Electoral College versus Popular Vote: Should the U.S. Change Its Presidential Election Scheme?

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Motivation

- The 2016 U.S presidential election:

Candidate	Electoral College	Popular Vote
Donald J. Trump	304	45.93 %
Hillary Clinton	227	48.02 %

  - **Prevailing scheme**: Electoral College, the district (state) winner takes all the votes of the district (state)
  - **A natural alternative in debate of electoral reform**: Popular Vote, in each district (state), each candidate takes his/her own votes won

- Other election inversions in US history:
  - In 2000, **George W. Bush** (271, 49.73%) & Albert Gore (267, 50.27%)
  - In 1888, **Benjamin Harrison** (233, 49.59%) & Grover Cleveland (168, 50.41%)
  - In 1876, **Rutherford Hayes** (185, 49.47%) & Samuel Tilden (184, 51.53%)

Our Research in This Paper

- We contribute to this debate by attempting to provide an answer to the following question
- How does Popular Vote perform compared to Electoral College in the following 2 dimensions?
  - Candidates' winning chances
  - Chance of generating controversy/ election inversion: consistency across the two standards

Main Findings

- We consider a 2-candidate 3-district (state) simultaneous-move election competition
  - Two asymmetric candidates: Strong and weak
  - Three identical districts (states):
    - Each district (state) has a total of 1 vote
    - Both candidates simultaneously exert effort in all three districts (states)
    - In each district (state), the candidates share votes following a random distribution, which is contingent on their effort profile
- Electoral College gives the stronger candidate a better winning chance than Popular Vote;
- Electoral College generates less election inversions / controversy than Popular Vote;

Related Literature

- **In political science**:
  - Arlington and Branner (1984): possible disadvantages of the two schemes
  - Miller (2012): factors generating election inversion under Electoral College scheme
- **In economics**: multi-battle contests
  - Simultaneous move: Snyder (1989), Myerson (1993), Lizzeri and Persico (2001), Stromberg (2008), Roberson (2008)
  - Dynamics:l Harris and Vickers (1987), Strumpf (2002), Klumpp and Polborn (2006), Konrad and Kovenock (2009, 2012), Kovenock and Roberson (2009), Ryvkin (2011), Gelder (2014), Fu, Lu and Pan (2015), Häfner (2017), Klumpp, Konrad and Solomon (2019)
  - Prize structure: Feng and Lu (2018), Lu, Shen and Wang (2019)
  - Survey: Kovenock and Roberson (2010), Konrad (2012), Corchón and Serena (2017), Fu and Wu (2019)

Model

- Three districts (states):
  - $k \in \mathrm{D} = \{1, 2, 3\}$,
  - Each with 1 unit of votes, normalized
- Two candidates: S and W
  - Marginal effort cost: $c_W \geq c_S > 0$,
  - Effort in district (state) k: $x_{k,S}$ and $x_{k,W}$,
  - Shares of votes in district (state) k: $\left(s_k, S; 1 - s_k, W\right)$
  - Common value of winning the election: 1

Model

- Campaign technology:
  - In district (state) k, with the following probability, candidate S's share of votes s_k is randomly drawn following a uniform distribution $U[1/2, 1]$ (favorite state):

$$\frac{x_{k,S}}{x_{k,S} + x_{k,W}},$$

  - With the following probability, candidate S's share of votes s_k is randomly drawn following a uniform distribution $U[0, 1/2]$ (less-liked state):

$$\frac{x_{k,W}}{x_{k,S} + x_{k,W}},$$

- Winning rules under Electoral College and Popular Vote

Electoral College G^{EC}

- A pure strategy of candidate $i \in \{S, W\}$ is

$$x_i = (x_{1,i}, x_{2,i}, x_{3,i}) \in X_i = [0, 1/c_i]^3.$$

- Candidate $i \in \{S, W\}$'s problem:

