Efficient Bilateral Trade with Interdependent Values — the Use of Two-Stage Mechanisms

Takashi Kunimoto and Cuiling Zhang

Singapore Management University

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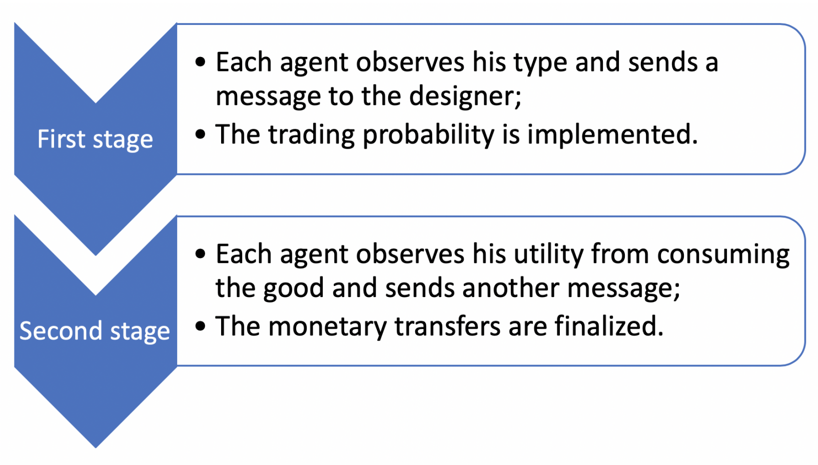
Introduction

- We study bilateral trade problem with interdependent values.
- Each agent receives different information about the value of the good, denoted by type $\theta_i \in \Theta_i$ which is a compact subset of $\mathbb{R}_+$.
- Types are independently distributed between agents.
- Each agent's valuation $\tilde{u}_i(\theta_i, \theta_{-i})$ depends on both θ_i and θ_{-i}.

Two-stage mechanisms proposed by Mezzetti (2004)



The Generalized Two-stage Groves mechanism

Mezzetti (2004) introduces the generalized two-stage Groves mechanism and shows that it always satisfies

- Bayesian incentive compatibility (BIC): Truthtelling in both stages constitutes an equilibrium strategy of a perfect Bayesian equilibrium;
- decision efficiency (EFF);
- ex post budget balance (BB).

Research Question

- Does the generalized two-stage Groves mechanism satisfy interim individual rationality (IIR) as well?
- If no, is there a different two-stage mechanism satisfying BIC, IIR, EFF and BB?

Preview of Our Results

- Under one-sided asymmetric information structure, the generalized two-stage Groves mechanism always satisfies IIR.
- Under two-sided asymmetric information structure,
  - we show by an example that it never satisfies IIR;
  - we propose the two-stage monotone mechanisms which satisfy IIR in a positive number of cases within the same example;
  - we characterize the existence of two-stage monotone mechanisms satisfying BIC, IIR, EFF and BB.

The Model

- Preferences of each agent $U_i : Q \times \Theta \times \mathbb{R} \to \mathbb{R}$ depend upon trading probability q, the type profile θ and his monetary transfer p_i:

$$\begin{aligned} U_1(q,\theta,p_1) &= u_1(q,\theta) + p_1 = (1-q)\tilde{u}_1(\theta) + p_1; \\ U_2(q,\theta,p_2) &= u_2(q,\theta) + p_2 = q\tilde{u}_2(\theta) + p_2, \end{aligned}$$

  where $u_i(q,\theta)$ is agent i's allocation payoff and $\tilde{u}_i(\theta)$ is his valuation.
- We assume that for any $\theta \in \Theta$, each agent i observes $u_i(q,\theta)$ after the outcome decision q is implemented, but before final transfers p are made.

The Model

Agents' outside option utilities are

$$U_1^O(\theta_1) = \int_{\Theta_2} \tilde{u}_1(\theta_1, \theta_2) dF_2(\theta_2) \text{ for all } \theta_1 \in \Theta_1$$

and

$$U_2^O(\theta_2) = 0 \text{ for all } \theta_2 \in \Theta_2.$$

The Generalized Revelation Principle

Two-stage mechanism (M^1, M^2, δ, τ)	Generalized revelation mechanism (Θ, Π, x, t)
Decision rule $\delta : M^1 \to [0,1]$; Transfer rule $\tau : M^1 \times M^2 \to \mathbb{R}^2$.	Decision rule $x : \Theta \to [0,1]$; Transfer rule $t : \Theta \times \Pi \to \mathbb{R}^2$.
Agent i's strategy $r_i = (r_i^1, r_i^2)$ where $r_i^1 : \Theta_i \to M_i^1$ and $r_i^2 : Q \times \Theta_i \times \Pi_i \to M_i^2$.	Decision rule: $x(\theta) = \delta(r^1(\theta))$; Transfer rule: $t_i(\theta, u) = \tau_i(r^1(\theta), r^2(\delta(\theta), \theta, u))$.

Any PBE outcome of a two-stage mechanism can be implemented as a PBE outcome of a generalized revelation mechanism in which trutelling in both stages constitutes an equilibrium strategy.

