Managing Investment and Production in Principal-Agent Relationships*

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Abstract

An agent's investment increases his future productivity but forgoes current production surplus. This paper studies dynamic contracting under this tradeoff, with moral hazard in production and adverse selection in investment gains. The principal's ability to observe the agent's investment (or not) affects how she deters the agent from *jointly* misrepresenting his private information and deviating from the recommended investment decision. When the principal can observe the agent's investment, there are less investing agents in equilibrium, investing agents get weaker production incentive, and non-investing agents receive time-stationary incentives; whereas under non-observability, non-investing agents are "punished" with lower incentive after a bad performance.

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As pointed out in the seminal work of Holmström and Milgrom (1991), "*multidimensional tasks are ubiquitous in the world of business*". For example, a college professor's responsibilities involve both teaching and doing research. Similarly, in many economic relationships, individuals have to balance activities that generate surplus (production) with activities that create learning/improvement (investment). The returns to investment are often derived only in the future. Hence, unlike the tradeoff between expanding effort on teaching and research, the tradeoff between investment and production is often dynamic, which is not captured by the canonical Holmström-Milgrom multitasking framework.

For example, a worker can undergo training to increase his future productivity, but training takes time away from his current production; a supplier can customize its equipment to cater to the needs of its buyers, but the process of familiarizing with the new equipment leads to low-quality supplies in the short run; a manager can explore for more profitable business ventures, but the exploration process allocates resources away from exploiting the profitability of the current projects. In all these instances, the decision to invest now or not depends on both the current and the future incentives on production. Despite its prevalence, the study of dynamic contracting in such situations has largely been neglected.

This paper provides a simple model to study the dynamic tradeoff between investment and current production, with contracting frictions arising from the investing party (agent) having both ex-ante private information regarding the value of his investment (adverse selection) and ex-post private information on his actions that affect the surplus generated (moral hazard). The focus is on understanding how the three issues – adverse selection, moral hazard, and multitasking between investment and production – both in isolation and in combination, affect the level of investment, the type of investing agents, the incentive structures, and the kind of distortions (if any) in equilibrium. In particular, I identify an important form of double deviation in such contracting problems, where the agent first misrepresents his ex-ante private information and then deviates from the recommended investment decision, and I illustrate how the contracting frictions are modulated in equilibrium, depending on whether the principal can observe the agent's investment decision or not.

The setup is a two-period principal-agent model.[1] To abstract away from other contracting frictions, the model sets aside limited liability and gives full commitment to the principal to offer long-term contracts on the outputs. In period 1, the agent chooses between investment or production (multitasking); in period 2, the agent can only engage in production. Production in each period generates a binary output and the probability of getting the high output is the agent's privately exerted effort (moral hazard). Investment, on the other hand, generates no output but it increases the marginal productivity of the agent's production effort in period 2. The increase in productivity in turn is increasing in a parameter that is privately known to the agent (adverse selection). This parameter, which is drawn from a continuum support and is simply referred to as the agent's type, can be interpreted in different ways. In the context of worker training, the type can be the worker's learning ability; in the context of supplier customizing its equipment, the type can be the quality of the new equipment; in the context of manager exploring for future ventures, the type can be the manager's private knowledge about the current climate for exploration. Note that the marginal productivity of effort of all types are ex-ante the same, so the period-1 investment can be viewed as making the agent's private information payoff relevant in period 2. For simplicity, the baseline model assumes away any direct cost of investment and investment intensity; Section 7.1 discusses how these features can be readily added.

I first show that the investment decision in the first best is a threshold rule – only agents with type above a threshold invest at the expense of forgoing period-1 production. Away from the first best, two kinds of distortions are possible, and both could arise *only* in the presence of both moral hazard and adverse selection (Proposition 2). The first is the distortion on the types of agent who invest in equilibrium (i.e. investment decision). The second is the distortion in efforts for production, which can be further categorized into distortions along the P*-path* (i.e. produce in both periods) and distortions along the I*-path* (i.e. invest in period 1 and produce only in period 2).

As a benchmark for the I-path, Section 3.3 considers the contracting problem

[1] I use the female pronoun for the principal and male pronoun for the agent.

in the absence of the production activity in period 1. This removes the multitasking aspect and reduces it to essentially a static problem of adverse selection with moral hazard where all types involved engage in the I-path. The principal thus screens the agent with a menu of bonus-pay contracts that consist of a fixed wage and a bonus for a high period-2 output. Since higher types get the bonus with a higher probability after exerting the same effort level, higher types would accept a contract with lower fixed wages in exchange for higher bonuses. Hence, to screen the agents, the optimal menu features a bonus schedule that is increasing in type and a fixed wage schedule that is decreasing in it. Analogous to results in standard screening problems, the higher types earn higher information rents, and there is distortion in effort incentive – hence distortion in effort exerted – for all types other than the highest type.

Bringing back the multitasking aspect in period 1, Section 4 considers the first main contracting environment where the agent's period-1 investment decision is also contractible. I first show that the investment decision is still a threshold rule in equilibrium (Proposition 4), but the threshold is not the same as in the first best. Moreover, there is no distortion in efforts for production along the P-path, but the kind of effort distortions in the I-path described in the I-path benchmark above remains here. This decreases the principal's relative profitability of the I-path and she thus induces less agents (relative to the first best) to invest in equilibrium. In Section 5, I illustrate how this equilibrium outcome can also be achieved when the agent's investment decision is only observable to the principal but is not contractible.

Next, Section 6 considers the second main environment where the agent's period-1 activity is neither contractible nor observable to the principal. Contracts in this case can still be categorized into P-contracts (which recommend taking the P-path) and I-contracts (which recommend taking the I-path), but the recommended path must be made incentive compatible for the agent in equilibrium. The investment decision in this case remains a threshold rule in equilibrium, but the interaction between multitasking and adverse selection is allowed to operate in full force now. To illustrate this, recall that in standard screening problems such as in second-degree price discrimination, the quantity consumed and the price paid by a buyer when he deviates to another type's contract is completely determined by the contract that the agent has chosen. In contrast, it is not clear in the context here what the "quantity" and "price" are when the agent takes the contract of another type, since the agent is free to take the P-path or I-path with any contract when the principal cannot observe his period-1 investment decision. In particular, the contract design faces the added complexity of having to deter the agent from *jointly* misrepresenting his type and deviating from the recommended path.

I show that the crucial form of such double deviations is what I term the *-*deviation*, where an agent takes a P-contract (not meant for him) and engages in the I-path with it. By itself, there is a systematic relationship between an agent's type and his *-deviation payoff that is analogous to a "single-crossing condition" – a higher type gets a higher marginal increase in his *-deviation payoff when the period-2 bonus after a low period-1 output of a P-contract is increased. However this single-crossing condition disappears when interacted with the truth-telling constraints that sort the higher types who are suppose to take the I-path – while a higher type has a higher *-deviation payoff, his payoff from his I-path contract is also higher due to the information rent needed to satisfy his truth-telling constraints. Hence it is not clear *a priori* if the constraints to deter *-deviations are more stringent for the higher or lower types.

Besides the added complexity in contract design as just described, I find three main differences in the equilibrium outcome. These differences, which are summarized below, speak to how the investment level and the incentive structures across different types of agents are affected as the contracting environments differ in the principal's ability to observe the agent's investment decision.

1. (P-contract) Under non-observability of agent's investment decision, the P-contracts "punish" the agent with a lower and less efficient period-2 bonus after a low period-1 output. On the other hand, under observability, the bonuses of the P-contracts are time-stationary and hence, the period-2 bonuses are independent of the period-1 output.

2. (I-contracts) The period-2 bonuses of the I-contracts are higher under non-observability of agent's investment decision than under observability. Hence, conditional on investing, non-observability leads to more efficient

effort levels exerted by the agent.

3. (Amount of investment) There is more investment in equilibrium under non-observability of agent's investment decision.

Intuitively, these differences arise due to the need to deter *-deviations when the principal cannot observe the agent's investment decision. First, since investment always leads to low output in period 1, the principal decreases the agent's gain from privately investing with a P-contract by punishing the agent with a lower period-2 bonus after a low period-1 output. Second, giving higher bonuses in the I-contracts increases the information rent and hence the on-path payoff of the agents with I-contracts, which in turn helps to deter *-deviations. Third, inducing more investment in equilibrium also increases the on-path payoffs of the agents with I-contracts. This is because the investment decision in equilibrium is a threshold rule, hence inducing more investment implies lowering the threshold type who invest in equilibrium, and having more low types in the screening problem increases the information rent for the high types. The implications of these three features are further discussed in Section 8.

The rest of the paper is organized as follows. The next section discusses the related literature. Section 2 sets up the baseline model and Section 3 provides benchmarks on the first best and the case where one form of contracting friction (adverse selection, moral hazard or multitasking) is absent. Combining all three forms of frictions, Section 4 considers the case where the agent's investment decision is contractible; Section 5 shows that the equilibrium outcome in Section 4 is also achievable with mere observability of the investment decision by the principal; and Section 6 considers the case where the agent's investment decision is neither contractible nor observable to the principal. Section 7 discusses some of the assumptions of the model, and finally Section 8 concludes.

1 Related Literature

Broadly, this paper combines elements of multitasking (Holmström and Milgrom, 1991), dynamic moral hazard and adverse selection.[2] Laffont and Tirole (1987,

[2]See Laffont and Martimort (2001) and Salanié (2005) for textbook treatments.

1988) are early contributions with both dynamic moral hazard and adverse selection but they are concern about the ratchet effect due to short-term contracting. Recent papers that consider long-term contracting include Sannikov (2007), Gershkov and Perry (2012), Garrett and Pavan (2015) and Halac, Kartik and Liu (2016); these papers study the interaction between incentives for effort and incentives for screening, but they do not consider multitasking. Manso (2011), Ederer (2013), Ferreira, Manso and Silva (2014) and Tan (2017) combine elements of multitasking and dynamic moral hazard but they do not have ex-ante asymmetric information. Thiele (2010), Meng and Tian (2013), and Bénabou and Tirole (2016) have both multitasking and adverse selection but they address very different issues from here.[3]

This paper is also related to contract design in agency problems when the contract has to serve the dual purpose of incentivizing information acquisition by the agent and then information elicitation from him – see for example, Crémer and Khalil (1992), Lewis and Sappington (1997), Crémer, Khalil and Rochet (1998*a*,*b*), Szalay (2005, 2009) and Iossa and Martimort (2015). The difference is that rather than motivating the agent to acquire a piece of ex-ante unknown information, the principal here is inducing the agent to make a piece of privately known information payoff-relevant via a period-1 investment.

That the principal's observability of the agent's investment decision significantly affects the equilibrium contracts and investment decisions is a consequence of how it allows the effects of moral hazard and multitasking to interact with adverse selection. Chade and Swinkels (2016) study a general model of adverse selection with moral hazard and consider when the two problems can be "decoupled" into finding a solution to the moral hazard problem for each type first, followed by solving the adverse selection problem while taking into account the solution to the moral hazard problem.[4] Here, there are two layers of hidden actions or "moral hazard". The first is the hidden action on the agent's effort (which I term the moral hazard aspect) and the second is on the agent's invest-

[3]Bénabou and Tirole (2016) consider competitive screening, whereas Thiele (2010) and Meng and Tian (2013) study the static screening problem in which the agent exerts a multi-dimensional effort that affects a single-dimensional contractible output.

[4]The setup in Chade and Swinkels (2016) is static and the agent's private information is on cost. Hence the model here is not encompassed by their setup but the intuition applies.

ment decision in period 1 (which I term the multitasking aspect). When the agent's investment decision is contractible (Section 4), there is only hidden action in the agent's effort. In this case, the principal's problem can be decoupled and, as Chade and Swinkels (2016) point out, the optimal contracts will have the familiar properties of the solution to standard adverse selection problems. When the agent's investment decision is not contractible (Section 6), the problem no longer decouples. In particular, the binding *-deviation implies that the incentive constraint that the agent does not misrepresent his type conditional on following the action recommendations (on both investment and effort) is insufficient.

2 The Model

2.1 The Environment

A principal (she) interacts with an agent (he) for two periods. Both players are risk-neutral, do not discount the future, have unlimited liability and per-period outside options normalized to zero. At $t = 1$, the agent chooses to engage in one of two available activities $\mathcal{A} \in \{P, I\}$, where P denotes *production* and I denotes *investment*. At $t = 2$, only production P is available.[5] Note that symbol $\mathcal{A}$ is used exclusively for the period-1 activity. Hence the agent has two possible paths. The *I-path* is when the agent chooses $\mathcal{A} = I$ and then produces in period 2; the *P-path* is when the agent produces in both periods.

The effort for production (i.e. activity P) in period t is denoted by $e_t \in [0, 1]$. The default marginal productivity of effort is $q < 1$. If it remains unchanged, then an effort e_t generates output $y_t = Y$ with probability qe_t and generates $y_t = 0$ with probability $1 - qe_t$. The cost of e_t is $\psi(e_t)$ where:

Assumption 1. *For $x \in [0, 1)$, $\psi(x)$ is continuously differentiable, strictly increasing and strictly convex. Moreover, $\psi(0) = \psi'(0) = 0$, $\lim_{x\to1} \psi(x) = \infty$, and $0 \leq \psi'''(x) \leq \frac{2[\psi''(x)]^2}{\psi'(x)}$ $\forall x > 0$.*[6]

[5] Investment I can also be made available in period 2 without altering any of the results since the agent will never engage in it in period 2.

[6] The Inada condition on $\psi(\cdot)$ ensures that the optimal production effort choice is interior; the bound on the third derivative is a technical condition that ensures that the screening

Investment (i.e. activity I) does not generate any output. However, if the agent engages in I in period 1, his marginal productivity of period-2 effort will increase to $q + \lambda$ (i.e. $\Pr[y_2 = Y | e_2, \mathcal{A} = I] = (q + \lambda) e_2$). The agent privately knows λ while the principal holds a commonly known prior on λ, which is represented by a distribution function F on $\left[0, \bar{\lambda}\right]$, with $\bar{\lambda} \leq 1 - q$.

Assumption 2. *F has an atom-less density f which is differentiable and strictly positive in its support, and $\frac{d}{d\lambda}\left(\frac{1-F(\lambda)}{f(\lambda)}\right) \leq 0$.*[7]

λ is the returns to investment and is termed the agent's type. For simplicity, I assume that there is no cost attached to investment (besides forgoing the production in period 1), and there is also no intensive margin on investment – the agent either engages in I or not. Section 7.1 discusses how investment costs and intensity can be added to the model without altering the general results.

2.2 Contracts

The principal can enforce long-term contracts on the output stream. Two main contracting environments are considered. In the first environment, which is denoted by E^c and considered in Section 4, the agent's period-1 activity $\mathcal{A}$ is also contractible; in the second environment, which is denoted by E^u and considered in Section 6, the principal *cannot* observe nor contract on $\mathcal{A}$. In Section 5, I discuss how the equilibrium outcomes do not differ between contractibility and mere observability of $\mathcal{A}$ by the principal.

Due to the asymmetric information during contracting, the principal offers a menu of contracts to screen the agent at the start of period 1. Since both players are risk neutral and do not discount the future, there is an isomorphism among many long-term contracts. Without loss of generality, I restrict attention to the class of long-term contracts that consist of a fixed wage and a non-negative bonus for output $y_t = Y$ in each period.

I denote period-1 payments in the contract by Roman alphabets and period-2 payments by Greek letters. Let a be the period-1 fixed wage and b be the period-

problem later is concave. Examples of functions that satisfy Assumption 1 include $\psi(x) = -\log(1-x) - x$, and $\psi(x) = \frac{x^{m_2}}{(1-x)^{m_1}}$ for any $m_1 \geq 1$ and $m_2 > m_1$.

[7] These are standard assumptions that ensure that the screening problem is well-behaved.

1 bonus; let $(\alpha_{y_1}, \beta_{y_1})$ be period-2 payments after a realization of y_1, where α_{y_1} is the fixed wage and β_{y_1} is the bonus. Hence a long-term contract consists of a set of payments (or wages) $\mathcal{W} = \{(a, b); (\alpha_Y, \beta_Y); (\alpha_0, \beta_0)\}$.[8] As a convention, the payments specified in the contract are from the principal to the agent; hence negative payments refer to transfers from the agent to the principal. If $\mathcal{A}$ is also contractible, then the contract also specifies $\mathcal{A} = \{P, I\}$, which entails a $-\infty$ payment if the agent accepts the contract but deviates from the $\mathcal{A}$-obligation.

The agent's payoff from a contract is the total expected payments from the principal less his costs of effort, and the principal's payoff from a contract is the total expected outputs less the total expected payments to the agent. For conciseness, whenever left unspecified, all payoffs refer to expected payoffs.

3 Benchmarks

3.1 First best

The per-period first best surplus under the P-path is $s^{P^{(FB)}} := \max_e \{qeY - \psi(e)\}$, where the per-period first best effort $e^{P^{(FB)}}$ is uniquely characterized by $qY = \psi'\left(e^{P^{(FB)}}\right)$. Hence the first best total P-path surplus is $S^{P^{(FB)}} := 2s^{P^{(FB)}}$.

Under the I-path, the first best period-2 surplus for type λ is $S^{I^{(FB)}}(\lambda) := \max_{e_2} \{(q+\lambda) e_2 Y - \psi(e_2)\}$,[9] where the first best post-investment effort $e_2^{I^{(FB)}}(\lambda)$ is uniquely characterized by $(q+\lambda) Y = \psi'\left(e_2^{I^{(FB)}}(\lambda)\right)$.

Proposition 1. *$S^{I^{(FB)}}(\lambda)$ is strictly increasing in λ. Hence there exists L^{FB} such that $S^{I^{(FB)}}(\lambda) \geq S^{P^{(FB)}}$ if and only if $\lambda \geq L^{FB}$.*

Proposition 1 follows immediately from the envelope theorem. It implies that the first best investment decision follows a threshold rule on the agent's type: the I-path (respectively P-path) is optimal if λ is higher (respectively lower) than L^{FB}. Henceforth, I assume that $0 < L^{FB} < \bar{\lambda}$.

