Productivity Investment, Power Law, and Welfare Gains from Trade

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Motivation

- In the trade literature, productivity distributions are mostly *exogenously given*.
- Trade only affects which parts of a productivity distribution are utilized via:
  - Firm selection, e.g., Melitz (2003).
  - Comparative advantage, e.g., Eaton and Kortum (2002).
- Empirically, trade liberalization affects firm-level productivities:
  - Improves within-plant productivity, and the effect is stronger for larger firms.
  - Competition: Pavcnik (2002) and Fernandes (2007).
  - Better market access: Aghion, Bergeaud, Lequien and Melitz (2018), and Bustos (2011).

This Paper

- studies a Melitz model with an additional stage where firms invest to determine their productivities.
- shows a general environment where power laws in productivity and firm size emerge.
- investigates the effects of productivity investment on the productivity distribution and welfare gains from trade.

In This Paper

- Productivity depends on both *luck/talent* and *effort*.
- Trade affects the later.
- We incorporate Sutton's (1991) idea on R&D into Melitz (2003):
  - Each firm pays an entry cost to obtain a distinct product.
  - Productivity is determined by efforts in productivity investment.
  - Firms are heterogeneous in the efficiency of productivity investment (say, talent).

Main Results: Power Law

- Power law: the tail probability follows a power function:

$$\Pr(S > s) \propto s^{-\zeta}, \text{ for } \zeta{>}0 \text{and large } s.$$

  Pareto distribution: power law holds for all $s > 0$.
- Empirical evidence: Axtell (2001) and Luttmer (2007).
- *Power laws in both productivity and firm size emerges when* both the demand and investment cost functions are *regularly varying*.
  - Greatly relaxes the model class that is consistent with power law.
  - Includes various non-CES preferences.
  - *Holds for almost any underlying firm heterogeneity*.

Main Results: Trade Cost and Productivity

Trade liberalization results in:

1. Tougher selection and a larger fraction of exporters, as in Melitz (2003).
2. Exporters invest more and become more productive.
3. Non-exporters invest less and become less productive because of import competition (exporters are now better).

Main Results: Welfare Gains from Trade

Arkolakis, Costinot and Rodriguez-Clare (2012, henceforth ACR) welfare formula:

$$\text{local formula: } d \ln W = \frac{1}{\varepsilon} d \ln \lambda,$$

$$\text{global formula: } \frac{W'}{W} = \left(\frac{\lambda'}{\lambda}\right)^{\frac{1}{\varepsilon}}.$$

In our model:

- The trade elasticity ε generically varies in τ.
- The local ACR formula applies, but not the global formula.
- The welfare gains are 36% higher than Melitz-Pareto framework.
- 31.3% of our welfare gains come from productivity investment.

Basic Setting

Consumer:

- Additive separable utility function: $U = \int_{\upsilon \in \Upsilon} u(q(\upsilon))\, d\upsilon$.
- Inverse demand: $p(\upsilon) = D(q(\upsilon); A)$, where $D'(q(\upsilon); A) < 0$ and A is endogenously determined.

Producer:

- Mostly the same as Melitz (2003): Monopolistic competitive, free entry, labor is the only input...
- Total labor force / country size: N.
- Each firm draws its type t, the probability to advance through quality ladder, from a given distribution.

Basic Setting: Investment

- Productivity investment involves in a continuum of procedures, each requires a worker (RA) to conduct a series of quality enhancing experiment.
  - Quality ladder of each procedure: $\{1, 2, 3, ...\}$.
  - With probability $\gamma \equiv 1 - t \in (0, 1)$, a worker fails to advance to the next quality level.
  - $\gamma \in (0, 1)$ follows a distribution with p.d.f. $f(\gamma)$.
  - Experiment ends when fails.

Basic Setting: Investment

- Specifically, by incorperating a continuum of k procedures:

$$\begin{aligned}\varphi =& B\left(k\sum_{y=1}^{\infty}(1-\gamma)^{y-1}\gamma y\right),\\ =& B\left(\frac{k}{\gamma}\right),\end{aligned}$$

where $B' > 0$, $B'' < 0$.

- The concavity is a result of management burdern.
- It is more convenient to work with labor cost

$$k = \gamma B^{-1}(\varphi) \equiv \gamma V(\varphi),$$

where $V' > 0$ and $V'' > 0$.

Basic Setting: Timing

Start with a closed economy:

1. Entry Stage: Each firm pays κ_e to enter, and then draws its γ from a given distribution.
2. Investment Stage: Each firm decides whether to invest, and if yes, the level of φ. The labor cost of investment is $\gamma V(\varphi)$
3. Production Stage: Each firm decides whether to produce, and if yes, the price and quantity. The labor cost of production is $q/\varphi + \kappa_D$.

