Efficient Bilateral Trade with Interdependent Values — the Use of Two-Stage Mechanisms

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Introduction

- We study bilateral trade problem with interdependent values.
- Each agent receives different information about the value of the good, denoted by type $\theta_i \in \Theta_i$ which is a compact subset of $\mathbb{R}_+$.
- Types are independently distributed between agents.
- Each agent’s valuation $\tilde{u}_i(\theta_i, \theta_{-i})$ depends on both $\theta_i$ and $\theta_{-i}$. 
Two-stage mechanisms proposed by Mezzetti (2004)

First stage
- Each agent observes his type and sends a message to the designer;
- The trading probability is implemented.

Second stage
- Each agent observes his utility from consuming the good and sends another message;
- The monetary transfers are finalized.
Mezzetti (2004) introduces the generalized two-stage Groves mechanism and shows that it always satisfies

- Bayesian incentive compatibility (BIC): Truth-telling in both stages constitutes an equilibrium strategy of a perfect Bayesian equilibrium;
- decision efficiency (EFF);
- ex post budget balance (BB).
Research Question

- Does the generalized two-stage Groves mechanism satisfy interim individual rationality (IIR) as well?
- If no, is there a different two-stage mechanism satisfying BIC, IIR, EFF and BB?
Preview of Our Results

- Under one-sided asymmetric information structure, the generalized two-stage Groves mechanism always satisfies IIR.
- Under two-sided asymmetric information structure,
  - we show by an example that it never satisfies IIR;
  - we propose the two-stage monotone mechanisms which satisfy IIR in a positive number of cases within the same example;
  - we characterize the existence of two-stage monotone mechanisms satisfying BIC, IIR, EFF and BB.
The Model

Preferences of each agent $U_i : Q \times \Theta \times \mathbb{R} \rightarrow \mathbb{R}$ depend upon trading probability $q$, the type profile $\theta$ and his monetary transfer $p_i$:

$$
U_1(q, \theta, p_1) = u_1(q, \theta) + p_1 = (1 - q)\tilde{u}_1(\theta) + p_1;
$$
$$
U_2(q, \theta, p_2) = u_2(q, \theta) + p_2 = q\tilde{u}_2(\theta) + p_2,
$$

where $u_i(q, \theta)$ is agent $i$'s allocation payoff and $\tilde{u}_i(\theta)$ is his valuation.

We assume that for any $\theta \in \Theta$, each agent $i$ observes $u_i(q, \theta)$ after the outcome decision $q$ is implemented, but before final transfers $p$ are made.
The Model

Agents’ outside option utilities are

\[ U_1^O(\theta_1) = \int_{\Theta_2} \tilde{u}_1(\theta_1, \theta_2) dF_2(\theta_2) \text{ for all } \theta_1 \in \Theta_1 \]

and

\[ U_2^O(\theta_2) = 0 \text{ for all } \theta_2 \in \Theta_2. \]
The Generalized Revelation Principle

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<td>$(M^1, M^2, \delta, \tau)$</td>
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<td>Decision rule $\delta : M^1 \rightarrow [0,1]$;</td>
<td>Decision rule $x : \Theta \rightarrow [0,1]$;</td>
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<td>Transfer rule $\tau : M^1 \times M^2 \rightarrow \mathbb{R}^2$.</td>
<td>Transfer rule $t : \Theta \times \Pi \rightarrow \mathbb{R}^2$.</td>
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<tr>
<td>Agent i’s strategy $r_i = (r^1_i, r^2_i)$ where $r^1_i : \Theta_i \rightarrow M^1_i$ and $r^2_i : Q \times \Theta_i \times \Pi_i \rightarrow M^2_i$.</td>
<td>Decision rule: $x(\theta) = \delta(r^1(\theta))$; Transfer rule: $t_i(\theta, u) = \tau_i(r^1(\theta), r^2(\delta(\theta), \theta, u))$.</td>
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Any PBE outcome of a two-stage mechanism can be implemented as a PBE outcome of a generalized revelation mechanism in which truthelling in both stages constitutes an equilibrium strategy.
The Generalized Two-stage Groves Mechanism

\((\Theta, \Pi, x^*, t^G)\)

For each agent \(i\), each type report \((\theta^r_i, \theta^r_{-i}) \in \Theta_i \times \Theta_{-i}\) and each payoff report \((u^r_i, u^r_{-i}) \in \Pi_i \times \Pi_{-i}\),

\[ t^G_i(\theta^r_i, \theta^r_{-i}; u^r_i, u^r_{-i}) = u^r_{-i} - h_i(\theta^r_i, \theta^r_{-i}) \]

where

\[
\begin{align*}
    h_i(\theta^r_i, \theta^r_{-i}) &= \frac{1}{2} \left[ \sum_{j=1}^{2} u_j(x^*(\theta^r), \theta^r) - \mathbb{E}_{-i} \left( \sum_{j=1}^{2} u_j(x^*(\theta^r_i, \theta_{-i}), \theta^r_i, \theta_{-i}) \right) \\
    &\quad + \mathbb{E}_{-(i+1)} \left( \sum_{j=1}^{2} u_j(x^*(\theta^r_{i+1}, \theta_{-(i+1)}), \theta^r_{i+1}, \theta_{-(i+1)}) \right) \right]
\end{align*}
\]

with \(\mathbb{E}_{-i}\) being the expectation operator over \(\theta_{-i}\) and \(\mathbb{E}_{-3} = \mathbb{E}_{-1}\).
One-sided asymmetric information

