TRIPS agreement had not been implemented. Trade liberalization by itself leads to faster rates of export-learning (χ_H increases from .0453 to .0587 and χ_L increases from .0133 to .0172). There is a redistribution of production away from northern firms that do not export (γ_{NH} decreases from .0225 to .0198 and γ_{NL} decreases from .7558 to .7115) towards exporting firms, low-productivity foreign affiliates, and low-productivity southern firms (γ_{XH} increases from .0163 to .0191, γ_{XL} increases from .1872 to .2307, γ_{FL} increases from .0021 to .0022 and γ_{IL} increases from .0120 to .0126). Surprisingly, consumers in both regions are made worse off by trade liberalization (u_0^N decreases from 302.2 to 285.2 and u_0^S decreases from 81.5 to 78.9). Trade liberalization directly decreases the prices of traded varieties in both regions. As exporting firms and multinational firms are owned by northern consumers, they benefit from the increase in market value of these firms. However, as exporters ex-

	(1) pre-TRIPS	(2) post-TRIPS	(3) $a_I \uparrow$	(4) $\tau \downarrow$
	$\tau = 1.54$ $a_I = 3.5$	$\tau = 1.33$ $a_I = 13.43$	$\tau = 1.54$ $a_I = 13.43$	$\tau = 1.33$ $a_I = 3.5$
w_N/w_S	2.19	1.90	1.90	2.22
δ	19.35	18.66	19.82	18.26
χ_H	.0453	.0564	.0431	.0587
χ_L	.0133	.0166	.0127	.0172
ϕ_H	.0128	.0259	.0294	.0109
ϕ_L	.0038	.0076	.0086	.0032
γ_{NH}	.0225	.0202	.0231	.0198
γ_{NL}	.7558	.7188	.7633	.7115
γ_{XH}	.0163	.0150	.0125	.0191
γ_{XL}	.1872	.2070	.1652	.2307
γ_{FH}	.0006	.0031	.0030	.0006
γ_{FL}	.0021	.0126	.0115	.0022
γ_{IH}	.0035	.0047	.0044	.0035
γ_{IL}	.0120	.0188	.0171	.0126
ι_S	.286	.074	.074	.286
L_{FH0}	.004	.015	.017	.003
L_{FL0}	.004	.018	.019	.003
f_N^N	.791	.757	.804	.743
f_{XH}^S	.326	.288	.297	.315
Y_F	.020	.069	.069	.019
Y_F^N	.006	.032	.021	.010
Y_F^S	.026	.089	.092	.025
e^N	2.59	2.27	2.27	2.63
e^S	1.00	1.00	1.00	1.00
e^N/e^S	2.59	2.27	2.27	2.63
P_0^N	.0086	.0077	.0072	.0092
P_0^S	.0123	.0104	.0102	.0127
$n_0^{1/(\sigma-1)}$	330.8	314.1	342.6	304.5
u_0^N	302.2	295.8	314.2	285.2
u_0^S	81.45	96.09	98.10	78.91

Table 1: Pre- and post-TRIPS benchmarks and two counterfactual scenarios.

pand production in response to trade liberalization, the northern wage rate increases and resources are drawn from innovation into production (δ decreases from 19.35 to 18.26). This is the key to why trade liberalization lowers consumer welfare. Less innovation results in less product variety ($n_0^{1/(\sigma-1)}$ decreases from 330.8 to 304.5) which puts upward pressure on the price indexes in both regions ($P_0^r \uparrow$).[31]

3.2.1 Labor reallocation across and within firms

We saw in the previous section that trade liberalization and stronger southern IPR protection have very different effects on consumer welfare. In this section, we take a closer look at how the labor market in each region responds to these policy changes. From the full employment of labor conditions (29) and (30) derived in Section 2.7 we obtain production and non-production employment for high and low productivity firms in each region. Northern non-production employment is captured by $L_{R0} = L_0 a_R g^{1+\beta}\delta$ (innovation). Southern non-production employment is $L_{Xz0} = L_0 a_X \delta \chi_z^{1+\beta} \gamma_{Nz}$ (export-learning) and $L_{Fz0} = L_0 a_F \delta \phi_z^{1+\beta} \gamma_{Xz}$ (MP-learning). We also look at how firms reallocate labor from production for the southern market to production for the northern market and vice versa. For all firms except foreign affiliates, aggregate labor demand from production for market r by firms of type j with productivity z is $L_{jz0}^r \equiv c_z X_{jz}^r L_0$ if market r is the home market, and $L_{jz0}^r \equiv \tau c_z X_{jz}^r L_0$ if market r is the export market. For foreign affiliates, we also take into account the production efficiency loss ξ.[32]

In Table 2, we present the results about employment shares across activities and firm types for the two benchmarks and the two counterfactuals. The results for the northern labor market are presented in the top panel, and the results for the southern labor market are in the bottom panel. Stronger southern IPR protection leads to more non-production employment in innovation (L_{R0} increases from .145 to .148). With more innovation, there are more newly invented northern varieties that are not yet introduced to the southern market. Therefore, employment in production by northern firms that do not export increases (L_{NL0}^N and L_{NH0}^N increase from .525 to .543 and from .053 to .056, respectively). As the speed of MP-learning increases with stronger IPR protection,

[31] Since more exported varieties from the North also means that more product varieties can be purchased by southern consumers, the welfare-decreasing effects of less innovation are less severe for the South (u_0^S just decreases slightly, from 81.5 to 78.9).

[32] For foreign affiliates, $L_{Fz0}^S \equiv \xi c_z X_{Fz}^S L_0$ and $L_{Fz0}^N \equiv \tau \xi c_z X_{Fz}^N L_0$.

	(1) pre-TRIPS	(2) post-TRIPS	(3) $a_I \uparrow$	(4) $\tau \downarrow$
	$\tau = 1.54$ $a_I = 3.5$	$\tau = 1.33$ $a_I = 13.43$	$\tau = 1.54$ $a_I = 13.43$	$\tau = 1.33$ $a_I = 3.5$
North $L_{N0} = 1$				
Non-production labor				
L_{R0}	.145	.140	.148	.137
Labor in production for home market North				
L^N_{NH0}	.053	.046	.056	.044
L^N_{NL0}	.525	.481	.543	.466
L^N_{XH0}	.039	.034	.030	.043
L^N_{XL0}	.130	.139	.118	.151
Labor in production for export market South				
L^S_{XH0}	.025	.032	.022	.035
L^S_{XL0}	.084	.128	.083	.124
South $L_{S0} = 2$. Numbers below are shares.				
Non-production labor				
L_{XH0}	.006	.009	.006	.009
L_{XL0}	.019	.026	.017	.028
L_{FH0}	.002	.007	.009	.002
L_{FL0}	.002	.009	.010	.002
Labor in production for home market South				
L^S_{FH0}	.013	.039	.044	.012
L^S_{FL0}	.013	.046	.050	.012
L^S_{IH0}	.400	.305	.342	.352
L^S_{IL0}	.396	.362	.388	.367
Labor in production for export market North				
L^N_{FH0}	.002	.010	.007	.003
L^N_{FL0}	.002	.012	.008	.004
L^N_{IH0}	.072	.079	.056	.103
L^N_{IL0}	.071	.094	.063	.107

Table 2: Demand for labor for pre- and post-TRIPS benchmarks and two counterfactual scenarios.

exporters step up their MP-activities and production employment by exporting firms decreases. The labor shares for exporters' production for the home market, L^N_{XH0} and L^N_{XL0} decrease from .039 to .030 and .130 to .118, respectively. The labor shares for production for the southern market also contract (L^S_{XH0} and L^S_{XL0} decrease from .025 to .022 and .084 to .083, respectively). That exporting firms contract in response to stronger IPR protection can be confirmed by looking at changes in aggregate demand in Table 3. Comparing Column 1 and Column 3, aggregate demand for exported varieties in both regions decrease ($X^r_{Xz} \downarrow$) and aggregate demand for MP varieties increase ($X^r_{Fz} \uparrow$). Recall that southern consumer expenditure is determined by wage income (normalized to 1). Therefore, even though southern consumer utility is higher with IPR protection, selling in the southern market does not become more lucrative for firms. (This would have been the case if for example southern consumers owned shares of multinational firms that rise in value with stronger IPR protection). In the southern labor market, the employment shares of imitating firms decrease. This labor is instead reallocated towards non-production and production employment in foreign affiliates. The share of employment in MP-learning increases from .004 in the pre-TRIPS benchmark to .019 in the counterfactual with stronger IPR protection.[33] The share of southern labor employed in foreign affiliate production for both markets increase.[34]

The counterfactual with trade liberalization that is not accompanied by any stronger IPR protection is presented in Column 4 of Table 2. With trade liberalization, sales in the southern market becomes relatively less important for multinationals and exports back to the northern market more relevant.[35] In the northern labor market, trade liberalization leads to a fall in the employment share of innovative R&D (L_{R0} falls from .145 to .137). Consequently, less labor is employed in non-exporting northern firms (L^N_{NH0} and L^N_{NL0} decrease from .053 to .044 and from .525 to .466, respectively). There are two reasons for this. First, with less innovation, there are fewer new varieties that have just been introduced to the northern market. Second, with lower trade costs, more northern firms find it worthwhile to learn how to export. Exporters employ a larger share of northern labor both for export market production and for home market production.[36] Trade lib-

[33]Total southern labor supply is $L^S_t = 2$ so the labor shares in Table 2 are scaled to sum to 1.

[34]L^S_{FH0} increases from .013 to .044, L^S_{FL0} from .013 to .050, L^N_{FH0} from .002 to .007, and L^N_{FL0} from .002 to .008.

[35]The employment share in foreign affiliate production for the southern market decreases from .026 to .024 while the employment share in foreign affiliate production for the northern market increases from .004 to .007. Likewise, for southern firms, the employment share in production for the southern market decreases from .796 to .719 while for the northern market, the employment share increases from .143 to .210.

[36]L^S_{XH0} increases from .025 to .035 and L^S_{XL0} increases from .084 to .124 while L^N_{XH0} increases from .039 to .043,

	(1) pre-TRIPS	(2) post-TRIPS	(3) $a_I \uparrow$	(4) $\tau \downarrow$
	$\tau = 1.54$ $a_I = 3.5$	$\tau = 1.33$ $a_I = 13.43$	$\tau = 1.54$ $a_I = 13.43$	$\tau = 1.33$ $a_I = 3.5$
Northern market				
X^N_{NH}	.0291	.0251	.0305	.0240
X^N_{NL}	.1750	.1604	.1811	.1554
X^N_{XH}	.0210	.0187	.0166	.0232
X^N_{XL}	.0433	.0462	.0392	.0504
X^N_{FH}	.0014	.0069	.0042	.0023
X^N_{FL}	.0008	.0050	.0029	.0015
X^N_{IH}	.0509	.0650	.0396	.0844
X^N_{IL}	.0308	.0472	.0275	.0538
Southern market				
X^S_{XH}	.0088	.0130	.0076	.0143
X^S_{XL}	.0181	.0322	.0180	.0311
X^S_{FH}	.0120	.0352	.0394	.0106
X^S_{FL}	.0073	.0255	.0273	.0067
X^S_{IH}	.4359	.3326	.3726	.3832
X^S_{IL}	.2641	.2414	.2585	.2444

Table 3: Aggregate demand (sales) by firm type and market.

eralization makes northern exporters devote more resources to production. This is confirmed by looking at changes in aggregate demand. Comparing Columns 1 and 4 in Table 3, in response to trade liberalization, aggregate demand for varieties sold only in the northern market decreases ($X^N_{Nz} \downarrow$) while aggregate demand for all other varieties rise. In the South, aggregate demand for imported varieties from the North increases ($X^S_{Xz} \uparrow$) while aggregate demand for all other varieties fall.