The Generalized Two-stage Groves Mechanism (Θ, Π, x^*, t^G)

For each agent i, each type report $(\theta_i^r, \theta_{-i}^r) \in \Theta_i \times \Theta_{-i}$ and each payoff report $(u_i^r, u_{-i}^r) \in \Pi_i \times \Pi_{-i}$,

$$t_i^G(\theta_i^r, \theta_{-i}^r; u_i^r, u_{-i}^r) = u_{-i}^r - h_i(\theta_i^r, \theta_{-i}^r)$$

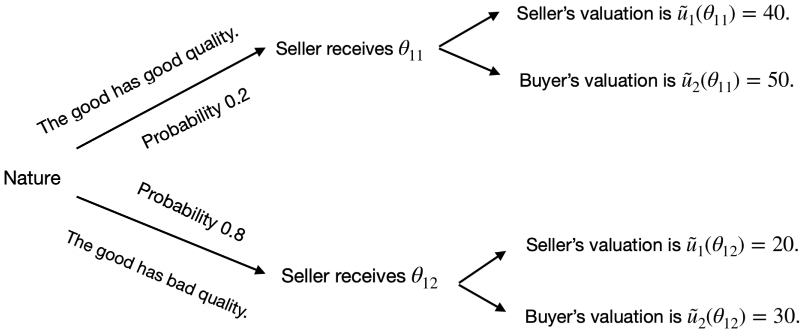
where

$$\begin{aligned} h_i(\theta_i^r, \theta_{-i}^r) &= \tfrac{1}{2}\Bigg[\textstyle\sum_{j=1}^2 u_j\left(x^*(\theta^r), \theta^r\right) - \mathbb{E}_{-i}\left(\sum_{j=1}^2 u_j\left(x^*(\theta_i^r, \theta_{-i}), \theta_i^r, \theta_{-i}\right)\right) \\ &\quad + \mathbb{E}_{-(i+1)}\left(\textstyle\sum_{j=1}^2 u_j\left(x^*(\theta_{i+1}^r, \theta_{-(i+1)}), \theta_{i+1}^r, \theta_{-(i+1)}\right)\right)\Bigg] \end{aligned}$$

with $\mathbb{E}_{-i}$ being the expectation operator over θ_{-i} and $\mathbb{E}_{-3} = \mathbb{E}_{-1}$.

One-sided asymmetric information

Example in Myerson's textbook (1991, page 489):



Note that it is always efficient to trade, i.e., $x^*(\theta_{11}) = x^*(\theta_{12}) = 1$.

Single-stage mechanisms fails.

Myerson (1991) verifies that in this example, no single-stage direct mechanism (x^*, t) satisfies BIC, IIR, EFF and BB.

$$
\begin{aligned}
IC_{\theta_{11}\to\theta_{12}} : 40\,(1 - x^*(\theta_{11})) + t_1(\theta_{11}) \quad &\geq \quad 40\,(1 - x^*(\theta_{12})) + t_1(\theta_{12}); \\
IC_{\theta_{12}\to\theta_{11}} : 20\,(1 - x^*(\theta_{12})) + t_1(\theta_{12}) \quad &\geq \quad 20\,(1 - x^*(\theta_{11})) + t_1(\theta_{11}).
\end{aligned}
$$

Since $x^*(\theta_{11}) = x^*(\theta_{12}) = 1$, then BIC implies $t_1(\theta_{11}) = t_1(\theta_{12})$.

$$
\begin{aligned}
IR_{\theta_{11}} &: \quad 40\,(1 - x^*(\theta_{11})) + t_1(\theta_{11}) \geq 40 \Rightarrow t_1(\theta_{11}) \geq 40; \\
IR_{\theta_{12}} &: \quad 20\,(1 - x^*(\theta_{12})) + t_1(\theta_{12}) \geq 20 \Rightarrow t_1(\theta_{11}) \geq 20.
\end{aligned}
$$

Then, seller' IIR constraints imply $t_1(\theta_{11}) \geq 40$.

$$
IR_{\bar{\theta}_2} : \quad 0.2\,(50x^*(\theta_{11}) + t_2(\theta_{11})) + 0.8\,(30x^*(\theta_{12}) + t_2(\theta_{12})) \geq 0.
$$

Finally, BB requires $t_2(\theta_{11}) = -t_1(\theta_{11})$ and $t_2(\theta_{12}) = -t_1(\theta_{12})$; then, buyer's IIR implies $t_1(\theta_{11}) \leq 34$, a contradiction.

The generalized two-stage Groves mechanism succeeds.

Claim 1

In Example 1, the generalized two-stage Groves mechanism (Θ, Π, x^*, t^G) *satisfies BIC, IIR, EFF and BB simultaneously.*

The generalized two-stage Groves mechanism succeeds.

Proof: For each $\theta_1^r \in \Theta_1$ and each $(u_1^r, u_2^r) \in \Pi_1 \times \Pi_2$,

$$\begin{aligned}
& t_1^G(\theta_1^r; u_1^r, u_2^r) \\
= \quad & u_2^r - \frac{1}{2}\left[\sum_{j=1}^2 u_j\left(x^*(\theta_1^r), \theta_1^r\right) - \mathbb{E}_2\left(\sum_{j=1}^2 u_j\left(x^*(\theta_1^r), \theta_1^r\right)\right) + \mathbb{E}_1\left(\sum_{j=1}^2 u_j\left(x^*(\theta_1), \theta_1\right)\right)\right] \\
= \quad & u_2^r - \frac{1}{2}\mathbb{E}_1\left(\tilde{u}_2(\theta_1)\right) \ \ (\because \forall \theta_1, x^*(\theta_1) = 1) \\
= \quad & u_2^r - 17
\end{aligned}$$

and

$$\begin{aligned}
& t_2^G(\theta_1^r; u_1^r, u_2^r) \\
= \quad & u_1^r - \frac{1}{2}\left[\sum_{j=1}^2 u_j\left(x^*(\theta_1^r), \theta_1^r\right) - \mathbb{E}_1\left(\sum_{j=1}^2 u_j\left(x^*(\theta_1), \theta_1\right)\right) + \mathbb{E}_2\left(\sum_{j=1}^2 u_j\left(x^*(\theta_1^r), \theta_1^r\right)\right)\right] \\
= \quad & u_1^r - \tilde{u}_2(\theta_1^r) + \frac{1}{2}\mathbb{E}_{-2}\left(\tilde{u}_2(\theta_1)\right) \ \ (\because \forall \theta_1, x^*(\theta_1) = 1) \\
= \quad & u_1^r - \tilde{u}_2(\theta_1^r) + 17.
\end{aligned}$$

Note that t_1^G is independent of u_1^r, and t_2^G is independent of u_2^r.