[8] There is a bit of redundancy in defining the set of payments this way. For example, the terms b and α_Y can be combined as one payment. But defining the set of payments this way – by having a bonus-pay contract at each history – helps streamline the exposition.

[9] Note that $S^{I^{(FB)}}(0) = s^{P^{(FB)}}$.

3.2 Absence of Adverse Selection or Moral Hazard

This subsection illustrates that inefficiency arises in this model only if both adverse selection and moral hazard are present.

Suppose there is no adverse selection: let λ be commonly known but neither effort nor $\mathcal{A}$ is observable so that the moral hazard and multitasking aspects are still present. In this case, the principal can "sell the firm" to the agent and achieve the first best payoff. Specifically, if $\lambda < L^{FB}$, the principal offers the contract: $a = \alpha_Y = \alpha_0 = -s^{P^{(FB)}}$; $b = \beta_Y = \beta_0 = Y$. If $\lambda \geq L^{FB}$, the principal offers the same set of bonuses but with fixed wages $a = \alpha_Y = 0$, and $\alpha_0 = -S^{I^{(FB)}}(\lambda)$ instead.[10] Since the agent is the residual claimant of the surplus, he exerts first best efforts on the optimal activity path. The total surplus generated is then extracted by the principal via the fixed wage.

Suppose there is no moral hazard next: e_t is contractible but λ is the agent's private information; contractibility of e_t implicitly implies that the principal can also enforce $\mathcal{A} = P$. The menu of contracts that achieves first best is the following: $b(\lambda) = \beta_Y(\lambda) = \beta_0(\lambda) = 0\ \forall\lambda$; for $\lambda < L^{FB}$, $a(\lambda) = \alpha_Y(\lambda) = \alpha_0(\lambda) = \psi\left(e^{P^{(FB)}}\right)$ with $e_1, e_2 = e^{P^{(FB)}}$; for $\lambda \geq L^{FB}$, $a(\lambda) = 0$ and $\alpha_0(\lambda) = \psi\left(e_2^{I^{(FB)}}(\lambda)\right)$ with $e_1 = 0$, $e_2 = e_2^{I^{(FB)}}(\lambda)$ and the recommendation that $\mathcal{A} = I$ (since there is no cost to engage in $\mathcal{A} = I$, the agent will obey the recommendation). Since the effort costs are independent of λ, the agent is indifferent between any contract in the menu and his truth-telling constraint is trivially satisfied.

In light of the arguments in the two preceding paragraphs:

Proposition 2. *The principal can earn the first best surplus if either adverse selection or moral hazard is absent.*[11]

3.3 Absence of Multitasking: Second Best I-Path

It is also useful to understand the interaction between adverse selection and moral hazard without the multitasking aspect in period 1. This subsection con-

[10] These contracts are not unique. In particular, the fixed wages can be spread out across periods without altering the incentives.

[11] Halac, Kartik and Liu (2016) has the same result under a setting without multitasking.

siders the contracting problem when only activity I is available in period 1.[12] For reasons which will be clear in the next section, I consider the principal maximizing her payoffs from inducing only agents of types $\lambda \in \left[\underline{\lambda}, \bar{\lambda}\right]$ to be involved, with $\underline{\lambda} \geq 0$ and exogenously given, and types $\lambda < \underline{\lambda}$ to take their outside options. To streamline the exposition, the set of wages in a contract is still denoted by $\mathcal{W}$, although only the terms a, α_0 and β_0 are relevant since all agents involved take the I-path.

To first handle the moral hazard problem in period 2, it is convenient to work with the agent's indirect utility. Denote:

$$\varepsilon\left(\beta,\lambda\right) := \underset{e\in[0,1]}{\arg\max}\left\{\left(q+\lambda\right)e\beta-\psi\left(e\right)\right\}, \tag{3.1}$$

$$u\left(\beta,\lambda\right) := \max_{e\in[0,1]}\left\{\left(q+\lambda\right)e\beta-\psi\left(e\right)\right\}. \tag{3.2}$$

Under bonus β, $u\left(\beta,\lambda\right)$ is the variable part of the indirect utility of an agent who has marginal productivity of effort $q+\lambda$, and $\varepsilon\left(\beta_0,\lambda\right)$ is his optimal effort. The agent's indirect utility under $\mathcal{W}$ is thus $a+\alpha_0+u\left(\beta_0,\lambda\right)$. By viewing the fixed wages $a+\alpha_0$ as a "transfer" and the bonus β_0 as an "allocation", this is a pure adverse selection problem with the agent having quasi-linear preferences.

Lemma 1. $\varepsilon\left(\beta,\lambda\right)$ and $u\left(\beta,\lambda\right)$ are continuously differentiable and strictly increasing in both their arguments. The agent's indirect utility function also satisfies the single-crossing condition: $\frac{\partial^2 u(\beta,\lambda)}{\partial\lambda\partial\beta} > 0\ \forall\lambda$.

By the revelation principle, it is without loss to consider the principal offering a menu of wages $\{\mathcal{W}(\lambda)\}_{\lambda\in\left[\underline{\lambda},\bar{\lambda}\right]}$ satisfying the relevant constraints. Let:

$$U^I\left(\mathcal{W},\lambda\right) := a+\alpha_0+u\left(\beta_0,\lambda\right). \tag{3.3}$$

[12] If only activity P is available in period 1, then the private information on λ is never payoff-relevant. The principal can then achieve first best by "selling the firm" to the agent through setting $a=\alpha_Y=\alpha_0=-s^{P^{(FB)}}$; $b=\beta_Y=\beta_0=Y$.

The menu $\{\mathcal{W}(\lambda)\}_{\lambda\in[\underline{\lambda},\bar{\lambda}]}$ is feasible if:

$$U^I(\mathcal{W}(\lambda),\lambda)\geq U^I(\mathcal{W}(\lambda'),\lambda) \quad ,\ \forall\lambda,\lambda'\geq\underline{\lambda}, \tag{3.4}$$

$$U^I(\mathcal{W}(\lambda),\lambda)\geq 0 \quad ,\ \forall\lambda\geq\underline{\lambda}, \tag{3.5}$$

$$U^I(\mathcal{W}(\lambda'),\lambda)\leq 0 \quad ,\ \forall\lambda<\underline{\lambda},\ \forall\lambda'\geq\underline{\lambda}. \tag{3.6}$$

(3.4) is the agent's truth-telling constraint, (3.5) is the participation constraint for $\lambda\geq\underline{\lambda}$, and (3.6) ensures that type $\lambda<\underline{\lambda}$ prefers his outside option. Let:

$$\Pi^I(\mathcal{W},\lambda):=-a-\alpha_0+(q+\lambda)\,\varepsilon(\beta_0,\lambda)\,[Y-\beta_0] \tag{3.7}$$

be the principal's payoff from a type-λ agent who optimally engages in the I-path under $\mathcal{W}$. The principal's contracting problem is program $\mathcal{P}^I$:

$$\max_{\mathcal{W}(\cdot)}\int_{\underline{\lambda}}^{\bar{\lambda}}\Pi^I(\mathcal{W}(\lambda),\lambda)f(\lambda)d\lambda \tag{3.8}$$
$$\text{subject to (3.4), (3.5) and (3.6).}$$

Lemma 2. $\{\mathcal{W}(\lambda)\}_{\lambda\in[\underline{\lambda},\bar{\lambda}]}$ *satisfies (3.4), (3.5) and (3.6) if and only if:*

$$U^I(\mathcal{W}(\lambda),\lambda)=\int_{\underline{\lambda}}^{\lambda}\frac{\partial u(\beta_0(l),l)}{\partial\lambda}dl,\quad \forall\lambda\in\left[\underline{\lambda},\bar{\lambda}\right],$$
$$\beta_0(\lambda)\text{ is non-decreasing in }\lambda,\quad \forall\lambda\in\left[\underline{\lambda},\bar{\lambda}\right].$$

Using standard arguments, (3.4) is equivalent to $U^I(\mathcal{W}(\lambda),\lambda)=U^I(\mathcal{W}(\underline{\lambda}),\underline{\lambda})+\int_{\underline{\lambda}}^{\lambda}\frac{\partial u(\beta_0(l),l)}{\partial\lambda}dl$ together with a monotonicity constraint on $\beta_0(\cdot)$. Since $U^I(\mathcal{W},\lambda)$ is continuous and increasing in λ, (3.6) is violated for types just below $\underline{\lambda}$ if $U^I(\mathcal{W}(\underline{\lambda}),\underline{\lambda})>0$. Hence (3.6) requires that $U^I(\mathcal{W}(\underline{\lambda}),\underline{\lambda})=0$. Substituting for $a(\lambda)+\alpha_0(\lambda)$, program $\mathcal{P}^I$ (3.8) becomes:

$$V^I(\underline{\lambda}):=\max_{\beta_0(\cdot)}\left\{\int_{\underline{\lambda}}^{\bar{\lambda}}\left[s(\beta_0(\lambda),\lambda)-\int_{\underline{\lambda}}^{\lambda}\frac{\partial u(\beta_0(l),l)}{\partial\lambda}dl\right]f(\lambda)d\lambda\right\} \tag{3.9}$$
$$\text{subject to }\beta_0(\cdot)\text{ being non-decreasing,}$$

where $s(\beta,\lambda):=[q+\lambda]\,\varepsilon(\beta,\lambda)\,Y-\psi(\varepsilon(\beta,\lambda))$ is the social surplus generated by type-λ agent under bonus β, and $\int_{\underline{\lambda}}^{\lambda}\frac{\partial u(\beta_0(l),l)}{\partial\lambda}dl$ is the information rent of type

λ under bonus schedule $\beta_0(\cdot)$.

Proposition 3. *The solution to program $\mathcal{P}^I$ (3.8) is unique. The optimal bonus schedule is $\beta^*(\cdot)$ which is characterized by:*

$$\frac{\partial\varepsilon\left(\beta^*(\lambda),\lambda\right)}{\partial\beta}\left(q+\lambda\right)\left(Y-\beta^*(\lambda)\right)=\frac{\partial^2 u\left(\beta^*(\lambda),\lambda\right)}{\partial\beta\partial\lambda}\left(\frac{1-F(\lambda)}{f(\lambda)}\right). \tag{3.10}$$

The corresponding optimal fixed wage schedule $a(\cdot)+\alpha_0(\cdot)$ is:

$$\alpha^*\left(\lambda|\underline{\lambda}\right)=\int_{\underline{\lambda}}^{\lambda}\frac{\partial u(\beta^*(l),l)}{\partial\lambda}dl-u\left(\beta^*(\lambda),\lambda\right). \tag{3.11}$$

$\beta^*(\cdot)$ is strictly increasing and $\beta^*\left(\bar{\lambda}\right)=Y$; $\alpha^*\left(\cdot|\underline{\lambda}\right)$ is strictly decreasing.

For any λ, the bonus that induces first best effort is Y. $\beta^*(\lambda)<Y$ for all $\lambda<\bar{\lambda}$ thus implies there is inefficiency for all types below $\bar{\lambda}$. Such distortions are a standard feature in the optimal menu in adverse selection problems. In the context here, notice first that under any bonus β_0, a higher type has a higher probability of getting the bonus. This advantage arises through two channels. First, for the same effort e_2, a higher type agent has a higher probability of getting β_0. Second, a higher type agent also optimally exerts higher e_2.

To satisfy the agent's participation constraint, a lower type agent then has to be given a higher fixed transfer. This creates incentives for agents to under-report their type; hence the high types have to be given information rent to satisfy their truth-telling constraints. To minimize the rent, the principal optimally distorts the bonuses for the low types to make their contracts less attractive to the high types, thus resulting in the distortion in incentives in (3.10). The left hand side of (3.10) is the marginal gain in social surplus $s(\beta_0,\lambda)$ from higher β_0; the right hand side is the marginal information rent provision from doing so.

Remark 1. The value of $\underline{\lambda}$ has no effect on $\beta^*(\cdot)$ other than on its domain. This is because under Assumptions 1 and 2, it suffices to consider the *local* trade-off between surplus creation and information rent minimization when determining $\beta^*(\cdot)$. However, the fixed wage schedule $\alpha^*\left(\cdot|\underline{\lambda}\right)$ is point-wise decreasing in $\underline{\lambda}$, which implies that a participating agent's payoff is decreasing in $\underline{\lambda}$.

4 Contracting under E^c

This section considers the first main contracting environment E^c where both activities are available and contractible in period 1. A contract here is $\kappa := \{\mathcal{A}, \mathcal{W}\}$ which specifies the period-1 activity $\mathcal{A}$ and the wages $\mathcal{W}$. Henceforth a contract that specifies $\mathcal{A}$ is termed a $\mathcal{A}$-contract. When a type-λ agents takes an I-contract with wages $\mathcal{W}$, his payoff is $U^I(\mathcal{W}, \lambda)$ in (3.3), and the principal's payoff is $\Pi^I(\mathcal{W}, \lambda)$ in (3.7). Let $U^P(\mathcal{W})$ and $\Pi^P(\mathcal{W})$ be the analogous agent's payoff and principal's payoff under a P-contract with wages $\mathcal{W}$;[13] note that the payoffs under P-path is independent of the agent's type

By the revelation principle, it suffices to consider the principal offering a menu of contracts $\{\kappa(\lambda)\}_{\lambda\in[0,\bar{\lambda}]}$ that satisfies the relevant constraints. The following class of menu will be important:

Definition 1. A menu of contracts $\{\kappa(\lambda)\}_{\lambda\in[0,\bar{\lambda}]}$ is a *threshold-menu* with threshold L if $\mathcal{A}(\lambda) = I \ \forall\lambda \geq L$ and $\mathcal{A}(\lambda) = P \ \forall\lambda < L$.

Proposition 4. *Under E^c, the optimal menu of contracts is a threshold-menu. If $\kappa = \{P, \mathcal{W}\}$ is in the optimal menu, then $U^P(\mathcal{W}) = 0$.*

Since an agent's payoff from a P-contract is independent of his type, any agent with a P-contract in equilibrium must get the lowest payoff among all agents; if not, an agent with a lower payoff can deviate to achieve the P-contract payoff regardless of his type. Under optimality, the principal gives the agent zero payoff under a P-contract. Moreover, from Lemma 1, the agent's payoff under an I-contract is strictly increasing in his type. Hence, if a lower type λ' accepts an I-contract, in which case he must be getting a non-negative payoff from it, a higher type λ'' will get a strictly positive payoff from the I-contract of type λ'. This means that λ'' cannot be getting a P-contract because P-contracts give zero payoff. This thus implies that the optimal menu is a threshold-menu.

[13]Formally, let $e_1^P(\mathcal{W}) := \underset{e_1\in[0,1]}{\arg\max}\left\{a + qe_1\left[b + \alpha_Y + u(\beta_Y, 0)\right] + (1 - qe_1)\left[\alpha_0 + u(\beta_0, 0)\right] - \psi(e_1)\right\}$. Then $U^P(\mathcal{W}) = a + qe_1^P(\mathcal{W})\left[b + \alpha_Y + u(\beta_Y, 0)\right] + (1 - qe_1^P(\mathcal{W}))\left[\alpha_0 + u(\beta_0, 0)\right] - \psi\left(e_1^P(\mathcal{W})\right)$; $\Pi^P(\mathcal{W}) = -a + qe_1^P(\mathcal{W})\left[Y - \alpha_Y + \varepsilon(\beta_Y, 0)\left[Y - \beta_Y\right]\right] + (1 - qe_1^P(\mathcal{W}))\left[-\alpha_0 + \varepsilon(\beta_0, 0)\left[Y - \beta_0\right]\right]$.

By the discussion above, it is without loss to restrict attention to threshold-menus and let all P-contracts in the optimal menu be the same contract with wages $\mathcal{W}^P$ (to be determined). A threshold-menu $\{\kappa(\lambda)\}_{\lambda\in[0,\bar{\lambda}]}$ with threshold L is feasible if:

$$
\begin{aligned}
U^P\left(\mathcal{W}^P\right) &= 0 && && (4.1)\\
0 &\geq U^I\left(\mathcal{W}(\lambda'),\lambda\right) && ,\ \forall\lambda<L,\ \forall\lambda'\geq L; && (4.2)\\
U^I\left(\mathcal{W}(\lambda),\lambda\right) &\geq 0 && ,\ \forall\lambda\geq L, && (4.3)\\
U^I\left(\mathcal{W}(\lambda),\lambda\right) &\geq U^I\left(\mathcal{W}(\lambda'),\lambda\right) && ,\ \forall\lambda,\lambda'\geq L, && (4.4)
\end{aligned}
$$

(4.1) comes from Proposition 4 and it satisfies the participation constraint for type $\lambda<L$; (4.2) then ensures type $\lambda<L$ does not take any of the I-contracts. Next, (4.3) is the participation constraint for type $\lambda\geq L$, which also ensures that he does not take the P-contract; and (4.4) ensures type $\lambda\geq L$ prefers his I-contract to another I-contract.

Hence the principal's optimal contracting problem is program $\mathcal{P}^c$:

$$
V^c := \max_{\mathcal{W}^P;\mathcal{W}(\cdot);L}\left\{\underbrace{\int_L^{\bar{\lambda}}\Pi^I\left(\mathcal{W}\left(\lambda\right),\lambda\right)f(\lambda)d\lambda}_{\text{from } I\text{-contracts}}+\underbrace{F\left(L\right)\Pi^P\left(\mathcal{W}^P\right)}_{\text{from } P\text{-contracts}}\right\} \qquad (4.5)
$$

subject to (4.1), (4.2), (4.3) and (4.4).

