Secret Sequential Advice

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Abstract

Two experts with career concerns sequentially recommend an action to a decision maker, with the second expert observing the first expert's recommendation. Should the Decision Maker (D) disclose who recommended what and when? We find that secrecy (weakly) dominates transparency in terms of better decisions. In particular: (i) only secrecy enables the second expert to, at times, partially communicate her information and its high precision level to D and swing the decision away from the first expert's recommendation; (ii) if the experts on average have high precision, then the second expert is effective only under secrecy.

We also show that the superiority of secrecy is further enhanced if either (1) the experts are allowed to revise their advice following each other's initial recommendation in a total of four stages (i.e., deliberate), or (2) the experts are allowed to make detailed recommendations (i.e., give advice and also report its quality) in a two-stage sequential game. Expanding the message space through gradualism under deliberation and under detailed recommendation, both generate the possibility of fully revealing equilibria, leading to informationally efficient choice by D. Moreover, full revelation equilibrium set is the same under either format.

JEL Classification: D82, D83, D23, C72. *Key Words:* Sequential advice, deliberations, detailed recommendation, gradualism, career concern, cheap talk, market reputation, transparency, secrecy, prior bias, lead opinion bias, conformity bias, signal revelation, beneficial herding, type revelation.

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1 Introduction

In many situations decisions are based on sequential advice given by multiple experts. Aid agencies get their field reports reviewed by specialists from the academia. Museums solicit advice from art connoisseurs before acquiring a piece of art. Patients consult a second specialist to get an opinion on a preliminary advice received for a major surgery. In many such cases, due to time constraint, the decision maker does not have the luxury of getting opinion of each expert reviewed by at least another. Furthermore, the advice provided may not be *able* to adequately transmit all the relevant information that an expert may be *willing* to transmit. This may be due to imperfections in language or comprehension. For example, a project appraisal report may not include the intricate properties of its statistical estimators. Our primary focus in this paper will be on a scenario where two experts sequentially provide cheap talk advice in two stages. The advice is constrained by the messages available.

Our secondary focus will be on two alternative procedures of advice – deliberation and detailed recommendation. Under deliberation experts have the option of revising their recommendations in light of the opinion of others.[1] Deliberations are also sequential in nature with commentators giving their views, endorsing or countering prevailing opinions. Prolonged deliberations may allow experts to potentially transfer all their information, overcoming language and comprehension imperfections. In monetary policy committees, select members debate over the inflation target or interest rate. The head of states such as the Prime Minister, or the leader of a major political party, may rely on their trusted advisors, who give their conflicting views about the suitability of important decisions in open parliamentary sessions or closed-door meetings.[2] Detailed recommendation captures the case where experts are not constrained by limitations in language or comprehension. Before undertaking large projects, firms routinely solicit *detailed* project appraisal reports from consultants. An important difference between these last two formats is that detailed recommendation requires half the number of stages than under deliberation.

We ask whether a decision maker, in order to select a decision corresponding with an unknown *state*, should make the experts' recommendations public, by disclosing who made what recommendations when (transparency), or maintain secrecy by informing the public of

[1] This is of secondary focus because the main result here is in accordance with the intuition derived from the analysis of the sequential advice model.

[2] See Jan. 4, 2010 report, "Democratic leaders plan secret health reform deliberations"; source: http://www.usnews.com/opinion/blogs/peter-roff/2010/01/04/democratic-leaders-plan-secret-health-reform-deliberations.

only the summary decision. The true state will eventually become known. One principal assumption will be that the experts have "career concerns," an idea in organizational economics introduced by Holmstrom (1999): they care mainly about the perception of outsider(s), i.e. the public, about their ability in predicting the true state. In many consulting services, often the pre-agreed payment for the specific service is fixed. But the experts giving the consulting advice improve or worsen their future earnings by creating a perception that they are good or bad experts.[3]

When the decision maker has the final authority to make a decision, he may base his decision not just on the number, but also the order of expert recommendations. Especially when experts can see and, hence, learn from each other's recommendation. In choosing to take the same decision following the same recommendations, but in different orders, the decision maker may be wasting valuable information. In this regard, sequentiality of advice introduces a new dynamic not prevalent in simultaneous decision making with an exogenous rule, e.g., in one-shot voting.

Under the two-stage recommendation process, with constrained message sets, our main result is that a Bayesian decision maker should always weakly prefer secrecy over transparency (Propositions 4 and 5). One key reason for this is that secrecy enables late recommendations to often be decisive even if early recommendations come from fairly high (average) quality experts (Corollary 3); under transparency an early opinion is more likely to stifle valuable late views (Proposition 1). In addition, only secrecy creates a positive chance for the second expert's intrinsic high ability to be communicated to the decision maker, thus helping him make better decisions (Proposition 2). When the restriction on message sets is removed we have two additional results: under secret deliberation as well as detailed recommendation, we show that at times all experts are able to reveal all their information (Proposition 6). Surprisingly, the scope of information revelation is the same under both these formats of communication.

Interactions among three principal forces shape the experts' information revelation incentives: the the prior bias, the lead opinion bias, and conformity bias. The first and second biases are exogenous in our setup and impact on the experts' beliefs about the likely state.

[3]Career concern and the motivation to impress the public need not be a distant target. For politicians (Congressmen in the USA), it could be that they are appealing to a higher level audience – heads of important committees in the House or the Senate, electorate who may win him or her a Senate seat in future election, etc. 'Skills' and 'achievements' (e.g., "sponsoring major pieces of legislation, delivering famous speeches, casting decisive votes on important issues" etc.) are reckoned to be important considerations in influencing a politician's career prospects (see Diermer, Keane and Merlo, 2005).

If one state is favored over the other in terms of the prior beliefs, then we say that there is a *prior bias*. For any given prior bias favoring a specific state, the *lead opinion bias* suggests how seriously the second expert should view the first expert's recommendation. As the experts' average precision levels improve, so does the lead opinion bias. The third bias, if and when it exists (and it can arise only endogenously), is induced by the beliefs of the outsider. Typically, when an expert's recommendation (actual or perceived) does not match the realized state, the outsider updates his beliefs towards the expert having low predictive skills. If such downward revision were to be greater when the alternative to status quo is found to have been wrongly predicted, one would expect experts to sometimes recommend in favor of the status quo even if they expect it to occur with probability less than one-half. This we call the *conformity bias*. It inclines experts toward recommending the decision favored by the prior.

Under the two-stage recommendation scenario, with constrained message sets, transparency at times allows the second expert to overturn a recommendation made against the favored state. But under secrecy, the second expert may also be able to overturn a recommendation made in support of the favored state. For small prior biases, the lead opinion bias comes into play. Under transparency, when the lead opinion bias is large the second expert herds and thus becomes redundant. Under secrecy, however, conformity bias comes into play and acts against the lead opinion bias, making the second expert's recommendation relevant over a larger range of parameters (Propositions 3 and 1, and Lemma 7). Even in the absence of conformity bias, under secrecy, the second expert may be able to sometimes communicate, through *partial herding*, all relevant information – her signal about the state as well as its quality when the quality is high (Proposition 2). Surprisingly, this is so even when advice is constrained by a limited message set. For these reasons, the decision maker favors secrecy over transparency.

Since secrecy allows for better information revelation, one would like to know whether both the experts can reveal all their private information – observed signals and their quality – if the message set is made richer. We consider two cases. Under deliberation, experts go back and forth in a total of four stages: expert 1 recommends a decision in stage 1, followed by expert 2's recommendation in stage 2, then a chance for expert 1 in stage 3 to revise her original recommendation, and finally a chance for expert 2 in stage 4 to do the same. Under detailed recommendation, the first expert recommends an action together with a suggestion on the level of its precision based on her ability. The second expert then follows with the same routine. As it has been noted above, allowing multi-dimensional information to be transmitted gradually, in a more interactive and piecemeal manner, or as a one-time opportunity, does not make any difference to their potency for full revelation. Our result in Proposition 6 therefore suggests that what is important for revelation is not gradualism per say but the fact that the experts can speak with greater scope.

■ **Related literature.** Our paper follows the sequential cheap-talk advice literature started by Scharfstein and Stein (1990), and studied extensively by Ottaviani and Sorensen (2001; 2006a,b,c). These papers focused on how financial experts, who care about their reputation in predicting assets' returns or an unknown *state*, tend to herd in their recommendations, or conform to some prior expectation of the unknown state.[4] The papers are not concerned about the relative merits of secrecy and transparency.

Also related are the papers of Levy (2007a,b), and Visser and Swank (2007). In both these papers, information is transmitted through voting. Votes can be thought of as messages which are not capable of transmitting the entire content of information. In this sense, our two-stage recommendation scenario is similar. The difference is that sequencing allows the second expert to learn from the first expert's action.

Levy analyzes a committee decision model using voting. Three experts, motivated by career concerns, simultaneously and independently vote on an action each, and the decision is determined by a given voting rule (unanimity or majority rule). The main argument is that with secretive voting experts are more likely to conform to pre-existing biases either in the voting rule or in the prior, while transparency often leads to contrarian voting. One of Levy's main findings is that under the unanimity rule, secretive voting may *sometimes* induce better decisions than a transparent procedure.[5,6]

Visser and Swank study a somewhat different model where career-concerned experts with private signals about the suitability of a public project engage in simultaneous information exchange, followed by voting. Smart experts observe the accurate information whereas dumb experts observe completely uninformative signal, and the exerts do not know whether they are dumb or smart. The authors find that transparency aligns experts' interests better with

[4]Austen-Smith (1993) studied the sequential referral mechanism involving biased experts on the question of suitability of a legislative bill. One of the concerns was whether sequential referral is better than simultaneous referral. Our setting is different in that the experts are disinterested in what decision gets implemented, nor are we concerned with the issue of sequential vs. simultaneous advice.

[5]The question of transparency has been analyzed in other applications also by Sibert (2003), Gersbach and Hahn (2008), Seidmann (2011), among others.

[6]Earlier, Prat (2005) also advocated secrecy in a principal-agent contracting model. He shows that making an agent's action observable can hurt the principal as the agent might ignore valuable information of her own and instead choose an action to conform to behaviors expected of a more able agent.

the first-best (or public) objective. Under secrecy, experts' behaviors are characterized by conformity and strategic suppression of individual information. Our results on the superiority of secret advice and/or deliberation contrast with their result on the superiority of transparent deliberation. Besides simultaneity of information exchange and voting (as opposed to our sequential advice/deliberation), Visser and Swank made an important assumption, that the project's implementation by itself would indicate to the outside world that the majority of experts got the correct signal and hence could agree to implement the project (instead of not adopting the project due to disagreements); as such the market does not get to observe the ex-post accuracy of the decision taken. Thus there is a group bias (especially by those driven by reputation concerns) towards conformity. Transparency, on the other hand, kills off the incentive to wrongly implement the project because any disagreement in the information exchange will already have hurt the group's collective reputation. In contrast, in our setup (and also in Levy) the market gets to see the actual state (i.e., whether the correct decision has been taken) and hence conformism, per say, does not enhance reputation.

Issues of deliberations in decision making have been analyzed in other voting models. Austen-Smith and Feddersen (2002) study simultaneous cheap-talk deliberations before a committee votes to elect one of two alternatives. Jackson and Tan (2013) analyze a simultaneous move binary voting model with a prior deliberations phase added in, where multiple experts simultaneously announce their privately observed verifiable signals.[7] The works of Austen-Smith and Feddersen, and Jackson and Tan are primarily about information aggregation for an exogenously *given* deliberation protocol. The distinctive feature of our work, different from these papers are: (i) our modelling of alternative forms of sequential deliberation (back-and-forth deliberation and gradual revelation vs. detailed recommendation); and (ii) our decision rule is Bayesian.

Our paper is also related to a small, but growing, literature on mechanism design without commitment (Bergemann and Pavan, 2015). Bester and Strausz (2001) show that full information revelation is not always possible when there is no commitment. A companion paper (Bester and Strausz, 2000), by means of an example, shows that when there is more than one agent (experts in our case), all equilibrium outcomes cannot be induced through mechanisms whose message space is bounded by the number of types. Horner and Skrzypacz (2016) study a persuasion (i.e., non-commitment) model between an agent and a firm where

[7]Wolinsky (2002) considered how allowing partial communication between two experts with different pieces of information relevant for a decision, before the experts communicate their information to a decision maker, helps making a better decision. The experts in Wolinsky are biased but not career-concerned.

the agent with some private information that might be valuable to the firm is induced to reveal her information gradually, despite a potential hold-up problem due to the firm running away with the revealed information. In our model we also do not have commitment. The communication protocols (of secrecy and transparency), along with the number of communication stages (message space), can be viewed as specific mechanisms. Furthermore, in our set-up, outside evaluations of experts are affected by informational externality, types are multi-dimensional and there is no monetary transfers made by the decision maker. Our two-stage results shed light on the possibility of full information revelation by certain types. The necessary and sufficient condition provided in Proposition 6 sheds light on the possibility, and complexity, of full information revelation by all experts of all types.

Finally, our result on the "equivalence" between deliberation and detailed recommendation can also be related to a result in Dekel and Piccione (2000), who had shown in a symmetric binary option, unanimity voting model that the set of voting equilibria is the same irrespective of the structure of sequential voting.

In the next two sections we present the decision maker's problem, the advice protocols and a technical result on partition of expert types. The core analysis is developed in Sections 4–7, followed by conclusions. Section 7 is different in its focus from the earlier sections (Sections 4–6), as it considers communication with expanded message space. Sections 6 and 7 also present some of the key results. The proofs are relegated to Appendix A.

2 Decision maker's problem

A decision maker, D, has to solicit recommendations (advice) from two experts. There is an outside observer O, to be referred as the public or the "market", whose evaluation of the experts' abilities confers the only benefits (payoffs) on the experts.

Formally, two experts make their recommendations to D sequentially about a payoff relevant state $\omega \in \{a, b\}$. Throughout e is the generic label for an expert, with the first mover referred as i and second mover as j. The two experts, D and O share a common prior on the states that favor state a: $\Pr(a) = q$, where $q \in (\frac{1}{2}, 1)$ will be referred as the ***prior bias***.

Each expert privately observes a signal $s_e \in \{\alpha, \beta\}$. Let

$$\Pr(s_e = \alpha \mid \omega = a) = \Pr(s_e = \beta \mid \omega = b) = t_e$$

be the quality of expert e's signal, that we call e's precision level (or ability) $t_e \in \{\xi, \lambda\} \subset (0,1)^2$, with $\frac{1}{2} < \xi < \lambda < 1$. Experts are privately informed about their abilities that are i.i.d., with $\Pr(t_e = \lambda) = \theta$, $0 < \theta < 1$ for $e = i, j$. Let

$$k \equiv \theta\lambda + (1-\theta)\xi$$

be the prior that any expert will observe the correct signal. The index k will also measure the influence of expert i, i.e., the first mover, on expert j's recommendation and will be referred as the ***lead opinion bias***.

Define

$$T_i = T_j = \left\{(\alpha, \xi), (\alpha, \lambda), (\beta, \xi), (\beta, \lambda)\right\},$$

where the elements of T_i and T_j represent the private information (types) of experts i and j, and are denoted by τ_i and τ_j.

An expert is randomly drawn by D with probability $\frac{1}{2}$ to move first. Let $H_i = \left\{\tau_i\right\}$ denote the first expert's information set and $\nu_i : H_i \to \{A, B\}$ denote her pure behavioral strategy. The recommendation is seen by D and expert j. Let $H_j = \left\{\tau_j, \nu_i\right\}$ denote the second expert's information set and $\nu_j : H_j \to \{A, B\}$ be her pure behavioral strategy. This recommendation is also observed by D. Abusing notation, denote the recommendation profile $(\nu_i, \nu_j(\nu_i))$ by ν, where $\nu \in V \equiv \{A, B\} \times \{A, B\}$. At times, it will be convenient to work with pure strategies $V_i : T_i \to \{A, B\}$ and $V_j : T_j \times \{A, B\} \to \{A, B\}$. Denote the set of strategies as:

$$\mathbf{V}_e = \left\{(\nu_e(\alpha, \xi), \nu_e(\alpha, \lambda), \nu_e(\beta, \xi), \nu_e(\beta, \lambda)) \mid \nu_e(s_e, t_e) \in \{A, B\}\right\}. \tag{1}$$

Though we do not explicitly specify the first expert's recommendation in $\mathbf{V}_j$, it will be clear from our analysis. For any element $\mathbf{v}$ of $\mathbf{V}_i$ or $\mathbf{V}_j$, let the number of A (B) recommendations be denoted as $\#A(\nu)$ ($\#B(\nu)$).

Note that each expert has four two-dimensional types. Furthermore, the message set $\{A, B\}$ is of cardinality two. In Section 7, with back-and-forth deliberation, the cardinality of the message set will expand to four.

D uses a **Bayesian decision rule** $d : V \to \{A, B\}$ to choose between actions A and B. After the decision, the *true* state is revealed and D receives a payoff $\pi_D(d, \omega)$, where

$$\begin{aligned} \pi_D(A, a) = \pi_D(B, b) &= 1, \\ \pi_D(B, a) = \pi_D(A, b) &= 0. \end{aligned} \tag{2}$$

Thus, action A (resp. B) is D's ideal decision in state a (resp. b).

All of the above, except realizations of types, states and signals, are common knowledge among experts, D and O.

We now state the two protocols and the payoffs of the experts.

[***Transparency* or $\wp = t$**] O *observes* d*, the state of the world* ω *and the sequence of moves (which expert moves first and which second) as well as the recommendations made by the experts. In particular,* O *observes a realization of the outcome,* $(\nu_i, \nu_j, d, \omega)$*, and Bayes-updates his beliefs regarding the experts' abilities denoted by* $\Pr(t_i \mid \nu_i, \nu_j, d, \omega)$*. The expected abilities of* i *and* j*, as well as their payoffs, are*

$$\begin{aligned} E^{\wp=t}(t_i|\nu_i,\nu_j,d,\omega) &= \Pr(t_i=\lambda \mid \nu_i,\nu_j,d,\omega)\lambda + \Pr(t_i=\xi \mid \nu_i,\nu_j,d,\omega)\xi, \\ E^{\wp=t}(t_j|\nu_i,\nu_j,d,\omega) &= \Pr(t_j=\lambda \mid \nu_i,\nu_j,d,\omega)\lambda + \Pr(t_j=\xi \mid \nu_i,\nu_j,d,\omega)\xi. \end{aligned} \tag{3}$$

[***Secrecy* or $\wp = s$**] O *only observes the decision maker's decision and the true realization of the state,* (d, ω)*, and Bayes-updates expert* e*'s ability to* $\Pr(t_e \mid d, \omega)$*.*[8] *The expected ability of* e *is*

$$E^{\wp=s}(t_e|d,\omega) = \Pr(t_e=\lambda \mid d,\omega)\lambda + \Pr(t_e=\xi \mid d,\omega)\xi. \tag{4}$$

This is also each expert's payoff.

D does not offer any explicit monetary rewards to the experts. The market pays the experts based on their expected absolute abilities.[9] These expectations depend on what the market can observe, i.e. on the protocol of advice.

Let $\mu_i^{\wp} = \Pr(\omega|\tau_i)$ and $\mu_j^{\wp} = \Pr(\omega, \tau_i|\nu_i, \tau_j)$ denote expert i and j's beliefs about ω, and in the case of j also about i's type, conditional on the expert's private information. Let $\mu_D^{\wp} = \Pr(\omega|\nu_i, \nu_j)$ denote D's updated belief conditional on the recommendations. O's beliefs are denoted by $\mu_O^{\wp=t} = \Pr(t_e = \lambda|\nu_i, \nu_j, d, \omega)$ and $\mu_O^{\wp=s} = \Pr(t_e = \lambda|d, \omega)$ under transparency and secrecy, respectively.

This ends the description of the two games, conditional on the protocols – transparency and secrecy. Our concept of equilibrium is that of a perfect Bayesian equilibrium in pure strategies.

Definition 1. *A* perfect Bayesian equilibrium (PBE) *of the game induced under the protocol*

[8] From O's point of view, e is now a generic label.

[9] We do not consider winner-take-all contest between the experts. Such analysis but without the consideration of optimal transparency protocol appears in Ottaviani and Sorensen (2006b,c).

$\wp$ is a profile of (pure behavioral) strategies and beliefs,

$$\left(\nu_i^*(.),\, \nu_j^*(.),\, d^*(.,.);\; \mu_i^{\wp},\, \mu_j^{\wp},\, \mu_D^{\wp}, \mu_O^{\wp}\right),$$

for all histories H_i and H_j, such that the strategies are sequentially rational given beliefs, and the beliefs are derived applying Bayes' rule wherever possible.

Recall that $E_i^{\wp}$ and $E_j^{\wp}$ ($E_i^{\wp} = E_j^{\wp}$ in the case of secrecy) are the expectations over expert abilities t_i, t_j, respectively, that the outsider O estimates. Note that $E_i^{\wp}$ and $E_j^{\wp}$ also define the experts' terminal payoffs in the game. These expectations are derived through $\mu_O^{\wp}$, which in turn is a part of equilibrium.[10] Thus, our Bayesian game is quite different from standard games where players' payoffs in the terminal nodes are taken **as given (or fixed)**, rather than endogenously determined in equilibrium.

Communication games always have equilibria under which recommendations are ignored. Such equilibria are called babbling equilibria. We will restrict our attention primarily to equilibria under which the expert moving first does not babble.

Definition 2. *An equilibrium of the sequential advice game, in short DE, induced under the protocol $\wp$ is a* PBE of $\wp$ *where the first expert does not babble.*

Under signal revealing strategies (i.e., an expert recommends A (B) when her signal is α (β)), we say that the expert reports truthfully. We now state a stronger version of DE.

Definition 3. *A strong sequential advice equilibrium, in short SDE, of the game induced under the protocol $\wp$ is a* DE of $\wp$ *where the first expert reports truthfully.*

D's problem is to choose an optimal protocol to maximize his ex-ante expected payoff from eventual decision making. Given the multiplicity of equilibria in communication games, we take a mechanism design approach.

Definition 4. D *chooses a protocol $\wp$ over $\wp'$ if for every profile of parameters $(q, \theta, \xi, \lambda)$, there exists some* PBE *in the game induced by $\wp$ under which the payoff of D is (weakly) greater than his payoff under all* PBE *in the game induced by $\wp'$, and strictly greater for some parameters.*

[10] This implies if strategies (and associated beliefs) were to change, for the same terminal nodes the market's expectation of experts' abilities are also likely to change.

This ends the description of the decision maker's two-step decision problem.[11]

The following assumption will be maintained throughout the paper.[12]

Assumption 1. $1/2 < q < \xi < \lambda < 1$.

That is, even the low-ability expert's signal is more informative than the unrefined (prior) information. Assumption 1 along with the definition of k implies the following fact:

Fact 1. $\frac{\lambda}{1-\lambda} > \frac{k}{1-k} > \frac{\xi}{1-\xi} > \frac{q}{1-q} > 1$.

3 Bias, beliefs, and partitioning k and q

As mentioned in the Introduction, the interaction between the prior bias q and the lead opinion bias k, will influence D's choice between transparency and secrecy (see the second part of Section 6). Since our main results pertain to *SDE*, we illustrate how q and k influence the experts' beliefs under the condition that the first expert recommends her signal.

We say that signal α (β) favors a corresponding state if conditional on signal (and possibly other observables), the expert assigns to state a (b) a probability greater than $\frac{1}{2}$.

In Appendix A we show that, both signals of the first expert, i.e. expert i, favor their respective states. This is due to Fact 1.

Now consider expert j who moves second. Under an *SDE,* she would deduce the first expert's signal, but not her ability, from the observed recommendation. If the second expert's signal *matches* that of the first, then the signal favors its corresponding state (see (A.2)).

When the first expert's signal is β and the second expert observes α, then the signal favors the corresponding state when the expert is of high ability λ. This "*non-herding*" belief occurs for two reasons: (i) prior favors a ($q > \frac{1}{2}$), and (ii) average precision k of the first expert is less than the high precision of the second expert (see (A.3)).

