Firm Investment Decisions under Hyperbolic Discounting

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April 11, 2016

Abstract

This paper constructs a model of corporate investment decisions under hyperbolic discounting of present values. The hyperbolic discounted present value can be interpreted as reflecting irrational myopic preferences or, as this paper demonstrates, reduced-form implications of corporate agency issues. Both cases result in an underinvestment problem for the firm, but the firm valuation criteria differ. We show that imposing revenue-neutral dividend taxes or investment subsidies by an outside authority can overcome the firm's underinvestment problem and consequently increase all periods' present value of dividends. Lastly, under a multi-period extension with Cobb-Douglas return functions, this paper shows quantitative implications of our model.

Keywords: Present bias; hyperbolic discounting; corporate investment; underinvestment; dividend taxation; investment subsidy; agency problem; asymmetric information

JEL classification: D03; D21; D92; E22; G02; G31; G38

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1. Introduction

The notion of short-termism behavior among corporations has been widely discussed. Practitioners of finance and policymakers often cite short-termism as a major constraint on value-enhancing corporate investment projects (e.g. Graham et al. 2005). Short-termism also features prominently in public policy debates on corporate taxation.[2] However, despite the wide attention received, theoretical underpinnings for the linkage between short-termism and corporate investment remain extremely sparse. This paper attempts to fill in the gap by constructing a theoretical model of corporate investment decisions under short-termism and analyzing its associated policy implications.

In particular, we introduce hyperbolic discounting to corporate investment decisions. We present a multi-period model under which the firm makes an investment decision in each period to maximize the present value of its dividend stream. The firm invests in one project that yields return in the final period. We show that a firm exhibiting hyperbolic discounting preferences faces an underinvestment problem, i.e. there exists another feasible investment plan that improves all periods' present values. Therefore, Pareto-improving policies by an outside authority, such as the government, may be justified. In the second part of this paper, we show that adopting revenue-neutral dividend taxes or investment subsidies can mitigate the firm's underinvestment problem and thus increase all periods' present value of dividends.

This paper is related to a few strands of literature. First, it directly addresses the issue of short-termism in economics. Experimental and introspective evidence have long suggested that animal and human behavior are short-term oriented and that their discount functions are closer to hyperbolic than exponential (Ainslie 1992; Loewenstein and Prelec 1992). Decades ago, Stroz (1956), Phelps and Pollak (1968) and Laibson (1994) have begun to apply the theory of hyperbolic discounting to consumer's consumption-saving decision problems.[3] Laibson (1996, 1997) further shows that consumers with hyperbolic discounting preferences face undersaving problems, resulting in implications that explain US household saving patterns.

In parallel, the literature in behavioral finance has also suggested that corporate decisions are short-term oriented, and such myopic decisions can result in suboptimal equilibrium (see Stein 1988, 1989; Porter 1992; Bebchuk and Stole 1993; Stein 2003). These theories on corporate short-termism have focused on agency conflicts between corporate managers and stockholders. Corporate managers may underinvest due to pressures from boosting earnings

[2]For examples of these debates, see Barton and Wiseman (2014), Denning (2014), and Lazonick (2015). Policies to address corporate short-termism have also been discussed extensively in the most recent US presidential election debates.

[3]Short-term discounting has also been linked to cognitive ability (Benjamin, Brown, and Shapiro 2013).

as reflected in stock values. The agency view to myopia, which maintains rationality, is distinct from the irrational managerial myopia view. The latter explains short-termism as a form of irrational intrinsic behavior arising from time inconsistency.

As will be shown in detail, this paper's framework is able to capture both views. Hyperbolic discounting can be interpreted directly as the time-inconsistent preference of investors and managers. However, this paper also shows that firm value under hyperbolic discounting can be interpreted as the reduced-form version of the myopic manager's preferences under Stein (1989)'s agency conflict setting. Under this agency approach, the reduced-form hyperbolic preferences are neither the manager nor investors' intrinsic preferences. Therefore, when evaluating firm value, time-consistent exponential preferences would be the relevant criteria. This paper shows that when exponential preferences are used to assess firm value, then the firm experiences a more severe underinvestment problem. This also implies that a policy that improves all hyperbolic present values is also an improvement based on exponential present value evaluation. Thus, an important feature of our model is that it could explain corporate myopia arising from agency problems, but yet shows that even under no agency conflicts, time inconsistency could in itself be a source of myopia that induces underinvestment. The underinvestment behavior resulting from our framework is consistent with both theories, with differing firm valuation criteria.

The underinvestment problem in our framework arises jointly from (a) the present-biased discounting functions and (b) the supermodularity of the return function. Supermodularity implies that the marginal return of investment in one period is increasing in the investment level of another period. With this property, lower investment in an earlier period raises the incentive to decrease investment in a later period. In the earlier period, present bias causes the firm to pay more dividends and invest less. Subsequently, from the perspective of the later period, the investment cutback in the early period induces the firm to invest less in the later period by supermodularity. Altogether, this implies that the firm's investment decisions are suboptimal in terms of both periods' present values.

In terms of normative implications, this paper provides perspectives on the optimality of dividend taxation and investment subsidies.[4] Under hyperbolic preferences, dividend taxes may increase investment and has the ability to address the underinvestment problem.[5]

[4]These two types of policies are commonly introduced to boost investment during periods of recessions. For example, in 2003, the US Congress passed the Jobs and Growth Tax Relief and Reconciliation Act, for which increasing investments was a justification for dividend tax cuts included in the package.

[5]The literature on dividend taxation has debated whether dividend tax cuts exerts a significant effect on investment. Some argue that if corporations finance marginal investment through new stocks, dividend tax cuts would increase investment (Chetty and Saez 2004; Poterba and Summers 1995). On the other hand, if marginal investment is financed through retained earnings, then dividend taxes would not affect investment (Auerbach 1979; Bradford 1981).

Specifically, we consider dividend taxation that are revenue-neutral, as the collected dividend taxes are returned to the firm with lump-sum subsidies. Even with this revenue-neutral policy, we show that such an intervention improves the firm's present values in all periods. This type of Pareto-improving multi-period taxation is distinct from Pigouvian-style lump-sum transfers, in which taxation in the current period without taxation in other periods inevitably lowers the firm's present value. Therefore, tax policies in all investment periods are necessary for Pareto-improving investment. We also introduce an investment subsidy policy, in which the government provides proportional investment subsidies and collects lump-sum taxes of equal value. Under the same logic as that of dividend taxation, the investment subsidy boosts investment as a result of lower costs.

The main analysis in this paper is based on a three-period model with general return functions. However, we also show that the three-period model can be extended into a T-period model (where $T \geq 3$) with the Cobb-Douglas return function. Under this stylized framework, we show that present bias, in general, induces greater decreases in investment the later the period. This phenomenon results from the supermodularity property - lower earlier-period investment decreases the marginal return of later-period investment, providing an additional incentive for the firm to decrease investment in the later periods.

In the final section of this paper, we conduct a numerical analysis of our model with the Cobb-Douglas return function and show a number of quantitative implications. For parameter values that match historical data on the US annual real interest rate, return on invested capital, and the project horizon of an average firm, we show that the extent of underinvestment resulting from present bias may be substantial. An increase in present bias by 20-25 percent from the benchmark case of no present bias induces a reduction in investment of up to 30-50 percent. The quantitative implications of our model are broadly in line with that of the empirical findings of Asker, Farre-Mensa, and Ljungqvist (2015), and suggest that this theoretical framework may be a useful benchmark for understanding the impact of short-termism on investment decisions.

The rest of the paper proceeds as follows. In Section 2, we present the set-up of the theoretical framework. In Section 3, we solve for the subgame-perfect Nash equilibrium firm investment levels in our model, define underinvestment, and show that in equilibrium, the firm faces an underinvestment problem. Section 4 compares our multi-period investment model with a consumption-savings model under hyperbolic discounting. Sections 5 and 6 consider policy solutions to the underinvestment problem. In particular, section 5 shows that a revenue-neutral increase in dividend taxes can overcome the underinvestment problem. Section 6 shows that investment subsidies can also achieve this purpose. Section 7 shows how our results fit under both the irrationality and agency views of myopia. Section 8 extends the

three-period framework to a T-period model with the Cobb-Douglas return function. Last but not least, section 9 shows quantitative implications of the model. Section 10 concludes.

2. Three-period model

We first introduce a three-period model of corporate investment decisions under the hyperbolic discounting framework. The firm makes an investment decision in each period to maximize the present value of dividend streams. x_1 and x_2 denote exogenous cash flows in periods 1 and 2, respectively. The firm chooses to undertake investments of amounts i_1 and i_2 in periods 1 and 2. The return from investments is realized in period 3 and takes on the function $f(i_1, i_2)$. The return function satisfies the Inada conditions and is strictly supermodular (i.e., $\partial^2 f/\left(\partial i_1 \partial i_2\right) > 0$).[6] The firm's dividends are denoted as $d_1 = x_1 - i_1$ in period 1, $d_2 = x_2 - i_2$ in period 2, and $d_3 = f(i_1, i_2)$ in period 3. We assume that x_1 and x_2 are large enough to avoid the negative dividend.

We apply the popular β, δ functional form in assessing the firm's present values. The present values in periods 1, 2 and 3 are given as

$$PV_1 = d_1 + \beta_1 \left(\delta d_2 + \delta^2 d_3\right),$$

$$PV_2 = d_2 + \beta_2 \delta d_3,$$

and

$$PV_3 = d_3.$$

where β_1 and β_2 are hyperbolic discounting factors in periods 1 and 2, respectively; and δ is the long-term discounting factor. In the traditional hyperbolic discounting model, β_1 and β_2 are identical, which implies that the decision maker has the same incremental discounting rate between today and the future periods. However, as will be shown in section 7, hyperbolic discounting preferences can be interpreted as the reduced-form of manager's preferences under agency conflicts with asymmetric information. In this case, β_1 and β_2 would no longer be interpreted as parameters reflecting intrinsic irrational myopia, but rather as market-driven myopia. In this paper, we consider both interpretations of hyperbolic discounting preferences.

With this β, δ functional form, $\beta_1 = \beta_2 = 1$ corresponds to exponential discounting, while

[6]The Inada conditions are that the return function is strictly increasing, continuously differenciable, $f(0, i_2) = 0$, $f(i_1, 0) = 0$, $f(0, 0) = 0$, $\partial^2 f/\partial^2 i_1 < 0$, $\partial^2 f/\partial^2 i_2 < 0$, $\lim_{i_1 \to 0} \partial f/\partial i_1 = \infty$, $\lim_{i_2 \to 0} \partial f/\partial i_2 = \infty$, $\lim_{i_1 \to \infty} \partial f/\partial i_1 = 0$ and $\lim_{i_2 \to \infty} \partial f/\partial i_2 = 0$. These limiting conditions are necessary to ensure interior solutions.

$\beta_1, \beta_2 \in (0,1)$ reflects present bias. In other word, β_1 and β_2 are excess discount factors between the current and the next period.

The use of extra discounting of present values to incorporate short-termism has been established by corporate finance research. This approach is grounded on vast empirical evidence that shows corporate discount rates are higher than those implied by efficient markets (King 1972; Poterba and Summers 1995; Miles 1993; Haldane and Davies 2011). Recently, Budish, Roin, and Williams (2015) defined a benchmark discount rate based on the real interest rate and risk factors. They defined short-termism as an exponential discount rate that is strictly greater than the benchmark discount rate. In our model, the extra discounting applies only to the current and the immediate future period, which deviates from the exponential discounting assumption.

We assume that the firm is sophisticated, as defined by knowing how its preferences change over time.[7] The sophisticated firm in period 1 knows how the firm in period 2 makes decisions, given the period-1 decision. Therefore, the equilibrium can be derived in a recursive way. The firm in period 2 chooses i_2 to maximize PV_2, conditional on i_1:

$$\max_{i_2|i_1} \ (x_2 - i_2) + \beta_2 \delta f(i_2, i_1). \tag{1}$$

From the maximization problem (1), we have an implicit function of i_2 in terms of i_1, denoted as $\widehat{i_2}(i_1)$. Even though in most cases, closed-form solutions for $\widehat{i_2}(i_1)$ do not exist, we know that $\widehat{i_2}(i_1)$ is a well-defined and strictly increasing function due to concavity and strict supermodularity of the return function. Thus, with $\widehat{i_2}(i_1)$, the sophisticated firm chooses i_1 to maximize PV_1:

$$\max_{i_1} (x_1 - i_1) + \beta_1 \delta \left(x_2 - \widehat{i_2}(i_1) \right) + \beta_1 \delta^2 f \left(\widehat{i_2}(i_1), i_1 \right). \tag{2}$$

3. Underinvestment problem

Having presented the set-up of the model, we analyze how the myopic firm would make suboptimally low levels of investment in equilibrium. In other words, there exist other investment plans that induce higher firm values in all periods.

Mathematically, the underinvestment problem arises jointly from present-bias ($\beta_1, \beta_2 < 1$) and supermodularity of the return function ($\partial^2 f / \partial i_1 \partial i_2 > 0$). Supermodularity of the return function means that the marginal return of one period's investment increases in the

[7] The behavior of present-biased agents can often be different depending on whether they are aware (sophisticated) or unaware (naive) of their self-control problems. O'Donoghue and Rabin (1999, 2015) carefully compare the decision and welfare differences between naive and sophisticated agents.

other period's investment. Consequently, this induces the choice function, $\widehat{i}_2(i_1)$, to be increasing. This implies that the firm will have stronger (weaker) incentive to invest more if investment level is higher (lower) in the past. With present-bias ($\beta < 1$), the firm in period 1 will pay out higher level of dividends and, consequently, undertake lower level of investment from the perspective of period 2. Due to the low level of period-1 investment, period-2 investment is also low because $\widehat{i}_2(i_1)$ is an increasing function. Therefore, low investment levels in both periods result in an underinvestment problem.