Example in Myerson’s textbook (1991, page 489):

Note that it is always efficient to trade, i.e., $x^*(\theta_{11}) = x^*(\theta_{12}) = 1$. 
Single-stage mechanisms fails.

Myerson (1991) verifies that in this example, no single-stage direct mechanism \((x^*, t)\) satisfies BIC, IIR, EFF and BB.

\[
IC_{\theta_{11} \rightarrow \theta_{12}} : 40 \left( 1 - x^*(\theta_{11}) \right) + t_1(\theta_{11}) \geq 40 \left( 1 - x^*(\theta_{12}) \right) + t_1(\theta_{12}); \\
IC_{\theta_{12} \rightarrow \theta_{11}} : 20 \left( 1 - x^*(\theta_{12}) \right) + t_1(\theta_{12}) \geq 20 \left( 1 - x^*(\theta_{11}) \right) + t_1(\theta_{11}).
\]

Since \(x^*(\theta_{11}) = x^*(\theta_{12}) = 1\), then BIC implies \(t_1(\theta_{11}) = t_1(\theta_{12})\).

\[
IR_{\theta_{11}} : 40 \left( 1 - x^*(\theta_{11}) \right) + t_1(\theta_{11}) \geq 40 \Rightarrow t_1(\theta_{11}) \geq 40; \\
IR_{\theta_{12}} : 20 \left( 1 - x^*(\theta_{12}) \right) + t_1(\theta_{12}) \geq 20 \Rightarrow t_1(\theta_{11}) \geq 20.
\]

Then, seller’s IIR constraints imply \(t_1(\theta_{11}) \geq 40\).

\[
IR_{\theta_2} : 0.2 \left( 50x^*(\theta_{11}) + t_2(\theta_{11}) \right) + 0.8 \left( 30x^*(\theta_{12}) + t_2(\theta_{12}) \right) \geq 0.
\]

Finally, BB requires \(t_2(\theta_{11}) = -t_1(\theta_{11})\) and \(t_2(\theta_{12}) = -t_1(\theta_{12})\); then, buyer’s IIR implies \(t_1(\theta_{11}) \leq 34\), a contradiction.
The generalized two-stage Groves mechanism succeeds.

Claim 1

*In Example 1, the generalized two-stage Groves mechanism* $(\Theta, \Pi, x^*, t^G)$ *satisfies BIC, IIR, EFF and BB simultaneously.*
The generalized two-stage Groves mechanism succeeds.

Proof: For each $\theta_1^r \in \Theta_1$ and each $(u_1^r, u_2^r) \in \Pi_1 \times \Pi_2$,

$$t_1^G(\theta_1^r; u_1^r, u_2^r) = u_2^r - \frac{1}{2} \left[ \sum_{j=1}^{2} u_j (x^*(\theta_1^r), \theta_1^r) - \mathbb{E}_2 \left( \sum_{j=1}^{2} u_j (x^*(\theta_1^r), \theta_1^r) \right) + \mathbb{E}_1 \left( \sum_{j=1}^{2} u_j (x^*(\theta_1), \theta_1) \right) \right]$$

$$= u_2^r - \frac{1}{2} \mathbb{E}_1 (\tilde{u}_2(\theta_1)) \text{ (because } \forall \theta_1, x^*(\theta_1) = 1)$$

$$= u_2^r - 17$$

and

$$t_2^G(\theta_1^r; u_1^r, u_2^r) = u_1^r - \frac{1}{2} \left[ \sum_{j=1}^{2} u_j (x^*(\theta_1^r), \theta_1^r) - \mathbb{E}_1 \left( \sum_{j=1}^{2} u_j (x^*(\theta_1), \theta_1) \right) + \mathbb{E}_2 \left( \sum_{j=1}^{2} u_j (x^*(\theta_1^r), \theta_1^r) \right) \right]$$

$$= u_1^r - \tilde{u}_2(\theta_1^r) + \frac{1}{2} \mathbb{E}_2 (\tilde{u}_2(\theta_1)) \text{ (because } \forall \theta_1, x^*(\theta_1) = 1)$$

$$= u_1^r - \tilde{u}_2(\theta_1^r) + 17.$$

Note that $t_1^G$ is independent of $u_1^r$, and $t_2^G$ is independent of $u_2^r$. 
The generalized two-stage Groves mechanism succeeds.