3.3 The implications of decreasing ξ, a_F and a_X

Our benchmark iceberg MP cost $\xi = 1.211$ is taken from Tintelnot (2016)'s general equilibrium calibration of his model for 12 OECD countries. He finds that for German foreign affiliates in

L^N_{XL0} increases from .130 to .151.

OECD countries, the iceberg MP cost rises with distance. We consider that North-South MP may imply a higher ξ than for North-North MP. Also, from the pre-TRIPS period (1990-1995) to the post-TRIPS period (2005-2007), technological advancement could have lead to lower communication and monitoring costs between headquarters and affiliates.

In Table 4, we study the implications of ξ decreasing over time from $\xi = 1.30$ in Column 1 (a high variable MP cost) to $\xi = 1.10$ in Column 2 (a low variable MP cost) to $\xi = 1$ in Column 3 (no communication and monitoring costs at all between the parent firm and its foreign affiliate). All other parameter values are the same as in the pre-TRIPS benchmark. We see that both northern and southern consumer welfare increase with falling variable MP costs. The smaller the iceberg productivity loss from communication and monitoring, the higher are the rates of MP-learning ($\phi_H = .0109$ and $\phi_L = .0032$ with $\xi = 1.30$, while $\phi_H = .0192$ and $\phi_L = .0056$ with $\xi = 1$).

Tintelnot (2016) finds that there are substantial entry costs for MP, and suggests a gravity pattern where entry costs for MP rise with distance. During the 1990-2007 time period, changes such as improvement in economic stability and FDI-promoting policies in host countries could have lead to lower entry costs for MP. In Columns 4-6 of Table 4, we study the implications of a_F decreasing over time from $a_F = 100$ (a high entry cost for MP) to $a_F = 10$ (a low entry cost for MP) to $a_F = 5$ (a very low entry cost for MP).

When a_F decreases, this leads to a smaller FDI inflow to the South ($w_S \sum_z L_{Fz0}$ becomes very small). Just as with stronger southern IPR protection, when there is more MP due to lower communication and monitoring costs ($\xi \downarrow$) or lower entry cost for MP ($a_F \downarrow$), there is a redistribution of production from the North to the South as more exporters become multinationals and produce in the South (ϕ_H and ϕ_L increase). This puts downward pressure on the northern wage rate and frees up resources for innovation ($\delta \uparrow$ and $n_0^{1/(\sigma-1)} \uparrow$). Newly invented varieties produced in the North are produced at a lower cost due to lower wages. The price index falls in both regions. Therefore, consumer welfare increase in both regions – more so in the South since southern consumers do not experience the drop in consumer expenditure that northern consumers experience (w_N/w_S decreases, δ increases and u_0^r increases from Column 1 to 3 and from Column 4 to 6).

Importantly, by only lowering the entry cost for MP, it is not possible to capture the observed large increase in R&D expenditure by non-OECD foreign affiliates (since L_{F0} decreases). Lowering the iceberg MP cost from a high level $\xi = 1.30$ to $\xi = 1$ generates a less than three-fold increase in L_{F0}, which should be compared with our moderate decrease in the imitation rate ι from .29 to .07 in Table 1, which generates a more than four-fold increase in L_{F0}. Stronger IPR protection in host countries is central for capturing the MP-activities of non-OECD foreign affiliates.

In Columns 7-9 of Table 4, we study the implications of a_X decreasing over time from $a_X = 20$ (a high entry cost for exporting) to $a_F = 2$ (a low entry cost for exporting) to $a_F = 0.16$ (a very low entry cost for exporting).[37] Contrary to our main exercise with trade liberalization in the form of lower iceberg trade costs, lower entry costs for exporting improves southern welfare (u_0^S increases from 72.60 to 85.63 to 92.05). With lower entry costs for exporting, a much larger share of firms in the North become exporters. From $a_X = 20$ to $a_X = 2$ to $a_X = 0.16$ the share of non-exporters among firms in the North (f_N^N) decreases from 91.3 percent to 66.3 percent to 19.2 percent. So many firms enter the southern market via exporting that there is a huge increase in product variety for southern consumers. There is still an increase in the relative wage (from $w_N/w_S = 2.13$ in Column 7 to 2.34 in Column 9) and a decrease in innovation (δ decreases from 19.96 in Column 7 to 18.03 in Column 9). These factors worsens southern consumer welfare but the dominant effect for the South is the increase in product variety as many more exporters gain access to the southern market.

In the online appendix, we study the effects of stronger IPR protection and trade liberalization for each of the cases discussed in this section. For each of the ξ, a_F and a_X cases, the counterfactuals of stronger IPR protection and trade liberalization generate qualitatively the same results: stronger southern IPR protection improves consumer welfare in both regions and trade liberalization worsens consumer welfare in both regions.

3.4 Solving the model with a R&D subsidy to innovation

The Table 1 result that trade liberalization lowers consumer welfare is surprising. Trade liberalization gives northern firms stronger incentives to become exporters. Labor resources are drawn from innovation into production, bidding up the northern wage rate and making innovation more costly. This reduction in innovation ($\tau \downarrow \Rightarrow \delta \downarrow$) is the key to understanding why trade liberalization can make consumers worse off in both regions. Because there are positive knowledge spillovers

[37] With $\chi_z = +\infty$ in (5), we obtain $\gamma_{Nz} = 0$. With a very fast export learning rate, there are no non-exporters in steady state equilibrium. Everyone exports.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\xi = 1.30$	$\xi = 1.10$	$\xi = 1$	$a_F = 100$	$a_F = 10$	$a_F = 5$	$a_X = 20$	$a_X = 2$	$a_X = 0.16$
w_N/w_S	2.30	2.06	1.94	2.55	2.00	1.91	2.13	2.23	2.34
δ	19.33	19.39	19.44	19.31	19.41	19.46	19.96	18.94	18.03
χ_H	.0459	.0446	.0440	.0470	.0444	.0440	.0167	.0865	.7124
χ_L	.0135	.0131	.0129	.0138	.0130	.0129	.0049	.0254	.2091
ϕ_H	.0109	.0158	.0192	.0076	.0174	.0207	.0209	.0099	.0061
ϕ_L	.0032	.0046	.0056	.0022	.0051	.0061	.0061	.0029	.0018
γ_{NH}	.0224	.0227	.0229	.0222	.0227	.0228	.0322	.0157	.0028
γ_{NL}	.7538	.7582	.7603	.7499	.7587	.7602	.8713	.6344	.1845
γ_{XH}	.0169	.0154	.0146	.0181	.0150	.0143	.0076	.0228	.0358
γ_{XL}	.1910	.1820	.1768	.1982	.1798	.1755	.0763	.3049	.7456
γ_{FH}	.0005	.0007	.0008	.0004	.0008	.0009	.0005	.0007	.0007
γ_{FL}	.0018	.0025	.0030	.0013	.0027	.0032	.0014	.0026	.0040
γ_{IH}	.0031	.0041	.0048	.0024	.0045	.0050	.0027	.0038	.0037
γ_{IL}	.0104	.0144	.0169	.0076	.0157	.0181	.0080	.0151	.0229
ι_S	.286	.286	.286	.286	.286	.286	.286	.286	.286
L_{FH0}	.003	.006	.008	.006	.003	.002	.005	.003	.002
L_{FL0}	.003	.006	.009	.009	.003	.002	.005	.004	.003
f_N^N	.787	.796	.801	.780	.798	.802	.913	.663	.192
f_{XH}^S	.330	.321	.314	.337	.317	.311	.357	.294	.211
Y_F	.027	.027	.037	.020	.020	.020	.020	.019	.019
e^N	2.72	2.43	2.29	3.01	2.37	2.26	2.52	2.64	2.77
e^S	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
P_0^N	.0090	.0080	.0075	.0100	.0078	.0074	.0081	.0089	.0098
P_0^S	.0130	.0114	.0107	.0145	.0111	.0105	.0138	.0117	.0109
$n_0^{1/(\sigma-1)}$	330.3	331.8	333.1	329.9	332.4	333.7	345.9	320.9	299.3
u_0^N	301.7	303.0	304.1	301.5	303.2	304.1	311.7	295.8	281.8
u_0^S	77.06	87.48	93.57	68.83	89.99	95.00	72.60	85.63	92.05

Table 4: Columns 1-3: Varying the iceberg cost for MP (ξ). Columns 4-6: Varying the entry cost for MP (a_F). Columns 7-9: Varying the entry cost for exporting (a_X). All other parameters as described in the pre-TRIPS benchmark ($\tau = 1.54$, $a_I = 3.5$, $\xi = 1.211$, $a_F = 26.6$, $a_X = 4.8$).

connected with innovative R&D ($\theta = 0.72$), firms can do too little R&D in equilibrium and trade liberalization can aggrevate this problem by diverting more resources from R&D to production. The individual firm does not take into account the positive spillover on other innovating firms when it decides how much resources to allocate to innovation.

We will now explore whether the welfare effects of trade liberalization change when innovative R&D is subsidized. Let s_R denote the fraction of the firm's cost of innovative R&D that is subsidized by the government. As in Segerstrom (1998) we assume that the government finances the subsidy s_R by means of lump-sum taxation. Free entry in the North implies that (23) becomes

$$qv_{NLt} + (1-q)\, v_{NHt} = (1-s_R)\, \frac{w^N a_R g^\beta}{n_t^\theta}$$

where the right-hand side now reflects the lower cost of innovation due to the subsidy. The northern no-arbitrage condition is $(q\pi_{NLt} + (1-q)\,\pi_{NHt}) \,/\, (\rho + \theta g) = (1-s_R)\, w^N a_R g^\beta / n_t^\theta$ and it follows that the steady-state northern no-arbitrage condition is

$$\frac{1}{\sigma - 1}\left(\frac{qc_L X_{NL}^N}{\gamma_{NL}\,(\rho+\theta g)} + \frac{(1-q)\, c_H X_{NH}^N}{\gamma_{NH}\,(\rho+\theta g)}\right) = (1-s_R)\, a_R g^\beta \delta.$$

The left-hand side is the market size-adjusted expected benefit from innovation and the right-hand side the now lower market size-adjusted cost of innovation.