$$\max \pi_i^{EC} = p_i^{EC} - c_i \cdot (x_{1,i} + x_{2,i} + x_{3,i}),$$

where

$$p_S^{EC} = \frac{x_{1,S}x_{2,S}x_{3,S} + x_{1,S}x_{2,S}x_{3,W} + x_{1,S}x_{2,W}x_{3,S} + x_{1,W}x_{2,S}x_{3,S}}{(x_{1,S} + x_{1,W})(x_{2,S} + x_{2,W})(x_{3,S} + x_{3,W})};$$

$$p_W^{EC} = 1 - p_S^{EC}.$$

- Snyder (1989) and Klumpp and Pulborn (2006) analyze this game with symmetric players

Electoral College G^{EC}

Definition 2.1

If candidate i chooses $x \in \mathbb{R}_+$ according to some cumulative distribution function Λ_i and sets $x_{k,i} = x$ for all k, then he is playing a *uniform* strategy. An equilibrium is called a *uniform equilibrium* if both candidates adopt uniform strategies.

Definition 2.2

An equilibrium is *non-zero* if no candidate plays a strategy in which zero effort is exerted in some district (state) with probability one .

Lemma 2.3

The equilirbium for election game G^{EC} is non-zero.

Electoral College G^{EC}

Proposition 2.4

Let $c_{EC} = \sqrt{7} - 2 < 1$. *In* G^{EC}, *the following statements hold:*

(a) *When* $c_{EC} \le c \le 1$, *uniform pure-strategy equilibrium:*

$$x^*_{k,i} = \frac{2c^2}{c_i \cdot (1+c)^4}, \forall i \in \{S, W\}, \forall k \in \{1,2,3\};$$

(b) *When* $0 < c < c_{EC}$, *uniform semi-pure-strategy equilibrium: Candidate* S *exerts*

$$\hat{x}_{k,S} = \frac{p^{EC}}{3c_S} \cdot \frac{3c_{EC}^2 + c_{EC}^3}{(1+c_{EC})^3}, \forall k \in \{1,2,3\};$$

W *stays inactive with prob.* $1 - p^{EC}$ *where* $p^{EC} = c/c_e$, *and otherwise exerts*

$$\hat{x}_{k,W} = \frac{1}{3c_W} \cdot \frac{3c_{EC}^2 + c_{EC}^3}{(1+c_{EC})^3}, \forall k \in \{1,2,3\}.$$

Electoral College G^{EC}

Sketch of proof:

- Schur concavity: Given opponent taking a uniform strategy, it's optimal for a candidate to play a uniform strategy
- Expected payoff maximization in single dimensional space

Proposition 2.5

The equilibrium for G^{EC} is unique.

- Interchangeability of the equilibria of the election game G^{EC}.

Electoral College G^{EC}

- For high ratio c, i.e. less asymmetric, candidates' effort ratio is constant.
- For small c, i.e, more asymmetric,
  - the probability that the weak candidate remains active increases with c;
  - the weak candidate always has zero expected payoff;
  - when the weak candidate is active, the candidates' effort ratio is constant, i.e. $\frac{1}{c_{CE}}$

Popular Vote G^{PV}

- A pure strategy of candidate $i \in \{S, W\}$ is

$$y_i = (y_{1,i}, y_{2,i}, y_{3,i}) \in Y_i = [0, 1/c_i]^3.$$

- Candidate $i \in \{H, L\}$'s problem:

$$\max \pi_i^{PV} = p_i^{PV} - c_i \cdot (y_{1,i} + y_{2,i} + y_{3,i}),$$

where

$$p_S^{PV} = \frac{y_{1,S}y_{2,S}y_{3,S} + \frac{5}{6}\left(y_{1,S}y_{2,S}y_{3,W} + y_{1,S}y_{2,W}y_{3,S} + y_{1,W}y_{2,S}y_{3,S}\right)}{(y_{1,S} + y_{1,W})(y_{2,S} + y_{2,W})(y_{3,S} + y_{3,W})} + \frac{\frac{1}{6}\left(y_{1,S}y_{2,W}y_{3,W} + y_{1,W}y_{2,W}y_{3,S} + y_{1,W}y_{2,S}y_{3,W}\right)}{(y_{1,S} + y_{1,W})(y_{2,S} + y_{2,W})(y_{3,S} + y_{3,W})},$$