When the first expert's signal is deduced as α and the second expert receives β, the latter's signal *does not* favor the corresponding state when the expert is of low ability ξ (see (A.4)). This is because of the prior bias favoring a and the first expert's average precision being higher than the second expert's low ability. Thus, low-ability expert's signal not prevailing over the prior bias arises endogenously.

[11] The reader may be concerned that in the protocol chosen there may be an equilibrium which is worse than all equilibria in the other protocol and strictly worse under some parameter values. This won't happen in our environment because the worst equilibrium in both protocols is the babbling equilibrium.

[12] The assumption helps reducing the set of equilibria. Our qualitative results do not change if $q \geq \xi$.

So far, the relative weights of q and k have not mattered. However, they *will* matter in two cases.

In case (i), the first expert's signal (s_i) is deduced to be β, the second expert is of ability $t_j = \xi$ with signal $s_j = \alpha$. Then, conditional on (s_i, s_j, t_j),

$$\Pr(a \mid \beta, \alpha, \xi) = \frac{q\xi(1-k)}{q\xi(1-k)+(1-q)(1-\xi)k} \geq \frac{1}{2} \quad \text{if and only if} \quad \frac{\xi}{1-\xi} \geq \frac{k}{1-k}\frac{1-q}{q}. \tag{5}$$

In case (ii) we have $(s_i, s_j, t_j) = (\alpha, \beta, \lambda)$ and,

$$\Pr(b \mid \alpha, \beta, \lambda) = \frac{(1-q)\lambda(1-k)}{q(1-\lambda)k+(1-q)\lambda(1-k)} \geq \frac{1}{2} \quad \text{if and only if} \quad \frac{\lambda}{1-\lambda} \geq \frac{k}{1-k}\frac{q}{1-q}. \tag{6}$$

To see when signals favor their respective states, or when the inequalities $\frac{\xi}{1-\xi} \geq \frac{k}{1-k}\frac{1-q}{q}$ and $\frac{\lambda}{1-\lambda} \geq \frac{k}{1-k}\frac{q}{1-q}$ are satisfied, let $k(\xi)$ and $k(\lambda)$ be the values of k when the first and second inequalities bind. Define:

$$\phi(k) = \frac{k}{1-k}, \qquad k(\xi) = \frac{q\xi}{q\xi + (1-q)(1-\xi)}, \qquad k(\lambda) = \frac{(1-q)\lambda}{(1-q)\lambda + q(1-\lambda)}. \tag{7}$$

Since $\frac{q}{1-q} > 1$, the graph of $\frac{q}{1-q}\phi(k)$ $(\frac{1-q}{q}\phi(k))$ lies above (below) $\phi(k)$ (Fig. 1).[13] Also, given $q > \frac{1}{2}$ it is easy to see that $\xi < k(\xi)$ and $k(\lambda) < \lambda$, but $k(\lambda)$ may exceed or be less than $k(\xi)$. In particular, Panels 1, 2 and 3 capture the following situations:[14]

$$k(\lambda) \leq \xi < \lambda \leq k(\xi), \tag{8}$$

$$\xi \leq k(\lambda) \leq k(\xi) < \lambda, \tag{9}$$

$$\xi < k(\xi) < k(\lambda) < \lambda. \tag{10}$$

These partitions will be important in deriving equilibria under secrecy and comparing across protocols.

Let

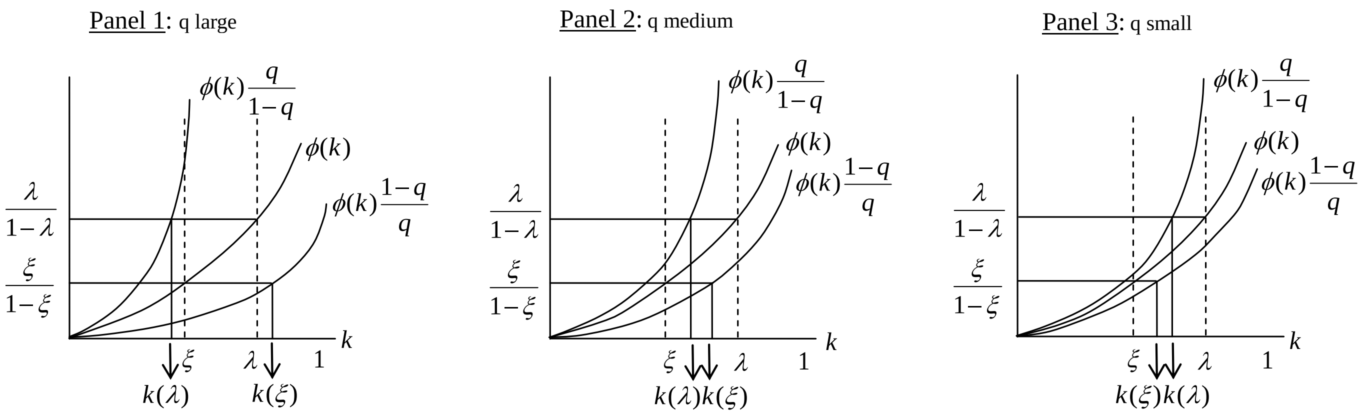
$$r \equiv (\frac{\lambda}{1-\lambda})/(\frac{\xi}{1-\xi}) > 1$$

be the "(relative) merit of the high ability expert". We now fix the following two definitions

[13] Keeping ξ and λ constant, k increases from ξ to λ as θ varies from 0 to 1. The function $\phi(k)$ is increasing and convex with $\phi(0) = 0$ and $\lim_{k\to 1}\phi(k) = \infty$. Therefore, $k(\xi)$ and $k(\lambda)$ exist such that $\frac{\xi}{1-\xi} = \phi(k(\xi))\frac{1-q}{q}$ and $\frac{\lambda}{1-\lambda} = \phi(k(\lambda))\frac{q}{1-q}$.

[14] It is easy to rule out $k(\lambda) \leq \xi < k(\xi) < \lambda$.

Figure 1: Partitioning k & q



to be repeatedly referred in the rest of this paper:

Definition 5. *The* ***lead opinion bias****,* k*, is* <u>*small*</u> *or* <u>*large*</u> *if, respectively,*

$$\mathsf{k} \leq \mathsf{k}(\xi), \quad \textit{or} \quad \mathsf{k} > \mathsf{k}(\xi).$$

Definition 6. *The* ***prior bias****,* q*, will be called* <u>*small*</u>*,* <u>*medium*</u> *or* <u>*large*</u> *if, respectively,*

$$\frac{\mathsf{q}}{1-\mathsf{q}} < \sqrt{\mathsf{r}}, \qquad \sqrt{\mathsf{r}} \leq \frac{\mathsf{q}}{1-\mathsf{q}} < \mathsf{r}, \qquad \textit{or} \qquad \frac{\mathsf{q}}{1-\mathsf{q}} \geq \mathsf{r}.$$

These ranges are specified in terms of the prior odds ratio for ease of comparison with the ability index. An important point to note here is that as q increases, so does $\mathsf{k}(\xi)$. This means, a strong prior bias is likely to weaken the importance of the lead opinion bias with k switching from being large to small.

The following lemma, to be studied with Fig. 1, is easy to verify:

Lemma 1.

(i) *[****Panel 1****] Inequalities (8) will hold if and only if the prior bias is* <u>*large*</u>*;*

(ii) *[****Panel 2****] Inequalities (9) hold if and only if the prior bias is* <u>*medium*</u>*;*

(iii) *[****Panel 3****] Inequalities (10) hold if and only if the prior bias is* <u>*small*</u>*.*

4 Transparency: Revelation hurdles

D's objective is to maximize the probability that the decision chosen corresponds with the state. An expert's payoff derives from O's beliefs about the expert's ability. Under transparency, O gets to see not only the realized state but also who recommended what and when. Since O is only interested in the expert's ability, this information is sufficient for forming beliefs; D's decision becomes redundant.

Recall from (1) that the strategy set of expert $e \in \{i, j\}$ is

$$\mathbf{V}_e = \{(\nu_e(\alpha, \xi), \nu_e(\alpha, \lambda), \nu_e(\beta, \xi), \nu_e(\beta, \lambda)) \mid \nu_e(s_e, t_e) \in \{A, B\}\}.$$

We highlight the following strategies in $\mathbf{V}_e$.

Truthful recommendation: We say that an expert $e \in \{i, j\}$ truthfully recommends her signal if her strategy belongs to $\mathbf{V}_e^s \subset \mathbf{V}_e$, where

$$\mathbf{V}_e^s = \{(A, A, B, B)\}.$$

Babbling: $\mathbf{V}_e^b$ is the set of strategies, where expert $e \in \{i, j\}$ is said to babble:

$$\mathbf{V}_e^b = \{(A, A, A, A), (B, B, B, B)\}.$$

With only two possible signals, the contrarian strategy (B, B, A, A) is equivalent to truthful recommendation.[15] Hence, without loss of generality, we drop this recommendation profile from the strategy sets. We can now start presenting the formal results.

Lemma 2.

(i) *There exists a* PBE *where the first expert truthfully recommends her signal.*

(ii) *There is no* PBE *with* $\nu_i \in \mathbf{V}_i \backslash \{\mathbf{V}_i^s \cup \mathbf{V}_i^b\}$.

(iii) *There is always a* PBE *in which expert* i*, who moves first, babbles.*

Thus there are *only* two kinds of equilibria, one where first expert truthfully recommends her signal and the other where she babbles. Under truthful recommendation, O's expectation

[15] Sometimes *contrarian* is used differently to mean the second expert recommending contrary to the first expert's recommendation. From the context the meaning should be clear.

about the expert's ability is greater if the recommendation were to match the state. Since signals are informative about the state for both precision levels (see previous section), the first expert reveals her signal.

Consider now the continuation equilibria involving the second expert. We first rule out certain strategies in equilibrium.

Lemma 3. *If* $\nu_j(\nu_i)$ *is an equilibrium strategy in the continuation game following a recommendation of* ν_i*, then* $\nu_j(\nu_i) \notin \mathbf{V}_j \backslash \{\mathbf{V}_j^s \cup \mathbf{V}_j^b\}$.

Lemma 4. $\nu_j(\nu_i) \in \mathbf{V}_j^s = \{(A, A, B, B)\}$ *is a continuation equilibrium strategy following a recommendation of* ν^1 *if and only if: the first expert babbles or*

(i) *the first expert observes* β *and* $\nu_i = B$*; and*

(ii) k *is small, i.e.,* $k \in [\xi, k(\xi)]$.

Note that for a large prior bias, condition (ii) of Lemma 4 is always satisfied (see (i) of Lemma 1).

Equilibria, under transparency, are influenced by the lead opinion bias and therefore solely driven by "learning". If the first expert were to babble, the second expert would learn nothing from first expert's recommendation, so it is optimal to recommend her signal. If the first expert were to reveal her signal and recommend A, the importance of the second expert's signal is washed out if she were to be of low ability with signal β. This in turn makes the second expert (of low ability) herd, making truthful recommendation impossible. Given Lemma 3, then the only equilibrium strategy for the second expert is to babble. Similarly, if the first expert were to reveal her signal and recommend B, then if the lead opinion bias, k, were to be large, the second expert of ability ξ with signal α would herd with the first expert. Given the impossibility of truthful recommendation, Lemma 3 then implies the second expert would babble. But if k were small, a signal of α would still be informative. Then, the second expert recommending truthfully would be an equilibrium strategy. Finally, the following lemma should be obvious.

Lemma 5. $\nu_j(\nu_i) \in \mathbf{V}_j^b$ *is a continuation equilibrium strategy for all* $\nu_i \in \{A, B\}$.

Lemmas 2–5 characterize the set of equilibrium strategies for the experts. They imply that the following types of equilibria exist: (i) both experts babble; (ii) first expert babbles and second expert recommends truthfully; (iii) first expert recommends truthfully and second expert always babbles; and (iv) first expert recommends truthfully and second expert recommends truthfully only if the first recommendation is B. Equilibria (iii) and (iv) are SDE and will be of special interest to us. We state them explicitly in Proposition 1.

Combining Lemmas 2–5 we have:

Proposition 1 (Transparency: Equilibrium characterization)**.** *Under transparency an* SDE *equilibrium exists. The experts' strategies in* SDE *equilibria are as follows:*

1. *In all* SDE*:*

(i) If $\nu_i = A$*, then* $\nu_j(\nu_i) \in \mathbf{V}_j^b$*.*

(ii) If $\nu_i = B$ *and* k *is small, then* $\nu_j(\nu_i) \in \mathbf{V}_j^s \cup \mathbf{V}_j^b$*. If* $\nu_i = B$ *and* k *is large, then* $\nu_j(\nu_i) \in \mathbf{V}_j^b$*.*

2. *In all* SDE*, following a recommendation of* A *by the first expert,* D *chooses* A*. Following a recommendation of* B*,* D*'s decision will depend on the continuation equilibrium. For the babbling equilibrium,* D *chooses* B*. For the truthful recommendation equilibrium,* $d(B,B) = B$*,* $d(B,A) = A$*.*

Under transparency there does not exist any equilibrium where both experts, regardless of their abilities, will recommend their signals. Also, the experts' abilities never get revealed. The only chance for the second expert to stand out (i.e., $\nu_j(\nu_i) \in \mathbf{V}_j^s$) is if the first expert recommends B.

To summarize, Proposition 1 shows the limitations of transparency. Market pressure leads to informative advice by the second expert only when the prior bias is large, otherwise we need that the average ability of the first mover is not too high. Furthermore, such advice is used only to over-rule a recommendation B by the first expert.

5 Secrecy and revelation

In this section we present two classes of equilibria under secrecy.

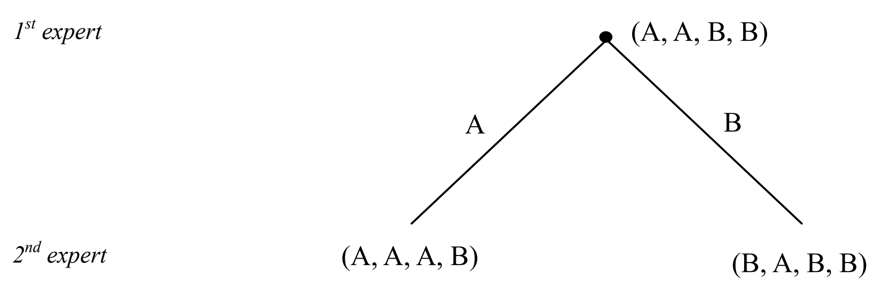
Under transparency only two biases mattered: the prior bias q and the lead opinion bias k. Under secrecy there is also a third bias, the conformity bias, that can arise endogenously due to O's beliefs. Specifically, O could believe that the low-ability type has a greater propensity to make a wrong prediction when the true state is a, the one favored by the prior, than when the true state is b, the less favored one. This biases the experts towards recommending A. Let x' (y'') be the experts' payoffs when D's decision matches (does not match) state a. Similarly, let x'' (y') be the experts' payoffs when D's decision matches (does not match) state b.[16] We say that there is ***conformity bias*** if

$$x' - y'' > x'' - y'.$$

In the first class of equilibria there will be no conformity bias, whereas the second class will exhibit this bias.

■ **Herding and partial type revelation.** We first present an equilibrium (see Fig. 2), where the second expert's advice is shaped *only* by the lead opinion bias and the prior bias. To help understand the figure, recall that an element of $\mathbf{V}_e$, $e \in \{i, j\}$ is a quadruple $(v_e(\alpha, \xi), v_e(\alpha, \lambda), v_e(\beta, \xi), v_e(\beta, \lambda))$. We call this a partial type revealing equilibrium because only the second expert is able to reveal her type and that too in certain cases. For example if the first expert were to recommend B, the second expert's *high* type gets revealed only if she were to recommend A (i.e. if she were to get signal α). A recommendation of B, neither reveals the signal nor ability.

Figure 2: Partial type revelation strategies



Proposition 2 (Partial type revelation)**.**

(i) *Consider the case of small prior bias* q *as in Lemma 1 (Panel 3 in Fig. 1) and* $k \in [k(\xi), k(\lambda)]$ *(so* k *is large but not "too large"). Then the following recommendation strategies can be supported as an* SDE *under secrecy:* $v_j(A) = (A, A, A, B)$, $v_j(B) = (B, A, B, B)$.

[16] These payoffs, identical for both experts, will be derived in the proof of Proposition 3.

(ii) *For the particular equilibrium in* **(i)***, conformity bias does not arise.*

In this equilibrium, the second expert's high ability, λ, is revealed when she recommends contrary to the first expert's recommendation. The beliefs, not specified here, will be derived in the proof.

On the left-hand branch of Fig. 2, the lead opinion bias induces the low-ability second expert who observes signal β to recommend non-truthfully the decision A. This is irrespective of the value of q, as even the worst prior ($q \approx 1/2$) agrees with the lead opinion (see (A.4)). Contrast this with a high-ability expert who gets signal β and sees a first recommendation of A. She knows that her ability is higher than the average ability of the first expert. This by itself is not sufficient for her to totally disregard the first recommendation. The weight she puts on the information content of the first expert depends on the prior bias q. When q is small (as in Panel 3), she puts relatively less weight on the information content and goes with her own signal β to recommend B; thus even a large lead opinion bias fails to prevail when the prior bias is weak. So for low enough q, ***failure*** *of full signal revelation* ***enables*** *partial type revelation.*

On the right-hand branch, given a small prior bias q, the low-ability second expert who observes signal α gets sufficiently influenced by a large lead opinion bias. For her, the expected type of the first expert is reasonably high (k above $k(\xi)$). Unable to ignore the first expert's recommendation of B, she then herds. For the high-ability expert who gets signal α, the expected precision of the first expert is not "too large" (k below $k(\lambda)$) relative to her own precision. Thus she recommends her signal. Therefore, again we see that a small prior bias q leads to partial type revelation.

Proposition 2 is quite significant in relation to the previous works on transparency of expert advice. The result is new and very different from what we know, say, from the one-shot voting model of Levy (2007a). In her framework, partial type revelation is *not* possible. Moreover, as can be seen from the proof, here partial type revelation happens with complete absence of the conformity bias, a bias known to play a prominent role in group decisions.

■ **Signal revelation.** Now consider the second class of equilibria where both experts recommend their signals. Then, for recommendation profiles (A, A), (A, B), (B, A) and (B, B), D knows that the corresponding signals are (α, α), (α, β), (β, α) and (β, β). D's posteriors are then:

$$\begin{array}{rcl} \Pr(\omega = a \mid A, A) = & \frac{qk^2}{qk^2+(1-q)(1-k)^2} & > \frac{1}{2}, \\ \Pr(\omega = a \mid B, A) = & q & > \frac{1}{2}, \\ \Pr(\omega = a \mid A, B) = & q & > \frac{1}{2}, \\ \Pr(\omega = a \mid B, B) = & \frac{q(1-k)^2}{q(1-k)^2+(1-q)k^2} & < \frac{1}{2}. \end{array} \tag{11}$$

These probabilities are calculated using Tables A.2 and A.3 in Appendix A.[17]

Lemma 6 (D's decision under signal revelation)**.** *Let the recommendations reveal signals.* D *selects* B *if and only if both experts recommend* B*; otherwise* D *selects* A*.*

Recall that under transparency, the first expert is decisive when she recommends A. There, following a recommendation of A, the second expert babbled. Here, on the other hand, the first expert is decisive when she recommends A, inspite of the second expert revealing her signal. This occurs because of two reasons: (i) the prior bias favors a; and (ii) the second expert does not reveal her ability, even partially.

We now proceed to determine conditions under which the experts will recommend their signals. This we do by providing a uniform threshold for the experts' signal accuracy, $t_i/(1-t_i)$ and $t_j/(1-t_j)$. Let

$$c(k) = \frac{1-k}{k}\frac{q}{1-q}\frac{1+k}{2-k}, \qquad c^{-1}(k) = \frac{k}{1-k}\frac{1-q}{q}\frac{2-k}{1+k}.$$

Note that $c(k)$ and $c^{-1}(k)$ are respectively the graphs $\phi(k)\frac{q}{1-q}$ and $\phi(k)\frac{1-q}{q}$ in Fig. 1, but moved up and down as follows:

$$c(k) = \frac{1}{\phi(k)}\frac{q}{1-q}\frac{1+k}{2-k}, \quad c^{-1}(k) = \phi(k)\frac{1-q}{q}\frac{2-k}{1+k}.$$

Note $c(k)$ is decreasing in k and therefore $c^{-1}(k)$ is increasing in k. Furthermore,

$$c(1) = 0, \ \lim_{k\to 0} c(k) = \infty, \ \text{and} \quad c^{-1}(0) = 0, \ \lim_{k\to 1} c^{-1}(k) = \infty.$$

[17] For example consider $\Pr(\omega = a \mid A, A)$. Since experts reveal their signals, $\Pr(\omega = a \mid A, A) = \Pr(\omega = a \mid \alpha, \alpha)$. This probability is a ratio where the numerator is the sum of all column entries in row two of Table A.2. In the denominator we have this sum plus the sum of all column entries in row two of Table A.3. This ratio is q. The other probabilities can be derived similarly.

Therefore we can define $\underline{k}(\xi)$ and $\bar{k}(\xi)$ such that

$$c(\underline{k}(\xi)) = \xi/(1-\xi) = c^{-1}(\bar{k}(\xi)),$$

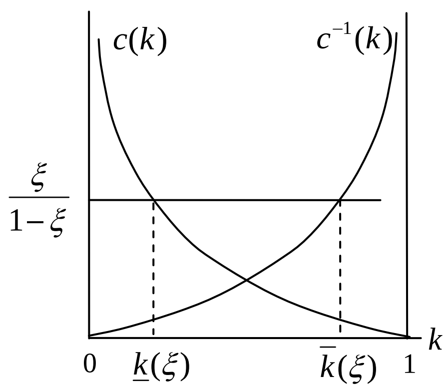
where $\underline{k}(\xi) < \bar{k}(\xi)$ and,

$$\lambda/(1-\lambda) > \xi/(1-\xi) \geq \max\{c(k), c^{-1}(k)\} \quad \text{for } \underline{k}(\xi) \leq k \leq \bar{k}(\xi).$$

Thus, for k in the range $[\underline{k}(\xi), \bar{k}(\xi)]$, the signal accuracy will always lie above the threshold $\max\{c(k), c^{-1}(k)\}$, as depicted in Fig. 3. As k changes, the threshold also changes. This means a given pair of signal accuracy can alter from being good enough to insufficient (for signal revelation) if the average quality of experts becomes too high.

The above (moving) threshold will be influenced by all three biases – conformity bias, lead opinion bias, and the prior. Exactly how the three biases play out appears in the formal proof of the following result. The intuitions are discussed below.

Figure 3: Critical k values



Proposition 3 (Signal revelation)**.** *Under secrecy, there exists an* SDE *where the experts' strategies are as follows: For all* $v^1 \in \{A, B\}$,

(i) *if* $\xi \leq k \leq \bar{k}(\xi)$, *then* $v_j(v_i) \in \mathbf{V}_j^s$; **(Revelation)**

(ii) *if* $k > \bar{k}(\xi)$, *then* $v_j(v_i) \in \mathbf{V}_j^b$. **(Babbling)**

Recall that by definition, under an SDE the first expert always recommends her signal. We now state a technical result.

Lemma 7. $\underline{k}(\xi) < \xi$, *and* $\bar{k}(\xi) > k(\xi)$.

Since $\bar{k}(\xi) > k(\xi)$, the following result is immediate from Proposition 3:

Corollary 1. *If the bias* q *is large as in Lemma 1, then under secrecy there is always a full signal revelation equilibrium.*

Recall from Proposition 1 that under transparency, in all SDE, the second expert babbles following a first period recommendation of A. Proposition 3 tells us that under secrecy, there exists an SDE for which this is not so. Under this SDE, when the prior bias is large, both types always recommend their signals. This is also true when the prior bias is medium or small, provided that $\xi \leq k \leq \bar{k}(\xi)$.