Backward Induction

- Operating profit from production:

$$\pi(\varphi) = pq - \varphi^{-1}q - \kappa_D = D(q; A)\, q - \varphi^{-1}q - \kappa_D.$$

- Given γ, choose φ to maximize the profit net of investment cost:

$$\Pi(\varphi) = \pi(\varphi) - \gamma V(\varphi).$$

- Free entry condition:

$$\int_0^{\gamma_D} \Pi(\widetilde{\varphi}(\gamma); \gamma)\, dF(\gamma) = \kappa_e.$$

Illustrative Example

- **CES demand**: $q = A^{\frac{1}{\sigma}} p^{-\frac{1}{\sigma}}$.
- **Power function**: $C(\varphi) = \gamma V(\varphi) = \gamma \varphi^{\beta}$
- Optimal productivity:

$$\widetilde{\varphi}(\gamma) = \left[\frac{A\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}}{\beta} \right]^{\frac{1}{\theta}} \gamma^{-\frac{1}{\theta}},$$

  where $\theta \equiv \beta - \sigma + 1 > 0$.

- Let $\widetilde{\gamma}(\varphi)$ denote the inverse function of $\widetilde{\varphi}(\cdot)$.
- Note that $\frac{\partial \widetilde{\gamma}(\varphi)}{\partial \varphi} < 0$ and $\lim_{\varphi \to \infty} \widetilde{\gamma}(\varphi) = 0$.

Illustrative Example

- $\kappa_D > 0$ implies existence of selection cutoff φ_D and therefore γ_D above which firms exit.
- Productivity distribution:

$$g(\varphi) = \frac{f(\widetilde{\gamma}(\varphi))}{F(\gamma_D)} A \frac{\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}}{\beta} \theta \varphi^{-\theta-1}.$$

- If $\lim_{\gamma \to 0} f(\gamma) = K > 0$, then

$$\frac{g(\varphi)}{\varphi^{-\theta-1}} \approx \frac{K}{F(\gamma_D)} A \frac{\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}}{\beta} \theta.$$

- Special case: uniform γ results in Pareto φ.

Smooth Variation

Definition

Definition 1. A function $v(x)$ is a *regularly varying function* if and only if there is some $\alpha \in \mathbb{R}$ such that $v(x)$ can be expressed as

$$v(x) = x^{\alpha} l(x),$$

where $l(x)$ is a slowly varying function, i.e., for any $\zeta > 0$,

$$\lim_{x \to \infty} \frac{l(\zeta x)}{l(x)} = 1.$$

Definition

Definition 2. A *smoothly varying function* is a infinitely differentiable regularly varying function $v(x)$, such that for all $n \geq 1$

$$\lim_{x \to \infty} \frac{x^n v^{(n)}(x)}{v(x)} = \alpha(\alpha - 1) \ldots (\alpha - n + 1),$$

where $v^{(n)}(x)$ denotes for the n-th derivative of $v(x)$.

Smooth Variation

Assumption 1:

1. The inverse demand for each variety is a smoothly varying function $p = D(q; A) \equiv q^{-\frac{1}{\sigma}} Q(q; A)$, where $\sigma > 1$.
2. The investment cost is a smoothly varying function $c(\varphi) = \gamma V(\varphi) \equiv \gamma \varphi^{\beta} L(\varphi)$, where $\beta > 1$.
3. There exist positive constants C_Q and C_L such that $\lim_{q \to \infty} Q(q; A) = C_Q$ and $\lim_{\varphi \to \infty} L(\varphi) = C_L$.

- Greatly extended the model class from CES.
- Includes many demand functions, e.g., bipower demand in Mrazova and Neary 2017 and CREMR in Mrazova, Neary and Parenti 2017.

Equilibrium Existence

- For equilibrium existence, we assume:
  Assumption 2: $\theta \equiv \beta - \sigma + 1 > \sigma - 1$ and the inverse demand function D is such that $\lim_{q \to \underline{q}} s(q) = D(q; A)\, q < \infty$.
- The optimal productivity $\varphi^* = \widetilde{\varphi}(\gamma) \equiv \widetilde{\gamma}^{-1}(\gamma)$ exists, which is decreasing in γ and such that $\lim_{\varphi \to \infty} \widetilde{\gamma}(\varphi) = 0$.