and $p_W^{PV} = 1 - p_S^{PV}$

Popular Vote G^{PV}

Proposition 2.6

Let $c_{PV} = \frac{\sqrt{13}-3}{2} < c_{EC}$. *In* G^{PV},

1. (a) *when* $c_{PV} \leq c \leq 1$, *uniform pure strategy equilibrium:*

$$y^*_{k,i} = \frac{1}{c_i} \cdot \frac{c(1+8c+c^2)}{6(1+c)^4}, \forall i \in \{S, W\}, \forall k \in \{1,2,3\};$$

2. (b) *if* $0 < c < c_{PV}$, *uniform semi-pure-strategy equilibrium: Let* $p^{PV} = c/c_{PV}$. *S plays a pure strategy equilibrium, exerting*

$$\hat{y}^*_{k,S} = \frac{p^{PV}}{3c_S} \cdot \frac{1+\frac{5}{2}p^{PV}c^{-1}+\frac{1}{2}(p^{PV}c^{-1})^2}{(1+p^{PV}c^{-1})^3}, \forall k \in \{1,2,3\}.$$

W plays, with probability p^{PV}, *uniform strategy*

$$\hat{y}^*_{k,W} = \frac{1}{3c_W} \cdot \frac{1+\frac{5}{2}p^{PV}c^{-1}+\frac{1}{2}(p^{PV}c^{-1})^2}{(1+p^{PV}c^{-1})^3}, \forall k \in \{1,2,3\},$$

and stays inactive with probability $1 - p^{PV}$.

Popular Vote G^{PV}

- Proof procedure is similar to that of Proposition 2.4
- Equilibrium is unique
- For high ratio c, i.e. less asymmetric, candidates' effort ratio is constant.
- For small c, i.e, more asymmetric,
  - the probability that the weak candidate remains active increases with c;
  - the weak candidate always has zero expected payoff;
  - when the weak candidate is active, the candidates' effort ratio is constant, i.e. $\frac{1}{c_{PV}}$

Electoral College versus Popular Vote: Winning Chances

Theorem 2.7

The stronger candidate always has a weakly higher winning chance under Electoral College than under Popular Vote.

- Under Electoral College, S's winning chance

$$p_S^{EC*} = \begin{cases} 1 - \frac{c^3+3c^2}{(1+c)^3}, & \text{if } c_{EC} \le c \le 1; \\ 1 - \frac{c}{c_{EC}} \cdot \frac{c_{EC}^3+3c_{EC}^2}{(1+c_{EC})^3}, & \text{if } 0 \le c < c_{EC}. \end{cases} \tag{1}$$

- Under Popular Vote, S's winning chance

$$p_S^{PV*} = \begin{cases} 1 - \frac{c^3+\frac{5}{2}c^2+\frac{1}{2}c}{(1+c)^3}, & \text{if } c_{PV} \le c \le 1; \\ 1 - \frac{c}{c_{PV}} \cdot \frac{c_{PV}^3+\frac{5}{2}c_{PV}^2+\frac{1}{2}c_{PV}}{(1+c_{PV})^3}, & \text{if } 0 \le c < c_{PV}. \end{cases} \tag{2}$$