The results from this exercise are presented in Table 5. Columns 1 and 2 reproduce the pre-TRIPS benchmark and the counterfactual with trade liberalization (from $\tau = 1.54$ to $\tau = 1.33$). In columns 3 and 4 we present the results from a pre-TRIPS benchmark and a counterfactual with trade liberalization when there innovative R&D is subsidized at the rate $s_R = 0.55$. Columns 5 and 6 present the results from the same exercise but with a higher subsidy rate $s_R = 0.85$. As seen earlier, without any innovative R&D subsidy, trade liberalization worsens consumer welfare in both regions. With a subsidy of 0.55, trade liberalization leads to higher consumer welfare in the South. However, northern consumer welfare is still worsened by trade liberalization. With a subsidy of 0.85 or higher, trade liberalization is welfare-improving for consumers in both regions. On the one hand, with lower trade costs, consumers in both regions benefit from lower prices on imported varieties. On the other hand, without the subsidy to innovation in the North, so much resources are allocated to production in the North that innovation suffers and consumers experience less product

	(1) pre-TRIPS $s_R = 0$ $\tau = 1.54$	(2) $\tau \downarrow$ $s_R = 0$ $\tau = 1.33$	(3) pre-TRIPS $s_R = 0.55$ $\tau = 1.54$	(4) $\tau \downarrow$ $s_R = 0.55$ $\tau = 1.33$	(5) pre-TRIPS $s_R = 0.85$ $\tau = 1.54$	(6) $\tau \downarrow$ $s_R = 0.85$ $\tau = 1.33$
w_N/w_S	2.19	2.22	2.60	2.52	4.50	3.78
δ	19.35	18.26	38.04	36.45	74.51	73.91
χ_H	.0453	.0587	.0221	.0306	.0053	.0093
χ_L	.0133	.0172	.0064	.0090	.0016	.0027
ϕ_H	.0128	.0109	.0148	.0122	.0249	.0182
ϕ_L	.0038	.0032	.0043	.0036	.0073	.0054
γ_{NH}	.0225	.0198	.0298	.0266	.0388	.0363
γ_{NL}	.7558	.7115	.8470	.8109	.9279	.9076
γ_{XH}	.0163	.0191	.0102	.0131	.0028	.0049
γ_{XL}	.1872	.2307	.1012	.1363	.0254	.0446
γ_{FH}	.0006	.0006	.0004	.0005	.0002	.0003
γ_{FL}	.0021	.0022	.0013	.0015	.0005	.0007
γ_{IH}	.0035	.0035	.0026	.0027	.0012	.0015
γ_{IL}	.0120	.0126	.0075	.0083	.0032	.0041
ι_S	.286	.286	.286	.286	.286	.286
L_{FH0}	.0041	.0033	.0068	.0057	.0102	.0097
L_{FL0}	.0041	.0034	.0058	.0051	.0081	.0075
f_N^N	.791	.743	.886	.847	.971	.949
f_{XH}^S	.326	.315	.359	.349	.378	.381
Y_F	.020	.019	.021	.020	.022	.022
Y_F^N	.006	.010	.008	.010	.014	.015
Y_F^S	.026	.025	.026	.026	.027	.027
e^N	2.59	2.63	3.00	2.91	4.93	4.14
e^S	1.00	1.00	1.00	1.00	1.00	1.00
e^N/e^S	2.59	2.63	3.00	2.91	4.93	4.14
P_0^N	.0086	.0092	.0039	.0039	.0025	.0021
P_0^S	.0122	.0127	.0056	.0056	.0031	.0028
$n_0^{1/(\sigma-1)}$	330.8	304.5	869.3	817.7	2271	2245
u_0^N	302.2	285.2	777.5	747.9	2000	2008
u_0^S	81.45	78.91	177.1	178.2	322.2	357.5

Table 5: Pre-TRIPS benchmark with trade liberalization and R&D subsidy

variety. The R&D subsidy can correct for this by preventing product variety from falling so much that the negative welfare effect from less innovation dominates the positive welfare effect from lower prices. A smaller R&D subsidy of 0.55 suffices to change the welfare results for the South. This is because trade liberalization expands the set of varieties that southern consumers can buy, which mitigates some of the welfare-worsening effect of less innovation.

The results in this section represent an example of the "theory of the second best." Because of the positive knowledge spillovers connected with innovative R&D, there can be too little innovation in equilibrium. The first best means of dealing with this source of market failure is for the government to subsidize R&D. But in the absence of a R&D subsidy, higher trade costs represent a second best solution because higher trade costs lead to more innovation ($\tau \uparrow \Rightarrow \delta \uparrow$).

4 Concluding Comments

We have developed a dynamic general equilibrium trade model with heterogeneous firms that incorporates issues specific to North-South multinational production (MP). Firms in the North engage in innovative R&D to develop new product varieties and then learn their productivities. Firms in the North can engage in export-learning R&D to access the southern market. They can then engage in MP-learning R&D to learn how to produce their products in the lower-wage South. Once any foreign affiliate of a northern firm starts producing in the South, it faces the risk of imitation from southern firms. Stronger IPR protection in the South (TRIPS) is modelled as a decrease in this imitation rate. We calibrate the model to match general trends in the global economy since the early 1990s and explore the effects of stronger IPR protection (TRIPS) and trade liberalization.

We find that stronger IPR protection in the South (TRIPS) induces foreign affiliates of northern firms to increase their R&D expenditures and results in a faster rate of technology transfer within these multinational firms, consistent with the empirical evidence in Branstetter, Fisman and Foley (2006). As a result of stronger IPR protection, more product varieties end up being produced in the South and exports of new products increase, consistent with the empirical evidence in Branstetter, Fisman, Foley and Saggi (2011). TRIPS also stimulate innovative R&D spending by northern firms and result in faster economic growth in the South, consistent with the empirical evidence in Gould and Gruben (1996). Stronger IPR protection, lower communication costs between parents and foreign affiliates, and lower entry costs for MP all lead to an increase in the share of world GDP produced via MP, consistent with data from UNCTAD (2012). Consequently, there is more production employment by foreign affiliates in the South and more innovative R&D employment by parent firms in the North, consistent with the facts documented in Arkolakis et al (2014) for the time period 1999-2009. When we solve the model numerically for plausible parameter values, we find that MP-promoting policies such as stronger IPR protection lead to higher long-run consumer welfare in both regions. In contrast, trade liberalization leads to more export-learning and actually lowers long-run consumer welfare in both regions by diverting northern resources away from innovative activities.

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Online Appendix: Solving the Model with Alternative Parameter Values

Counterfactuals for different levels of ξ, a_F and a_X

In Tables 6-8, we examine the robustness of our earlier findings by looking at the two counterfactuals (stronger southern IPR protection and trade liberalization) but with different levels of ξ (the variable MP cost), a_F (the entry cost for MP), a_X (the entry cost for exporting). For all the results that we present, the constraint $w_N/w_S > \tau\xi$ holds, so foreign affiliates export to the North.

In Table 6, we present the pre-TRIPS benchmark and the two counterfactuals with $\xi = 1$ (no variable cost for MP), $\xi = 1.10$ (a low variable cost for MP), and $\xi = 1.30$ (a high variable cost for MP). The two counterfactuals with stronger IPR protection and trade liberalization generate qualitatively same results as before. It is a robust finding that stronger southern IPR protection improves consumer welfare and trade liberalization worsens consumer welfare in both regions.

In Table 7, we present the pre-TRIPS benchmark and the two counterfactuals with $a_F = 100$ in Columns 1-3 (a high entry cost for MP), $a_F = 10$ in Columns 4-6 (a low entry cost for MP) and $a_F = 5$ in Columns 7-9 (a very low entry cost for MP). The counterfactuals of stronger IPR protection and trade liberalization generate qualitatively the same results in all three cases: stronger southern IPR protection improves consumer welfare and trade liberalization worsens consumer welfare in both regions.

In Table 8, we present the pre-TRIPS benchmark and the two counterfactuals with $a_X = 20$ (a high entry cost for exporting), $a_X = 2$ (a low entry cost for exporting), and $a_X = 0.16$ (a very low entry cost for exporting). Again, the results from our counterfactuals of stronger IPR protection and trade liberalization are qualitatively the same as before: stronger southern IPR protection improves consumer welfare and trade liberalization worsens consumer welfare in both regions.

Strength of IPR protection in the South

In our benchmark analysis, we chose parameter values so that the model replicated the seven-fold increase in R&D expenditure by non-OECD foreign affiliates of US manufacturing firms from before TRIPS (1990-1995) to after TRIPS (2005-2007) based on two policy changes: trade liberalization ($\tau \downarrow$) and stronger IPR protection ($a_I \uparrow$). But there are other changes over the time period that could explain part of the observed increase in R&D activities of foreign affiliates. For example, the monitoring and communication costs between parents and foreign affiliates could have decreased ($\xi \downarrow$) or the costs of setting up multinational production facilities could have decreased ($a_F \downarrow$). If these or other changes are partly responsible for the increase in R&D expenditure of foreign affiliates, then the actual increase in a_I could be smaller than in our benchmark analysis.

In Table 9, we study what happens if the TRIPS agreement by itself is associated with a smaller increase in a_I than our earlier results suggest. Starting from the pre-TRIPS benchmark in Column 1, we study what happens when a_I gradually increases from 3.5 to 1000 holding τ fixed at the pre-TRIPS level 1.54.[38] If the TRIPS agreement only increased a_I by half as much as our benchmark

[38]For columns 2,4 and 5, by setting $\tau = 1.33$ as in the post-TRIPS benchmark we find the values of a_I that generate a two-fold, four-fold (benchmark) and six-fold increase in L_{F0}. These a_I-values are then chosen along with $\tau = 1.54$ to study the effects of a gradual increase in a_I.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\xi = 1$			$\xi = 1.10$			$\xi = 1.30$		
	$\tau = 1.54$	$\tau = 1.54$	$\tau = 1.33$	$\tau = 1.54$	$\tau = 1.54$	$\tau = 1.33$	$\tau = 1.54$	$\tau = 1.54$	$\tau = 1.33$
	$a_I = 3.5$	$a_I = 13.43$	$a_I = 3.5$	$a_I = 3.5$	$a_I = 13.43$	$a_I = 3.5$	$a_I = 3.5$	$a_I = 10$	$a_I = 3.5$
w_N/w_S	1.94	1.72	1.95	2.06	1.81	2.08	2.30	2.01	2.34
δ	19.44	20.07	18.28	19.39	19.94	18.26	19.33	19.61	18.27
χ_H	.0440	.0416	.0572	.0446	.0424	.0579	.0459	.0441	.0594
χ_L	.0129	.0122	.0168	.0131	.0125	.0170	.0135	.0129	.0174
ϕ_H	.0192	.0404	.0167	.0158	.0346	.0136	.0109	.0220	.0092
ϕ_L	.0056	.0119	.0049	.0046	.0102	.0040	.0032	.0065	.0027
γ_{NH}	.0229	.0235	.0201	.0227	.0232	.0199	.0224	.0228	.0196
γ_{NL}	.7603	.7689	.7163	.7582	.7660	.7141	.7538	.7599	.7095
γ_{XH}	.0146	.0108	.0172	.0154	.0117	.0181	.0169	.0140	.0197
γ_{XL}	.1768	.1520	.2192	.1820	.1587	.2250	.1910	.1745	.2348
γ_{FH}	.0008	.0035	.0009	.0007	.0032	.0007	.0005	.0021	.0005
γ_{FL}	.0030	.0145	.0032	.0025	.0130	.0027	.0018	.0075	.0019
γ_{IH}	.0048	.0052	.0049	.0041	.0048	.0042	.0031	.0041	.0031
γ_{IL}	.0169	.0216	.0183	.0144	.0193	.0153	.0104	.0151	.0108
ι_S	.286	.074	.286	.286	.074	.286	.286	.074	.286
L_{FH0}	.008	.028	.007	.006	.022	.005	.003	.011	.002
L_{FL0}	.009	.034	.008	.006	.026	.005	.003	.011	.002
f_N^N	.801	.814	.754	.796	.809	.749	.787	.798	.739
f_{XH}^S	.314	.284	.304	.321	.290	.310	.330	.309	.319
Y_F	.037	.124	.037	.027	.093	.027	.015	.042	.015
e^N	2.29	2.07	2.32	2.43	2.16	2.46	2.72	2.39	2.77
e^S	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
P_0^N	.0075	.0064	.0081	.0080	.0068	.0086	.0090	.0077	.0097
P_0^S	.0107	.0090	.0110	.0114	.0096	.0118	.0130	.0110	.0134
$n_0^{1/(\sigma-1)}$	333.1	348.5	305.1	331.8	345.4	304.6	330.3	337.4	304.7
u_0^N	304.1	321.4	286.0	303.0	317.5	285.5	301.7	308.5	285.3
u_0^S	93.57	111.4	90.85	87.48	104.6	84.84	77.06	91.10	74.62

