Public-Private Mix of Health Expenditure: A Political Economy and Quantitative Analysis*

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Abstract

This paper constructs a simple overlapping generations model to examine how the choice of public and private health expenditure is affected by preferences and economic factors under majority voting. In the model, agents with heterogeneous income decide how much to consume, save, and spend on health care, and vote for the income tax to be used to finance public health expenditure. Agents’ survival probabilities are endogenously determined by a CES composite of public and private health expenditure. We establish the existence and uniqueness of the voting equilibrium as well as some qualitative properties. We calibrate the model to Canadian data to conduct a quantitative analysis. Our results suggest that the public-private mix of health expenditure is quite sensitive to the degree of substitutability between private and public health and the relative effectiveness of them. Using a sample of advanced democratic countries, we further infer these two parameters and construct the shares of public health in total health expenditure for each country, and find that the predicted values match the data reasonably well.

**JEL code:** D7, H51, I1

**Keywords:** Public-private mix, Health expenditure, Majority voting, Overlapping generations model
1 Introduction

Achieving a good health status in the overall population is one of the most important goals in every society, not only because it enhances people’s life expectancy by reducing both the mortality and morbidity rates, but also because it can improve workers’ labor productivity and hence allow a society to consume more output. There are various factors that can improve health status and life expectancy. Among those most important factors are public and private expenditure on health care. The former includes expenditure on public health programs that provide public hospitals, immunization, disease control and diagnostic health screening, invest in new medical facilities, and promote healthy environment through, e.g., reducing air and water pollutions. The latter refers to private expenditure on healthy food, preventive medicines and vitamins, preventive and diagnostic health screenings, etc.

Although the total spending on public and private health care has been rising, and consequently health status has been improving in most countries, there are considerable variations in the mixture of public and private health spending over time and across countries. For instance, the share of public health in total health expenditure has increased by more than 10 percent since the 1970s in the U.S., Austria, Greece, and Japan, while it has decreased by more than 10 percent in the Czech Republic, Norway and the UK. Across OECD countries, the share of public health ranges from less than 50 percent (e.g., the U.S.) to more than 80 percent (e.g., Denmark, Norway, Sweden, Japan, the UK, the Czech Republic) in the 2000s.

This paper aims to shed light on the interaction between public and private expenditure on health care in a society. Understanding this relationship can help policy makers to allocate resources to health care more efficiently so as to achieve better health outcomes. This is particularly relevant for discussions regarding the role of private and public sectors in financing health care and their implications for efficiency and equity of the society.

In particular, we study how the public-private mix of health expenditure is chosen by people in a democratic society collectively, when people can choose between public and private health spending in improving their life expectancies. We construct an overlapping
generations model to explore how public and private health spending are determined by utility-maximizing agents with heterogeneous income through majority voting, and how their decisions are shaped by the degree of substitutability between public and private health, income distribution and other economic factors, as well as preferences. Furthermore, the model is calibrated to conduct a quantitative exercise to investigate how well the model can explain the observed differences in the mixture of health expenditure across a group of advanced democratic countries.

In the model, agents live for two periods: young adulthood and old adulthood. In young adulthood, agents receive exogenous heterogeneous income, and they decide how much to consume, to save for old adulthood, and to spend on health care, and they vote for the income tax to be used to finance public health expenditure. Agents’ survival probabilities to old adulthood are endogenously determined by a CES composite of the public and private health expenditure. We establish the existence and uniqueness of the voting equilibrium as well as some qualitative properties of the equilibrium and derive the equations that implicitly determine the equilibrium majority choice of tax rate. Instructed by the equilibrium equations, we then calibrate the model and conduct a quantitative analysis.

The baseline values for parameters are calibrated to match moments of the Canadian data, including several important moments that capture the life expectancy, relative size and composition of health expenditure in Canada. The comparative static results suggest that the size of public health spending relative to national income and the share of public health in total health expenditure are quite sensitive to the degree of substitutability between private and public health and the parameters in the CES function that indicate the relative effectiveness of public and private health. We further infer these two parameters for each country using country-specific data, and construct the model predicted shares of public health in total health expenditure for each country in the sample. The results show that the predicted mixture of health expenditure matches the data quite well for the majority of countries, with an overall correlation of 0.38 between predicted shares of public health and
corresponding data values. We then discuss several factors the model abstracts from that may have important implications for the public-private mix of health expenditure, such as demographic structure, pricing of health care services, and the composition of funds within public and private financing of health care.