To demonstrate the underinvestment problem, we will show the existence of an equilibrium that solves the two maximization problems defined in periods 1 and 2. Next, we show that marginal increases in both periods' investments from the equilibrium investment level can improve the firm's value in all three periods, which implies that the firm is facing an underinvestment problem. In the following section, we will show that there exists tax and subsidy policies that address this issue by inducing an increase in investment and thus a rise in the firm's value in all periods.

The following proposition shows that there exists an equilibrium(s) from the firm's maximization problem:

Proposition 1 *There exists a subgame perfect Nash equilibrium* $\left(i_1^*, \widehat{i}_2(i_1)\right)$ *such that* $\widehat{i}_2(i_1)$ *solves the period-2 maximization problem, conditional on* i_1*; and* i_1^* *solves the period-1 maximization problem by replacing* i_2 *with* $\widehat{i}_2(i_1)$*.*

Proof. The first order condition from the maximization problem (1) is

$$-1 + \beta_2 \delta f_2(i_1, i_2) = 0. \tag{3}$$

The second order condition from the maximization problem (1) is

$$\beta_2 \delta f_{22}(i_1, i_2) < 0. \tag{4}$$

By the first and second order conditions, we know that for any value of $i_1 > 0$, there exists a unique $i_2 > 0$ that solves equation (3). We define $\widehat{i}_2(i_1)$, which solves the first order condition in (3), such that

$$-1 + \beta_2 \delta f_2\left(i_1, \widehat{i}_2(i_1)\right) = 0. \tag{5}$$

Implicitly differentiating equation (5) with respect to i_1, we have

$$\beta_2 \delta f_{12}\left(i_1, \widehat{i}_2(i_1)\right) + \beta_2 \delta f_{22}\left(i_1, \widehat{i}_2(i_1)\right) \widehat{i}_2'(i_1) = 0.$$

which in turn equivalently is

$$\widehat{i}_2'(i_1) = -\frac{f_{12}\left(i_1, \widehat{i}_2(i_1)\right)}{f_{22}\left(i_1, \widehat{i}_2(i_1)\right)} > 0. \tag{6}$$

The firm maximizes the following in period 1:

$$PV_1 = (x_1 - i_1) + \beta_1\delta\left(x_2 - \widehat{i}_2(i_1)\right) + \beta_1\delta^2 f\left(i_1, \widehat{i}_2(i_1)\right).$$

By the Inada conditions and that $\widehat{i}_2'(i_1) > 0$, the optimal solution for i_1^* is neither zero nor infinite. Because $PV_1\left(i_1, \widehat{i}_2(i_1)\right)$ is a smooth function of i_1, by the mean value theorem there is an interior solution i_1^* in which the first order condition is zero and the second order condition is negative.[8] The first order condition is

$$-1 - \beta_1\delta\widehat{i}_2'(i_1) + \beta_1\delta^2\left(f_1 + f_2\widehat{i}_2'(i_1)\right) = 0. \tag{7}$$

The second order condition is

$$\beta_1\delta\widehat{i}_2''(i_1) + \beta_1\delta^2\left(f_{11} + 2f_{12}\widehat{i}_2'(i_1) + f_{22}\left(\widehat{i}_2'(i_1)\right)^2\right) + \beta_1\delta^2 f_2\widehat{i}_2''(i_1) \leq 0. \tag{8}$$

■

Next, we will show that the firm's equilibrium decision is suboptimal and the firm experiences the underinvestment problem. We define the underinvestment problem as follows:

Definition 1 *At the equilibrium investment levels (i_1^*, i_2^*), the firm faces the underinvestment problem if there exists $(i_1', i_2') \gg 0$ such that*

$$i_1' > i_1^*,\ i_2' > i_2^*,$$

$$\overline{PV}_1(i_1', i_2') > \overline{PV}_1(i_1^*, i_1^*),$$

$$\overline{PV}_2(i_1', i_2') > \overline{PV}_2(i_1^*, i_1^*),$$

and

$$\overline{PV}_3(i_1', i_2') > \overline{PV}_3(i_1^*, i_1^*),$$

[8] If $\widehat{i}_2(i_1)$ is linear, the second derivative of $PV_1\left(i_1, \widehat{i}_2(i_1)\right)$ with respect to i_1 is strictly negative globally and, therefore, a unique solution is guaranteed. However, in general, $\widehat{i}_2(i_1)$ is not linear and in a special case, there can be multiple equilibria. Even though multiple maximum equilibria exist, at the equilibrium the first and second order conditions are well-defined by the mean value theorem.

where

$$\overline{PV}_1(i_1, i_2) = (x_1 - i_1) + \beta_1\delta(x_2 - i_2) + \beta_1\delta^2 f(i_1, i_2),$$

$$\overline{PV}_2(i_1, i_2) = (x_2 - i_2) + \beta_2\delta f(i_1, i_2),$$

and

$$\overline{PV}_3(i_1, i_2) = f(i_1, i_2).$$

Definition 1 states that the firm has the underinvestment problem if there exists another investment plan (i_1', i_2') such that (a) it is strictly higher than the equilibrium investment level (i_1^*, i_2^*) and (b) its associated present values are strictly higher than those of the equilibrium investment decisions. The following proposition shows that based on Definition 1, the firm has an underinvestment problem at the equilibrium:

Proposition 2 *The firm faces an underinvestment problem.*

Proof: The proof of Proposition 2 will be based on the following two lemmas. Lemmas 1 and 2 investigate whether the present value functions $\overline{PV}_1(i_1, i_2)$ and $\overline{PV}_2(i_1, i_2)$ increase or decrease in small variations in (i_1, i_2) at the equilibrium. The present value in period 3, $\overline{PV}_3(i_1, i_2)$, trivially increases in (i_1, i_2).

Lemma 1 *At the equilibrium investment plan (i_1^*, i_2^*), we have*

$$\frac{\partial\overline{PV}_1}{\partial i_1} < 0 \quad \textit{and} \quad \frac{\partial\overline{PV}_1}{\partial i_2} > 0.$$

Proof of Lemma 1: Taking the partial derivative $\overline{PV}_1$ with respect to i_1 at the equilibrium (i_1^*, i_2^*), we have

$$\frac{\partial\overline{PV}_1}{\partial i_1}|_{(i_1,i_2)=(i_1^*,i_2^*)} = -1 + \beta_1\delta^2 f_1. \tag{9}$$

From (7) and (9), we have

$$\frac{\partial\overline{PV}_1}{\partial i_1}|_{(i_1,i_2)=(i_1^*,i_2^*)} = -1 + \beta_1\delta^2 f_1 = \beta_1\delta\widehat{i_2'}(i_1)(1 - \delta f_2). \tag{10}$$

From (3) and (10), we have

$$\frac{\partial\overline{PV}_1}{\partial i_1}|_{(i_1,i_2)=(i_1^*,i_2^*)} = (\beta_2 - 1)\beta_1\delta^2\widehat{i_2'}(i_1)f_2 < 0. \tag{11}$$

Taking the derivative of $\overline{PV}_1$ with respect to i_2 at equilibrium (i_1^*, i_2^*), we have

$$\frac{\partial\overline{PV}_1}{\partial i_2}|_{(i_1,i_2)=(i_1^*,i_2^*)} = -\beta_1\delta + \beta_1\delta^2 f_2 = \beta_1\delta(-1 + \delta f_2). \tag{12}$$

From (3) and (12), we have

$$\frac{\partial \overline{PV}_1}{\partial i_2}|_{(i_1^*,i_2^*)} = \beta_1 \delta \left(-\beta_2 \delta f_2 + \delta f_2\right) = \beta_1 \delta^2 \left(1-\beta_2\right) f_2 > 0. \tag{13}$$

End of Proof of Lemma 1.

Lemma 2 *At the equilibrium investment plan* (i_1^*, i_2^*)*, we have*

$$\frac{\partial \overline{PV}_2}{\partial i_1} > 0 \quad \textit{and} \quad \frac{\partial \overline{PV}_2}{\partial i_2} = 0.$$

Proof of Lemma 2: Taking the partial derivative of $\overline{PV}_2$ with respect to i_2 at equilibrium (i_1^*, i_2^*), we have

$$\frac{\partial \overline{PV}_2}{\partial i_1}|_{(i_1,i_2)=(i_1^*,i_2^*)} = \beta_2 \delta f_1 > 0. \tag{14}$$

The partial derivative of $\overline{PV}_2$ with respect to i_2 is the first order condition (3). Therefore, we have

$$\frac{\partial \overline{PV}_2}{\partial i_2}|_{(i_1,i_2)=(i_1^*,i_2^*)} = 0. \tag{15}$$

End of Proof of Lemma 2.

From Lemmas 1 and 2, in a small open set around the investment equilibrium (i_1^*, i_2^*), there are four different regions, as depicted in Figure 1. Region I is the area where all three present values are higher than those associated with equilibrium investment. Furthermore, in none of the regions is there an overinvestment situation, in which there would exist a lower investment level that leads to Pareto-improving present values in all periods.

End of Proof of Proposition 2.

From Lemma 2, we know that i_2 must be increasing in order to raise $\overline{PV}_2$ at the equilibrium. From Lemma 1, we know that an increase in i_1 decreases $\overline{PV}_1$ but increases $\overline{PV}_2$. Therefore, i_2 must be increasing sufficiently compared to an increase in i_1 for $\overline{PV}_1$ to be increasing. From equation (11) and (13), we have the following:

$$\frac{\partial \overline{PV}_1}{\partial i_1}\Delta i_1 + \frac{\partial \overline{PV}_1}{\partial i_2}\Delta i_2 = di_1 \left(\left(\beta_2 - 1\right)\beta_1 \delta^2 \widehat{i_2'}(i_1) f_2\right) + di_2 \beta_2 \delta^2 \left(1-\beta_1\right) f_2 \tag{16}$$

In order for (16) to be strictly positive, $\Delta i_2 / \Delta i_1$ needs to satisfy the following inequality:

$$\frac{\Delta i_2}{\Delta i_1} > \widehat{i_2'}(i_1) > 0 \tag{17}$$

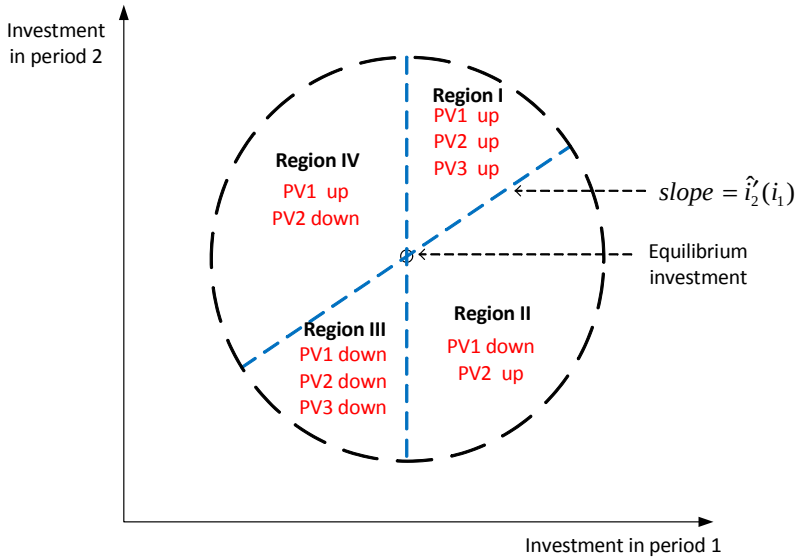

Figure 1: Four regions around the equilibrium investment

Inequality (17) will be used in showing the existence of Pareto-improving tax-policies in the following sections.

As an example in which $f(i_1, i_2) = 20i_1^{1/4}i_2^{1/6}$, $\beta_1 = \beta_2 = 0.6$ and $\delta = 0.9$, the indifference curves of $\overline{PV}_1(i_1, i_2)$ and $\overline{PV}_2(i_1, i_2)$ are plotted in Figure 2.[9] The surrounding region by the two indifference curves is the Pareto-superior region (Region I). The main goal of policies that are introduced in the following two sections is to move the equilibrium investment plan into region I in Figure 2.

4. Comparison to hyperbolic consumption-savings models

This paper applies the β, δ functional form (hyperbolic discounting), which is popularized by Laibson (1997), to firm investment decision problems. Even though one might interpret investment in our model as savings in the Laibson-style model, our model differs from it in two respects. The first is that investments generate return in the last period, while in the Laibson-style model, savings are liquidated in the immediate next period. Secondly, in our model, investments across two periods are supermodular (complementary-oriented), while savings in the Laibson-style model are perfect substitutes. We show that in the consumption-savings model with concave utility functions, savings are mathematically *submodular*.

[9] With a Cobb-Douglas return function, there exists a closed form solution for the equilibrium investment (See section 8). In this example, we have $(i_1^*, i_2^*) = (4.69, 3.22)$.