Table 6: Varying the variable cost for MP (ξ): Pre-TRIPS benchmark and counterfactuals. For $\xi = 1.30$, stronger IPR protection is represented by $a_I = 10$. For $a_I = 13.43$, the restriction $w_N > \xi\tau w_S$ is not satisfied.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$a_F = 100$			$a_F = 10$			$a_F = 5$		
		$a_I \uparrow$	$\tau \downarrow$		$a_I \uparrow$	$\tau \downarrow$		$a_I \uparrow$	$\tau \downarrow$
	$\tau = 1.54$	$\tau = 1.54$	$\tau = 1.33$	$\tau = 1.54$	$\tau = 1.54$	$\tau = 1.33$	$\tau = 1.54$	$\tau = 1.54$	$\tau = 1.33$
	$a_I = 3.5$	$a_I = 13.43$	$a_I = 3.5$	$a_I = 3.5$	$a_I = 5$	$a_I = 3.5$	$a_I = 3.5$	$a_I = 3.8$	$a_I = 3.5$
w_N/w_S	2.55	2.27	2.55	2.00	1.87	2.06	1.91	1.87	1.99
δ	19.31	19.57	18.30	19.41	19.54	18.27	19.46	19.50	18.28
χ_H	.0470	.0447	.0604	.0444	.0438	.0578	.0440	.0439	.0575
χ_L	.0138	.0131	.0177	.0130	.0129	.0170	.0129	.0129	.0169
ϕ_H	.0076	.0154	.0068	.0174	.0236	.0142	.0207	.0225	.0161
ϕ_L	.0022	.0045	.0020	.0051	.0069	.0042	.0061	.0066	.0047
γ_{NH}	.0222	.0227	.0195	.0227	.0229	.0199	.0228	.0229	.0200
γ_{NL}	.7499	.7577	.7061	.7587	.7611	.7142	.7602	.7608	.7154
γ_{XH}	.0181	.0155	.0207	.0150	.0136	.0180	.0143	.0139	.0174
γ_{XL}	.1982	.1827	.2412	.1798	.1721	.2242	.1755	.1733	.2207
γ_{FH}	.0004	.0019	.0004	.0008	.0013	.0008	.0009	.0010	.0008
γ_{FL}	.0013	.0067	.0014	.0027	.0048	.0028	.0032	.0036	.0031
γ_{IH}	.0024	.0029	.0024	.0045	.0052	.0043	.0050	.0052	.0048
γ_{IL}	.0076	.0099	.0082	.0157	.0191	.0159	.0181	.0192	.0178
ι_S	.286	.074	.286	.286	.200	.286	.286	.263	.286
L_{FH0}	.006	.022	.005	.003	.004	.002	.002	.002	.001
L_{FL0}	.009	.022	.005	.003	.005	.002	.002	.002	.001
f_N^N	.780	.791	.733	.798	.803	.749	.802	.803	.752
f_{XH}^S	.337	.321	.323	.317	.306	.309	.311	.308	.305
Y_F	.020	.069	.019	.020	.093	.019	.020	.074	.019
e^N	3.01	2.70	3.02	2.37	2.22	2.44	2.26	2.22	2.35
e^S	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
P_0^N	.0100	.0087	.0106	.0078	.0072	.0086	.0074	.0073	.0082
P_0^S	.0145	.0125	.0147	.0111	.0102	.0117	.0105	.0103	.0112
$n_0^{1/(\sigma-1)}$	329.9	336.3	305.5	332.4	335.7	304.7	333.7	334.6	305.1
u_0^N	301.5	309.3	285.8	303.2	305.9	285.4	304.1	304.7	285.7
u_0^S	68.83	79.98	68.07	89.99	97.65	85.62	95.00	97.24	89.19

Table 7: Varying the entry cost for MP (a_F): Pre-TRIPS benchmark (all parameters as in the pre-TRIPS benchmark except for a_F) and counterfactuals. In columns 5 and 8, the values of a_I are chosen to ensure that the constraint $w^N/w^S > \tau\xi$ is satisfied.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$a_X = 20$ $\tau = 1.54$ $a_I = 3.5$	$\tau = 1.54$ $a_I = 13.43$	$\tau = 1.33$ $a_I = 3.5$	$a_X = 2$ $\tau = 1.54$ $a_I = 3.5$	$\tau = 1.54$ $a_I = 13.43$	$\tau = 1.33$ $a_I = 3.5$	$a_X = 0.16$ $\tau = 1.54$ $a_I = 3.5$	$\tau = 1.54$ $a_I = 13.43$	$\tau = 1.33$ $a_I = 3.5$
w_N/w_S	2.13	1.90	2.18	2.23	1.91	2.25	2.34	1.95	2.32
δ	19.96	20.36	19.11	18.94	19.45	17.73	18.03	18.55	16.73
χ_H	.0167	.0162	.0215	.0865	.0818	.1131	.7124	.6578	.9691
χ_L	.0049	.0047	.0063	.0254	.0240	.0332	.2091	.1931	.2844
ϕ_H	.0209	.0478	.0170	.0099	.0229	.0087	.0061	.0147	.0060
ϕ_L	.0061	.0140	.0050	.0029	.0067	.0026	.0018	.0043	.0017
γ_{NH}	0322	.0325	.0301	.0157	.0163	.0132	.0028	.0030	.0021
γ_{NL}	.8713	.8740	.8498	.6344	.6464	.5748	.1845	.1966	.1429
γ_{XH}	.0076	.0054	.0096	.0228	.0183	.0254	.0358	.0309	.0365
γ_{XL}	.0763	.0648	.0974	.3049	.2737	.3635	.7456	.6999	.7866
γ_{FH}	.0005	.0021	.0005	.0007	.0034	.0007	.0007	.0036	.0006
γ_{FL}	.0014	.0073	.0015	.0026	.0148	.0028	.0040	.0243	.0041
γ_{IH}	.0027	.0031	.0028	.0038	.0050	.0038	.0037	.0054	.0037
γ_{IL}	.0080	.0109	.0083	.0151	.0221	.0158	.0229	.0362	.0234
ι_S	.286	.074	.286	.286	.074	.286	.286	.074	.286
L_{FH0}	.005	.020	.004	.003	.015	.003	.002	.010	.002
L_{FL0}	.005	.021	.004	.004	.019	.003	. 003	.019	.003
f_N^N	.913	.919	.890	.663	.681	.600	.192	.208	.149
f_{XH}^S	.357	.316	.355	.294	.271	.280	.211	.197	.205
Y_F	.020	.070	.020	.019	.069	.019	.019	.068	.019
e^N	2.52	2.26	2.58	2.64	2.29	2.67	2.77	2.77	2.75
e^S	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
P_0^N	.0081	.0070	.0086	.0089	.0074	.0096	.0098	.0080	.0105
P_0^S	.0138	.0118	.0140	.0117	.0095	.0121	.0109	.0085	.0114
$n_0^{1/(\sigma-1)}$	345.9	356.0	325.2	320.9	333.2	292.1	299.3	311.5	268.8
u_0^N	311.7	322.7	298.6	295.8	308.2	277.0	281.8	293.9	261.5
u_0^S	72.60	84.61	71.30	85.63	105.1	82.47	92.05	117.0	87.94

Table 8: Varying the entry cost for exporting (a_X): Pre-TRIPS benchmark (all parameters as in pre-TRIPS benchmark except for a_X) and counterfactuals.

results suggest (a_I increases from 3.5 to 8.465 instead of from 3.5 to 13.43), then southern welfare increases by more than half as much as our benchmark results suggest (u_0^S increases from 81.45 in to 93.92, instead of from 81.45 to 98.10). The case for TRIPS benefiting developing countries is stronger than our benchmark identification of its effects. Even if we have exaggerated by a factor 2, there still are considerable long-run welfare gains for southern consumers.

In column 5 of Table 9, we solve for a value of a_I that together with $\tau = 1.33$ generates a six-fold increase in L_{F0}. This corresponds to the observed ten-fold increase in the FDI inflow going to developing countries and transition economies from 1990 to 2005 (*UNCTAD FDI Statistics*).[39] Similarly to our benchmark scenario, stronger IPR protection (a_I increasing to 21.35) leads to higher long-run southern consumer welfare (u_0^S increases from 81.45 to 100.7). There is more product variety ($n_0^{1/(\sigma-1)} \uparrow$) and prices are lower as more production is done in the low-wage South.

However, the properties of the model do change somewhat if we look at very high values of a_I, such as $a_I = 80$ and $a_I = 1000$. Such strong IPR protection generates very fast MP-learning rates and there is so much innovation that the northern wage rate increases again. For very strong IPR protection ($a_I = 1000$), the northern wage rate is even higher than in the pre-TRIPS benchmark ($w_N/w_S = 2.36 > 2.19$). A higher northern wage rate translates into higher prices for the newly invented varieties that southern consumers import from the North. Even though there are more varieties ($n_0^{1/(\sigma-1)}$ increases from 330.8 to 396.7), those new varieties that are produced in the North are more expensive to southern consumers and we find that southern consumer welfare eventually decreases on the margin (from $u_0^S = 101.3$ when $a_I = 80$ to $u_0^S = 96.81$ when $a_I = 1000$).

Solving The Model

In this appendix, calculations done to solve the model are spelled out in more detail.

Households

The static consumer optimization problem is

$$\max_{x_t(\cdot)} \int_0^{n_t} x_t(\omega)^\alpha d\omega \qquad \text{s.t.} \qquad \dot{y}(\omega) = p_t(\omega)x_t(\omega),\ y(0) = 0,\ y(n_t) = e_t.$$

where $y(\omega)$ is a new state variable and $\dot{y}(\omega)$ is the derivative of y with respect to ω. The Hamiltonian function for this optimal control problem is

$$H = x_t(\omega)^\alpha + \gamma(\omega)p_t(\omega)x_t(\omega)$$

[39]During the same time period, the world population grew by 23.4 percent using the annual 1.4 percent annual population growth rate. From 1990 to 2005, there was a 38.4 percent increase in the US GDP implicit price deflator (Federal Reserve Bank of St Louis, 2011). Multiplying the observed FDI inflow in 1990 by the population growth and inflation over the period generates the expected FDI inflow in 2005 in the absence of any policy changes.

	(1) pre-TRIPS	(2) $2X \uparrow L_{F0}$	(3)	(4) $4X \uparrow L_{F0}$	(5) $6X \uparrow L_{F0}$	(6)	(7)
	$\tau = 1.54$ $a_I = 3.5$	$a_I = 6.89$	$a_I = 8.465$	$a_I = 13.43$	$a_I = 21.35$	$a_I = 80$	$a_I = 1000$
w_N/w_S	2.19	1.99	1.95	1.90	1.89	2.04	2.36
δ	19.35	19.53	19.61	19.82	20.10	21.06	21.97
χ_H	.0453	.0441	.0438	.0432	.0424	.0389	.0339
χ_L	.0133	.0130	.0129	.0127	.0124	.0114	.0099
ϕ_H	.0128	.0203	.0229	.0294	.0368	.0624	.0986
ϕ_L	.0038	.0059	. 0067	.0086	.0108	.0183	.0289
γ_{NH}	.0225	.0228	.0229	.0231	.0233	.0242	.0256
γ_{NL}	.7558	.7598	.7609	.7633	.7661	.7789	.7981
γ_{XH}	.0163	.0144	.0138	.0125	.0114	.0084	.0058
γ_{XL}	.1872	.1762	.1729	.1652	.1569	.1303	.1006
γ_{FH}	.0006	.0015	.0019	.0030	.0043	.0084	.0113
γ_{FL}	.0021	.0054	.0069	.0115	.0175	.0383	.0571
γ_{IH}	.0035	.0043	.0044	.0044	.0041	.0021	.0002
γ_{IL}	.0120	.0156	.0164	.0171	.0164	.0096	.0011
ι_S	.286	.145	.118	.074	.047	.013	.001
L_{FH0}	.004	.009	.011	.017	.025	.055	.100
L_{FL0}	.004	.010	.012	.019	.029	.074	.148
f_N^N	.791	.799	.800	.804	.806	.813	.825
f_{XH}^S	.326	.312	.307	.297	.287	.263	.244
Y_F	.020	.038	.045	.069	.104	.270	.584
e^N	2.59	2.36	2.32	2.27	2.28	2.56	3.10
e^S	1.00	1.00	1.00	1.00	1.00	1.00	1.00
P_0^N	.0086	.0077	.0075	.0072	.0071	.0072	.0078
P_0^S	.0123	.0109	.0106	.0102	.0099	.0099	.0103
$n_0^{1/(\sigma-1)}$	330.8	335.2	337.2	342.6	349.5	373.5	396.7
u_0^N	302.2	306.3	308.2	314.2	322.6	357.0	399.5
u_0^S	81.45	91.52	93.92	98.10	100.7	101.3	96.81

