The Structure of Dominant Strategy Incentive Compatible Mechanisms

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Abstract. Dominant strategy implementability is considered when the number of alternatives is finite, utilities are quasilinear, and an individual's type is a vector whose components are his valuations of the alternatives. The Rockafellar–Rochet Theorem shows that a necessary and sufficient condition for the dominant strategy implementability of an allocation function defined on an arbitrary type space is that for every one-person mechanism obtained by fixing the types of the other individuals, all cycles in the corresponding allocation graph have nonnegative length. For any such one-person mechanism, the geometry of the partition of the type space into subsets that are assigned the same alternative is analyzed using polyhedra called difference sets. Under relative weak conditions, it is shown that it is necessary for many two-arc cycles in the allocation graph to have zero length. Situations are identified in which it is necessary for all cycle lengths in this graph to be zero, which is equivalent to the vertices of the difference sets restricted to suitably normalized type vectors coinciding. For an arbitrary type space, it is also shown that any one-person dominant strategy implementable allocation function (i) can be extended to the unrestricted domain and (ii) that it is the solution to an affine maximization problem.