In the equilibrium stated in Proposition 3, conformity bias comes into play. Recall that x' (y'') are the experts' payoffs when D's decision matches (does not match) state a and x'' (y') be the experts' payoffs when D's decision matches (does not match) state b. It turns out that, in equilibrium, $[x' - y''] = \frac{1+k}{2-k}[x'' - y']$. Since $\frac{1+k}{2-k} > 1$, we have $[x' - y''] > [x'' - y']$. Hence, by definition we have conformity bias – a bias which was absent in all previous equilibria.

We can now understand the results which follow when the prior bias is not large. Consider the case where the first expert recommends B (so that the second expert is pivotal). If the second expert who is of low ability observes α, her posterior on state a may go below $\frac{1}{2}$.[18] However, even though her posterior on state a is low, due to conformity bias her expected *payoff* from recommending A may be greater than the payoff from recommending B. In such situations, by recommending honestly and triggering A, she avails the insurance of conformity bias. On the other hand if she observes β, whether she is of low or high ability, she is emboldened by the first expert's opinion (lead opinion bias) and therefore recommends B despite the conformity bias. The safety net of the conformity bias when in possession of α signal and the support of the lead opinion bias in case of β signal are the two contributory factors for signal revelation under secrecy, that might not be possible under the full glare of transparency.

[18] If $\phi(k)\frac{1-q}{q}\frac{2-k}{1+k} \leq \frac{\xi}{1-\xi}$ so that $k \leq \bar{k}(\xi)$, it is possible that $\phi(k)\frac{1-q}{q} > \frac{\xi}{1-\xi}$; see (5) for the posterior. This may happen despite prior favoring a.

What is rather surprising about Proposition 3, however, is that for signal revelation to happen k must not exceed $\bar{k}(\xi)$: having "too good" experts spoils it! Notice that conformity bias again comes into play but this time it is washed out by the lead opinion bias. To see this, consider the case when k is large. As k increases, the conformity bias effect (as previously observed in the relation $[x' - y''] = \frac{1+k}{2-k}[x'' - y']$) increases. But for k above $\bar{k}(\xi)$ the conformity bias is swamped by the lead opinion bias effect due to $k/(1-k)$. So following a first expert recommendation of B, the second expert who observes signal α would recommend B too. The increase in k makes her updated beliefs in favor of state a very low. Conformity bias is no longer sufficient for her to recommend A. The following observation is already implicit in Proposition 3:

Corollary 2. *If the experts are "too good," the decision maker may as well do away with the second expert even if the expert is costless.*

The above corollary is meant only in reference to signal revelation. By bringing together Propositions 2 and 3, one can see that there is an overlap of partial type revelation with full signal revelation when: $\{k : k(\xi) \leq k \leq k(\lambda)\} \cap \{k : \xi \leq k \leq \bar{k}(\xi)\} \neq \emptyset$ (i.e. when q is small). Hence, under secrecy there are multiple SDE.[19] A natural question then is how do these equilibria rank? In partial type revelation equilibrium there is some amount of herding, whereas in full signal revelation D loses out on the important information that the second expert could be of high ability. An answer is provided in the next section.

6 Transparency or secrecy?

To evaluate the relative merits of two protocols, it is sufficient to consider only one class of equilibria under secrecy, i.e., the one in Proposition 3. Given that $\bar{k}(\xi) > k(\xi)$ we conclude, from Proposition 1 and Proposition 3, that signal revelation occurs under secrecy over a larger parameter space. The main difference between the two mechanisms comes in the form of revelation incentives of a low (ξ) ability expert who observes the signal α, when preceded

[19]Multiplicity of equilibria is not surprising; there is always a babbling equilibrium.

by a B-recommendation by the first expert.[20] Under transparency, signal revelation requires:

$$\begin{aligned} \Pi_j^t(B, A, \alpha, \xi) = \Pr(a \mid \beta, \alpha, \xi)\gamma \quad &+ \quad \Pr(b \mid \beta, \alpha, \xi)\gamma' \\ &\geq \quad \Pr(a \mid \beta, \alpha, \xi)\gamma' + \Pr(b \mid \beta, \alpha, \xi)\gamma = \Pi_j^t(B, B, \alpha, \xi) \\ \text{or,} \quad \Pr(a \mid \beta, \alpha, \xi)\big[\gamma - \gamma'\big] \quad &\geq \quad \Pr(b \mid \beta, \alpha, \xi)\big[\gamma - \gamma'\big], \end{aligned}$$

where $\gamma = E_j^t(B, A, a) = E_j^t(B, B, b)$, $\gamma' = E_j^t(B, A, b) = E_j^t(B, B, a)$ (see Lemma 4 proof). That is, the second expert will recommend truthfully if her chance of being right is higher than that of being wrong, i.e., $\Pr(a \mid \beta, \alpha, \xi) \geq 1/2$. The term $[\gamma - \gamma']$, which is the gain in one's perceived ability from being an accurate predictor over being inaccurate, is the <u>same</u> whether the revealed state is a or b and thus drops out. In our terminology, this means that there is no conformity bias.

In contrast, under secrecy, a similar payoff comparison makes truthful recommendation optimal if (see Proposition 3 proof):

$$\Pr(a|s_i = \beta, s_j = \alpha, \xi)\big[x' - y''\big] \geq \Pr(b|s_i = \beta, s_j = \alpha, \xi)\big[x'' - y'\big].$$

Here due to conformity bias (i.e., $\big[x' - y''\big] > \big[x'' - y'\big]$) even if the state a is less likely, the low level expert j chooses to recommend decision A as she gets a higher payoff. This expands the truthful recommendation range of k beyond $k(\xi)$ to $\bar{k}(\xi)$.

Hence, restricting ourselves to only that class of secrecy equilibria in Proposition 3, we have the following result. For $k \in [\xi, k(\xi)]$, transparency and secrecy provide the same payoff to D. For $k \in (k(\xi), \bar{k}(\xi)]$, D is strictly better off under secrecy. Due to Definition 4, we then have:

Proposition 4 (Choice of protocol)**.** D *prefers secrecy over transparency.*

Proposition 4 is an overall ranking of the two protocols. Next we turn to the question posed at the end of Section 5: comparing D's payoffs in the two types of equilibrium under the secrecy protocol.

Proposition 5 (Value of information: signal vs. type)**.** *Consider the secret advice protocol. In the parameter range* $\{k : k(\xi) \leq k \leq k(\lambda)\} \cap \{k : \xi \leq k \leq \bar{k}(\xi)\} \neq \emptyset$*, the partial*

[20] As we already know, under transparency if the first expert recommends A the only continuation equilibrium possible is that of babbling by the second expert. And under secrecy, the first expert recommending A makes the second expert's recommendation inconsequential for D's decision.

type revelation equilibrium strictly payoff dominates (from D*'s point of view), the full signal revelation equilibrium.*

The reason for this payoff dominance can be seen by inspecting the recommendation sequences and the decisions in the respective equilibria. The decisions differ only for the sequence (A, B), with the final decision being A in the signal revealing equilibrium whereas the decision is B in the partial type revealing equilibrium. When the decisions differs, D not only learns the true signal of the second expert he also learns that it is coming from a high-ability expert. For the sequence (B, A), D's decision is the same under both equilibria. However, under the partial type revealing equilibrium, D is more confident of his decision knowing that the truthful A recommendation by the second expert is from the high-ability type. These two facts, together, improve D's ex-ante payoffs.

■ **Summary of comparison**. Recall from Definition 6 that the prior bias q, is *small*, *medium* or *large* if, respectively, $\frac{q}{1-q} < \sqrt{r}$, $\sqrt{r} \leq \frac{q}{1-q} < r$ and $\frac{q}{1-q} \geq r$. We now provide a summary of our comparison in terms of the prior q and the lead opinion bias k.

When $k \in [\xi, k(\xi)]$, the partial type revealing equilibrium under secrecy does not exist. D's maximum equilibrium payoff under transparency and secrecy are the same (see the proof of Proposition 4). Recall from Lemma 1 that if the prior bias, q, is *large* then k can only lie in this range. So when the prior bias is large, D is indifferent between transparency and secrecy.

Let q be not large. When $k \in [\xi, k(\xi)]$, D is indifferent between secrecy and transparency as noted above. When $k \in (k(\xi), \lambda]$, the second expert is redundant under transparency as she babbles (Proposition 1).

Now consider *medium* values of q. When $k \in (k(\xi), \bar{k}(\xi)]$, while the second expert babbles under transparency, she reveals her signal under secrecy (Proposition 3). This leads to D strictly preferring secrecy over transparency (proof of Proposition 4). Under *medium* values of q, we have $k(\xi) \geq k(\lambda)$ (see (9) and Lemma 1). Hence, under secrecy, the partial type revealing equilibrium does not exist ((i) of Proposition 2).

Finally consider *small* values of q. For $k \in (k(\xi), k(\lambda)]$, the second expert babbles under transparency but partially reveals her type under secrecy (Proposition 2). As a result, D's maximum equilibrium payoff under secrecy is strictly larger than that under transparency (follows from Propositions 4 and 5). For $k \in (k(\xi), \bar{k}(\xi)]$, we also have the signal revealing equilibrium under secrecy.

When q is *small*, the intersection of $(k(\xi), k(\lambda)]$ and $(k(\xi), \bar{k}(\xi)]$ is non-empty. Let $z = \max\{k(\lambda), \bar{k}(\xi)\}$. When q is *small* and $k \in (k(\xi), z]$, then under secrecy, D's payoff under the partial type revealing equilibrium is greater than that under the signal revealing equilibrium (Proposition 5).

The above summary distills into one of the main insights of our paper:

Corollary 3. *Suppose* q *is small, and* $k(\xi) < k \leq z$*. Then second expert can be meaningful only under the secrecy protocol.*

In the context of transparent (sequential) debates with heterogenous experts of ***known*** abilities, Ottaviani and Sorensen (2001) already pointed out why having too good a first expert might render the second expert's opinion meaningless. Our above observation goes well beyond Ottaviani-Sorensen, in answering the broader question of transparency vs. secrecy when the experts' abilities are private information. Also in contrast to Ottaviani-Sorensen's experts, our second expert is of the same expected quality as the first expert.

7 Deliberation, detailed advice and revelation of types

Under secrecy, we saw that partial type revelation was possible. But can *both* experts reveal their entire type (signal and precision level)? We show that this is possible if experts are allowed to deliberate through back-and-forth messages or alternatively to talk more meaningfully. We do not pose the same question for transparency. It is clear from Section 4 that since payoffs are a function of the expected type, under transparency a low ability expert would mimic the recommendations of the high ability expert.

Consider a communication format under secrecy where there are four stages. In each stage an expert recommends an action, i.e., an element from $\{A, B\}$. The experts and D observe all recommendations but O does not. A recommendation profile is a four-dimensional vector with co-ordinates belonging to $\{A, B\}$. Following the recommendation, D chooses $d \in \{A, B\}$. As before, O observes d and ω and forms expectations about the experts' types. These expectations are the payoffs of the experts. We shall be interested in the existence of an equilibrium under which *both* experts reveal their entire two dimensional type. We consider two communication formats: *deliberation* and *detailed recommendation.*[21]

Note that the experts get to send two messages, which we call recommendations. Since messages do not have any content in terms of meaning, we can let experts choose "reports" from $\{\alpha, \beta\} \times \{\xi, \lambda\}$ instead of "recommendations" from $\{A, B\} \times \{A, B\}$. Thus, unlike in the

[21] For the latter, the communication effectively reduces to only two stages.

previous sections, each expert now has access to messages with the richness to convey the entire content of information.

Under deliberation, in stages one and three, expert i reports and in stages two and four, expert j reports. The first report is about the signal and the second report is about the signal's precision. Under detailed recommendation, in stage one expert i makes a recommendation together with an indication of its precision (i.e., first two stages rolled into one), followed by a similar communication in stage two (last two stages combined) by expert j.

We consider only truth-telling equilibria, so D can correctly deduce the signal and precision level of both the experts from the communication. Call such an equilibrium a *fully revealing equilibrium.*

Conditional on the posterior formed on the basis of a profile of reports, D makes the choice d. In the case of deliberation, Table 1 lists the report profiles under a fully revealing equilibrium (assuming that it exists). In the case of detailed recommendation, since strategies reveal types, one should ignore the "Report Profile" column and simply view the next column as the deduced profile of types. In both formats of communication, in the continuation game following recommendations (or reports), the actions of D and O would be the same in equilibrium. The optimal d is derived from "Types Deduced by D". Since this is a straightforward derivation, we simply state the optimal choice of d in the last two columns.

When q is large D's choice of B depends only on the first report of each expert, i.e., only on the experts' signals of β. Since this is similar to the two-stage case studied earlier, we focus on the case where q is not large. So let $\frac{q}{1-q} < r$.

We now come to O's beliefs. Consider the case where q is not large. Conditional on (d, ω), O's beliefs are

$$\begin{aligned}
\Pr(\lambda \mid A, a) &= \frac{\theta\lambda[1+\theta(1-\lambda)]}{\theta\lambda[1+\theta(1-\lambda)]+(1-\theta)[k+(1-\theta)\xi(1-\xi)]}, \\
\Pr(\lambda \mid A, b) &= \frac{\theta(1-\lambda)[1+\theta\lambda)]}{\theta(1-\lambda)[1+\theta\lambda)]+(1-\theta)[(1-k)+(1-\theta)\xi(1-\xi)]}, \\
\Pr(\lambda \mid B, b) &= \frac{\theta\lambda[(1-\theta)+\theta\lambda]}{\theta\lambda[(1-\theta)+\theta\lambda]+(1-\theta)[(1-\theta)\xi^2+\theta\lambda]}, \\
\Pr(\lambda \mid B, a) &= \frac{\theta(1-\lambda)[(1-\theta)+\theta(1-\lambda)]}{\theta(1-\lambda)[(1-\theta)+\theta(1-\lambda)]+(1-\theta)[(1-\theta)(1-\xi)^2+\theta(1-\lambda)]}.
\end{aligned} \tag{12}$$

Define

$$\begin{aligned}
g' &= \Pr(\lambda \mid A, a), \\
h'' &= \Pr(\lambda \mid B, a), \\
g'' &= \Pr(\lambda \mid B, b), \\
h' &= \Pr(\lambda \mid A, b).
\end{aligned} \tag{13}$$

Table 1:

	Report Profile	Types Deduced by D	d	q large	q not large
1.	$(\alpha, \alpha, \lambda, \lambda)$	$\{(\alpha, \lambda), (\alpha, \lambda)\}$	A		
2.	$(\alpha, \alpha, \lambda, \xi)$	$\{(\alpha, \lambda), (\alpha, \xi)\}$	A		
3.	$(\alpha, \alpha, \xi, \lambda)$	$\{(\alpha, \xi), (\alpha, \lambda)\}$	A		
4.	$(\alpha, \alpha, \xi, \xi)$	$\{(\alpha, \xi), (\alpha, \xi)\}$	A		
5.	$(\alpha, \beta, \lambda, \xi)$	$\{(\alpha, \lambda), (\beta, \xi)\}$	A		
6.	$(\alpha, \beta, \lambda, \lambda)$	$\{(\alpha, \lambda), (\beta, \lambda)\}$	A		
7.	$(\alpha, \beta, \xi, \xi)$	$\{(\alpha, \xi), (\beta, \xi)\}$	A		
8.	$(\alpha, \beta, \xi, \lambda)$	$\{(\alpha, \xi), (\beta, \lambda)\}$		A	B
9.	$(\beta, \alpha, \xi, \lambda)$	$\{(\beta, \xi), (\alpha, \lambda)\}$	A		
10.	$(\beta, \alpha, \xi, \xi)$	$\{(\beta, \xi), (\alpha, \xi)\}$	A		
11.	$(\beta, \alpha, \lambda, \lambda)$	$\{(\beta, \lambda), (\alpha, \lambda)\}$	A		
12.	$(\beta, \alpha, \lambda, \xi)$	$\{(\beta, \lambda), (\alpha, \xi)\}$		A	B
13.	(β, β, ξ, ξ)	$\{(\beta, \xi), (\beta, \xi)\}$	B		
14.	$(\beta, \beta, \xi, \lambda)$	$\{(\beta, \xi), (\beta, \lambda)\}$	B		
15.	$(\beta, \beta, \lambda, \xi)$	$\{(\beta, \lambda), (\beta, \xi)\}$	B		
16.	$(\beta, \beta, \lambda, \lambda)$	$\{(\beta, \lambda), (\beta, \lambda)\}$	B		

Let

$$Q = \max\left\{ \left(\frac{1-\xi}{\xi} \Big/ \frac{\xi}{1-\xi}\right), \left(\frac{\xi}{1-\xi} \Big/ \frac{\lambda}{1-\lambda}\right) \right\}.$$

The proof of the following Proposition is in Appendix A.

Proposition 6 (Deliberation and detailed advice)**.** *Let the recommendation protocol be secrecy, and* $q/(1-q) < r$*. A fully revealing equilibrium exists under deliberation or detailed recommendation if and only if*

$$\frac{q}{1-q}(g' - h'') \geq (g'' - h') \geq \frac{q}{1-q} \cdot Q \cdot (g' - h'').$$

Proposition 6 informs us that for a fully revealing equilibrium to exist we need $g' - h'' \geq 0$ and $g'' - h' \geq 0$ (as $Q < 1$). However, conformity bias, i.e. $g' - h'' \geq g'' - h'$, is not necessary.

It is somewhat surprising that detailed recommendation does not improve upon deliberation as far as fully revealing equilibria are concerned. In a parallel result, Dekel and Piccione (2000) had shown in a sequential binary-option unanimity voting model that the voting equilibria, and thus information aggregation, did not depend on the particular order of sequential voting. Our model is very different – (i) cheap talk and independent decision maker, (ii) experts have multiple opportunities to communicate their information (as opposed to one-time vote), (iii) experts are intrinsically disinterested in the decision, etc. Our deliberation/detailed recommendation can sometimes overcome fully the herding incentive, whereas herding persists in Dekel and Piccione's sequential voting. Casual intuition weighs in for deliberation to induce more herding. This suggests that one should study the two communication formats under other equilibria as well – a task we leave for future research.

■ **An Example.** Throughout we round off to approximate values. Let $\lambda = \frac{9}{10}$ and $\xi = \frac{6}{10}$. Recall that for a fully revealing equilibrium we need

$$\frac{q}{1-q}(g' - h'') \geq (g'' - h') \geq \frac{q}{1-q} \cdot Q \cdot (g' - h''),$$

where

$$Q = \max\left\{\left(\frac{1-\xi}{\xi} / \frac{\xi}{1-\xi}\right), \left(\frac{\xi}{1-\xi} / \frac{\lambda}{1-\lambda}\right)\right\}.$$

Given our parameter specification,

$$Q = \frac{4}{9}.$$

Now,

$$g' - h'' = 100\theta \frac{\theta^2 + \theta - 2}{3\theta^4 + 24\theta^3 + 116\theta^2 + 272\theta - 448},$$

and we have that $g' - h'' \geq 0$ for all $\theta \in [0, 1]$. Also,

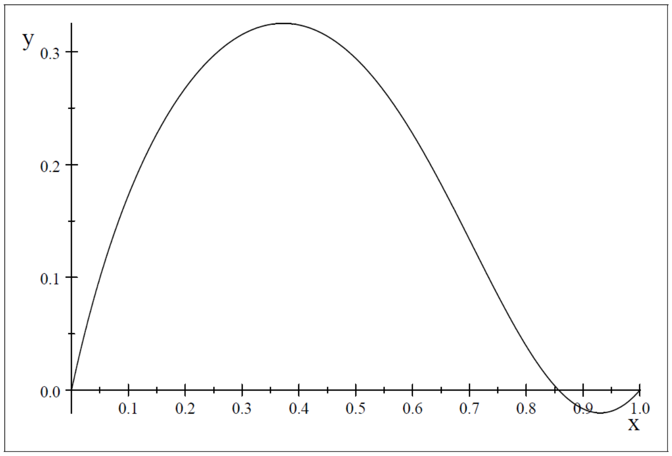
$$g'' - h' = 100\theta \frac{7\theta^2 - 13\theta + 6}{-441\theta^4 + 1512\theta^3 - 1492\theta^2 + 336\theta + 256}.$$

Note as $g'' - h' < 0$ for $\theta \in (.855, 1)$, a fully revealing equilibrium does not exist for large lead opinion biases; see Fig. 4.

Now let $\frac{q}{1-q} = \frac{51}{49}$. So,

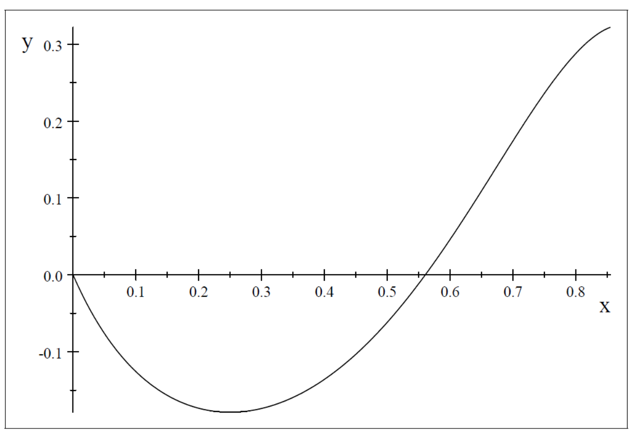
$$\begin{aligned} & \frac{51}{49}(g' - h'') - (g'' - h') \\ = \; & (\frac{51}{49})100\theta \frac{\theta^2 + \theta - 2}{3\theta^4 + 24\theta^3 + 116\theta^2 + 272\theta - 448} - 100\theta \frac{7\theta^2 - 13\theta + 6}{-441\theta^4 + 1512\theta^3 - 1492\theta^2 + 336\theta + 256} \\ = \; & -\frac{2000}{49}\theta \frac{-1176\theta^6 + 2415\theta^5 + 1031\theta^4 - 11\,982\theta^3 + 23\,760\theta^2 - 19\,328\theta + 5280}{1323\theta^8 + 6048\theta^7 + 19\,344\theta^6 - 20\,640\theta^5 - 444\,592\theta^4 + 1038\,080\theta^3 - 789\,504\theta^2 + 80\,896\theta + 114\,688}. \end{aligned}$$

Figure 4:



The graph of $\frac{51}{49}(g' - h'') - (g'' - h')$ is plotted in Fig. 5.

Figure 5:

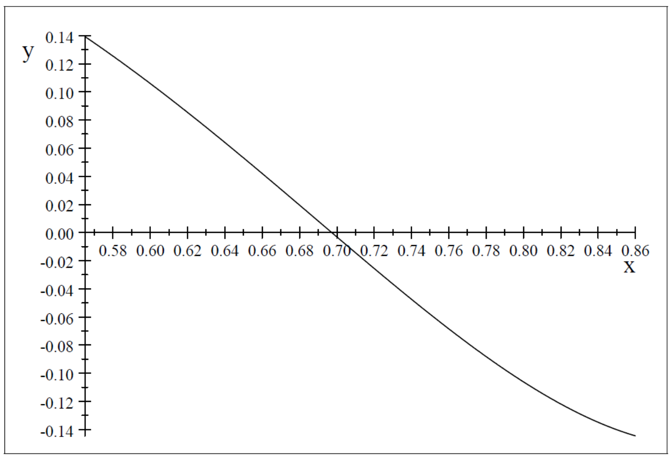


For $\theta \in (0.0.565)$, we have that $\frac{51}{49}(g' - h'') < (g'' - h')$, so because of this anti-conformity bias a fully revealing equilibrium does not exist for low lead opinion bias. Now consider

$$
\begin{aligned}
& (g'' - h') - \frac{51}{49}\frac{4}{9}(g' - h'') \\
= \; & 100\theta \frac{7\theta^2 - 13\theta + 6}{-441\theta^4 + 1512\theta^3 - 1492\theta^2 + 336\theta + 256} - (\frac{51}{49})(\frac{4}{9}) 100\theta \frac{\theta^2 + \theta - 2}{3\theta^4 + 24\theta^3 + 116\theta^2 + 272\theta - 448} \\
= \; & -\frac{500}{147}\theta \frac{6615\theta^6 - 10\,773\theta^5 + 2962\theta^4 + 72\,724\theta^3 - 224\,328\theta^2 + 224\,864\theta - 72\,064}{1323\theta^8 + 6048\theta^7 + 19\,344\theta^6 - 20\,640\theta^5 - 444\,592\theta^4 + 1038\,080\theta^3 - 789\,504\theta^2 + 80\,896\theta + 114\,688}.
\end{aligned}
$$

The graph of $(g'' - h') - \frac{51}{49}\frac{4}{9}(g' - h'')$ is plotted in Fig. 6.