Power Law of Productivity

Proposition

Under Assumptions 1 and 2, if

$$\lim_{\gamma \to 0} f(\gamma) = K > 0,$$

then the distributions of productivity φ *and firm size* $s \equiv pq$ *are approximately*

$$g(\varphi) \approx \frac{K}{F(\gamma_D)} \frac{C_Q^{\sigma}}{C_L} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} \frac{\theta}{\beta} \varphi^{-\theta - 1},$$

$$g(s) \approx \frac{K}{F(\gamma_D)} \frac{C_Q^{\frac{\beta}{\rho}}}{C_L} \left(\frac{\sigma - 1}{\sigma}\right)^{\beta} \frac{\theta}{\beta \sigma} s^{-\frac{\theta}{\sigma - 1} - 1}.$$

$n+1$ (Asymmetric) Countries

Easily extended to $n+1$ asymmetric country case:

Proposition

Under Assumptions 1 and 2, suppose that $\theta_{ij} \equiv \beta_i + 1 - \sigma_j > 0$ for all $(i, j) \in \{0, 1, 2, ..., n\}$, if

$$\lim_{\gamma \to 0} f_i(\gamma) = K_i > 0,$$

then the productivity distribution of aribitrary country i satisfies power law with a tail index $\min\{\theta_{i0}, ..., \theta_{in}\}$.

The tail index is determined by the country with least product differentiation (highest σ).

The Effects of Trade Costs on Productivity Distribution

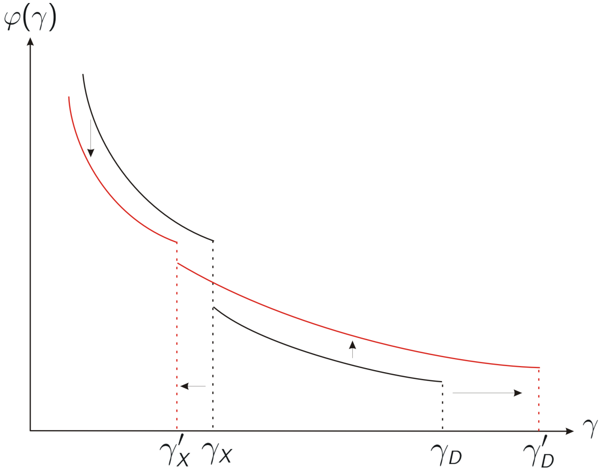
- For tractability we consider the following settings:
  - $n+1$ symmetric countries (wages are normalized to 1).
  - CES utility.
  - Power function investment cost.
- Optimal productivities:

$$\tilde{\varphi}(\gamma) = \begin{cases} \kappa_D^{\frac{1}{\beta}} \gamma_D^{\frac{\sigma-1}{\beta\theta}} \left(\frac{\theta}{\sigma-1}\right)^{-\frac{1}{\beta}} \gamma^{-\frac{1}{\theta}} & \text{if } \gamma \in (\gamma_X, \gamma_D] \\ \phi \kappa_D^{\frac{1}{\beta}} \gamma_D^{\frac{\sigma-1}{\beta\theta}} \left(\frac{\theta}{\sigma-1}\right)^{-\frac{1}{\beta}} \gamma^{-\frac{1}{\theta}} & \text{if } \gamma \in [0, \gamma_X] \end{cases},$$

where $\phi \equiv \left(1 + n\tau^{1-\sigma}\right)^{\frac{1}{\theta}} > 1$ denotes the productivity advantage of exporters.

The Effect of Variable Trade Cost

Figure: The Effect of Increasing τ



Welfare Gains from Trade

- Our model does not directly fit ACR framework, as the technological choice in their model is multiplicative to other components of marginal cost (trade cost, wages, exogenously given component of productivity).
- For a **general distribution** of γ, $F(.)$, the welfare gains from trade follows the local ACR formula:

$$\frac{d \ln W}{d \ln \tau} = \frac{1}{\varepsilon} \frac{d \ln \lambda}{d \ln \tau} = \lambda - 1,$$

  where the trade elasticity ε is generally a function of τ, and λ is the domestic expenditure share.
- If $\kappa_X = 0$ so all firms export, the elasticity becomes a constant

$$\varepsilon^{\kappa_X = 0} = 1 - \sigma < 0.$$

Welfare Gains from Trade

- To highlight the role of productivity investment, we will compare with Melitz-Pareto:
  - $g^{MP}(\varphi) = \theta^{MP} \varphi^{-\theta^{MP}-1}$ where $\theta^{MP} > \sigma - 1$ is exogenously given and equals the trade elasticity.
- We focus on uniformly distributed γ: the resulting distribution of φ is piecewise Pareto.