The contributions of this paper are two-fold. The first contribution is a theoretical one. The existing literature on the mixture of public and private expenditure on human capital mainly focuses on education (see e.g., De la Croix and Doepke, 2009; and Corcoran and Evans, 2010). Our paper is one of the few studies that aims to explain the mixture of public and private expenditure on health care. Our work is related to a theoretical literature on the coexistence of private and public provision of health care, e.g., see Epple and Romano (1996) and Gouveia (1997). Following the strand of literature on the socialization of commodities, they focus on the public provision of a private good –health care– through majority voting in a static micro-theoretic context. In their models, private and public health care are treated as perfect substitutes, and they directly enter the utility function as an ordinary consumption good. Our model also uses a voting mechanism to study the public and private mix of health expenditure.\footnote{Our focus is on the public and private expenditure on health care, rather than the public and private provision of health care, although these two are closely linked.} We carry out the analysis in a dynamic macro-theoretic context that emphasizes the role of health care in improving life expectancy and allows for general substitutability between public and private health spending. We establish the existence and uniqueness of the voting equilibrium as well as some qualitative properties, in particular, a negative relationship between the majority choice of tax rate and the degree of income inequality.

Lahiri and Richardson (2008) develop a similar political economy model as ours in which individuals vote on the division of tax revenues between public health and a lump sum transfer payment. Their focus, however, is on how public and private health spending impact on wealth inequality in the long run. As the existence and uniqueness of a voting equilibrium cannot be analytically established, their analysis is primarily based on numerical simulations.
Bethencourt and Galasso (2008) also present a political economy model that incorporates both public and private health spending, but they address the political complementarities between public health and social security. Another political economy model by Kifmann (2005) focuses on explaining the existence of public health insurance system in the absence of complete markets to insure against income risks.

This paper also relates to a large literature that incorporates health and endogenous mortality into a standard growth model to examine their implications for growth, poverty, inequality of income or wealth, and so on. Most of this literature considers one type of health expenditure, either private health in the forms of physical resources, time and human capital (Grossman, 1972; Blackburn and Cipriani, 2002; Leung, Zhang, and Zhang, 2004; Chakraborty and Das, 2005; and Castelló-Climent and Doménech, 2008) or public health (Chakraborty, 2004; Ása and Pueyo, 2006; and Osang and Sarkar, 2008). A few studies consider the roles of both public and private health expenditure, such as Zhang, Zhang and Leung (2006), Tang and Zhang (2007), Bhattacharya and Qiao (2007), and Gupta and Vermeulen (2010). Again, the focus is on the developmental implications of health spending rather than the mixture of health spending. In particular, when public health is considered, the public policy involved (such as the tax rate) is exogenously given rather than being a collective choice. Besides, most of the studies with endogenous life-expectancy, such as Blackburn and Cipriani (2002), Chakraborty (2004), do not have disparities in health status across agents, while our model generates heterogeneous life-expectancy as private health spending varies with income, which is heterogenous.

The second contribution is a quantitative one. The aforementioned literature on health is largely purely theoretical. Our paper provides the first quantitative study on the public-private mix of health expenditure, and the quantitative results are reasonably good. The model implies a distribution of private health expenditure that is much more skewed than the distribution of income, which is qualitatively consistent with the data. The computed income elasticities suggest that health care is a normal good, which is in line with the
empirical literature. The estimated elasticities of substitution between public and private health for some countries (such as the U.S. and the UK) receive some empirical support from country-specific studies. Finally, there is a relatively close match between the predicted shares of public health in total health spending and the corresponding data values for the group of 22 advanced democratic countries. These results suggest that our model provides a promising framework to study the determination of public and private health spending. The quantitative results also provide important insights for empirical work on health. The model has identified several important factors for the size and composition of health spending, such as the substitutability between public and private health spending and their relative effectiveness in improving health outcome. These factors are largely ignored in the empirical literature on health, possibly due to their unobservability in the data. Our quantitative exercise provides a way to deduce the values for these important factors, which may be utilised in relevant empirical work.

Our study belongs to an emerging quantitative literature on health related issues, such as De Nardi, French, and Jones (2010), Jung and Tran (2010), and Hsu and Lee (2011). None of them, however, focuses on the public-private mix of health expenditure.

The rest of the paper proceeds as follows. Section 2 describes the model and establishes the analytical results. Section 3 presents the quantitative exercise, and Section 4 concludes.

2 The Model

In this section, we first describe the environment of the model economy, then solve the individuals’ optimization problem, and finally characterise the majority voting equilibrium. All proofs are provided in the Appendix.
2.1 The Environment

We consider an overlapping generations economy with infinite number of periods. The economy is populated with a large number of agents who potentially live for two periods: young and old adulthood. In young adulthood, agents receive exogenous incomes and make decisions on their consumption and private health expenditure as well as saving for old age, and they vote for taxes that finance public health expenditure. Each young adult gives birth to one offspring, and thus the population size is constant. In their old adulthood, agents simply consume what they have and exit the economy. All economic and political decisions are made in young adulthood.

Young adults in the same generation at time \( t \) are differentiated by their income, \( y_{i,t} \), according to an exogenous probability distribution function \( F_t(\cdot) \), where \( i \) refers to the \( i \)th young adult. The mean income at time \( t \) is thus given by \( \bar{y}_t \equiv \int y dF_t(y) \).