Proof (Cont’d): Suppose seller reports $\theta_1^r$ instead of his true type $\theta_1$ and each agent reports the true allocation payoff. Then seller receives the following utility:

$$u_1(x^*(\theta_1^r), \theta_1) + t_1^G(\theta_1^r; u_1(x^*(\theta_1^r), \theta_1), u_2(x^*(\theta_1^r), \theta_1)) = u_1(x^*(\theta_1^r), \theta_1) + u_2(x^*(\theta_1^r), \theta_1) - 17 (\because u_2^r = u_2(x^*(\theta_1^r), \theta_1)) = 0 + \tilde{u}_2(\theta_1) - 17 (\because \forall \theta_1, x^*(\theta_1) = 1),$$

which is independent of his first-stage report $\theta_1^r$. So, seller has no incentive to deviate and together truth-telling in both stages constitutes a PBE; hence, BIC is satisfied.
The generalized two-stage Groves mechanism succeeds.

Proof (Cont’d): BB is satisfied on equilibrium path because for each \( \theta_1 \in \Theta_1 \),

\[
\begin{align*}
t_1^G(\theta_1; u_1, u_2) + t_2^G(\theta_1; u_1, u_2) &= (u_2(x^*(\theta_1), \theta_1) - 17) + (u_1(x^*(\theta_1), \theta_1) - \tilde{u}_2(\theta_1) + 17) \\
&= (\tilde{u}_2(\theta_1) - 17) + (0 - \tilde{u}_2(\theta_1) + 17) \quad (\because \forall \theta_1, x^*(\theta_1) = 1) \\
&= 0,
\end{align*}
\]

where \( u_1 = u_1(x^*(\theta_1), \theta_1) \) and \( u_2 = u_2(x^*(\theta_1), \theta_1) \).
The generalized two-stage Groves mechanism succeeds.

Proof (Cont’d): Agents’ interim expected utility from participating in the generalized two-stage Groves mechanism are

\[ U_1^G(\theta_{11}) = u_1(x^*(\theta_{11}), \theta_{11}) + t_1^G(\theta_{11}; u_1, u_2) = \tilde{u}_2(\theta_{11}) - 17 = 33; \]
\[ U_1^G(\theta_{12}) = u_1(x^*(\theta_{12}), \theta_{12}) + t_1^G(\theta_{12}; u_1, u_2) = \tilde{u}_2(\theta_{12}) - 17 = 13; \]

and

\[ U_2^G(\bar{\theta}_2) = \mathbb{E}_1 \left[ u_2(x^*(\theta_1), \theta_1) + t_2^G(\theta_1; u_1, u_2) \right] \]
\[ = \mathbb{E}_1 \left[ u_2(x^*(\theta_1), \theta_1) + u_1(x^*(\theta_1), \theta_1) - \tilde{u}_2(\theta_1) + 17 \right] \]
\[ = \mathbb{E}_1 \left[ \tilde{u}_2(\theta_1) + 0 - \tilde{u}_2(\theta_1) + 17 \right] \quad (\because \forall \theta_1, x^*(\theta_1) = 1) \]
\[ = 17. \]

Hence,

\[ U_1^G(\theta_{11}) < U_1^O(\theta_{11}) = \tilde{u}_1(\theta_{11}) = 40; \]
\[ U_1^G(\theta_{12}) < U_1^O(\theta_{12}) = \tilde{u}_1(\theta_{12}) = 20; \]
\[ U_2^G > U_2^O = 0. \]
The generalized two-stage Groves mechanism succeeds.

Proof (Cont’d): Then, a lump-sum transfer $l$ must be imposed from buyer to seller so that everyone is better off after participation, i.e.,

\[
U_G^1(\theta_{11}) + l \geq U_O^1(\theta_{11}) \quad \Rightarrow \quad 33 + l \geq 40;
\]
\[
U_G^1(\theta_{12}) + l \geq U_O^1(\theta_{12}) \quad \Rightarrow \quad 13 + l \geq 20;
\]
\[
U_G^2 - l \geq U_O^2 \quad \Rightarrow \quad 17 - l \geq 0,
\]

hence, $7 \leq l \leq 17$. In conclusion, the generalized two-stage Groves mechanism satisfies BIC, IIR, EFF and BB.
The generalized two-stage Groves mechanism succeeds.