Comparison with Melitz-Pareto

Price index of our model (PI):

$$P^{1-\sigma} = M_e \left[\int_{\gamma_X}^{\gamma_D} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \widetilde{\varphi}(\gamma)^{\sigma-1} \, dF(\gamma) + \int_0^{\gamma_X} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \widetilde{\varphi}(\gamma)^{\sigma-1} \, dF(\gamma) \right]$$
$$+ nM_e \int_0^{\gamma_X} \tau^{1-\sigma} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \widetilde{\varphi}(\gamma)^{\sigma-1} \, dF(\gamma).$$

Price index of Melitz-Pareto (MP):

$$\left(P^{MP}\right)^{1-\sigma} = M_e^{MP} \int_{\varphi_D^{MP}}^{\infty} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \varphi^{\sigma-1} dG(\gamma)$$
$$+ nM_e \int_{\varphi_X^{MP}}^{\infty} \tau^{1-\sigma} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \varphi^{\sigma-1} dG(\gamma).$$

Quantitative Analysis

Calibration:

- Set $\sigma \approx 4.33$ using the median of price markups in the US documented by Feenstra and Weinstein (2017).
- Using US data from Penn World Table 9.0, $\lambda \approx 0.853$ and $n = 3$.
- Set trade elasticity $\varepsilon = 4.63$ following Simonovska and Waugh (2014). This implies that $\theta^{MP} = 4.63$.
- Set $\delta \equiv \gamma_X / \gamma_D = 0.18$ following Bernard, Jensen, Redding, and Schott (2007).
- Calibrate $(\tau, \beta, \kappa_X / \kappa_D)$ to match $(\lambda, \varepsilon, \delta)$.

Quantitative Analysis

Parameter	τ	β	$\frac{\kappa_X}{\kappa_D}$
Value	2.097	7.838	0.572

(a) Calibrated Parameters

Model	$\frac{d \ln W}{d \ln \tau}$	$\left(\frac{d \ln W}{d \ln \tau}\right)_{prod}$	$\left(\frac{d \ln W}{d \ln \tau}\right)_{dir}$	$\left(\frac{d \ln W}{d \ln \tau}\right)_{ext}$	$\frac{W}{W_{\tau \to \infty}}$
PI	-0.147	-0.046	-0.063	-0.038	1.035
MP	-0.108	NA	-0.078	-0.030	1.025

(b) Welfare Gains from Trade

Quantitative Analysis

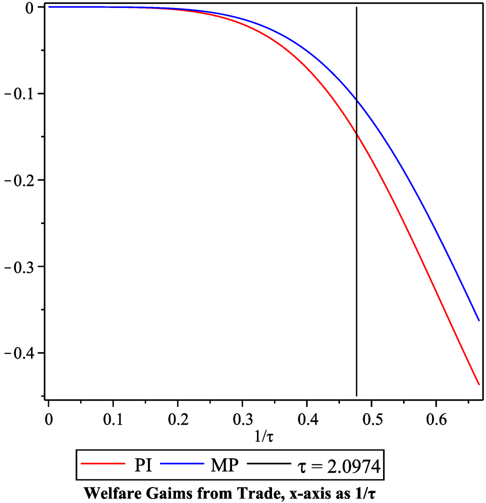


Figure: $d \ln W / d \ln \tau = \lambda - 1$

Quantitative Analysis

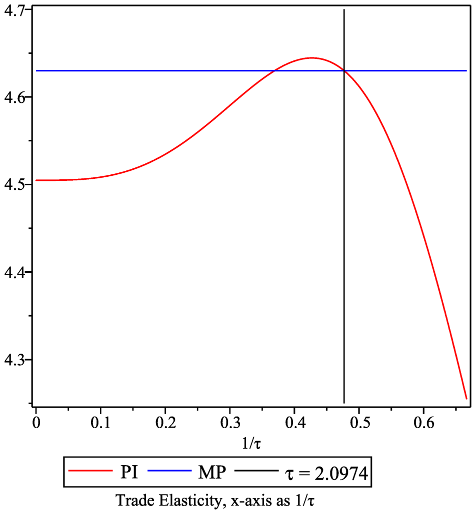


Figure: $|\varepsilon| = (1-\lambda)^{-1}\, d\ln\lambda / d\ln\tau$

Conclusion

- Using regular variation to relax the environment in which power laws emerge. Holds for arbitrary number of asymmetric countries.
- Delivers how trade affects productivity distribution.
- studies the role of productivity investment in welfare gains from trade.