Survival in young adulthood is certain, but survival in old adulthood endogenously depends upon an agent’s health status in young adulthood. That is, we assume that young adult \( i \)’s survival probability to old adulthood, \( p_{i,t} \in (0,1) \), is a function of the health capital acquired in young adulthood, \( \hat{H}_{i,t} \):

\[
p_{i,t} = \frac{1}{p} \left( \hat{H}_{i,t} \right),
\]

where \( \partial p_{i,t} / \partial \hat{H}_{i,t} > 0, \ \partial^2 p_{i,t} / \partial \hat{H}_{i,t}^2 < 0, \ p_{i,t}(0) = 0 \) and \( \lim_{\hat{H}_{i,t} \to \infty} p_{i,t} = 1/\kappa < 1 \). The health capital is defined as a CES composite of public and private health expenditure:

\[
\hat{H}_{i,t} = (\phi_H H_t^p + \phi_h h_{i,t}^p)^{1/\rho},
\]

where \( H_t \) denotes per capita public health expenditure in period \( t \), \( h_{i,t} \) is the private health expenditure of agent \( i \) in period \( t \). The parameters \( \phi_H \in (0,1) \) and \( \phi_h \equiv 1 - \phi_H \) indicate the effectiveness of public and private health expenditure in forming health capital, respectively,
and $\rho \in [0, 1]$ measures the elasticity of substitution between public and private health expenditure, which is a constant given by $\varepsilon \equiv 1/(1 - \rho)$.

The restriction that $\rho \in [0, 1]$ is needed to establish the existence of a voting equilibrium in later sections. It implies that public and private health spending are substitutable in contributing to an individual’s health status, with the elasticity of substitution $\varepsilon \in [1, \infty)$.\footnote{The empirical work of McAvinchey and Yannopoulos (1994) finds that private and public health care are substitutes.}

The degree of substitution depends on the institutional arrangements of the public and private provision as well as financing of health care in a society, which can vary substantially across countries. The health technology specified in (2) is a reduced-form representation of the structure of public and private health care in a society which allows for a general degree of substitutability between these two.\footnote{Intuitively, public and private health can be substitutable due to the diversity of tastes and needs. For instance, individuals may prefer private hospitals over public hospitals because private hospitals provide faster access or nicer environment than public hospitals do. In the case of using publicly financed capital in private hospital constructions and equipments, public and private health are more complementary.}

Note that when $\rho = 0$ ($\varepsilon = 1$), the health capital takes the Cobb-Douglas form, and when $\rho = 1$ ($\varepsilon = \infty$), the health capital takes the linear form and public and private health spending are perfectly substitutable.

The assumption in (1) implies that an agent’s mortality later in life is determined by her health status in young adulthood, which depends on her own choices of health expenditure. This assumption is extensively used in the literature that links health and longevity, such as Hall and Jones (2007) and Chakraborty and Das (2005).\footnote{An alternative assumption is to assume that an agent’s mortality depends on her parents’ decisions rather than her own, see Blackburn and Cipriani (2002) and Castelló-Climent and Doménech (2008) for examples.}

The lifetime utility of agent $i$ at time $t$ is defined over her consumption in young adulthood, $c_{i,t} \in R_+$, and consumption in old adulthood, $d_{i,t+1} \in R_+$.\footnote{The utility from death is assumed to be zero. In our calibration, the values of $c_{i,t}$ and $d_{i,t+1}$ are well above 1 such that the utilities from survival are positive.}

\begin{equation}
U_{it} = \ln (c_{i,t}) + \beta p_{i,t} \ln (d_{i,t+1}),
\end{equation}
old adulthood.

Agent \( i \) draws income \( y_{i,t} \) from the exogenous distribution, pays income taxes at uniform rate \( \tau_t \), spends her disposable income on consumption in young adulthood, private savings, and private health spending. To deal with the mortality risk, we follow the strand of literature (e.g., Chakraborty, 2004) that assumes a perfectly competitive annuities market for private savings. This implies that the gross rate of return on private savings is given by \( R_{t+1} = \frac{(1 + r_{t+1})}{\bar{p}_t} \), where \( \bar{p}_t \) is the average survival probability, and \( 1 + r_{t+1} \) is the exogenous gross interest rate. In old adulthood, agent \( i \) simply consumes her private saving and any interest income earned. The budget constraints of agent \( i \) in young and old adulthood, respectively, are given by:

\[
\begin{align*}
c_{i,t} + s_{i,t} + h_{i,t} &= (1 - \tau_t)y_{i,t}, \\
d_{i,t+1} &= R_{t+1}s_{i,t}.
\end{align*}
\]

Given a tax rate \( \tau_t \), agent \( i \)'s utility maximization problem is to choose \( s_{i,t} \) and \( h_{i,t} \) to maximize (3) subject to (4) and (5).