Electoral College versus Popular Vote: Winning Chances

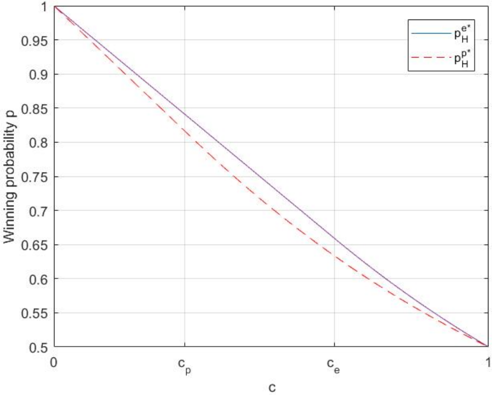


Figure: Stronger candidate's winning probability

Electoral College versus Popular Vote: Winning Chances

Intuitions:

- Recall S's share of vote in a district (state) can follow $U[1/2, 1]$ (favorite state, State G), or $U[0, 1/2]$ (less-liked state, State B).
- 4 possible combinations: $\{G, G, G\}$, $\{B, B, B\}$, $\{G, G, B\}$ and $\{G, B, B\}$
- S wins under EC iff $\{G, G, G\}$ or $\{G, G, B\}$ happen in the 3 districts (states); S definitely wins under PV when $\{G, G, G\}$ happens, he can also win if $\{G, G, B\}$ and $\{G, B, B\}$ happen
- When $c > c_{EC}$, both EC and PV induce pure strategy with same effort ratio, the probability of G is the same across the two schemes: $P_G = \frac{c_W}{c_S + c_W} > \frac{1}{2}$. Under EC, S wins with prob. $P_G^3 + 3P_G^2(1 - P_G)$. Under PV, S wins with prob. $P_G^3 + 3P_G^2(1 - P_G)P_S(G, G, B) + 3P_G(1 - P_G)^2 P_S(G, B, B)$.
- Note $3P_G^2(1 - P_G) > 3P_G(1 - P_G)^2$ as $P_G > \frac{1}{2}$ and $P_S(G, G, B) + P_S(G, B, B) = 1$.

Electoral College versus Popular Vote: Winning Chances

When $c > c_{EC}$, pure strategy equilibrium for both schemes:

$$P_S^{EC*} - P_S^{PV*} = \left(1 - \frac{c^3 + 3c^2}{(1+c)^3}\right) - \left(1 - \frac{c^3 + \frac{5}{2}c^2 + \frac{1}{2}c}{(1+c)^3}\right)$$
$$= \underbrace{\frac{c^3 + \frac{5}{2}c^2 + \frac{1}{2}c}{(1+c)^3} - \frac{c^3 + 3c^2}{(1+c)^3}}_{\text{rounding effect } > 0}.$$

Electoral College versus Popular Vote: Winning Chances

Intuitions:

- When $c_{PV} < c < c_{EC}$, we have PV induces pure strategy equilibrium, and prob. of G under EC is $P_G = \frac{c_W}{c_S + c_W} = \frac{1}{1+c} > \frac{1}{2}$. Under PV, S wins with prob. $P_G^3 + 3P_G^2(1 - P_G)P_S(G, G, B) + 3P_G(1 - P_G)^2 P_S(G, B, B)$.
- EC induces semi-pure strategy equilibrium. S wins with prob. $(1 - p^{EC}) \cdot 1 + p^{EC}[\tilde{P}_G^3 + 3\tilde{P}_G^2(1 - \tilde{P}_G)]$ where $\tilde{P}_G = \frac{1}{1+c_{EC}}$ is the prob. of S getting state G conditional on W is active.
- Note $P_G^3 + 3P_G^2(1 - P_G)P_S(G, G, B) + 3P_G(1 - P_G)^2 P_S(G, B, B) <$ $P_G^3 + 3P_G^2(1 - P_G)$ since $P_G > \frac{1}{2}$ and $P_S(G, G, B) + P_S(G, B, B) = 1$.
- The wanted result obtains, if we can show $P_G^3 + 3P_G^2(1 - P_G) \leq (1 - p^{EC}) \cdot 1 + p^{EC}[\tilde{P}_G^3 + 3\tilde{P}_G^2(1 - \tilde{P}_G)]$, which says that under EC, S gets $\{G, G, G\}$ and $\{G, G, B\}$ more often
- Unfortunately this does not hold