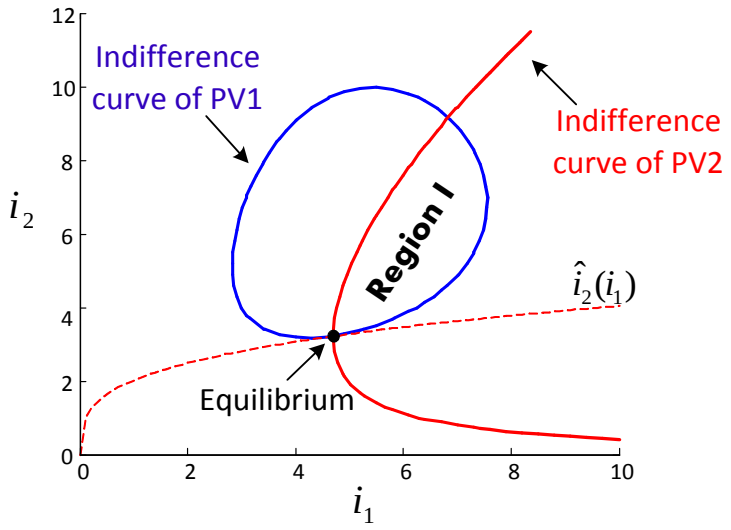

Figure 2: Equilibrium and Pareto-superior region

The supermodularity property across different periods' investments is a natural assumption, as otherwise there would be no reason to invest in both periods. Under submodularity, the firm would decide to invest in only one period, the one with the higher marginal return. Therefore, the assumption that the firm invests in multiple periods for one project, by itself, implies that investments are supermodular. The supermodular property is widely adopted across many areas of economics. One example is in macroeconomic growth theory between productivity (or technology) and capital. Among models in growth theory, productivity and capital typically take on the Cobb-Douglas form (i.e., multiplicatively separable). For example, in the endogenous growth model developed by Romer (1986, 1990), technology can be improved through long-term R&D investment, and capital can be accumulated through short-term investment. In our model, i_1 and i_2 might be interpreted as long-term R&D investment and short-term capital investment, respectively.[10]

Coming back to the consumption-savings model, it is possible to interpret d_1, d_2, and d_3 as consumptions and $i_1(= s_1)$ and $i_2(= s_2)$ as savings that would be liquidated in period 3. We can define $d_3 = Rs_1 + rs_2$, where R is a long-term gross interest rate and r is a short-term gross interest rate. We also interpret x_1 and x_2 as consumer's exogenous income flows. The perfect substitution between i_1 and i_2 does not provide any convexity and therefore there does not exist an interior solution if the utility function is linear. Therefore, in the consumption-saving model we define consumer's utility function as $u(d_t)$, where $u(\cdot)$ is strictly increasing and strictly concave. Then, the utility function in the third period is $u(d_3) = u\left(Rs_1 + rs_2\right)$,

[10]Examples of other papers that have been grounded on supermodularity in firm investment settings include Bloom, Bond, and VanReenen (2007); Dixit (1997); and Eberly and Mieghem (1997).

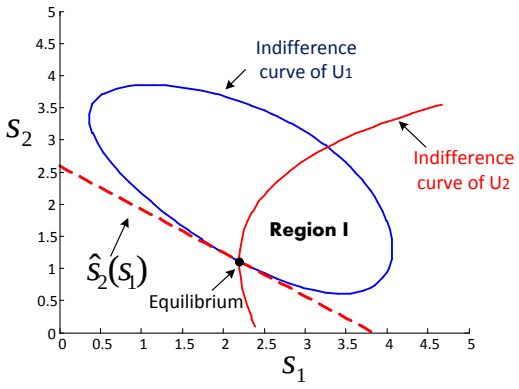

Figure 3: Equilibrium and Pareto-superior region in consumption-savings problems

which represents that the two savings are submodular, i.e., $\partial u(d_3)^2/(\partial s_1 \partial s_2) < 0$. This submodularity induces the savings choice function $\widehat{s}_2(s_1)$ to be decreasing, which actually mitigates the underinvestment problem derived from hyperbolic discounting. The lower value of s_1 increases s_2 by the choice function and therefore the consumer can even have an oversavings problem in terms of s_2. This stands in contrast to our model, in which the firm never has an overinvestment problem (all investment plans in Region I are strictly higher than the equilibrium investment plans.)

To visualize the role of sub and super-modularity, we present a consumption-savings example where the utility function is $u(d) = \ln d$, the hyperbolic discount factor is $\beta_1 = \beta_2 = 0.6$, the regular discount factor is $\delta = 0.9$, exogenous incomes are $x_1 = 14$, $x_2 = 8$, the long-term interest rate is $R = 1$ and the short-term interest rate is $r = 1$. Figure 3 shows that the choice function $\widehat{s}_2(s_1)$ is negatively sloped and consequently, the Pareto-superior region is extended into the region where the period-2 investment is negative. Also, the portion of positive investment area in Region I is smaller than that in our model.

Rather than these consumption-savings results, the firm investment problem in our paper has a strictly underinvestment problem due to the supermodularity property. That is, if another investment plan brings higher firm value in all three periods, the investment levels of that plan must be higher than that of the equilibrium investment plan. The following corollary addresses this issue:

Corollary 1 *If* $\overline{PV}_1(i_1', i_2') > \overline{PV}_1(i_1^*, i_1^*), \overline{PV}_2(i_1', i_2') > \overline{PV}_2(i_1^*, i_1^*)$, *and* $\overline{PV}_3(i_1', i_2') > \overline{PV}_3(i_1^*, i_1^*)$ *where* (i_1^*, i_1^*) *is the equilibrium investment plan, it must be that* $i_1' > i_1^*$ *and* $i_2' > i_2^*$.

Proof. See the proof of Proposition 2. ■

5. Dividend taxation

In the previous section, we have shown that myopic corporate decisions result in an underinvestment problem. Now, we move on to policy implications and examine whether outside authorities' intervention can improve the firm's value. For this normative question, we assume that the authority has no exogenous expenditures so that the tax policy is balanced. The collected amount of dividend taxes would be returned to the firm in the form of lump-sum subsidies. We show that even with a revenue-neutral tax policy, the firm's value can be improved.

We examine the effects of proportional dividend taxes on the firm's dividend/investment decisions and present values. Let there be a proportional dividend tax rate τ_t and a lump-sum transfer s_t in period t. The firm's budget sets are

$$(1+\tau_1)\,d_1 + i_1 = x_1 + s_1,$$

$$(1+\tau_2)\,d_2 + i_2 = x_2 + s_2,$$

and

$$d_3 = f(i_1, i_2).$$

Since the government has no exogenous expenditure to finance, its budget constraints satisfy $s_t = \tau_t d_t^*$, where d_t^* is the equilibrium dividend in period t. The tax policies τ_1 and τ_2 are fully anticipated and affect both period-1 and period-2 decisions.

For the proof of the existence of Pareto-improving policies, we consider infinitesimal changes of two periods' tax policies at $(\tau_1, \tau_2) = 0$ in order to guarantee the existence of an equilibrium. In Proposition 1, we have shown that without tax policies, there exists an equilibrium in which the first and second order conditions are satisfied. The result in Proposition 1 also implies the existence of an equilibrium with $(\tau_1, \tau_2) = 0$. However, for any strictly positive tax policy $(\tau_1, \tau_2) > 0$, the existence of an equilibrium is not guaranteed, and therefore we need to focus on local analysis in which small changes in tax-policies are considered.

Imposing dividend taxes decreases the marginal cost of investment relative to that of dividends. Because the collected tax is returned as a lump-sum subsidy, an increase in taxes has a substitution effect but not an income effect.[11] The substitution effect, in general,

[11] It may seem trivial that an increase in the dividend tax in period t causes a decrease in dividend and increase in investment in the same period. However, our context also accounts for the ability of the tax policy in one period to affect the firm's decision in another period. Because of this intertemporal effect, investment is not necessarily increasing in dividend taxes in the same period. This will be shown for the case of period-2 dividend taxation in this section.

decreases the level of dividend and increase the level of investment. The following lemma shows that an increase in τ_1 increase both i_1^* and i_2^*.

Lemma 3 *At the equilibrium of* $(\tau_1, \tau_2) = 0$*, a (finite) increase in* τ_1 *increases the equilibrium investments in both periods, that is*

$$0 \leq \frac{di_1^*}{d\tau_1} < \infty \text{ and } 0 \leq \frac{di_2^*}{d\tau_1} < \infty. \tag{18}$$

We also have

$$\frac{di_2^*}{d\tau_1} / \frac{di_1^*}{d\tau_1} = \widehat{i}_2'(i_1). \tag{19}$$

Proof. The present value in period 1 is

$$\begin{aligned} PV_1 &= \frac{x_1 - i_1 + s_1}{1+\tau_1} \\ &\quad + \beta_1\delta\left(\frac{x_2 - \widehat{i}_2(i_1) + s_2}{1+\tau_2}\right) + \beta_1\delta^2 f\left(i_1, \widehat{i}_2(i_1)\right). \end{aligned} \tag{20}$$

The first order condition from (20) is

$$-\frac{1}{1+\tau_1} - \beta_1\delta\frac{\widehat{i}_2'(i_1)}{1+\tau_2} + \beta_1\delta^2 f_1 + \beta_1\delta^2 f_2 \widehat{i}_2'(i_1) = 0. \tag{21}$$

Where $(\tau_1, \tau_2) = (0, 0)$, the first-order condition in (21) is equivalent to (7) in the proof of Proposition 1. The second order condition from (21) is

$$\begin{aligned} &-\beta_1\delta\frac{\widehat{i}_2''(i_1)}{1+\tau_2} + \beta_1\delta^2 f_{11} + 2\beta_1\delta^2 f_{12}\widehat{i}_2'(i_1) \\ &+\beta_1\delta^2 f_{22}\left(\widehat{i}_2'(i_1)\right)^2 + \beta_1\delta^2 f_2\widehat{i}_2''(i_1) \\ \leq\ & 0. \end{aligned} \tag{22}$$

Where $(\tau_1, \tau_2) = (0, 0)$, the second-order condition in (22) is equivalent to (8) in the proof of Proposition 1. Implicitly differentiating (21) with respect to τ_1, we have

$$\begin{aligned} &\frac{1}{(1+\tau_1)^2}d\tau_1 - \beta_1\delta\frac{\widehat{i}_2''(i_1)}{1+\tau_2}di_1 \\ &+\beta_1\delta^2 f_{11}di_1 + 2\beta_1\delta^2 f_{12}\widehat{i}_2'(i_1)di_1 \\ &+\beta_1\delta^2 f_{22}\left(\widehat{i}_2'(i_1)\right)^2 di_1 + \beta_1\delta^2 f_2\widehat{i}_2''(i_1)di \\ =\ & 0. \end{aligned} \tag{23}$$

By equation (23) and the second order condition (22), we have

$$0 \leq \frac{di_1^*}{d\tau_1} < \infty. \tag{24}$$

By (24) and that $\widehat{i}_2'(i_1) > 0$, we have

$$0 \leq \frac{di_2^*}{d\tau_1} < \infty, \tag{25}$$

and

$$\frac{di_2^*}{d\tau_1} / \frac{di_1^*}{d\tau_1} = \widehat{i}_2'(i_1). \tag{26}$$

■

Lemma 3 shows that period-1 dividend taxation increases investment levels in both periods, and the ratio of the marginal increases of the two periods' investments is equal to $\widehat{i}_2'(i_1)$. The increasing rate $\widehat{i}_2'(i_1)$ implies that if only period-1 dividend taxation is imposed, the Pareto-superior investment plan cannot be achieved (see inequality (17)). Therefore, we also need period-2 dividend taxation. The substitution effect from higher period-2 dividend taxes can increase the choice function $\widehat{i}_2(i_1)$, but does not directly increase the equilibrium period-2 investment, i_2^*. The change of the choice function $\widehat{i}_2(i_1)$ affects the period-1 investment choice, and the period-1 investment choice will affect the period-2 investment, i_2^*, through the choice function $\widehat{i}_2(i_1)$. Therefore, whether the two periods' investments increase or decrease from period-2 taxation is not a trivial question. Nevertheless, we can derive the possible range of investment changes by period-2 taxation, which is sufficient to show the existence of Pareto-improving tax policies.[12]

Lemma 4 *At the equilibrium of* $(\tau_1, \tau_2) = 0$*, the following inequality is satisfied:*

$$\widehat{i}_2'(i_1)\frac{di_1}{d\tau_2} < \frac{di_2}{d\tau_2}. \tag{27}$$

Proof. The present value in period 2 is

$$PV_2 = \frac{x_2 - i_2 + s_2}{1 + \tau_2} + \beta_2 \delta f(i_1, i_2). \tag{28}$$

[12]We conjecture that depending on the elasticity of substitution between the two periods' investments, the period-1 investment can increase or decrease from period-2 taxation. For higher values of the elasticity of substitution, increases in period-2 taxation might decrease the period-1 investment because the increased period-1 investment (by period-2 taxation) can substitute for period-1 investment. If the elasticity is small, the reverse result would be expected. Further studies on this issue are necessary.