Public health expenditure, \( H_t \), is financed by income taxes collected from young adults in period \( t \). Government budgets are balanced in every period:

\[ H_t = \tau_t \bar{y}_t. \]

The tax rates prevailing in each period are endogenously determined by a majority voting mechanism. That is, in period \( t \), each young adult votes on her preferred tax rate and the collective choice of the tax rate, \( \tau_t \), is determined by the majority rule. Note that old adults do not have incentives to vote.
2.2 Individual’s Optimization for Given Tax Rate

Consider the following parametric form for the survival probability \( p_{i,t} \) that is strictly increasing and strictly concave in health capital:\(^6\)

\[
p_{i,t} = p \left( \frac{\hat{H}_{i,t}}{1 + \kappa \hat{H}_{i,t}} \right), \quad \kappa > 1,
\]

and the CES form of health capital in (2) with \( \rho \in [0, 1] \). Agent \( i \)'s survival probability depends positively on her choice of private health expenditure:

\[
\frac{\partial p_{i,t}}{\partial h_{i,t}} = \frac{\phi_h \psi_{i,t}^{\rho-1} h_{i,t}^{\rho-1}}{(1 + \kappa \hat{H}_{i,t})^2} > 0,
\]

with \( \psi_{i,t} \equiv \phi_h H_t^\rho + \phi_h h_{i,t}^\rho \). Agent \( i \)'s utility maximization problem yields the following equations:

\[
s_{i,t} = \frac{\beta p_{i,t}}{1 + \beta p_{i,t}} \left( (1 - \tau_t) y_{i,t} - h_{i,t} \right),
\]

\[
\frac{1}{(1 - \tau_t) y_{i,t} - s_{i,t} - h_{i,t}} = \beta \frac{\phi_h \hat{H}_{i,t} h_{i,t}^{\rho-1}}{\psi_{i,t} \left( 1 + \kappa \hat{H}_{i,t} \right)^2} \ln \left( R_{t+1} s_{i,t} \right).
\]

Combining these two equations gives the equation that implicitly determines \( h_{i,t} \)

\[
\frac{(1 + \beta p_{i,t}) \psi_{i,t} h_{i,t}^{1-\rho}}{(1 - \tau_t) y_{i,t} - h_{i,t}} = \beta \frac{\phi_h \hat{H}_{i,t}}{1 + \kappa \hat{H}_{i,t}} \ln \left( R_{t+1} \frac{\beta p_{i,t}}{1 + \beta p_{i,t}} \left( (1 - \tau_t) y_{i,t} - h_{i,t} \right) \right).
\]

It is obvious from (8) that an increase in the probability of survival increases private savings, as young adults who expect to live longer are effectively more patient and more willing to save for old adulthood consumption. This result is consistent with empirical findings. Hurd, McFadden and Gan (1998) and Tsai, Chu and Chung (2000), for example, find evidence in household data that an increase in prospective longevity leads to higher

\(^6\)A similar functional form for the probability of survival is assumed in Chakraborty (2004).
saving rates.

Although there is no explicit solution for $h_{i,t}$ from (10), we are able to obtain some interesting analytical results. It is shown that under certain condition, namely Condition 1 in the Appendix, we have the following results.

**Proposition 1** Under Condition 1, the following properties hold:

(i) Private health expenditure increases with private income, that is \( \frac{\partial h_{i,t}}{\partial y_{i,t}} > 0 \).

(ii) Private health expenditure decreases with income tax rate, that is \( \frac{\partial h_{i,t}}{\partial \tau_t} < 0 \).

(iii) Individual’s probability of survival increases with private income, that is \( \frac{\partial p_{i,t}}{\partial y_{i,t}} > 0 \).

Part (i) implies that private health is a normal good. This result is in line with the empirical findings that private health is a luxury good with income elasticity larger than one (see e.g., Andersen and Benham, 1970; Scanlon, 1980; and Parker and Wong, 1997). It is also supported by a more recent study in Zhang and Soukup (2012) who find that higher income households have higher health expenditure than lower income households in the U.S.

In our model, private health is also affected by income taxes that finance public health. The negative relationship between private health and income tax rate in Part (ii) comes from two forces. First, a higher income tax rate reduces the disposable income. And second, a higher income tax rate leads to higher public health expenditure which crowds out private health expenditure, as private health and public health expenditure are substitutable with \( \rho \in (0, 1) \).

Part (iii) shows that given the tax rate, survival probabilities increase with private incomes because the probability of survival increases with private health spending and private health is a normal good. This result is consistent with the positive effect of income on life expectancy found in the empirical literature (e.g., see Preston, 1975; Pritchett and Summers, 1996; and Cutler et al, 2006, among others).