The objective function in (4.5) consists of the payoffs from the I-contracts given to types $\lambda\geq L$ (i.e. integral term) and the payoffs from the P-contract for $\lambda<L$ (i.e. $F\left(L\right)\Pi^P\left(\mathcal{W}^P\right)$). To avoid the trivial solution where the principal does not induce any investment in equilibrium, I assume that $V^c>S^{P(FB)}$.[14]

To solve program $\mathcal{P}^c$, first notice that $\mathcal{W}^P$ can be arbitrarily chosen as long as it satisfies (4.1). Optimality then implies choosing it to achieve the first best P-path surplus: $\Pi^P\left(\mathcal{W}^P\right)=S^{P(FB)}$.

Next, consider the optimal menu of I-contracts for a fixed threshold L. Notice that (4.2) to (4.4) are equivalent to (3.4) to (3.6) with $L=\underline{\lambda}$. Hence this problem

[14]If $V^c=S^{P(FB)}$, then the solution is trivially that the principal "sells the firm" to the agent at the "price" $S^{P(FB)}$.

is program $\mathcal{P}^I$ in (3.8), and the solution is the menu $\{\alpha^*(\lambda|L), \beta^*(\lambda)\}_{\lambda\in[L,\bar{\lambda}]}$ in Proposition 3. Given that the optimal menu is unique for each L, one can then maximize the principal's indirect payoff over the threshold L to get the solution to program $\mathcal{P}^c$.

Proposition 5. *The optimal threshold L^c of program $\mathcal{P}^c$ in (4.5) is uniquely characterized by:*

$$s(\beta^*(L^c), L^c) - \frac{\partial u(\beta^*(L^c), L^c)}{\partial\lambda}\left(\frac{1-F(L^c)}{f(L^c)}\right) = S^{P^{(FB)}}, \tag{4.6}$$

with $L^c > L^{FB}$. The optimal menu of contracts $\{\{\mathcal{A}^c(\lambda), \mathcal{W}^c(\lambda)\}\}_{\lambda\in[0,\bar{\lambda}]}$ is:

- *For $\lambda \geq L^c$: $\mathcal{A}^c(\lambda) = I$; $\beta_0^c(\lambda) = \beta^*(\lambda)$; $a^c(\lambda) + \alpha_0^c(\lambda) = \alpha^*(\lambda|L^c)$.*
- *For $\lambda < L^c$: $\mathcal{A}^c(\lambda) = P$; $a^c(\lambda) = \alpha_Y^c(\lambda) = \alpha_0^c(\lambda) = -s^{P^{(FB)}}$; $b^c(\lambda) = \beta_Y^c(\lambda) = \beta_0^c(\lambda) = Y$.*[15]

In the presence of asymmetric information, the P-path is still carried out efficiently every period with the principal getting all the surplus from the relationship. However, the distortion in the I-path surplus described in Section 3.3 is also present here. Hence, relative to the first best benchmark considered in Section 3.1, the I-path becomes less profitable for the principal while the profitability of the P-path remains unchanged. This is reflected in how the principal chooses the optimal threshold L^c in (4.6). The term on the right is the first best P-path surplus, whereas the term on the left is the virtual surplus generated by the lowest type I-path agent, which is strictly less than the first best I-path surplus $S^{I^{(FB)}}(L^c)$ due to the distortion in incentives (i.e. $\beta^*(L^c) < Y$) and the information rent given to the agent. As a result, there is "under-investment" as some agents with types in $\left(L^{FB}, L^c\right)$ also engage in the P-path in equilibrium.

[15]The optimal P-contracts are not unique. Any contract that induces the agent to provide the first best effort level $e^{P^{(FB)}}$ every period and also gives him zero payoff will qualify to be an optimal P-contract here.

5 Observable but Non-contractible $\mathcal{A}$

Suppose now that $\mathcal{A}$ is only observable but not contractible by the principal, and the principal can offer contracts at the start of every period. Then there exists a perfect Bayesian equilibrium that is outcome equivalent (in terms of the agent's actions and both players' payoffs) to that of under the optimal menu in E^c as characterized in Proposition 5.

The principal's equilibrium strategy is the following. In period 1, the principal offers the menu of period-1 contracts: $a(\lambda) = -s^{P(FB)}$, $b(\lambda) = Y$ for $\lambda < L^c$; and $a(\lambda) = b(\lambda) = 0$ for $\lambda \geq L^c$. In period 2, if $y_1 = Y$, the principal offers $\alpha_Y = -s^{P(FB)}$ and $\beta_Y = Y$; if $y_1 = 0$ and the principal observed $\mathcal{A} = P$, the principal offers $\alpha_0 = -s^{P(FB)}$, $\beta_0 = Y$; if $y_1 = 0$ and the principal observed $\mathcal{A} = I$, the principal offers the menu of contracts with bonus schedule $\beta_0(\lambda) = \beta^*(\lambda)$ and fixed wage schedule $\alpha_0(\lambda) = \alpha^*(\lambda|L^c)$ for only $\lambda \in \left[L^c, \bar{\lambda}\right]$. It is readily verified that one of the agent's best response to this strategy results in the same actions as when he was facing the long-term contracts in Proposition 5, and that this principal's strategy is sequentially rational for her given the agent's best response to it; hence this is an equilibrium.

This result implies two things. First, the outcome under E^c can be achieved with just mere observability of $\mathcal{A}$; hence the differences between the equilibrium outcomes under E^c and E^u (to be considered next) can be completely attributed to the principal's ability to observe $\mathcal{A}$ or not. Second, since the equilibrium constructed above only requires that the principal can enforce spot contracts, this implies that the lack of commitment power to enforce long term contracts does not create any loss to the principal when $\mathcal{A}$ is contractible or observable.

6 Contracting under E^u.

This section considers environment E^u where the agent's period-1 activity $\mathcal{A}$ is neither contractible nor observable to the principal. To streamline exposition with E^c, a contract here is still $\kappa = \{\mathcal{A}, \mathcal{W}\}$, but $\mathcal{A}$ has to be made incentive compatible for the agent; this is without loss by the revelation principle.

6.1 The Contracting Problem

Notice that the arguments for Proposition 4 holds verbatim under E^u. Hence, as in E^c, it is also without loss under E^u to restrict attention to threshold-menus and let all P-contracts in the menu be the same. However, in contrast to E^c, incentive compatibility under E^u must also take into account that the agent can take a P-contract but engage in the I-path with it, or vice versa.

Consider a threshold-menu $\{\kappa(\lambda)\}_{\lambda\in[0,\bar{\lambda}]}$ with threshold L. Let $\mathcal{W}^P = \left\{ \left(a^P, b^P\right); \left(\alpha_Y^P, \beta_Y^P\right); \left(\alpha_0^P, \beta_0^P\right) \right\}$ be the set of wages of the P-contract meant for types $\lambda < L$; and let $\mathcal{W}^I(\lambda) = \left\{\left(a^I(\lambda), b^I(\lambda)\right); \left(\alpha_Y^I(\lambda), \beta_Y^I(\lambda)\right); \left(\alpha_0^I(\lambda), \beta_0^I(\lambda)\right)\right\}$ be the set of wages of the menu of I-contracts for types $\lambda \geq L$. Since an I-contract should deter an agent from engaging in the P-path with it, it is without loss to set $b^I(\lambda) = -\infty, \alpha_Y^I(\lambda) = \beta_Y^I(\lambda) = 0$; the relevant terms to be determined are $a^I(\lambda)$, $\alpha_0^I(\lambda)$ and $\beta_0^I(\lambda)$. The menu is feasible under E^u if:

$$
\begin{aligned}
U^P\left(\mathcal{W}^P\right) &= 0 && && (6.1)\\
0 &\geq U^I\left(\mathcal{W}^I(\lambda'), \lambda\right) && , \forall\lambda < L, \forall\lambda' \geq L, && (6.2)\\
0 &\geq U^I\left(\mathcal{W}^P, \lambda\right) && , \forall\lambda < L, && (6.3)\\
0 &\geq U^P\left(\mathcal{W}^I(\lambda')\right) && , \forall\lambda' \geq L; && (6.4)
\end{aligned}
$$

$$
\begin{aligned}
U^I\left(\mathcal{W}^I(\lambda), \lambda\right) &\geq U^I\left(\mathcal{W}^I(\lambda'), \lambda\right) && , \forall\lambda, \lambda' \geq L, && (6.5)\\
U^I\left(\mathcal{W}^I(\lambda), \lambda\right) &\geq U^P\left(\mathcal{W}^I(\lambda')\right) && , \forall\lambda, \lambda' \geq L, && (6.6)\\
U^I\left(\mathcal{W}^I(\lambda), \lambda\right) &\geq U^I\left(\mathcal{W}^P, \lambda\right) && , \forall\lambda \geq L, && (6.7)\\
U^I\left(\mathcal{W}^I(\lambda), \lambda\right) &\geq 0 && , \forall\lambda \geq L. && (6.8)
\end{aligned}
$$

Constraints (6.1) to (6.4) are for types $\lambda < L$ who are given the P-contract. (6.1) comes from Proposition 4 and satisfies the participation constraint; (6.2) deters the agent from taking any of the I-contracts to engage in the I-path; (6.3) deters the agent from using the P-contract to engage in the I-path; and (6.4) deters the agent from using any of the I-contract to engage in the P-path.

Constraints (6.5) to (6.8) are for types $\lambda \geq L$ who are given I-contracts.

(6.5) deters the agent from taking another I-contract to engage in the I-path; (6.6) deters the agent from taking any of the I-contracts (including his own) to engage in the P-path; (6.7) deters the agent from taking the P-contract to engage in the I-path; and (6.8) is the participation constraint which also deters the agent from taking the P-contract to engage in the P-path.

Lemma 3. *A threshold-menu* $\{\kappa(\lambda)\}_{\lambda\in[0,\bar{\lambda}]}$ *with threshold* L *satisfies constraints (6.1) to (6.8) if and only if:*

$$U^P\left(\mathcal{W}^P\right) = 0. \tag{6.9}$$

$$\text{For all } \lambda \geq L,\ U^I\left(\mathcal{W}^I(\lambda), \lambda\right) = \int_L^{\lambda} \frac{\partial u\left(\beta_0^I(l), l\right)}{\partial \lambda} dl. \tag{6.10}$$

$$\text{For all } \lambda \geq L,\ \beta_0^I(\lambda) \text{ is non-decreasing.} \tag{6.11}$$

$$\text{For all } \lambda \geq L,\ U^I\left(\mathcal{W}^I(\lambda), \lambda\right) \geq a^P + \alpha_0^P + u\left(\beta_0^P, \lambda\right). \tag{6.12}$$

(6.9) is (6.1). (6.10) and (6.11) are sorting constraints for agents who engage in the I-path in equilibrium – see Lemma 2 in Section 3.3. (6.12) is constraint (6.7), which deters high-type agents from taking the P-contract to engage in the I-path. The proof of Lemma 3 illustrates how constraints (6.2) to (6.8) are subsumed by (6.10) to (6.12).

Next, I illustrate that the optimal $\mathcal{W}^P$ can be reduced to just optimally determining the terms b^P and β_0^P. Consider the class of wage set $\bar{\mathcal{W}}(b, \beta_0)$ defined by:

$$\bar{\mathcal{W}}(b, \beta_0) = \left\{\left(\underbrace{-u(b,0)}_{=a}, b\right); \left(\underbrace{-u(Y,0)}_{=\alpha_Y}, \underbrace{Y}_{=\beta_Y}\right); \left(\underbrace{-u(\beta_0,0)}_{\alpha_0}, \beta_0\right)\right\} \tag{6.13}$$

$\bar{\mathcal{W}}(b, \beta_0)$ is completely determined by the terms b and β_0, and it has the feature that the agent's period-2 payoff along the P-path is always zero and that $U^P\left(\bar{\mathcal{W}}(b, \beta_0)\right) = 0$ for any b and β_0.

Lemma 4. *If* $\mathcal{W}^P = \left\{\left(a^P, b^P\right); \left(\alpha_Y^P, \beta_Y^P\right); \left(\alpha_0^P, \beta_0^P\right)\right\}$ *is the* P*-contract in an optimal menu, then* $\beta_Y^P = Y$*, and there exists* $\hat{b}^P$ *such that:*

1. $-u\left(\hat{b}^P, 0\right) - u\left(\beta_0^P, 0\right) = a^P + \alpha_0^P$.

2. $\Pi^P\left(\bar{\mathcal{W}}\left(\hat{b}^P,\beta_0^P\right)\right)=\Pi^P\left(\mathcal{W}^P\right)$.

Hence $\bar{\mathcal{W}}\left(\hat{b}^P,\beta_0^P\right)$ *is also an optimal* P*-contract.*

Proof. $\hat{b}^P=b^P+\alpha_Y^P-\alpha_0^P+u\left(Y,0\right)-u\left(\beta_0^P,0\right)$. Details are in Online Appendix. □

Since β_Y^P does not affect (6.10) to (6.12), β_Y^P must be the first best level Y in the optimal P-contract to maximize surplus. Lemma 4.1 implies that if $\mathcal{W}^P$ satisfies (6.12), $\bar{\mathcal{W}}\left(\hat{b}^P,\beta_0^P\right)$ also satisfies (6.12). Since (6.10) and (6.11) concern only the I-contracts and $\bar{\mathcal{W}}\left(\hat{b}^P,\beta_0^P\right)$ always satisfies (6.9), it is also feasible. Lemma 4.2 then implies that if $\mathcal{W}^P$ is optimal, $\bar{\mathcal{W}}\left(\hat{b}^P,\beta_0^P\right)$ is also optimal.

By Lemma 4, it is without loss to restrict attention to the class of P-contracts of the form $\bar{\mathcal{W}}\left(\cdot\right)$ in (6.13). The principal's optimal contracting problem is thus program $\mathcal{P}^u$:

$$V^u:=\max_{b^P,\beta_0^P;\mathcal{W}^I(\cdot);L}\left\{\underbrace{\int_L^{\bar{\lambda}}\Pi^I\left(\mathcal{W}^I(\lambda),\lambda\right)f(\lambda)d\lambda}_{\text{from } I\text{-contracts}}+\underbrace{F\left(L\right)\Pi^P\left(\bar{\mathcal{W}}\left(b^P,\beta_0^P\right)\right)}_{\text{from } P\text{-contracts}}\right\} \tag{6.14}$$
$$\text{subject to (6.10) to (6.12).}$$

To avoid trivial solutions with only P-contract or only I-contracts in equilibrium:

Assumption 3. $V^u>\max\left\{S^{P(FB)},V^I\left(0\right)\right\}$ *where* $V^I\left(\cdot\right)$ *is defined in (3.9).*

6.2 *-Deviations

Comparing with program in $\mathcal{P}^c$ in (4.5), program $\mathcal{P}^u$ in (6.14) has an additional constraint (6.12) which deters *-deviations:

Definition 2. An agent is said to be engaging in *-*deviation* if he takes a P-contract and chooses $\mathcal{A}=I$.

This subsection discusses the implications of the *-deviation. Combining (6.12) with (6.10), one gets:

$$\int_L^{\lambda}\frac{\partial u\left(\beta_0^I\left(x\right),x\right)}{\partial\lambda}dx\ \geq\ a^P+\alpha_0^P+u\left(\beta_0^P,\lambda\right)\quad\forall\lambda\geq L. \tag{IC$_u^*$}$$

Under E^c, the optimal menu of I-contracts and the optimal P-contract can be chosen *separately*. Hence the P-contract there is optimally set to achieve the first best P-path surplus, while the period-2 bonus schedule $\beta_0^I(\cdot)$ for the I-contracts is set at $\beta^*(\cdot)$ which achieves the constrained-optimal I-path surplus as in Section 3.3. Under E^u, this separability disappears due to (IC_u^*).

Let $\mathcal{D}\left(\lambda; b^P, \beta_0^P\right)$ be type-λ's *-deviation payoff under P-contract $\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)$:

$$\mathcal{D}\left(\lambda; b^P, \beta_0^P\right) := -u\left(b^P, 0\right) - u\left(\beta_0^P, 0\right) + u\left(\beta_0^P, \lambda\right).$$

(IC_u^*) holds for type λ if and only if $\mathcal{IC}^*\left(\lambda; \left\{b^P, \beta_0^P, \beta_0^I(\cdot), L\right\}\right) \geq 0$, where:

$$\mathcal{IC}^*\left(\lambda; \left\{b^P, \beta_0^P, \beta_0^I(\cdot), L\right\}\right) := \int_L^\lambda \frac{\partial u\left(\beta_0^I(l), l\right)}{\partial \lambda} dl - \mathcal{D}\left(\lambda; b^p, \beta_0^P\right).$$

Lemma 5. *The effects of the contracts on constraint (IC_u^*) are as follows:*

1. $\frac{\partial}{\partial \beta_0^P}\mathcal{D}\left(\lambda; b^P, \beta_0^P\right) > 0$; *and* $\frac{\partial}{\partial \beta_0^P}\mathcal{IC}^*\left(\lambda; \left\{b^P, \beta_0^P, \beta_0^I(\cdot), L\right\}\right) < 0$.
2. $\frac{\partial}{\partial b^P}\mathcal{D}\left(\lambda; b^P, \beta_0^P\right) < 0$, *and* $\frac{\partial}{\partial b^P}\mathcal{IC}^*\left(\lambda; \left\{b^P, \beta_0^P, \beta_0^I(\cdot), L\right\}\right) > 0$.
3. $\frac{\partial}{\partial L}\mathcal{IC}^*\left(\lambda; \left\{b^P, \beta_0^P, \beta_0^I(\cdot), L\right\}\right) < 0$.
4. $\frac{\partial}{\partial \lambda}\mathcal{D}\left(\lambda; b^P, \beta_0^P\right) > 0$.
5. $\frac{\partial}{\partial \lambda}\mathcal{IC}^*\left(\lambda; \left\{b^P, \beta_0^P, \beta_0^I(\cdot), L\right\}\right) \geq 0$ *if and only if* $\beta_0^I(\lambda) \geq \beta_0^P$.
6. $\frac{\partial^2}{\partial \beta_0^P \partial \lambda}\mathcal{D}\left(\lambda; b^P, \beta_0^P\right) > 0$; *and* $\frac{\partial^2}{\partial b^P \partial \lambda}\mathcal{D}\left(\lambda; b^P, \beta_0^P\right) = 0$

Proof. Follows from straightforward differentiation. □

Higher β_0^P increases the *-deviation payoff and causes constraint (IC_u^*) to be more stringent (Lemma 5.1). On the other hand, higher b^P increases the agent's period-1 variable payoff along the P-path; in turn, the period-1 fixed wage a^P is more negative to extract that surplus away from the agent. The more negative a^P decreases the *-deviation payoff and relaxes (IC_u^*) (Lemma 5.2).

