Conflicting Interest Groups, Contentious Public Goods, and Cooperation

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Abstract

This paper investigates how the arms race and cooperation effects caused by the political activities of conflicting groups affect the cooperation–noncooperation choice of allied members by using a two-state game model where the preferences of the groups on the issue are divergent but the size of the groups is the same. A natural conjecture is that cooperation within a group is desirable to internalize the free-riding behavior of group members. It is shown that if the members of a group are less crucially dependent on an issue compared with the members of their rival group, their cooperative behavior may be desirable owing to the cooperation effect. On the contrary, if they are more crucially dependent on the issue, their noncooperative behavior may be desirable owing to the arms race effect, which is a negative response from the adversarial group. Hence, a Nash equilibrium is likely where members of the latent group cooperate but those of the vital group do not cooperate.

Key words: conflicting interest groups, contentious public goods, different preferences, cooperation effect, arms race effect

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1. Introduction

Spending on political activities by an interest group is a good example of public goods provision where one agent's supply of lobbying activities may be a substitute for another allied agent's supply within the same interest group. It is widely recognized that each member of an interest group has an incentive to free ride on spending on the political activities of other allied members. In their classical paper, Olson and Zeckhauser (1966) applied the theory of private provision of public goods to allied agents and concluded that all allied agents gain when they determine the level of spending on lobbying activities cooperatively. When the size of the group is large, the gains from cooperation would also be large. They highlighted the importance of allied cooperation in setting the spending on lobbying activities. We call this the cooperation effect.

Conflicting interest groups often compete on the same issue. When two or more conflicting interest groups engage in lobbying activities, an adversary's spending would rise in response to an increase in spending by the alliance. We call this the arms race effect, which would apply to various areas of contentious public goods such as political activities, environmental issues, and defense spending. Bruce (1990) first pointed out that cooperation among allies in setting their defense spending is not necessarily welfare improving because of the arms race effect. Ihori (2000) showed that when the number of countries in one bloc is larger than that in another bloc, the countries in the larger bloc might be better off by cooperating than by not cooperating even if there is a negative spillover from the adversarial smaller bloc. In this sense, group size divergence does matter in cooperation–noncooperation issues.

In reality, a latent group such as consumers and a vital group such as producers would have both different sizes and preferences. Consider, for example, the issue of whether a particular market is opened up to foreign producers. For the relevant consumer group, the number of members is large but the policy issue is not vital, whereas for the producer group in domestic competition, the number of members is small but the policy issue is vital. We could also consider an interregional transfer between urban and rural areas. It seems that the issue is not vital for people in urban areas but is crucial for those in rural areas. It is often observed that a small number of vital agents form a powerful and well-organized pressure group to cooperate, whereas a large number of latent agents do not always form a powerful pressure group. Two explanations could hold for this. First, since the size of the former group is small, it may be easy for them to internalize the free-riding incentives. Second, since the former group has a vital interest in the issue, a cooperative outcome would be desirable.

It is useful to investigate the effects of divergent sizes and different preferences separately. As a vital group often consists of a small number of members, a cooperative outcome could be attained from the divergent size effect. In the present paper, the two conflicting interest groups have different preferences over the relevant issue but the size of the groups is the same. Thus, we do not consider differences in group size in this paper. We examine the plausibility of the conventional conjecture that a vital group cooperates whereas a latent group does not. By developing a simple multi-agent model of two conflicting interest groups with the same size, we explore an interesting counterexample; a plausible Nash outcome is that the latent group cooperates but the vital group does not cooperate.

For example, we may apply this analytical result to the case of an intergenerational redistribution issue such as public pension reform. If the population is stationary, the group size of the elderly and the working people would be almost the same, and hence, we may ignore the differences in group size on this issue. On the contrary, the elderly and the working generations may have different preferences on pension reform. We may assume that the elderly have a vital interest on the issue but the younger generations do not. In reality, the elderly do not often organize a strong political body to seek more pension benefits whereas working people usually organize a strong political body such as a labor union, which would resist paying more contributions to support pension benefits. Even if the elderly form the vital group in this case, it seems that they do not often have an incentive to organize a strong political party, the counterpart of a labor union.

The organization of this paper is as follows. Section 2 develops a simple analytical framework of a two-stage game. Section 3 examines the pay-offs of noncooperative and cooperative Nash solutions in the second stage of the game. Section 4 investigates a plausible outcome by comparing the cooperative and noncooperative payoffs. Finally, section 5 concludes the paper. Our analysis provides one reason why people in the vital groups do not cooperate and those in the latent groups cooperate.