The first order condition from (28) is

$$-\frac{1}{1+\tau_2}+\beta_2\delta f_2(i_1,i_2)=0. \tag{29}$$

Implicitly differentiating (29) with respect to τ_2, we have

$$\frac{d\tau_2}{(1+\tau_2)^2}+\beta_2\delta f_{22}(i_1,i_2)di_2=0,$$

and, equivalently,

$$\frac{d\widehat{i}_2(i_1;\tau_2)}{d\tau_2}d\tau_2=-\frac{1}{(1+\tau_2)^2\beta_2\delta f_{22}}>0. \tag{30}$$

The maximization problem of period-1 present value can be expressed as

$$\max_{i_1,i_2}\ \overline{PV}_1(i_1,i_2),$$

subject to

$$\widehat{i}_2(i_1)=i_2. \tag{31}$$

Taking a total derivative of equation (31) with respect to τ_2, we have

$$\frac{d\widehat{i}_2(i_1)}{d\tau_2}+\widehat{i}_2'(i_1)\frac{di_1}{d\tau_2}=\frac{di_2}{d\tau_2}. \tag{32}$$

Because $\frac{d\widehat{i}_2(i_1)}{d\tau_2}>0$ from (30), we have

$$\widehat{i}_2'(i_1)\frac{di_1}{d\tau_2}<\frac{di_2}{d\tau_2}.$$

■

Lemma 4 indicates that period-2 taxation induces the equilibrium investment to move above the $\widehat{i}_2(i_1)$ curve (i.e., $\widehat{i}_2'(i_1)\frac{di_1}{d\tau_2}<\frac{di_2}{d\tau_2}$). Inequality (27) does not imply whether period-1 and 2 investments increase or decrease. From Lemmas 3 and 4, the existence of Pareto-improving dividends taxation policies is shown in the following proposition:

Proposition 3 *There exists positive Pareto-improving proportional dividend taxes* $(\tau_1,\tau_2)\gg 0$.

Proof: In the proof, we consider small changes in dividend taxes at $(\tau_1,\tau_2)=0$. Because there exists an equilibrium at $(\tau_1,\tau_2)=0$, there is also an open set $T\subset\mathbb{R}^2$ such that T includes $(0,0)$ and that an equilibrium exists for any $(\tau_1,\tau_2)\in T$. Therefore, there

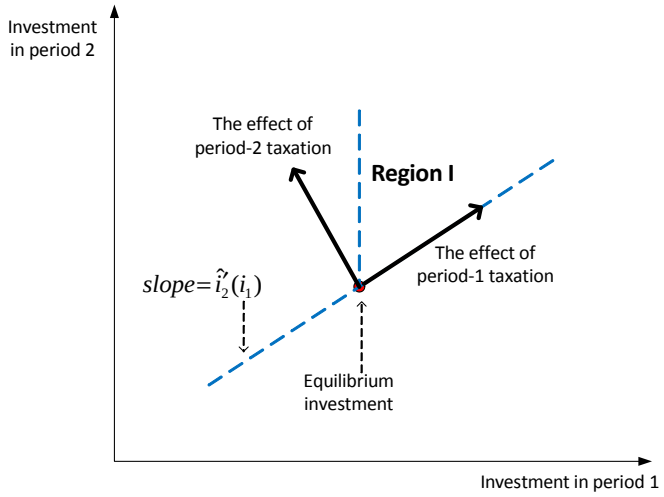

Figure 4: Pareto-improving tax policies

still exists a unique equilibrium with small variations in (τ_1, τ_2). Lemma 3 indicates that period-1 taxation induces both periods' investment to move along the $\widehat{i}_2'(i_1)$-line in Figure 4 (see (19) in Lemma 3). Inequality (27) in Lemma 4 implies that period-2 taxation induces the investments in both periods to move above the $\widehat{i}_2'(i_1)$-line in Figure 4. Therefore, by combining dividend taxations in both periods, the equilibrium investment can move into the Pareto-superior region (region I). Mathematically, this means that at the equilibrium $(\tau_1, \tau_2) = (0, 0)$, there exists a positive constant a such that

$$\widehat{i}_2'(i_1^*) < \frac{\frac{di_2^*}{d\tau_1} + a\frac{di_2^*}{d\tau_2}}{\frac{di_1^*}{d\tau_1} + a\frac{di_1^*}{d\tau_2}} < \infty.$$

The end of Proof of Proposition 3.

Figure 5 also describes how dividend taxation policies can Pareto-improve the firm's values. As an example in which $f(i_1, i_2) = 20i_1^{1/4}i_2^{1/6}, \beta_1 = \beta_2 = 0.6$ and $\delta = 0.9$, the equilibrium investment with $(\tau_1, \tau_2) = (10\%, 40\%)$ is indicated in Figure 5. An increase in period-1 tax can move the equilibrium point along the $\widehat{i}_2(i_1)$ curve. Without period-2 taxation, the period-1 taxation cannot improve the period-1 present value (the period-1 present value becomes even lower along the $\widehat{i}_2(i_1)$ curve). Together with period 1 and 2's tax policies, the equilibrium can move into the Pareto-superior region.

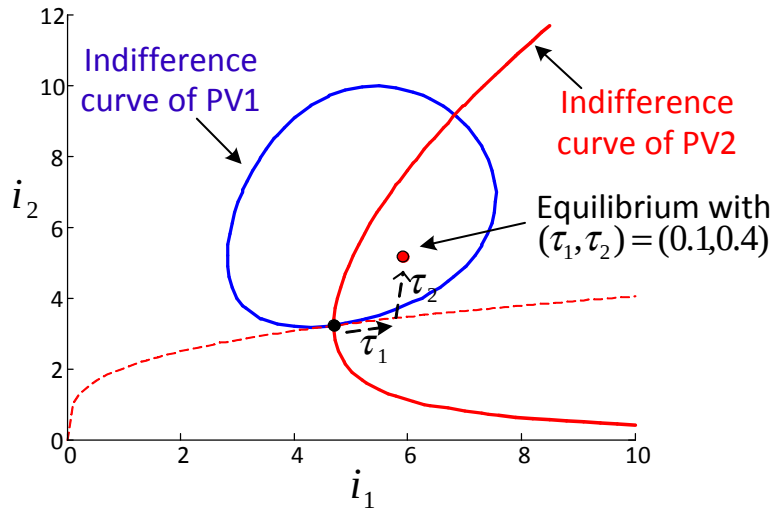

Figure 5: Pareto-improving dividend taxation

6. Investment subsidy

In this section, we show that investment subsidies can also improve the firm's value in all periods. As in the previous section, we assume that the outside authority adopts revenue-neutral policies. Therefore, lump-sum taxes in the same amount of the investment subsidies will be imposed in the same period. The firm's budget constraints under investment subsidies are

$$d_1 + (1 - \theta_1)\, i_1 = x_1 - \eta_1,$$

$$d_2 + (1 - \theta_2)\, i_2 = x_2 - \eta_2,$$

and

$$d_3 = f(i_1, i_2),$$

where θ_t and η_t are the proportional subsidy rate and the lump sum tax in period t, respectively. Since the outside authority has no exogenous expenditure to finance, its budget constraints satisfy $\eta_t = \theta_t i_t^*$ for $t = 1, 2$, where i_t^* is the equilibrium investment level in period t.

Following the same logic as the dividend-taxation case in Section 5, an increase in (θ_1, θ_2) decreases the cost of investment relative to the cost of dividends. By the substitution effect, an increase in (θ_1, θ_2) induces higher equilibrium investment, and therefore, higher present value.

Proposition 4 *There exists positive Pareto-improving proportional investment subsidies*

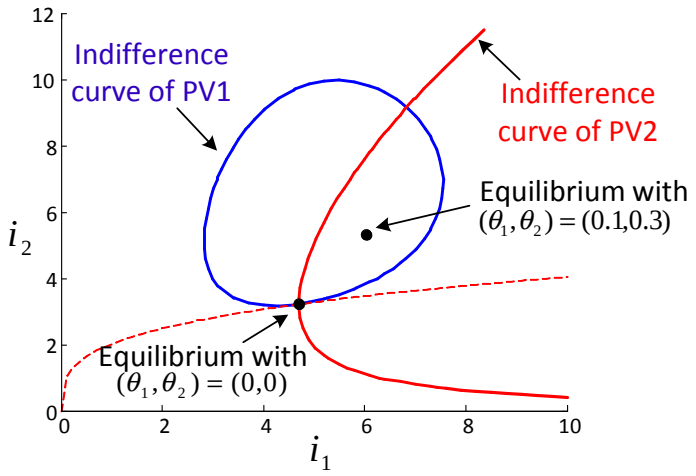

Figure 6: Pareto-improving investment subsidy

$(\theta_1, \theta_2) \gg 0$.

Proof. We do not state the detailed proof of Proposition 4, because the same logic as the proof of Proposition 3 applies. The increase in investment subsidies raises the cost of dividend payout and decreases the cost of investment, which is mathematically equivalent to the case of an increase in dividend taxation. ■

Figure 6 describes how investment subsidy policies can Pareto-improve the firm's values. As an example in which $f(i_1, i_2) = 20 i_1^{1/4} i_2^{1/6}$, $\beta_1 = \beta_2 = 0.6$ and $\delta = 0.9$, the equilibrium investment with $(\theta_1, \theta_2) = (10\%, 30\%)$ is indicated in Figure 6.

7. Agency problems and hyperbolic discounting

One of the most popular approaches in explaining managerial myopia is agency problems resulting from information asymmetry, under which investors and shareholders have incomplete information about the manager's internal decisions. Based on Stein (1988, 1989)'s work, under efficient and rational stock markets, investors naturally infer future stock prices based on previous dividend payouts. The manager's preferences are assumed to be dependent on both the firm's current stock price as well as on its long-term value. This provides the manager an incentive to boost current stock prices in order to increase her utility. Grounded on Stein's theory, Asker, Farre-Mensa, and Ljungqvist (2015) empirically compare investment levels of publicly-listed firms with that of private firms. Holding firm size, industry characteristics, and investment opportunities constant, they show that on average, public firms invest 45% less than private firms over the period 2001–2011.

This section incorporates Stein's (1989) agency-problem model to the multi-period investment model and shows that even when the investors and managers exhibit the typical exponential discounted time preferences, asymmetric information would lead to an investment plan that is the same as that under hyperbolic discounted preferences. The main premise of Stein's theory is that investors are not able to observe the manager's actions and earnings directly. Since investors have incomplete information on cash flow, they infer future cash flows (future earning) based on current dividend payouts. To induce higher investors' expectation of future earnings, the manager has an incentive to increase dividend payouts by decreasing investment. When investors have higher expectations about future earnings, current stock price rises and subsequently the manager's utility increases.

To incorporate Stein's agency-problem framework to our paper, we need to define cash flows as a random variable. Specifically, we assume that the cash flow x_t (the earning from the previous project) is incomplete information to the manager and the market. For $t \geq 2$, we have

$$x_t = z_t + \varepsilon_t, \tag{33}$$

where z_t and ε_t represent permanent and transitory components of earnings, respectively. The ε_t's are independent across periods with mean zero and variance $\sigma_\varepsilon^2 \, (= 1/h_\varepsilon)$. z_t follows a random walk: $z_t = z_{t-1} + u_t$, where u_t are a sequence of independent mean zero normal variates with variance $\sigma_u^2 \, (= 1/h_u)$. The manager and the market share prior beliefs about z_1. That prior is normally distributed with mean m_1 and variance $\delta_1^2 (= 1/h_1)$.

The assumptions about dividend payouts in each period are the following: $d_1 = x_1 - i_1$, and $d_2 = x_2 - i_2$, and $d_3 = x_3 + R(i_1, i_2)$. Assuming that investors are risk neutral, we can define their wealth based on exponential discounting time preferences as $V_1 = d_1 + \delta E_1 \left[d_2 + \delta d_3 \right], V_2 = d_2 + \delta E_2 \left[d_3 \right]$, and $V_3 = d_3$, where E_1 and E_2 represent the expectations of future earnings in periods 1 and 2, respectively.

The market price of the firm's stock is the investor's expected valuation. We define the stock price in period t as the discounted sum of all dividend payouts since period $t+1$. Therefore, we have $P_1 = E_1^I \left[\delta d_2 + \delta^2 d_3 \right]$ and $P_2 = E_2^I \left[\delta d_3 \right]$, where E_1^I and E_2^I represent the investors' expectations in periods 1 and 2, respectively.

We also define the manager's preferences in the same way as in Stein (1989).

$$V_1^M = E_1 \left[d_1 + \pi P_1 + (1 - \pi) \left(\delta d_2 + \delta^2 d_3 \right) \right] \tag{34}$$

$$V_2^M = E_2 \left[d_2 + \pi P_2 + (1 - \pi) \delta d_3 \right] \tag{35}$$

$$V_3^M = d_3 \tag{36}$$

where P_t is the estimated stock price by investors at period t and $(1-\pi)$ represents the fraction of the manager's stock ownership at market value. Stein (1989, p.659) proposes several interpretations of the positive value of π. Even though managers want to hold the stock for the longer them, they face a probability π of takeover in each period (see also Stein 1988). Another possibility is that funding requirements might force the manager to go to the stock market and issue new stocks. Where the exogenous cash flow is deterministic, the dividend payouts are perfectly estimated and therefore the managers' preferences become the same as those of the investors following exponential discounting. In other words, without information uncertainty, there is no difference across managers' preferences, stock prices, and investors' preferences.