Having less I-path agents (i.e. higher L) decreases the agent's information rent from the I-path contracts (see Remark 1) and hence causes (IC_u^*) to be more stringent (Lemma 5.3). The *-deviation payoff is increasing in the agent's type

(Lemma 5.4), and whether (IC_u^*) is more or less stringent as the type changes depends on the relative value of $\beta_0^I(\lambda)$ and β_0^P (Lemma 5.5).

Lemma 5.6 implies that there is a systematic relationship between an agent's type and the marginal effect of β_0^P on his *-deviation payoff. Hence, by itself, the *-deviation satisfies the "single-crossing condition" whereby a higher type gets a higher marginal increase in his *-deviation payoff as β_0^P increases. However, this sorting condition disappears when interacted with the truth-telling constraints amongst agents who are given I-contracts – while a higher type agent has a higher *-deviation payoff, his payoff from his I-contract is also higher due to the truth-telling constraints. Hence it is not clear if it is the higher or lower types who have greater incentives to engage in *-deviation. This is illustrated in Lemma 5.5 where the sign of $\frac{\partial}{\partial\lambda}\mathcal{IC}^*\left(\lambda;\left\{b^P,\beta_0^P,\beta_0^I(\cdot),L\right\}\right)$ depends on the contracts. This then creates the possibility of pooling of contracts for intermediate types.

6.3 Optimal Menu

The additional constraint (IC_u^*) in E^u is consequential. In particular, at least type $\bar{\lambda}$ has a profitable *-deviation under the optimal contracts in E^c.

Proposition 6. $\mathcal{IC}^*\left(\bar{\lambda};\{Y,Y,\beta^*(\cdot),L^c\}\right) < 0$. *Hence the optimal menu of contracts under E^c in Proposition 5 is not feasible under E^u.*

Next, from Lemma 5.2, the principal can relax constraint (IC_u^*) at the expense of period-1 surplus by offering an excessively high b^P. The loss in surplus from doing so increases at a decreasing rate for sufficiently high b^P – that is, the principal's payoff is convex in b^P for sufficiently high b^P. To prevent a non-concave maximization problem, I impose further restrictions on the bounds of the higher derivatives of the cost function $\psi(\cdot)$ in Assumption 4. This assumption assures that when the principal's payoff is convex in b^P, b^P is also high enough such that constraint (IC_u^*) does not bind, which in turn implies that the principal will never choose a b^P high enough that her payoff is convex in it.

Assumption 4. *For any $x \in [0,1)$, $\psi(\cdot)$ also satisfies $\psi^{(4)}(x) \leq \frac{2\left[\psi^{(3)}(x)\right]^2}{\psi^{(2)}(x)}$;*

$\psi^{(3)}(x) \leq \frac{\left[\psi^{(2)}(x)\right]^2}{q\left(x\psi'(x)-\psi(x)\right)}$*; and* $\psi^{(3)}(x)\left[\psi'(x) - q\Big(x\psi'(x) - \psi(x)\Big)\right] \leq \left[\psi^{(2)}(x)\right]^2$.[16]

Proposition 7. *Under Assumptions 3 and 4, program* $\mathcal{P}^u$ *in (6.14) has a unique solution. Let the optimal threshold be* L^u*, the optimal* P*-contract be* $\bar{\mathcal{W}}\left(b^{P(u)}, \beta_0^{P(u)}\right)$*, and the optimal* I*-contract period-2 bonus schedule be* $\beta_0^{I(u)}(\cdot)$*:*

1. *P-contract:* $b^{P(u)} > Y$ *and* $\beta_0^{P(u)} < Y$.[17]

2. *I-contracts:*

$$\beta_0^{I(u)}(\lambda) = \begin{cases} \beta^*(\lambda) & , \text{ if } \lambda \geq \lambda_2 \\ \beta_0^{P(u)} & , \text{ if } \lambda \in [\lambda_1, \lambda_2] \\ \beta^{**}(\lambda) & , \text{ if } \lambda \in [L^u, \lambda_1] \end{cases}$$

with $\lambda_2 := \beta^{*-1}\left(\beta_0^{P(u)}\right) \in \left(L^u, \bar{\lambda}\right)$ *and* $\lambda_1 \in [L^u, \lambda_2)$,[18] *and* $\beta^{**}(\lambda)$ *is characterized by:*

$$\frac{\partial \varepsilon\left(\beta^{**}(\lambda), \lambda\right)}{\partial \beta}(q+\lambda)\left(Y - \beta^{**}(\lambda)\right) = \frac{\partial^2 u\left(\beta^{**}(\lambda), \lambda\right)}{\partial \beta \partial \lambda}\left(\frac{1 - F(\lambda) - \zeta}{f(\lambda)}\right), \tag{6.15}$$

where ζ *(characterized in the proof) is strictly positive. Moreover:*

- $\beta_0^I(\cdot)$ *is continuous.*
- $\beta^{**}(\cdot)$ *is strictly increasing and* $\beta^{**}(\lambda) > \beta^*(\lambda)\ \forall \lambda$.
- $\mathcal{IC}^*\left(\lambda; \left\{b^{P(u)}, \beta_0^{P(u)}, \beta_0^{I(u)}(\cdot), L^{(u)}\right\}\right) = 0\ \forall \lambda \in [\lambda_1, \lambda_2]$.

3. *Optimal threshold:* $L^u < L^c$ *and is characterized by:*

$$s\left(\beta^{**}\left(L^u\right), L^u\right) - \frac{\partial u\left(\beta^{**}\left(L^u\right), L^u\right)}{\partial \lambda}\left(\frac{1 - F\left(L^u\right) - \zeta}{f\left(L^u\right)}\right) = \Pi^P\left(\bar{\mathcal{W}}\left(b^{P(u)}, \beta_0^{P(u)}\right)\right). \tag{6.16}$$

[16] $\psi^{(n)}(x)$ is the n-th derivative. The functions given in fn. 6 would also satisfy the first two conditions in Assumption 4. However, the third condition requires the right value of q. For example, $\psi(x) = -\log(1-x) - x$ satisfies it for $q > 0.6$; $\psi(x) = \frac{x^2}{1-x}$ satisfies it for $q > 0.5$; for any q, there are sufficiently high values of m_2 such that $\psi(x) = \frac{x^{m_2}}{(1-x)}$ satisfies it.

[17] The characterization of $b^{P(u)}$ and $\beta^{P(u)}$ are in the proof.

[18] $\beta^{*-1}(\cdot)$ is the inverse function of $\beta^*(\cdot)$.

Proposition 7 is the formal statement for the three differences due to the unobservability of $\mathcal{A}$ as stated in the introduction. Each of these features arises to help satisfy constraint (IC_u^*) and deter *-deviations.

P-contract: If $\mathcal{A} = I$, $y_1 = 0$. Hence, the principal "punishes" the agent with a lower period-2 bonus after $y_1 = 0$ (i.e. $\beta_0^{P(u)} < Y$), which decreases the agent's gain from privately investing. This lower bonus also decreases the period-2 surplus after $y_1 = 0$. In equilibrium, the principal gets all the surplus under a P-contract, so the principal offers a higher period-1 bonus (i.e. $b^{P(u)} > Y$) to increase the likelihood of getting to the higher period-2 surplus.

I-contracts: A higher bonus for λ increases the information rent of all types above λ. By offering a (weakly) higher bonus schedule under E^u (see Figure 6.1), the principal increases the agent's truth-telling payoff to offset his gain from a *-deviation. The reasoning behind the form of the bonus schedule is given in Section 6.3.1.

Investment level: Increasing the proportion of I-path agents in equilibrium (i.e. $L^u < L^c$) increases the agent's information rent (see Remark 1), which in turn helps to deter *-deviations. This feature that there is more investment in equilibrium when the principal cannot observe the agent's investment decision has also been noted in Riordan (1990). In Riordan (1990), the agent refuses to invest because he expects the principal to expropriate the investment gains ex-post; hence, by committing not to observe the investment, the resulting information asymmetry regarding the investment outcome allows the agent to capture rent ex-post, which then feeds back as his ex-ante incentive to invest. Tan (2017) considers how the incentive structure should be appropriately designed to complement the non-observability environment. In contrast, in this paper, the agent's ex-ante incentive for investment is always present because he has ex-ante private information about his returns to investment. Instead, the feature that investment increases with non-observability arises here because non-observability hinders the principal's ability to sort agents who are supposed to invest from those who are not supposed to. The higher investment under non-observability then arises via the optimal incentive design, in particular, as an artifact to increase the on-path payoffs of the investing agents.

6.3.1 Discussion on $\beta_0^{I(u)}(\cdot)$

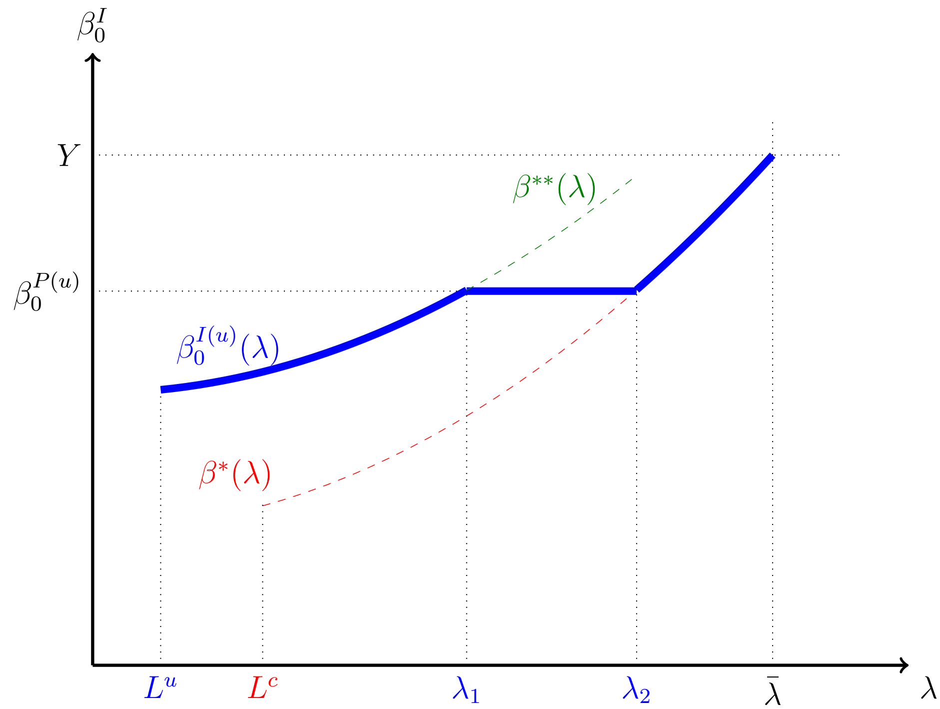

Figure 6.1: Optimal Bonus Schedule $\beta_0^{I(u)}(\cdot)$ in Proposition 7.

This subsection elaborates on the form of $\beta_0^{I(u)}(\cdot)$ which is illustrated in Figure 6.1. From Section 3.3, $\beta^*(\cdot)$ is the pointwise optimal $\beta_0^I(\cdot)$ schedule that satisfies (6.10) and (6.11). But under P-contract $\bar{\mathcal{W}}\left(b^{P(u)}, \beta_0^{P(u)}\right)$ and threshold L^u, it violates constraint (IC_u^*). To see this, first notice that:

$$\arg\min_{\lambda}\left\{\mathcal{IC}^*\left(\lambda; \left\{b^{P(u)}, \beta_0^{P(u)}, \beta^*(\cdot), L^{(u)}\right\}\right)\right\} = \lambda_2 := \beta^{*-1}\left(\beta_0^{P(u)}\right). \quad (6.17)$$

From Lemma 5.5, $\frac{\partial}{\partial\lambda}\mathcal{IC}^*\left(\lambda_2; \left\{b^{P(u)}, \beta_0^{P(u)}, \beta^*(\cdot), L^{(u)}\right\}\right) = 0$. Since $\beta^*(\cdot)$ is strictly increasing, $\frac{\partial}{\partial\lambda}\mathcal{IC}^*\left(\lambda; \left\{b^{P(u)}, \beta_0^{P(u)}, \beta^*(\cdot), L^{(u)}\right\}\right)$ is strictly negative for $\lambda < \lambda_2$ and strictly positive for $\lambda > \lambda_2$. (6.17) thus implies that (IC_u^*) is violated most severely at λ_2 under the bonus schedule $\beta^*(\cdot)$.

To find the optimal bonus schedule, one can begin with the non-feasible $\beta^*(\cdot)$ and try to restore feasibility with minimal modification at every point, starting with ensuring that constraint (IC_u^*) is satisfied at (the most severe type) λ_2 by increasing λ_2's rent. The schedule for types above λ_2 does not affect the rent of λ_2 (see (6.10)), so no modification should be made on the schedule for them. Instead, to increase the rent of λ_2, the schedule below λ_2 must be raised.

Since the schedule must be non-decreasing everywhere, the schedule below λ_2 cannot be raised above $\beta^*(\lambda_2) = \beta_0^{P(u)}$. Consider a $\lambda_1 < \lambda_2$ and raise the schedule for all $\lambda \in [\lambda_1, \lambda_2]$ to $\beta_0^I(\lambda) = \beta_0^{P(u)}$. The schedule for λ below λ_1 is then optimally determined, analogously to (3.10), by the trade-off between surplus creation and information rent provision, which is reflected in $\beta^{**}(\cdot)$ in (6.15). Comparing (6.15), with (3.10), the additional term ζ in (6.15) reflects the property that the schedule at λ_1 is $\beta_0^{P(u)}$, which is higher than $\beta^*(\lambda_1)$.

The optimal schedule is then determined by optimally choosing λ_1. To minimize modification from $\beta^*(\cdot)$, λ_1 should be chosen as close to λ_2 as possible (i.e. small "bunching" region). On the other hand, λ_1 needs to be sufficiently far from λ_2 to give type λ_2 sufficient rent to satisfy constraint (IC_u^*). Hence λ_1 is chosen exactly at the point in which constraint (IC_u^*) binds for type λ_2.

By the continuity of $\mathcal{IC}^*(\cdot)$, when constraint (IC_u^*) was violated for λ_2 under schedule $\beta^*(\cdot)$, constraint (IC_u^*) was also violated for types close to λ_2, albeit not as severely. The modification on $\beta^*(\cdot)$ described above would also ensure that feasibility for all these types are restored. It can be verified that an increase in the schedule increases the rent for the higher type at a faster rate. Hence, since the violation of (IC_u^*) for types above λ_2 was not as severe as for λ_2, constraint (IC_u^*) become slack for all types above λ_2 under the modified schedule. On the other hand, for types below λ_2, the increase in their rent is slower than for λ_2, but the violation of constraint (IC_u^*) for these types was also not as severe as for λ_2. It turns out that under the modified schedule, constraint (IC_u^*) would bind for all types in $[\lambda_1, \lambda_2]$, and it will be slack for types below λ_1.

7 Discussions

This section discusses some of the assumptions of the model.

7.1 Generalization of Model

The moral hazard aspect of the contracting problem is conveniently subsumed by working with the agent's indirect utility function. Hence the model can be readily generalized in various directions as long as the resulting analog of the indirect utility have the properties of $u(\beta, \lambda)$ in (3.2). Here are some examples.

General forms of investment gain. Let $\Pr[Y|e] = \rho(e, 0)$ without investment and let $\Pr[Y|e] = \rho(e, \lambda)$ after investment; $\rho(e, \lambda) = (q + \lambda) e$ in the baseline model. The analog of (3.2) is $u(\beta, \lambda) = \max_{e \in [0,1]} \{\rho(e, \lambda) \beta - \psi(e)\}$ now.

Investment cost and intensity. Suppose the agent also chooses investment effort $i \in [0, 1]$ with cost $C(i)$ if he chooses $\mathcal{A} = I$, so that $\Pr[y_2 = Y|e_2, i] = (q + \lambda i) e_2$ (i.e. i represents the investment intensity). The analog of (3.2) is $u(\beta, \lambda) = \max_i \left\{ \max_{e \in [0,1]} \{(q + \lambda i) e\beta - \psi(e)\} - C(i) \right\}$, and the principal's payoff will depend on the agent's optimal investment effort choice as well now.

Investment for cost reduction. Suppose the marginal productivity of effort is always q but investment lowers the cost of effort – cost of e_2 is $\psi\left(\frac{e_2}{1+\lambda}\right)$ if $\mathcal{A} = I$ and is $\psi(e_2)$ if $\mathcal{A} = P$. The analog of (3.2) is $u(\beta, \lambda) = \max_{e \in [0,1]} \left\{ q\beta - \psi\left(\frac{e}{1+\lambda}\right) \right\}$.