So far, we have characterised the individual’s problem for a given tax rate. Next we characterise the preferred tax rate of each voter and the equilibrium tax rate under majority voting.
2.3 The Majority Choice of Tax Rate

The preferred tax rate by agent $i$, denoted as $\tau_{i,t}$, maximizes her indirect utility $V_{i,t}$, where $V_{i,t}$ is obtained by substituting (8) into the utility function (3):

$$V_{i,t} = \ln \left( \frac{(1 - \tau_{i,t}) y_{i,t} - h_{i,t}}{1 + \beta p_{i,t}} \right) + \beta p_{i,t} \ln(R_{t+1} \frac{\beta p_{i,t}}{1 + \beta p_{i,t}} ((1 - \tau_{i,t}) y_{i,t} - h_{i,t})).$$

That is $\partial V_{i,t}/\partial \tau_{i,t} = 0$, which simplifies to

$$H_{i,t} = \left( \frac{\phi_H \bar{y}_{i,t}}{\phi_h y_{i,t}} \right)^{\frac{1}{\tau_{i,t}}} h_{i,t},$$

where $H_{i,t} \equiv \tau_{i,t} \bar{y}_{i,t}$, and $h_{i,t}$ is implicitly determined by (10) as a function of $y_{i,t}$ and $\tau_{i,t}$.

An agent $i$’s preferred tax rate, $\tau_{i,t}$, is unique because the left-hand side of (12) is strictly increasing in $\tau_{i,t}$ and the right-hand side is strictly decreasing in $\tau_{i,t}$ ($h_{i,t}$ is strictly decreasing in $\tau_{i,t}$ as stated in Part (ii) of Proposition 1). We can also show that under certain condition (see the Appendix) the preferred tax rates decrease with private incomes, that is, $\partial \tau_{i,t}/\partial y_{i,t} < 0$. This negative relationship between agents’ preferred tax rates and their incomes follows from the property that private health expenditure increases with private incomes (Proposition 1 (i)). It also comes from the redistributive role of public health across heterogenous agents within a generation. Because the coverage of public health is universal while the contributions are proportional to private incomes, people with lower incomes would prefer higher tax rates.

As the choice of voters is over a single dimension—the income tax rate, to show the existence of a majority voting equilibrium, we need to show that the indirect utilities, as defined in (11), are single-peaked in tax rates. Proposition 2 establishes that the single-peakedness of voters’ preference holds such that a majority voting equilibrium exists. The majority choice of the tax rate is simply the preferred tax rate of the median voter (the young adult with median income), which is unique.
Proposition 2  Under Condition 1, there exists a unique voting equilibrium and the equilibrium tax rate, $\tau_{m,t}$, is the preferred tax rate of the median voter.

Therefore, the unique equilibrium tax rate, $\tau_{m,t}$, satisfies $\partial V_{m,t}/\partial \tau_{m,t} = 0$, and hence it is implicitly determined by

$$\frac{h_{m,t}}{H_{m,t}} = \left( \frac{\phi_h y_{m,t}}{\phi_h y_t} \right)^{\frac{1}{1-\rho}}, \tag{13}$$

where $H_{m,t} = \tau_{m,t} \bar{y}_t$. Note that $h_{m,t}$ is implicitly determined by

$$\frac{(1 + \beta p_{m,t}) \psi_{m,t} h_{m,t}^{1-\rho}}{(1 - \tau_{m,t}) y_{m,t} - h_{m,t}} = \beta \frac{\phi_h \hat{H}_{m,t}}{(1 + H_{m,t})} \ln \left( \frac{R_{t+1} \beta p_{m,t}}{1 + \beta p_{m,t} ((1 - \tau_{m,t}) y_{m,t} - h_{m,t})} \right), \tag{14}$$

where $\psi_{m,t} = \phi_H H_{m,t}^\rho + \phi_h h_{m,t}^\rho$, and the survival probability, $p_{m,t}$, using the parametric form of $p(\cdot)$ in (7), is given by

$$p_{m,t} = \frac{1}{\kappa + \frac{1}{\phi_H + \phi_h \left( \frac{h_{m,t}}{H_{m,t}} \right)^{1/\rho} H_{m,t}}}. \tag{15}$$

Therefore $\tau_{m,t}$ is fully characterised by Eq. (13) to (15). We also establish some analytical results concerning the size of public health relative to national income, indicated by $\tau_{m,t}$, and the public-private mixture in health expenditure, indicated by the ratio of median private health expenditure to per capita public health expenditure, $h_{m,t}/H_{m,t}$. These are summarized in Proposition 3 below.

Proposition 3  With the equilibrium characterized by Eq. (13) to (15), the following properties hold.

(i) $\tau_{m,t}$ decreases with $y_{m,t}/\bar{y}_t$, given $\bar{y}_t$.

(ii) $h_{m,t}/H_{m,t}$ increases with $y_{m,t}/\bar{y}_t$.

(iii) $h_{m,t}/H_{m,t}$ increases with $\phi_h/\phi_H$.