**Theorem 1**
When only the seller has a non-trivial set of types and the buyer has only one type, the generalized two-stage Groves mechanism \((\Theta, \Pi, x^*, t^G)\) always satisfies BIC, IIR, EFF and BB.
Two-sided asymmetric information

Example 2

- Both agents’ types are uniformly distributed on the unit interval $[0, 1]$;

- $\tilde{u}_1(\theta_1, \theta_2) = \theta_1 + \gamma_1 \theta_2$ and $\tilde{u}_2(\theta_1, \theta_2) = \theta_2 + \gamma_2 \theta_1$ where $\gamma_1, \gamma_2 > 0$. 
Two-sided asymmetric information

Example 2 (Cont’d)

Case (i): $0 < \gamma_2 \leq \gamma_1 < 1$

Case (ii): $0 < \gamma_1 < \gamma_2 < 1$

Case (iii): $0 < \gamma_2 \leq 1 \leq \gamma_1$

Case (iv): $0 < \gamma_1 < 1 < \gamma_2$
Two-sided asymmetric information

Example 2 (Cont’d)

Case (v): when $1 < \gamma_2 \leq \gamma_1$

$\theta_2 = \frac{\gamma_2 - 1}{\gamma_1 - 1} \theta_1$

Case (vi): when $1 < \gamma_1 < \gamma_2$

$\theta_2 = \frac{\gamma_2 - 1}{\gamma_1 - 1} \theta_1$
The generalized two-stage Groves mechanism fails.

Claim 2

In Example 2, the generalized two-stage Groves mechanism $(\Theta, \Pi, x^*, t^G)$ violates IIR in all cases.

Remark

In Example 2, the economy as a whole is worse off after participation; hence, it is impossible to make everyone better off through welfare redistribution.
Two-stage monotone mechanisms

Definition 2
A two-stage mechanism \((\Theta, \Pi, x^*, t)\) is monotone if the following properties are satisfied:

1. \(t_2(\theta_1^r, \theta_2^r; u_1^r, u_2^r) \leq 0\) for all \((\theta_1^r, \theta_2^r)\) and \((u_1^r, u_2^r)\);
2. if \(x^*(\theta_1^r, \theta_2^r) = 1\), then \(|t_2(\theta_1^r, \theta_2^r; u_1^r, u_2^r)| \leq \tilde{u}_2(\theta_1^r, \theta_2^r)\).
3. if \(\hat{\theta}_2^r > \theta_2^r\) and \(x(\theta_1^r, \hat{\theta}_2^r) = x(\theta_1^r, \theta_2^r) = 1\), then
   \(|t_2(\theta_1^r, \hat{\theta}_2^r; u_1^r, u_2^r)| > |t_2(\theta_1^r, \theta_2^r; u_1^r, u_2^r)|\).
Two-stage monotone mechanisms

Claim 3

In Example 2, the generalized two-stage Groves mechanism $(\Theta, \Pi, x^*, t^G)$ is not monotone.

Remark

In the generalized two-stage Groves mechanism, either buyer receives subsidies or buyer’s payment is not strictly increasing in buyer’s type report.
Two-stage monotone mechanisms succeed in Case (i) and (iii).

Claim 4

In Example 2, there exists a two-stage monotone mechanism satisfying BIC, IIR, EFF and BB in the following two cases: (i) \(0 < \gamma_2 \leq \gamma_1 < 1\); (iii) \(0 < \gamma_2 \leq 1 \leq \gamma_1\); in all the other cases, two-stage monotone mechanisms violate BIC.
Two stage monotone mechanisms succeed in Case (i) and (iii).

Recall

Case (i): $0 < \gamma_2 \leq \gamma_1 < 1$

Case (iii): $0 < \gamma_2 \leq 1 \leq \gamma_1$

\[
\theta_2 = \frac{1 - \gamma_2}{1 - \gamma_1} \cdot \theta_1
\]
Two-stage monotone mechanisms succeed in Case (i).

Proof: Case (i): $0 < \gamma_2 \leq \gamma_1 < 1$

Consider the following mechanism $(\Theta, \Pi, x^*, t^S)$:

$$t^S_1(\theta^r_1, \theta^r_2; u^r_1, u^r_2) = \begin{cases} u^r_2 & \text{if } x^*(\theta^r_1, \theta^r_2) = 1 \text{ and } u^r_2 = u_2(x^*(\theta^r_1, \theta^r_2), \theta_1, \theta_2) \\ -\psi & \text{if } x^*(\theta^r_1, \theta^r_2) = 1 \text{ and } u^r_2 \neq u_2(x^*(\theta^r_1, \theta^r_2), \theta_1, \theta_2) \\ 0 & \text{if } x^*(\theta^r_1, \theta^r_2) = 0 \end{cases}$$

and

$$t^S_2(\theta^r_1, \theta^r_2; u^r_1, u^r_2) = \begin{cases} -u_2(x^*(\theta^r_1, \theta^r_2), \theta^r_1, \theta^r_2) & \text{if } x^*(\theta^r_1, \theta^r_2) = 1 \\ 0 & \text{if } x^*(\theta^r_1, \theta^r_2) = 0 \text{ and } u^r_1 = u_1(x^*(\theta^r_1, \theta^r_2), \theta_1, \theta_2) \\ -\psi & \text{if } x^*(\theta^r_1, \theta^r_2) = 0 \text{ and } u^r_1 \neq u_1(x^*(\theta^r_1, \theta^r_2), \theta_1, \theta_2) \end{cases}$$

where $\psi > 0$. It is monotone. If each agent reports the truth in both stages, then