2. Analytical Framework

Consider a simple competition model in which two groups compete for political power. Assume that there are $n + n$ agents and two opposing interest groups, α and β, in the economy. Each interest group consists of n allied agents. Agent i's utility function is given by

$$U^i = U^i(c_i, G) \tag{1}$$

where U^i is the welfare of agent i, c_i the private consumption of agent i, and G the benefit of a particular policy of agent i; G could reflect a degree of deregulation for a given market, or be regarded as public goods mainly for group α and as public bads for group β. An increase in G would benefit interest group α but hurt interest group β. In order to win the political game, both groups may spend resources to lobby legislators, influence voters, etc., and persuade them to vote to either raise G or reduce G. Thus, one group's public good is another's public bad, and each group can take action to shift the total quantity toward its own preferred level.

We formulate that the actual level of G is determined by

$$G = G(\sum_{i\in\alpha} g_i, \sum_{i\in\beta} g_i) = \sum_{i\in\alpha} g_i - \sum_{i\in\beta} g_i \tag{2}$$

where g_i is the amount of lobbying activities provided by agent i. Following the seminal studies of Tullock (1980) and Becker and Mulligan (1998), the outcome of political conflict/contest between groups α and β is summarized by a modified version of contest success function, Eq.(2). The conflict/contest involved is presumably complicated, but a key factor used to determine the "output" of the conflict/contest is the "input" expended by the players. Function (2) is a reduced-form end result of what may be a very complicated process of electoral voting, legislative decision-making, and executive branch initiatives in a democratic regime, or some complicated process of political influence in a nondemocratic regime. In this reduced-form end result, the size of G directly depends on the amount spent by both groups to gain political influence. We formulate that the outcome of the political conflict/contest is a function of the difference between the political spending of players. More pressure by group α increases the size of G, whereas more pressure by group β decreases it. In order to simplify our analysis, we assume that it is the net of the pressures applied by the groups that determines the actual policy size, G.[1] Function (2) exhibits the property of homogeneity of degree one such that the same

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[1] A contest success function usually yields the probability of winning or losing. This formulation may then be justified if both the contesting groups are concerned with the expected division of tax revenue. Alternatively, following Long and Vousden (1987) and others, the contest success function may be given a non-probabilistic interpretation: players expend resources to compete for a share of divisible rent rather than for the entire indivisible rent, and therefore the relative share of vested rights is allocated according to the relative size of the players' political efforts. Our formulation follows the latter approach in that the initial level of vested rights is affected by the net pressure, G.

proportional increase or decrease in g_α and g_β raises the conflict/contest outcome by the same magnitude.

If we denote by A the initial level of vested rights for agents, $2A$ would mean the total amount of "pie." An increase in g_α at the given g_β results in an increased distribution of pie, $2A$, in favor of group α but against group β, and vice versa. Put in another way, a given amount of $2A$ is allocated according to the net pressure G. If G > 0, group α can get more than A, and vice versa.

For simplicity, we assume that the utility function is specified in a Cobb–Douglas form for each agent.

$$U^{\alpha i} = c_{\alpha i}^{1-\theta}(A+G)^{\theta} \tag{3-1}$$

$$U^{\beta i} = c_{\beta i}^{1-\mu}(A-G)^{\mu} \tag{3-2}$$

where superscript (subscript) α or β denotes the interest group that each agent belongs to. Variable θ is assumed to be relatively small for group α, whereas μ is assumed to be relatively large for group β. Also, $1 > \mu > \theta > 0$. G is not vital for group α, but it is vital for group β. We call group α the latent group and group β the vital group. Condition $A > 0$ is incorporated into Eq. (3-2) so that $A-G > 0$. It is true that the Cobb–Douglas functional form is very restrictive. However, in order to obtain concrete results and provide a counterexample against the conventional conjecture, this formulation is useful as a first step of this research.

Agent i's budget constraint can be given by

$$c_i + g_i = Y_i \tag{4}$$

where Y_i is the exogenously given income of agent i. For simplicity we assume that $Y_\alpha = Y_\beta = Y$. Production technologies are linear and identical across all agents, and units are chosen such that the constant marginal rate of transformation between c_i and g_i for all agents is unity.

If the issue is whether a particular market is opened up to foreign producers, group α may represent a general consumer group and group β a special producer group. In the example of public pension reform, group α is the younger generation group and β the elderly group. G may reflect the degree of privatization of public pension, or $-G$ may capture the size of pension benefits. In order to exclude the effects of differences in group size, we assume that both groups have the same number of members. By doing so, we focus on the implications of differences in preferences.[2]

The structure of the game is as follows:

Stage I: An agent of each interest group determines whether to cooperate or not within the group.