The generalized two-stage Groves mechanism succeeds.

Proof (Cont'd): Suppose seller reports θ_1^r instead of his true type θ_1 and each agent reports the true allocation payoff. Then seller receives the following utility:

$$\begin{aligned} & u_1(x^*(\theta_1^r), \theta_1) + t_1^G\left(\theta_1^r; u_1(x^*(\theta_1^r), \theta_1), u_2(x^*(\theta_1^r), \theta_1)\right) \\ = \; & u_1(x^*(\theta_1^r), \theta_1) + u_2(x^*(\theta_1^r), \theta_1) - 17 \; (\because u_2^r = u_2(x^*(\theta_1^r), \theta_1)) \\ = \; & 0 + \tilde{u}_2(\theta_1) - 17 \; (\because \forall \theta_1, x^*(\theta_1) = 1), \end{aligned}$$

which is independent of his first-stage report θ_1^r. So, seller has no incentive to deviate and together truthtelling in both stages constitutes a PBE; hence, BIC is satisfied.

The generalized two-stage Groves mechanism succeeds.

Proof (Cont'd): BB is satisfied on equilibrium path because for each $\theta_1 \in \Theta_1$,

$$\begin{aligned} & t_1^G(\theta_1; u_1, u_2) + t_2^G(\theta_1; u_1, u_2) \\ = \; & (u_2(x^*(\theta_1), \theta_1) - 17) + (u_1(x^*(\theta_1), \theta_1) - \tilde{u}_2(\theta_1) + 17) \\ = \; & (\tilde{u}_2(\theta_1) - 17) + (0 - \tilde{u}_2(\theta_1) + 17) \;\; (\because \forall \theta_1, x^*(\theta_1) = 1) \\ = \; & 0, \end{aligned}$$

where $u_1 = u_1(x^*(\theta_1), \theta_1)$ and $u_2 = u_2(x^*(\theta_1), \theta_1)$.

The generalized two-stage Groves mechanism succeeds.

Proof (Cont'd): Agents' interim expected utility from participating in the generalized two-stage Groves mechanism are

$$\begin{aligned} U_1^G(\theta_{11}) &= u_1(x^*(\theta_{11}), \theta_{11}) + t_1^G(\theta_{11}; u_1, u_2) = \tilde{u}_2(\theta_{11}) - 17 = 33; \\ U_1^G(\theta_{12}) &= u_1(x^*(\theta_{12}), \theta_{12}) + t_1^G(\theta_{12}; u_1, u_2) = \tilde{u}_2(\theta_{12}) - 17 = 13; \end{aligned}$$

and

$$\begin{aligned} U_2^G(\bar{\theta}_2) &= \mathbb{E}_1\left[u_2(x^*(\theta_1), \theta_1) + t_2^G(\theta_1; u_1, u_2)\right] \\ &= \mathbb{E}_1\left[u_2(x^*(\theta_1), \theta_1) + u_1(x^*(\theta_1), \theta_1) - \tilde{u}_2(\theta_1) + 17\right] \\ &= \mathbb{E}_1\left[\tilde{u}_2(\theta_1) + 0 - \tilde{u}_2(\theta_1) + 17\right] \;\; (\because \forall \theta_1, x^*(\theta_1) = 1) \\ &= 17. \end{aligned}$$

Hence,

$$\begin{aligned} U_1^G(\theta_{11}) &< U_1^O(\theta_{11}) = \tilde{u}_1(\theta_{11}) = 40; \\ U_1^G(\theta_{12}) &< U_1^O(\theta_{12}) = \tilde{u}_1(\theta_{12}) = 20; \\ U_2^G &> U_2^O = 0. \end{aligned}$$

The generalized two-stage Groves mechanism succeeds.

Proof (Cont'd): Then, a lump-sum transfer I must be imposed from buyer to seller so that everyone is better off after participation, i.e.,

$$\begin{aligned} U_1^G(\theta_{11}) + I \geq U_1^O(\theta_{11}) &\Rightarrow 33 + I \geq 40; \\ U_1^G(\theta_{12}) + I \geq U_1^O(\theta_{12}) &\Rightarrow 13 + I \geq 20; \\ U_2^G - I \geq U_2^O &\Rightarrow 17 - I \geq 0, \end{aligned}$$

hence, $7 \leq I \leq 17$. In conclusion, the generalized two-stage Groves mechanism satisfies BIC, IIR, EFF and BB.

The generalized two-stage Groves mechanism succeeds.

Theorem 1

When only the seller has a non-trivial set of types and the buyer has only one type, the generalized two-stage Groves mechanism (Θ, Π, x^*, t^G) *always satisfies BIC, IIR, EFF and BB.*

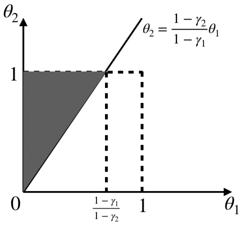
Two-sided asymmetric information

Example 2

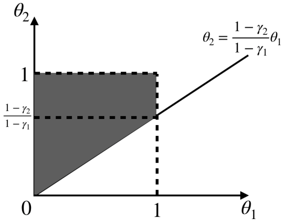
- Both agents' types are uniformly distributed on the unit interval $[0, 1]$;
- $\tilde{u}_1(\theta_1, \theta_2) = \theta_1 + \gamma_1\theta_2$ and $\tilde{u}_2(\theta_1, \theta_2) = \theta_2 + \gamma_2\theta_1$ where $\gamma_1, \gamma_2 > 0$.