Table 9: Varying IPR protection.

where $\gamma(\omega)$ is the costate variable. The costate equation $\partial H/\partial y = 0 = -\dot{\gamma}(\omega)$ implies that $\gamma(\omega)$ is constant across ω. $\partial H/\partial x = \alpha x_t(\omega)^{\alpha-1} + \gamma \cdot p_t(\omega) = 0$ implies that

$$x_t(\omega) = \left(\frac{\alpha}{-\gamma \cdot p_t(\omega)}\right)^{1/(1-\alpha)}.$$

Substituting this back into the budget constraint yields

$$\begin{aligned} e_t &= \int_0^{n_t} p_t(\omega) x_t(\omega) d\omega = \int_0^{n_t} p_t(\omega) \left(\frac{\alpha}{-\gamma \cdot p_t(\omega)}\right)^{1/(1-\alpha)} d\omega \\ &= \left(\frac{\alpha}{-\gamma}\right)^{1/(1-\alpha)} \int_0^{n_t} p_t(\omega)^{\frac{1-\alpha-1}{1-\alpha}} d\omega. \end{aligned}$$

Now $\sigma \equiv 1/(1-\alpha)$ implies that $1-\sigma = (1-\alpha-1)/(1-\alpha) = -\alpha/(1-\alpha)$, so

$$\frac{e_t}{\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega} = \left(\frac{\alpha}{-\gamma}\right)^{1/(1-\alpha)}.$$

It immediately follows that the consumer demand function is

$$x_t(\omega) = \frac{p_t(\omega)^{-\sigma} e_t}{P_t^{1-\sigma}} \tag{3}$$

where $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ is an index of consumer prices.

Substituting this consumer demand function back into the consumer utility function yields

$$u_t = \left[\int_0^{n_t} x_t(\omega)^{\alpha} d\omega\right]^{\frac{1}{\alpha}} = \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha} e_t^{\alpha}}{P_t^{(1-\sigma)\alpha}} d\omega\right]^{\frac{1}{\alpha}} = e_t \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha}}{P_t^{(1-\sigma)\alpha}} d\omega\right]^{\frac{1}{\alpha}}.$$

Taking into account that $-\sigma\alpha = -\alpha/(1-\alpha) = 1-\sigma$, consumer utility can be simplified further to

$$u_t = \frac{e_t}{P_t^{1-\sigma}} \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{\alpha}} = \frac{e_t}{P_t^{1-\sigma}} \left[P_t^{1-\sigma}\right]^{\frac{1}{\alpha}} = \frac{e_t}{P_t^{1-\sigma}} P_t^{-\sigma} = \frac{e_t}{P_t}$$

or

$$\ln u_t = \ln e_t - \ln P_t.$$

The individual household takes the prices of all products as given, as well as how prices change over time, so the $\ln P_t$ term can be ignored in solving the household's dynamic optimization problem. This problem simplifies to:

$$\max_{e_t} \int_0^{\infty} e^{-(\rho-g_L)t} \ln e_t \, dt \qquad \text{s.t.} \qquad \dot{\tilde{a}}_t = w_t + r_t \tilde{a}_t - g_L \tilde{a}_t - e_t,$$

where $\tilde{a}_t$ represents the asset holding of the representative consumer, w_t is the wage rate and r_t is the interest rate.

The Hamiltonian function for this optimal control problem is

$$H = e^{-(\rho - g_L)t} \ln e_t + \lambda_t \left[w_t + r_t \tilde{a}_t - g_L \tilde{a}_t - e_t \right]$$

where λ_t is the relevant costate variable. The costate equation $-\dot{\lambda}_t = \partial H / \partial \tilde{a}_t = \lambda_t \left[r_t - g_L \right]$ implies that

$$\frac{\dot{\lambda}_t}{\lambda_t} = g_L - r_t.$$

$\partial H / \partial e_t = e^{-(\rho - g_L)t} \left(1/e_t\right) - \lambda_t = 0$ implies that $e^{-(\rho - g_L)t} \left(1/e_t\right) = \lambda_t$. Taking logs of both sides yields $-\left(\rho - g_L\right) t - \ln e_t = \ln \lambda_t$ and then differentiating with respect to time yields

$$-\left(\rho - g_L\right) - \frac{\dot{e}_t}{e_t} = \frac{\dot{\lambda}_t}{\lambda_t} = g_L - r_t.$$

It immediately follows that

$$\frac{\dot{e}_t}{e_t} = r_t - \rho. \tag{4}$$

Steady-State Dynamics

We will now derive some steady-state equilibrium implications of the model.

The export-learning rate is $\chi_z \equiv (\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt})/n_{Nzt}$. It is constant over time in any steady-state equilibrium since

$$\begin{aligned} \chi_z &\equiv \frac{\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Nzt}} = \frac{\dot{n}_{Xzt}}{n_{Xzt}} \frac{n_{Xzt}/n_t}{n_{Nzt}/n_t} + \frac{\dot{n}_{Fzt}}{n_{Fzt}} \frac{n_{Fzt}/n_t}{n_{Nzt}/n_t} + \frac{\dot{n}_{Izt}}{n_{Izt}} \frac{n_{Izt}/n_t}{n_{Nzt}/n_t} \\ &= g \frac{\gamma_{Xz}}{\gamma_{Nz}} + g \frac{\gamma_{Fz}}{\gamma_{Nz}} + g \frac{\gamma_{Iz}}{\gamma_{Nz}}. \end{aligned}$$

The FDI rate is $\phi_z \equiv (\dot{n}_{Fzt} + \dot{n}_{Izt})/n_{Xzt}$. It is constant over time in any steady-state equilibrium since

$$\phi_z \equiv \frac{\dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Xzt}} = \frac{\dot{n}_{Fzt}}{n_{Fzt}} \frac{n_{Fzt}/n_t}{n_{Xzt}/n_t} + \frac{\dot{n}_{Izt}}{n_{Izt}} \frac{n_{Izt}/n_t}{n_{Xzt}/n_t} = g \frac{\gamma_{Fz}}{\gamma_{Xz}} + g \frac{\gamma_{Iz}}{\gamma_{Xz}}.$$

The imitation rate of foreign affiliates is $\iota_S \equiv \dot{n}_{Izt}/n_{Fzt}$. It is constant over time in steady-state equilibrium since

$$\iota_S \equiv \frac{\dot{n}_{Izt}}{n_{Fzt}} = \frac{\dot{n}_{Izt}}{n_{Izt}} \frac{n_{Izt}/n_t}{n_{Fzt}/n_t} = g \frac{\gamma_{Iz}}{\gamma_{Fz}}.$$

We can now solve for γ_{Nz}. By differentiating the variety condition for z-productivity firms

$q_z n_t = n_{Nzt} + n_{Xzt} + n_{Fzt} + n_{Izt}$, we obtain that

$$\begin{aligned} q_z \dot{n}_t &= \dot{n}_{Nzt} + \dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt} \\ q_z \frac{\dot{n}_t}{n_t} &= \frac{\dot{n}_{Nzt} + \dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}}{n_t} \\ q_z g &= \frac{\dot{n}_{Nzt}}{n_{Nzt}} \frac{n_{Nzt}}{n_t} + \frac{\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Nzt}} \frac{n_{Nzt}}{n_t} \\ q_z g &= g \gamma_{Nz} + \chi_z \gamma_{Nz} \end{aligned}$$

and solving for γ_{Nz} yields

$$\gamma_{Nz} = q_z \frac{g}{g + \chi_z}, \qquad (z = H, L). \tag{5}$$

To solve for γ_{Xz}, note that

$$\chi_z = \frac{\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Nzt}} = \frac{\dot{n}_{Xzt}}{n_{Xzt}} \frac{n_{Xzt}/n_t}{n_{Nzt}/n_t} + \frac{\dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Xzt}} \frac{n_{Xzt}/n_t}{n_{Nzt}/n_t} = (g + \phi_z) \frac{\gamma_{Xz}}{\gamma_{Nz}}$$

from which it follows that $\gamma_{Xz} = \gamma_{Nz} \chi_z / (g + \phi_z)$. Inserting the steady-state expression for γ_{Nz} (5) yields

$$\gamma_{Xz} = q_z \frac{\chi_z}{g + \chi_z} \frac{g}{g + \phi_z}, \qquad (z = H, L). \tag{6}$$

To solve for γ_{Fz}, note that

$$\phi_z = \frac{\dot{n}_{Fzt} + \dot{n}_{Izt}}{n_{Xzt}} = \frac{\dot{n}_{Fzt}}{n_{Fzt}} \frac{n_{Fzt}/n_t}{n_{Xzt}/n_t} + \frac{\dot{n}_{Izt}}{n_{Fzt}} \frac{n_{Fzt}/n_t}{n_{Xzt}/n_t} = (g + \iota_S) \frac{\gamma_{Fz}}{\gamma_{Xz}}$$

from which it follows that $\gamma_{Fz} = \gamma_{Xz} \phi_z / (g + \iota_S)$. Inserting the steady-state expressions for γ_{Xz} from (6) yields

$$\gamma_{Fz} = q_z \frac{\chi_z}{g + \chi_z} \frac{\phi_z}{g + \phi_z} \frac{g}{g + \iota_S}, \qquad (z = H, L). \tag{7}$$

To solve for γ_{Iz}, note that

$$\iota_S \equiv \frac{\dot{n}_{Izt}}{n_{Fzt}} = \frac{\dot{n}_{Izt}}{n_{Izt}} \frac{n_{Izt}/n_t}{n_{Fzt}/n_t} = g \frac{\gamma_{Iz}}{\gamma_{Fz}},$$

from which it follows that $\gamma_{Iz} = (\iota_S / g) \gamma_{Fz}$. Inserting the steady-state expressions for γ_{Fz} from (7) yields

$$\gamma_{Iz} = q_z \frac{\chi_z}{g + \chi_z} \frac{\phi_z}{g + \phi_z} \frac{\iota_S}{g + \iota_S}, \qquad (z = H, L). \tag{8}$$

Product Markets

A northern firm with productivity z earns the flow of domestic profits

$$\pi_{Nzt} = \left(p^N_{Nz} - c_z w^N\right) x^N_{Nzt} L^N_t$$

where x^N_{Nzt} is the quantity demanded by the typical northern consumer of the product produced by a northern firm with productivity z. From the earlier demand function, it follows that $x^N_{Nzt} = \left(p^N_{Nz}\right)^{-\sigma} e^N / \left(P^N_t\right)^{1-\sigma}$. Hence, we can write a northern firm's profit flow as:

$$\pi_{Nzt} = \left(p^N_{Nz} - c_z w^N\right) \frac{\left(p^N_{Nz}\right)^{-\sigma} e^N L^N_t}{\left(P^N_t\right)^{1-\sigma}}.$$