Stage II: The agent determines her spending on political activities, and a Nash equilibrium is obtained.

Since all the agents in an interest group are identical, they behave in the same way. We do not consider the free-riding behavior within each group in the cooperative case[3]. Therefore, each agent may commit herself to the cooperation decision in the cooperative case. Thus, in stage I the agents uniformly decide whether to cooperate or not within the group. When cooperation is chosen, each group determines a representative who will then decide the per-capita contribution from inside the group in stage II. In case of noncooperation, each member will determine her own contribution noncooperatively in stage II. We do not consider cooperation between the conflicting two groups.

Cornes and Rubbelke (2012) use a formulation similar to G as Eq. (2) and investigate the contentious public characteristics. They present conditions under which the existence of a unique noncooperative equilibrium is retained and analyze its normative and comparative static properties. They showed an interesting twist on the proposition of neutrality: resource growth may be entirely dissipated by conflict over a public characteristic (see section 3.1).

3. Second Stage

3.1 Noncooperative Case

G is determined as a Nash equilibrium of a "game" between two interest groups. First, we investigate the noncooperative case where each agent determines her own spending on lobbying activities in stage II, treating the rest of the allied agents' spending on lobbying activities as given. In other words, alliances within the same

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[2] Ihori (2000) analyzed the arms race effect in a case of differences in group size.

[3] It is important to explore mechanisms that emerge to address the free-rider problem in a setting in which no central authority exists to induce cooperation. Guttman (1978) showed how extending the voluntary-contribution model to a two-stage setting could have profound consequences by using the special case of quasi-linear preferences. Danziger and Schnytzer (1991) showed that the result could be generalized to more preferences and any number of agents. Boadway et al. (2007) considered the scope for noncooperative multistage games in a standard voluntary contribution setting to find improvement in the efficiency of public good supply. They showed that in some cases the free-rider problem could be overcome fully.

group do not make any cooperative decisions with respect to spending on allied lobbying activities but behave at Nash conjectures.[4]

We now derive a reaction function of agent i of interest group α. Agent i maximizes (3-1) subject to her budget constraint

$$c_{\alpha i} + G_\alpha + A = Y_\alpha + (n-1)g_{\alpha j} - ng_\beta + A \tag{5}$$

taking the lobbying activities of other agents $g_{\alpha j}, g_\beta$ as given. Here, $g_{\alpha j}$ denotes the spending on lobbying activities by agent j ($\neq$ i) of interest group α, and g_β denotes the spending on lobbying activities by any identical agent of interest group β.

From the first-order condition, we have

$$g_{\alpha i} + (n-1)g_{\alpha j} - ng_\beta + A = \theta[Y_\alpha + (n-1)g_{\alpha j} - ng_\beta + A]$$

Since all the agents of interest group α are identical, we have $g_{\alpha i} = g_{\alpha j} = g_\alpha$ at any solution. Substituting $g_{\alpha i} = g_{\alpha j} = g_\alpha$ into the above equation, we finally obtain

$$g_\alpha = \frac{1}{n-(n-1)\theta}[\theta Y_\alpha - (1-\theta)A + (1-\theta)ng_\beta] \tag{6}$$

which is a reduced reaction function of each agent belonging to interest group α.

Variable g_α is an increasing function of the agent's own income and the spending on lobbying activities by the rival agents. An increase in θ will raise g_α, which is intuitively plausible. Eq. (6) also includes the arms race response of g_α to g_β,

$$dg_\alpha / dg_\beta = \frac{(1-\theta)n}{n-(n-1)\theta},$$

which is positive and decreasing with θ from 1 at θ = 0 to 0 at θ = 1. In other words, if θ is small, an increase in g_β will induce a large increase in g_α. The intuition is as follows. When θ is small, a change in the real income of group α would not affect the demand for G to a great extent, and hence, a decrease in real

[4] As discussed in detail by Bergstrom et al. (1986) and Andreoni (1988), a nonnegativity constraint on providing public goods may well be binding as a solution if the number of rival agents becomes large. In order to present the results in the simplest way and in their strongest form, we consider only the case where the nonnegativity constraints are nonbinding in equilibrium.

income due to an increase in g_β induces little decrease in the demand for *G*. On the contrary, it would reduce private consumption to a great extent, resulting in a large increase in g_α.

Similarly, the reaction function of agent i of interest group β can be given as

$$g_\beta = \frac{1}{n-(n-1)\mu}[\mu Y_\beta - (1-\mu)A + (1-\mu)ng_\alpha] \tag{7}$$

An increase in μ would raise g_β, which is intuitively plausible.