Figure 6:



We see that $g'' - h' \geq \frac{51}{49}\frac{4}{9}(g' - h'')$ only in the range $\theta \in (0.565, 0.698)$. So when $\frac{q}{1-q} = \frac{51}{49}$, a fully revealing equilibrium exists only when $\theta \in (0.565, 0.698)$.

8 Conclusion

Career-concern motive suggests that politicians, experts, and any advisor with special predictive talent who want to progress further should like the "market" to know about their skills. So when presented with an opportunity to engage in debates and deliberation, any organization should create a platform through which their motivated advisors give them the best advice possible.

Modelling the sequential nature of soliciting advice in real life, this paper shows that for more informed decisions it is better to block out the actual advice with their ordering or deliberations from public view. Since under secrecy outsiders cannot associate specific recommendations to individual experts, herding incentives could be mitigated leading to full signal revelation. Moreover, sometimes the decision maker may even strictly prefer inducing partial herding (over full signal revelation) to be able to get more precise information about one of the recommender's expert skills.

When it comes to back-and-forth deliberation, not just sequential advice, secret protocol may help transmit the full information about the experts' signals as well as signal qualities to the decision maker. This delivers the best possible decisions. But deliberation may also fail to transmit the desired information due to problems associated with the existence of a fully revealing equilibrium (i.e., when the condition in Proposition 6 fails to hold). Besides, time constraints on deliberations may render deliberations ineffective, in which case secret sequential advice, from constrained message sets, seems a natural procedure to follow. Finally, as far as full revelation is concerned, detailed recommendation does not seem to outperform back and forth deliberation.

This work fills a gap in the optimal design of transparency of decision making in organizations and political decisions by studying alternative disclosure protocols when career-concerned experts deliberate over issues.

A Appendix

■ **Section 3 materials.** The joint distribution of signal and state, given t_e, is as follows:

Table A.1: Joint distribution of signal and state

	a	b
α	qt_e	$(1-q)(1-t_e)$
β	$q(1-t_e)$	$(1-q)t_e$

An expert infers the state from her signal using Bayes' rule, e.g., $\Pr(\omega = a \mid s_e = \alpha, t_e) = \frac{qt_e}{qt_e+(1-q)(1-t_e)}$. We assume that the distribution of the experts' signals conditional on the state are independent.

Below we report the joint distribution over state, signals and expert abilities. Given our assumption on independence (of both signals and abilities), we have (to facilitate reading we divide the distribution into two tables, and for only the following tables let $q' = (1-q)$):

Table A.2: Joint distribution of state, signals and abilities

	$t_i = \lambda, t_j = \lambda$	$t_i = \lambda, t_j = \xi$	$t_i = \xi, t_j = \lambda$	$t_i = \xi, t_j = \xi$
a, α, α	$q\theta^2\lambda^2$	$q\theta(1-\theta)\lambda\xi$	$q\theta(1-\theta)\lambda\xi$	$q(1-\theta)^2\xi^2$
a, α, β	$q\theta^2\lambda(1-\lambda)$	$q\theta(1-\theta)\lambda(1-\xi)$	$q\theta(1-\theta)\xi(1-\lambda)$	$q(1-\theta)^2\xi(1-\xi)$
a, β, α	$q\theta^2\lambda(1-\lambda)$	$q\theta(1-\theta)(1-\lambda)\xi$	$q\theta(1-\theta)(1-\xi)\lambda$	$q(1-\theta)^2(1-\xi)\xi$
a, β, β	$q\theta^2(1-\lambda)^2$	$q\theta(1-\theta)(1-\lambda)(1-\xi)$	$q\theta(1-\theta)(1-\xi)(1-\lambda)$	$q(1-\theta)^2(1-\xi)^2$

Table A.3: Joint distribution of state, signals and abilities

	$t_i = \lambda, t_j = \lambda$	$t_i = \lambda, t_j = \xi$	$t_i = \xi, t_j = \lambda$	$t_i = \xi, t_j = \xi$
b, α, α	$q'\theta^2(1-\lambda)^2$	$q'\theta(1-\theta)(1-\lambda)(1-\xi)$	$q'\theta(1-\theta)(1-\xi)(1-\lambda)$	$q'(1-\theta)^2(1-\xi)^2$
b, α, β	$q'\theta^2\lambda(1-\lambda)$	$q'\theta(1-\theta)(1-\lambda)\xi$	$q'\theta(1-\theta)(1-\xi)\lambda$	$q'(1-\theta)^2(1-\xi)\xi$
b, β, α	$q'\theta^2\lambda(1-\lambda)$	$q'\theta(1-\theta)\lambda(1-\xi)$	$q'\theta(1-\theta)\xi(1-\lambda)$	$q'(1-\theta)^2\xi(1-\xi)$
b, β, β	$q'\theta^2\lambda^2$	$q'\theta(1-\theta)\lambda\xi$	$q'\theta(1-\theta)\lambda\xi$	$q'(1-\theta)^2\xi^2$

The beliefs of expert i who moves first, conditional on her ability and signal are given by $\Pr(\omega \mid s_i, t_i)$ as follows (refer Table A.1):

$$\begin{aligned}
\Pr(a \mid \alpha, t_i) &= \frac{qt_i}{qt_i+(1-q)(1-t_i)} && > \frac{1}{2}, \\
\Pr(b \mid \alpha, t_i) &= \frac{(1-q)(1-t_i)}{qt_i+(1-q)(1-t_i)} && < \frac{1}{2}, \\
\Pr(a \mid \beta, t_i) &= \frac{q(1-t_i)}{q(1-t_i)+(1-q)t_i} && < \frac{1}{2}, \\
\Pr(b \mid \beta, t_i) &= \frac{(1-q)t_i}{q(1-t_i)+(1-q)t_i} && > \frac{1}{2}\ .
\end{aligned} \tag{A.1}$$

The inequalities follow from Assumption 1 and Fact 1. Note that despite the prior bias in favor of state a ($q > \frac{1}{2}$), signal β reverses this belief for either ability of expert.

Next consider expert j who moves second. Suppose she were to deduce the first expert's signal, but not her ability, from the observed recommendation. She would then update her beliefs conditional on (s_i, s_j, t_j), as follows (Assumption 1 and Fact 1 are used to establish the inequalities):

$$\begin{aligned}
\Pr(a \mid \alpha, \alpha, t_j) &= \frac{qt_jk}{qt_jk+(1-q)(1-t_j)(1-k)} && > \frac{1}{2}, \\
\Pr(b \mid \alpha, \alpha, t_j) &= \frac{(1-q)(1-t_j)(1-k)}{qt_jk+(1-q)(1-t_j)(1-k)} && < \frac{1}{2}, \\
\Pr(a \mid \beta, \beta, t_j) &= \frac{q(1-t_j)(1-k)}{(1-q)t_jk+q(1-t_j)(1-k)} && < \frac{1}{2}, \\
\Pr(b \mid \beta, \beta, t_j) &= \frac{(1-q)t_jk}{(1-q)t_jk+q(1-t_j)(1-k)} && > \frac{1}{2}.
\end{aligned} \tag{A.2}$$

These four probabilities indicate that when the second expert's signal matches that of the first, then irrespective of ability, her posterior on state a (b) is higher than that on b (a) when she sees signal α (β).

When the first expert's signal is deduced as β and the second expert of ability λ receives signal α, we have:

$$\begin{aligned} \Pr(a \mid \beta, \alpha, \lambda) &= \tfrac{q\lambda(1-k)}{q\lambda(1-k)+(1-q)(1-\lambda)k} > \tfrac{1}{2}, \\ \Pr(b \mid \beta, \alpha, \lambda) &= \tfrac{(1-q)(1-\lambda)k}{q\lambda(1-k)+(1-q)(1-\lambda)k} < \tfrac{1}{2}. \end{aligned} \tag{A.3}$$

When the first expert's signal is deduced as α and the second expert of ability ξ receives signal β, we have:

$$\begin{aligned} \Pr(a \mid \alpha, \beta, \xi) &= \tfrac{q(1-\xi)k}{q(1-\xi)k+(1-q)\xi(1-k)} > \tfrac{1}{2}, \\ \Pr(b \mid \alpha, \beta, \xi) &= \tfrac{(1-q)\xi(1-k)}{q(1-\xi)k+(1-q)\xi(1-k)} < \tfrac{1}{2}. \end{aligned} \tag{A.4}$$

■ **The proofs**

Proof Lemma 2. (i) Let

$$\gamma_i = \frac{\theta\lambda}{k} \cdot \lambda + \frac{(1-\theta)\xi}{k} \cdot \xi, \qquad \gamma_i' = \frac{\theta(1-\lambda)}{1-k} \cdot \lambda + \frac{(1-\theta)(1-\xi)}{1-k} \cdot \xi.$$

By Assumption 1, we have:

$$\gamma_i > \gamma_i'.$$

Assume that expert 1 reveals her signal (we will have to show that this is indeed so in equilibrium). Since O's beliefs depend on i's recommendation, which he observes, the second expert's recommendation does not matter. Then,

$$\begin{aligned} E_i^t(A, \nu_j, a) = E_i^t(B, \nu_j, b) = E_i^t(A, a) = E_i^t(B, b) = \gamma_i, \\ E_i^t(A, \nu_j, b) = E_i^t(B, \nu_j, a) = E_i^t(A, b) = E_i^t(B, a) = \gamma_i'. \end{aligned}$$

Hence, as the second expert's recommendation does not matter,

$$\begin{aligned} \Pi_i^t(A, \nu_j(A), \alpha, t_i) &= \Pr(a|\alpha, t_i)\gamma_i + (1 - \Pr(a|\alpha, t_i))\gamma_i', \\ \Pi_i^t(B, \nu_j(B), \alpha, t_i) &= \Pr(a|\alpha, t_i)\gamma_i' + (1 - \Pr(a|\alpha, t_i))\gamma_i, \\ \Pi_i^t(A, \nu_j(A), \beta, t_i) &= \Pr(a|\beta, t_i)\gamma_i + (1 - \Pr(a|\beta, t_i))\gamma_i', \\ \Pi_i^t(B, \nu_j(B), \beta, t_i) &= \Pr(a|\beta, t_i)\gamma_i' + (1 - \Pr(a|\beta, t_i))\gamma_i. \end{aligned}$$

For i to reveal her signal, we need to show that:

$$\Pi_i^t(A, v_j(A), \alpha, t_i) \geq \Pi_i^t(B, v_j(B), \alpha, t_i),$$

$$\Pi_i^t(B, v_j(B), \beta, t_i) \geq \Pi_i^t(A, v_j(A), \beta, t_i)$$

or,

$$\Pr(a|\alpha, t_i)\gamma_i + (1 - \Pr(a|\alpha, t_i))\gamma_i' \geq \Pr(a|\alpha, t_i)\gamma_i' + (1 - \Pr(a|\alpha, t_i))\gamma_i,$$

$$\Pr(a|\beta, t_i)\gamma_i' + (1 - \Pr(a|\beta, t_i))\gamma_i \geq \Pr(a|\beta, t_i)\gamma_i + (1 - \Pr(a|\beta, t_i))\gamma_i'$$

or,

$$\Pr(a|\alpha, t_i)(\gamma_i - \gamma_i') \geq \Pr(b|\alpha, t_i))(\gamma_i - \gamma_i'),$$

$$\Pr(b|\beta, t_i)(\gamma_i - \gamma_i') \geq \Pr(a|\beta, t_i)(\gamma_i - \gamma_i')$$

or, as $\gamma_i > \gamma_i'$,

$$\Pr(a|\alpha, t_i) \geq \Pr(b|\alpha, t_i),$$

$$\Pr(b|\beta, t_i) \geq \Pr(a|\beta, t_i)$$

which is true since $\Pr(a|\alpha, t_i) = \frac{t_i q}{t_i q + (1-t_i)(1-q)} > \frac{(1-t_i)(1-q)}{t_i q + (1-t_i)(1-q)} = \Pr(b|\alpha, t_i)$, or equivalently $t_i q > (1 - t_i)(1 - q)$, or equivalently $q + t_i > 1$ given that $q > 1/2$ and $t_i > q$ (Assumption 1), and likewise for the second inequality. So the first expert recommends her signal in equilibrium.

(ii) See the proof of Lemma 3 below where the same strategies by expert j (following any specific recommendation v_i) will be eliminated. The method of eliminations here will follow the same principles. The only difference is that for expert i there is no prior history except the null history, which simplifies the conditional probability calculations.

(iii) Expert i babbling can be supported in equilibrium by assuming that any deviation, say when i recommends B given that the posited equilibrium strategy is (A, A, A, A) (say), is punished by the outsider's belief that i must be of ability ξ. This is worse than the pooling ability k with which i will be perceived if she plays her equilibrium strategy. ■

Proof of Lemma 3. **Case 1**: Let $v_j(v_i) \in \mathbf{V}_j^{st} = \{\mathbf{v} \mid \mathbf{v} \in \mathbf{V}_j; \#A(v) = 1 \text{ or } \#B(v) = 1\}$. Consider (i) $v_j(v_i) = (A, A, A, B)$, or (ii) $v_j(v_i) = (A, A, B, A)$. The other possibilities can be analyzed similarly.

(i) Suppose $v_j(v_i) = (A, A, A, B)$. Then, $E_j^t(v_i, B, a) = E_j^t(v_i, B, b) = \lambda$. So

$$\Pi_j^t(v_i, B, \alpha, t_j) = \lambda,$$

and

$$E_j^t(\nu_i, A, a) = \left(\frac{\theta\lambda}{\theta\lambda + (1-\theta)\xi + (1-\theta)(1-\xi)}\right)\lambda + \left(\frac{(1-\theta)\xi + (1-\theta)(1-\xi)}{\theta\lambda + (1-\theta)\xi + (1-\theta)(1-\xi)}\right)\xi < \lambda,$$
$$E_j^t(\nu_i, A, b) = \left(\frac{\theta(1-\lambda)}{(1-\theta\lambda - (1-\theta)\xi) + (1-\theta)\xi}\right)\lambda + \left(\frac{(1-\theta)\xi + (1-\theta)(1-\xi)}{(1-\theta\lambda - (1-\theta)\xi) + (1-\theta)\xi}\right)\xi < \lambda.$$

So, for both abilities $t_j \in \{\xi, \lambda\}$ who observe α, recommending B is better than recommending A as:

$$\Pi_j^t(\nu_i, A, \alpha, t_j) = \Pr(a|\nu_i, \alpha, t_j) \cdot E_j^t(\nu_i, A, a) + \Pr(b|\nu_i, \alpha, t_j) \cdot E_j^t(\nu_i, A, b) < \lambda = \Pi_j^t(\nu_i, B, \alpha, t_j).$$

Thus, $\nu_j(\nu_i) = (A, A, A, B)$ cannot be an equilibrium strategy.

(ii) Now suppose $\nu_j(\nu_i) = (A, A, B, A)$. Then, $E_j^t(\nu_i, B, a) = E_j^t(\nu_i, B, b) = \xi$. Using arguments similar to above, one can see that

$$\Pi_j^t(\nu_i, B, \beta, \xi) = \xi < \Pi_j^t(\nu_i, A, \beta, \xi),$$

so $\nu_j(\nu_i) = (A, A, B, A)$ cannot be an equilibrium strategy.

Case 2: Consider $\nu_j(\nu_i) = (A, B, A, B)$. Then

$$E_j^t(\nu_i, B, a) = E_j^t(\nu_i, B, b) = \lambda, \qquad E_j^t(\nu_i, A, a) = E_j^t(\nu_i, A, b) = \xi$$
$$\text{and} \qquad \Pi_j^t(\nu_i, A, \alpha, \xi) = \xi < \lambda = \Pi_j^t(\nu_i, B, \alpha, \xi).$$

So $\nu_j(\nu_i) = (A, B, A, B)$ cannot be an equilibrium strategy. The strategy $\nu_j(\nu_i) = (B, A, B, A)$ can be eliminated likewise.

Case 3: Let $\nu_j(\nu_i) \in \mathbf{V}_j^{sc} = \{(B, A, A, B), (A, B, B, A)\}$. Consider the case where $\nu_j(\nu_i) = (B, A, A, B)$. Then,

$$E_j^t(\nu_i, B, b) = E_j^t(\nu_i, A, a) = \left(\frac{\theta\lambda}{\theta\lambda + (1-\theta)(1-\xi)}\right)\lambda + \left(\frac{(1-\theta)(1-\xi)}{\theta\lambda + (1-\theta)(1-\xi)}\right)\xi \equiv \eta,$$
$$E_j^t(\nu_i, B, a) = E_j^t(\nu_i, A, b) = \left(\frac{\theta(1-\lambda)}{\theta(1-\lambda) + (1-\theta)\xi}\right)\lambda + \left(\frac{(1-\theta)\xi}{\theta(1-\lambda) + (1-\theta)\xi}\right)\xi \equiv \eta'.$$

Note that

$$\eta > \eta',$$
$$\text{since} \qquad \frac{\theta\lambda}{\theta\lambda + (1-\theta)(1-\xi)} > \frac{\theta(1-\lambda)}{\theta(1-\lambda) + (1-\theta)\xi}, \quad \text{i.e.,} \quad \frac{\lambda}{1-\lambda} > \frac{1-\xi}{\xi},$$

given Assumption 1 and $\xi > q > 1/2$.

Also,

$$\begin{aligned}\Pi_j^t(\nu_i, B, \alpha, \xi) &= \Pr(a \mid \nu_i, \alpha, \xi)\eta' + \Pr(b \mid \nu_i, \alpha, \xi)\eta, \\ \Pi_j^t(\nu_i, A, \alpha, \xi) &= \Pr(a \mid \nu_i, \alpha, \xi)\eta + \Pr(b \mid \nu_i, \alpha, \xi)\eta'.\end{aligned}$$

In equilibrium we need $\Pi_j^t(\nu_i, B, \alpha, \xi) \geq \Pi_j^t(\nu_i, A, \alpha, \xi)$, which is true if and only if (as $\eta > \eta'$):

$$\Pr(b \mid \nu_i, \alpha, \xi) \geq \Pr(a \mid \nu_i, \alpha, \xi).$$

If the first expert babbled then $\Pr(\omega \mid \nu_i, \alpha, \xi) = \Pr(\omega \mid \alpha, \xi)$. So due to (A.1) the above cannot hold. Now suppose the first expert recommends truthfully. But then, due to (A.2) (see also (5)), it must be the case that the first expert observed β and recommended $\nu^1 = B$ as she recommends her signal. (If the first expert observed α and recommended her signal, then that would have meant $\Pr(b \mid \alpha, \alpha, t_j) \geq \Pr(a \mid \alpha, \alpha, t_j)$, contradicting the first two inequalities in (A.2) for $t_j = \xi$.) Now,

$$\begin{aligned}\Pi_j^t(\nu_i, A, \beta, \xi) &= \Pr(a \mid \beta, \beta, \xi)\eta + \Pr(b \mid \beta, \beta, \xi)\eta', \\ \Pi_j^t(\nu_i, B, \beta, \xi) &= \Pr(a \mid \beta, \beta, \xi)\eta' + \Pr(b \mid \beta, \beta, \xi)\eta.\end{aligned}$$

In equilibrium we need $\Pi_j^t(\nu_i, A, \beta, \xi) \geq \Pi_j^t(\nu_i, B, \beta, \xi)$, which is true if and only if (as $\eta > \eta'$):

$$\Pr(a \mid \beta, \beta, \xi) \geq \Pr(b \mid \beta, \beta, \xi).$$

But then this contradicts the last two inequalities in (A.2) for $t_j = \xi$. So $\nu_j(\nu_i) = (B, A, A, B)$ cannot be an equilibrium strategy.

Next consider the case where $\nu_j(\nu_i) = (A, B, B, A)$. Then,

$$\begin{aligned}E_j^t(\nu_i, B, a) &= E_j^t(\nu_i, A, b) = \left(\frac{\theta\lambda}{\theta\lambda + (1-\theta)(1-\xi)}\right)\lambda + \left(\frac{(1-\theta)(1-\xi)}{\theta\lambda + (1-\theta)\xi}\right)\xi \equiv \eta, \\ E_j^t(\nu_i, B, b) &= E_j^t(\nu_i, A, a) = \left(\frac{\theta(1-\lambda)}{\theta(1-\lambda) + (1-\theta)\xi}\right)\lambda + \left(\frac{(1-\theta)\xi}{\theta(1-\lambda) + (1-\theta)\xi}\right)\xi \equiv \eta'.\end{aligned}$$

As shown previously,

$$\eta > \eta'.$$

Also,

$$\begin{aligned}\Pi_j^t(\nu_i, A, \alpha, \xi) &= \Pr(a \mid \nu_i, \alpha, \xi)\eta' + \Pr(b \mid \nu_i, \alpha, \xi)\eta, \\ \Pi_j^t(\nu_i, B, \alpha, \xi) &= \Pr(a \mid \nu_i, \alpha, \xi)\eta + \Pr(b \mid \nu_i, \alpha, \xi)\eta'.\end{aligned}$$

In equilibrium we need $\Pi_j^t(\nu_i, A, \alpha, \xi) \geq \Pi_j^t(\nu_i, B, \alpha, \xi)$, which is true if and only if (as $\eta > \eta'$):

$$\Pr(b \mid \nu_i, \alpha, \xi) \geq \Pr(a \mid \nu_i, \alpha, \xi).$$

Again, if the first expert babbled then $\Pr(\omega \mid \nu_i, \alpha, \xi) = \Pr(\omega \mid \alpha, \xi)$. So due to (A.1) the above cannot hold. Now suppose the first expert recommends truthfully. But then due to (A.2) it must be the case that the first expert observed β and recommended $\nu_i = B$. Now,

$$\begin{aligned}
\Pi_j^t(\nu_i, B, \beta, \xi) &= \Pr(a \mid \beta, \beta, \xi)\eta + \Pr(b \mid \beta, \beta, \xi)\eta', \\
\Pi_j^t(\nu_i, A, \beta, \xi) &= \Pr(a \mid \beta, \beta, \xi)\eta' + \Pr(b \mid \beta, \beta, \xi)\eta.
\end{aligned}$$

In equilibrium we need $\Pi_j^t(\nu_i, B, \beta, \xi) \geq \Pi_j^t(\nu_i, B, \beta, \xi)$, which is true if and only if (as $\eta > \eta'$):

$$\Pr(a \mid \beta, \beta, \xi) \geq \Pr(b \mid \beta, \beta, \xi).$$

But then again this contradicts the last two inequalities in (A.2) for $t_j = \xi$. So $\nu_j(\nu_i) = (A, B, B, A)$ cannot be an equilibrium strategy. ■

Proof of Lemma 4. If the first expert babbled then $\Pr(\omega \mid \nu_i, s_j, t_j) = \Pr(\omega \mid s_j, t_j)$. Then, as in (i) of Lemma 1, $\nu_j(\nu_i) = (A, A, B, B)$ is an equilibrium strategy.