Even though market investors do not observe the manager's actions, they are able to infer them through the following observations,

$$o_1 \equiv x_1 = d_1 + i_1^* \quad \text{and} \quad o_2 \equiv x_2 = d_1 + i_2^*, \tag{37}$$

where i_1^* and i_2^* are the equilibrium investment decisions in periods 1 and 2, respectively. As will be shown in Proposition 5, i_1^* and i_2^*, are not affected by these observations. This is because the expectation of future cash flows by the Bayesian learning process is linearly related to current and past cash flows (see equations (38) and (39)). This further implies that in the first order conditions of the manager's investment decisions, the observations $\{o_1, o_2\}$ would be cancelled out. Therefore, both the manager and the stock market know the Nash equilibrium investment plan (i_1^*, i_2^*) such that the manager cannot fool the market. Nevertheless, the manager and investors are trapped into behaving myopically because the manager's decision on increasing investment beyond the Nash equilibrium is recognized as a decrease in cash flow by investors. Stein (1989) described this situation as analogous to the prisoner's dilemma in the sense that the efficient equilibrium is not sustained as a Nash equilibrium.[13]

Through the observation of $\{o_1, o_2\}$, the market learns about earnings, z_t. Then, the posterior distribution of z_t will remain normal and we have the following expectations:

$$E_1[x_2|o_1] = E_1[x_3|o_1] = E_1[z_2|o_1] = (1-\mu_1)\, m_1 + \mu_1 o_1, \tag{38}$$

and

$$E_2[x_3|o_1, o_2] = E_1[z_3|o_1, o_2] = (1-\mu_2) m_2 + \mu_2 o_2, \tag{39}$$

[13]The main assumption in Fudenberg and Tirole (1986), Stein (1988, 1989), and Holmström (1999) is that the manager's internal decisions, such as investment, are not directly observable by the investors or shareholders. Therefore, the stock market uses past and current dividend payouts to make a rational forecast of future earnings.

where

$$m_2 = (1-\mu_1)\, m_1 + \mu_1 o_1,$$

$$\mu_1 = \frac{h_\varepsilon}{h_1 + h_\varepsilon} < 1 \quad \text{and} \quad \mu_2 = \frac{(h_\varepsilon/h_u) + \mu_1}{1 + (h_\varepsilon/h_u) + \mu_1} < 1. \tag{40}$$

Equations (38-40) indicate that if the variance of the transitory noise is low (i.e., the precision of h_ε is high), expectation of future earnings would be sensitive to the observations. In this case, the stock price is more dependent on past dividend payouts such that the manager will behave more myopically.

We now show that under Stein's setting with asymmetric information, the corresponding manager's investment decisions are equivalent to that under hyperbolic discounted firm value, expressed in the following proposition:

Proposition 5 *The reduced form of the manager's maximization problem under information asymmetry leads to equivalent investment plans as that under hyperbolic discounting time preferences.*

Proof. In period 2, the manager maximizes the following problem

$$\max_{i_2|i_1} (x_2 - i_2) + \pi P_2 + (1-\pi)\,\delta\,(x_3 + R(i_1, i_2))\,. \tag{41}$$

From the maximization problem of (41), we have the choice function $\widehat{i}_2(i_1)$. The choice function is known to both the manager and the stock market (investors). In the maximization problem, the choice function is not affected by previous and future cash flows.

The first-order condition of the maximization problem in (41) is

$$-1 + \pi \frac{dP_2}{di_2} + (1-\pi)\,\delta R_2(i_1, i_2) = 0. \tag{42}$$

where we have

$$\frac{dP_2}{di_2} = -\delta\mu_2 + \delta R_2(i_1, i_2), \tag{43}$$

because an increase in investment results in a decrease in dividend, which consequently results in a decrease in observation o_2 (see equation (37).

From the first order condition (42), we have

$$\frac{\delta}{1 + \pi\delta\mu_2} R(i_1, i_2) = 1. \tag{44}$$

Defining

$$\beta_2 = \frac{1}{1 + \pi\delta\mu_2} < 1, \tag{45}$$

the maximization problem in (41) and the first-order condition (42) are equivalent to those under hyperbolic discounted time preferences.

In period 1, given the choice function $\widehat{i}_2(i_1)$, the manager has the following maximization problem

$$\max_{i_1} \left[\begin{array}{c} (x_1 - i_1) + \pi P_1 + (1-\pi)\,\delta \left(x_2 - \widehat{i}_2(i_1) \right) \\ + (1-\pi)\,\delta^2 \left(x_3 + R(i_1, \widehat{i}_2(i_1)) \right) \end{array} \right] \tag{46}$$

Because we have

$$\frac{dP_1}{di_1} = -\delta\mu_1 - \delta^2\mu_1 + \delta^2 R_1 + \delta^2 R_2 \widehat{i}_2'(i_1), \tag{47}$$

the first-order condition of (46) is

$$-1 - \pi\left(\delta + \delta^2\right)\mu_1 - \delta\widehat{i}_2'(i_1) + \delta^2 R_1 + \delta^2 R_2 \widehat{i}_2'(i_1) = 0.$$

Defining β_2 as

$$\beta_2 = \frac{1}{1 + \pi\left(\delta + \delta^2\right)\mu_1} < 1,$$

we have the following first-order condition,

$$-1 - \beta_2\delta\widehat{i}_2'(i_1) + \beta_2\delta^2 f_1 + \beta_2\delta^2 f_2 \widehat{i}_2'(i_1) = 0, \tag{48}$$

which is equivalent to that of hyperbolic discounting time preferences. ■

Proposition 5 indicates that in the presence of the agency problem with asymmetric information, the manager's investment decision can be derived from hyperbolic discounting time preferences. Specifically, defining the hyperbolic discount factors $\beta_1 = \left(1 + \pi\delta\left(1+\delta\right)\mu_1\right)^{-1}$ and $\beta_2 = \left(1 + \pi\delta\mu_2\right)^{-1}$, the corresponding hyperbolic discounting preferences are also able to explain the manager's myopic behavior under the agency problem.[14]

Under agency problems, the firm's value must be evaluated by exponential discounting, rather than the manager's hyperbolic reduced-form utilities. Therefore, a natural question that arises is whether investment decisions based on hyperbolic discounting also implies

[14] In this finite period model, the hyperbolic discounting factors β_1 and β_2 are not identical. However, in the steady-state of an infinite-period model described in Stein (1989) and Holmström (1999), the derived hyperbolic discounting factors could be identical across all periods. Specifically, in the infinite model, the steady state β^* is given by

$$\beta^* = \left(1 + \frac{\pi\delta\mu^*}{1-\delta}\right) \text{ where } \mu^* = \frac{1}{2}\left(\sqrt{h_\varepsilon^2/h_u^2 + 4h_\varepsilon/h_u} - h_u/h_u\right).$$

See equation (19) in Holmström (1999) for deriving μ^*.

underinvestment in term of exponential discounting preferences.[15] The following lemma shows that the equilibrium investment plan derived from hyperbolic preferences also results in underinvestment problems based on the original exponential preferences.

Lemma 5 *Assume that there are two dividend payout plans,* (d_1^*, d_2^*, d_2^*) *and* (d_1', d_2', d_2')*. If the hyperbolic discounted present values in all three periods based on* (d_1', d_2', d_2') *is strictly higher than those based on* (d_1^*, d_2^*, d_2^*)*, then the exponential discounted present values under the former plan is strictly higher than those under the latter plan, that is,*

$$d_1' + \delta d_2' + \delta^2 d_3' > d_1^* + \delta d_2^* + \delta^2 d_3^*, \tag{49}$$

and

$$d_2' + \delta d_3' > d_2^* + \delta d_3^*, \tag{50}$$

Proof. We have

$$d_1' + \beta_1 \delta d_2' + \beta_1 \delta^2 d_3' > d_1^* + \beta_1 \delta d_2^* + \beta_1 \delta^2 d_3^*, \tag{51}$$

$$d_2' + \beta_2 \delta d_3' > d_2^* + \beta_2 \delta d_3^*, \tag{52}$$

and

$$d_3' > d_3^*. \tag{53}$$

Inequality (52) can be expressed as

$$d_2' + \delta d_3' - (1 - \beta_2)\, \delta d_3' > d_2^* + \delta d_3^* - (1 - \beta_2)\, \delta d_3^*. \tag{54}$$

Multiplying $(1 - \beta_2)$ in inequality (53) and adding it to inequality (54), we have

$$d_2' + \delta d_3' > d_2^* + \delta d_3^*. \tag{55}$$

Inequality (51) can be expressed as

$$\begin{aligned} & d_1' + \delta d_2' + \delta^2 d_3' - (1 - \beta_1)\, \delta \left(d_2' + \delta d_3'\right) \\ > \; & d_1^* + \delta d_2^* + \delta^2 d_3^* - (1 - \beta_1)\, \delta \left(d_2^* + \delta d_3^*\right). \end{aligned} \tag{56}$$

[15] O'Donoghue and Rabin (1999) have argued that policy effectiveness should be evaluated with unbiased discounted values. They proposed a long-run value function from a prior perspective, in which the agent weighs all future periods based on unbiased exponential discounting. This long-term perspective criterion is widely used in the literature for analyzing policy implications. For policy evaluations based on the long-run criterion, see O'Donoghue and Rabin (1999, 2003, 2006), Krusell and Smith (2002), Diamond and Koszegi (2003) and Guo and Krause (2015). In the corporate finance context, unbiased present value has been interpreted as shareholders' present value, whereas biased present value refers to that of corporate managers.

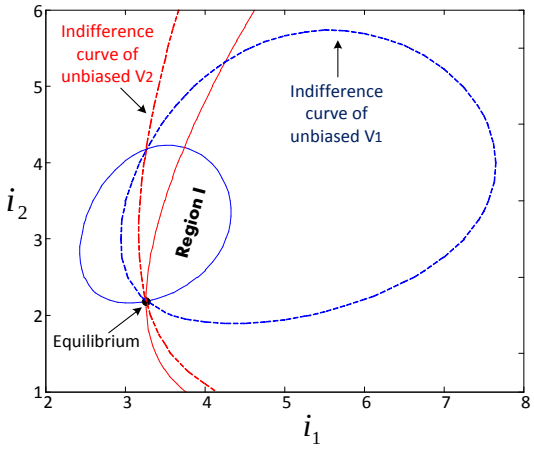

Figure 7: Pareto-superior regions with biased and unbiased present value

Multiplying $(1-\beta_1)\,\delta$ in inequality (55) and adding it to inequality (56), we have

$$d_1' + \delta d_2' + \delta^2 d_3' > d_1^* + \delta d_2^* + \delta^2 d_3^*.$$

■

Lemma 5 implies that if there is an underinvestment problem based on the welfare function of hyperbolic preferences, there would also be an underinvestment problem based on the original exponential preferences.[16] Specifically, Lemma 5 indicates that at any investment plan, the Pareto-superior region (Region I) defined for the hyperbolic firm value is smaller than and strictly included by the Pareto-superior region defined for the exponential (i.e. unbiased) firm value. Figure 7 shows this in an example of $f(i_1, i_2) = 12 i_1^{1/4} i_2^{1/6}$, $\beta_1 = \beta_2 = 0.75$ and $\delta = 0.95$. The dashed curves in Figure 7 represent the indifference curves of unbiased (i.e., $\beta = 1$) present values in periods 1 and 2.Therefore, with agency problems, the firm suffers from two underinvestment problems: one is the internal decisions conflict due to hyperbolic discounting time preferences (i.e., reduced form of manager's preferences) and the other is from the preference difference between hyperbolic and exponential.

Propositions 3 and 4 show that outside authority's policies can result in Pareto-improvement of hyperbolic discounted values in all periods. Lemma 5 shows that the Pareto-superior region based on exponentially discounted values includes the region based on hyperbolic discounted values. Therefore, we can conclude that if a policy improves biased present values in all three periods, it also improves the exponential present values. This is shown in the following corollary:

[16] However, the reverse is not true. See Kang (2015).

Corollary 2 *There exist positive dividend taxes* (τ_1, τ_2) *that improve the firm's unbiased values. There exist positive investment proportional subsidies* (θ_1, θ_2) *that improve the firm's unbiased values.*

Proof. Directly from Propositions 3 and 4, and Lemma 5. ■

8. Multi-period case with Cobb-Douglas return function

We have shown that a firm with hyperbolic preferences faces the underinvestment problem in a three-period model. In this section, we introduce a multi-period model with Cobb-Douglas return function under quasi-hyperbolic discounted present values.[17,18] The main purpose of this extension is to create a more realistic model with multiple investment decisions and facilitate applications of our theoretical model with empirical data. We will show that in this multi-period Cobb-Douglas setting, the firm also faces an underinvestment problem if $\beta < 1$ (i.e. present bias). In Section 3, we have shown that the three-period model can be reduced into a two-period maximization problem by plugging a choice function into the original three-period model. In the same way, a four-period model can be reduced into a three-period model with a choice function of the last-period investment, and so on. In this section, we show that a multi-period model with Cobb-Douglas return function can be solved in a recursive way, in which we consecutively reduce the T period model into $T-1$, $T-2$, and down to a 3-period model.

This recursive approach is feasible with a Cobb-Douglas return function because the derived return function in a reduced model is also a Cobb-Douglas function: any T-period model (where $T \geq 3$) can be reduced into a 3-period model with another Cobb-Douglas return function. This "preservation" property of the Cobb-Douglas return function is not satisfied under other return functions, such as non-Cobb-Douglas CES functions. Using the main result of this section, we present examples showing how investment levels are changing over time for different values of β. Finally, we will show that with Cobb-Douglas return functions,

[17] In a three-period model, there is no mathematical distinction between hyperbolic discounting and quasi-hyperbolic discounting. However, over more than three periods, these two discounting concepts are different. Psychologists first proposed hyperbolic discounting, but economic theorists more frequently use quasi-hyperbolic discounting time preferences, mainly due to computational convenience.