Henceforth, we call agent α (or β) the representative agent of interest group α (or β). In Figure 1, curve Y represents agent α's reaction curve, and curve X represents agent β's reaction curve. Both curves are upward sloping. Spending on lobbying activities is a strategic complement reflecting the arms race between rival interest groups. An intersection of both curves N represents the noncooperative Nash equilibrium point.

From Eqs. (6) and (7), the lobbying activities for both agents can be respectively given as

$$g_\alpha = \frac{[n-(n-1)\mu][\theta Y_\alpha - (1-\theta)A] + (1-\theta)n[\mu Y_\beta - (1-\mu)A]}{[n-(n-1)\theta][n-(n-1)\mu] - (1-\theta)(1-\mu)n^2} \tag{8-1}$$

$$g_\beta = \frac{[n-(n-1)\theta][\mu Y_\beta - (1-\mu)A] + (1-\mu)n[\theta Y_\alpha - (1-\theta)A]}{[n-(n-1)\theta][n-(n-1)\mu] - (1-\theta)(1-\mu)n^2} \tag{8-2}$$

When the marginal valuation of the issue for the rival interest group μ increases, the spending on lobbying activities g_α also increases. Since an increase in the marginal valuation of the rival interest group β would raise its political spending, g_β, this will raise threats to the agents of the other interest group α, giving rise to the arms race effect. Thus, both g_α and g_β would increase with θ and μ. G is also increasing with θ but is decreasing with μ.

Thus, we have

$$G + A = \frac{\theta A[2(n-\mu)+\mu]}{n(\theta+\mu-2\theta\mu)+\mu\theta},$$

$$c_\alpha = \frac{(1-\theta)A[2(n-\mu)+\mu]}{n(\theta+\mu-2\theta\mu)+\mu\theta},$$

$$U^\alpha = \frac{(1-\theta)^{1-\theta}\theta^\theta A[2(n-\mu)+\mu]}{[n(\theta+\mu-2\theta\mu)+\mu\theta]^2},$$

$$U^{\beta}=\frac{(1-\mu)^{1-\mu}\mu^{\mu}A[2(n-\theta)+\theta]}{[n(\theta+\mu-2\theta\mu)+\mu\theta]^{2}} \tag{9}$$

As shown in Eq. (9), the marginal rate of substitution of $G+A$ with respect to c_{α} equals 1, which is the marginal cost of providing political pressure. This is the well-known result of noncooperative solutions on public goods within a group. The same applies to agent β. Eq. (9) also suggests that the welfare of each agent decreases with a rival's marginal valuation of the issue, whereas it may well increase with the agent's own marginal valuation of the issue. This is intuitively plausible.

The actual level of policy, G, increases with the number of allied agents of group α; this is consistent with McGuire (1974). It is also interesting to note that the real variables including G, c, and U are independent of Y if $Y_{\alpha}=Y_{\beta}=Y$. More precisely, the net income, $n(Y_{\alpha}-Y_{\beta})$, matters in the provision of contentious public goods. As shown in Cornes and Rubbelke (2012), a net increase in the resources available to the economy $dY_{\alpha}=dY_{\beta}>0$ may have no real consequences in the provision of contentious public goods. This is the so-called super neutrality result, which holds in a general functional form of the utility function. It should also be noted that a redistribution between two conflicting groups does have a real impact. A giving group loses, whereas a receiving group gains. On the contrary, a redistribution within a group does not have real effects. The conventional neutrality result holds within each group.

3.2 Allied Cooperation

We now consider a cooperative case where the allied agents cooperate within their interest group. Note that there is still no cooperation (or negotiation) between the two conflicting groups.

Consider the joint optimization problem of representative agent α. Adding Eq. (4) up to n and considering $g_{\alpha i}=g_{\alpha j}=g_{\alpha}$, the agent's consolidated budget constraint may be written as

$$nc_{\alpha}+G_{\alpha}+A=nY_{\alpha}-ng_{\beta}+A \tag{10}$$

Thus, agent α jointly maximizes (3-1) subject to the above consolidated budget constraint (10), taking g_β as given. From the first-order condition, we have

$$ng_\alpha - ng_\beta + A = \theta(nY_\alpha - ng_\beta + A)$$

Thus, the reaction function of agent α can be given as

$$g_\alpha = \theta Y_\alpha - \frac{1-\theta}{n}A + (1-\theta)g_\beta \tag{11}$$

Eq. (11) implies that $\frac{\partial g_\alpha}{\partial g_\beta} = 1-\theta > 0$, which decreases with θ. This property is qualitatively the same as in the noncooperative case. When θ is low and the latent group α does not recognize the benefit of G to a great extent, it is desirable for group α not to change $A + G$ to a great extent. In order to reduce $A + G$ to a small extent, group α would raise its spending on political activities to a great extent when group β increases its spending on political activities.