1. if $x^*(\theta_1, \theta_2) = 0$, $t^S_1(\theta_1, \theta_2; u_1, u_2) = t^S_2(\theta_1, \theta_2; u_1, u_2) = 0$;

2. if $x^*(\theta_1, \theta_2) = 1$, $t^S_1(\theta_1, \theta_2; u_1, u_2) = -t^S_2(\theta_1, \theta_2; u_1, u_2) = u_2(x^*(\theta_1, \theta_2); \theta_1, \theta_2) = \tilde{u}_2(\theta_1, \theta_2)$. 

Two-stage monotone mechanisms succeed in Case (i).

Proof (Cont’d):

- Since $t_1^S$ is independent of $u_1^r$ and $t_2^S$ is independent of $u_2^r$, each agent has no incentive to deviate in the second stage.
- We assume that buyer always reports the truth in the first stage and show that seller has no incentive to deviate in the first stage. Recall

Case (i): $0 < \gamma_2 \leq \gamma_1 < 1$

There are two cases: (a) $\theta_1 < (1 - \gamma_1)/(1 - \gamma_2)$; (b) $\theta_1 \geq (1 - \gamma_1)/(1 - \gamma_2)$. 

![Diagram showing the relationship between $\theta_1$, $\theta_2$, and the condition $\theta_2 = \frac{1-\gamma_2}{1-\gamma_1} \theta_1$.](image-url)
Two-stage monotone mechanisms succeed in Case (i).

Proof (Cont’d): (a) If seller’s true type is $\theta_1 < (1 - \gamma_1)/(1 - \gamma_2)$:

- his expected utility under truthtelling is

$$\int_{0}^{\frac{1-\gamma_2}{1-\gamma_1} \theta_1} (\tilde{u}_1(\theta_1, \theta_2) + 0) \, d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1} \theta_1}^{1} (0 + \tilde{u}_2(\theta_1, \theta_2)) \, d\theta_2.$$

- If he deviates to $0 < \theta'_1 < (1 - \gamma_1)/(1 - \gamma_2)$, his expected utility becomes

$$\int_{0}^{\frac{1-\gamma_2}{1-\gamma_1} \theta'_1} (\tilde{u}_1(\theta_1, \theta_2) + 0) \, d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1} \theta'_1}^{1} (0 - \psi) \, d\theta_2.$$

because if trade occurs, buyer’s second-stage report becomes $u'_2 = u'_2(x^*(\theta'_1, \theta_2), \theta_1, \theta_2) = \tilde{u}_2(\theta_1, \theta_2) \neq \tilde{u}_2(\theta'_1, \theta_2)$

- Since $\psi > 0$, seller’s highest expected utility after deviation is $\int_{0}^{1} \tilde{u}_1(\theta_1, \theta_2) \, d\theta_2$. However, it is still lower than truthtelling.
Two-stage monotone mechanisms succeed in Case (i).

Proof (Cont’d): (a) If seller’s true type is $\theta_1 < (1 - \gamma_1)/(1 - \gamma_2)$:

- if seller deviates to $\theta_1^r > (1 - \gamma_1)/(1 - \gamma_2)$, trade never occurs and seller’s expected utility becomes

$$\int_0^1 (\tilde{u}_1(\theta_1, \theta_2) + 0) \, d\theta_2,$$

which is lower than truth-telling.

- In conclusion, seller has no incentive to deviate when his true type is $\theta_1 < (1 - \gamma_1)/(1 - \gamma_2)$. 

Two-stage monotone mechanisms succeed in Case (i).

Proof (Cont’d): (b) If seller’s true type is $\theta_1 > (1 - \gamma_1)/(1 - \gamma_2)$,

- his expected utility under truthtelling is
  $$\int_0^1 (\tilde{u}_1(\theta_1, \theta_2) + 0) \, d\theta_2.$$  
- if he deviates to $(1 - \gamma_1)/(1 - \gamma_2) < \theta_1^r < 1$, trade never occur and seller obtains the same expected utility.
- if he deviates to $0 < \theta_1^r < (1 - \gamma_1)/(1 - \gamma_2)$, his expected utility becomes
  $$\int_0^{\frac{1-\gamma_2}{1-\gamma_1} \theta_1^r} (\tilde{u}_1(\theta_1, \theta_2) + 0) \, d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1} \theta_1^r}^1 (0 - \psi) \, d\theta_2,$$

because if trade occurs, buyer’s second-stage report becomes $u_2^r = u_2^r(x^*(\theta_1^r, \theta_2), \theta_1, \theta_2) = \tilde{u}_2(\theta_1, \theta_2) \neq \tilde{u}_2(\theta_1^r, \theta_2)$. Since $\psi > 0$, it is always lower than truthtelling.