Now let the first expert recommend truthfully and let $\nu_j(\nu_i) = (A, A, B, B)$ (we will have to show that this is an equilibrium). Then,

$$\begin{aligned}
E_j^t(\nu_i, B, b) &= E_j^t(\nu_i, A, a) = \left(\frac{\theta\lambda}{\theta\lambda + (1-\theta)\xi}\right)\lambda + \left(\frac{(1-\theta)\xi}{\theta\lambda + (1-\theta)\xi}\right)\xi \equiv \gamma, \\
E_j^t(\nu_i, B, a) &= E_j^t(\nu_i, A, b) = \left(\frac{\theta(1-\lambda)}{\theta(1-\lambda) + (1-\theta)(1-\xi)}\right)\lambda + \left(\frac{(1-\theta)(1-\xi)}{\theta(1-\lambda) + (1-\theta)(1-\xi)}\right)\xi \equiv \gamma'.
\end{aligned}$$

These outside evaluations of j's ability, under transparency, do not depend on the first expert's recommendation as per the projected strategy of j.

Note that,

$$\gamma > \gamma'.$$

Now first consider when j observes signal β and she is of ability ξ. Her expected payoffs are:

$$\begin{aligned}
\Pi_j^t(\nu_i, B, \beta, \xi) &= \Pr(a \mid \nu_i, \beta, \xi)\gamma' + \Pr(b \mid \nu_i, \beta, \xi)\gamma, \\
\Pi_j^t(\nu_i, A, \beta, \xi) &= \Pr(a \mid \nu_i, \beta, \xi)\gamma + \Pr(b \mid \nu_i, \beta, \xi)\gamma'.
\end{aligned}$$

In equilibrium we need $\Pi_j^t(\nu_i, B, \beta, \xi) \geq \Pi_j^t(\nu_i, A, \beta, \xi)$, which is true if and only if (as $\gamma > \gamma'$):

$$\Pr(b \mid \nu_i, \beta, \xi) \geq \Pr(a \mid \nu_i, \beta, \xi). \tag{A.5}$$

This can hold only if the first expert were to observe β and recommend B, otherwise (A.4) would be violated; this establishes the necessity of part (i) condition of the lemma. That (A.5) will be satisfied if $s_i = \beta$ and $\nu_i = B$ follows from (A.2).

Similarly, the condition for truthful recommendation by j for $t_j = \lambda$ and $s_j = \beta$ is given by

$$\Pr(b \mid \nu_i, \beta, \lambda) \geq \Pr(a \mid \nu_i, \beta, \lambda),$$

which, for $s_i = \beta$ and $\nu_i = B$, will be satisfied, given (A.2).

Next suppose j observes signal α. Then for $t_j = \xi$,

$$\begin{aligned}
\Pi_j^t(\nu_i, A, \alpha, \xi) &= \Pr(a \mid \beta, \alpha, \xi)\gamma + \Pr(b \mid \beta, \alpha, \xi)\gamma', \\
\Pi_j^t(\nu^1, B, \alpha, \xi) &= \Pr(a \mid \beta, \alpha, \xi)\gamma' + \Pr(b \mid \beta, \alpha, \xi)\gamma.
\end{aligned}$$

The equilibrium requirement $\Pi_j^t(\nu_i, A, \alpha, \xi) \geq \Pi_j^t(\nu_i, B, \alpha, \xi)$, given $\gamma > \gamma'$, is equivalent to:

$$\Pr(a \mid \beta, \alpha, \xi) \geq \Pr(b \mid \beta, \alpha, \xi) \quad \text{i.e.,} \quad \Pr(a \mid \beta, \alpha, \xi) \geq \frac{1}{2} \quad \text{i.e.,} \quad \frac{q\xi(1-k)}{q\xi(1-k) + (1-q)(1-\xi)k} \geq \frac{1}{2}$$

$$\text{i.e.,} \qquad k \leq \frac{q\xi}{q\xi + (1-q)(1-\xi)} = k(\xi), \tag{A.6}$$

establishing the necessity requirement of part (ii) of the lemma. Sufficiency is completed with the next incentive compatibility verification.

Truthful recommendation by expert j for $t_j = \lambda$ and $s_j = \alpha$ requires

$$\Pr(a \mid \beta, \alpha, \lambda) \geq \Pr(b \mid \beta, \alpha, \lambda),$$

which is satisfied given (A.3). ■

Proof of Proposition 2. $(\mathbf{i})$ The assumption $k(\xi) \leq k \leq k(\lambda)$ is equivalent to (see Panel 3 of Fig. 1):

$$\frac{\xi}{1-\xi}\frac{q}{1-q} \leq \frac{k}{1-k} \leq \frac{1-q}{q}\frac{\lambda}{1-\lambda}\,. \tag{A.7}$$

D's beliefs for the proposed strategies would be as follows (the tuple (AB) etc. are ordered pairs where the first coordinate denotes the first expert's advice):

$$\begin{aligned}
\Pr(a \mid AB) &= \tfrac{q(1-\lambda)k}{(1-q)\lambda(1-k)+q(1-\lambda)k} \quad \leq \tfrac{1}{2}\,, \\
\Pr(a \mid AA) &= \tfrac{qk[k+(1-\theta)(1-\xi)]}{qk[k+(1-\theta)(1-\xi)]+(1-q)(1-k)[1-k+(1-\theta)\xi]} \quad > \tfrac{1}{2}\,, \\
\Pr(a \mid BA) &= \tfrac{q\lambda(1-k)}{q\lambda(1-k)+(1-q)(1-\lambda)k} \quad > \tfrac{1}{2}\,, \\
\Pr(a \mid BB) &= \tfrac{q(1-k)[1-k+(1-\theta)\xi]}{q(1-k)[1-k+(1-\theta)\xi]+(1-q)k[k+(1-\theta)(1-\xi)]} \quad < \tfrac{1}{2}.
\end{aligned} \tag{A.8}$$

The first and third inequalities follow given the RHS inequality of (A.7), and the second and fourth inequalities follow given the LHS inequality of (A.7).

Let $d(v_i, v_j)$ be D's decision. Given D's beliefs (A.8), we have

$$d(AB) = B, \quad d(AA) = A, \quad d(BA) = A, \quad d(BB) = B.$$

Thus, if D's decision is A, then O knows that the recommendation profile is either (AA) or (BA). Conditional on observing D's decision A and $\omega = a$, O's beliefs about the experts' abilities are as follows:[22]

$$\begin{aligned}
\Pr(t = \lambda \mid A, a) &= \tfrac{\theta\lambda}{\theta\lambda+(1-\theta)k}, \qquad & \Pr(t = \lambda \mid A, b) &= \tfrac{\theta(1-\lambda)}{\theta(1-\lambda)+(1-\theta)(1-k)}, \\
\Pr(t = \lambda \mid B, b) &= \tfrac{\theta\lambda}{\theta\lambda+(1-\theta)k}, \qquad & \Pr(t = \lambda \mid B, a) &= \tfrac{\theta(1-\lambda)}{\theta(1-\lambda)+(1-\theta)(1-k)}.
\end{aligned} \tag{A.9}$$

Let us consider the second expert's payoff:

$$\Pi_j^s(v_i, v_j, s_j, t_j) = \Pr(a|v_i, s_j, t_j)E_j^s(v_i, v_j, d(v_i, v_j), a) + \Pr(b|v_i, s_j, t_j)E_j^s(v_i, v_j, d(v_i, v_j), b),$$

[22] To illustrate how the beliefs are derived, let us consider $\Pr(t = \lambda|A, a)$. Note that $d = A$ if and only if the recommendation profile is (AA) or (BA). Given $\omega = a$, the probability of the event (AA) is $k[k + (1 - \theta)(1 - \xi)]$ and that of (BA) is $(1 - k)\theta\lambda$. So the probability of (AA) or (BA), given $\omega = a$, is $k[k + (1 - \theta)(1 - \xi)] + (1 - k)\theta\lambda = k(1 - \theta) + \theta\lambda$.

The probability of the event that the recommendation profile is (AA) or (BA) and a randomly selected expert is of ability λ, given $\omega = a$, is $\frac{1}{2}[\theta\lambda(k + (1 - \theta)(1 - \xi)) + \theta(1 - \lambda)\theta\lambda] + \frac{1}{2}[k\theta\lambda + (1 - k)\theta\lambda] = \theta\lambda$. So, $\Pr(t = \lambda \mid A, a) = \frac{\theta\lambda}{\theta\lambda+(1-\theta)k}$. The other beliefs follow similarly.

where

$$\begin{aligned}
E^s_j(A,A,A,a) &= \left[\tfrac{\theta\lambda}{\theta\lambda+(1-\theta)k}\right](\lambda-\xi)+\xi = E^s_j(B,A,A,a),\\
E^s_j(A,A,A,b) &= \left[\tfrac{\theta(1-\lambda)}{\theta(1-\lambda)+(1-\theta)(1-k)}\right](\lambda-\xi)+\xi = E^s_j(B,A,A,b),\\
E^s_j(B,B,B,b) &= \left[\tfrac{\theta\lambda}{\theta\lambda+(1-\theta)k}\right](\lambda-\xi)+\xi = E^s_j(A,B,B,b),\\
E^s_j(B,B,B,a) &= \left[\tfrac{\theta(1-\lambda)}{\theta(1-\lambda)+(1-\theta)(1-k)}\right](\lambda-\xi)+\xi = E^s_j(A,B,B,a).
\end{aligned} \tag{A.10}$$

So, consider the second expert of ability t_j with signal s_j who sees a recommendation A. Then,

$$\begin{aligned}
\Pi^s_j(A,A,s_j,t_j) &= \left[\Pr(a|\alpha,s_j,t_j)\left\{\frac{\theta\lambda}{\theta\lambda+(1-\theta)k}\right\}+\Pr(b|\alpha,s_j,t_j)\left\{\frac{\theta(1-\lambda)}{\theta(1-\lambda)+(1-\theta)(1-k)}\right\}\right](\lambda-\xi)+\xi,\\
\Pi^s_j(A,B,s_j,t_j) &= \left[\Pr(a|\alpha,s_j,t_j)\left\{\frac{\theta(1-\lambda)}{\theta(1-\lambda)+(1-\theta)(1-k)}\right\}+\Pr(b|\alpha,s_j,t_j)\left\{\frac{\theta\lambda}{\theta\lambda+(1-\theta)k}\right\}\right](\lambda-\xi)+\xi.
\end{aligned}$$

Since $\frac{\theta\lambda}{\theta\lambda+(1-\theta)k} > \frac{\theta(1-\lambda)}{\theta(1-\lambda)+(1-\theta)(1-k)}$ and $\Pr(a|\alpha,\alpha,t_j) > 1/2$ (by (A.2)), following a recommendation of A it is optimal for both abilities of the second expert to recommend A, when she gets signal α. Due to (A.4), $\Pr(a|\alpha,\beta,\xi) > 1/2$. So, it is optimal for the second expert of ability ξ who gets signal β to herd and recommend A. Due to (6) and the condition that $\frac{\lambda}{1-\lambda} \geq \frac{q}{1-q}\frac{k}{1-k}$, we have $\Pr(b|\alpha,\beta,\lambda) > 1/2$. So, following an A-recommendation it is optimal for the second expert of ability λ, who gets signal β, to truthfully recommend B.

Now consider the second expert of ability t_j with signal s_j who sees a recommendation B. Then,

$$\begin{aligned}
\Pi^s_j(B,B,s_j,t_j) &= \left[\Pr(a|\beta,s_j,t_j)\left\{\frac{\theta(1-\lambda)}{\theta(1-\lambda)+(1-\theta)(1-k)}\right\}+\Pr(b|\beta,s_j,t_j)\left\{\frac{\theta\lambda}{\theta\lambda+(1-\theta)k}\right\}\right](\lambda-\xi)+\xi,\\
\Pi^s_j(B,A,s_j,t_j) &= \left[\Pr(a|\beta,s_j,t_j)\left\{\frac{\theta\lambda}{\theta\lambda+(1-\theta)k}\right\}+\Pr(b|\beta,s_j,t_j)\left\{\frac{\theta(1-\lambda)}{\theta(1-\lambda)+(1-\theta)(1-k)}\right\}\right](\lambda-\xi)+\xi.
\end{aligned}$$

Again due to (A.2), $\Pr(b|\beta,\beta,t_j) > 1/2$. So, following a recommendation of B, it is optimal for the second expert to recommend B when she gets signal β. Due to (A.3), following a recommendation of B, it is optimal for the second expert to truthfully recommend A when she is of ability λ and gets signal α. Finally, consider the second expert of ability ξ who gets a signal α. From (5), she will herd and recommend B if and only if

$$\frac{(1-q)(1-\xi)k}{q\xi(1-k)+(1-q)(1-\xi)k} \geq \frac{1}{2} \quad \Leftrightarrow \quad \frac{k}{1-k} \geq \frac{q}{1-q}\frac{\xi}{1-\xi},$$

which is satisfied given (A.7).

Let us now consider the first expert and suppose she recommends A. Given the second expert's recommendation strategy, the first expert knows that if the second expert were to get signal α, then irrespective of her ability she would recommend A; if she were to get signal β and her ability were ξ she would recommend A, and if her ability were λ she would recommend B. Hence, if $\omega = a$, then the probability of the second expert recommending A would be $k + (1-\theta)(1-\xi)$, and of recommending B would be $\theta(1-\lambda)$. And if $\omega = b$, then the probability of the second expert recommending A would be $(1-k) + (1-\theta)\xi$ and of recommending B would be $\theta\lambda$. Hence, the first expert's expected payoff from recommending A is

$$\Pi_i^s(A, s_i, t_i) = \Pr(a|s_i, t_i)\left\{[k + (1-\theta)(1-\xi)]\rho + \theta(1-\lambda)\rho'\right\} + \Pr(b|s_i, t_i)\left\{[(1-k) + (1-\theta)\xi]\rho' + \theta\lambda\rho\right\},$$
$$\text{where from (A.9)}, \ \ \rho = \frac{\theta\lambda}{\theta\lambda + (1-\theta)k}, \quad \rho' = \frac{\theta(1-\lambda)}{\theta(1-\lambda) + (1-\theta)(1-k)}.$$

Similarly, if the first expert were to recommend B, then conditional on $\omega = a$ the second expert would recommend A with probability $\theta\lambda$ and recommend B with probability $(1-k) + (1-\theta)\xi$; conditional on $\omega = b$ the second expert would recommend A with probability $\theta(1-\lambda)$ and B with probability $k + (1-\theta)(1-\xi)$. Thus,

$$\Pi_i^s(B, s_i, t_i) = \Pr(a|s_i, t_i)\left\{\theta\lambda\rho + [(1-k) + (1-\theta)\xi]\rho'\right\} + \Pr(b|s_i, t_i)\left\{\theta(1-\lambda)\rho' + [k + (1-\theta)(1-\xi)]\rho\right\}.$$

Hence,

$$\Pi_i^s(A, s_i, t_i) - \Pi_i^s(B, s_i, t_i) = \Pr(a|s_i, t_i)\{(1-\theta)(\rho - \rho')\} - \Pr(b|s_i, t_i)\{(1-\theta)(\rho - \rho')\}.$$

Therefore, as $\rho - \rho' > 0$ and $\theta < 1$, we have

$$\Pi_i^s(A, s_i, t_i) \geq \Pi_i^s(B, s_i, t_i) \quad \text{if and only if} \quad \Pr(a|s_i, t_i) \geq \Pr(b|s_i, t_i).$$

Since $\Pr(a|\alpha, t_i) > \Pr(b|\alpha, t_i)$ and $\Pr(b|\beta, t_i) > \Pr(a|\beta, t_i)$ for any $t_i \in \{\xi, \lambda\}$, the first expert recommends A when she gets signal α and recommends B when she gets signal β.

Thus, under the stated condition (A.7), the proposed equilibrium will exist. It follows from Lemma 1 that for *small* q, condition (A.7) is met.

(**ii**) It is easy to see from (A.10) that $E_j^s(.,.,A,a) - E_j^s(.,.,B,a) = E_j^s(.,.,B,b) - E_j^s(.,.,A,b)$, so there will be no conformity bias generated in the partial type revelation equilibrium. ■

Proof of Proposition 3. For the first part of the proof we shall assume that the experts recommend their signals in equilibrium and then show that such an equilibrium indeed exists under the stated parameter restrictions. The only information that O will have about the recommendations is through d. Recall, given that the experts recommend their signals, D selects B only if two recommendations are in favor of B; otherwise D selects A (Lemma 6). Therefore when $d = A$, O knows that one of three pairs of signals, (α, α), (α, β), (β, α), could have resulted. When $d = B$, O knows that (β, β) resulted. O's relevant posteriors are then calculated, using Tables A.2 and A.3, as follows:[23]

$$\begin{aligned}
\Pr(t=\lambda \mid A, a) &= \frac{\theta[\lambda + (1-\lambda)(\theta\lambda + (1-\theta)\xi)]}{\theta[\lambda + (1-\lambda)(\theta\lambda + (1-\theta)\xi)] + (1-\theta)[\xi + (1-\xi)(\theta\lambda + (1-\theta)\xi)]} = \frac{\theta[\lambda + (1-\lambda)k]}{k(2-k)},\\
\Pr(t=\lambda \mid A, b) &= \frac{\theta[(1-\lambda) + \lambda(1-\theta\lambda - (1-\theta)\xi)]}{\theta[(1-\lambda) + \lambda(1-\theta\lambda - (1-\theta)\xi)] + (1-\theta)[(1-\xi) + \xi(1-\theta\lambda - (1-\theta)\xi)]} = \frac{\theta(1-\lambda k)}{1-k^2},\\
\Pr(t=\lambda \mid B, b) &= \frac{\theta\lambda}{\theta\lambda + (1-\theta)\xi} = \frac{\theta\lambda}{k},\\
\Pr(t=\lambda \mid B, a) &= \frac{\theta(1-\lambda)}{\theta(1-\lambda) + (1-\theta)(1-\xi)} = \frac{\theta(1-\lambda)}{1-k}.
\end{aligned}$$

Also, $\Pr(t = \xi \mid A, a) = 1 - \Pr(t = \lambda \mid A, a)$, and likewise for the remaining posteriors.

Define

$$\begin{aligned}
x' &= \Pr(t=\lambda \mid A, a)\lambda + (1 - \Pr(t=\lambda \mid A, a))\xi,\\
y' &= \Pr(t=\lambda \mid A, b)\lambda + (1 - \Pr(t=\lambda \mid A, b))\xi,\\
x'' &= \Pr(t=\lambda \mid B, b)\lambda + (1 - \Pr(t=\lambda \mid B, b))\xi,\\
y'' &= \Pr(t=\lambda \mid B, a)\lambda + (1 - \Pr(t=\lambda \mid B, a))\xi.
\end{aligned}$$

Now, these are O's expectations of expert abilities given any state and D's decision.

Let the second expert j recommend her signal. Consider the first expert i's strategy who has observed signal α. If she recommends A then irrespective of what the second expert recommends, $d = A$. So expert i receives a payoff:[24]

$$\Pi_i^s(A, \alpha, t_i) = \Pr(a|\alpha, t_i)x' + \Pr(b|\alpha, t_i)y' = \frac{qt_i}{H(t_i)}x' + \frac{(1-q)(1-t_i)}{H(t_i)}y',$$

where $H(t_i) \equiv qt_i + (1-q)(1-t_i)$.

[23] To alert the reader, here the first conditioning variable is the decision d.

[24] $\Pr(a|\alpha, t_i) = \frac{\Pr(\alpha|a,t_i)\cdot\Pr(a,t_i)}{\Pr(\alpha|a,t_i)\cdot\Pr(a,t_i)+\Pr(\alpha|b,t_i)\cdot\Pr(b,t_i)} = \frac{qt_i\cdot\Pr(t_i)}{qt_i\cdot\Pr(t_i)+(1-q)(1-t_i)\cdot\Pr(t_i)} = \frac{qt_i}{qt_i+(1-q)(1-t_i)}$.

If she recommends B then d depends on whether j observes α or β. This payoff can be written as:

$$\begin{aligned}
\Pi_i^s(B,\alpha,t_i) &= \Pr(a|s_i=\alpha,t_i)\Big[\sum_{t_j=\lambda,\xi}\Pr(s_j=\alpha|a,t_j)\cdot\Pr(t_j)x' + \sum_{t_j=\lambda,\xi}\Pr(s_j=\beta|a,t_j)\cdot\Pr(t_j)y''\Big] \\
&\quad +\Pr(b|s_i=\alpha,t_i)\Big[\sum_{t_j=\lambda,\xi}\Pr(s_j=\alpha|b,t_j)\cdot\Pr(t_j)y' + \sum_{t_j=\lambda,\xi}\Pr(s_j=\beta|b,t_j)\cdot\Pr(t_j)x''\Big] \\
&= \Pr(a|\alpha,t_i)\Big[\Big\{\Pr(\alpha|a,\lambda)\theta+\Pr(\alpha|a,\xi)(1-\theta)\Big\}x' + \Big\{\Pr(\beta|a,\lambda)\theta+\Pr(\beta|a,\xi)(1-\theta)\Big\}y''\Big] \\
&\quad +\Pr(b|\alpha,t_i)\Big[\Big\{\Pr(\alpha|b,\lambda)\theta+\Pr(\alpha|b,\xi)(1-\theta)\Big\}y' + \Big\{\Pr(\beta|b,\lambda)\theta+\Pr(\beta|b,\xi)(1-\theta)\Big\}x''\Big] \\
&= \Pr(a|\alpha,t_i)\Big[\Big\{\lambda\theta+\xi(1-\theta)\Big\}x' + \Big\{(1-\lambda)\theta+(1-\xi)(1-\theta)\Big\}y''\Big] \\
&\quad +\Pr(b|\alpha,t_i)\Big[\Big\{(1-\lambda)\theta+(1-\xi)(1-\theta)\Big\}y' + \Big\{\lambda\theta+\xi(1-\theta)\Big\}x''\Big] \\
&= \frac{qt_i}{H(t_i)}\Big[kx'+(1-k)y''\Big] + \frac{(1-q)(1-t_i)}{H(t_i)}\Big[kx''+(1-k)y'\Big].
\end{aligned}$$

Substituting terms and with some algebra we obtain:

$$\begin{aligned}
&\Pi_i^s(A,\alpha,t_i)\geq\Pi_i^s(B,\alpha,t_i)\Leftrightarrow qt_i\Big[(1-k)(x'-y'')\Big]\geq(1-q)(1-t_i)\Big[k(x''-y')\Big] \\
&\Leftrightarrow \frac{t_i}{1-t_i}\geq\frac{k}{1-k}\frac{1-q}{q}\frac{x''-y'}{x'-y''}\Leftrightarrow\frac{t_i}{1-t_i}\geq c^{-1}(k),
\end{aligned}$$

since

$$\frac{x''-y'}{x'-y''}=\frac{2-k}{1+k}>0. \tag{A.11}$$

Let i observe signal β. Then,

$$\begin{aligned}
\Pi_i^s(B,\beta,t_i) &= \Pr(a|s_i=\beta,t_i)\Big[\sum_{t_j=\lambda,\xi}\Pr(s_j=\alpha|a,t_j)\cdot\Pr(t_j)x' + \sum_{t_j=\lambda,\xi}\Pr(s_j=\beta|a,t_j)\cdot\Pr(t_j)y''\Big] \\
&\quad +\Pr(b|s_i=\beta,t_i)\Big[\sum_{t_j=\lambda,\xi}\Pr(s_j=\alpha|b,t_j)\cdot\Pr(t_j)y' + \sum_{t_j=\lambda,\xi}\Pr(s_j=\beta|b,t_j)\cdot\Pr(t_j)x''\Big] \\
&= \Pr(a|\beta,t_i)\Big[\Big\{\Pr(\alpha|a,\lambda)\theta+\Pr(\alpha|a,\xi)(1-\theta)\Big\}x' + \Big\{\Pr(\beta|a,\lambda)\theta+\Pr(\beta|a,\xi)(1-\theta)\Big\}y''\Big] \\
&\quad +\Pr(b|\beta,t_i)\Big[\Big\{\Pr(\alpha|b,\lambda)\theta+\Pr(\alpha|b,\xi)(1-\theta)\Big\}y' + \Big\{\Pr(\beta|b,\lambda)\theta+\Pr(\beta|b,\xi)(1-\theta)\Big\}x''\Big] \\
&= \Pr(a|\beta,t_i)\Big[\Big\{\lambda\theta+\xi(1-\theta)\Big\}x' + \Big\{(1-\lambda)\theta+(1-\xi)(1-\theta)\Big\}y''\Big] \\
&\quad +\Pr(b|\beta,t_i)\Big[\Big\{(1-\lambda)\theta+(1-\xi)(1-\theta)\Big\}y' + \Big\{\lambda\theta+\xi(1-\theta)\Big\}x''\Big] \\
&= \frac{q(1-t_i)}{J(t_i)}[kx'+(1-k)y''] + \frac{(1-q)t_i}{J(t_i)}[(1-k)y'+kx''],
\end{aligned}$$

where $J(t_i) \equiv (1-q)t_i + q(1-t_i)$. On the other hand,

$$\Pi_i^s(A,\beta,t_i) = \Pr(a|\beta,t_i)x' + \Pr(b|\beta,t_i)y' = \frac{q(1-t_i)}{J(t_i)}x' + \frac{(1-q)t_i}{J(t_i)}y'.$$

It is easy to check that

$$\begin{aligned} \Pi_i^s(B,\beta,t_i) \geq \Pi_i^s(A,\beta,t_i) \quad &\Leftrightarrow \quad (1-q)t_i\big[k(x''-y')\big] \geq q(1-t_i)\big[(1-k)(x'-y'')\big] \\ &\Leftrightarrow \quad \frac{t_i}{1-t_i} \geq c(k). \end{aligned}$$

But $\frac{t_i}{1-t_i} \geq c^{-1}(k)$ and $\frac{t_i}{1-t_i} \geq c(k)$ if and only if $\underline{k}(\xi) \leq k \leq \bar{k}(\xi)$.