Note that if $n = 1$, Eq. (11) reduces to Eq. (6). When $n > 1$, dg_α / dg_β, the slope of the reaction function, $(1-\theta)$, is less than $(1-\theta)$ n/ $\{n-(n-1)\theta\}$ in the noncooperative case. Thus, when the adversarial group β raises the level of political pressure, the agents of rival group α would react by spending less in the cooperative case than in the noncooperative case. This is because in the cooperative case, a member of group α would incorporate the positive reaction of other allied members when g_β increases, resulting in a smaller increase in g_α than in the noncooperative case, where she does not consider the positive reaction of other allied members. We also need to note that g_α is higher in the cooperative case than in the noncooperative case at the same level of g_β, since a cooperative behavior can internalize the free-riding motive. If $\theta = 1$, Eq. (11) again reduces to Eq. (6). In other words, if θ is large, the gap between cooperative g_α and noncooperative g_α becomes small.

Similarly, the reaction function of agent β in the cooperative case can be given as

$$g_\beta = \mu Y_\beta - \frac{1-\mu}{n}A + (1-\mu)g_\alpha \tag{12}$$

Again, if $n = 1$, Eq. (12) reduces to Eq. (7). If μ is large, the gap between cooperative g_β and noncooperative g_β is small.

There are three cases where cooperation is chosen in at least one group. First, let us investigate the cooperative case where all the agents of interest group α as well as interest group β cooperate. In Figure 1, curve S represents the reaction curve of agent α when all the agents of interest group α cooperate, and curve T represents the reaction curve of agent β when all the agents of interest group β cooperate. The intersection of curves S and T, denoted by point C, corresponds to the cooperative case where both interest groups α and β cooperate respectively. The cooperative equilibrium levels of g_α, g_β are respectively given as

$$g_\alpha = \frac{\theta Y_\alpha + \mu(1-\theta)Y_\beta - (1-\theta)A(2-\mu)/n}{\mu+\theta-\mu\theta} \tag{13-1}$$

$$g_\beta = \frac{\mu Y_\beta + \theta(1-\mu)Y_\alpha - (1-\mu)A(2-\theta)/n}{\mu+\theta-\mu\theta} \tag{13-2}$$

An increase in θ or μ raises the political activities of both groups, g_α, g_β, and G is increasing with θ but decreasing with μ. An increase in μ lowers the welfare of group α by reducing *G*, and an increase in θ lowers the welfare of group β by raising G. Thus, we have

$$G + A = \frac{\theta A(2-\mu)}{\mu+\theta-\mu\theta}, \qquad c_\alpha = \frac{(1-\theta)A(2-\mu)}{(\mu+\theta-\mu\theta)n},$$
$$U^\alpha = \frac{(1-\theta)^{1-\theta}\theta^\theta}{(\mu+\theta-\mu\theta)^2}\frac{A(2-\mu)}{n^{1-\theta}},$$
$$U^\beta = \frac{(1-\mu)^{1-\mu}\mu^\mu}{(\mu+\theta-\mu\theta)^2}\frac{A(2-\theta)}{n^{1-\mu}} \tag{14}$$

As shown in Eq. (14), the total marginal rate of substitution of $G + A$ with respect to c_α equals 1, which is the marginal cost of providing political pressure. This condition is nothing but the Samuelson rule on public goods within the group, which is the well-known result of cooperative solution. Although the Samuelson rule holds for each interest group, it does not apply to the overall world. Since there is no cooperation between two conflicting groups, the cooperative solution within the group here cannot attain the first-best result. The second-best theory suggests that the second-best utility is not necessarily higher at the cooperative solution than at the noncooperative solution.

We may also consider the partial cooperative case where the agents of interest group α do not cooperate whereas the agents of interest group β cooperate. In this case, agent α's reaction curve can be given as Eq. (6) while agent β's reaction curve can be given as Eq. (12). We may also consider the partial cooperative case where the agents of interest group α cooperate while the agents of interest group β do not cooperate. We need to note that the net income $n(Y_\alpha - Y_\beta)$ matters in the provision of contentious public goods. Thus, the neutrality and super-neutrality results hold in the three cooperative cases as well.

4. First Stage

Table 1 indicates the hypothetical payoffs of four cases in the second stage of the game: (1) either group α or group β does not cooperate at point N, (2) group α cooperates while group β does not cooperate at point P, (3) group α does not cooperate while group β cooperates at point Q, and (4) both groups α and β cooperate at point C.