7.2 Investment to Reduce Asymmetric Information

In the baseline model, the agent's private information on λ becomes payoff-relevant only after investment. Hence investment creates asymmetric information in the relationship. One can imagine scenarios where, instead, there is ex-ante payoff-relevant asymmetric information and investment serves to eliminate it. For example, workers might enter the firm with privately known production ability, and training (i.e. investment) brings everyone to the same level.[19]

To formalize this, consider a model where in the absence of investment, the probability of getting the high output Y with effort e is λe where $\lambda \in \left[0, \bar{\lambda}\right]$ is the agent's privately known type. After investment, regardless of type, the marginal productivity of effort becomes $\bar{\lambda}$; hence, the value of training is *decreasing* in the

[19] A familiar context where this is applicable is the first-year curriculum in economics Ph.D programs. Incoming students enter with heterogeneous backgrounds and one of the aims of the structured first year courses is to bring the students to the required level of knowledge before they embark on their individual research.

agent's type. The rest of the model remains unchanged from the baseline model.

Many qualitative features of the baseline model remain. First, the first best assignment is still a threshold rule where types *below* the threshold invest before production while types *above* the threshold produce in both periods (cf. Proposition 1). Next, using the same argument, inefficiency arises only in the presence of both moral hazard and adverse selection (cf. Proposition 2). Moreover, under the second best, because the agent's I-path payoff is now type-independent while his P-path payoff is increasing in his type, the optimal menu is an analogous threshold-menu in which types above a threshold are assigned P-contracts while types below the threshold are assigned I-contracts (cf. Proposition 4).

When $\mathcal{A}$ is contractible, the P-contracts and the I-contracts can be determined separately and there is no distortion along the I-path; however there is distortion along the P-path as the principal screens the agent with a menu of P-contracts. When $\mathcal{A}$ is not observable by the principal, the separation does not exist due to the possibility of *-deviation – agents who are supposed to take an I-contract might take a P-contract but engage in the I-path.[20] There is no distortion in the I-path in equilibrium;[21] however, the *-deviation would require further (downward) distortions on the P-path relative to when $\mathcal{A}$ is contractible.

8 Concluding Remarks

This paper studies contracting under the dynamic tradeoff faced by the agent when multitasking between production and investment, which is not captured in

[20] Given how the "role" of I-path and P-path are seemingly reversed here relatively to the baseline model, one might think that the analogous *-deviation in the modified model is to deter agents, who are supposed to take a P-contract, from taking the I-contract and then engaging on the P-path with it. But such a deviation is not profitable because it is without loss to let the period-1 bonus of any I-contract to be $-\infty$. Hence, no agent will produce in period 1 under a I-contract and since investment is costless, any agent who takes a I-contract always follows the I-path.

[21] Because the agent's payoff from the I-path is type-independent, an agent who is given a I-contract must have the lowest payoff among all the agents. Moreover, any agent who takes an I-contract will always engage in the I-path (fn. 20). Hence, any I-contract that gives the agent, who takes the I-path with it, the lowest payoff among all agents will be feasible. Under optimality, this implies setting the I-contract to achieve first best I-path surplus and then fully extracting it for the principal.

the canonical multitasking framework, with frictions arising jointly from adverse selection in the agent's investment again and moral hazard on the production effort. The results illustrate how these contracting frictions, both in isolation and in combination, affect the investment level, the incentive structure and the inefficiency in equilibrium.

The paper also highlights the differences arising from the principal's ability to observe the agent's investment decision or not. These results have implications on the investment levels and the incentive structures observed across different principal-agent environments. Consider the size or organizational structure of firms as an example of variation in the principal's ability to observe the agent's investment. A larger and more decentralized firm would have less ability to monitor its workers' investment on firm-specific skills and thus corresponds to the E^u environment.

The results would then suggest that larger or more decentralized firms should have more investment and innovation from its workers. Moreover, the P-contract takers can be interpreted as the low-level employees while the I-contract takers are the management-level employees whose marginal productivity is higher. Hence, the performance pay of managers in larger and more decentralized firms should be more powerful; on the other hand, the performance pay of low-level employees in larger and more decentralized firms should be linked across time – in particular, bad performance will lead to less powerful incentive in the future – whereas the performance pay of low-level employees in smaller and more centralized firms should be time-stationary.[22]

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[22] Acemoglu et al. (2007) and Kastl, Martimort and Piccolo (2013) document a positive correlation between decentralization and investment level, but the rationale provided is that decentralization provides ex-ante incentives for employees to innovate – see Riordan (1990), Tan (2017) and discussion in Section 6.3. I am not aware of any paper that documents the variation in performance pay of different job scopes across different firm sizes and organizational structures. This exercise should prove to be a fruitful avenue for future research.

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A Appendix

A.1 For Section 3

A.1.1 Proof of Lemma 1

Proof. $\varepsilon(\beta,\lambda)$ in (3.1) is characterized by FOC $(q+\lambda)\beta = \psi'(\varepsilon(\beta,\lambda))$. Dropping the arguments and after simplifications, the derivatives relevant for subsequent analysis are: $\frac{\partial\varepsilon}{\partial\beta} = \frac{q+\lambda}{\psi''(\varepsilon)} > 0$, $\frac{\partial\varepsilon}{\partial\lambda} = \frac{\beta}{\psi''(\varepsilon)} > 0$, $\frac{\partial^2\varepsilon}{\partial\beta^2} = -\frac{(q+\lambda)^2\psi'''(\varepsilon)}{[\psi''(\varepsilon)]^3} < 0$, $\frac{\partial^2\varepsilon}{\partial\lambda^2} = -\frac{\beta^2\psi'''(\varepsilon)}{[\psi''(\varepsilon)]^3} < 0$, $\frac{\partial^2\varepsilon}{\partial\lambda\partial\beta} = \frac{[\psi''(\varepsilon)]^2-\psi'(\varepsilon)\cdot\psi'''(\varepsilon)}{[\psi''(\varepsilon)]^3}$; $\frac{\partial u}{\partial\beta} = (q+\lambda)\varepsilon > 0$, $\frac{\partial u}{\partial\lambda} = \varepsilon\beta > 0$, $\frac{\partial^2 u}{\partial\beta^2} = \frac{(q+\lambda)^2}{\psi''(\varepsilon)} > 0$, $\frac{\partial^2 u}{\partial\lambda^2} = \beta\frac{\partial\varepsilon}{\partial\lambda} = \frac{\beta^2}{\psi''(\varepsilon)} > 0$, $\frac{\partial^2 u}{\partial\beta\partial\lambda} = \varepsilon + \beta\frac{\partial\varepsilon}{\partial\beta} = \varepsilon + \frac{\psi'(\varepsilon)}{\psi''(\varepsilon)} > 0$, $\frac{\partial^3 u}{\partial\beta\partial\lambda^2} = \beta\left(\frac{2[\psi''(\varepsilon)]^2-\psi'''(\varepsilon)\cdot\psi'(\varepsilon)}{[\psi''(\varepsilon)]^3}\right) \geq 0$, $\frac{\partial^3 u}{\partial\beta^2\partial\lambda} = (q+\lambda)\left(\frac{2[\psi''(\varepsilon)]^2-\psi'''(\varepsilon)\cdot\psi'(\varepsilon)}{[\psi''(\varepsilon)]^3}\right) \geq 0$. □

A.1.2 Proof of Lemma 2

Proof. From standard argument (e.g. Salanié (2005) Chapter 2.3.1), (3.4), is satisfied if and only if $U^I(\mathcal{W}(\lambda),\lambda) = U^I(\mathcal{W}(\underline{\lambda}),\underline{\lambda}) + \int_{\underline{\lambda}}^{\lambda} \frac{\partial u(\beta_0(l),l)}{\partial \lambda} dl$ and $\beta_0(\cdot)$ is non-decreasing. Since $U^I(\mathcal{W},\lambda)$ is continuous and increasing in λ, if $U^I(\mathcal{W}(\underline{\lambda}),\underline{\lambda}) > 0$, then there exists $\lambda' < \underline{\lambda}$ such that $U^I(\mathcal{W}(\underline{\lambda}),\lambda') > 0$, which violates (3.6). Hence $U^I(\mathcal{W}(\underline{\lambda}),\underline{\lambda}) \leq 0$. But (3.5) implies $U^I(\mathcal{W}(\underline{\lambda}),\underline{\lambda}) \geq 0$; hence $U^I(\mathcal{W}(\underline{\lambda}),\underline{\lambda}) = 0$. It is readily verified that under this, both (3.5) and (3.6) are satisfied. $\square$

A.1.3 Proof of Proposition 3

Proof. Doing integration by parts on (3.9), one obtains $\int_{\underline{\lambda}}^{\bar{\lambda}} v(\beta_0(\lambda),\lambda) f(\lambda) d\lambda$ where $v(\beta,\lambda) := s(\beta,\lambda) - \frac{\partial u(\beta,\lambda)}{\partial \lambda} J(\lambda)$, and $J(\lambda) = \frac{1-F(\lambda)}{f(\lambda)}$. I will consider the relaxed problem without the monotonicity constraint on $\beta_0(\cdot)$ first and check ex-post that it is not violated. Doing point-wise optimization, the FOC is:

$$\frac{\partial v(\beta_0,\lambda)}{\partial \beta} = 0 \iff (q+\lambda)(Y-\beta_0)\frac{\partial \varepsilon(\beta_0,\lambda)}{\partial \beta} - \frac{\partial^2 u(\beta_0,\lambda)}{\partial \beta \partial \lambda} J(\lambda) = 0. \quad \text{(A.1)}$$

(A.1) characterizes $\beta^*(\cdot)$ in (3.10). The SOC is $\frac{\partial^2 v(\beta_0,\lambda)}{\partial \beta^2} = (q+\lambda)\Bigg((Y-\beta_0)\frac{\partial^2 \varepsilon(\beta_0,\lambda)}{\partial \beta^2} - \frac{\partial \varepsilon(\beta_0,\lambda)}{\partial \beta}\Bigg) - \frac{\partial^3 u(\beta_0,\lambda)}{\partial \beta^2 \partial \lambda} J(\lambda) < 0$. From (A.1), $Y - \beta_0 = \frac{\frac{\partial^2 u(\beta_0,\lambda)}{\partial \beta \partial \lambda} J(\lambda)}{(q+\lambda)\frac{\partial \varepsilon(\beta_0,\lambda)}{\partial \beta}}$. With this and some tedious algebra, $\frac{\partial^2 v(\beta_0,\lambda)}{\partial \beta \partial \lambda} = J(\lambda)\left[\frac{\psi''(\varepsilon(\beta_0,\lambda))}{\psi'(\varepsilon(\beta_0,\lambda))}\varepsilon(\beta_0,\lambda)\right]\frac{\partial^3 u(\beta_0,\lambda)}{\partial \beta \partial \lambda^2} - \frac{\partial u^2(\beta_0,\lambda)}{\partial \beta \partial \lambda} J'(\lambda) > 0$. By implicit function theorem, $\frac{d\beta^*(\lambda)}{d\lambda} > 0$. The fixed wage schedule in (3.11) is obtained from Lemma 2; by Leibniz rule, $\frac{d}{d\lambda}\alpha^*(\lambda|\underline{\lambda}) = -\frac{\partial u(\beta^*(\lambda),\lambda)}{\partial \beta}\beta^{*\prime}(\lambda) < 0$. $\square$

A.2 For Sections 4 and 6

A.2.1 The Optimal Contracting Problem under E^c and E^u

Let $\mathcal{I}(\mathcal{A})$ be 1 if $\mathcal{A} = P$, and 0 if $\mathcal{A} = I$. Abusing notation for $U^P(\cdot)$, a redundant argument λ is added to it. Hence under $\kappa = \{\mathcal{A},\mathcal{W}\}$, the payoff of a type-$\lambda$ agent is $\bar{U}(\kappa,\lambda) := \mathcal{I}(\mathcal{A})U^P(\mathcal{W},\lambda) + (1-\mathcal{I}(\mathcal{A}))U^I(\mathcal{W},\lambda)$. The principal's corresponding payoff is $\bar{\Pi}(\kappa,\lambda) = \mathcal{I}(\mathcal{A})\left[\Pi^P(\mathcal{W})\right] + (1-\mathcal{I}(\mathcal{A}))\left[\Pi^I(\mathcal{W},\lambda)\right]$. The principal's optimal contracting problem is:

$$\max_{\kappa(\cdot)} \int_0^{\bar{\lambda}} \bar{\Pi}\left(\kappa(\lambda), \lambda\right) f(\lambda) d\lambda \qquad \text{subject to}$$

$$\bar{U}\left(\kappa\left(\lambda\right), \lambda\right) \geq 0 \quad \forall \lambda \tag{A.2}$$

and

$$(\text{for } E^c) \qquad \bar{U}\left(\kappa\left(\lambda\right), \lambda\right) \geq \bar{U}\left(\kappa\left(\lambda'\right), \lambda\right) \qquad \forall \lambda \neq \lambda', \ \forall \lambda. \tag{A.3}$$

$$(\text{for } E^u) \qquad \bar{U}\left(\kappa\left(\lambda\right), \lambda\right) \geq U^{\mathcal{A}}\left(\mathcal{W}(\lambda'), \lambda\right) \qquad \forall \lambda \neq \lambda', \ \forall \lambda, \ \forall \mathcal{A} \in \{I, P\} \tag{A.4}$$

In contrast to (A.3), (A.4) also takes into account the fact that under E^u, the agent is free to engage in either the P-path or I-path under any contract.

A.2.2 Proof of Proposition 4

Let $\{\kappa(\lambda)\}_{\lambda \in [0, \bar{\lambda}]} = \{\{\mathcal{W}(\lambda), \mathcal{A}(\lambda), \}\}_{\lambda \in [0, \bar{\lambda}]}$ be a solution to the principal's contracting problem above. Proposition 4 follows from Lemma A.1 to A.2 below.

Lemma A.1. *Under both E^c and E^u, $\forall \lambda$ such that $\mathcal{A}(\lambda) = P$, $\bar{U}\left(\kappa(\lambda), \lambda\right) = 0$.*

Proof. Suppose that $\mathcal{A}(\lambda) = P$ but $\bar{U}\left(\kappa(\lambda), \lambda\right) = x > 0$. For any λ', $\bar{U}\left(\kappa(\lambda), \lambda\right) = \bar{U}\left(\kappa(\lambda), \lambda'\right) \leq \bar{U}\left(\kappa(\lambda'), \lambda'\right)$ where the inequality is due to (A.3) or (A.4) for type λ'. Hence type λ has the lowest payoff among all agents. Suppose the principal decreases $a(\lambda')$ by x $\forall \lambda'$; the principal is strictly better off under this new menu. Neither (A.3) nor (A.4) is affected; (A.2) is still satisfied for λ which means it is satisfied for all $\lambda' \in \left[0, \bar{\lambda}\right]$. This contradicts the optimality of $\{\kappa(\lambda)\}_{\lambda \in [0, \bar{\lambda}]}$. $\square$

Lemma A.2. *Let $\lambda'' > \lambda'$. Under both E^c and E^u, if $\mathcal{A}(\lambda'') = P$, $\mathcal{A}(\lambda') = P$.*

Proof. Suppose for contradiction that $\mathcal{A}(\lambda'') = P$ but $\mathcal{A}(\lambda') = I$. Hence $\bar{U}\left(\kappa(\lambda'), \lambda'\right)$ $= a\left(\lambda'\right) + \alpha_0\left(\lambda'\right) + u\left(\beta_0(\lambda'), \lambda'\right) < a\left(\lambda'\right) + \alpha_0\left(\lambda'\right) + u\left(\beta_0(\lambda'), \lambda''\right) = U^I\left(\mathcal{W}(\lambda'), \lambda''\right)$. Since $\mathcal{A}(\lambda'') = P$, $\bar{U}\left(\kappa(\lambda''), \lambda''\right) = 0$ from Lemma A.1. This implies $0 = \bar{U}\left(\kappa(\lambda''), \lambda''\right) \geq U^I\left(\mathcal{W}(\lambda'), \lambda''\right) > \bar{U}\left(\kappa(\lambda'), \lambda'\right)$ which contradicts (A.2) for λ'. $\square$

A.2.3 Proof of Proposition 5

Proof. As established in the main text, $\mathcal{W}^P$ is set such that $\Pi^P\left(\mathcal{W}^P\right) = S^{P(FB)}$; the proposed P-contract satisfies this. Next, consider the program which fixes the threshold of program $\mathcal{P}^c$ in (4.5): $V^c(L) := \max_{\mathcal{W}(\cdot)} \left\{ \int_L^{\bar{\lambda}} \Pi^I(\mathcal{W}(\lambda), \lambda) f(\lambda) d\lambda + F(L) S^{P^{(FB)}} \right\}$ subject to (4.2) to (4.4). Notice that $V^c(L) = V^I(L) + F(L) S^{P^{(FB)}}$; hence the solution is $\beta^*(\cdot)$ and $\alpha^*(\cdot|L)$, with $V^c(L) = \int_L^{\bar{\lambda}} v(\beta^*(\lambda), \lambda) f(\lambda) d\lambda + F(L) S^{P^{(FB)}}$. By envelope theorem, $\frac{dV^c(L)}{dL} = \left[S^{P^{(FB)}} - v(\beta^*(L), L)\right] f(L)$. $f(\cdot)$ is strictly positive in $\left[0, \bar{\lambda}\right]$ which implies that $V^c(L)$ has a stationary point at L^c characterized by $S^{P^{(FB)}} = v(\beta^*(L^c), L^c)$ (i.e. (4.6)). Moreover $\frac{dv(\beta^*(\lambda),\lambda)}{d\lambda} = \frac{\partial v(\beta^*(\lambda),\lambda)}{\partial\lambda}$ since $\frac{\partial v(\beta^*(\lambda),\lambda)}{\partial\beta} = 0$.