Two-sided asymmetric information

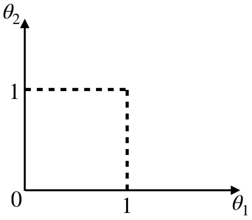
Example 2 (Cont'd)



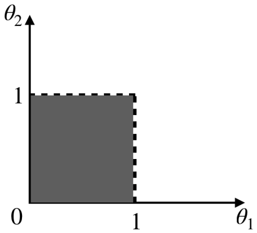
Case (i): $0 < \gamma_2 \leq \gamma_1 < 1$



Case (ii): $0 < \gamma_1 < \gamma_2 < 1$



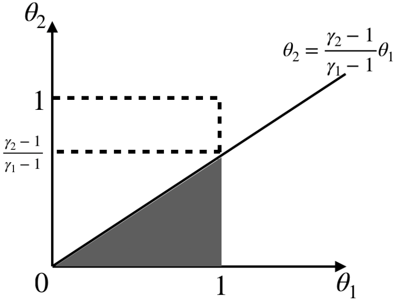
Case (iii): $0 < \gamma_2 \leq 1 \leq \gamma_1$



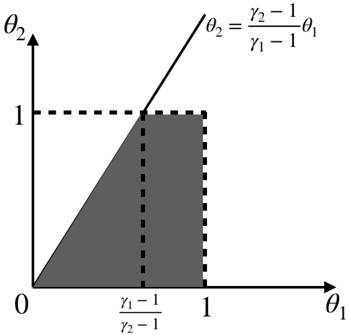
Case (iv): $0 < \gamma_1 < 1 < \gamma_2$

Two-sided asymmetric information

Example 2 (Cont'd)



Case (v): when $1 < \gamma_2 \leq \gamma_1$



Case (vi): when $1 < \gamma_1 < \gamma_2$

The generalized two-stage Groves mechanism fails.

Claim 2

In Example 2, the generalized two-stage Groves mechanism (Θ, Π, x^*, t^G) *violates IIR in all cases.*

Remark

In Example 2, the economy as a whole is worse off after participation; hence, it is impossible to make everyone better off through welfare redistribution.

Two-stage monotone mechanisms

Definition 2

A two-stage mechanism (Θ, Π, x^*, t) is *monotone* if the following properties are satisfied:

1. $t_2(\theta_1^r, \theta_2^r; u_1^r, u_2^r) \leq 0$ for all (θ_1^r, θ_2^r) and (u_1^r, u_2^r);
2. if $x^*(\theta_1^r, \theta_2^r) = 1$, then $|t_2(\theta_1^r, \theta_2^r; u_1^r, u_2^r)| \leq \tilde{u}_2(\theta_1^r, \theta_2^r)$.
3. if $\hat{\theta}_2^r > \theta_2^r$ and $x(\theta_1^r, \hat{\theta}_2^r) = x(\theta_1^r, \theta_2^r) = 1$, then $|t_2(\theta_1^r, \hat{\theta}_2^r; u_1^r, u_2^r)| > |t_2(\theta_1^r, \theta_2^r; u_1^r, u_2^r)|$.

Two-stage monotone mechanisms

Claim 3

In Example 2, the generalized two-stage Groves mechanism (Θ, Π, x^*, t^G) *is not monotone.*

Remark

In the generalized two-stage Groves mechanism, either buyer receives subsidies or buyer's payment is not strictly increasing in buyer's type report.

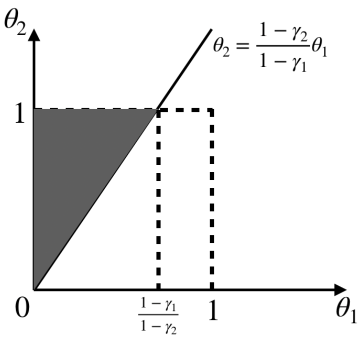
Two-stage monotone mechanisms succeed in Case (i) and (iii).

Claim 4

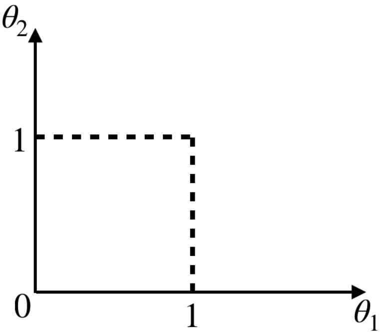
In Example 2, there exists a two-stage monotone mechanism satisfying BIC, IIR, EFF and BB in the following two cases: (i) $0 < \gamma_2 \leq \gamma_1 < 1$*; (iii)* $0 < \gamma_2 \leq 1 \leq \gamma_1$*; in all the other cases, two-stage monotone mechanisms violate BIC.*

Two stage monotone mechanisms succeed in Case (i) and (iii).

Recall



Case (i): $0 < \gamma_2 \leq \gamma_1 < 1$



Case (iii): $0 < \gamma_2 \leq 1 \leq \gamma_1$

Two-stage monotone mechanisms succeed in Case (i).