Now, we can investigate the Nash equilibrium by comparing four possible payoffs at the second stage: the noncooperative payoffs where no agents cooperate and the three cooperative payoffs where at least some allied cooperation occurs in interest groups α and/or β.

In order to internalize the positive spillover effect between members within the same interest group, the agents of the same interest group, say α, should choose a representative, or have an agreement to determine lobbying activities cooperatively. By doing so, the interest group's spending on lobbying activities is stimulated and benefits all the agents of the group. This is the cooperation effect. However, the members of the rival group β would react by raising their lobbying activities, which would hurt the agents of group α. We call this the negative spillover of the arms race effect. If the negative spillover due to the arms race effect outweighs the positive spillover due to the cooperation effect, such cooperation hurts group α. This possibility was first pointed by Bruce (1990) in the study of national defense. Ihori (2000) showed that the cooperation effect might well dominate the arms race effect when the number of allied members is larger than the number of rival members.

By excluding the differences in group size, the present paper focuses only on the differences in preferences between the two groups; θ is smaller than μ. Let us first investigate the optimal strategy of group α in stage I. Suppose group β cooperates. As shown in Table 1, it is desirable for group α to cooperate if and only if

$$D = n^{1-\theta}(\mu+\theta) - \mu\theta n^{1-\theta} - (\theta + \mu n - \theta n \mu)$$
$$= \mu n(n^{-\theta} - 1)(1-\theta) + \theta(n^{1-\theta} - 1) < 0$$

Since $n^{-1} < n^{-\theta} < 1$, this sign could be negative when μ is relatively large. Table 2 (i) suggests that D becomes negative if $\mu > 0.5$ for $n = 3$. In other words, if μ is relatively large, group α gains by cooperating within the group.

Suppose now that group β does not cooperate. Then, it is desirable for group α to cooperate if and only if

$$E = \mu n^{1-\theta} + \theta n^{2-\theta} - \theta n^{2-\theta}\mu - n(\theta + \mu - 2\theta\mu) - \mu\theta$$
$$= \mu n(n^{-\theta} - 1) + \theta n(n^{1-\theta} - 1) + \theta\mu(2n - 1 - n^{2-\theta}) < 0$$

This sign could be negative when μ is relatively large, since $2n < n^{2-\theta} + 1$ for a small θ. Table 2 (ii) shows that E becomes negative if $\mu \geq 0.9$ for n = 3. Table 2 (iii) further shows that E becomes negative if $\mu \geq 0.8$ for $n = 6$. In other words, if μ is relatively large, it is always desirable for group α to cooperate within the group. Hence, cooperation is the dominant strategy for group α when μ is relatively large.

Table 3 compares the payoffs of group β in the cooperative case and noncooperative case. Table 3 (i) shows that D is positive if $\theta \leq 0.7$ for $n = 3$. In other words, if θ is relatively small, group β gains by not cooperating within the group. Table 3 (ii) shows that E is positive if $\theta \leq 0.5$ for $n = 3$. Table 3 (iii) further shows that E becomes positive if $\theta \leq 0.3$ for $n = 6$. In other words, if θ is relatively small, it is desirable for group β not to cooperate within the group. Therefore, noncooperation is the dominant strategy for group β when θ is relatively small. Hence, a Nash outcome is likely that the latent group cooperates and the vital group does not cooperate.

The intuition is as follows. When μ is relatively large, the effect of an increase in g_α on g_β is small. In such a case, an increase in the political activities of group α due to cooperation within the group would not stimulate political activities in the rival group β to a great extent. This is because the gap between cooperative g_β and noncooperative g_β would be small when μ is relatively large. Hence, the arms race effect is small and group α gains by cooperating. Furthermore, when θ is relatively small, the negative spillover from an increase in the political activities of group β would not hurt group α to a great extent. In such a case, group α would not lose to a great extent from an arms race reaction by rival group β. In other words, if θ is relatively small, the cooperation effect dominates the arms race effect for group α. Qualitatively, the opposite mechanism would apply to group β if μ is relatively large. That is, the arms race effect may dominate the cooperation effect for group β.

In the real economy, it is often observed that a small number of agents form a powerful and well-organized pressure group to cooperate within groups, whereas a large number of agents do not always build a powerful pressure group. It is true that group size divergence has an important role since it affects the size of the cooperation effect. In addition to this, a natural conjecture is that since the former group has a vital interest in the issue, a cooperative strategy would be desirable although it stimulates political activities of the rival group, and it would be desirable for the latter group not to cooperate since it has a latent interest in the issue. One could argue that if the issue is vital, cooperation becomes desirable.