(iv) $h_{m,t}/H_{m,t}$ decreases (increases) with $1/(1 - \rho)$ when $(\phi_h y_{m,t})/(\phi_H \bar{y}_t)$ is less than one (greater than one ).
Property (i) and (ii) imply that when income inequality is higher (lower $y_{m,t}/\bar{y}_t$), a majority of voters tend to favor a higher income tax rate or a higher level of public health as well as a lower ratio of private health to public health expenditure. This result is consistent with a large literature that models majority voting over the tax rate; see Meltzer and Richard (1981) and Krusell and Rios-Rull (1999) for examples. In those models, tax revenues are used to finance some type(s) of redistributive expenditure such as transfer payments and pensions. In our model, public health, financed by tax revenues, also plays a redistributive role within a generation. Property (iii) is self-intuitive, as a higher value of $\phi_H/\phi_h$ implies a more effective role of private health expenditure. Property (iv) implies that when the relative effectiveness of private health to public health ($\phi_h/\phi_H$) is not too big ($y_{m,t}/\bar{y}_t$ is typically less than 1 in the data), a majority of voters would be more likely to substitute public health for private health expenditure with a higher elasticity of substitution ($1/(1 - \rho)$).

It is difficult to derive further analytical results. Instead, we calibrate the parameters and conduct a quantitative analysis for a sample of advanced democratic countries.

3 Quantitative Exercise

3.1 Baseline Calibration

Parameters of the model include: the discount factor ($\beta$), the interest rate ($r$), the parameter in the parametric form of survival probability ($\kappa$), the parameter measuring the relative effectiveness of public and private health in the CES function of health capital ($\phi_H, \phi_h = 1 - \phi_H$), and the parameter measuring the degree of substitution between public and private health ($\rho$). We calibrate these parameters to match certain characteristics of the Canadian data (2000-2009). Canada is chosen for its well-established universal public health care system as well as data availability.

First, we set a period in the model to be 30 years and young adulthood starts at age 31, that is, young adulthood is from 31 to 60 years old, and old adulthood is from 61 to
90 years old. Then \( \beta \) and \( r \) can be set to match the average annual real interest rate in Canada, which is around 2.51\% according to *World Development Indicator-2011*. That is, 
\[
 r = (1 + 0.0251)^{30} - 1, \quad \text{and} \quad \beta = 1/(1 + r).
\]

The parameters \( \kappa, \rho \) and \( \phi_H \) are not directly deducible from the data. We calibrate them jointly using the three equations that characterize the solution of the model, namely, Eq. (13)-(15). Note that the value of public health expenditure \( H \) also needs to be calibrated, as it is not unit-free and cannot be determined from the data.\(^7\) We calibrate \( \kappa, \rho, \phi_H, \) and \( H \) to match four moments of the Canadian data.

The first moment is the average median survival probability, \( p_m \). In the model \( p_{m,t} \) and \( h_{m,t} \) refer to the survival probability and the private health expenditure of the median voter (the individual who has the median income) in period \( t \). Recall that private health is a normal good and the survival probability is strictly increasing in private health expenditure, so \( h_{m,t} \) and \( p_{m,t} \) are the median private health expenditure and median survival probability in period \( t \) as well. Using the data from *World Health Statistics - 2011*, we find that the median age at which people would die, conditional on that they have survived over 60 years but under 90 years, is 83.526. So \( p_m \) is set to \((83.526 - 60)/(90 - 60) = 0.784.\(^8\)

The second moment is the ratio of average annual public health to national income, represented by \( \tau_m \equiv H/\bar{y} \) in the model, which is 6.83\% according to *OECD Health Dataset - 2011*. The third moment is the average annual share of public health expenditure in total health expenditure, \( H/(H + \bar{h}) \), where \( \bar{h} \) denotes the average private health expenditure.\(^9\)

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\(^7\)The calibrated value of \( H \) is later used to determine a scale factor to scale down average incomes across countries.

\(^8\)World Health Organization provides a life-table for all countries. The life time is 100 years and the table reports data by age intervals \( x \) to \( x + n \), where \( x = 0, 1, 5, ..., 95 \) and 100. The life-table considers a sample of 100000 people for each country and reports age-specific death rates calculated from data on death among individuals aged between \( x \) and \( x + n \).

\(^9\)Total expenditure on health is defined by OECD Health Data 2011 as the sum of public and private health expenditure. Public expenditure on health is health expenditure incurred by public funds. This includes promoting health and preventing disease, curing illness, caring for persons require nursing care, publicly-financed gross capital formation in health facilities plus capital transfers to the private sector for hospital construction and equipment, expenditure on medical goods. Private expenditure on health care is privately funded part of total health expenditure. Private sources of funds include out-of-pocket payments (both over-the-counter and cost-sharing), private insurance programmes, charities and occupational health care.
This moment is equal to 70.14 percent for Canada according to OECD Health Dataset - 2011. The fourth moment is the ratio of median to mean private health expenditure, \( h_{m}/\bar{h} \), which is 0.6962 according to National Household Survey from Statistics Canada.\(^{10} \) Besides, the income inequality measure that is taken as exogenous, \( y_{m}/\bar{y} \), is equal to 0.8659 according to OECD.Stat Extracts - 2011. Given the moments above, the ratio of median private health expenditure to per capita public health, \( h_{m}/H \), appearing in Eq. (13) to (15), is obtained as \( h_{m}/H = (h_{m}/\bar{h}) (\bar{h}/H) \).