- In conclusion, seller has no incentive to deviate.
Two-stage monotone mechanisms succeed in Case (i).

Proof (Cont’d):

► We assume that seller always reports the truth in the first stage and show that buyer has no incentive to deviate in the first stage.

► If buyer reports his true type $\theta_2$, his expected utility is

$$\int_0^{\frac{1-\gamma_1}{1-\gamma_2} \theta_2} (\tilde{u}_2(\theta_1, \theta_2) - \tilde{u}_2(\theta_1, \theta_2)) \, d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2} \theta_2}^1 (0 + 0) \, d\theta_1 = 0.$$ 

► If buyer deviates to $\theta_2^r \neq \theta_2$, his expected utility becomes

$$\int_0^{\frac{1-\gamma_1}{1-\gamma_2} \theta_2^r} (\tilde{u}_2(\theta_1, \theta_2) - \tilde{u}_2(\theta_1, \theta_2^r)) \, d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2} \theta_2^r}^1 (0 - \psi) \, d\theta_1$$

$$= \int_0^{\frac{1-\gamma_1}{1-\gamma_2} \theta_2^r} (\theta_2 - \theta_2^r) \, d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2} \theta_2^r}^1 (0 - \psi) \, d\theta_1,$$

because if no trade occurs, seller’s second-stage report becomes $u_1^r = u_1(x^*(\theta_1, \theta_2^r), \theta_1, \theta_2) = \tilde{u}_1(\theta_1, \theta_2) \neq \tilde{u}_1(\theta_1, \theta_2^r).$
Two-stage monotone mechanisms succeed in Case (i).

- Recall that if buyer deviates to \( \theta_2^r \neq \theta_2 \), his expected utility becomes
  \[
  \int_0^{\frac{1-\gamma_1}{1-\gamma_2}} \theta_2^r (\theta_2 - \theta_2^r) d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2}}^1 (0 - \psi) d\theta_1.
  \]

- Buyer will not deviate to \( \theta_2^r = \theta_2^{\text{max}} = 1 \), because his expected utility becomes negative which is worse than truth-telling.

- To stop buyer from deviating, the penalty \( \psi \) must be large enough, that is, for any \( 0 \leq \theta_2 \leq 1 \) and \( 0 \leq \theta_2^r < 1 \),
  \[
  0 \geq \int_0^{\frac{1-\gamma_1}{1-\gamma_2}} \theta_2^r (\theta_2 - \theta_2^r) d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2}}^1 (0 - \psi) d\theta_1
  \]

  \[
  \Rightarrow \psi \geq \frac{(1 - \gamma_1)(\theta_2 - \theta_2^r)\theta_2^r}{(1 - \gamma_2) - (1 - \gamma_1)\theta_2^r}.
  \]

- It suffices to set
  \[
  \psi \geq \frac{1 - \gamma_1}{\gamma_1 - \gamma_2}.
  \]
Two-stage monotone mechanisms succeed in Case (i).

- BB is satisfied because on equilibrium path,
  - if \( x^*(\theta_1, \theta_2) = 0 \), then \( t^S_1(\theta_1, \theta_2; u_1, u_2) = t^S_2(\theta_1, \theta_2; u_1, u_2) = 0 \);
  - if \( x^*(\theta_1, \theta_2) = 1 \), then \( t^S_1(\theta_1, \theta_2; u_1, u_2) = -t^S_2(\theta_1, \theta_2; u_1, u_2) = u_2(x^*(\theta_1, \theta_2); \theta_1, \theta_2) = \tilde{u}_2(\theta_1, \theta_2) \).

- Seller obtains a higher expected utility after participation than the outside option because for all \( \theta_1 \in \Theta_1 \),
  \[
  \int_0^{\frac{1-\gamma_2}{1-\gamma_1}} \theta_1 \left( \tilde{u}_1(\theta_1, \theta_2) + 0 \right) d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1}}^1 \left( 0 + \tilde{u}_2(\theta_1, \theta_2) \right) d\theta_2 \\
  > \int_0^1 \tilde{u}_1(\theta_1, \theta_2) d\theta_2 \\
  = U^{O}_1(\theta_1).
  \]

- Buyer is indifferent between participation and outside option because his expected utility after participation is zero.

- Therefore, IIR is also satisfied.
Two-stage monotone mechanisms succeed in Case (iii).

Case (iii): $0 < \gamma_2 \leq 1 \leq \gamma_1$

▶ We use the same mechanism $(\Theta, \Pi, x^*, t^S)$ as in Case (i).

▶ Recall 

Case (iii): $0 < \gamma_2 \leq 1 \leq \gamma_1$

- Since $t_1^S$ is independent of $u_1^r$ and $t_2^S$ is independent of $u_2^r$, each agent has no incentive to deviate in the second stage.
Two-stage monotone mechanisms succeed in Case (iii).