Our analytical result suggests that the above conjecture would not necessarily be valid if the divergent group size effect is controlled for. We have shown that the arms race effect is large for the vital group but not for the latent group. Hence, the arms race effect would dominate for the vital group whereas the cooperation effect would dominate for the latent group. The cooperative behavior of the vital group might stimulate the offsetting political activities of the latent rival group to a great extent, hurting the vital group very much. The opposite mechanism would apply to the latent group. Hence, a Nash outcome may well be that the latent group cooperates and the vital group does not cooperate. In reality, the interest groups on the intergenerational issue of public pension reform may have such features.

5. Conclusion

This paper investigated the cooperation and arms race effects of noncooperative and cooperative spending on the political activities of the allied members of two rival interest groups with the same group size. We have shown that if the two rival groups' preferences on an issue are different, a Nash outcome is likely that the latent group cooperates and the vital group does not cooperate. As for the vital group, the noncooperative supply of political activities is close to the cooperative level so that the gains from cooperative behavior may not be large, and hence, the noncooperative (free-riding) behavior of each member would not hurt the vital group to a great extent compared with the cooperative choice. Moreover, it could benefit the other allied members by depressing the arms race effect. On the contrary, as for the latent group, the noncooperative (free-riding) behavior would hurt each member to a great extent since the gap between the cooperative and noncooperative levels of political activities are large. The vital group may lose much by cooperation since it would induce considerable arms race activities from the latent agents. We have shown that the arms race effect may be dominant for the vital group and that the cooperative effect may be dominant for the latent group.

In the case of public pension reform, it is often observed that the elderly do not organize a strong political body to seek more benefits whereas the working generations organize a powerful political body such as a labor union, which would resist paying more contributions to support pension benefits. Although the Cobb–Douglas formulation is rather restrictive, our simple model may explain this outcome by providing a numerical example. It is true that this model does not endogenize political group membership. This is a topic of interest for future research.

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Table 1

		Group β	
		N	C
Group α	N	$\frac{2(n-\mu)+\mu}{\Delta}, \frac{2(n-\theta)+\theta}{\Delta^*}$	$\frac{2-\mu}{\Phi}, \frac{2n(1-\theta)+\theta}{n^{1-\theta}\Phi^*}$
	C	$\frac{2n(1-\mu)+\mu}{n^{1-\theta}\Lambda}, \frac{2-\theta}{\Lambda^*}$	$\frac{2-\mu}{n^{1-\theta}\Gamma}, \frac{2-\theta}{n^{1-\mu}\Gamma^*}$

Notes: N means noncooperation, and C means cooperation.

$$\Delta \equiv \frac{[n(\theta+\mu-2\theta\mu)+\theta\mu]^2}{(1-\theta)^{1-\theta}\theta^{\theta}A}$$

$$\Delta^* \equiv \frac{[n(\theta+\mu-2\theta\mu)+\theta\mu]^2}{(1-\mu)^{1-\mu}\mu^{\mu}A}$$

$$\Phi \equiv \frac{(\theta+\mu n-n\theta\mu)^2}{(1-\theta)^{1-\theta}\theta^{\theta}A}$$

$$\Phi^* \equiv \frac{(\theta+\mu n-n\theta\mu)^2}{(1-\mu)^{1-\mu}\mu^{\mu}A}$$

$$\Lambda \equiv \frac{(n\theta+\mu-n\theta\mu)^2}{(1-\theta)^{1-\theta}\theta^{\theta}A}$$

$$\Lambda^* \equiv \frac{(n\theta+\mu-n\theta\mu)^2}{(1-\mu)^{1-\mu}\mu^{\mu}A}$$

$$\Gamma \equiv \frac{(\theta+\mu-\theta\mu)^2}{(1-\theta)^{1-\theta}\theta^{\theta}A}$$

$$\Gamma^* \equiv \frac{(\theta+\mu-\theta\mu)^2}{(1-\mu)^{1-\mu}\mu^{\mu}A}$$

Table 2 (i). Value of D for n = 3

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.12	-0.16	-0.19	-0.20	-0.20	-0.18
0.8		-0.11	-0.13	-0.14	-0.14	-0.12
0.7			-0.07	-0.08	-0.07	-0.07
0.6				-0.01	-0.01	-0.01
0.5					0.05	0.05
0.4						0.11
(μ)						

Table 2 (ii). Value of E for n = 3

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.08	-0.11	-0.12	-0.13	-0.13	-0.11
0.8		-0.00	0.00	0.01	0.01	0.01
0.7			0.12	0.14	0.15	0.14
(μ)						