Maximizing π_{Nzt} with respect to p^N_{Nz} yields the first-order condition

$$\frac{\partial \pi_{Nzt}}{\partial p^N_{Nz}} = \left[(1-\sigma)\left(p^N_{Nz}\right)^{-\sigma} + \sigma c_z w^N \left(p^N_{Nz}\right)^{-\sigma-1}\right] \frac{e^N L^N_t}{\left(P^N_t\right)^{1-\sigma}} = 0,$$

which implies that $(1-\sigma)\left(p^N_{Nz}\right)^{-\sigma} + \sigma c_z w^N \left(p^N_{Nz}\right)^{-\sigma-1} = 0$ since $e^N L^N_t / \left(P^N_t\right)^{1-\sigma} \neq 0$. Dividing by $\left(p^N_{Nz}\right)^{-\sigma}$ yields $\sigma c_z w^N / p^N_{Nz} = \sigma - 1$ or

$$p^N_{Nz} = \frac{\sigma c_z w^N}{\sigma - 1} = \frac{c_z w^N}{\alpha}.$$

To demonstrate the second equality, first note that $\sigma \equiv 1/(1-\alpha)$ implies that $\sigma - 1 = (1-(1-\alpha)) / (1-\alpha) = \alpha/(1-\alpha)$. It follows that $\sigma/(\sigma-1) = (1/(1-\alpha)) / (\alpha/(1-\alpha)) = 1/\alpha$. Plugging the prices back into the profit expression, we obtain

$$\begin{aligned}
\pi_{Nzt} &= \left(p^N_{Nz} - c_z w^N\right) \frac{\left(p^N_{Nz}\right)^{-\sigma} e^N L^N_t}{\left(P^N_t\right)^{1-\sigma}} \\
&= \left(\frac{c_z w^N}{\alpha} - c_z w^N\right) \frac{\left(p^N_{Nz}\right)^{-\sigma} e^N L^N_t}{\left(P^N_t\right)^{1-\sigma}} \\
&= \frac{c_z w^N}{\sigma - 1} \left[\frac{\left(p^N_{Nz}\right)^{-\sigma} e^N L^N_t}{\left(P^N_t\right)^{1-\sigma}}\right] \\
&= \frac{c_z w^N}{\sigma - 1} \left[\frac{\left(p^N_{Nz}\right)^{-\sigma} e^N L^N_t n_{Nzt}}{\left(P^N_t\right)^{1-\sigma} L_t}\right] \frac{L_t}{\frac{n_{Nzt}}{n_t} n_t}.
\end{aligned}$$

Now $\gamma_{Nz} \equiv n_{Nzt}/n_t$ is constant over time and $X^N_{Nz} \equiv \left(p^N_{Nz}\right)^{-\sigma} e^N L^N_t n_{Nzt} / \left(P^N_t\right)^{1-\sigma} L_t$ is constant over time since $\left(P^N_t\right)^{1-\sigma}$ grows at the same rate g as n_{Nzt}. Thus we can write π_{Nzt} more simply as:

$$\pi_{Nzt} = \left[\frac{c_z w^N X^N_{Nz}}{(\sigma - 1)\gamma_{Nz}}\right] \frac{L_t}{n_t}. \tag{11}$$

An exporter earns the flow of global profits

$$\pi_{Xzt} = \left(p^N_{Xz} - c_z w^N\right) x^N_{Xzt} L^N_t + \left(p^S_{Xz} - \tau c_z w^N\right) x^S_{Xzt} L^S_t$$

where $x^N_{Xzt} = \left(p^N_{Xz}\right)^{-\sigma} e^N / \left(P^N_t\right)^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the exporter's product and $x^S_{Xzt} = \left(p^S_{Xz}\right)^{-\sigma} e^S / \left(P^S_t\right)^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the exporter's product. Hence, we can write an exporter's global profit flow as:

$$\pi_{Xzt} = \left(p_{Xz}^N - c_z w^N\right) \frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t\right)^{1-\sigma}} + \left(p_{Xz}^S - \tau c_z w^N\right) \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}}.$$

Maximizing π_{Xzt} with respect to p_{Xz}^N yields the first-order condition

$$\frac{\partial \pi_{Xzt}}{\partial p_{Xz}^N} = \left[(1-\sigma)\left(p_{Xz}^N\right)^{-\sigma} + \sigma c_z w^N \left(p_{Xz}^N\right)^{-\sigma-1}\right] \frac{e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} = 0,$$

which implies that $(1-\sigma)\left(p_{Xz}^N\right)^{-\sigma} + \sigma c_z w^N \left(p_{Xz}^N\right)^{-\sigma-1} = 0$ since $e^N L_t^N / \left(P_t^N\right)^{1-\sigma} \neq 0$. Dividing by $\left(p_{Xz}^N\right)^{-\sigma}$ yields $\sigma c_z w^N / p_{Xz}^N = \sigma - 1$ or

$$p_{Xz}^N = \frac{\sigma c_z w^N}{\sigma - 1} = \frac{c_z w^N}{\alpha}.$$

Similarly, maximizing π_{Xzt} with respect to p_{Xz}^S yields the first-order condition

$$\frac{\partial \pi_{Xzt}}{\partial p_{Xz}^S} = \left[(1-\sigma)\left(p_{Xz}^S\right)^{-\sigma} + \sigma \tau c_z w^N \left(p_{Xz}^S\right)^{-\sigma-1}\right] \frac{e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} = 0,$$

which implies that $(1-\sigma)\left(p_{Xz}^S\right)^{-\sigma} + \sigma \tau c_z w^N \left(p_{Xz}^S\right)^{-\sigma-1} = 0$. Dividing by $\left(p_{Xz}^S\right)^{-\sigma}$ yields $\sigma \tau c_z w^N / p_{Xz}^S = \sigma - 1$ or

$$p_{Xz}^S = \frac{\sigma \tau c_z w^N}{\sigma - 1} = \frac{\tau c_z w^N}{\alpha}.$$

Plugging the prices back into the profit expression, we obtain

$$\begin{aligned}
\pi_{Xzt} &= \left(p_{Xz}^N - c_z w^N\right) \frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} + \left(p_{Xz}^S - \tau c_z w^N\right) \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} \\
&= \left(\frac{c_z w^N}{\alpha} - c_z w^N\right) \frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} + \left(\frac{\tau c_z w^N}{\alpha} - \tau c_z w_N\right) \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} \\
&= \frac{c_z w^N}{\sigma - 1} \left[\frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} + \tau \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}}\right] \\
&= \frac{c_z w^N}{\sigma - 1} \left[\frac{\left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N n_{Xzt}}{\left(P_t^N\right)^{1-\sigma} L_t} + \tau \frac{\left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S n_{Xzt}}{\left(P_t^S\right)^{1-\sigma} L_t}\right] \frac{L_t}{\frac{n_{Xzt}}{n_t} n_t}.
\end{aligned}$$

Now $\gamma_{Xz} \equiv n_{Xzt}/n_t$ is constant over time, $X_{Xz}^N \equiv \left(p_{Xz}^N\right)^{-\sigma} e^N L_t^N n_{Xzt} / \left(P_t^N\right)^{1-\sigma} L_t$ is constant over time since $\left(P_t^N\right)^{1-\sigma}$ grows at the same rate g as n_{Xzt}, and $X_{Xz}^S \equiv \left(p_{Xz}^S\right)^{-\sigma} e^S L_t^S n_{Xzt}$ / $\left(P_t^S\right)^{1-\sigma} L_t$ is constant over time since $\left(P_t^S\right)^{1-\sigma}$ grows at the same rate g as n_{Xzt}. Thus we can write π_{Xzt} more simply as:

$$\pi_{Xzt} = \left[\frac{c_z w^N \left(X_{Xz}^N + \tau X_{Xz}^S\right)}{(\sigma-1)\gamma_{Xz}}\right]\frac{L_t}{n_t}. \tag{12}$$

A foreign affiliate earns the flow of global profits:

$$\pi_{Fzt} = \left(p_{Fz}^S - \xi c_z w^S\right) x_{Fzt}^S L_t^S + \left(p_{Fz}^N - \tau\xi c_z w^S\right) x_{Fzt}^N L_t^N$$

where $x_{Fzt}^S = \left(p_{Fz}^S\right)^{-\sigma} e^S / \left(P_t^S\right)^{1-\sigma}$ is the quantity demanded by the typical southern consumer of the foreign affiliate's product and $x_{Fzt}^N = \left(p_{Fz}^N\right)^{-\sigma} e^N / \left(P_t^N\right)^{1-\sigma}$ is the quantity demanded by the typical northern consumer of the foreign affiliate's product. Hence, we can write a foreign affiliate's profit flow as

$$\pi_{Fzt} = \left(p_{Fz}^S - \xi c_z w^S\right)\frac{\left(p_{Fz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} + \left(p_{Fz}^N - \tau\xi c_z w^S\right)\frac{\left(p_{Fz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}}.$$

Maximizing π_{Fzt} with respect to p_{Fz}^S yields the first-order condition

$$\frac{\partial \pi_{Fzt}}{\partial p_{Fz}^S} = \left[(1-\sigma)\left(p_{Fz}^S\right)^{-\sigma} + \sigma\xi c_z w^S \left(p_{Fz}^S\right)^{-\sigma-1}\right]\frac{e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} = 0$$

which implies that $(1-\sigma)\left(p_{Fz}^S\right)^{-\sigma} + \sigma\xi c_z w^S \left(p_{Fz}^S\right)^{-\sigma-1} = 0$. Dividing by $\left(p_{Fz}^S\right)^{-\sigma}$ yields $\sigma\xi c_z w^S / p_{Fz}^S = \sigma - 1$ or

$$p_{Fz}^S = \frac{\sigma\xi c_z w^S}{\sigma-1} = \frac{\xi c_z w^S}{\alpha}.$$

Similarly, maximizing π_{Fzt} with respect to p_{Fz}^N yields the first-order condition

$$\frac{\partial \pi_{Fzt}}{\partial p_{Fz}^N} = \left[(1-\sigma)\left(p_{Fz}^N\right)^{-\sigma} + \sigma\tau\xi c_z w^S \left(p_{Fz}^N\right)^{-\sigma-1}\right]\frac{e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} = 0,$$

which implies that $(1-\sigma)\left(p_{Fz}^N\right)^{-\sigma} + \sigma\tau\xi c_z w^S \left(p_{Fz}^N\right)^{-\sigma-1} = 0$. Dividing by $\left(p_{Fz}^N\right)^{-\sigma}$ yields $\sigma\tau\xi c_z w^S / p_{Fz}^N = \sigma - 1$ or

$$p_{Fz}^N = \frac{\sigma\tau\xi c_z w^S}{\sigma-1} = \frac{\tau\xi c_z w^S}{\alpha}.$$

When the inequality $\tau\xi w_S < w_N$ holds, each foreign affiliate exports to the northern market. The trade cost and MP cost parameters τ and ξ cannot be too high. Plugging the prices back into the profit expression, we obtain

$$\begin{aligned}
\pi_{Fzt} &= \left(\frac{\xi c_z w^S}{\alpha} - \xi c_z w^S\right) \frac{\left(p_{Fz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} + \left(\frac{\tau \xi c_z w^S}{\alpha} - \tau \xi c_z w^S\right) \frac{(p_{Fz}^N)^{-\sigma} e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}} \\
&= \frac{\xi c_z w^S}{\sigma - 1} \left[\frac{\left(p_{Fz}^S\right)^{-\sigma} e^S L_t^S}{\left(P_t^S\right)^{1-\sigma}} + \tau \frac{\left(p_{Fz}^N\right)^{-\sigma} e^N L_t^N}{\left(P_t^N\right)^{1-\sigma}}\right] \\
&= \frac{\xi c_z w^S}{\sigma - 1} \left[\frac{\left(p_{Fz}^S\right)^{-\sigma} e^S L_t^S n_{Fzt}}{\left(P_t^S\right)^{1-\sigma} L_t} + \tau \frac{\left(p_{Fz}^N\right)^{-\sigma} e^N L_t^N n_{Fzt}}{\left(P_t^N\right)^{1-\sigma} L_t}\right] \frac{L_t}{\frac{n_{Fzt}}{n_t} n_t}.
\end{aligned}$$