- We assume that buyer always reports truthfully in the first stage and show that seller has no incentive to deviate.
- If seller reports his true type $\theta_1$, his expected utility is
  \[ \int_0^1 (\tilde{u}_1(\theta_1, \theta_2) + 0) \, d\theta_2. \]
- If he deviates, it is still efficient not to trade and his expected utility is the same.
- Hence, seller has no incentive to deviate.
Two-stage monotone mechanisms succeed in Case (iii).

▶ We assume that seller always reports truthfully in the first stage and show that buyer has no incentive to deviate.

▶ If buyer reports his true type $\theta_2$, his expected utility is zero because it is efficient not to trade and he pays nothing.

▶ If buyer deviates to $\theta_r^2 \neq \theta_2$, buyer’s expected utility becomes

$$\int_0^1 (0 - \psi) d\theta_1 = -\psi < 0,$$

because trade never occurs and seller’s second-stage report becomes $u_1^r = u_1(x^*(\theta_1, \theta_r^2), \theta_1, \theta_2) = \tilde{u}_1(\theta_1, \theta_2) \neq \tilde{u}_1(\theta_1, \theta_r^2)$.

▶ Hence, buyer has no incentive to deviate in the first stage and BIC is satisfied.
Two-stage monotone mechanisms succeed in Case (iii).

- BB is satisfied because no trade, no pay.
- IIR is satisfied because everyone’s expected utility is the same as the outside option.
Two-stage monotone mechanisms violate BIC in Case (ii).

- We assume that seller always reports the true type in the first stage and both agents report their allocation payoffs truthfully in the second stage. Recall

\[
\text{Case (ii): } 0 < \gamma_1 < \gamma_2 < 1
\]

- If buyer’s true type is \((1 - \gamma_2)/(1 - \gamma_2) \leq \theta_2 \leq 1\), buyer obtains the following expected utility under truthtelling:

\[
U_2(\theta_2; \theta_2) = \int_0^1 (\tilde{u}_2(\theta_1, \theta_2) + t_2(\theta_1, \theta_2; u_1, u_2)) \, d\theta_1.
\]
Two-stage monotone mechanisms violate BIC in Case (ii).

- If he deviates to $(1 - \gamma_2)/(1 - \gamma_2) \leq \theta_2^r < \theta_2$, his expected utility becomes the following:

  $$U_2(\theta_2; \theta_2^r) = \int_0^1 (\tilde{u}_2(\theta_1, \theta_2) + t_2(\theta_1, \theta_2^r; u_1, u_2)) d\theta_1,$$

- By monotonicity,

  $$|t_2(\theta_1, \theta_2; u_1, u_2)| > |t_2(\theta_1, \theta_2^r; u_1, u_2)|,$$

  or equivalently,

  $$t_2(\theta_1, \theta_2; u_1, u_2) < t_2(\theta_1, \theta_2^r; u_1, u_2) \leq 0.$$

- Therefore,

  $$U_2(\theta_2; \theta_2^r) = \int_0^1 (\tilde{u}_2(\theta_1, \theta_2) + t_2(\theta_1, \theta_2^r; u_1, u_2)) d\theta_1$$

  $$> \int_0^1 (\tilde{u}_2(\theta_1, \theta_2) + t_2(\theta_1, \theta_2; u_1, u_2)) d\theta_1$$

  $$= U_2(\theta_2; \theta_2).$$

leading to a contradiction against BIC.
Two-stage monotone mechanisms violate BIC in Case (v).

- We assume that seller always reports the true type in the first stage and both agents report their allocation payoffs truthfully in the second stage. Recall

\[ \text{Case (v): } 1 < \gamma_2 \leq \gamma_1 \]

- If buyer’s true type is \((\gamma_2 - 1)/(\gamma_1 - 1) \leq \theta_2 \leq 1\), buyer obtains the following expected utility under truthtelling:

\[
\int_0^1 (0 + t_2(\theta_1, \theta_2; u_1, u_2)) \, d\theta_1 \leq 0,
\]

by monotonicity.
Two-stage mechanisms violate BIC in Case (v).

- If buyer deviates to $\theta_2^r = 0$, it is always efficient to trade and buyer’s expected utility becomes

$$\int_0^1 (\tilde{u}_2(\theta_1, \theta_2) + t_2(\theta_1, \theta_2^r; u_1^r, u_2^r)) \, d\theta_1$$

$$> \int_0^1 (\tilde{u}_2(\theta_1, \theta_2^r) + t_2(\theta_1, \theta_2^r; u_1^r, u_2^r)) \, d\theta_1$$

($\because \theta_2 > \theta_2^r$ and $\tilde{u}_2$ is a strictly increasing function.)

$$\geq \int_0^1 (\tilde{u}_2(\theta_1, \theta_2^r) - \tilde{u}_2(\theta_1, \theta_2^r)) \, d\theta_1$$