Electoral College versus Popular Vote: Election Inversion Rate

When $c_{PV} < c \le c_{EC}$, semi-mixed strategy equilibrium for EC, and pure strategy equilibrium for PV:

$$
\begin{aligned}
& P_S^{EC*} - P_S^{PV*} \\
=& \left(1 - \frac{c}{c_e} \cdot \frac{c_e^3 + 3c_e^2}{(1+c_e)^3}\right) - \left(1 - c \cdot \frac{c^2 + \frac{5}{2}c + \frac{1}{2}}{(1+c)^3}\right) \\
=& c \cdot \frac{c^2 + \frac{5}{2}c + \frac{1}{2}}{(1+c)^3} - \frac{c}{c_e} \cdot \frac{c_e^3 + 3c_e^2}{(1+c_e)^3} \\
=& \underbrace{c \cdot \frac{c^2 + \frac{5}{2}c + \frac{1}{2}}{(1+c)^3} - \frac{c}{c_e} \cdot \frac{c_e^3 + \frac{5}{2}c_e^2 + \frac{1}{2}c_e}{(c_e+1)^3}}_{\text{strategic effect without rounding } > 0} + \underbrace{\frac{c}{c_e} \cdot \frac{c_e^3 + \frac{5}{2}c_e^2 + \frac{1}{2}c_e}{(c_e+1)^3} - \frac{c}{c_e} \cdot \frac{c_e^3 + 3c_e^2}{(1+c_e)^3}}_{\text{rounding effect} > 0}.
\end{aligned}
$$

Electoral College versus Popular Vote: Election Inversion Rate

Definition 2.8

Given a strategy profile, the inversion rate is defined as the probability that the two election schemes produce different winners.

Theorem 2.9

Electoral College generates weakly lower inversion rate than Popular Vote, i.e.,

$$P^{IR,EC} \leq P^{IR,PV}.$$

Specifically,

1. *when $c_{EC} \leq c \leq 1$, both schemes have the same inversion rate.*
2. *when $0 < c < c_{EC}$, Electoral College has lower inversion rate than Popular Vote.*

Electoral College versus Popular Vote: Election Inversion Rate

When $c_{EC} \leq c \leq 1$:

- Under Electoral College, the inversion rate is

$$P^{IR,EC} = \frac{1}{6} \cdot \frac{3c + 3c^2}{(1+c)^3} = \frac{1}{2} \cdot \frac{c}{(1+c)^2}.$$

- Under Popular Vote, the inversion rate is

$$P^{IR,PV} = \frac{c}{2(1+c)^2}.$$

Electoral College versus Popular Vote: Election Inversion Rate

When $c_{PV} \leq c < c_{EC}$:

- Under Electoral College, the inversion rate is

$$P^{IR,EC} = \frac{c}{c_{EC}} \cdot \frac{c_{EC}}{2(1+c_{EC})^2}.$$

- Under Popular Vote, the inversion rate is

$$P^{IR,PV} = \frac{c}{2(1+c)^2}.$$

- The difference is

$$P^{IR,EC} - P^{IR,PV} = \frac{c}{2(1+c_{EC})^2} - \frac{c}{2(1+c)^2} < 0.$$

Electoral College versus Popular Vote: Election Inversion Rate

When $0 < c < c_{PV}$:

- Under Electoral College, the inversion rate is

$$P^{IR,EC} = \frac{c}{2} \cdot \frac{1}{(1+c_{EC})^2}.$$

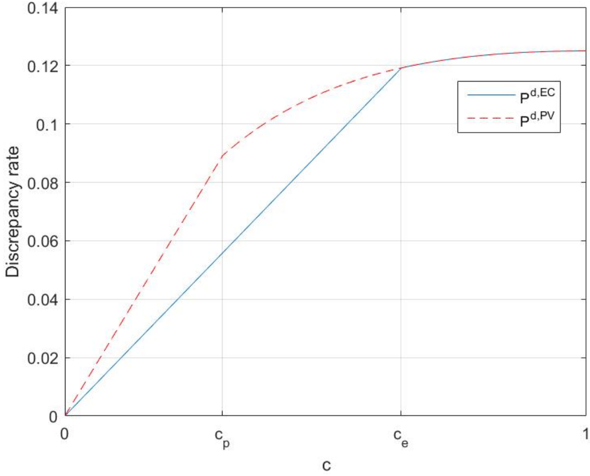
- Under Popular Vote, the inversion rate is

$$P^{IR,PV} = \frac{c}{2} \cdot \frac{1}{(1+c_{PV})^2}.$$

- The difference is

$$P^{IR,EC} - P^{IR,PV} = \frac{c}{2} \cdot \frac{1}{(1+c_{EC})^2} - \frac{c}{2} \cdot \frac{1}{(1+c_{PV})^2} < 0.$$

Electoral College versus Popular Vote: Election Inversion Rate



Electoral College versus Popular Vote: Election Inversion Rate

Intuitions:

- Recall 4 possible combinations in the perspective of S: $\{G, G, G\}$, $\{B, B, B\}$, $\{G, G, B\}$ and $\{G, B, B\}$
- Election inversion occurs iff $\{G, G, B\}$ and $\{G, B, B\}$
- When $c > c_{EC}$, $\{G, G, B\}$ and $\{G, B, B\}$ happen with same probabilities under EC and PV
- When $c_{PV} < c < c_{EC}$, PV induces pure-strategy equilibrium, and EC induces semi-pure-strategy equilibrium. EC leads to higher probability for $\{G, G, G\}\&\{B, B, B\}$, and lower probabilities for $\{G, G, B\}$ and $\{G, B, B\}$
- When $c < c_{PV}$, similar

Asymmetric districts (states): no dominant district

Example 3.1

Consider an election game with N_k votes in district (state) k. N_3 is normalized to 1, $N_2 = 0.8$, and $N_1 = 0.4$. Candidates' effort costs ratio is $c = 0.8$.

- Under Electoral College: no dominant district (state) $\Rightarrow$ a candidate still has to win at least two districts (states) to win the election:

$$p_S^{EC,asy} = \frac{1+3c}{(1+c)^3} \approx 0.583.$$

- Under Popular Vote: still fight in all of the three districts (states):

$$p_S^{PV,asy} = \frac{1}{(1+c)^3} + 2.325\frac{c}{(1+c)^3} + 0.675\frac{c^2}{(1+c)^3} \approx 0.564.$$

- Compare the two winning probabilities, we have

$$p_S^{EC,asy} - p_S^{PV,asy} \approx 0.019 > 0.$$

Asymmetric districts (states): dominant district

Example 3.2

Consider an election game with N_k votes in district (state) k. N_3 is normalized to 1, $N_2 = 0.5$, and $N_1 = 0.4$. Candidates' effort costs ratio is $c = 0.3$.

- Under Electoral College: district (state) 3 is a dominant district (state) $\Rightarrow$ a candidate still has to win at least two districts so as to win the election:

$$p_S^{EC,asy} = \frac{1}{1+c} \approx 0.7692.$$

- Under Popular Vote: still fight in all of the three districts (states):

$$p_S^{p,asy} \approx 0.8024.$$

Concluding Remarks

- We formulate a stylized model of political campaign, in which two asymmetric candidates compete in three identical districts (states).
- We compare Electoral College and Popular Vote in this model
- Equilibria are fully characterized
- Electoral College elects the stronger candidate with higher probability than Popular Vote
- Electoral College generates lower election inversion rate than Popular Vote
- Robustness check

Possible Extensions

- More general vote-generating distributions
- More districts (states), and more general asymmetry across districts (states)
- Temporal structure
- Team competition

Thank you very much!