The calibration procedure is as follows. For a given \( H \) and \( \bar{p} \) (such that \( R = (1+r)/\bar{p} \)), \( \kappa \), \( \rho \) and \( \phi_{H} \) are solved jointly from (13)-(15). We assume that the distribution of income is a log-normal distribution with parameters \( \mu \) and \( \sigma \), and calibrate \( \mu \) and \( \sigma \) to match the mean income and income inequality in the Canadian data, that is, \( \mu = \ln(\bar{y} \cdot (y_{m}/\bar{y})) \), where \( \bar{y} \) is the scaled mean income given by \( H/\tau_{m} \), and \( \sigma = \sqrt{2 \ln(\bar{y}/y_{m})} \). Then we draw 20,000 income realizations from this distribution, and for each income draw, \( y_{i,t} \), we solve the corresponding private health expenditure, \( h_{i,t} \), from (10) to get \( p_{i,t} \). A new value of \( \bar{p} \) is calculated as the average of \( p_{i,t} \)'s. If it is different from the \( \bar{p} \) given, we use this new value and repeat the process described above until \( \bar{p} \) converges. Once \( \bar{p} \) converges, we check whether the computed value of \( H/(H + \bar{h}) \), where \( \bar{h} \) is the average of \( h_{i,t} \)'s solved corresponding to each income draw, matches the share of public health in total health expenditure in the data. If they are different, another value of \( H \) is chosen and the whole process is repeated, until the computed \( H/(H + \bar{h}) \) matches its data counterpart.\(^{11} \)

Table 1 summarizes the calibrated parameter values and the data moments used for calibration. The calibration implies that the maximum survival probability to age 90, which would be achieved when the health capital approaches infinity, is given by \( 1/\kappa = 0.85 \). The calibrated value of \( \rho \) implies an elasticity of substitution between public and private health of 4.85, suggesting that public and private health spending are substitutable to a relatively

\(^{10} \) National Household Survey is not publicly available and is purchased from Statistics Canada. Available at: http://cansim2.statcan.gc.ca/

\(^{11} \) The computed \( H/(H + \bar{h}) \) is 72.23%, close to its data value 70.14%. All other moments are exactly matched.
Table 1: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>$1/(1 + 2.51%)$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Parameter in the survival probability</td>
<td>1.1784</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Degree of substitution between public and private health</td>
<td>0.7936</td>
</tr>
<tr>
<td>$\phi_H$</td>
<td>Effectiveness of public health in health production</td>
<td>0.5267</td>
</tr>
<tr>
<td>$H$</td>
<td>Public health expenditure</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moments</th>
<th>Description</th>
<th>Data Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>Interest rate</td>
<td>2.51%</td>
</tr>
<tr>
<td>$p_{m,t}$</td>
<td>Median survival probability</td>
<td>0.784</td>
</tr>
<tr>
<td>$\tau_m \equiv H/\bar{y}$</td>
<td>Ratio of public health to national income</td>
<td>6.83%</td>
</tr>
<tr>
<td>$H/(H + \bar{h})$</td>
<td>Share of public health in total health expenditure</td>
<td>70.14%</td>
</tr>
<tr>
<td>$h_m/\bar{h}$</td>
<td>Ratio of median to mean private health expenditure</td>
<td>0.6962</td>
</tr>
<tr>
<td>$y_m/\bar{y}$</td>
<td>Ratio of median to mean income</td>
<td>0.8659</td>
</tr>
</tbody>
</table>

high degree. The value of $\phi_H$ is a bit higher than 0.5, suggesting that public health spending is slightly more effective than private health spending in contributing to the society’s health capital. Due to the lack of relevant empirical or quantitative studies, we cannot compare the calibrated values for $\rho$ and $\phi_H$ with other studies. Nevertheless, these values seem realistic for the Canadian economy.

Under the baseline calibration, the income elasticities of public health and total health expenditure with respect to national income are computed as 0.69 and 0.74, respectively, suggesting that health care are not luxury goods at aggregate level. At the individual level, the income elasticities of private health expenditures ($h_{i,t}$) with respect to individual incomes ($y_{i,t}$) are 2.09 on average (see Figure 1 for the distribution of individual income elasticities), suggesting that private health care is a luxury good for individuals.\(^\text{12}\) The income elasticity of health expenditure has been an important subject addressed in the empirical literature. Our quantitative results are in line with empirical findings. For instance, the income elasticity of public health expenditure for Canada is estimated to be 0.77 by Di Matteo and Di Matteo (1998), and studies on private medical care like eyeglasses and plastic surgery find income

\(^\text{12}\)To compute the income elasticities, we numerically calculate the derivatives of public health expenditure and total health expenditure with respect to average income. To compute the individual income elasticities, we numerically calculate the derivatives of individual private health expenditures with respect to individual incomes for 20,000 random income draws.
elasticities that are substantially greater than one (see e.g., Andersen and Benham, 1970; Scanlon, 1980; and Parker and Wong, 1997).