Proof: Case (i): $0 < \gamma_2 \leq \gamma_1 < 1$
Consider the following mechanism (Θ, Π, x^*, t^S):

$$t_1^S(\theta_1^r, \theta_2^r; u_1^r, u_2^r) = \begin{cases} u_2^r & \text{if } x^*(\theta_1^r, \theta_2^r) = 1 \text{ and } u_2^r = u_2(x^*(\theta_1^r, \theta_2^r), \theta_1, \theta_2) \\ -\psi & \text{if } x^*(\theta_1^r, \theta_2^r) = 1 \text{ and } u_2^r \neq u_2(x^*(\theta_1^r, \theta_2^r), \theta_1, \theta_2) \\ 0 & \text{if } x^*(\theta_1^r, \theta_2^r) = 0 \end{cases}$$

and

$$t_2^S(\theta_1^r, \theta_2^r; u_1^r, u_2^r) = \begin{cases} -u_2(x^*(\theta_1^r, \theta_2^r), \theta_1^r, \theta_2^r) & \text{if } x^*(\theta_1^r, \theta_2^r) = 1 \\ 0 & \text{if } x^*(\theta_1^r, \theta_2^r) = 0 \text{ and } u_1^r = u_1(x^*(\theta_1^r, \theta_2^r), \theta_1, \theta_2) \\ -\psi & \text{if } x^*(\theta_1^r, \theta_2^r) = 0 \text{ and } u_1^r \neq u_1(x^*(\theta_1^r, \theta_2^r), \theta_1, \theta_2) \end{cases}$$

where $\psi > 0$. It is monotone. If each agent reports the truth in both stages, then

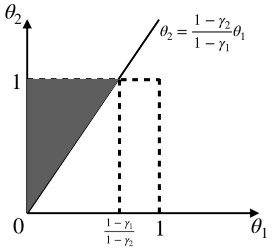
1. if $x^*(\theta_1, \theta_2) = 0$, $t_1^S(\theta_1, \theta_2; u_1, u_2) = t_2^S(\theta_1, \theta_2; u_1, u_2) = 0$;
2. if $x^*(\theta_1, \theta_2) = 1$, $t_1^S(\theta_1, \theta_2; u_1, u_2) = -t_2^S(\theta_1, \theta_2; u_1, u_2) = u_2(x^*(\theta_1, \theta_2); \theta_1, \theta_2) = \tilde{u}_2(\theta_1, \theta_2)$.

Two-stage monotone mechanisms succeed in Case (i).

Proof (Cont'd):

- Since t_1^S is independent of u_1^r and t_2^S is independent of u_2^r, each agent has no incentive to deviate in the second stage.
- We assume that buyer always reports the truth in the first stage and show that seller has no incentive to deviate in the first stage. Recall

Case (i): $0 < \gamma_2 \le \gamma_1 < 1$



- There are two cases: (a) $\theta_1 < (1-\gamma_1)/(1-\gamma_2)$; (b) $\theta_1 \ge (1-\gamma_1)/(1-\gamma_2)$.

Two-stage monotone mechanisms succeed in Case (i).

Proof (Cont'd): (a) If seller's true type is $\theta_1 < (1-\gamma_1)/(1-\gamma_2)$:

- his expected utility under truthtelling is

$$\int_0^{\frac{1-\gamma_2}{1-\gamma_1}\theta_1} (\tilde{u}_1(\theta_1,\theta_2)+0)\,d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1}\theta_1}^1 (0+\tilde{u}_2(\theta_1,\theta_2))\,d\theta_2.$$

- If he deviates to $0 < \theta_1^r < (1-\gamma_1)/(1-\gamma_2)$, his expected utility becomes

$$\int_0^{\frac{1-\gamma_2}{1-\gamma_1}\theta_1^r} (\tilde{u}_1(\theta_1,\theta_2)+0)\,d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1}\theta_1^r}^1 (0-\psi)d\theta_2.$$

  because if trade occurs, buyer's second-stage report becomes $u_2^r = u_2^r(x^*(\theta_1^r,\theta_2),\theta_1,\theta_2) = \tilde{u}_2(\theta_1,\theta_2) \neq \tilde{u}_2(\theta_1^r,\theta_2)$
- Since $\psi > 0$, seller's highest expected utility after deviation is $\int_0^1 \tilde{u}_1(\theta_1,\theta_2)d\theta_2$. However, it is still lower than truthtelling.

Two-stage monotone mechanisms succeed in Case (i).

Proof (Cont'd): (a) If seller's true type is $\theta_1 < (1-\gamma_1)/(1-\gamma_2)$:

- if seller deviates to $\theta_1^r > (1-\gamma_1)/(1-\gamma_2)$, trade never occurs and seller's expected utility becomes

$$\int_0^1 (\tilde{u}_1(\theta_1, \theta_2) + 0)\, d\theta_2,$$

  which is lower than truthtelling.
- In conclusion, seller has no incentive to deviate when his true type is $\theta_1 < (1-\gamma_1)/(1-\gamma_2)$.

Two-stage monotone mechanisms succeed in Case (i).

Proof (Cont'd): (b) If seller's true type is $\theta_1 > (1-\gamma_1)/(1-\gamma_2)$,

- his expected utility under truthtelling is

$$\int_0^1 \left(\tilde{u}_1(\theta_1, \theta_2) + 0\right) d\theta_2.$$

- if he deviates to $(1-\gamma_1)/(1-\gamma_2) < \theta_1^r < 1$, trade never occur and seller obtains the same expected utility.
- if he deviates to $0 < \theta_1^r < (1-\gamma_1)/(1-\gamma_2)$, his expected utility becomes

$$\int_0^{\frac{1-\gamma_2}{1-\gamma_1}\theta_1^r} \left(\tilde{u}_1(\theta_1, \theta_2) + 0\right) d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1}\theta_1^r}^1 (0 - \psi) d\theta_2,$$

  because if trade occurs, buyer's second-stage report becomes $u_2^r = u_2^r(x^*(\theta_1^r, \theta_2), \theta_1, \theta_2) = \tilde{u}_2(\theta_1, \theta_2) \neq \tilde{u}_2(\theta_1^r, \theta_2)$. Since $\psi > 0$, it is always lower than truthtelling.
- In conclusion, seller has no incentive to deviate.