Now consider the second expert j. If she sees a first period recommendation of A then she knows that $d = A$, so she is indifferent between recommending A and recommending B. Thus, recommending her signal is a best response for j.

Suppose j sees recommendation B. She knows that i recommended her signal, so $s_i = \beta$. Let j observe signal α. Then,

$$\begin{aligned} \Pi_j^s(A, s_i=\beta, s_j=\alpha, t_j) &= \Pr(a|s_i=\beta, s_j=\alpha, t_j)x' + \Pr(b|s_i=\beta, s_j=\alpha, t_j)y', \\ \Pi_j^s(B, s_i=\beta, s_j=\alpha, t_j) &= \Pr(a|s_i=\beta, s_j=\alpha, t_j)y'' + \Pr(b|s_i=\beta, s_j=\alpha, t_j)x'', \end{aligned}$$

where $\Pr(a|s_i = \beta, s_j = \alpha, t_j) = \frac{qt_j(1-k)}{qt_j(1-k)+(1-q)(1-t_j)k}$ and $\Pr(b|s_i = \beta, s_j = \alpha, t_j) = 1 - \Pr(a|s_i = \beta, s_j = \alpha, t_j)$ (see (A.3) and (5)). It is then easy to verify that

$$\begin{aligned} &\Pi_j^s(A, s_i=\beta, s_j=\alpha, t_j) \geq \Pi_j^s(B, s_i=\beta, s_j=\alpha, t_j) \\ \Leftrightarrow\ &\Pr(a|s_i=\beta, s_j=\alpha, t_j)\big[x'-y''\big] \geq \Pr(b|s_i=\beta, s_j=\alpha, t_j)\big[x''-y'\big] \\ \Leftrightarrow\ &qt_j(1-k)\big[x'-y''\big] \geq (1-q)(1-t_j)k\big[x''-y'\big] \\ \Leftrightarrow\ &\frac{t_j}{1-t_j} \geq c^{-1}(k). \end{aligned} \tag{A.12}$$

Let j observe signal β. Then,

$$\begin{aligned} \Pi_j^s(B, s_i=\beta, s_j=\beta, t_j) &= \Pr(a|s_i=\beta, s_j=\beta, t_j)y'' + \Pr(b|s_i=\beta, s_j=\beta, t_j)x'', \\ \Pi_j^s(A, s_i=\beta, s_j=\beta, t_j) &= \Pr(a|s_i=\beta, s_j=\beta, t_j)x' + \Pr(b|s_i=\beta, s_j=\beta, t_j)y', \end{aligned}$$

where $\Pr(a|s_i = \beta, s_j = \beta, t_j) = \frac{q(1-t_j)(1-k)}{(1-q)t_jk+q(1-t_j)(1-k)}$ and $\Pr(b|s_i = \beta, s_j = \beta, t_j) = 1 -$

$\Pr(a|s_i = \beta, s_j = \beta, t_j)$ (see (A.2)). Now it can be verified that

$$\begin{aligned}
& \Pi_j^s(B, s_i = \beta, s_j = \beta, t_j) \geq \Pi_j^s(A, s_i = \beta, s_j = \beta, t_j) \\
\Leftrightarrow\ & \Pr(a|s_i = \beta, s_j = \beta, t_j)\left[y'' - x'\right] \geq \Pr(b|s_i = \beta, s_j = \beta, t_j)\left[y' - x''\right] \\
\Leftrightarrow\ & q(1 - t_j)(1 - k)\left[y'' - x'\right] \geq (1 - q)t_j k\left[y' - x''\right] \\
\Leftrightarrow\ & \frac{t_j}{1 - t_j} \geq \frac{1 - k}{k} \frac{q}{1 - q} \frac{x' - y''}{x'' - y'} \ \Leftrightarrow\ \frac{t_j}{1 - t_j} \geq c(k).
\end{aligned}$$

But then again, $\frac{t_j}{1-t_j} \geq c^{-1}(k)$ and $\frac{t_j}{1-t_j} \geq c(k)$ if and only if $\underline{k}(\xi) \leq k \leq \bar{k}(\xi)$.

When $k > \bar{k}(\xi)$ we will construct an SDE under which the second expert babbles. In fact, such an SDE exists for all parameters k.

Let

$$\begin{aligned}
x &\equiv \frac{\theta\lambda}{\theta\lambda + (1 - \theta)\xi}\lambda + \frac{(1 - \theta)\xi}{\theta\lambda + (1 - \theta)\xi}\xi, \\
y &\equiv \frac{\theta(1 - \lambda)}{\theta(1 - \lambda) + (1 - \theta)(1 - \xi)}\lambda + \frac{(1 - \theta)(1 - \xi)}{\theta(1 - \lambda) + (1 - \theta)(1 - \xi)}\xi.
\end{aligned}$$

Here x is the expected accuracy of an expert's signal who has made a correct recommendation based on her signal alone, and y is similarly defined for an expert who has made an inaccurate recommendation. Thus, x and y are the market's imputed value of an expert's skill based on the accuracy of her recommendation, when the recommendation is based only on her own signal.

Due to Assumption 1,

$$x > k > y.$$

To calculate equilibrium beliefs about the experts' abilities, suppose the experts follow their respective strategies as specified above (the optimality of strategies to be verified later). Then, for recommendation pairs (A, A) and (A, B), the decision maker will choose $d = A$ (Lemma 6). If the first expert recommends B (i.e. for recommendation pairs (B, A) and (B, B)), the decision maker will select $d = B$ based only on the first expert's recommendation;

the second recommendation is uninformative. Hence for O the beliefs are as follows:

$$\begin{aligned}
\Pr(t=\lambda \mid A, a) = \quad \Pr(t=\lambda \mid B, b) &= \quad \tfrac{\theta\lambda}{\theta\lambda+(1-\theta)\xi}, \\
\Pr(t=\xi \mid A, a) = \quad \Pr(t=\xi \mid B, b) &= \quad \tfrac{(1-\theta)\xi}{\theta\lambda+(1-\theta)\xi}, \\
\Pr(t=\lambda \mid A, b) = \quad \Pr(t=\lambda \mid B, a) &= \quad \tfrac{\theta(1-\lambda)}{\theta(1-\lambda)+(1-\theta)(1-\xi)}, \\
\Pr(t=\xi \mid A, b) = \quad \Pr(t=\xi \mid B, a) &= \quad \tfrac{(1-\theta)(1-\xi)}{\theta(1-\lambda)+(1-\theta)(1-\xi)}.
\end{aligned}$$

Again, the beliefs are applicable to both experts, given that neither the experts' identities nor the timing of moves are revealed.

Now consider expert i who moves first. Let her observe α. If she recommends A, she receives

$$\Pi_i^s(A, \alpha, t_i) = \frac{qt_i}{qt_i + (1-q)(1-t_i)}x + \frac{(1-q)(1-t_i)}{qt_i + (1-q)(1-t_i)}y.$$

If she recommends B, her payoff is

$$\Pi_i^s(B, \alpha, t_i) = \frac{(1-q)(1-t_i)}{qt_i + (1-q)(1-t_i)}x + \frac{qt_i}{qt_i + (1-q)(1-t_i)}y.$$

Since $qt_i > (1-q)(1-t_i)$ and $x > y$, we have $\Pi_i^s(A, \alpha, t_i) > \Pi_i^s(B, \alpha, t_i)$. Now suppose she observes β. If she recommends B, her payoff is

$$\Pi_i^s(B, \beta, t_i) = \frac{(1-q)t_i}{(1-q)t_i + q(1-t_i)}x + \frac{q(1-t_i)}{(1-q)t_i + q(1-t_i)}y.$$

If she recommends A, she receives

$$\Pi_i^s(A, \beta, t_i) = \frac{q(1-t_i)}{(1-q)t_i + q(1-t_i)}x + \frac{(1-q)t_i}{(1-q)t_i + q(1-t_i)}y.$$

As $\frac{t_i}{1-t_i} > \frac{q}{1-q}$, and $x > y$, we have $\Pi_i^s(B, \beta, t_i) > \Pi_i^s(A, \beta, t_i)$. Hence, it is strictly optimal for i to recommend her signal, irrespective of her ability. Now, given a first period recommendation of A, the second expert j knows that $d = A$ (Lemma 6). Her payoff remains unchanged whether she recommends A or B. So j recommending her signal is optimal. Similarly, if the first recommendation is B, j's recommendation is immaterial and $d = B$. Hence again, j's payoff remains unchanged whether she recommends A or B. So it is optimal for j to babble. ■

Proof of Lemma 7. The proof will rely on Figs. 1 and 3.

$$\underline{k}(\xi) < \xi \underbrace{\Leftrightarrow}_{\text{Fig. 3}} c(\underline{k}(\xi)) > c(\xi) \Leftrightarrow \frac{\xi}{1-\xi} > \frac{1-\xi}{\xi}\frac{q}{1-q}\frac{1+\xi}{2-\xi}.$$

Start with the RHS of the above inequality to show that

$$\frac{1-\xi}{\xi}\frac{q}{1-q}\frac{1+\xi}{2-\xi} \underbrace{<}_{\text{since } \xi>1/2} \frac{1-\xi}{\xi}\frac{q}{1-q}\frac{\xi}{1-\xi} = \frac{q}{1-q} \underbrace{<}_{\text{since } \xi>q} \frac{\xi}{1-\xi},$$

establishing our first claim.

Next,

$$\bar{k}(\xi) > k(\xi) \underbrace{\Leftrightarrow}_{\text{Fig. 3}} c^{-1}(\bar{k}(\xi)) > c^{-1}(k(\xi)) \underbrace{\Leftrightarrow}_{\text{Fig. 3}} \frac{\xi}{1-\xi} > \frac{k(\xi)}{1-k(\xi)}\frac{1-q}{q}\frac{2-k(\xi)}{1+k(\xi)}$$

$$\Leftrightarrow \frac{\xi}{1-\xi} > \phi(k(\xi))\frac{1-q}{q}\frac{2-k(\xi)}{1+k(\xi)} \underbrace{=}_{\text{Fig. 1}} \frac{\xi}{1-\xi}\frac{2-k(\xi)}{1+k(\xi)} \Leftrightarrow 1 > \frac{2-k(\xi)}{1+k(\xi)}$$

$$\Leftrightarrow 1 + k(\xi) > 2 - k(\xi) \Leftrightarrow k(\xi) > \frac{1}{2},$$

which is true. This establishes the second claim. ■

Proof of Proposition 4. Under the transparency protocol, Lemmas 2 through 5 state that the following PBE exist: (i) both experts babble; (ii) one expert babbles and one recommends truthfully; (iii) the first expert recommends truthfully and the second expert babbles if the first recommendation is A and recommends truthfully if the first recommendation is B, provided that the bias q is large, or the bias q is medium or small and $k \in [\xi, k(\xi)]$. The payoff to D under (i) is q, under (ii) is k, and under (iii) is $q[k^2 + 2k(1-k)] + (1-q)k^2$ (one can calculate this from Tables A.1 and A.2, given the appropriate equilibrium strategies). By Assumption 1, $k > q$. Since $2q > 1$, we have $q[k^2 + 2k(1-k)] + (1-q)k^2 > k$.

Thus for large medium or small prior bias with $k \in [\xi, k(\xi)]$, the maximum equilibrium payoff of D is $q[k^2 + 2k(1-k)] + (1-q)k^2$. When the prior bias is medium/small and $k > k(\xi)$, D's maximum equilibrium payoff is k.

Under the secrecy SDE in Proposition 3 the payoff of D is $q[k^2 + 2k(1-k)] + (1-q)k^2$ when the prior bias is large, medium or small and $k \in [\xi, \bar{k}(\xi)]$. When the prior bias is medium/small and $k > \bar{k}(\xi)$, the first expert recommends her signal and the second expert babbles. The payoff of D then is k.

Since by Lemma 7 we have $\bar{k}(\xi) > k(\xi)$, D chooses secrecy over transparency. ■

Proof of Proposition 5. We need to show that

$$q[k^2+k(1-\theta)(1-\xi)+\lambda(1-k)]+(1-q)[\lambda(1-k)+k^2+k(1-\theta)(1-\xi)] > q[k^2+2k(1-k)]+(1-q)k^2.$$

The LHS is D's ex-ante payoff in the partial type revelation equilibrium, whereas the RHS is the payoff in the full signal revelation equilibrium.

We have

$$\begin{aligned}
&\frac{1-q}{q}\frac{k}{1-k} \geq \frac{\xi}{1-\xi} \quad \Rightarrow \quad k(1-\xi) \geq qk(1-\xi) + q\xi(1-k) \\
&\Rightarrow \quad k(1-\xi)(1-\theta) \geq qk(1-\xi)(1-\theta) + q\xi(1-k)(1-\theta) \\
&\Rightarrow \quad k(1-\xi)(1-\theta) + \lambda(1-k) \\
&\qquad \geq \lambda(1-k) + qk(1-\xi)(1-\theta) + q\xi(1-k)(1-\theta) \\
&\qquad = \theta\lambda(1-k) + (1-\theta)\lambda(1-k) + qk(1-\xi)(1-\theta) + q\xi(1-k)(1-\theta) \\
&\qquad = q\theta\lambda(1-k) + (1-q)\theta\lambda(1-k) + (1-\theta)\lambda(1-k) + q(1-\theta)(1-\xi)k + q(1-\theta) \\
&\qquad = qk(1-k) + [(1-q)\theta\lambda + (1-\theta)\lambda](1-k) - q\theta(1-\lambda)k + q\theta(1-\lambda)k + q(1-\theta)(1-\xi)k \\
&\text{(after adding and subtracting } q\theta(1-\lambda)k) \\
&\qquad = 2qk(1-k) + [(1-q)\theta\lambda + (1-\theta)\lambda](1-k) - q\theta(1-\lambda)k.
\end{aligned}$$

Now note that if $[(1-q)\theta\lambda + (1-\theta)\lambda](1-k) - q\theta(1-\lambda) \geq 0$, then we are done. Let us then verify

$$\begin{aligned}
[(1-q)\theta\lambda + (1-\theta)\lambda](1-k) - q\theta(1-\lambda) &\geq 0 \\
\Leftrightarrow \lambda(1-k) - q\theta\lambda(1-k) - q\theta(1-\lambda)k &\geq 0 \\
\Leftrightarrow (1-q\theta)\lambda(1-k) &\geq q\theta(1-\lambda)k \\
\Leftrightarrow \frac{\lambda}{1-\lambda} &\geq \frac{q\theta}{1-q\theta}\frac{k}{1-k},
\end{aligned}$$

which is true because $\frac{\lambda}{1-\lambda} \geq \frac{q}{1-q}\frac{k}{1-k} \geq \frac{q\theta}{1-q\theta}\frac{k}{1-k}$ (as $\frac{q}{1-q} \geq \frac{q\theta}{1-q\theta}$). ■

Proof of Proposition 6. We have already stated O's beliefs and D's choice in the proposed equilibrium. It remains to show that the experts' strategies constitute an equilibrium. We first start with deliberation and then conclude with detailed recommendation.

Deliberation

Fix equilibrium beliefs: each expert believes that the other expert truthfully reveals her type in the process of deliberation. Also fix the continuation equilibrium strategies of D (as in Table 1) and the beliefs of O (as given by g', h', g'' and h'').

Stage 4: We start with the last stage where j makes her last report. She can change D's decision only if the first three reports were (α, β, β) or (β, α, β) (see rows 7, 8, 11 and 12 in Table 1).

Case 1. Let the first three reports be (α, β, β). Given equilibrium beliefs, j knows i's type to be (α, ξ). Suppose the second expert is of type (β, ξ). Her posterior on a, conditional on (α, ξ) and (β, ξ), is q. By reporting ξ she gets a payoff of $[qg' + (1-q)h'](\lambda - \xi) + \xi$ and by reporting λ she gets $[qh'' + (1-q)g''](\lambda - \xi) + \xi$. So she reports ξ if and only if

$$\frac{q}{1-q}(g' - h'') \geq (g'' - h'). \tag{A.13}$$

If the second expert is of type (β, λ), a similar calculation shows that she reports λ if and only if

$$\frac{q}{1-q}\frac{\xi}{1-\xi}(g' - h'') \leq \frac{\lambda}{1-\lambda}(g'' - h'). \tag{A.14}$$

Combining (A.13) and (A.14), we get

$$\frac{q}{1-q}(g' - h'') \geq (g'' - h') \geq \frac{q}{1-q}\left(\frac{\xi}{1-\xi}/\frac{\lambda}{1-\lambda}\right)(g' - h''). \tag{A.15}$$

If the second expert is of type (α, λ) or (α, ξ) we let her recommend whatever is optimal for her. We shall show later on (Stage 2 analysis) that such types would *not* have reported β in the second stage (as it is in this equilibrium path).

Case 2. Let the first three reports be (β, α, β). Expert j knows that the first expert's type is (β, λ). Let j's type be (α, λ). Then she reports λ if and only if (A.13) holds. If j is of type (α, ξ), then $\Pr(a \mid (\beta, \lambda), (\alpha, \xi)) = \frac{q\xi(1-\lambda)}{q\xi(1-\lambda)+(1-q)(1-\xi)\lambda}$ and she reports λ if and only if (A.14) holds. If the second expert is of type (β, ξ) or (β, λ), we allow her to make any optimal recommendation. We shall show (see Stage 2 analysis below) that these types would have reported β instead of α (as required in this case).

Result: As $\left(\frac{\xi}{1-\xi}/\frac{\lambda}{1-\lambda}\right) < 1$, (A.15) implies that

$$g' - h'' \geq 0 \quad \text{and} \quad g'' - h' \geq 0.$$

Stage 3: Let (A.15) hold. By equilibrium supposition, i and j truthfully report their signals in stages 1 and 2. Also j truthfully reports her ability in Stage 4, as (A.15) holds. In Stage 3, we would like to consider under what conditions would i reveal her ability. Four cases are to be considered. In stages 1 and 2 the reported signals are: (i) α and β; (ii) β and α; (iii) α and α; (iv) β and β.

Case 1. Let i and j reveal signal α and β in stages 1 and 2. Let i's ability be ξ and consider if she were to report it truthfully. In Stage 4, if j reports her ability as λ then $d = B$ and if she reports her ability as ξ then $d = A$ (see rows 8 and 7 of Table 1). So, $d = B$ with probability $\Pr(a, \lambda \mid \beta, (\alpha, \xi)) + \Pr(b, \lambda \mid \beta, (\alpha, \xi))$ and $d = A$ with probability $\Pr(a, \xi \mid \beta, (\alpha, \xi)) + \Pr(b, \xi \mid \beta, (\alpha, \xi))$. Let

$$\begin{aligned} H &= \Pr(a, \lambda \mid \beta, (\alpha, \xi)) + \Pr(b, \lambda \mid \beta, (\alpha, \xi)) + \Pr(a, \xi \mid \beta, (\alpha, \xi)) + \Pr(b, \xi \mid \beta, (\alpha, \xi)) \\ &= \Pr(\beta, (\alpha, \xi)). \end{aligned}$$

So by reporting ability ξ, expert j gets a payoff of

$$\left[\Pr(a, \lambda \mid \beta, (\alpha, \xi))h'' + \Pr(b, \lambda \mid \beta, (\alpha, \xi))g'' + \Pr(a, \xi \mid \beta, (\alpha, \xi))g' + \Pr(b, \xi \mid \beta, (\alpha, \xi))h'\right](\lambda - \xi) + \xi.$$

Using Tables A.2 and A.3, this can be written as

$$\begin{aligned} &\frac{1}{H}\Big[(q\theta(1-\theta)(1-\lambda)\xi)h'' + ((1-q)\theta(1-\theta)\lambda(1-\xi))g'' \\ &+(q(1-\theta)^2(1-\xi)\xi)g' + ((1-q)(1-\theta)^2\xi(1-\xi))h'\Big](\lambda - \xi) + \xi. \end{aligned} \tag{A.16}$$

If instead, i were to falsely report her ability as λ, then $d = A$ with probability one (see rows 5 and 6 in Table 1). Her payoff then is

$$\begin{aligned} &\left[\Pr(a \mid \beta, (\alpha, \xi))g' + \Pr(b \mid \beta, (\alpha, \xi))h'\right](\lambda - \xi) + \xi \\ = &\left[\Pr(a, \lambda \mid \beta, (\alpha, \xi))g' + \Pr(b, \lambda \mid \beta, (\alpha, \xi))h' + \Pr(a, \xi \mid \beta, (\alpha, \xi))g' + \Pr(b, \xi \mid \beta, (\alpha, \xi))h'\right](\lambda - \xi) + \xi. \end{aligned} \tag{A.17}$$

We can write this payoff (using Tables A.2 and A.3) as

$$\frac{1}{H}\Big[(q\theta(1-\theta)(1-\lambda)\xi)g' + ((1-q)\theta(1-\theta)\lambda(1-\xi))h' + (q(1-\theta)^2(1-\xi)\xi)g' + ((1-q)(1-\theta)^2\xi(1-\xi))h'\Big](\lambda-\xi)+\xi.$$

The necessary and sufficient condition for truthful revelation is then

$$\begin{aligned}
& \frac{1}{H}\Big[(q\theta(1-\theta)(1-\lambda)\xi)h'' + ((1-q)\theta(1-\theta)\lambda(1-\xi))g'' \\
& +(q(1-\theta)^2(1-\xi)\xi)g' + ((1-q)(1-\theta)^2\xi(1-\xi))h'\Big](\lambda-\xi)+\xi \\
\geq \; & \frac{1}{H}\Big[(q\theta(1-\theta)(1-\lambda)\xi)g' + ((1-q)\theta(1-\theta)\lambda(1-\xi))h' \\
& +(q(1-\theta)^2(1-\xi)\xi)g' + ((1-q)(1-\theta)^2\xi(1-\xi))h'\Big](\lambda-\xi)+\xi,
\end{aligned}$$

or,

$$(g''-h') \geq \frac{q}{1-q}\left(\frac{\xi}{1-\xi}/\frac{\lambda}{1-\lambda}\right)(g'-h''). \tag{A.18}$$

Now suppose i and j reveal signals α and β in stages 1 and 2, but i's ability is λ, i.e., she is of type (α, λ). If she reports λ in Stage 3 then $d = A$ with probability one (see rows 5 and 6 of Table 1). Suppose she reports ξ instead. Then $d = B$ if j reports her type as λ in Stage 4 (see row 8 of Table 1). This happens with probability $\Pr(a, \lambda \mid \beta, (\alpha, \lambda)) + \Pr(b, \lambda \mid \beta, (\alpha, \lambda))$. Otherwise, if j reports ξ then $d = A$ (row 7 of Table 1). This happens with probability $\Pr(a, \xi \mid \beta, (\alpha, \lambda)) + \Pr(b, \xi \mid \beta, (\alpha, \lambda))$. Let $H' = \Pr(\beta, (\alpha, \lambda))$. Then the necessary and sufficient condition for truthful revelation is

$$\begin{aligned}
& \frac{1}{H'}\Big[(q\theta^2(1-\lambda)\lambda)g' + ((1-q)\theta^2\lambda(1-\lambda))h' \\
& +(q(1-\theta)\theta(1-\xi)\lambda)g' + ((1-q)(1-\theta)\theta\xi(1-\lambda))h'\Big](\lambda-\xi)+\xi \\
\geq \; & \frac{1}{H'}\Big[(q\theta^2(1-\lambda)\lambda)h'' + ((1-q)\theta^2\lambda(1-\lambda))g'' \\
& +(q(1-\theta)\theta(1-\xi)\lambda)g' + ((1-q)(1-\theta)\theta\xi(1-\lambda))h'\Big](\lambda-\xi)+\xi,
\end{aligned}$$

or,

$$\frac{q}{1-q}(g'-h'') \geq (g''-h'). \tag{A.19}$$

In what follows, the steps are similar to what has been done above. Hence, we shall simply state the necessary and sufficient conditions for truthful revelation.