$$\begin{aligned}
\left.\frac{\partial v(\beta,\lambda)}{\partial\lambda}\right|_{\beta=\beta^*(\lambda)} &= \left.\frac{\partial s(\beta,\lambda)}{\partial\lambda} - \frac{\partial^2 u(\beta,\lambda)}{\partial\lambda^2} J(\lambda) - \frac{\partial u(\beta,\lambda)}{\partial\lambda} J'(\lambda)\right|_{\beta=\beta^*(\lambda)} \\
&\geq \left.[q+\lambda]\frac{\partial\varepsilon(\beta,\lambda)}{\partial\lambda}[Y-\beta] - \frac{\partial^2 u(\beta,\lambda)}{\partial\lambda^2} J(\lambda)\right|_{\beta=\beta^*(\lambda)} \\
&= \left.[q+\lambda][Y-\beta]\frac{\partial\varepsilon(\beta,\lambda)}{\partial\lambda}\left(1 - \frac{\beta\frac{\partial\varepsilon(\beta,\lambda)}{\partial\beta}}{\varepsilon(\beta,\lambda) + \beta\frac{\partial\varepsilon(\beta,\lambda)}{\partial\beta}}\right)\right|_{\beta=\beta^*(\lambda)} > 0
\end{aligned}$$

The last equality uses $J(\lambda) = \frac{(q+\lambda)(Y-\beta^*(\lambda))\frac{\partial\varepsilon(\beta^*(\lambda),\lambda)}{\partial\beta}}{\frac{\partial^2 u(\beta^*(\lambda),\lambda)}{\partial\beta\partial\lambda}}$ from (A.1); and (because $\frac{\partial u}{\partial\lambda} = \varepsilon\beta$) $\frac{\partial^2 u(\beta,\lambda)}{\partial\lambda^2} = \beta\frac{\partial\varepsilon(\beta,\lambda)}{\partial\lambda}$ and $\frac{\partial^2 u(\beta,\lambda)}{\partial\lambda\partial\beta} = \varepsilon(\beta,\lambda) + \beta\frac{\partial\varepsilon(\beta,\lambda)}{\partial\beta}$. $\frac{dv(\beta^*(\lambda),\lambda)}{d\lambda} > 0$ implies that the stationary point of $V(L)$ is uniquely L^c. Moreover, $\frac{d^2}{dL^2}V^c(L^c) = -\frac{dv(\beta^*(L^c),L^c)}{d\lambda} f(L^c) < 0$ which implies that L^c is a local maximal. Since the stationary point is unique, the local optimal is a global optimal. Finally, $S^{I^{(FB)}}\left(L^{FB}\right) = S^{P^{(FB)}} = v(\beta^*(L^c), L^c) < S^{I^{(FB)}}(L^c)$ implies that $L^{FB} < L^c$. □

A.2.4 Proof of Lemma 3

Proof. Using the same arguments as in Lemma 2, (6.2), (6.5) and (6.8) are satisfied if and only if (6.10) and (6.11) hold. Noting that $U^I(\mathcal{W}, \lambda)$ is increasing in λ, and $U^P\left(\mathcal{W}^I(\lambda')\right) = a^I(\lambda') + \alpha^I(\lambda') + u\left(\beta^I(\lambda'), 0\right) = U^I\left(\mathcal{W}^I(\lambda'), 0\right)$,[23]

[23] The agent optimally engages in the P-path with $\mathcal{W}^I(\lambda')$ by first exerting $e_1 = 0$ to avoid getting $b^I(\lambda') = -\infty$; in period 2, since the agent has not undergone training in period 1, he is equivalent to an agent with type $\lambda = 0$ and his period-2 payoff is thus $\alpha_0^I(\lambda') + u\left(\beta_0^I(\lambda'), 0\right)$.

(6.5) implies (6.6). (6.7) is (6.12). (6.10) implies that $U^I\left(\mathcal{W}^I(L),L\right)=0$, hence (6.7) for L implies (6.3); (6.6) for L and $U^I\left(\mathcal{W},\lambda\right)$ is increasing in λ imply (6.4). $\square$

A.2.5 Proof of Proposition 6

Proof. $u\left(Y,\lambda\right)=s\left(Y,\lambda\right)=S^{I^{(FB)}}\left(\lambda\right)\ \forall\lambda$. Moreover $s\left(Y,0\right)=s^{P^{(FB)}}$. Hence $2s\left(Y,0\right)=s\left(Y,L^{FB}\right)<s\left(Y,L^c\right)=u\left(Y,L^c\right)$. In turn, $u\left(Y,\bar{\lambda}\right)-2u\left(Y,0\right)>u\left(Y,\bar{\lambda}\right)-u\left(Y,L^c\right)\implies\mathcal{D}\left(\bar{\lambda};Y,Y\right)>\int_{L^c}^{\bar{\lambda}}\frac{\partial u(Y,l)}{\partial\lambda}dl>\int_{L^c}^{\bar{\lambda}}\frac{\partial u(\beta^*(l),l)}{\partial\lambda}dl$. $\square$

A.2.6 Proof of Proposition 7

Consider program $\mathcal{P}^u\left(L\right)$, which is program $\mathcal{P}^u$ in (6.14) for a fixed $L\in\left(0,\bar{\lambda}\right)$.

$$V^u\left(L\right)=\max_{\beta_0^I,b^P,\beta_0^P;\nu}\left\{\int_L^{\bar{\lambda}}v\left(\beta_0^I(\lambda),\lambda\right)f(\lambda)d\lambda\ +\ F\left(L\right)\Pi^P\left(\bar{\mathcal{W}}\left(b^P,\beta_0^P\right)\right)\right\}\quad\text{(A.5)}$$

subject to

$$\forall\lambda\geq L:\quad\beta_0^{I'}(\lambda)=\nu\left(\lambda\right)\ ;\ \nu\left(\lambda\right)\geq0\ ;\ \text{ and }\ \mathcal{IC}^*\left(\lambda;\left\{b^P,\beta_0^P,\beta_0^I\left(\cdot\right),L\right\}\right)\geq0$$

Constraint (6.9) is satisfied under $\bar{\mathcal{W}}\left(b^P,\beta_0^P\right)$. Substituting constraint (6.10) into (6.14) and doing integration by parts gives the objective above. Constraint (6.11) is written as the first two constraints,[24] and constraint (6.12) as (IC_u^*).

Lemma A.3. *The solution to program $\mathcal{P}^u\left(L\right)$ is unique. Under the solution*

[24] Since $\beta_0^I(\cdot)$ is monotonic, it is differentiable almost everywhere.

$\left\{\beta_0^I, b^P, \beta_0^P; \nu\right\}$, *there exist continuous functions* γ, μ *and* η *such that:*

$$\gamma(\lambda) + \eta(\lambda) = 0 \tag{A.6}$$

$$\eta(\lambda) \geq 0 \quad and \quad \eta(\lambda)\nu(\lambda) = 0 \tag{A.7}$$

$$\frac{\partial s\left(\beta_0^I(\lambda),\lambda\right)}{\partial \beta} f(\lambda) - \left[1 - F(\lambda) - \int_\lambda^{\bar{\lambda}} \mu(l)\, dl\right] \frac{\partial^2 u\left(\beta_0^I(\lambda),\lambda\right)}{\partial \lambda \partial \beta} = -\gamma'(\lambda) \tag{A.8}$$

$$\gamma(L) = \gamma\left(\bar{\lambda}\right) = 0 \tag{A.9}$$

$$\mu(\lambda) \geq 0 \quad and \quad \mu(\lambda)\left[\int_L^{\bar{\lambda}} \frac{\partial u\left(\beta_0^I(l),l\right)}{\partial \lambda} dl - \mathcal{D}\left(\lambda; b^P, \beta_0^P\right)\right] = 0 \tag{A.10}$$

$$-\int_L^{\bar{\lambda}} \mu(\lambda)\left[\frac{\partial u\left(\beta_0^P,\lambda\right)}{\partial \beta} - \frac{\partial u\left(\beta_0^P,0\right)}{\partial \beta}\right] d\lambda + \frac{\partial \Pi^P\left(\bar{\mathcal{W}}\left(b^P,\beta_0^P\right)\right)}{\partial \beta_0} F(L) = 0 \tag{A.11}$$

$$\left(\int_L^{\bar{\lambda}} \mu(\lambda)\, d\lambda\right) \frac{\partial u\left(b^P,0\right)}{\partial \beta} + \frac{\partial \Pi^P\left(\bar{\mathcal{W}}\left(b^P,\beta_0^P\right)\right)}{\partial b} F(L) = 0 \tag{A.12}$$

Proof. See Online Appendix B.2.[25] □

Lemma A.4. *If* $\beta_0^I(\cdot)$ *and* $\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)$ *are feasible for program* $\mathcal{P}^u(L)$ *and* $\mathcal{IC}^*\left(\hat{\lambda}; \left\{b^P, \beta_0^P, \beta_0^I(\cdot), L\right\}\right) = 0$ *for type* $\hat{\lambda} \in \left(L, \bar{\lambda}\right)$*, then* $\beta_0^I\left(\hat{\lambda}\right) = \beta_0^P$*.*

Proof. $\beta_0^I\left(\hat{\lambda}\right) > \beta_0^P \implies \frac{\partial}{\partial \lambda}\mathcal{IC}^*\left(\hat{\lambda}; \left\{b^P, \beta_0^P, \beta_0^I(\cdot), L\right\}\right) > 0$, which implies that $\mathcal{IC}^*\left(\lambda; \left\{b^P, \beta_0^P, \beta_0^I(\cdot), L\right\}\right) < 0$ for $\lambda < \hat{\lambda}$ (but close to $\hat{\lambda}$), thus violating (IC_u^*). Analogously, if $\beta_0^I\left(\hat{\lambda}\right) < \beta_0^P$, then ($\text{IC}_u^*$) is violated for types just above $\hat{\lambda}$. □

Lemma A.5. *If* $\frac{\partial s(\beta(\lambda),\lambda)}{\partial \beta} - g(\lambda) \frac{\partial^2 u(\beta(\lambda),\lambda)}{\partial \lambda \partial \beta} = 0$*,* $g(\lambda) > 0$ *and* $g'(\lambda) \leq 0$ $\forall \lambda$*, then* $\beta'(\lambda) > 0$*.*

[25] Intuitively, fixing the values of b^P and β_0^P, program $\mathcal{P}^u(L)$ is an optimal control problem. Let β_0^I be the state variable, ν be the control variable, γ be the co-state variables for β_0^I, η be the shadow price for the constraint that $\nu \geq 0$, and μ be the shadow price for constraint (IC_u^*). The Hamiltonian for the auxiliary optimal control problem is:

$$\begin{aligned}\mathcal{H} =& s\left(\beta_0^I(\lambda), \lambda\right) f(\lambda) - \frac{\partial u\left(\beta_0^I(\lambda), \lambda\right)}{\partial \lambda}\left[1 - F(\lambda)\right] f(\lambda) + \gamma(\lambda)\nu(\lambda) + \eta(\lambda)\nu(\lambda) \\ &+ \left(\int_\lambda^{\bar{\lambda}} \mu(l)\, dl\right) \frac{\partial u\left(\beta_0^I(\lambda), \lambda\right)}{\partial \lambda} - \mu(\lambda)\mathcal{D}\left(\lambda; b^P, \beta_0^P\right).\end{aligned}$$

The second line is obtained from an integration by parts on $\int_L^{\bar{\lambda}} \mu(\lambda)\left[\int_L^{\lambda} \frac{\partial u\left(\beta_0^I(l),l\right)}{\partial \lambda} dl - \mathcal{D}\left(\lambda; b^P, \beta_0^P\right)\right] d\lambda$; see the Online Appendix B.2 for details. (A.6) to (A.10) characterize the necessary and sufficient conditions for optimality – see Seierstad and Sydsaeter (1987) for reference. Conditions (A.11) and (A.12) are then respectively the first order conditions for β_0^P and b^P. The proof of Lemma A.3 uses Lagrangian arguments to establish that these conditions are jointly sufficient for optimality.

Proof. Replace $J(\lambda)$ by $g(\lambda)$ in the proof of Proposition 3. $\square$

Lemma A.6. *For any $b \geq 0$, $\frac{\partial \Pi^P\left(\bar{\mathcal{W}}(b,\beta_0)\right)}{\partial \beta_0} > 0$ implies $\beta_0 < Y$. For any $\beta_0 \leq Y$, $\frac{\partial \Pi^P\left(\bar{\mathcal{W}}(b,\beta_0)\right)}{\partial b} < 0$ implies $b > Y$. $\frac{\partial \Pi^P\left(\bar{\mathcal{W}}(Y,Y)\right)}{\partial \beta_0} = \frac{\partial \Pi^P\left(\bar{\mathcal{W}}(Y,Y)\right)}{\partial b} = 0$.*

Proof. See (B.1), (B.2) and (B.3) in Online Appendix. $\square$

Lemma A.7. *Let $\hat{L}$ be such that $\mathcal{IC}^*\left(\bar{\lambda}; \left\{Y, Y, \beta^*(\cdot), \hat{L}\right\}\right) = 0$.*[26] *The solution to program $\mathcal{P}^u(L)$ in (A.5) is as follows:*

1. *If $L \leq \hat{L}$, then $b^P = \beta_0^P = Y$ and $\beta_0^I(\lambda) = \beta^*(\lambda)\ \forall \lambda \in \left[L, \bar{\lambda}\right]$.*
2. *If $L > \hat{L}$, then $b^P > Y$, $\beta_0^P < Y$, and*

$$\beta_0^I(\lambda) = \begin{cases} \beta^*(\lambda) & , \ \text{if } \lambda \geq \lambda_2 \\ \beta_0^P & , \ \text{if } \lambda \in [\lambda_1, \lambda_2] \\ \beta^{**}(\lambda) & , \ \text{if } \lambda \in [L, \lambda_1] \end{cases}$$

with $\lambda_2 := \beta^{*-1}\left(\beta_0^P\right) \in \left(L, \bar{\lambda}\right)$, $\lambda_1 \in [L, \lambda_2)$; $\beta^{**}(\lambda)$ is characterized by:

$$\frac{\partial s\left(\beta^{**}(\lambda), \lambda\right)}{\partial \beta} = \frac{\partial^2 u\left(\beta^{**}(\lambda), \lambda\right)}{\partial \beta \partial \lambda}\left(\frac{1 - F(\lambda) - \zeta}{f(\lambda)}\right), \tag{A.13}$$

where $\zeta > 0$ is characterized in the proof. Moreover:[27]

- *$\beta_0^I(\cdot)$ is continuous.*
- *$\beta^{**}(\cdot)$ is strictly increasing and $\beta^{**}(\lambda) > \beta^*(\lambda)\ \forall \lambda$.*
- *$\mathcal{IC}^*\left(\lambda; \left\{b^P, \beta_0^P, \beta_0^I(\cdot), L\right\}\right) = 0\ \forall \lambda \in [\lambda_1, \lambda_2]$.*

Proof. (A.6) and (A.7) imply that $\gamma(\lambda) = 0$ when $\beta_0^{I'}(\lambda) = \nu(\lambda) > 0$.

Consider $L \leq \hat{L}$ first. Under $b^P = \beta_0^P = Y$, $\mathcal{IC}^*(\lambda; \{Y, Y, \beta^*(\cdot), L\}) > 0$ $\forall \lambda < \bar{\lambda}$;[28] from (A.10), $\mu(\lambda) = 0\ \forall \lambda < \bar{\lambda}$. Lemma A.6 then implies (A.11) and

[26] $\hat{L}$ is possibly negative. If $\hat{L} < 0$, then it is irrelevant; henceforth assume that $\hat{L} > 0$.

[27] The solution $\beta_0^I(\cdot)$, b^P, and β_0^P, and the values of λ_1, λ_2 and ζ will all depend on the value of L, but the argument is dropped to ease notations.

[28] Recall that $\mathcal{IC}^*$ is strictly increasing in λ and strictly decreasing in L – see Lemma 5.

(A.12) are satisfied. Since $\beta^{*'}(\lambda) > 0$, $\gamma(\lambda) = 0\ \forall\lambda$ and hence $\gamma'(\lambda) = 0\ \forall\lambda$; (A.8) and (A.9) are thus satisfied. This proves point 1.

Consider $L > \hat{L}$ next. From Lemma A.4, if (IC^*_u) binds, it is over *one* connected interval. Denote this interval by $[\lambda_1, \lambda_2]$; Lemma A.4 also implies $\beta^I_0(\lambda) = \beta^P_0\ \forall\lambda \in [\lambda_1, \lambda_2]$. (A.10) implies $\int_\lambda^{\bar{\lambda}} \mu(l)\,dl$ is decreasing; and $\int_\lambda^{\bar{\lambda}} \mu(l)\,dl = 0$ $\forall\lambda \geq \lambda_2$, $\int_\lambda^{\bar{\lambda}} \mu(l)\,dl > 0\ \forall\lambda \in (\lambda_1, \lambda_2)$, $\int_\lambda^{\bar{\lambda}} \mu(l)\,dl = \zeta > 0\ \forall\lambda \leq \lambda_1$ where ζ is to be determined. Using Lemma A.6, $\beta^P_0 < Y$ follows from $\mu(\lambda) > 0$ for $\lambda \in [\lambda_1, \lambda_2]$ and (A.11); $b^P > Y$ follows from $\int_L^{\bar{\lambda}} \mu(l)\,dl = \zeta > 0$ and (A.12).