Table 2 (iii). Value of E for n = 6

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.31	-0.40	-0.48	-0.51	-0.50	-0.44
0.8		-0.03	-0.01	0.03	0.07	0.11
0.7			0.46	0.57	0.64	0.66
(μ)						

Table 3 (i). Value of D for n = 3

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.00	0.04	0.08	0.11	0.15	0.19
0.8		0.07	0.14	0.22	0.29	0.37
0.7			0.20	0.31	0.41	0.51
0.6				0.37	0.50	0.062
0.5					0.55	0.69
0.4						0.70
(μ)						

Table 3 (ii). Value of E for n = 3

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.05	-0.03	-0.01	0.01	0.03	0.05
0.8		-0.05	-0.01	0.02	0.06	0.09
0.7			-0.02	0.03	0.08	0.13
0.6				0.04	0.10	0.16
(μ)						

Table 3 (iii). Value of E for n = 6

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.21	-0.16	-0.11	-0.06	-0.02	0.03
0.8		-0.29	-0.20	-0.11	-0.02	0.07
0.7			-0.27	-0.14	-0.02	0.11
0.6				-0.16	-0.00	0.15
(μ)						

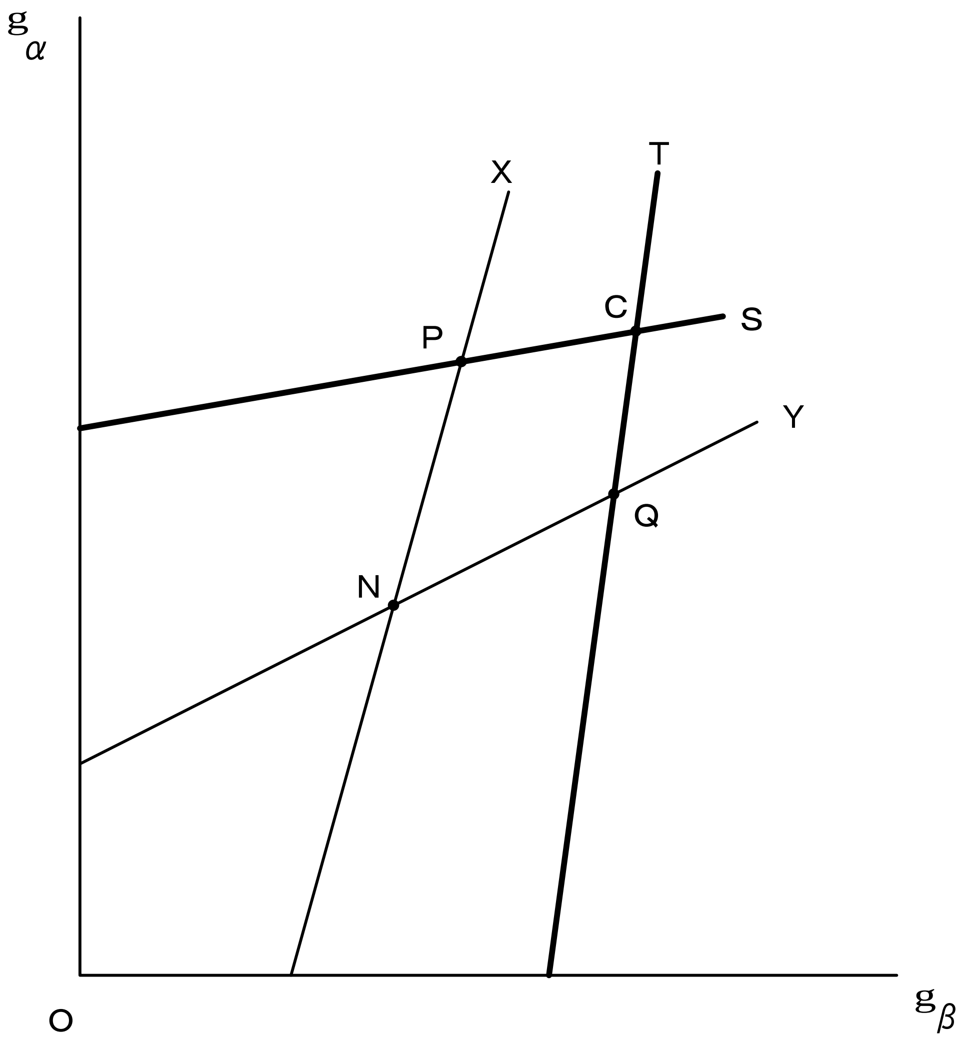

Figure 1 Four Nash Equilibria