Two-stage monotone mechanisms succeed in Case (i).

Proof (Cont'd):

- We assume that seller always reports the truth in the first stage and show that buyer has no incentive to deviate in the first stage.
- If buyer reports his true type θ_2, his expected utility is

$$\int_0^{\frac{1-\gamma_1}{1-\gamma_2}\theta_2} \left(\tilde{u}_2(\theta_1,\theta_2) - \tilde{u}_2(\theta_1,\theta_2)\right) d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2}\theta_2}^{1} (0+0) d\theta_1 = 0.$$

- If buyer deviates to $\theta_2^r \neq \theta_2$, his expected utility becomes

$$\begin{aligned} & \int_0^{\frac{1-\gamma_1}{1-\gamma_2}\theta_2^r} \left(\tilde{u}_2(\theta_1,\theta_2) - \tilde{u}_2(\theta_1,\theta_2^r)\right) d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2}\theta_2^r}^{1} (0-\psi) d\theta_1 \\ = & \int_0^{\frac{1-\gamma_1}{1-\gamma_2}\theta_2^r} (\theta_2 - \theta_2^r) d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2}\theta_2^r}^{1} (0-\psi) d\theta_1, \end{aligned}$$

because if no trade occurs, seller's second-stage report becomes $u_1^r = u_1(x^*(\theta_1,\theta_2^r),\theta_1,\theta_2) = \tilde{u}_1(\theta_1,\theta_2) \neq \tilde{u}_1(\theta_1,\theta_2^r)$.

Two-stage monotone mechanisms succeed in Case (i).

- Recall that if buyer deviates to $\theta_2^r \neq \theta_2$, his expected utility becomes

$$\int_0^{\frac{1-\gamma_1}{1-\gamma_2}\theta_2^r} (\theta_2 - \theta_2^r) d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2}\theta_2^r}^1 (0 - \psi) d\theta_1.$$

- Buyer will not deviate to $\theta_2^r = \theta_2^{max} = 1$, because his expected utility becomes negative which is worse than truthtelling.
- To stop buyer from deviating, the penalty ψ must be large enough, that is, for any $0 \leq \theta_2 \leq 1$ and $0 \leq \theta_2^r < 1$,

$$\begin{aligned} 0 &\geq \int_0^{\frac{1-\gamma_1}{1-\gamma_2}\theta_2^r} (\theta_2 - \theta_2^r) d\theta_1 + \int_{\frac{1-\gamma_1}{1-\gamma_2}\theta_2^r}^1 (0 - \psi) d\theta_1 \\ \Rightarrow \psi &\geq \frac{(1-\gamma_1)(\theta_2 - \theta_2^r)\theta_2^r}{(1-\gamma_2) - (1-\gamma_1)\theta_2^r}. \end{aligned}$$

- It suffices to set

$$\psi \geq \frac{1-\gamma_1}{\gamma_1 - \gamma_2}.$$

Two-stage monotone mechanisms succeed in Case (i).

- BB is satisfied because on equilibrium path,
  - if $x^*(\theta_1, \theta_1) = 0$, then $t_1^S(\theta_1, \theta_2; u_1, u_2) = t_2^S(\theta_1, \theta_2; u_1, u_2) = 0$;
  - if $x^*(\theta_1, \theta_2) = 1$, then $t_1^S(\theta_1, \theta_2; u_1, u_2) = -t_2^S(\theta_1, \theta_2; u_1, u_2) = u_2(x^*(\theta_1, \theta_2); \theta_1, \theta_2) = \tilde{u}_2(\theta_1, \theta_2)$.
- Seller obtains a higher expected utility after participation than the outside option because for all $\theta_1 \in \Theta_1$,

$$
\begin{aligned}
& \int_0^{\frac{1-\gamma_2}{1-\gamma_1}\theta_1} (\tilde{u}_1(\theta_1, \theta_2) + 0)\, d\theta_2 + \int_{\frac{1-\gamma_2}{1-\gamma_1}\theta_1}^{1} (0 + \tilde{u}_2(\theta_1, \theta_2))\, d\theta_2 \\
> & \int_0^1 \tilde{u}_1(\theta_1, \theta_2) d\theta_2 \\
= & \; U_1^O(\theta_1).
\end{aligned}
$$

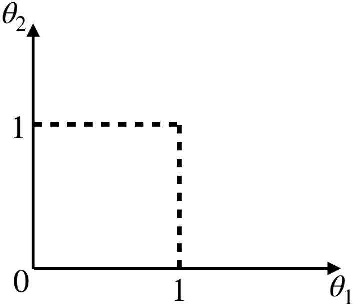
- Buyer is indifferent between participation and outside option because his expected utility after participation is zero.
- Therefore, IIR is also satisfied.

Two-stage monotone mechanisms succeed in Case (iii).

Case (iii): $0 < \gamma_2 \leq 1 \leq \gamma_1$

- We use the same mechanism (Θ, Π, x^*, t^S) as in Case (i).
- Recall

Case (iii): $0 < \gamma_2 \leq 1 \leq \gamma_1$



- Since t_1^S is independent of u_1^r and t_2^S is independent of u_2^r, each agent has no incentive to deviate in the second stage.

Two-stage monotone mechanisms succeed in Case (iii).

- We assume that buyer always reports truthfully in the first stage and show that seller has no incentive to deviate.
- If seller reports his true type θ_1, his expected utility is

$$\int_0^1 (\tilde{u}_1(\theta_1, \theta_2) + 0)\, d\theta_2.$$

- If he deviates, it is still efficient not to trade and his expected utility is the same.
- Hence, seller has no incentive to deviate.