I show that $[\lambda_1, \lambda_2]$ is the only "bunching" interval. Denote $\bar{v}(\beta, \lambda, x) := s(\beta, \lambda) - \left(\frac{1-F(\lambda)-x}{f(\lambda)}\right)\frac{\partial u(\beta,\lambda)}{\partial\lambda}$. Suppose, for contradiction, there exists $[l_1, l_2]$, $l_1 > \lambda_2$ such that $\beta^I_0(l_1) = \beta^I_0(l_2)$. By continuity of $\gamma(\cdot)$, $\gamma'(l_1) = \gamma'(l_2) = 0$ since the regions just below l_1 and just above l_2 are non-bunching regions. (A.8) implies that $\frac{\partial\bar{v}\left(\beta^I_0(l_2),l_2,0\right)}{\partial\beta} = 0$ and $\frac{\partial\bar{v}\left(\beta^I_0(l_1),l_1,0\right)}{\partial\beta} = 0$. Lemma A.5 implies $\beta^I_0(l_1) < \beta^I_0(l_2)$ (contradiction). Next, suppose, for contradiction, there exists $[l_1, l_2]$, $l_2 < \lambda_1$ such that $\beta^I_0(l_1) = \beta^I_0(l_2)$. Analogous to previously, (A.8) implies that $\frac{\partial\bar{v}\left(\beta^I_0(l_2),l_2,\zeta\right)}{\partial\beta} = 0$ and $\frac{\partial\bar{v}\left(\beta^I_0(l_1),l_1,\zeta\right)}{\partial\beta} = 0$. For $l \in \{l_1, l_2\}$, $1 - F(l) - \zeta \geq 0$ and $\frac{d}{dl}\left(\frac{1-F(l)-\zeta}{f(l)}\right) \leq 0$.[29] Lemma A.5 thus implies $\beta^I_0(l_1) < \beta^I_0(l_2)$ (contradiction).

Since $[\lambda_1, \lambda_2]$ is the only bunching interval, $\gamma'(\lambda) = 0\ \forall\lambda \geq \lambda_2$ and $\lambda \leq \lambda_1$. (A.8) implies that $\beta^I_0(\lambda) = \beta^*(\lambda)$ for $\lambda \geq \lambda_2$, which means $\lambda_2 = \beta^{*-1}\left(\beta^P_0\right)$; and $\beta^I_0(\lambda) = \beta^{**}(\lambda)$ in (A.13) for $\lambda \leq \lambda_1$. Since (IC^*_u) binds at λ_1, λ_1 and ζ are jointly determined by $\mathcal{IC}^*\left(\lambda_1; \left\{b^P, \beta^P_0, \beta^{**}_0(\cdot), L\right\}\right) = 0$ and $\frac{\partial\bar{v}\left(\beta^I_0(\lambda_1),\lambda_1,\zeta\right)}{\partial\beta} = 0$.

From Lemma A.5, $\beta^{**}(\cdot)$ is strictly increasing. To show $\beta^{**}(\lambda) > \beta^*(\lambda)$, first notice that $\frac{\partial s(\beta_0,\lambda)}{\partial\beta} - \left[\frac{1-F(\lambda)}{f(\lambda)}\right]\frac{\partial^2 u(\beta_0,\lambda)}{\partial\lambda\partial\beta}$ is 0 for $\beta_0 = \beta^*_0(\lambda)$ from (A.1), and it is strictly positive for $\beta_0 < \beta^*_0(\lambda)$. Suppose for a contradiction that $\beta^{**}(\lambda) \leq \beta^*(\lambda)$. This implies that $\frac{\partial s\left(\beta^{**}_0(\lambda),\lambda\right)}{\partial\beta} - \left[\frac{1-F(\lambda)}{f(\lambda)}\right]\frac{\partial^2 u\left(\beta^{**}_0(\lambda),\lambda\right)}{\partial\lambda\partial\beta} \geq 0$. But (A.13) implies $\frac{\partial s\left(\beta^{**}_0(\lambda),\lambda\right)}{\partial\beta} - \left[\frac{1-F(\lambda)}{f(\lambda)}\right]\frac{\partial^2 u\left(\beta^{**}_0(\lambda),\lambda\right)}{\partial\lambda\partial\beta} = -\frac{\zeta}{f(\lambda)}\frac{\partial^2 u\left(\beta^{**}_0(\lambda),\lambda\right)}{\partial\lambda\partial\beta} < 0$ (contradiction). $\square$

[29] If $1 - F(l) - \zeta < 0$, [LHS (A.8)] $> 0 = \gamma'(l)$ (contradiction). $\frac{d}{dl}\left(\frac{1-F(l)-\zeta}{f(l)}\right) = \zeta\frac{f'(l)}{f(l)^2} + \left[\frac{-f(l)^2-[1-F(l)]f'(l)}{f(l)^2}\right]$. The second term is non-positive by Assumption 2. So if $f'(l) \leq 0$, the result follows. If $f'(l) \geq 0$, since $\zeta \leq 1 - F(l)$, $\frac{d}{dl}\left(\frac{1-F(l)-\zeta}{f(l)}\right) \leq [1 - F(l)]\frac{f'(l)}{f(l)^2} + \left[\frac{-f(l)^2-[1-F(l)]f'(l)}{f(l)^2}\right] = -1 < 0$.

Lemma A.8. *$V^u(L)$ is differentiable in $\left[0, \bar{\lambda}\right]$.*

- *If $L \leq \hat{L}$, $\frac{dV^u(L)}{dL} = \left[\Pi^P\left(\bar{\mathcal{W}}(Y,Y)\right) - v\left(\beta^*(L), L\right)\right] f(L)$.*
- *If $L > \hat{L}$, $\frac{dV^u(L)}{dL} = \left[\Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right) - \left(v\left(\beta^{**}(L), L\right) - \left(\frac{\int_L^{\bar{\lambda}} \mu(l)dl}{f(L)}\right)\frac{\partial u(\beta^{**}(L),L)}{\partial \lambda}\right)\right] f(L)$, where b^P and β_0^P are part of the solution of $\mathcal{P}^u(L)$.*

Proof. See Online Appendix B.3. ☐

Lemma A.9. *L^u is characterized by (6.16) with $\hat{L} < L^u < L^c$.*

Proof. The optimal L exists since $\left[0, \bar{\lambda}\right]$ is compact. Under Assumption 3, $L^u \in \left(0, \bar{\lambda}\right)$ which implies that $\frac{dV^u(L^u)}{dL} = 0$. If $L^u \leq \hat{L}$, the solution is the same as under E^c which contradicts Proposition 6. So $L^u > \hat{L}$ with $\int_{L^u}^{\bar{\lambda}} \mu(l)\, dl > 0$. To show that $L^u < L^c$, let $\hat{v}(\beta, \lambda) = v(\beta, \lambda) + \left(\frac{\int_{L^u}^{\bar{\lambda}} \mu(l)dl}{f(\lambda)}\right)\frac{\partial u(\beta,\lambda)}{\partial \lambda}$; note that $\beta^{**}(\lambda) = \arg\max_{\beta} \{\hat{v}(\beta, \lambda)\}$ and $\hat{v}(\beta, \lambda) > v(\beta, \lambda)\ \forall \beta, \lambda$. Suppose, for a contradiction, $L^u \geq L^c$. From $\frac{dV^u(L^u)}{dL} = 0$:

$$\begin{aligned}
\Pi^P\left(\bar{\mathcal{W}}\left(b^{P(u)}, \beta_0^{P(u)}\right)\right) &= \hat{v}\left(\beta^{**}(L^u), L^u\right) \geq \hat{v}\left(\beta^*(L^u), L^u\right) > v\left(\beta^*(L^u), L^u\right) \\
&\geq v\left(\beta^*(L^c), L^c\right) = \Pi^P\left(\bar{\mathcal{W}}(Y,Y)\right) = S^{P(FB)}
\end{aligned}$$

The last inequality follows from $L^u \geq L^c$ and $\frac{dv(\beta^*(\lambda),\lambda)}{d\lambda} > 0$ – see proof of Proposition 5. This implies $\Pi^P\left(\bar{\mathcal{W}}\left(b^{P(u)}, \beta_0^{P(u)}\right)\right) > S^{P(FB)}$ (contradiction). ☐

Proposition 7 follows from Lemma A.7 and A.9.

B Online Appendix

This Online Appendix provides the proofs of Lemma 4, Lemma A.3 and Lemma A.8.

B.1 Proof of Lemma 4

Proof. Let $\mathcal{W}^P = \left\{\left(a^P, b^P\right); \left(\alpha_Y^P, \beta_Y^P\right); \left(\alpha_0^P, \beta_0^P\right)\right\}$ be an optimal P-contract. That $\beta_Y^P = Y$ is established in the main text. Let $\hat{b}^P = \left(b^P + \alpha_Y^P - \alpha_0^P\right) + u\left(Y, 0\right) - u\left(\beta_0^P, 0\right)$. Since the period-2 bonuses of $\mathcal{W}^P$ and $\bar{\mathcal{W}}\left(\hat{b}^p, \beta_0^P\right)$ are the same, the two contracts induce the same period-2 effort from the agent. Moreover, under $\mathcal{W}^P$, the agent chooses e_1 to maximize $qe_1\Big[b^P + \alpha_Y^P + u\left(\beta_Y^P, 0\right) - \alpha_0^P - u\left(\beta_0^P, 0\right)\Big] - \psi\left(e_1\right)$; under $\bar{\mathcal{W}}\left(\hat{b}^p, \beta_0^P\right)$, the agent chooses e_1 to maximize $qe_1\hat{b}^P - \psi\left(e_1\right)$. Hence the two contracts also induce the same period-1 effort. $U^P\left(\bar{\mathcal{W}}\left(b, \beta_0\right)\right) = 0 = U^P\left(\mathcal{W}^P\right)$ then implies $\Pi^P\left(\bar{\mathcal{W}}\left(\hat{b}^P, \beta_0^P\right)\right) = \Pi^P\left(\mathcal{W}^P\right)$ (point 2). For point 1, $U^P\left(\mathcal{W}^P\right) = 0$ implies that:

$$\begin{aligned}
a^P + \alpha_0^P &= -\max_{e_1}\left\{u\left(\beta_0^P, 0\right) + qe_1\left[b^P + \alpha_Y^P + u\left(\beta_Y^P, 0\right) - \alpha_0^P - u\left(\beta_0^P, 0\right)\right] - \psi\left(e_1\right)\right\} \\
&= -\max_{e_1}\left\{u\left(\beta_0^P, 0\right) + qe_1\hat{b}^P - \psi\left(e_1\right)\right\} \\
&= -\left[u\left(\beta_0^P, 0\right) + u\left(\hat{b}^P, 0\right)\right].
\end{aligned}$$

□

B.2 Proof of Lemma A.3

I first map the problem to the one in Luenberger (1969) Section 8 (p. 216):

$$\begin{gathered}
\max \; Obj\left(x\right) \\
\text{subject to:} \quad x \in \Omega, \; G\left(x\right) \geq z
\end{gathered}$$

where Ω is a convex subset of a vector space X, Obj is a real-valued concave functional on Ω, and G is a convex mapping from Ω into a normed space Z having positive cone P, and z in an arbitrary vector in Z.

First, I show that it is without loss to restrict attention on b^P and β_0^P from a compact set.

Lemma B.1. *If $b^{P(u)}$ and $\beta_0^{P(u)}$ are part of a solution to program $\mathcal{P}^u(L)$ in (A.5), then $\beta_0^{P(u)} \leq Y$ and $b^{P(u)} \leq \bar{b}$, where*

$$\bar{b} := \max\left\{\tilde{b}, Y + s(Y,0)\right\},$$

and $\tilde{b}$ is defined as the b^p such that $\mathcal{D}\left(\bar{\lambda}; \tilde{b}, Y\right) = 0$.

Proof. Since $U^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right) = 0$, $\Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)$ is the total production surplus. Hence we can write:

$$\Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right) = q\varepsilon\left(b^P,0\right)Y - \psi\left(\varepsilon\left(b^P,0\right)\right) + q\varepsilon\left(b^P,0\right)s(Y,0) + \left[1 - q\varepsilon\left(b^P,0\right)\right]s\left(\beta_0^P,0\right).$$

This implies that

$$\frac{\partial \Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)}{\partial \beta_0} = \left[1 - q\varepsilon\left(b^P,0\right)\right]\frac{\partial s\left(\beta_0^P,0\right)}{\partial \beta}, \tag{B.1}$$

which is zero if and only if $\beta_0^P = Y$; moreover

$$\frac{\partial^2 \Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)}{\partial (\beta_0)^2} = \left[1 - q\varepsilon\left(b^P,0\right)\right]\frac{\partial^2 s\left(\beta_0^P,0\right)}{\partial (\beta)^2} < 0. \tag{B.2}$$

Hence $\Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)$ is maximized at $\beta_0^P = Y$ and is strictly decreasing in β_0^P for $\beta_0^P > Y$. Recall that $\frac{\partial \mathcal{D}\left(\lambda; b^P, \beta_0^P\right)}{\partial \beta_0^P} > 0$. Hence increasing β_0^P beyond Y worsens constraint (IC_u^*) and the objective; it is thus never optimal to set $\beta_0^P > Y$.

Next,

$$\begin{aligned}\frac{\partial \Pi^P\left(\bar{\mathcal{W}}\left(b^P,\beta_0^P\right)\right)}{\partial b} &= \left(q\left[Y+s\left(Y,0\right)-s\left(\beta_0^P,0\right)\right]-\psi'\left(\varepsilon\left(b^P,0\right)\right)\right)\frac{\partial \varepsilon\left(b^P,0\right)}{\partial \beta}\\ &= q\left[Y+s\left(Y,0\right)-s\left(\beta_0^P,0\right)-b^p\right]\frac{\partial \varepsilon\left(b^P,0\right)}{\partial \beta}. \qquad \text{(B.3)}\end{aligned}$$

When $\beta_0^P \leq Y$, $b^P > \tilde{b}$ implies that $\mathcal{D}\left(\lambda; b^P, \beta_0^P\right) \leq 0 \ \forall \lambda \in \left[L, \bar{\lambda}\right]$; this implies that increasing b^P beyond $\tilde{b}$ does not help to relax constraint (IC_u^*). $b^P \geq Y + s\left(Y,0\right) \geq Y + s\left(Y,0\right) - s\left(\beta_0^P, 0\right)$ implies that $\frac{\partial \Pi^P\left(\bar{\mathcal{W}}\left(b^P,\beta_0^P\right)\right)}{\partial b} < 0$. Hence choosing $b^P > \bar{b}$ is always suboptimal when $\beta_0^P \leq Y$. $\square$

Next, let $\Gamma^L : \left[L, \bar{\lambda}\right] \longrightarrow [0,\infty)$ and let $\left\{\beta_0^I, U, b^P, \beta_0^P, \nu\right\} \in \Phi^L := \Gamma^L \times \Gamma^L \times [0, Y] \times \left[0, \bar{b}\right] \times \Gamma^L$. Program $\mathcal{P}^u\left(L\right)$ in (A.5) can be written as:

$$V^u\left(L\right) = \max_{\beta_0^I, U, b^P, \beta_0^P, \nu}\left\{\int_L^{\bar{\lambda}}\left[s\left(\beta_0^I(\lambda),\lambda\right)-U\left(\lambda\right)\right]f\left(\lambda\right)d\lambda + F\left(L\right)\Pi^P\left(\bar{\mathcal{W}}\left(b^P,\beta_0^P\right)\right)\right\} \qquad \text{(B.4)}$$

subject to:

SA-(6.10)	$U\left(\lambda\right) \geq \int_L^\lambda \frac{\partial u\left(\beta_0^I(l),l\right)}{\partial \lambda}dl \quad, \forall \lambda \geq L$	$, [\rho]$
SA-(6.11-a$^+$)	$\beta_0^{I\prime}\left(\lambda\right) \geq \nu\left(\lambda\right) \quad, \forall \lambda \geq L$	$, [\gamma^+]$
SA-(6.11-a$^-$)	$-\beta_0^{I\prime}\left(\lambda\right) \geq -\nu\left(\lambda\right) \quad, \forall \lambda \geq L$	$, [\gamma^-]$
SA-(6.11-b)	$\nu\left(\lambda\right) \geq 0 \quad, \forall \lambda \geq L$	$, [\eta]$
SA-(6.12)	$U\left(\lambda\right) \geq \mathcal{D}\left(\lambda; b^P, \beta_0^P\right) \quad, \forall \lambda \geq L$	$, [\mu]$

The first constraint $U\left(\lambda\right) \geq \int_L^\lambda \frac{\partial u\left(\beta_0^I(l),l\right)}{\partial \lambda}dl$ is constraint (6.10), where $U\left(\lambda\right)$ is the payoff of type λ. It is without loss to write it as an inequality like this, because higher U decreases the objective while higher β_0^I increases it; hence the constraint will always hold with equality at the optimal.

Let:

$$
\begin{array}{rcl}
X & = & \Phi^L \\
\Omega & = & \Phi^L \\
Z & = & \left\{ \begin{array}{c} \{\rho, \gamma^+, \gamma^-, \eta, \mu\} \in \left(\Gamma^L\right)^5 \\ \text{where the norm is } ||z|| = \sup\limits_{\lambda \in \left[L, \bar{\lambda}\right]} |z(\lambda)| \text{ for } z = \rho, \gamma^+, \gamma^-, \eta, \mu \end{array} \right\} \\
P & = & \left\{ \begin{array}{c} \{\rho, \gamma^+, \gamma^-, \eta, \mu\} \in Z \\ \text{with } z(\lambda) \geq 0 \ \forall \lambda \in \left[L, \bar{\lambda}\right] \text{ for } z = \rho, \gamma^+, \gamma^-, \eta, \mu \end{array} \right\}
\end{array}
$$

with the *Obj* given by (B.4), and G by the constraints SA-(6.10), SA-(6.11-a$^+$), SA-(6.11-a$^-$), SA-(6.11-b) and SA-(6.12), which is a convex mapping. Moreover, since $s(\beta, \lambda)$ is strictly concave in β, the following lemma establishes that the objective is strictly concave in Φ^L:

Lemma B.2. *Under Assumption 4,* $\Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)$ *is strictly concave for* $\left(b^P, \beta^P\right) \in \left[0, \bar{b}\right] \times [0, Y]$.