Case 2. Suppose i and j reveal signal β and α in stages 1 and 2. Let i be of ability ξ. The necessary and sufficient condition for truthful revelation is the same as (A.19). If i is of ability λ, then the condition is the same as (A.18).

Case 3. Let i and j reveal signal β and β in stages 1 and 2. Then $d = B$ with probability one. The ability reports that follow do not matter (see rows 13 through 16 of Table 1). Hence i reveals her ability in the third stage.

Case 4. Similarly if i and j reveal signal α and α in stages 1 and 2. Then $d = A$ with probability one (see rows 1 through 4 of Table 1), and i reveals her type in Stage 3.

Hence, combining (A.18) and (A.19), the necessary and sufficient conditions for i to truthfully reveal her ability in the third stage are the same as in (A.15).

Stage 2: Let condition (A.15) hold and let expert i truthfully report her signal in Stage 1. As (A.15) holds, i truthfully reports her ability in Stage 3. We need to consider two cases: (i) i reveals her signal as β; (ii) i reveals her signal as α.

Case 1. Suppose i reveals her signal as β. So j knows i's signal. The following four sub-cases now need to be considered.

Sub-case (i): Suppose j's signal is α and she is of ability ξ. If she reveals her signal in Stage 2 then, as (A.15) holds, she will reveal her ability in Stage 4. Her payoff then is as given in (A.16). If instead she were to report β, then $d = B$ with probability one (rows 13 through 16 of Table 1). Then, the necessary and sufficient condition for truthful revelation is the same as (A.19).

Sub-case (ii): Suppose j's signal is α and she is of ability λ. If j reports her signal truthfully in Stage 2 then, as she truthfully reports her ability λ in Stage 4, $d = A$ with probability one (rows 9 and 11 of Table 1). If instead she reports β then $d = B$ with probability one (rows 13 through 16 of Table 1). In the former case her payoff is

$$\frac{1}{H'}\Big[(q\theta^2(1-\lambda)\lambda)g' + ((1-q)\theta^2\lambda(1-\lambda))h'$$
$$+(q(1-\theta)\theta(1-\xi)\lambda)g' + ((1-q)(1-\theta)\theta\xi(1-\lambda))h'\Big](\lambda-\xi)+\xi,$$

and in the latter case her payoff is

$$\frac{1}{H'}\Big[(q\theta^2(1-\lambda)\lambda)h'' + ((1-q)\theta^2\lambda(1-\lambda))g''$$
$$+(q(1-\theta)\theta(1-\xi)\lambda)h'' + ((1-q)(1-\theta)\theta\xi(1-\lambda))g''\Big](\lambda-\xi)+\xi.$$

The necessary and sufficient condition for truthful revelation is then

$$\frac{q}{1-q}\left(\frac{\lambda}{1-\lambda}\Big/\frac{k}{1-k}\right)(g'-h'') \geq (g''-h'). \tag{A.20}$$

As $\left(\frac{\lambda}{1-\lambda}/\frac{k}{1-k}\right) > 1$, $g''-h'$ and $g'-h'' \geq 0$, this condition is satisfied if (A.15) is satisfied.

Sub-case (iii): Suppose j's signal is β and she is of ability ξ. If she reveals her signal in Stage 2 then $d = B$ with probability one (rows 13 through 16 of table 1). Her payoff then is

$$\frac{1}{\Pr(\beta,(\beta,\xi))}\Big[(q\theta(1-\theta)(1-\lambda)(1-\xi))h'' + ((1-q)\theta(1-\theta)\lambda\xi)g'' \\ +(q(1-\theta)^2(1-\xi)^2)h'' + ((1-q)(1-\theta)^2\xi^2)g''\Big](\lambda-\xi)+\xi. \tag{A.21}$$

Now instead let j misreport her signal as α. If i were to be of type ξ then $d = A$ (rows 9 and 10). If i were to be of type λ, then $d = A$, only if j were to misreport her type again as λ in Stage 4 (row 11). Expert j's fourth stage payoff would then be $q\theta(1-\theta)(1-\lambda)(1-\xi))g' + (1-q)\theta(1-\theta)\lambda\xi h'$. On the other hand, by reporting ξ she would get $q\theta(1-\theta)(1-\lambda)(1-\xi))h'' + (1-q)\theta(1-\theta)\lambda\xi g''$. Since (A.15) holds, and $\xi > \frac{1}{2}$, we have that

$$(g''-h') \geq \frac{q}{1-q}\left(\frac{\xi}{1-\xi}\Big/\frac{\lambda}{1-\lambda}\right)(g'-h'') \geq \frac{q}{1-q}\left(\frac{1-\xi}{\xi}\Big/\frac{\lambda}{1-\lambda}\right)(g'-h'').$$

Hence j reports ξ in Stage 4 when i reports λ in Stage 3 and as a result $b = B$. So j's second stage payoff from reporting α is

$$\frac{1}{\Pr(\beta,(\beta,\xi))}\Big[(q\theta(1-\theta)(1-\lambda)(1-\xi))h'' + ((1-q)\theta(1-\theta)\lambda\xi)g'' \\ +(q(1-\theta)^2(1-\xi)^2)g' + ((1-q)(1-\theta)^2\xi^2)h'\Big](\lambda-\xi)+\xi. \tag{A.22}$$

The necessary and sufficient condition for truthful revelation is then (using (A.21) and (A.22))

$$(g''-h') \geq \frac{q}{1-q}\left(\frac{1-\xi}{\xi}\Big/\frac{\xi}{1-\xi}\right)(g'-h''). \tag{A.23}$$

Sub-case (iv): Suppose j's signal is β and she is of ability λ. If she reveals her signal in

Stage 2 then $d = B$ with probability one (rows 13 through 16 of table 1). Her payoff then is

$$\frac{1}{\Pr(\beta,(\beta,\lambda))}\Big[(q\theta(1-\theta)(1-\lambda)^2)h'' + ((1-q)\theta(1-\theta)\lambda^2)g'' \\ +(q(1-\theta)^2(1-\xi)(1-\lambda))h'' + ((1-q)(1-\theta)^2\xi\lambda)g''\Big](\lambda-\xi)+\xi. \quad \text{(A.24)}$$

Instead, let j misreport her signal as α. If i were to be of type ξ then $d = A$ (rows 9 and 10). If i were to be of type λ, then $d = A$, only if j were to report her type as λ in Stage 4 (row 11). Expert j's fourth stage payoff would then be $q\theta(1-\theta)(1-\lambda)^2g' + (1-q)\theta(1-\theta)\lambda^2h'$. On the other hand, by reporting ξ she would get $q\theta(1-\theta)(1-\lambda)^2h'' + (1-q)\theta(1-\theta)\lambda^2g''$. Since (A.17) holds, and $\frac{\xi}{1-\xi} > \frac{1-\lambda}{\lambda}$, we have that

$$\begin{aligned}(g''-h') &\geq \frac{q}{1-q}\left(\frac{\xi}{1-\xi}/\frac{\lambda}{1-\lambda}\right)(g'-h'') \\ &\geq \frac{q}{1-q}\left(\frac{1-\lambda}{\lambda}/\frac{\lambda}{1-\lambda}\right)(g'-h'').\end{aligned}$$

Hence j reports ξ in Stage 4 when i reports λ in Stage 3 and as a result $b = B$. So j's second stage payoff from reporting α is

$$\frac{1}{\Pr(\beta,(\beta,\lambda))}\Big[(q\theta(1-\theta)(1-\lambda)^2)h'' + ((1-q)\theta(1-\theta)\lambda^2)g'' \\ +(q(1-\theta)^2(1-\xi)(1-\lambda)g' + ((1-q)(1-\theta)^2\xi\lambda)h'\Big](\lambda-\xi)+\xi. \quad \text{(A.25)}$$

The necessary and sufficient condition for truthful revelation is then (using (A.24) and (A.25))

$$(g''-h') \geq \frac{q}{1-q}\left(\frac{1-\xi}{\xi}/\frac{\lambda}{1-\lambda}\right)(g'-h''), \quad \text{(A.26)}$$

which is satisfied if (A.23) is satisfied (as $\left(\frac{1-\xi}{\xi}/\frac{\xi}{1-\xi}\right) > \left(\frac{1-\xi}{\xi}/\frac{\lambda}{1-\lambda}\right)$).

Let

$$Q = \max\left\{\left(\frac{1-\xi}{\xi}/\frac{\xi}{1-\xi}\right), \left(\frac{\xi}{1-\xi}/\frac{\lambda}{1-\lambda}\right)\right\}. \quad \text{(A.27)}$$

Then, (A.15) along with (A.23) give us

$$\frac{q}{1-q}(g'-h'') \geq (g''-h') \geq \frac{q}{1-q}\cdot Q\cdot(g'-h''). \quad \text{(A.28)}$$

Case 2. Suppose i reveals her signal as α. The following four sub-cases now need to be considered. The analysis is similar to the ones above and hence we simply report the results.

Sub-case (i): Suppose j's signal is β and she is of ability ξ. If she reveals her signal in Stage 2 then as (A.15) holds she will also reveal her ability in Stage 4 and $d = A$ (rows 5 and 7 in Table 1). If she misreports her signal as α, then $d = A$ (rows 1 through 4 in Table 1). Thus she has no incentive to misreport her signal.

Sub-case (ii): Suppose j's signal is β and she is of ability λ. If she reveals her signal in Stage 2 then as (A.15) holds she will also reveal her ability in Stage 4 and $d = A$ if i reveals ability λ in Stage 3, otherwise $d = B$. If j misreports her signal as α, then $d = A$ with probability one. Using arguments similar to those in *Case 1*, it can be shown that the necessary and sufficient condition for truthful revelation is (A.15).

Sub-case (iii): Suppose j's signal is α and she is of ability ξ. If she reveals her signal in Stage 2 then $d = A$. If she misreports her signal as β then $d = B$, only if i reports ξ in Stage 3 and j misreports her ability as λ in Stage 4. Her payoff then is $q(1-\theta)^2\xi^2h'' + (1-q)(1-\theta)^2(1-\xi)^2g''$. Instead if j were to report ξ, her payoff would be $q(1-\theta)^2\xi^2g' + (1-q)(1-\theta)^2(1-\xi)^2h'$. But the latter payoff is larger as (A.15) holds. So she would report ξ and hence $d = A$. Since $d = A$, irrespective of j's signal report in Stage 2, she has no incentive to misreport her signal.

Sub-case (iv): Suppose j's signal is α and she is of ability λ. Again as (A.15) holds, it can be shown that j has no incentive to misreport.

Stage 1: Expert i reports on her signal. Let (A.28) hold. There are four cases to consider.

Case 1. Let i be of type (α, λ). If she reports α in Stage 1, then as (A.28) holds (A.15) holds and so she reports λ in Stage 3. So $d = A$ and i's Stage 1 payoff from reporting α is

$$\begin{aligned}\frac{1}{\Pr(\alpha,\lambda)}\Big[&q\theta^2\lambda^2g' + (1-q)\theta^2(1-\lambda)^2h' \\ &+ q\theta(1-\theta)\xi\lambda g' + (1-q)\theta(1-\theta)(1-\xi)(1-\lambda)h' \\ &+ q\theta^2(1-\lambda)\lambda g' + (1-q)\theta^2\lambda(1-\lambda)h' \\ &+ q\theta(1-\theta)(1-\xi)\lambda g' + (1-q)\theta(1-\theta)\xi(1-\lambda)h'\Big](\lambda-\xi) + \xi. \end{aligned} \tag{A.29}$$

If instead i were to report β in Stage 1, then if j reports β in Stage 2, $d = B$. If j were to report α in Stage 2, then i could report either λ or ξ in Stage 3 to get (respective) third stage payoffs of

$$\frac{1}{\Pr(\alpha,(\alpha,\lambda))}\Big[(q\theta^2\lambda^2 g' + (1-q)\theta^2(1-\lambda)^2 h' \\ + q\theta(1-\theta)\xi\lambda h'' + (1-q)\theta(1-\theta)(1-\xi)(1-\lambda)g''\Big](\lambda-\xi)+\xi, \quad (A.30)$$

and

$$\frac{1}{\Pr(\alpha,(\alpha,\lambda))}\Big[(q\theta^2\lambda^2 g' + (1-q)\theta^2(1-\lambda)^2 h' \\ + q\theta(1-\theta)\xi\lambda g' + (1-q)\theta(1-\theta)(1-\xi)(1-\lambda)h'\Big](\lambda-\xi)+\xi. \quad (A.31)$$

As $\frac{q}{1-q}\frac{\xi}{1-\xi}\frac{\lambda}{1-\lambda}(g'-h'') \geq \frac{q}{1-q}(g'-h'')$ and $\frac{q}{1-q}(g'-h'') \geq (g''-h')$ (from (A.28)) the payoff in (A.30) is greater than that in (A.31). So in Stage 3 i will report ξ, if j were to report α in Stage 2 and $d = A$. With this in place, the payoff of i from reporting β in Stage 1 is

$$\frac{1}{\Pr(\alpha,\lambda)}\Big[q\theta^2\lambda^2 g' + (1-q)\theta^2(1-\lambda)^2 h' \\ + q\theta(1-\theta)\xi\lambda g' + (1-q)\theta(1-\theta)(1-\xi)(1-\lambda)h' \\ + q\theta^2(1-\lambda)\lambda h'' + (1-q)\theta^2\lambda(1-\lambda)g'' \\ + q\theta(1-\theta)(1-\xi)\lambda h'' + (1-q)\theta(1-\theta)\xi(1-\lambda)g''\Big](\lambda-\xi)+\xi. \quad (A.32)$$

But as $\frac{q}{1-q}\left(\frac{\lambda}{1-\lambda}/\frac{k}{1-k}\right)\frac{\lambda}{1-\lambda}(g'-h'') \geq \frac{q}{1-q}(g'-h'')$ and $\frac{q}{1-q}(g'-h'') \geq (g''-h')$ (from (A.28)), the payoff in (A.29) is greater than that in (A.32). So type (α,λ) will report her signal in Stage 1.

Case 2. Let i be of type (α,ξ). If she reports α in Stage 1, then as (A.28) holds (A.15) holds and so she reports ξ in Stage 3. So $d = B$ only if j is of type (β,λ). Then, i's Stage 1 payoff from reporting α is

$$\frac{1}{\Pr(\alpha,\xi)}\Big[q\theta(1-\theta)\lambda\xi g' + (1-q)\theta(1-\theta)(1-\lambda)(1-\xi)h' \\ + q(1-\theta)^2\xi^2 g' + (1-q)(1-\theta)^2(1-\xi)^2 h' \\ + q\theta(1-\theta)(1-\lambda)\xi h'' + (1-q)\theta(1-\theta)\lambda(1-\xi)g'' \\ + q\theta(1-\theta)(1-\xi)\xi g' + (1-q)\theta(1-\theta)\xi(1-\xi)h'\Big](\lambda-\xi)+\xi. \quad (A.33)$$

Suppose instead i reported β in Stage 1. Then if j were to report β in Stage 2, $d = B$. If j were to report α in Stage 2, then i could report either λ or ξ in Stage 3 to get (respective) third stage payoffs of

$$\frac{1}{\Pr(\alpha, \xi)}\Big[q\theta(1-\theta)\lambda\xi g' + (1-q)\theta(1-\theta)(1-\lambda)(1-\xi)h' \\ +q(1-\theta)^2\xi^2 h'' + (1-q)(1-\theta)^2(1-\xi)^2 g''\Big](\lambda - \xi) + \xi, \tag{A.34}$$

and

$$\frac{1}{\Pr(\alpha, \xi)}\Big[q\theta(1-\theta)\lambda\xi g' + (1-q)\theta(1-\theta)(1-\lambda)(1-\xi)h' \\ +q(1-\theta)^2\xi^2 g' + (1-q)(1-\theta)^2(1-\xi)^2 h'\Big](\lambda - \xi) + \xi. \tag{A.35}$$

As $(g'' - h') \geq 0$ (from (A.28)), the latter payoff is larger and hence i reports ξ in Stage 3. Hence, the payoff of i from reporting β in Stage 1 is

$$\frac{1}{\Pr(\alpha, \xi)}\Big[q\theta(1-\theta)\lambda\xi g' + (1-q)\theta(1-\theta)(1-\lambda)(1-\xi)h' \\ + q(1-\theta)^2\xi^2 g' + (1-q)(1-\theta)^2(1-\xi)^2 h' \\ + q\theta(1-\theta)(1-\lambda)\xi h'' + (1-q)\theta(1-\theta)\lambda(1-\xi)g'' \\ +q\theta(1-\theta)(1-\xi)\xi h'' + (1-q)\theta(1-\theta)\xi(1-\xi)g''\Big](\lambda - \xi) + \xi. \tag{A.36}$$

As, $\frac{q}{1-q}(g' - h'') \geq (g'' - h')$ (from (A.28)), we have (A.33) is larger than (A.36). So (α, ξ) will report her signal in Stage 1.

Case 3. Let i be of type (β, ξ). If she reports β in Stage 1, then as (A.28) holds (A.15) holds she reports ξ in Stage 3. So $d = B$ if j reports β in Stage 2, otherwise $d = A$. Then, i's Stage 1 payoff from reporting β is

$$\frac{1}{\Pr(\beta, \xi)}\Big[q\theta(1-\theta)\lambda(1-\xi)g' + (1-q)\theta(1-\theta)(1-\lambda)\xi h' \\ + q(1-\theta)^2\xi(1-\xi)g' + (1-q)(1-\theta)^2(1-\xi)\xi h' \\ + q\theta(1-\theta)(1-\lambda)(1-\xi)h'' + (1-q)\theta(1-\theta)\lambda\xi g'' \\ +q(1-\theta)^2(1-\xi)^2 h'' + (1-q)(1-\theta)^2\xi^2 g''\Big](\lambda - \xi) + \xi. \tag{A.37}$$

If instead i were to report α in Stage 1. Then if j were to report α in Stage 2, $d = A$. If j were to report β in Stage 2, then i could report either λ or ξ in Stage 3 to get (respective) third stage payoffs of

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta(1-\theta)(1-\lambda)(1-\xi)g' + (1-q)\theta(1-\theta)\lambda\xi h' \\ + q(1-\theta)^2(1-\xi)^2 g' + (1-q)(1-\theta)^2\xi^2 h'\Big](\lambda-\xi) + \xi, \tag{A.38}$$

and

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta(1-\theta)(1-\lambda)(1-\xi)g' + (1-q)\theta(1-\theta)\lambda\xi h' \\ + q(1-\theta)^2(1-\xi)^2 h'' + (1-q)(1-\theta)^2\xi^2 g''\Big](\lambda-\xi) + \xi. \tag{A.39}$$

As $(g''-h') \geq \frac{q}{1-q}\cdot Q\cdot(g'-h'')$ (from (A.28)), the latter payoff is larger and hence i reports ξ in Stage 3. Hence, the payoff of i from reporting α in Stage 1 is

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta(1-\theta)\lambda(1-\xi)g' + (1-q)\theta(1-\theta)(1-\lambda)\xi h' \\ + q(1-\theta)^2\xi(1-\xi)g' + (1-q)(1-\theta)^2(1-\xi)\xi h' \\ + q\theta(1-\theta)(1-\lambda)(1-\xi)g' + (1-q)\theta(1-\theta)\lambda\xi h' \\ + q(1-\theta)^2(1-\xi)^2 h'' + (1-q)(1-\theta)^2\xi^2 g''\Big](\lambda-\xi) + \xi. \tag{A.40}$$

As $\left(\frac{1-\xi}{\xi}/\frac{\xi}{1-\xi}\right) > \left(\frac{1-\xi}{\xi}/\frac{\lambda}{1-\lambda}\right)$ and $(g''-h') \geq \frac{q}{1-q}\cdot Q\cdot(g'-h'')$ (from (A.28)), we have (A.37) is larger than (A.40). So (β,ξ) will report her signal in Stage 1.

Case 4. Let i be of type (β,λ). If she reports β in Stage 1, then as (A.28) holds (A.15) holds she reports λ in Stage 3. So $d = A$ if j is of type (α,λ). Then, i's Stage 1 payoff from reporting β is

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta^2\lambda(1-\lambda)g' + (1-q)\theta^2(1-\lambda)\lambda h' \\ + q\theta(1-\theta)\xi(1-\lambda)h'' + (1-q)\theta(1-\theta)(1-\xi)\lambda g'' \\ + q\theta^2(1-\lambda)^2 h'' + (1-q)\theta^2\lambda^2 g'' \\ + q\theta(1-\theta)(1-\xi)(1-\lambda)h'' + (1-q)\theta(1-\theta)\xi\lambda g''\Big](\lambda-\xi) + \xi. \tag{A.41}$$

If instead i were to report α in Stage 1. Then if j were to report α in Stage 2, $d = A$. If j were to report β in Stage 2, then i could report either λ or ξ in Stage 3 to get (respective) third stage payoffs of

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta^2(1-\lambda)^2 g' + (1-q)\theta^2\lambda^2 h' \\ +q\theta(1-\theta)(1-\xi)(1-\lambda)g' + (1-q)\theta(1-\theta)\xi\lambda h'\Big](\lambda-\xi)+\xi, \tag{A.42}$$

and

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta^2(1-\lambda)^2 g' + (1-q)\theta^2\lambda^2 h' \\ +q\theta(1-\theta)(1-\xi)(1-\lambda)h'' + (1-q)\theta(1-\theta)\xi\lambda g''\Big](\lambda-\xi)+\xi. \tag{A.43}$$

As $\left(\frac{1-\xi}{\xi}/\frac{\xi}{1-\xi}\right) > \left(\frac{1-\xi}{\xi}/\frac{\lambda}{1-\lambda}\right)$ and $(g''-h') \geq \frac{q}{1-q}\cdot Q\cdot(g'-h'')$ (from (A.28)), the latter payoff is larger and hence i reports ξ in Stage 3. Hence, the payoff of i from reporting α in Stage 1 is

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta^2\lambda(1-\lambda)g' + (1-q)\theta^2(1-\lambda)\lambda h' \\ + q\theta(1-\theta)\xi(1-\lambda)h'' + (1-q)\theta(1-\theta)(1-\xi)\lambda g'' \\ + q\theta^2(1-\lambda)^2 g' + (1-q)\theta^2\lambda^2 h' \\ +q\theta(1-\theta)(1-\xi)(1-\lambda)h'' + (1-q)\theta(1-\theta)\xi\lambda g''\Big](\lambda-\xi)+\xi. \tag{A.44}$$

As, $\left(\frac{1-\xi}{\xi}\right)^2 > \left(\frac{1-\lambda}{\lambda}\right)^2$ (or $\left(\frac{1-\xi}{\xi}/\frac{\xi}{1-\xi}\right) > \left(\frac{1-\lambda}{\lambda}/\frac{\lambda}{1-\lambda}\right)$) and $(g''-h') \geq \frac{q}{1-q}\cdot Q\cdot(g'-h'')$ (from (A.28)), we have (A.41) is larger than (A.44). So (β,λ) will report her signal in Stage 1.