[18] The quasi-hyperbolic discounting funtions are applied in various economic models, such as the contract design model of Dellavigna and Malmendier (2004), the repeated games model of Chade, Prokopovych and Smith (2008), the mechansim design problems of Gilpatric (2008) and principal-agent problems with moral hazard by Yilmaz (2013).

present bias combined with supermodularity decrease late-period investments more than early-period investments.

Consider the Cobb-Douglas return function in a T-period model. The return function is given by

$$f(i_1, i_2...., i_{T-1}) = z \prod_{s=1}^{a_s} i_s^{a_s}, \tag{57}$$

where

$$z > 0,\ a_j > 0 \text{ for all } j \in \{1, ..., t-1\}, \text{ and } \sum_{j=1}^{T-1} a_j < 1.$$

With quasi-hyperbolic discounting, proposed by Laibson (1997), the present value of period t is defined as

$$\begin{aligned} PV_t &= (x_t - i_t) + \beta \sum_{s=1}^{T-1-t} \delta^s (x_{t+s} - i_{t+s}) + \beta\delta^{T-t} f(i_1, i_2...., i_{T-1}) \\ \text{where } t &\in \{1, 2, ..., T-1\} \end{aligned}$$

and

$$PV_t = f(i_1, i_2...., i_{T-1}) \quad \text{where } t = T,$$

where x_t is an exogenous cash flow in period t. We assume that the cash flow in each period is large enough to avoid negative dividends. If $\beta = 1$, then (β, δ) present values are simply exponential discounting. However, $\beta < 1$ implies present-biased present values. Thus, the firm gives more relative weight to the period-t dividend in period t than it did in any period prior to t. In the multi-period model, the hyperbolic discounting factor β can also be interpreted as irrational myopia or reduced-form implication of corporate agency issues. We assume that β is constant across time in this section, a setting that seems more applicable to when the hyperbolic discounting factor is a result of irrational myopic preferences. However, as mentioned in section 7, even with asymmetric information, an infinite-period model results in the steady-state constant β^*.

With this multi-period Cobb-Douglas return function, there exists an equilibrium and the equilibrium possesses an underinvestment problem. The following Proposition addresses this issue:

Proposition 6 *For any finite period $T \geq 3$, there exists a unique equilibrium under a Cobb-Douglas return function. At the equilibrium, the firm faces an underinvestment problem.*

Proof. See Appendix A. ■

Proposition 6 shows that for any Cobb-Douglas return function and for any finite number of periods, there exists an equilibrium investment plan and the equilibrium decisions possess

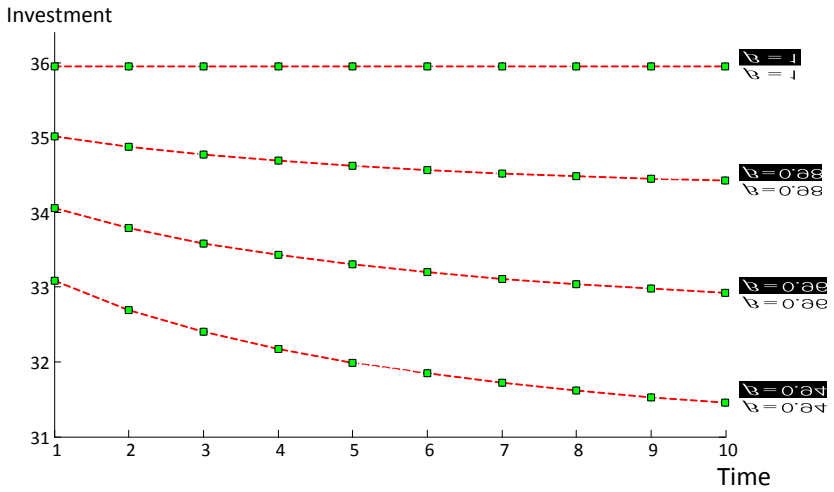

Figure 8: Investment over time

an underinvestment problem if $\beta < 1$. The equilibrium investment plan can be analytically and recursively solved from equations (71–72) in the proof of Proposition 6. In Figure 8, we show the investment plans across different values of β in a model with 11 periods. The example is based on the return function, $f(i_1, ..., i_{10}) = 100 \prod_{s=1}^{10} i_s^{a_s}$ where $a_1 = 0.1$ and $a_{t-1} = a_t \delta$ for $t = \{2, ..., 10\}$. The discount rate is $\delta = 0.9$.

Figure 8 clearly shows that investment is decreasing (constant) in time if $\beta < 1$ ($\beta = 1$). In general, the existence of present bias decreases all periods' investment levels. However, due to the supermodularity property of the return function (i.e. marginal product of one period's investment is positively related to the levels of other periods' investments), present bias uneven impacts investments across different time periods. The low investment levels from the earlier periods will decrease the marginal return of later period investments, and thus the firm in the later-period will have an incentive to decrease investment further. The combination of present bias and supermodularity causes the later-period investment to be even lower compared to earlier-period investments. On the other hand, the supermodularity property affects the earlier-period investments differently. The firm in the earlier period knows that the low investment in the current period will decease future investments and also knows that low future investment will decrease the marginal product of current investment. Therefore, the supermodularity property provides the early-period firm an incentive to decrease investment less intensively than in the later period. The following proposition shows that present bias disproportionately affects the later-period investment as compared to the earlier-period investment.

Proposition 7 *For any finite period $T \geq 3$ with a Cobb-Douglas return function satisfying $a_t = a_{t-1}\delta$ for all $t \in \{2, ..., T\}$, investment levels are strictly decreasing (constant) over time if $\beta < 1$ ($\beta = 1$).*

Proof. See Appendix B. ■

In Proposition 7, in order to show the investment decisions across time, we need a benchmark case. We consider a special case of $\beta = 1$, where investment is constant across time in the model. The condition for the constant investment stream is $\delta \left(\frac{\partial f}{\partial i_{t-1}}\right) / \frac{\partial f}{\partial i_t} = 1$, which is the condition $a_t = a_{t-1}\delta$ with a Cobb-Douglas return function. Then, where $\beta = 1$, the ratio of marginal product to marginal cost is identical for all periods, and consequently equilibrium investment levels are identical across time. If $\beta < 1$, investment is decreasing over time, which is shown in Proposition 7.

However, the key equation in the proof of Proposition 7, equation (81), not only applies to the Cobb-Douglas return function, but also any return function in a three-period model. Equation (81) in the three-period model with general production functions is

$$\delta \left(\frac{\partial f}{\partial i_1}\right) / \frac{\partial f}{\partial i_2} = 1 - \delta \widehat{i}_2'(i_1)\,(1 - \beta)\,. \tag{58}$$

In equation (58), we know that the two properties of $\widehat{i}_2'(i_1) > 0$ (due to supermodularity) and $\beta < 1$ (due to present bias) induce the marginal product of period-2 investment to be relatively higher than that of period-1 investment. The higher marginal product in period-2 equilibrium investment implies disproportionately lower level of period-2 equilibrium investment due to diminishing marginal product of the return function. Therefore, we can conclude that supermodularity in general amplifies present-bias-induced underinvestment problems for later-stage investment decisions.

9. Quantitative implications

This section derives quantitative implications from our model by attempting to assess the impact of short-termism on the magnitude of underinvestment, as well as how tax policies can address this issue. We calibrate the model with the Cobb-Douglas return function.[19] As

[19]Not only is the Cobb-Douglas return function commonly used in economics research, but it also has a number of other important properties. First, it satisfies the limiting conditions, which implies that zero investment in any given period would cause the project to fail, i.e., yield zero return. Secondly, the choice of the Cobb-Douglas return function makes the model calibration more tractable, as its parameters has a clear connection to the several major empirical data series (see equation (63)). The choice of Cobb-Douglas function implies the value of 1 for the elasticity of substitution. While 1 is a reasonable assumption for the

in Section 8, we assume that investment levels are constant across periods when corporate decisions are not myopic ($\beta = 1$). Table 1 summarizes how the parameters can be calibrated from empirical data:

Parameter	**Interpretation/Derivation**
$T-1$	= Investment period
$1/\delta$	= Annual gross real interest rate
z	= Productivity
$\left(\sum_{j=1}^{T-1} a_j\right)^{-1}$	$= \frac{\text{Present value of return}}{\text{Present value of investment}}$ = ROIC+1
a_t	$= \frac{1}{ROIC} \frac{1-\delta}{1-\delta^{T-1}} \delta^{t-1}$

Table 1. Parameters and Calibration

The values of T and δ can be directly derived from data on project investment duration and annual real interest rate. The percentage change in investment and net present value resulting from lower β (i.e. greater present bias) are not affected by the productivity of the Cobb-Douglas return function, denoted as z in (57). Thus, calibrating z is not necessary.

The value for $\sum_{j=1}^{T-1} a_j$ can be derived from the return on invested capital (ROIC), which is shown in the following formula. Equation (59) shows the relationship among ROIC, the present value of investment and the present value of project return in the benchmark case of $\beta = 1$:

$$\begin{aligned} ROIC + 1 &= \frac{\text{Present value of total revenue}}{\text{Present value of total investment}} \\ &= \frac{\delta^{T-1} f\left(i_1, ..., i_{T-1}\right)}{i_1 + \delta i_2 + ... + \delta^{T-2} i_{T-1}}. \end{aligned} \tag{59}$$

Given the assumption of constant investment over time, $i = i_1 = i_2 = ... = i_{T-1}$, equation (59) can be expressed as

$$ROIC + 1 = \frac{\delta^{T-1}}{1 + \delta + ... + \delta^{T-2}} z i^{a_1 + a_2 + ... + a_{T-1} - 1}. \tag{60}$$

From the first-order conditions, we have

$$-\left(1 + \delta + ... + \delta^{T-2}\right) + \left(a_1 + a_2 + ... + a_{T-1}\right) \delta^{T-1} z i^{a_1 + a_2 + ... + a_{T-1} - 1} = 0 \tag{61}$$

elasticity of substitution based on the existing literature, further studies are needed in order to more fully understand the elasticity of substitution for investment choices over time.

From (60) and (61), we have

$$ROIC + 1 = \left(\sum_{j=1}^{T-1} a_j\right)^{-1} \tag{62}$$

Since we have $a_t = a_{t-1}\delta$ by the assumption of constant investment over time (see Proposition 7) from (62), we can derive a_t :

$$a_t = \frac{1}{ROIC}\frac{1-\delta}{1-\delta^{T-2}}\delta^{t-1} \tag{63}$$

From (63), we know that parameters $(a_1, ..., a_{T-1})$ relate to data on the real interest rate, the duration of investment, and the ROIC ratio. In the U.S., the annual real interest rate from 2006 to 2015 ranged from 1.2 to 4 percent. We choose 2 percent as the parameter value for the real interest rate. Based on Porter (2008), the average annual return on invested capital in the U.S. from 1992 to 2006 is 14.9 percent after corporation taxes, and, for the industries engaged in long-term investments such as semiconductors and medical instruments, the value for ROIC is approximately 21%. We choose the parameter value for ROIC to be 21 percent. For investment horizons, we assume $T = 6$, which implies that the investment gestation period before return realization is 5 years.[20]

The following table summarizes the empirical parameter choices for the calibration:

	Data Choice
Investment period	5 years
Return on investment (ROIC)	21%
Annual real interest rate	2%

Table 2. Empirical parameter choices

For the values of $\beta = 1, 0.9, 0.8, 0.7$, the figure in Figure 9 plots the firm investment decisions over the period of 5 years. We set investment where $\beta = 1$ to be 100% in the graph. The graph shows that corporate investment decisions are sensitive to the degree of short-termism. For $\beta = 0.8$ (i.e. exhibiting present bias of 20 percent relative to no present bias), there is an approximately 45 percent reduction in investment. This would be in line with the empirical estimates of Asker, Farre-Mensa and Ljungqvist (2015), who conclude that short-termism, as measured by the agency gap between public and private firms, contributed to investment reduction of up to 45 percent during the sample period

[20]Even though the time horizons of specific projects vary across industries, in practice, a five-year horizon is commonly adopted in corporate project evaluations (Jacobs and Shivdasani 2012)

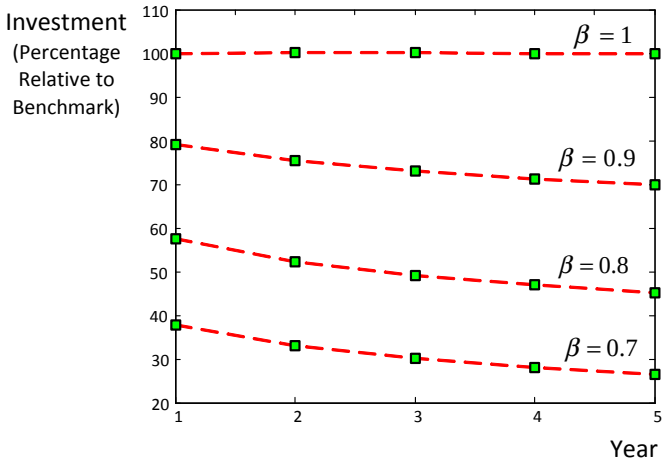

Figure 9: Short-termism and investment

2001-2011. Experimental and field data show the individual's myopic parameter β is around 0.7 (see Angeletos et al. 2001 and Laibson et al. 2007).[21] Even considering that the degree of corporate myopia may differ from the individual's, the value of 0.8 is not a particularly low one. We have also conducted numerical analysis for different values of $T \in [3, 11]$, $ROIC \in [1.1\%, 1.6\%]$, and real interest rate $\in [1\%, 6\%]$. For these parameter ranges, the amount of investment reduction is within $[30\%, 65\%]$ where β is 0.8, and within $[50\%, 90\%]$ where β is 0.7.[22]

The main results of our paper are that short-termism decreases investment as well as both biased and unbiased net present values. In our model, the existence of agency problems is not necessary for the firm to experience the underinvestment problem. This means that even if the manager and the shareholders have aligned incentives, the underinvestment problem would still arise as long as they share short-term oriented objectives. Nevertheless, as shown in section 7, this framework is inclusive of the case in which agency problems exist, a potentially more reasonable assumption based on existing corporate finance research. The result in Lemma 5 implies that asymmetric information between managers and investors

[21] This means that if a consumer equally prefers \$100 (for example) t years later and \$120 $t+1$ years later where $t \geq 2$, the consumer would be indifferent between \$100 today and \$171 ($\approx 120 \times (1/\beta)$) a year later. Because the consumer's degree of myopia is not necessarily the same as that of the firm, further research is needed in this area.