($\because x^*(\theta_1, \theta^r) = 1$ implies $t_2(\theta_1, \theta_2^r; u_1^r, u_2^r) \geq -\tilde{u}_2(\theta_1, \theta_2^r)$

$$= 0;$$

hence, buyer obtains a higher expected utility after deviation and BIC is violated.
The general results in two-sided asymmetric information

Assumption 1
\[ \int_{\Theta_1} x^*(\theta_1, \theta_2) dF_1(\theta_1) < 1 \text{ for all } \theta_2 < \theta_2^{\max}. \]

Theorem 3
When both agents have non-trivial sets of types, there exists a two-stage monotone mechanism satisfying BIC, IIR, EFF and BB if and only if Assumption 1 is satisfied.
Assumption 1 is satisfied in Case (i) and (iii) in Example 2.

Recall that in Case (i) and (iii), there exists a two-stage monotone mechanism satisfying BIC, IIR, EFF and BB.

Case (i): \(0 < \gamma_2 \leq \gamma_1 < 1\)

Case (iii): \(0 < \gamma_2 \leq 1 \leq \gamma_1\)
Assumption 1 is violated in the other cases in Example 2.

Case (ii): $0 < \gamma_1 < \gamma_2 < 1$

Case (iv): $0 < \gamma_1 < 1 < \gamma_2$

Case (v): when $1 < \gamma_2 \leq \gamma_1$

Case (vi): when $1 < \gamma_1 < \gamma_2$
How restrictive is Assumption 1?
Consider linear valuation function \( u_i(\theta_i, \theta_{-i}) = \theta_i + \gamma_i \theta_{-i} \) where \( \gamma_i > 0 \). Then

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Different cases</th>
<th>Is Assumption 1 satisfied?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \gamma_1 &lt; 1, \gamma_2 &lt; 1, ) and ( (1 - \gamma_2)/(1 - \gamma_1) \geq \theta_2^{max}/\theta_1^{max} )</td>
<td>✓</td>
</tr>
<tr>
<td>2</td>
<td>( \gamma_1 &lt; 1, \gamma_2 &lt; 1, ) and ( (1 - \gamma_2)/(1 - \gamma_1) &lt; \theta_2^{max}/\theta_1^{max} )</td>
<td>✗</td>
</tr>
<tr>
<td>3</td>
<td>( \gamma_1 \geq 1 ) and ( \gamma_2 \leq 1 )</td>
<td>✓</td>
</tr>
<tr>
<td>4</td>
<td>( \gamma_1 &lt; 1 ) and ( \gamma_2 &gt; 1 )</td>
<td>✗</td>
</tr>
<tr>
<td>5</td>
<td>( \gamma_1 &gt; 1, \gamma_2 &gt; 1, ) and ( \theta_1^{min} = 0 )</td>
<td>✓</td>
</tr>
<tr>
<td>6</td>
<td>( \gamma_1 &gt; 1, \gamma_2 &gt; 1, ) ( \theta_1^{min} &gt; 0 ) and ( (\gamma_2 - 1)/(\gamma_1 - 1) &lt; \theta_2^{min}/\theta_1^{min} )</td>
<td>✓</td>
</tr>
<tr>
<td>7</td>
<td>( \gamma_1 &gt; 1, \gamma_2 &gt; 1, ) ( \theta_1^{min} &gt; 0 ) and ( (\gamma_2 - 1)/(\gamma_1 - 1) \geq \theta_2^{min}/\theta_1^{min} )</td>
<td>✗</td>
</tr>
</tbody>
</table>
What if Assumption 1 is violated?

▷ Does there exist a two-stage non-monotone mechanism satisfying BIC, IIR, EFF and BB? Yes!

▷ Example: \( \tilde{u}_1(\theta_1, \theta_2) = \theta_1 + 0.5\theta_2 \) and \( \tilde{u}_2(\theta_1, \theta_2) = \theta_2 + 3\theta_1 \) for all \((\theta_1, \theta_2) \in [0, 1]^2\).

▷ Note that \( \tilde{u}_2(\theta_1, \theta_2) - \tilde{u}_1(\theta_1, \theta_2) = 0.5\theta_2 + 2\theta_1 \geq 0 \) for all \((\theta_1, \theta_2) \in \Theta_1 \times \Theta_2\). Hence, Assumption 1 is violated.

▷ There exists a two-stage mechanism with the fixed-payment scheme \( \bar{t}_1 = -\bar{t}_2 = 1.25 \) satisfying BIC, IIR, EFF and BB.
Concluding Remarks

- Under one-sided asymmetric information structure, the generalized two-stage Groves mechanism always satisfies IIR.
- Under two-sided asymmetric information structure,
  - we show by an example that it never satisfies IIR;
  - we propose the two-stage monotone mechanisms which satisfy IIR in a positive number of cases within the same example;
  - we characterize the existence of two-stage monotone mechanisms satisfying BIC, IIR, EFF and BB.