Proof. We have established in (B.2) that $\frac{\partial^2 \Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)}{\partial(\beta_0)^2} < 0 \ \forall b^P, \beta_0^P > 0$. Next:

$$
\frac{\partial^2 \Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)}{\partial b^2} = -q\frac{\partial \varepsilon\left(b^P, 0\right)}{\partial \beta} + q\left[Y + s(Y, 0) - s\left(\beta_0^P, 0\right) - b^p\right]\frac{\partial^2 \varepsilon\left(b^P, 0\right)}{\partial \beta^2}. \tag{B.5}
$$

Suppose first that $\tilde{b} \geq Y + s(Y, 0)$. Hence $b^P \leq \bar{b} = \tilde{b}$. For any $b^P \leq \tilde{b}$, $\mathcal{D}\left(\bar{\lambda}; \tilde{b}, Y\right) \geq 0$, which implies

$$
\varepsilon\left(b^P, 0\right)\psi'\left(\varepsilon\left(b^P, 0\right)\right) - \psi\left(\varepsilon\left(b^P, 0\right)\right) \leq u\left(Y, \bar{\lambda}\right) - u(Y, 0) < Y + s(Y, 0) - s\left(\beta^P, 0\right). \tag{B.6}
$$

The last inequality follows from $u\left(Y, \bar{\lambda}\right) = s\left(Y, \bar{\lambda}\right) < Y$, $s(Y, 0) > 0$ and $u(Y, 0) = s(Y, 0) \geq s\left(\beta_0^P, 0\right)$ since $\beta_0^P \leq Y$. From (B.5), we have strict concavity if and only if:

$$b^p + \frac{\frac{\partial \varepsilon\left(b^P,0\right)}{\partial \beta}}{\frac{\partial^2 \varepsilon\left(b^P,0\right)}{\partial \beta^2}} < Y + s\left(Y,0\right) - s\left(\beta_0^P,0\right) \tag{B.7}$$

$$\iff \quad \frac{\psi'\left(\varepsilon\left(b^P,0\right)\right)}{q} - \frac{\left(\frac{q}{\psi''\left(\varepsilon\left(b^P,0\right)\right)}\right)}{\left(\frac{q^2\psi'''\left(\varepsilon\left(b^P,0\right)\right)}{\left[\psi''\left(\varepsilon\left(b^P,0\right)\right)\right]^3}\right)} < Y + s\left(Y,0\right) - s\left(\beta_0^P,0\right)$$

$$\iff \quad \frac{\psi'\left(\varepsilon\left(b^P,0\right)\right)\psi'''\left(\varepsilon\left(b^P,0\right)\right) - \left[\psi''\left(\varepsilon\left(b^P,0\right)\right)\right]^2}{q\psi'''\left(\varepsilon\left(b^P,0\right)\right)} < Y + s\left(Y,0\right) - s\left(\beta_0^P,0\right) \tag{B.8}$$

Note that the last condition of Assumption 4 is equivalent to:

$$\frac{\psi'\left(\varepsilon\left(b^P,0\right)\right)\psi'''\left(\varepsilon\left(b^P,0\right)\right) - \left[\psi''\left(\varepsilon\left(b^P,0\right)\right)\right]^2}{q\psi'''\left(\varepsilon\left(b^P,0\right)\right)} \leq \varepsilon\left(b^P,0\right)\psi'\left(\varepsilon\left(b^P,0\right)\right) - \psi\left(\varepsilon\left(b^P,0\right)\right). \tag{B.9}$$

(B.6) and (B.9) jointly imply that (B.8) holds.

Next, suppose that $\tilde{b} \leq Y + s\left(Y,0\right)$. Hence $b^P \leq \bar{b} = Y + s\left(Y,0\right)$. If $b^P \leq Y$, since $\beta_0^P \leq Y$ and $\frac{\partial^2 \varepsilon\left(b^P,0\right)}{\partial \beta^2} < 0$, (B.5) is strictly negative and we are done. Suppose $b^P \in \left[Y, Y + s\left(Y,0\right)\right]$ now. Since $Y + s\left(Y,0\right) - b^P \geq 0$, (B.7) holds if:

$$\frac{\frac{\partial \varepsilon\left(b^P,0\right)}{\partial \beta}}{\frac{\partial^2 \varepsilon\left(b^P,0\right)}{\partial \beta^2}} < -s\left(\beta_0^P,0\right)$$

$$\iff \quad \frac{\left[\psi''\left(\varepsilon\left(b^P,0\right)\right)\right]^2}{\psi'''\left(\varepsilon\left(b^P,0\right)\right)} > qs\left(\beta_0^P,0\right) \tag{B.10}$$

Under the first condition of Assumption 4, $\frac{\left[\psi''(x)\right]^2}{\psi'''(x)}$ is strictly increasing. Since $\beta_0^P \leq Y$ and we are only considering $b^P \geq Y$, (B.10) holds for if

$$\frac{\left[\psi''\left(\varepsilon\left(Y,0\right)\right)\right]^2}{\psi'''\left(\varepsilon\left(Y,0\right)\right)} > qs\left(Y,0\right)$$

$$\iff \quad \frac{\left[\psi''\left(\varepsilon\left(Y,0\right)\right)\right]^2}{\psi'''\left(\varepsilon\left(Y,0\right)\right)} > q\Big[\left(\varepsilon\left(Y,0\right)\right)\psi'\left(\varepsilon\left(Y,0\right)\right) - \psi\left(\varepsilon\left(Y,0\right)\right)\Big] \tag{B.11}$$

(B.11) is obtained from noting that $s\left(Y,0\right) = u\left(Y,0\right) = q\left(\varepsilon\left(Y,0\right)\right)Y - \psi\left(\varepsilon\left(Y,0\right)\right)$ and $qY = \psi'\left(\varepsilon\left(Y,0\right)\right)$. Under the second condition of Assumption 4, (B.11) always holds.

□

Lemma B.3. *For any $L \in \left(0, \bar{\lambda}\right)$, the solution to program $\mathcal{P}^u(L)$ is unique.*

Proof. Suppose for a contradiction that $\hat{\phi} = \left\{\hat{\beta}_0^I, \hat{U}, \hat{b}^P, \hat{\beta}_0^P, \hat{\nu}\right\}$ and $\tilde{\phi} = \left\{\tilde{\beta}_0^I, \tilde{U}, \tilde{b}^P, \tilde{\beta}_0^P, \tilde{\nu}\right\}$ are both solutions to $\mathcal{P}^u(L)$. Since the set of feasible $\phi \in \Phi^L$ satisfying all the constraints of $\mathcal{P}^u(L)$ is a convex set, $\phi^\theta = \theta\hat{\phi} + (1-\theta)\tilde{\phi}$ is also feasible for any $\theta \in (0,1)$. But since the objective is strictly concave, ϕ^θ does better than both $\hat{\phi}$ and $\tilde{\phi}$ (contradiction). □

With the Lagrangian multipliers stated at the right of each constraint, the Lagrangian function of program $\mathcal{P}^u(L)$ in (B.4) is:

$$\begin{aligned}
&\mathcal{L}^L\left(\beta_0^I, U, b^P, \beta_0^P, \nu \middle| \rho, \gamma, \eta, \mu\right) \\
:= &\int_L^{\bar{\lambda}} \left[s\left(\beta_0^I(\lambda), \lambda\right) - U(\lambda)\right] f(\lambda)\, d\lambda + \left[F(L)\, \Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)\right] \\
&+ \int_L^{\bar{\lambda}} \rho(\lambda) \left[U(\lambda) - \int_L^{\lambda} \frac{\partial u\left(\beta_0^I(l), l\right)}{\partial \lambda} dl\right] d\lambda \qquad \text{(B.12)} \\
&+ \int_L^{\bar{\lambda}} \gamma(\lambda) \left[\nu(\lambda) - \beta_0^{I\prime}(\lambda)\right] d\lambda \qquad \text{(B.13)} \\
&+ \int_L^{\bar{\lambda}} \eta(\lambda)\, \nu(\lambda)\, d\lambda \\
&+ \int_L^{\bar{\lambda}} \mu(\lambda) \left[U(\lambda) - \mathcal{D}\left(\lambda; b^P, \beta_0^P\right)\right] d\lambda
\end{aligned}$$

where $\gamma = \gamma^+ - \gamma^-$.

Lemma B.4. *$\phi^L = \left\{\beta_0^I, U, b^P, \beta_0^P, \nu\right\} \in \Psi^L$ is a solution to program $\mathcal{P}^u(L)$ if there exist Lagrangian multiplier functions ρ, γ, η and μ such that:*

$$\mathcal{L}^L\left(\phi^L \middle| \tilde{\rho}, \tilde{\gamma}, \tilde{\eta}, \tilde{\mu}\right) \geq \mathcal{L}^L\left(\phi^L \middle| \rho, \gamma, \eta, \mu\right) \geq \mathcal{L}^L\left(\tilde{\phi}^L \middle| \rho, \gamma, \eta, \mu\right) \quad \forall \tilde{\phi}^L \in \Phi^L,\ \forall \left\{\tilde{\rho}, \tilde{\gamma}, \tilde{\eta}, \tilde{\mu}\right\} \in \left(\Gamma^L\right)^4.$$

Proof. Follows from Luenberger (1969) Theorem 1 (p. 220). □

This thus implies that the maximization of the Lagrangian is sufficient for optimality for program $\mathcal{P}^u(L)$. The Berge's maximum theorem and the uniqueness of the solution (Lemma B.3) jointly imply that the Lagrangian multiplier functions for the solution is continuous. The next lemma provides the necessary and sufficient conditions for the maximization of the Lagrangian.

Lemma B.5. *We have* $\mathcal{L}^L\left(\beta_0^I, U, b^P, \beta_0^P, \nu \middle| \rho, \gamma, \eta, \mu\right) \geq \mathcal{L}^L\left(\tilde{\phi} \middle| \rho, \gamma, \eta, \mu\right)$ $\forall \tilde{\phi} \in \Phi^L$ *if and only if*

$$\delta\mathcal{L}^L\left(\beta_0^I, U, b^P, \beta_0^P, \nu; h_{\beta^I}, h_U, h_{b^P}, h_{\beta^P}, h_\nu \middle| \rho, \gamma, \eta, \mu\right) \leq 0, \ \ \forall \left\{h_{\beta^I}, h_U, h_{b^P}, h_{\beta^P}, h_\nu\right\} \in \Phi^L \tag{B.14}$$

and

$$\delta\mathcal{L}^L\left(\beta_0^I, U, b^P, \beta_0^P, \nu; \beta_0^I, U, b^P, \beta_0^P, \nu \middle| \rho, \gamma, \eta, \mu\right) = 0 \tag{B.15}$$

where $\delta T(x; h)$ *is the Gateaux differential of* T *at* x *with direction* h.[30]

Proof. Follows from Luenberger (1969) Lemma 1 (p. 227).[31] □

Doing integration by parts, lines (B.12) and (B.13) become respectively:

$$\int_L^{\bar{\lambda}} \rho(\lambda) U(\lambda) d\lambda - \int_L^{\bar{\lambda}} \left(\int_\lambda^{\bar{\lambda}} \rho(l) dl\right) \frac{\partial u\left(\beta_0^I(\lambda), \lambda\right)}{\partial \lambda} d\lambda$$

and

$$\int_L^{\bar{\lambda}} \gamma(\lambda) \nu(\lambda) d\lambda + \int_L^{\bar{\lambda}} \gamma'(\lambda) \beta_0^I(\lambda) d\lambda - \left[\gamma\left(\bar{\lambda}\right) \beta_0^I\left(\bar{\lambda}\right) - \gamma(L) \beta_0^I(L)\right].$$

[30] Let X be a vector space and Y be a normed space. Given a transformation $T : \Omega \to R$ where $\Omega \subset X$ and $R \subset Y$, let $x \in \Omega$ and h be arbitrary in X. If the limit

$$\delta T(x; h) = \lim_{k \to 0} \frac{1}{k} \left[T(x + kh) - T(x)\right]$$

exists, then it is called the Gateaux differential of T at x with direction h.

[31] That the Fréchet differentials in Lemma 1 of Luenberger (1969) can be replaced by the Gateaux differentials relies on a remark in p. 228 and that the Gateaux differentials here are linear.

The Gateaux differential of the Lagrangian is thus:

$$\delta\mathcal{L}^L\left(\beta_0^I, U, b^P, \beta_0^P, \nu; h_{\beta^I}, h_U, h_{b^P}, h_{\beta^P}, h_\nu \middle| \rho, \gamma, \eta, \mu\right)$$

$$= \int_L^{\bar{\lambda}} \left[\frac{\partial s\left(\beta_0^I(\lambda), \lambda\right)}{\partial \beta} f(\lambda) - \left(\int_\lambda^{\bar{\lambda}} \rho(l)\, dl\right) \frac{\partial^2 u\left(\beta_0^I(\lambda), \lambda\right)}{\partial\beta\partial\lambda} + \gamma'(\lambda) \right] h_{\beta^I}(\lambda)\, d\lambda \tag{B.16}$$

$$- \left[\gamma\left(\bar{\lambda}\right)\right] h_{\beta^I}\left(\bar{\lambda}\right) + \left[\gamma(L)\right] h_{\beta^I}(L) \tag{B.17}$$

$$+ \int_L^{\bar{\lambda}} \left[-f(\lambda) + \rho(\lambda) + \mu(\lambda) \right] h_U(\lambda)\, d\lambda \tag{B.18}$$

$$+ \left[F(L) \frac{\partial \Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)}{\partial b} - \int_L^{\bar{\lambda}} \mu(\lambda) \frac{\partial \mathcal{D}\left(\lambda; b^P, \beta_0^P\right)}{\partial b^P} d\lambda \right] h_{b^P} \tag{B.19}$$

$$+ \left[F(L) \frac{\partial \Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)}{\partial \beta_0} - \int_L^{\bar{\lambda}} \mu(\lambda) \frac{\partial \mathcal{D}\left(\lambda; b^P, \beta_0^P\right)}{\partial \beta_0^P} d\lambda \right] h_{\beta^P} \tag{B.20}$$

$$+ \int_L^{\bar{\lambda}} \left[\gamma(\lambda) + \eta(\lambda) \right] h_\nu(\lambda)\, d\lambda \tag{B.21}$$

Since (B.14) and (B.15) must hold for any $\left\{h_{\beta^I}, h_U, h_{b^P}, h_{\beta^P}, h_\nu\right\} \in \Phi^L$, every term inside each square bracket must be equal to 0.

(B.18) implies that $\rho(\lambda) = f(\lambda) - \mu(\lambda) = 0\ \forall\lambda$, which in turn implies that $\int_\lambda^{\bar{\lambda}} \rho(l)\, dl = 1 - F(\lambda) - \int_\lambda^{\bar{\lambda}} \mu(l)\, dl$. Substitute this into (B.16) and we get (A.8). (B.17) implies (A.9); (B.19) implies (A.12); (B.20) implies (A.11); (B.21) implies (A.6); and (A.7) and (A.10) are the complementary slackness conditions.

B.3 Proof of Lemma A.8

Proof. By the envelope theorem (e.g. Milgrom and Segal, 2002):

$$\frac{dV^u(L)}{dL} = \frac{\partial \mathcal{L}^L\left(\beta_0^I, U, b^P, \beta_0^P, \nu \middle| \rho, \gamma, \eta, \mu\right)}{\partial L}$$

if the derivative at the RHS exists, where the RHS is evaluated at the optimal solution. Using Leibniz rule:

$$
\begin{aligned}
&\frac{\partial \mathcal{L}^L\left(\beta_0^I, U, b^P, \beta_0^P, \nu \middle| \rho, \gamma, \eta, \mu\right)}{\partial L} \\
=&-\left[s\left(\beta_0^I(L), L\right) - U\left(L\right)\right] f\left(L\right) + \left[f\left(L\right) \Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right)\right] && \text{(B.22)} \\
&-\rho\left(L\right) U\left(L\right) + \left(\int_L^{\bar{\lambda}} \rho\left(l\right) dl\right) \frac{\partial u\left(\beta_0^I\left(L\right), L\right)}{\partial \lambda} && \text{(B.23)} \\
&-\gamma\left(L\right)\left[\nu\left(L\right) - \beta_0^{I'}\left(L\right)\right] && \text{(B.24)} \\
&-\eta\left(L\right)\nu\left(L\right) d\lambda && \text{(B.25)} \\
&-\mu\left(L\right)\left[U\left(L\right) - \mathcal{D}\left(L; b^P, \beta_0^P\right)\right] && \text{(B.26)}
\end{aligned}
$$

Lines (B.24) to (B.26) are zero by the complementary slackness conditions at optimality. Using $\rho\left(L\right) = f\left(L\right) - \mu\left(L\right)$ from (B.18), line (B.23) becomes

$$
\mu\left(L\right) U\left(L\right) - f\left(L\right) U\left(L\right) + \left(1 - F\left(L\right) - \int_L^{\bar{\lambda}} \mu\left(l\right) dl\right) \frac{\partial u\left(\beta_0^I\left(L\right), L\right)}{\partial \lambda}.
$$

Since $U\left(L\right) = 0$, combining this with (B.22) gives:

$$
\begin{aligned}
&\frac{\partial \mathcal{L}^L\left(\beta_0^I, U, b^P, \beta_0^P, \nu \middle| \rho, \gamma, \eta, \mu\right)}{\partial L} \\
=&f\left(L\right)\left[\Pi^P\left(\bar{\mathcal{W}}\left(b^P, \beta_0^P\right)\right) - \left(s\left(\beta_0^I(L), L\right) - \left[\frac{1 - F\left(L\right) - \int_L^{\bar{\lambda}} \mu\left(l\right) dl}{f\left(L\right)}\right] \frac{\partial u\left(\beta_0^I\left(L\right), L\right)}{\partial \lambda}\right)\right]
\end{aligned}
$$

The results then follow from Lemma A.7 and noting that $\int_L^{\bar{\lambda}} \mu\left(l\right) dl = 0$ for $L \leq \hat{L}$. □