Now $\gamma_{Fz} \equiv n_{Fzt}/n_t$ is constant over time, $X_{Fz}^S \equiv \left(p_{Fz}^S\right)^{-\sigma} e^S L_t^S n_{Fzt} / \left(P_t^S\right)^{1-\sigma} L_t$ is constant over time since $\left(P_t^S\right)^{1-\sigma}$ grows at the same rate g as n_{Fzt}, and $X_{Fz}^N \equiv \left(p_{Fz}^N\right)^{-\sigma} e^N L_t^N n_{Fzt} / \left(P_t^N\right)^{1-\sigma} L_t$ is constant over time since $\left(P_t^N\right)^{1-\sigma}$ grows at the same rate g as n_{Fzt}. Thus, we can write π_{Fzt} more simply as:

$$\pi_{Fzt} = \left[\frac{\xi c_z w^S \left(X_{Fz}^S + \tau X_{Fz}^N\right)}{(\sigma - 1)\gamma_{Fz}}\right] \frac{L_t}{n_t}. \tag{13}$$

A foreign affiliate's variety is imitated by southern firms at the exogenously given rate ι_S. Once the imitated technology is available to southern firms, competition drives down price to marginal cost and southern firms therefore earn zero profits. The quantity demanded by the typical southern consumer of a southern product is $x_{Izt}^S = \left(p_{Iz}^S\right)^{-\sigma} e^S / \left(P_t^S\right)^{1-\sigma}$ and $x_{Izt}^N = \left(p_{Iz}^N\right)^{-\sigma} e^N / \left(P_t^N\right)^{1-\sigma}$ is the quantity demanded by the typical northern consumer of a southern product. Since southern firms set price equal to marginal cost, we must have $p_{Iz}^S = c_z w^S$ and $p_{Iz}^N = \tau c_z w^S$.

R&D Incentives

For a northern firm, the relevant no-arbitrage condition is

$$v_{Nt} = \frac{q\,\pi_{NLt} + (1-q)\,\pi_{NHt}}{\rho + \theta g} = \frac{w^N a_R g^\beta}{n_t^\theta}.$$

Substituting for π_{NHt} and π_{NLt} yields

$$\begin{aligned}
\frac{q\, c_L w^N X_{NL}^N}{(\sigma-1)\gamma_{NL}(\rho+\theta g)} \frac{L_t}{n_t} + \frac{(1-q) c_H w^N X_{NH}^N}{(\sigma-1)\gamma_{NH}(\rho+\theta g)} \frac{L_t}{n_t} &= \frac{w^N a_R g^\beta}{n_t^\theta} \\
\frac{q\, c_L X_{NL}^N}{(\sigma-1)\gamma_{NL}(\rho+\theta g)} + \frac{(1-q) c_H X_{NH}^N}{(\sigma-1)\gamma_{NH}(\rho+\theta g)} &= a_R g^\beta \frac{n_t^{1-\theta}}{L_t}.
\end{aligned}$$

Thus the steady-state northern no-arbitrage condition is

$$\frac{1}{\sigma-1}\left(\frac{q\, c_L X_{NL}^N}{\gamma_{NL}(\rho+\theta g)} + \frac{(1-q) c_H X_{NH}^N}{\gamma_{NH}(\rho+\theta g)}\right) = a_R g^\beta \delta. \tag{26}$$

The no-arbitrage condition for the exporter simplifies to $\pi_{Xzt}/v_{Xzt} - \theta g = \rho$ or $v_{Xzt} = \pi_{Xzt}/(\rho+\theta g)$. Combining this expression with (24), the exporter no-arbitrage condition can be written as

$$v_{Xzt} - v_{Nzt} = \frac{\pi_{Xzt}}{\rho + \theta g} - \frac{\pi_{Nzt}}{\rho + \theta g} = \frac{w^S a_X \chi_z^\beta}{n_t^\theta}.$$

Using the profits for northern firms (11) and exporters (12), we can write this as:

$$\begin{aligned}
\frac{c_z w^N}{(\sigma - 1)} \frac{X_{Xz}^N + \tau X_{Xz}^S}{\gamma_{Xz} (\rho + \theta g)} \frac{L_t}{n_t} - \frac{c_z w^N X_{Nz}^N}{(\sigma - 1) \gamma_{Nz} (\rho + \theta g)} \frac{L_t}{n_t} &= \frac{w^S a_X \chi_z^\beta}{n_t^\theta} \\
\frac{c_z w}{(\sigma - 1)} \frac{X_{Xz}^N + \tau X_{Xz}^S}{\gamma_{Xz} (\rho + \theta g)} - \frac{c_z w X_{Nz}^N}{(\sigma - 1) \gamma_{Nz} (\rho + \theta g)} &= a_X \chi_z^\beta \frac{n_t^{1-\theta}}{L_t}.
\end{aligned}$$

It follows that the steady-state exporter no-arbitrage condition is

$$\frac{c_z w}{\sigma - 1} \left[\frac{X_{Xz}^N + \tau X_{Xz}^S}{\gamma_{Xz} (\rho + \theta g)} - \frac{X_{Nz}^N}{\gamma_{Nz} (\rho + \theta g)} \right] = a_X \chi_z^\beta \delta \tag{27}$$

where $w \equiv w_N / w_S$ is the northern relative wage.

The no-arbitrage condition for the foreign affiliate simplifies to $\pi_{Fzt}/v_{Fzt} - \theta g - \iota_S = \rho$ or $v_{Fzt} = \pi_{Fzt}/ (\rho + \theta g + \iota_S)$. Combining this expression with (25), the foreign affiliate no-arbitrage condition can be written as

$$\frac{\pi_{Fzt}}{\rho + \theta g + \iota_S} - \frac{\pi_{Xzt}}{\rho + \theta g} = \frac{w^S a_F \phi_z^\beta}{n_t^\theta}.$$

Using profits (12) and (13), we can write this as

$$\begin{aligned}
\frac{\xi c_z w^S}{\sigma - 1} \frac{X_{Fz}^S + \tau X_{Fz}^N}{\gamma_{Fz} (\rho + \theta g + \iota_S)} \frac{L_t}{n_t} - \frac{c_z w^N}{\sigma - 1} \frac{X_{Xz}^N + \tau X_{Xz}^S}{\gamma_{Xz} (\rho + \theta g)} \frac{L_t}{n_t} &= \frac{w_S a_F \phi_z^\beta}{n_t^\theta} \\
\frac{\xi c_z}{\sigma - 1} \frac{X_{Fz}^S + \tau X_{Fz}^N}{\gamma_{Fz} (\rho + \theta g + \iota_S)} - \frac{c_z w}{\sigma - 1} \frac{X_{Xz}^N + \tau X_{Xz}^S}{\gamma_{Xz} (\rho + \theta g)} &= a_F \phi_z^\beta \frac{n_t^{1-\theta}}{L_t}.
\end{aligned}$$

It follows that the steady-state foreign affiliate no-arbitrage condition is

$$\frac{c_z}{\sigma - 1} \left[\frac{\xi X_{Fz}^S + \tau \xi X_{Fz}^N}{\gamma_{Fz} (\rho + \theta g + \iota_S)} - \frac{w \left(X_{Xz}^N + \tau X_{Xz}^S \right)}{\gamma_{Xz} (\rho + \theta g)} \right] = a_F \phi_z^\beta \delta. \tag{28}$$

Labor Markets

In the South, labor is employed in export-learning R&D, MP-learning R&D, production by foreign affiliates and production by southern firms that have imitated foreign affiliates.

Each northern product variety introduced to the southern market via exports requires $a_X \chi_z^\beta / n_t^\theta$ units of labor, so total employment in export-learning R&D by firms is $\sum_z \left(a_X \chi_z^\beta / n_t^\theta \right) (\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt})$. Each variety transferred to the South by a foreign affiliate requires $a_F \phi_z^\beta / n_t^\theta$ units of labor, so total employment in MP-learning R&D is $\sum_z \left(a_F \phi_z^\beta / n_t^\theta \right) (\dot{n}_{Fzt} + \dot{n}_{Izt})$.

Turning to southern production, a foreign affiliate with productivity z uses $\xi c_z \left(p_{Fz}^S \right)^{-\sigma} e^S L_t^S /$

$\left(P_t^S\right)^{1-\sigma} + \tau\xi c_z \left(p_{Fz}^N\right)^{-\sigma} e^N L_t^N / \left(P_t^N\right)^{1-\sigma} = \xi c_z X_{Fz}^S L_t / n_{Fzt} + \tau\xi c_z X_{Fz}^N L_t / n_{Fzt}$ units of labor for each variety produced, and there are n_{Fzt} such varieties produced, so total employment in foreign affiliate production is $\sum_z \left(\xi c_z X_{Fz}^S L_t / n_{Fzt} + \tau\xi c_z X_{Fz}^N L_t / n_{Fzt}\right) n_{Fzt} = \sum_z \xi c_z \left[X_{Fz}^S + \tau X_{Fz}^N\right] L_t$.

A southern firm that has imitated a foreign affiliate with productivity z uses $c_z \left(p_{Iz}^S\right)^{-\sigma} e^S L_t^S / \left(P_t^S\right)^{1-\sigma} + \tau c_z \left(p_{Iz}^N\right)^{-\sigma} e^N L_t^N / \left(P_t^N\right)^{1-\sigma} = c_z X_{Iz}^S L_t / n_{Izt} + \tau c_z X_{Iz}^N L_t / n_{Izt}$ units of labor for each variety produced, and there are n_{Izt} such varieties produced, so total employment in southern production is $\sum_z \left(c_z X_{Iz}^S L_t / n_{Izt} + \tau c_z X_{Iz}^N L_t / n_{Izt}\right) n_{Izt} = \sum_z c_z \left[X_{Iz}^S + \tau X_{Iz}^N\right] L_t$.

As L_t^S denotes the labor supply in the South, full employment requires that

$$\begin{aligned} L_t^S &= \sum_{z=H,L} \frac{a_X \chi_z^\beta}{n_t^\theta} \left(\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}\right) + \frac{a_F \phi_z^\beta}{n_t^\theta} \left(\dot{n}_{Fzt} + \dot{n}_{Izt}\right) \\ &\quad + \xi c_z \left[X_{Fz}^S + \tau X_{Fz}^N\right] L_t + c_z \left[X_{Iz}^S + \tau X_{Iz}^N\right] L_t. \end{aligned}$$

Now using $\delta \equiv n_t^{1-\theta} / L_t$, $\chi_z \equiv \left(\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}\right) / n_{Nzt}$, $\phi_z \equiv \left(\dot{n}_{Fzt} + \dot{n}_{Izt}\right) / n_{Xzt}$ and $\iota_S = \dot{n}_{Izt} / n_{Fzt}$, southern R&D employment can be written as

$$\begin{aligned} &\sum_{z=H,L} \left[\frac{a_X \chi_z^\beta}{n_t^\theta} \left(\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}\right) + \frac{a_F \phi_z^\beta}{n_t^\theta} \left(\dot{n}_{Fzt} + \dot{n}_{Izt}\right)\right] \\ = &\sum_{z=H,L} \left[\frac{a_X \chi_z^\beta \left(\dot{n}_{Xzt} + \dot{n}_{Fzt} + \dot{n}_{Izt}\right)}{n_{Nzt}} \frac{n_{Nzt}}{n_t} \frac{n_t^{1-\theta}}{L_t} L_t + \frac{a_F \phi_z^\beta \left(\dot{n}_{Fzt} + \dot{n}_{Izt}\right)}{n_{Xzt}} \frac{n_{Xzt}}{n_t} \frac{n_t^{1-\theta}}{L_t} L_t\right] \\ = &\sum_{z=H,L} \left[a_X \chi_z^{1+\beta} \gamma_{Nz} \delta L_t + a_F \phi_z^{1+\beta} \gamma_{Xz} \delta L_t\right]. \end{aligned}$$