[22] Numerical results show that higher values of T, higher values of ROIC, and lower values of real interest rates cause greater reductions in investment. However, there is no analytical proof for these results. The numerical anlaysis in this section were performed with MATLAB 9. All MATLAB codes can be downloaded from minwook.host22.com/code/firm_investment_hyperbolic.

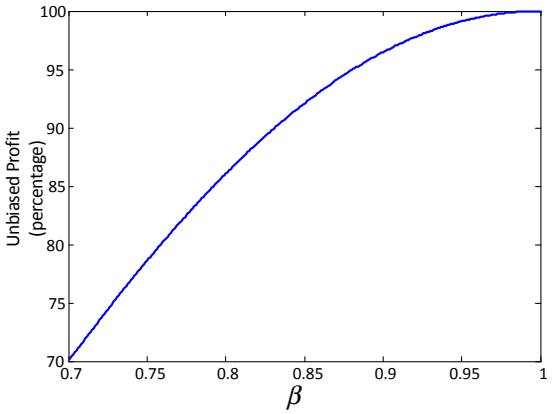

Figure 10: Short-termism and unbiased profits

necessarily decreases unbiased firm values and, therefore, decreases unbiased profit.[23]

Figure 10 shows how the unbiased profit decreases as β decreases, as we use the profit level with $\beta = 1$ as a numeraire (100%). The graph shows that where $\beta = 0.7$ (0.8), there would be about a 30% (14%) loss of unbiased profit relative to the profit level where $\beta = 1$. We consider the case where the outside authority imposes a proportional dividend taxation τ in each period. Figure 11 shows how the dividend taxation policy increases the firm's investment where $\beta = 0.8$. See Appendix C for the analytical derivation of investment decision under the dividend taxation policy. Figure 12 shows how revenue-neutral dividend taxation increases unbiased profit where $\beta = 0.8$. Where there is no dividend taxation i.e., $\tau = 0$, the profit loss due to short-termism under $\beta = 0.8$ is around 14%, i.e., the unbiased profit with $\beta = 0.8$ is 86% of the profit with $\beta = 1$. The graph shows that by implementing dividend taxation policy, the profit loss could be almost recovered up to 99.5%.

In this section, we have shown a benchmark calibration along with simulation results that exemplify an average firm for which long-term projects are part of its business operations. Naturally, these parameters would vary across different types of industries. This evaluation framework, however, can be flexibly adapted towards assessing underinvestment in specific sectors as well as potential policies (e.g. corporate tax policies) that are industry-specific. For example, our theory indicates that for investment projects with long gestation periods (typical in the pharmaceutical industry, for instance), short-termism generates more severe

[23]The unbiased profit is the net present value in period 1 with $\beta = 1$, that is

$$\text{Unbiased profit} = -i_1 - \delta i_2 - .... - \delta^{T-2} i_{T-1} + \delta^{T-1} f(i_1, ..., i_{T-1})$$

The expression "*unbiased* profit" implies that there is an agency problem where the decision maker (manager) has present-biased (i.e, $\beta < 1$) objectives, while profit and firm values are evaluated based on unbiased (i.e, $\beta = 1$) objective functions.

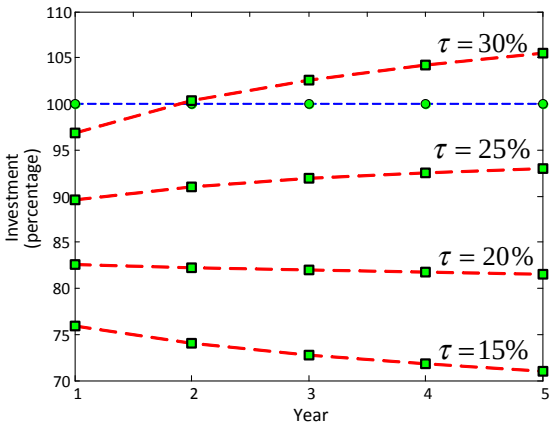

Figure 11: Dividend taxation and investment where $\beta = 0.8$

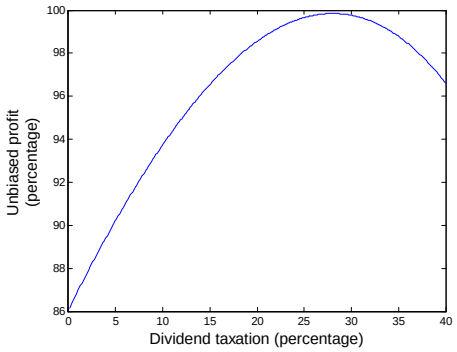

Figure 12: Dividend taxation and unbiased profit where $\beta = 0.8$

underinvestment problems. Bearing in mind the important heterogeneities across industries, the quantitative results derived in the benchmark model appear to be broadly in line with existing empirical evidence on the extent of short-termism's effect on corporate investment.

10. Conclusion

We construct a theoretical framework that incorporates hyperbolic discounting preferences into corporate investment decisions. In doing so, we rigorously establish the linkage between short-termism and underinvestment. In our three-period framework, the firm with present bias makes investment decisions that result in suboptimally low levels of investment, as defined by the existence of a higher-level investment plan that improves all periods' present value of dividends.

We then conduct two policy analyses that can overcome this underinvestment problem:

dividend taxation and investment subsidies. We show that revenue-neutral dividend taxes and investment subsidies can correct the market distortions imposed by present bias. In a finite multi-period extension of the model, we demonstrate that the underinvestment induced by hyperbolic discounting preferences is uneven across time. Finally, quantitative implications based on calibration of our model suggest that the effect of short-termism on corporate investment may be substantial, consistent with recent empirical evidence. The firm's underinvestment problem is more severe as time elapses. The analysis in this paper provides theoretical underpinnings for arguments in the policy arena that advocate corporate taxation as a method for addressing corporate short-termism.

Theories of myopia have been separately developed in two strands: 1) agency problems in the corporate finance literature, and 2) irrational decision-making behavior. Our paper is able to bridge these two theories in a unified framework. This paper indicates that the two main approaches can be modeled by the same mathematical framework – hyperbolic discounted time preferences, and that both of these theories lead to underinvestment problems. Therefore, the same types of policies can improve firm value regardless of whether managerial myopia is attributed to intrinsic nature or agency conflict.

More recently, empirical and survey evidence have demonstrated that short-termism is a prominent feature of corporations (Asker, Farre-Mensa and Ljungqvist 2015; Budish, Roin and Williams 2015; Poterba and Summers 1995). In particular, Asker, Farre-Mensa and Ljungqvist (2015) have directly investigated the effect of short-termism on corporate investment. They argue that public firms would invest substantially less than private firms, because the former are subject to short-termism arising from pressure on current share prices. The strong ownership in private firms, on the contrary, allows more effective monitoring of management to pursue long-term values. Consistent with their hypothesis, in a sample of US firms spanning 2001-2011, they show that the average annual gross fixed investment (as scaled by total assets) is 4.1% for public firms and 7.5% for private firms. This implies that short-termism may have contributed to investment reductions of up to 45 percent.

Despite these empirical evidence and the prevalent view that short-termism features importantly in manager's behavior, the theory of hyperbolic discounting has not been formally applied to corporate decisions. This stands in contrast to the large volume of literature that applied hyperbolic discounting preferences to consumers' decision. This paper contributes to theories of corporate short-termism by introducing the hyperbolic discounting framework to corporate investments, and shows that this framework can be meaningfully related to both existing empirical evidence and theoretical approaches.

Appendices

A. Proof of Proposition 6

To simplify the notation, we drop the cash flow x_t in the maximization problem. Because the exogenous cash flows are eliminated in the first order conditions, they do not affect the firm's investment decisions. In period $T-1$ and for any given $(i_1, i_2, ..., i_{T-2})$, the firm solves the following maximization problem:

$$\max_{i_{T-1}|\{i_s\}_{s=1}^{T-2}} -i_{T-1} + \beta\delta f\left(i_1, i_2...., i_{T-1}\right). \tag{64}$$

From maximization problem (64), we can derive a choice function $\widehat{i}_{T-1} : \mathbb{R}_{++}^{T-2} \to \mathbb{R}_{++}$ such that

$$\widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2}) = \left\{\beta\delta a_{T-1} f\left(i_1, ..., i_{T-2}, 1\right)\right\}^{\frac{1}{1-a_{T-1}}}. \tag{65}$$

The maximization problem in period $T-2$ is given as

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -i_{T-2} - \beta\delta\widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2}) + \beta\delta^2 f\left(i_1, i_2...., \widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2})\right), \tag{66}$$

which is equivalent in turn, to

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -i_{T-2} + \beta\delta\left\{-\widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2}) + \delta f\left(i_1, i_2...., \widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2})\right)\right\}. \tag{67}$$

Using (65), the expression inside $\{\cdot\}$ in (67) can be expressed as

$$\begin{aligned}
& \left\{-\beta\delta a_{T-1} f\left(i_1, ..., i_{T-2}, 1\right)\right\}^{\frac{1}{1-a_{T-1}}} + \delta f\left(i_1, i_2...., \widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2})\right) \\
= & -\left(\beta\delta a_{T-1}\right)^{\frac{1}{1-a_{T-1}}} f\left(i_1, ..., i_{T-2}, 1\right)^{\frac{1}{1-a_{T-1}}} \\
& +\delta f\left(i_1, ..., i_{T-2}, 1\right)\left\{\beta\delta a_{T-1} f\left(i_1, ..., i_{T-2}, 1\right)\right\}^{\frac{a_{T-1}}{1-a_{T-1}}} \\
= & \left(\delta\left(\beta\delta a_{T-1}\right)^{\frac{a_{T-1}}{1-a_{T-1}}} - \left(\beta\delta a_{T-1}\right)^{\frac{1}{1-a_{T-1}}}\right) f\left(i_1, ..., i_{T-2}, 1\right)^{\frac{1}{1-a_{T-1}}} \\
= & \left(\beta\delta a_{T-1}\right)^{\frac{1}{1-a_{T-1}}}\left(\frac{1-\beta a_{T-1}}{\beta a_{T-1}}\right) f\left(i_1, ..., i_{T-2}, 1\right)^{\frac{1}{1-a_{T-1}}}.
\end{aligned} \tag{68}$$

We define a function $f^{(T-2)} : \mathbb{R}_{++}^{T-2} \to \mathbb{R}_{++}$:

$$f^{(T-2)}\left(\{i_s\}_{s=1}^{T-2}\right) = \left(\beta\delta a_{T-1}\right)^{\frac{1}{1-a_{T-1}}}\left(\frac{1-\beta a_{T-1}}{\beta a_{T-1}}\right) f\left(i_1, ..., i_{T-2}, 1\right)^{\frac{1}{1-a_{T-1}}}, \tag{69}$$

which is also a Cobb-Douglas function that is strictly increasing and strictly concave. With the function in (69), the maximization problem in (66) can be expressed as

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -i_{T-2} + \beta\delta f^{(T-2)}\left(\{i_s\}_{s=1}^{T-2}\right), \tag{70}$$

which is defined in the same way as the maximization problem of (64). Therefore, we have the choice function $\widehat{i}_{T-2}$, given as

$$\widehat{i}_{T-2} = \left\{\beta\delta a^{(T-2)} f^{(T-2)}\left(i_1, ..., i_{T-3}, 1\right)\right\}^{\frac{1}{1-a^{(T-2)}}}, \tag{71}$$

where

$$a^{(k)} = \frac{\partial f^{(k)}\left(\{i_s\}_{s=1}^{k}\right)/\partial i_k}{f^{(k)}\left(\{i_s\}_{s=1}^{k}\right)}|_{\{i_1,..,i_k\}=\{1,...,1\}}. \tag{72}$$

In a recursive way, we define a function $f^{(k)} : \mathbb{R}_{++}^{k} \to \mathbb{R}_{++}$ such that

$$f^{(k)} = \left(\beta\delta a^{(k+1)}\right)^{\frac{1}{1-a^{(k+1)}}} \left(\frac{1-\beta a^{(k+1)}}{\beta a^{(k+1)}}\right) f^{(k+1)}\left(i_1, ..., i_k, 1\right)^{\frac{1}{1-a^{(k+1)}}}, \tag{73}$$

Then, the maximization problem in period k is expressed as

$$\max_{i_k|\{i_s\}_{s=1}^{k-1}} -i_k + \beta\delta f^{(k)}\left(\{i_s\}_{s=1}^{k}\right). \tag{74}$$

Since the function $f^{(k)}$ is strictly concave and strictly increasing, the maximization problem in period k has a unique solution for any given $\{i_s\}_{s=1}^{k-1}$. In period 1, we have a unique solution, i_1^*, from the maximization problem (74) with $k = 1$. This unique solution is used to get a unique solution i_2^* from the maximization problem with $k = 2$. Repeating this process, we have a unique investment equilibrium, $\{i_s^*\}_{s=1}^{T-1}$.