Hence the necessary and sufficient conditions for a fully revealing equilibrium under deliberation is (A.28), i.e.,

$$\frac{q}{1-q}(g'-h'') \geq (g''-h') \geq \frac{q}{1-q}\cdot Q\cdot(g'-h'').$$

Detailed recommendation

Fix equilibrium beliefs, i.e., each expert believes that the other expert truthfully reveals her type in the process of detailed recommendation. Also fix the continuation equilibrium strategies of D (as in Table 1) and the beliefs of O (as given by g', h', g'' and h'').

Agent j

Case 1. Let agent i reveal her type as (α, λ). From Table 1, it follows that agent j, irrespective of her type, cannot be strictly better off by misreporting her type.

Case 2. Let agent i reveal her type as (α, ξ).

If j is of type (β, ξ) and she reveals her type, then $d = A$ and she gets a payoff of $\frac{1}{\Pr(\beta,\xi)}[q(1-\theta)^2\xi(1-\xi)g' + (1-q)(1-\theta)^2(1-\xi)\xi h'](\lambda - \xi) + \xi$. Instead, she could induce $d = B$ by reporting (β, λ) and get $\frac{1}{\Pr(\beta,\xi)}[q(1-\theta)^2\xi(1-\xi)h'' + (1-q)(1-\theta)^2(1-\xi)\xi g''](\lambda-\xi) + \xi$. Hence, she truthfully reveals her type if and only if

$$\frac{q}{1-q}(g' - h'') \geq (g'' - h'). \tag{A.45}$$

If j is of type (β, λ) and she reveals her type, then $d = B$ and she gets a payoff of $\frac{1}{\Pr(\beta,\lambda)}[q(1-\theta)\theta\xi(1-\lambda)h'' + (1-q)(1-\theta)\theta(1-\xi)\lambda g''](\lambda - \xi) + \xi$. Instead, she could induce $d = A$ by reporting, say, (β, ξ) and get $\frac{1}{\Pr(\beta,\lambda)}[q(1-\theta)\theta\xi(1-\lambda)g' + (1-q)(1-\theta)\theta(1-\xi)\lambda h'](\lambda - \xi) + \xi$. Hence, she truthfully reveals her type if and only if

$$(g'' - h') \geq \frac{q}{1-q}\left(\frac{\xi}{1-\xi}/\frac{\lambda}{1-\lambda}\right)(g' - h''). \tag{A.46}$$

Combining (A.45) and (A.46) we get

$$\frac{q}{1-q}(g' - h'') \geq (g'' - h') \geq \frac{q}{1-q}\left(\frac{\xi}{1-\xi}/\frac{\lambda}{1-\lambda}\right)(g' - h''). \tag{A.47}$$

Result: (A.47) implies that

$$g' - h'' \geq 0 \quad \text{and} \quad g'' - h' \geq 0. \tag{A.48}$$

Let (A.47) hold. Now if j is of type (α, ξ) and she reveals her type, then $d = A$ and she gets a payoff of $\frac{1}{\Pr(\alpha,\xi)}[q(1-\theta)^2\xi^2 g' + (1-q)(1-\theta)^2(1-\xi)^2 h']$. Instead, she could induce $d = B$ by reporting (β, λ) and get $\frac{1}{\Pr(\alpha,\xi)}[q(1-\theta)^2\xi^2 h'' + (1-q)(1-\theta)^2(1-\xi)^2 g'']$. Since $\frac{\xi 2}{(1-\xi)^2} > 1$, due to (A.47) and (A.48), she reveals her type.

Finally if j is of type (α, λ) and she reveals her type, then $d = A$ and she gets a payoff of $\frac{1}{\Pr(\alpha,\lambda)}[q\theta(1-\theta)\xi\lambda g' + (1-q)\theta(1-\theta)(1-\xi)(1-\lambda)h']$. Instead, she could induce $d = B$ by reporting (β, λ) and get $\frac{1}{\Pr(\alpha,\lambda)}[q\theta(1-\theta)\xi\lambda h'' + (1-q)\theta(1-\theta)(1-\xi)(1-\lambda)g'']$. Since

$\frac{\xi}{1-\xi}\frac{\lambda}{1-\lambda} > 1$, due to (A.47) and (A.48), she reveals her type.

Case 3. Let (A.47) hold and let agent i reveal her type as (β, ξ).

If j is of type (α, λ) and she reveals her type, then $d = A$ and she gets a payoff of $\frac{1}{\Pr(\alpha,\lambda)}[q\theta(1-\theta)(1-\xi)\lambda g' + (1-q)\theta(1-\theta)\xi(1-\lambda)h'](\lambda-\xi)+\xi$. By reporting (β, λ) and inducing $d = B$ she gets $\frac{1}{\Pr(\alpha,\lambda)}[q\theta(1-\theta)(1-\xi)\lambda h'' + (1-q)\theta(1-\theta)\xi(1-\lambda)g''](\lambda-\xi)+\xi$. As (A.47) holds, she reveals her type.

If j is of type (α, ξ) and she reveals her type, then $d = A$ and she gets a payoff of $\frac{1}{\Pr(\alpha,\xi)}[q(1-\theta)^2\xi(1-\xi)g' + (1-q)(1-\theta)^2(1-\xi)\xi h'](\lambda-\xi)+\xi$. Instead, she could induce $d = B$ by reporting (β, λ) and get $\frac{1}{\Pr(\alpha,\xi)}[q(1-\theta)^2\xi(1-\xi)h'' + (1-q)(1-\theta)^2(1-\xi)\xi g''](\lambda-\xi)+\xi$. Hence, she truthfully reveals her type as (A.47) holds.

If j's type is (β, λ) and she reveals her type, then $d = B$ and she gets a payoff of $\frac{1}{\Pr(\beta,\lambda)}[q\theta(1-\theta)(1-\xi)(1-\lambda)h'' + (1-q)\theta(1-\theta)\xi\lambda g''](\lambda-\xi)+\xi$. By reporting (α, λ) and inducing $d = A$ she gets $\frac{1}{\Pr(\beta,\lambda)}[q\theta(1-\theta)(1-\xi)(1-\lambda)g' + (1-q)\theta(1-\theta)\xi\lambda h'](\lambda-\xi)+\xi$. Then due to (A.47) she reveals her type.

Finally, if j is of type (β, ξ) and she reveals her type, then $d = B$ and she gets a payoff of $\frac{1}{\Pr(\beta,\xi)}[q(1-\theta)^2(1-\xi)^2h'' + (1-q)(1-\theta)^2\xi^2g''](\lambda-\xi)+\xi$. By reporting (α, λ) and inducing $d = A$ she gets $\frac{1}{\Pr(\beta,\xi)}[q(1-\theta)^2(1-\xi)^2g' + (1-q)(1-\theta)^2\xi^2h'](\lambda-\xi)+\xi$. She therefore reports truthfully if and only if

$$(g'' - h') \geq \frac{q}{1-q}\left(\frac{1-\xi}{\xi}/\frac{\xi}{1-\xi}\right)(g' - h''). \tag{A.49}$$

Combining (A.47) and (A.49) we get

$$\frac{q}{1-q}(g' - h'') \geq (g'' - h') \geq \frac{q}{1-q} \cdot Q \cdot (g' - h''). \tag{A.50}$$

Case 4. Let (A.50) hold and let agent i reveal her type as (β, λ).

If j is of type (α, λ) and she reveals her type, then $d = A$ and she gets a payoff of $\frac{1}{\Pr(\alpha,\lambda)}[q\theta^2(1-\lambda)\lambda g' + (1-q)\theta^2\lambda(1-\lambda)h'](\lambda-\xi)+\xi$. By reporting (β, λ), say, and inducing $d = B$ she gets $\frac{1}{\Pr(\alpha,\lambda)}[q\theta^2(1-\lambda)\lambda h'' + (1-q)\theta^2\lambda(1-\lambda)g''](\lambda-\xi)+\xi$. Due to (A.50) she reveals her type.

If j is of type (α, ξ) and she reveals her type, then $d = B$ and she gets a payoff of $\frac{1}{\Pr(\alpha,\xi)}[q\theta(1-\theta)(1-\lambda)\xi h'' + (1-q)\theta(1-\theta)\lambda(1-\xi)g''](\lambda-\xi)+\xi$. Instead, she could induce $d = A$ by reporting (α, λ) and get $\frac{1}{\Pr(\alpha,\xi)}[q\theta(1-\theta)(1-\lambda)\xi g' + (1-q)\theta(1-\theta)\lambda(1-\xi)h'](\lambda-\xi)+\xi$. Hence, she truthfully reveals her type as (A.50) holds.

If j is of type (β, ξ) and she reveals her type, then $d = B$ and she gets a payoff of $\frac{1}{\Pr(\beta,\xi)}[q\theta(1-\theta)(1-\lambda)(1-\xi)h'' + (1-q)\theta(1-\theta)\lambda\xi g''](\lambda - \xi) + \xi$. By reporting (α, λ) and inducing $d = A$ she gets $\frac{1}{\Pr(\beta,\xi)}[q\theta(1-\theta)(1-\lambda)(1-\xi)g' + (1-q)\theta(1-\theta)\lambda\xi h'](\lambda - \xi) + \xi$. Since $\frac{1-\xi}{\xi}/\frac{\xi}{1-\xi} > \frac{1-\xi}{\xi}/\frac{\lambda}{1-\lambda}$, due to (A.50) and (A.48), j reveals her type.

Finally, if j's type is (β, λ) and she reveals her type, then $d = B$ and she gets a payoff of $\frac{1}{\Pr(\beta,\lambda)}[q\theta^2(1-\lambda)^2 h'' + (1-q)\theta^2\lambda^2 g''](\lambda - \xi) + \xi$. By reporting (α, λ) and inducing $d = A$ she gets $\frac{1}{\Pr(\beta,\lambda)}[q\theta^2(1-\lambda)^2 g' + (1-q)\theta^2\lambda^2 h'](\lambda - \xi) + \xi$. Since $\frac{1-\xi}{\xi}/\frac{\xi}{1-\xi} > \frac{1-\lambda}{\lambda}/\frac{\lambda}{1-\lambda}$ and (A.50) holds, j reveals her type.

Expert i

Let (A.50) hold. So expert j reveals her type. Consider the first expert i. Again there are four cases to consider.

Case 1. Suppose agent i is of type (α, λ). By revealing her type she induces $d = A$ and gets a payoff as given by (A.29).

If she were to report (α, ξ) instead, then $d = B$ only if expert j were to be of type (β, λ). Then, expert i's payoff is

$$\begin{aligned} &\frac{1}{\Pr(\alpha,\lambda)}\Big[q\theta^2\lambda^2 g' + (1-q)\theta^2(1-\lambda)^2 h' \\ &+ q\theta(1-\theta)\xi\lambda g' + (1-q)\theta(1-\theta)(1-\xi)(1-\lambda)h' \\ &+ q\theta^2(1-\lambda)\lambda h'' + (1-q)\theta^2\lambda(1-\lambda)g'' \\ &+ q\theta(1-\theta)(1-\xi)\lambda g' + (1-q)\theta(1-\theta)\xi(1-\lambda)h'\Big](\lambda - \xi) + \xi. \end{aligned} \tag{A.51}$$

Note that the difference in payoffs occurs only when j is of type (β, λ). (A.50) and (A.48) then imply that i is better off by reporting (α, λ).

Now suppose i were to report (β, ξ). Then her payoff would be

$$\begin{aligned} &\frac{1}{\Pr(\alpha,\lambda)}\Big[q\theta^2\lambda^2 g' + (1-q)\theta^2(1-\lambda)^2 h' \\ &+ q\theta(1-\theta)\xi\lambda g' + (1-q)\theta(1-\theta)(1-\xi)(1-\lambda)h' \\ &+ q\theta^2(1-\lambda)\lambda h'' + (1-q)\theta^2\lambda(1-\lambda)g'' \\ &+ q\theta(1-\theta)(1-\xi)\lambda h'' + (1-q)\theta(1-\theta)\xi(1-\lambda)g''\Big](\lambda - \xi) + \xi. \end{aligned} \tag{A.52}$$

Note that the difference in payoffs between (A.51) and (A.52) occurs only when j is of type

(β, ξ). Since $\frac{\lambda}{1-\lambda}/\frac{\xi}{1-\xi} > 1$, (A.50) and (A.48) imply that the payoff in (A.51) is greater than that in (A.52). So i prefers to report (α, ξ) over (β, ξ).

But we already saw that i prefers reporting (α, λ) to reporting (α, ξ). So i prefers reporting (α, λ) to reporting (β, ξ).

Finally, suppose i were to report (β, λ). Her payoff then is

$$\begin{aligned}&\frac{1}{\Pr(\alpha,\lambda)}\Big[q\theta^2\lambda^2 g' + (1-q)\theta^2(1-\lambda)^2 h' \\ &+ q\theta(1-\theta)\xi\lambda h'' + (1-q)\theta(1-\theta)(1-\xi)(1-\lambda)g'' \\ &+ q\theta^2(1-\lambda)\lambda h'' + (1-q)\theta^2\lambda(1-\lambda)g'' \\ &+ q\theta(1-\theta)(1-\xi)\lambda h'' + (1-q)\theta(1-\theta)\xi(1-\lambda)g''\Big](\lambda-\xi) + \xi. \qquad \text{(A.53)}\end{aligned}$$

Again due to (A.50) and (A.48) the payoff in (A.52) is larger than that in (A.53). Using the same transitivity argument we conclude that i will reveal her type (α, λ).

Case 2. Suppose agent i is of type (α, ξ). By revealing her type she gets a payoff as given by (A.33), i.e.,

$$\begin{aligned}&\frac{1}{\Pr(\alpha,\xi)}\Big[q\theta(1-\theta)\lambda\xi g' + (1-q)\theta(1-\theta)(1-\lambda)(1-\xi)h' \\ &+ q(1-\theta)^2\xi^2 g' + (1-q)(1-\theta)^2(1-\xi)^2 h' \\ &+ q\theta(1-\theta)(1-\lambda)\xi h'' + (1-q)\theta(1-\theta)\lambda(1-\xi)g'' \\ &+ q\theta(1-\theta)(1-\xi)\xi g' + (1-q)\theta(1-\theta)\xi(1-\xi)h'\Big](\lambda-\xi) + \xi.\end{aligned}$$

If instead she were to report (α, λ), her payoff is

$$\begin{aligned}&\frac{1}{\Pr(\alpha,\xi)}\Big[q\theta(1-\theta)\lambda\xi g' + (1-q)\theta(1-\theta)(1-\lambda)(1-\xi)h' \\ &+ q(1-\theta)^2\xi^2 g' + (1-q)(1-\theta)^2(1-\xi)^2 h' \\ &+ q\theta(1-\theta)(1-\lambda)\xi g' + (1-q)\theta(1-\theta)\lambda(1-\xi)h' \\ &+ q\theta(1-\theta)(1-\xi)\xi g' + (1-q)\theta(1-\theta)\xi(1-\xi)h'\Big](\lambda-\xi) + \xi. \qquad \text{(A.54)}\end{aligned}$$

Due to (A.50) and (A.48), the payoff in (A.33) is greater than that in (A.54) and so she is better off by reporting (α, ξ).

If she were to report (β, ξ), her payoff is

$$\frac{1}{\Pr(\alpha, \xi)}\Big[q\theta(1-\theta)\lambda\xi g' + (1-q)\theta(1-\theta)(1-\lambda)(1-\xi)h'$$
$$+ q(1-\theta)^2\xi^2 g' + (1-q)(1-\theta)^2(1-\xi)^2 h'$$
$$+ q\theta(1-\theta)(1-\lambda)\xi h'' + (1-q)\theta(1-\theta)\lambda(1-\xi)g''$$
$$+q\theta(1-\theta)(1-\xi)\xi h'' + (1-q)\theta(1-\theta)\xi(1-\xi)g''\Big](\lambda-\xi) + \xi.$$

Again, due to (A.50), the payoff in (A.33) is greater than that in (A.55)(???) and so she is better off by reporting (α, ξ).

If she were to report (β, λ), her payoff is

$$\frac{1}{\Pr(\alpha, \xi)}\Big[q\theta(1-\theta)\lambda\xi g' + (1-q)\theta(1-\theta)(1-\lambda)(1-\xi)h'$$
$$+ q(1-\theta)^2\xi^2 h'' + (1-q)(1-\theta)^2(1-\xi)^2 g''$$
$$+ q\theta(1-\theta)(1-\lambda)\xi h'' + (1-q)\theta(1-\theta)\lambda(1-\xi)g''$$
$$+q\theta(1-\theta)(1-\xi)\xi h'' + (1-q)\theta(1-\theta)\xi(1-\xi)g''\Big](\lambda-\xi) + \xi.$$

Again it can be seen that i is better off by reporting (β, ξ) instead of reporting (β, λ). Hence i reveals her type (α, ξ).

Case 3. Suppose agent i is of type (β, ξ).

By revealing her type she gets a payoff as given by (A.37), i.e.,

$$\frac{1}{\Pr(\beta, \xi)}\Big[q\theta(1-\theta)\lambda(1-\xi)g' + (1-q)\theta(1-\theta)(1-\lambda)\xi h'$$
$$+ q(1-\theta)^2\xi(1-\xi)g' + (1-q)(1-\theta)^2(1-\xi)\xi h'$$
$$+ q\theta(1-\theta)(1-\lambda)(1-\xi)h'' + (1-q)\theta(1-\theta)\lambda\xi g''$$
$$+q(1-\theta)^2(1-\xi)^2 h'' + (1-q)(1-\theta)^2\xi^2 g''\Big](\lambda-\xi) + \xi. \tag{A.55}$$

If she reports (β, λ), she would get

$$\frac{1}{\Pr(\beta, \xi)}\Big[q\theta(1-\theta)\lambda(1-\xi)g' + (1-q)\theta(1-\theta)(1-\lambda)\xi h'$$
$$+ q(1-\theta)^2\xi(1-\xi)h'' + (1-q)(1-\theta)^2(1-\xi)\xi g''$$
$$+ q\theta(1-\theta)(1-\lambda)(1-\xi)h'' + (1-q)\theta(1-\theta)\lambda\xi g''$$
$$+q(1-\theta)^2(1-\xi)^2 h'' + (1-q)(1-\theta)^2\xi^2 g''\Big](\lambda-\xi) + \xi. \tag{A.56}$$

As (A.50) holds, she is better off by reporting (β, ξ).

If she reports (α, ξ), she would get

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta(1-\theta)\lambda(1-\xi)g' + (1-q)\theta(1-\theta)(1-\lambda)\xi h' \\ + q(1-\theta)^2\xi(1-\xi)g' + (1-q)(1-\theta)^2(1-\xi)\xi h' \\ + q\theta(1-\theta)(1-\lambda)(1-\xi)g' + (1-q)\theta(1-\theta)\lambda\xi h' \\ + q(1-\theta)^2(1-\xi)^2 h'' + (1-q)(1-\theta)^2\xi^2 g''\Big](\lambda-\xi) + \xi. \tag{A.57}$$

Since $\frac{1-\xi}{\xi}/\frac{\xi}{1-\xi} > \frac{1-\xi}{\xi}/\frac{\lambda}{1-\lambda}$, due to (A.50) and (A.48), expert i is better off by reporting (β, ξ).

Finally, by reporting (α, λ) she gets

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta(1-\theta)\lambda(1-\xi)g' + (1-q)\theta(1-\theta)(1-\lambda)\xi h' \\ + q(1-\theta)^2\xi(1-\xi)g' + (1-q)(1-\theta)^2(1-\xi)\xi h' \\ + q\theta(1-\theta)(1-\lambda)(1-\xi)g' + (1-q)\theta(1-\theta)\lambda\xi h' \\ + q(1-\theta)^2(1-\xi)^2 g' + (1-q)(1-\theta)^2\xi^2 h'\Big](\lambda-\xi) + \xi. \tag{A.58}$$

Since (A.50) holds, she prefers reporting (α, ξ) over (α, λ). But as she prefers reporting (β, ξ) over (α, ξ), it is optimal for her to reveal her type.

Case 4. Suppose agent i is of type (β, λ).

By revealing her type she gets a payoff as given by (A.41), i.e.,

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta^2\lambda(1-\lambda)g' + (1-q)\theta^2(1-\lambda)\lambda h' \\ + q\theta(1-\theta)\xi(1-\lambda)h'' + (1-q)\theta(1-\theta)(1-\xi)\lambda g'' \\ + q\theta^2(1-\lambda)^2 h'' + (1-q)\theta^2\lambda^2 g'' \\ + q\theta(1-\theta)(1-\xi)(1-\lambda)h'' + (1-q)\theta(1-\theta)\xi\lambda g''\Big](\lambda-\xi) + \xi. \tag{A.59}$$

By reporting (β, ξ) she gets

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta^2\lambda(1-\lambda)g' + (1-q)\theta^2(1-\lambda)\lambda h' \\ + q\theta(1-\theta)\xi(1-\lambda)g' + (1-q)\theta(1-\theta)(1-\xi)\lambda h' \\ + q\theta^2(1-\lambda)^2 h'' + (1-q)\theta^2\lambda^2 g'' \\ + q\theta(1-\theta)(1-\xi)(1-\lambda)h'' + (1-q)\theta(1-\theta)\xi\lambda g''\Big](\lambda-\xi)+\xi. \tag{A.60}$$

Due to (A.50), she is better off by reporting (β, λ).

If she reports (α, ξ) she gets

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta^2\lambda(1-\lambda)g' + (1-q)\theta^2(1-\lambda)\lambda h' \\ + q\theta(1-\theta)\xi(1-\lambda)g' + (1-q)\theta(1-\theta)(1-\xi)\lambda h' \\ + q\theta^2(1-\lambda)^2 g' + (1-q)\theta^2\lambda^2 h' \\ + q\theta(1-\theta)(1-\xi)(1-\lambda)h'' + (1-q)\theta(1-\theta)\xi\lambda g''\Big](\lambda-\xi)+\xi. \tag{A.61}$$

Since $\frac{1-\xi}{\xi}/\frac{\xi}{1-\xi} > \frac{1-\lambda}{\lambda}/\frac{\lambda}{1-\lambda}$, due to (A.50) she does better by reporting (β, ξ) instead of (α, ξ).

If she reports (α, λ) she gets

$$\frac{1}{\Pr(\beta,\xi)}\Big[q\theta^2\lambda(1-\lambda)g' + (1-q)\theta^2(1-\lambda)\lambda h' \\ + q\theta(1-\theta)\xi(1-\lambda)g' + (1-q)\theta(1-\theta)(1-\xi)\lambda h' \\ + q\theta^2(1-\lambda)^2 g' + (1-q)\theta^2\lambda^2 h' \\ + q\theta(1-\theta)(1-\xi)(1-\lambda)g' + (1-q)\theta(1-\theta)\xi\lambda h'\Big](\lambda-\xi)+\xi. \tag{A.62}$$

Due to (A.50) she does better by reporting (α, ξ) rather than (α, λ). Hence, it is optimal for i to report (β, λ).

This concludes our proof. ■

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