It follows that

$$L_t^S = L_t \left[\sum_{z=H,L} a_X \delta \chi_z^{1+\beta} \gamma_{Nz} + a_F \delta \phi_z^{1+\beta} \gamma_{Xz} + c_z \left(\xi X_{Fz}^S + \tau\xi X_{Fz}^N + X_{Iz}^S + \tau X_{Iz}^N\right)\right]$$

and evaluating at time $t = 0$ yields the steady-state full employment of labor condition for the South:

$$L_0^S = L_0 \left[\sum_{z=H,L} a_X \delta \chi_z^{1+\beta} \gamma_{Nz} + a_F \delta \phi_z^{1+\beta} \gamma_{Xz} + c_z \left(\xi X_{Fz}^S + \tau\xi X_{Fz}^N + X_{Iz}^S + \tau X_{Iz}^N\right)\right]. \quad (30)$$

Aggregate Demand

We need to solve for steady-state values of the aggregate demand expressions X^N_{Nz}, X^N_{Xz}, X^S_{Xz}, X^S_{Fz}, X^N_{Fz}, X^S_{Iz} and X^N_{Iz}. The calculations

$$\begin{aligned}\frac{X^N_{Nz}}{X^N_{Fz}} &= \frac{\left[\left(p^N_{Nz}\right)^{-\sigma} e^N L^N_t n_{Nzt}\right] / \left[\left(P^N_t\right)^{1-\sigma} L_t\right]}{\left[\left(p^N_{Fz}\right)^{-\sigma} e^N L^N_t n_{Fzt}\right] / \left[\left(P^N_t\right)^{1-\sigma} L_t\right]} = \left(\frac{p^N_{Nz}}{p^N_{Fz}}\right)^{-\sigma} \frac{n_{Nzt}/n_t}{n_{Fzt}/n_t} \\ &= \left(\frac{c_z w^N/\alpha}{\tau\xi c_z w^S/\alpha}\right)^{-\sigma} \frac{\gamma_{Nz}}{\gamma_{Fz}} = \left(\frac{w}{\tau\xi}\right)^{-\sigma} \frac{q_z g/\left(g+\chi_z\right)}{q_z\chi_z\phi_z g/\left[\left(g+\chi_z\right)\left(g+\phi_z\right)\left(g+\iota_S\right)\right]},\end{aligned}$$

$$\begin{aligned}\frac{X^N_{Xz}}{X^N_{Fz}} &= \frac{\left[\left(p^N_{Xz}\right)^{-\sigma} e^N L^N_t n_{Xzt}\right] / \left[\left(P^N_t\right)^{1-\sigma} L_t\right]}{\left[\left(p^N_{Fz}\right)^{-\sigma} e^N L^N_t n_{Fzt}\right] / \left[\left(P^N_t\right)^{1-\sigma} L_t\right]} = \left(\frac{p^N_{Xz}}{p^N_{Fz}}\right)^{-\sigma} \frac{n_{Xzt}/n_t}{n_{Fzt}/n_t} \\ &= \left(\frac{c_z w^N/\alpha}{\tau\xi c_z w^S/\alpha}\right)^{-\sigma} \frac{\gamma_{Xz}}{\gamma_{Fz}} = \left(\frac{w}{\tau\xi}\right)^{-\sigma} \frac{q_z\chi_z g/\left[\left(g+\chi_z\right)\left(g+\phi_z\right)\right]}{q_z\chi_z\phi_z g/\left[\left(g+\chi_z\right)\left(g+\phi_z\right)\left(g+\iota_S\right)\right]},\end{aligned}$$

$$\begin{aligned}\frac{X^S_{Xz}}{X^S_{Fz}} &= \frac{\left[\left(p^S_{Xz}\right)^{-\sigma} e^S L^S_t n_{Xzt}\right] / \left[\left(P^S_t\right)^{1-\sigma} L_t\right]}{\left[\left(p^S_{Fz}\right)^{-\sigma} e^S L^S_t n_{Fzt}\right] / \left[\left(P^S_t\right)^{1-\sigma} L_t\right]} = \left(\frac{p^S_{Xz}}{p^S_{Fz}}\right)^{-\sigma} \frac{n_{Xzt}/n_t}{n_{Fzt}/n_t} \\ &= \left(\frac{\tau c_z w^N/\alpha}{\xi c_z w^S/\alpha}\right)^{-\sigma} \frac{\gamma_{Xz}}{\gamma_{Fz}} = \left(\frac{\tau w}{\xi}\right)^{-\sigma} \frac{q_z\chi_z g/\left[\left(g+\chi_z\right)\left(g+\phi_z\right)\right]}{q_z\chi_z\phi_z g/\left[\left(g+\chi_z\right)\left(g+\phi_z\right)\left(g+\iota_S\right)\right]},\end{aligned}$$

$$\begin{aligned}\frac{X^S_{Iz}}{X^S_{Fz}} &= \frac{\left[\left(p^S_{Iz}\right)^{-\sigma} e^S L^S_t n_{Izt}\right] / \left[\left(P^S_t\right)^{1-\sigma} L_t\right]}{\left[\left(p^S_{Fz}\right)^{-\sigma} e^S L^S_t n_{Fzt}/\right] / \left[\left(P^S_t\right)^{1-\sigma} L_t\right]} = \left(\frac{p^S_{Iz}}{p^S_{Fz}}\right)^{-\sigma} \frac{n_{Izt}/n_t}{n_{Fzt}/n_t} \\ &= \left(\frac{c_z w^S}{\xi c_z w^S/\alpha}\right)^{-\sigma} \frac{\gamma_{Iz}}{\gamma_{Fz}} = \left(\frac{\xi}{\alpha}\right)^{\sigma} \frac{q_z\chi_z\phi_z\iota_S/\left[\left(g+\chi_z\right)\left(g+\phi_z\right)\left(g+\iota_S\right)\right]}{q_z\chi_z\phi_z g/\left[\left(g+\chi_z\right)\left(g+\phi_z\right)\left(g+\iota_S\right)\right]}\end{aligned}$$

and

$$\begin{aligned}\frac{X^N_{Iz}}{X^N_{Fz}} &= \frac{\left[\left(p^N_{Iz}\right)^{-\sigma} e^N L^N_t n_{Izt}\right] / \left[\left(P^N_t\right)^{1-\sigma} L_t\right]}{\left[\left(p^N_{Fz}\right)^{-\sigma} e^N L^N_t n_{Fzt}\right] / \left[\left(P^N_t\right)^{1-\sigma} L_t\right]} = \left(\frac{p^N_{Iz}}{p^N_{Fz}}\right)^{-\sigma} \frac{n_{Izt}/n_t}{n_{Fzt}/n_t} \\ &= \left(\frac{\tau c_z w^S}{\tau\xi c_z w^S/\alpha}\right)^{-\sigma} \frac{\gamma_{Iz}}{\gamma_{Fz}} = \left(\frac{\xi}{\alpha}\right)^{\sigma} \frac{q_z\chi_z\phi_z\iota_S/\left[\left(g+\chi_z\right)\left(g+\phi_z\right)\left(g+\iota_S\right)\right]}{q_z\chi_z\phi_z g/\left[\left(g+\chi_z\right)\left(g+\phi_z\right)\left(g+\iota_S\right)\right]}\end{aligned}$$

imply that

$$X^N_{Nz} = X^N_{Fz}\left(\frac{\tau\xi}{w}\right)^{\sigma} \frac{\left(g+\phi_z\right)\left(g+\iota_S\right)}{\chi_z\phi_z},$$

$$X^N_{Xz} = X^N_{Fz}\left(\frac{\tau\xi}{w}\right)^{\sigma}\frac{g+\iota_S}{\phi_z},$$

$$X^S_{Xz} = X^S_{Fz}\left(\frac{\xi}{w\tau}\right)^{\sigma}\frac{g+\iota_S}{\phi_z},$$

$$X^S_{Iz} = X^S_{Fz}\left(\frac{\xi}{\alpha}\right)^{\sigma}\frac{\iota_S}{g},$$

and

$$X^N_{Iz} = X^N_{Fz}\left(\frac{\xi}{\alpha}\right)^{\sigma}\frac{\iota_S}{g}.$$

Finally, we need to express X^r_{FH} in terms of X^r_{FL}. The calculations

$$\begin{aligned}\frac{X^r_{FH}}{X^r_{FL}} &= \frac{\left[(p^r_{FH})^{-\sigma} e^r L^r_t n_{FHt}\right] / \left[(P^r_t)^{1-\sigma} L_t\right]}{\left[(p^r_{FL})^{-\sigma} e^r L^r_t n_{FLt}\right]\left[(P^r_t)^{1-\sigma} L_t\right]} = \left(\frac{p^r_{FH}}{p^r_{FL}}\right)^{-\sigma}\frac{n_{FHt}/n_t}{n_{FLt}/n_t} \\ &= \left(\frac{\xi c_H w^S/\alpha}{\xi c_L w^S/\alpha}\right)^{-\sigma}\frac{\gamma_{FH}}{\gamma_{FL}} = \left(\frac{c_H}{c_L}\right)^{-\sigma}\frac{q_H\chi_H\phi_H g/\left[(g+\chi_H)(g+\phi_H)(g+\iota_S)\right]}{q_L\chi_L\phi_L g/\left[(g+\chi_L)(g+\phi_L)(g+\iota_S)\right]}\end{aligned}$$

yields

$$X^r_{FH} = X^r_{FL}\left(\frac{c_H}{c_L}\right)^{-\sigma}\left(\frac{1-q}{q}\right)\left(\frac{g+\chi_L}{g+\chi_H}\right)\left(\frac{\chi_H}{\chi_L}\right)\left(\frac{g+\phi_L}{g+\phi_H}\right)\left(\frac{\phi_H}{\phi_L}\right)$$

where we have used $q_L = q$ and $q_H = 1-q$.

Asset Ownership and Consumer Expenditure

After having solved for steady-state consumer expenditures e^N and e^S, we can take the ratio

$$\begin{aligned}\frac{X^N_{FL}}{X^S_{FL}} &= \frac{\left[\left(p^N_{FL}\right)^{-\sigma} e^N L^N_t n_{FLt}\right] / \left[\left(P^N_t\right)^{1-\sigma} L_t\right]}{\left[(p^S_{FL})^{-\sigma} e^S L^S_t n_{FLt}\right] / \left[(P^S_t)^{1-\sigma} L_t\right]} = \left(\frac{p^N_{FL}}{p^S_{FL}}\right)^{-\sigma}\frac{e^N L^N_t}{e^S L^S_t}\frac{\left(P^S_t\right)^{1-\sigma}}{\left(P^N_t\right)^{1-\sigma}} \\ &= \left(\frac{\tau\xi c_L w^S/\alpha}{\xi c_L w^S/\alpha}\right)^{-\sigma}\frac{e^N L^N_t}{e^S L^S_t}\frac{\left(P^S_t\right)^{1-\sigma}}{\left(P^N_t\right)^{1-\sigma}} = \left(\frac{1}{\tau}\right)^{\sigma}\frac{e^N L^N_t}{e^S L^S_t}\frac{\left(P^S_t\right)^{1-\sigma}}{\left(P^N_t\right)^{1-\sigma}}.\end{aligned}$$

Evaluating at time $t = 0$ yields the steady-state asset condition

$$\frac{X^N_{FL}}{X^S_{FL}} = \left(\frac{1}{\tau}\right)^{\sigma}\frac{e^N L^N_0}{e^S L^S_0}\frac{\left(P^S_t\right)^{1-\sigma}}{\left(P^N_t\right)^{1-\sigma}}. \qquad (33)$$