Two-stage monotone mechanisms succeed in Case (iii).

- We assume that seller always reports truthfully in the first stage and show that buyer has no incentive to deviate.
- If buyer reports his true type θ_2, his expected utility is zero because it is efficient not to trade and he pays nothing.
- If buyer deviates to $\theta_2^r \neq \theta_2$, buyer's expected utility becomes

$$\int_0^1 (0 - \psi) d\theta_1 = -\psi < 0,$$

because trade never occurs and seller's second-stage report becomes $u_1^r = u_1(x^*(\theta_1, \theta_2^r), \theta_1, \theta_2) = \tilde{u}_1(\theta_1, \theta_2) \neq \tilde{u}_1(\theta_1, \theta_2^r)$.
- Hence, buyer has no incentive to deviate in the first stage and BIC is satisfied.

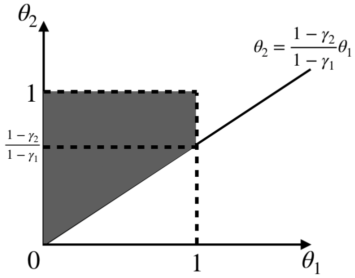
Two-stage monotone mechanisms succeed in Case (iii).

- BB is satisfied because no trade, no pay.
- IIR is satisfied because everyone's expected utility is the same as the outside option.

Two-stage monotone mechanisms violate BIC in Case (ii).

- We assume that seller always reports the true type in the first stage and both agents report their allocation payoffs truthfully in the second stage. Recall

Case (ii): $0 < \gamma_1 < \gamma_2 < 1$



- If buyer's true type is $(1-\gamma_2)/(1-\gamma_2) \leq \theta_2 \leq 1$, buyer obtains the following expected utility under truthtelling:

$$U_2(\theta_2; \theta_2) = \int_0^1 (\tilde{u}_2(\theta_1, \theta_2) + t_2(\theta_1, \theta_2; u_1, u_2))\, d\theta_1.$$

Two-stage monotone mechansims violate BIC in Case (ii).

- If he deviates to $(1-\gamma_2)/(1-\gamma_2) \leq \theta_2^r < \theta_2$, his expected utility becomes the following:

$$U_2(\theta_2; \theta_2^r) = \int_0^1 \left(\tilde{u}_2(\theta_1, \theta_2) + t_2(\theta_1, \theta_2^r; u_1, u_2)\right) d\theta_1,$$

- By monotonicty,

$$|t_2(\theta_1, \theta_2; u_1, u_2)| > |t_2(\theta_1, \theta_2^r; u_1, u_2)|,$$

or equivalently,

$$t_2(\theta_1, \theta_2; u_1, u_2) < t_2(\theta_1, \theta_2^r; u_1, u_2) \leq 0.$$

- Therefore,

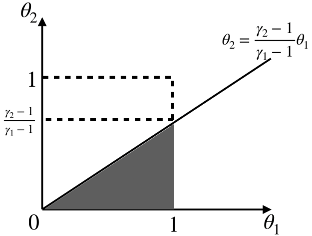
$$\begin{aligned} U_2(\theta_2; \theta_2^r) &= \int_0^1 \left(\tilde{u}_2(\theta_1, \theta_2) + t_2(\theta_1, \theta_2^r; u_1, u_2)\right) d\theta_1 \\ &> \int_0^1 \left(\tilde{u}_2(\theta_1, \theta_2) + t_2(\theta_1, \theta_2; u_1, u_2)\right) d\theta_1 \\ &= U_2(\theta_2; \theta_2). \end{aligned}$$

leading to a contradiction against BIC.

Two-stage monotone mechanisms violate BIC in Case (v).

- We assume that seller always reports the true type in the first stage and both agents report their allocation payoffs truthfully in the second stage. Recall

Case (v): $1 < \gamma_2 \leq \gamma_1$



- If buyer's true type is $(\gamma_2 - 1)/(\gamma_1 - 1) \leq \theta_2 \leq 1$, buyer obtains the following expected utility under truthtelling:

$$\int_0^1 (0 + t_2(\theta_1, \theta_2; u_1, u_2))\, d\theta_1 \leq 0,$$

by monotonicity.

Two-stage mechanisms violate BIC in Case (v).

- If buyer deviates to $\theta_2^r = 0$, it is always efficient to trade and buyer's expected utility becomes

$$
\begin{aligned}
& \int_0^1 \left(\tilde{u}_2(\theta_1, \theta_2) + t_2(\theta_1, \theta_2^r; u_1^r, u_2^r)\right) d\theta_1 \\
> & \int_0^1 \left(\tilde{u}_2(\theta_1, \theta_2^r) + t_2(\theta_1, \theta_2^r; u_1^r, u_2^r)\right) d\theta_1 \\
& (\because \theta_2 > \theta_2^r \text{ and } \tilde{u}_2 \text{ is a strictly increasing function.}) \\
\geq & \int_0^1 \left(\tilde{u}_2(\theta_1, \theta_2^r) - \tilde{u}_2(\theta_1, \theta_2^r)\right) d\theta_1 \\
& (\because x^*(\theta_1, \theta^r) = 1 \text{ implies } t_2(\theta_1, \theta_2^r; u_1^r, u_2^r) \geq -\tilde{u}_2(\theta_1, \theta_2^r) \\
= & \ 0;
\end{aligned}
$$

hence, buyer obtains a higher expected utility after deviation and BIC is violated.