Next, we move on to the underinvestment issue. From Proposition 2, we know that in period $T-2$, for any given $(i_1, i_2, ..., i_{T-3})$, the equilibrium investment

$$(i_{T-2}^*, i_{T-1}^*) = \left(\widehat{i}_{T-2}\left(\{i_s\}_{s=1}^{T-3}\}\right), \widehat{i}_{T-1}\left(\{i_s\}_{s=1}^{T-3}, \widehat{i}_{T-2}(\{i_s\}_{s=1}^{T-3}\}\right)\right) \tag{75}$$

is underinvestment in the sense that there is another investment plan (i'_{T-1}, i'_{T-2}), which (a) is strictly greater than the equilibrium investment level and (b) the corresponding present values in periods $T-2$, $T-1$ and T are strictly higher than those in the equilibrium. In the maximization problem in period $T-3$ and $T-2$, replacing the variable i_{T-1} with a choice function $\widehat{i}_{T-1}\left(\{i_s\}_{s=1}^{T-2}\}\right)$, we have a "derived" 3-period model with Cobb-Douglas function.

Specifically, in period $T-2$, the maximization problem is

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -i_{T-2} - \beta\delta\left\{\widehat{i}_{T-1} + \delta f(i_1, ..., i_{T-2}, \widehat{i}_{T-1})\right\}, \tag{76}$$

and in period $T-3$, the maximization problem is

$$\max_{i_{T-3}|\{i_s\}_{s=1}^{T-4}} -i_{T-3} - \beta\delta\widehat{i}_{T-2} - \beta\delta^2\left\{\widehat{i}_{T-1} + \delta f(i_1, ..., \widehat{i}_{T-2}, \widehat{i}_{T-1})\right\}, \tag{77}$$

where $\widehat{i}_{T-2} = \widehat{i}_{T-2}\left(\{i_s\}_{s=1}^{T-3}\}\right)$ and $\widehat{i}_{T-1} = \widehat{i}_{T-1}\left(\{i_s\}_{s=1}^{T-3}, \widehat{i}_{T-2}(\{i_s\}_{s=1}^{T-3}\}\right)$. We have shown that the term $\left\{\widehat{i}_{T-1} + \delta f(i_1, ..., \widehat{i}_{T-2}, \widehat{i}_{T-1})\right\}$ is well-defined, strictly increasing, and strictly concave (see (68)). Therefore, by Proposition 2, for any given $(i_1, ..., i_{T-4})$ the equilibrium investment plan

$$(i^*_{T-3}, i^*_{T-2}) = \left(\widehat{i}_{T-3}\left(\{i_s\}_{s=1}^{T-4}\right), \widehat{i}_{T-2}\left(\{i_s\}_{s=1}^{T-3}, \widehat{i}_{T-3}\left(\{i_s\}_{s=1}^{T-4}\right)\right)\right) \tag{78}$$

is underinvestment. From (75) and (78), we know that for any given $(i_1, ..., i_{T-4})$, the equilibrium investment $(i^*_{T-3}, i^*_{T-2}, i^*_{T-1})$ is underinvestment in the sense that there is another investment plan $(i'_{T-3}, i'_{T-2}, i'_{T-1})$ that induces higher present values in period $T, T-1, T-2$ and $T-3$. Repeating this process, we can show that the equilibrium investment $(i^*_1, i^*_2..., i^*_{T-1})$ is underinvestment.

B. Proof of Proposition 7

To simplify the notation, we drop the cash flow x_t in the maximization problem in the same way as in the proof of Proposition 6. The choice function $\widehat{i}_{T-1}(i_1, ..., i_{T-1})$ is strictly increasing in i_{T-1} from (65). We have shown that if the return function is supermodular, the choice function is strictly increasing in a three-period model (See (6) in Section 2). Given $(i_1, ..., i_{T-2})$, the maximization problem in period $T-1$ is

$$\max_{i_{T-1}|\{i_s\}_{s=1}^{T-2}} -i_{T-1} + \beta\delta f\left(i_1, i_2...., i_{T-1}\right),$$

and its first-order condition is

$$-1 + \beta\delta\frac{\partial f}{\partial i_{T-1}} = 0. \tag{79}$$

Given $(i_1, ..., i_{T-3})$, the maximization problem in period $T-2$ is

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -i_{T-2} - \beta\delta\widehat{i}_{T-1} + \beta\delta^2 f\left(i_1, i_2...., \widehat{i}_{T-1}\right),$$

and its first-order condition is

$$-1 - \beta\delta\frac{\partial\widehat{i}_{T-1}}{\partial i_{T-2}} + \beta\delta^2\frac{\partial f}{\partial i_{T-2}} + \beta\delta^2\frac{\partial f}{\partial i_{T-1}}\frac{\partial\widehat{i}_{T-1}}{\partial i_{T-2}} = 0. \tag{80}$$

From equations (79) and (80), we have

$$\delta\left(\frac{\partial f}{\partial i_{T-2}}\right)/\frac{\partial f}{\partial i_{T-1}} = 1 - \delta\frac{\partial\widehat{i}_{T-1}}{\partial i_{T-2}}(1-\beta). \tag{81}$$

Because we have $a_t = a_{t-1}\delta$, the left term in equation (81) is

$$\delta\left(\frac{\partial f}{\partial i_{T-2}}\right)/\frac{\partial f}{\partial i_{T-1}} = \delta\frac{i_{T-1}^{a_{T-1}}a_{T-2}i_{T-2}^{a_{T-2}-1}}{a_{T-1}i_{T-1}^{a_{T-1}-1}i_{T-2}^{a_{T-2}}} = \delta\frac{a_{T-2}}{a_{T-1}}\frac{i_{T-1}}{i_{T-2}} = \frac{i_{T-1}}{i_{T-2}}. \tag{82}$$

From (81) and (82), we have

$$\frac{i_{T-1}}{i_{T-2}} = 1 - \delta\frac{\partial\widehat{i}_{T-1}}{\partial i_{T-2}}(1-\beta), \tag{83}$$

which implies that for any given $(i_1, ..., i_{T-3})$, if $\beta < 1$ ($\beta = 1$), we have $i_{T-1} > i_{T-2}$ ($i_{T-1} = i_{T-2}$) because $\partial\widehat{i}_{T-1}/\partial i_{T-2} > 0$. In the same recursive way as in the proof of Proposition 6, plugging the choice function $\widehat{i}_{T-1}(\cdot)$ into the maximization problems in periods $T-4$ and $T-3$, we have the following equation

$$\frac{i_{T-2}}{i_{T-3}} = 1 - \delta\frac{\partial\widehat{i}_{T-2}}{\partial i_{T-3}}(1-\beta), \tag{84}$$

which also implies that for any given $(i_1, ..., i_{T-4})$, if $\beta < 1$ ($\beta = 1$), we have $i_{T-2} > i_{T-3}$ ($i_{T-2} = i_{T-3}$) because $\partial\widehat{i}_{T-2}/\partial i_{T-3} > 0$. Repeating this recursive analysis, we know that the equilibrium investment i_t is strictly decreasing (constant) in t if $\beta < 1$ ($\beta = 1$).

C. Multi-period Cobb-Douglas model with dividend taxation

We assume that in each period, the government imposes proportional dividend tax τ under revenue-neutral policy. In period $T-1$ and for any given $(i_1, i_2, ..., i_{T-2})$, the firm solves the following maximization problem:

$$\max_{i_{T-1}|\{i_s\}_{s=1}^{T-2}} -\frac{i_{T-1}}{1+\tau} + \beta\delta f\left(i_1, i_2...., i_{T-1}\right). \tag{85}$$

From maximization problem (85), we can derive a choice function $\widehat{i}_{T-1} : \mathbb{R}_{++}^{T-2} \to \mathbb{R}_{++}$ such that

$$\widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2}) = \left\{(1+\tau)\beta\delta a_{T-1} f\left(i_1, ..., i_{T-2}, 1\right)\right\}^{\frac{1}{1-a_{T-1}}}. \tag{86}$$

The maximization problem in period $T-2$ is given as

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -\frac{i_{T-2}}{1+\tau} + \beta\delta\left\{-\widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2}) + \delta f\left(i_1, i_2...., \widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2})\right)\right\}, \tag{87}$$

Using (86), the expression inside $\{\cdot\}$ in (87) can be expressed as

$$\begin{aligned}
& \left\{-(1+\tau)\beta\delta a_{T-1} f\left(i_1, ..., i_{T-2}, 1\right)\right\}^{\frac{1}{1-a_{T-1}}} + \delta f\left(i_1, i_2...., \widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2})\right) \\
= & -\left((1+\tau)\beta\delta a_{T-1}\right)^{\frac{1}{1-a_{T-1}}} f\left(i_1, ..., i_{T-2}, 1\right)^{\frac{1}{1-a_{T-1}}} \\
& +\delta f\left(i_1, ..., i_{T-2}, 1\right)\left\{\beta\delta a_{T-1} f\left(i_1, ..., i_{T-2}, 1\right)\right\}^{\frac{a_{T-1}}{1-a_{T-1}}} \\
= & \left(\delta\left(\beta\delta a_{T-1}\right)^{\frac{a_{T-1}}{1-a_{T-1}}} - \left((1+\tau)\beta\delta a_{T-1}\right)^{\frac{1}{1-a_{T-1}}}\right) f\left(i_1, ..., i_{T-2}, 1\right)^{\frac{1}{1-a_{T-1}}} \\
= & \left(\beta\delta a_{T-1}\right)^{\frac{1}{1-a_{T-1}}}\left(\frac{1-(1+\tau)\beta a_{T-1}}{\beta a_{T-1}}\right) f\left(i_1, ..., i_{T-2}, 1\right)^{\frac{1}{1-a_{T-1}}}.
\end{aligned} \tag{88}$$

We define a function $f^{(T-2)} : \mathbb{R}_{++}^{T-2} \to \mathbb{R}_{++}$:

$$f_\tau^{(T-2)}\left(\{i_s\}_{s=1}^{T-2}\right) = \left(\beta\delta a_{T-1}\right)^{\frac{1}{1-a_{T-1}}}\left(\frac{1-(1+\tau)\beta a_{T-1}}{\beta a_{T-1}}\right) f\left(i_1, ..., i_{T-2}, 1\right)^{\frac{1}{1-a_{T-1}}}. \tag{89}$$

With the function in (89), the maximization problem in (87) can be expressed as

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -\frac{i_{T-2}}{1+\tau} + \beta\delta f_\tau^{(T-2)}\left(\{i_s\}_{s=1}^{T-2}\right), \tag{90}$$

which is defined in the same way as the maximization problem of (85). Therefore, we have the choice function $\widehat{i}_{T-2}$, given as

$$\widehat{i}_{T-2} = \left\{(1+\tau)\,\beta\delta a^{(T-2)} f_\tau^{(T-2)}\left(i_1,...,i_{T-3},1\right)\right\}^{\frac{1}{1-a^{(T-2)}}}, \tag{91}$$

where

$$a^{(k)} = \frac{\partial f_\tau^{(k)}\left(\{i_s\}_{s=1}^k\right)/\partial i_k}{f_\tau^{(k)}\left(\{i_s\}_{s=1}^k\right)}|_{\{i_1,..,i_k\}=\{1,...,1\}}. \tag{92}$$

In a recursive way, we define a function $f_\tau^{(k)}: \mathbb{R}_{++}^k \to \mathbb{R}_{++}$ such that

$$f_\tau^{(k)} = \left(\beta\delta a^{(k+1)}\right)^{\frac{1}{1-a^{(k+1)}}} \left(\frac{1-(1+\tau)\,\beta a^{(k+1)}}{\beta a^{(k+1)}}\right) f_\tau^{(k+1)}\left(i_1,...,i_k,1\right)^{\frac{1}{1-a^{(k+1)}}}, \tag{93}$$

Then, the maximization problem in period k is expressed as

$$\max_{i_k|\{i_s\}_{s=1}^{k-1}} -\frac{i_k}{1+\tau} + \beta\delta f_\tau^{(k)}\left(\{i_s\}_{s=1}^k\right). \tag{94}$$

Since the function $f_\tau^{(k)}$ is strictly concave and strictly increasing, the maximization problem in period k has a unique solution for any given $\{i_s\}_{s=1}^{k-1}$. In period 1, we have a unique solution, i_1^*, from the maximization problem (94) with $k=1$. This unique solution is used to get a unique solution i_2^* from the maximization problem with $k=2$. Repeating this process, we have a unique investment equilibrium, $\{i_s^*\}_{s=1}^{T-1}$.

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