The general results in two-sided asymmetric information

Assumption 1

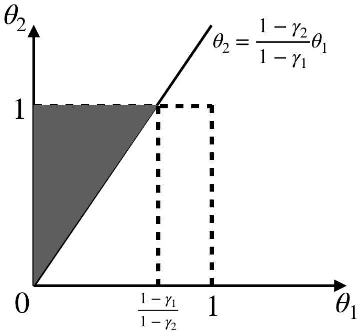
$\int_{\Theta_1} x^*(\theta_1, \theta_2) dF_1(\theta_1) < 1$ *for all* $\theta_2 < \theta_2^{max}$.

Theorem 3

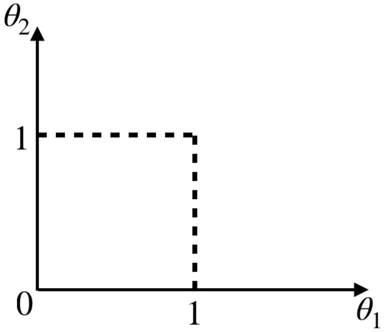
When both agents have non-trivial sets of types, there exists a two-stage monotone mechanism satisfying BIC, IIR, EFF and BB if and only if Assumption 1 is satisfied.

Assumption 1 is satisfied in Case (i) and (iii) in Example 2.

Recall that in Case (i) and (iii), there exists a two-stage monotone mechanism satisfying BIC, IIR, EFF and BB.

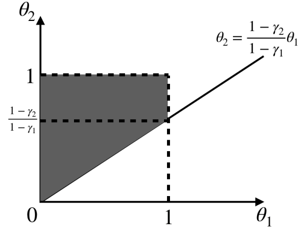


Case (i): $0 < \gamma_2 \leq \gamma_1 < 1$

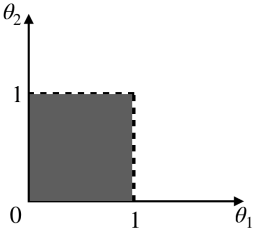


Case (iii): $0 < \gamma_2 \leq 1 \leq \gamma_1$

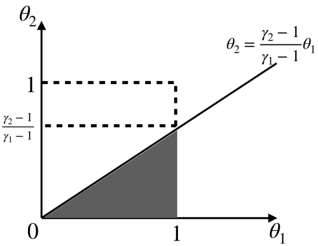
Assumption 1 is violated in the other cases in Example 2.



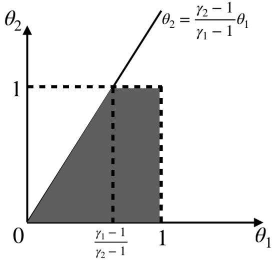
Case (ii): $0 < \gamma_1 < \gamma_2 < 1$



Case (iv): $0 < \gamma_1 < 1 < \gamma_2$



Case (v): when $1 < \gamma_2 \leq \gamma_1$



Case (vi): when $1 < \gamma_1 < \gamma_2$

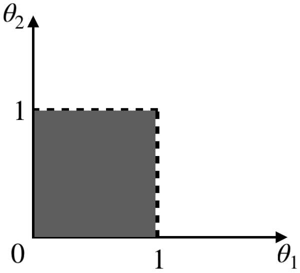
How restrictive is Assumption 1?

Consider linear valuation function $u_i(\theta_i, \theta_{-i}) = \theta_i + \gamma_i \theta_{-i}$ where $\gamma_i > 0$. Then

Numbers	Different cases	Is Assumption 1 satisfied?
1	$\gamma_1 < 1, \gamma_2 < 1$, and $(1-\gamma_2)/(1-\gamma_1) \geq \theta_2^{max}/\theta_1^{max}$	✓
2	$\gamma_1 < 1, \gamma_2 < 1$, and $(1-\gamma_2)/(1-\gamma_1) < \theta_2^{max}/\theta_1^{max}$	X
3	$\gamma_1 \geq 1$ and $\gamma_2 \leq 1$	✓
4	$\gamma_1 < 1$ and $\gamma_2 > 1$	X
5	$\gamma_1 > 1, \gamma_2 > 1$, and $\theta_1^{min} = 0$	✓
6	$\gamma_1 > 1, \gamma_2 > 1, \theta_1^{min} > 0$ and $(\gamma_2 - 1)/(\gamma_1 - 1) < \theta_2^{min}/\theta_1^{min}$	✓
7	$\gamma_1 > 1, \gamma_2 > 1, \theta_1^{min} > 0$ and $(\gamma_2 - 1)/(\gamma_1 - 1) \geq \theta_2^{min}/\theta_1^{min}$	X

What if Assumption 1 is violated?

- Does there exist a two-stage non-monotone mechanism satisfying BIC, IIR, EFF and BB? Yes!
- Example: $\tilde{u}_1(\theta_1, \theta_2) = \theta_1 + 0.5\theta_2$ and $\tilde{u}_2(\theta_1, \theta_2) = \theta_2 + 3\theta_1$ for all $(\theta_1, \theta_2) \in [0, 1]^2$.
- Note that $\tilde{u}_2(\theta_1, \theta_2) - \tilde{u}_1(\theta_1, \theta_2) = 0.5\theta_2 + 2\theta_1 \geq 0$ for all $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$. Hence, Assumption 1 is violated.
- There exists a two-stage mechanism with the fixed-payment scheme $\bar{t}_1 = -\bar{t}_2 = 1.25$ satisfying BIC, IIR, EFF and BB.



Concluding Remarks

- Under one-sided asymmetric information structure, the generalized two-stage Groves mechanism always satisfies IIR.
- Under two-sided asymmetric information structure,
  - we show by an example that it never satisfies IIR;
  - we propose the two-stage monotone mechanisms which satisfy IIR in a positive number of cases within the same example;
  - we characterize the existence of two-stage monotone mechanisms satisfying BIC, IIR, EFF and BB.