In our model, private health expenditure and hence the probability of surviving to old adulthood are heterogeneous across individuals. Most existing studies that model endogenous life-expectancy, such as Blackburn and Cipriani (2002), Chakraborty (2004) and Hall and Jones (2007), do not have disparities in health status across agents. Based on 20,000 income draws, Figure 1 plots the kernel density and cumulative distribution functions for income, private health expenditure, income elasticities of private health expenditure and survival probabilities in equilibrium. A notable feature from the figure is that the distribution of private health expenditure is much more skewed than the distribution of income, with about a quarter of the population having zero or close to zero private health expenditure. The
distribution of income elasticities of private health expenditure is also much more skewed than the distribution of income and for most individuals private health is a luxury good with income elasticity above one. However, the distribution of survival probabilities appears quite symmetric, due to the contribution of public health. These qualitative features are broadly consistent with the data, though we do not have individual level data to conduct a quantitative comparison.

3.2 Comparative Statics

Next we conduct a numerical exercise to investigate the comparative static properties of the model. The aim is to see how the majority choice of tax rate, which in the model measures the size of public health relative to national income, and the public-private mix of health expenditure respond to variations in the primitives of the model, in particular, how sensitive they are to each variation.

Specifically, we examine how the majority choice of tax rate, \( \tau_m \), the average survival probability, \( \bar{\rho} \), the ratio of median private health to public health expenditure, \( h_m/H \), and the share of public health in total health expenditure, \( H/(H + \bar{h}) \), respond to changes in parameters \( \beta \), \( \kappa \), \( \phi_H \), \( \rho \), and the statistics that characterise the distribution of income, \( y_m/\bar{y} \) and \( \bar{y} \). Baseline values for these parameters are the ones described in the calibration above. We consider 5, 10 and 15 percent variations of each parameter around its baseline value, with all other parameters kept at their baseline values. For each variation, \( h_m/H \) is determined by (13), and \( \tau_m \) is solved from (13) to (15). To get the average survival probability \( \bar{\rho} \) and the average private health expenditure \( \bar{h} \), we follow the same procedure as in the baseline calibration, then the share of public health in total health spending is calculated as \( H/(H + \bar{h}) \), where \( H \) equals the product of computed \( \tau_m \) and \( \bar{y} \).

Table 2 summarizes the results from the numerical exercise. It is found that the ratio of per capita public health to average income, \( \tau_m \), decreases with \( \kappa \), \( y_m/\bar{y} \), and \( \bar{y} \), and increases with \( \beta \), \( \rho \) and \( \phi_H \). In terms of the magnitude of change, \( \tau_m \) is most sensitive to variations in
Table 2: Variation in Tax and the Ratio for Alternative Parameters

<table>
<thead>
<tr>
<th>Majority Choice of Tax Rate ($\tau_m$)</th>
<th>$\beta$</th>
<th>$\kappa$</th>
<th>$\rho$</th>
<th>$\phi_H$</th>
<th>$\frac{h_m}{\bar{y}}$</th>
<th>$\bar{y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-15%$</td>
<td>0.0634</td>
<td>0.0773</td>
<td>0.0601</td>
<td>0.0359</td>
<td>0.0773</td>
<td>0.0721</td>
</tr>
<tr>
<td>$-10%$</td>
<td>0.0652</td>
<td>0.0741</td>
<td>0.0622</td>
<td>0.0477</td>
<td>0.0745</td>
<td>0.0707</td>
</tr>
<tr>
<td>$-5%$</td>
<td>0.0668</td>
<td>0.0710</td>
<td>0.0649</td>
<td>0.0590</td>
<td>0.0716</td>
<td>0.0695</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.0683</td>
<td>0.0683</td>
<td>0.0683</td>
<td>0.0683</td>
<td>0.0683</td>
<td>0.0683</td>
</tr>
<tr>
<td>$+5%$</td>
<td>0.0698</td>
<td>0.0657</td>
<td>0.0727</td>
<td>0.0750</td>
<td>0.0648</td>
<td>0.0672</td>
</tr>
<tr>
<td>$+10%$</td>
<td>0.0712</td>
<td>0.0634</td>
<td>0.0784</td>
<td>0.0791</td>
<td>0.0612</td>
<td>0.0661</td>
</tr>
<tr>
<td>$+15%$</td>
<td>0.0725</td>
<td>0.0612</td>
<td>0.0847</td>
<td>0.0812</td>
<td>0.0575</td>
<td>0.0651</td>
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</table>

<table>
<thead>
<tr>
<th>Average Survival Probability ($\bar{p}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-15%$</td>
</tr>
<tr>
<td>$-10%$</td>
</tr>
<tr>
<td>$-5%$</td>
</tr>
<tr>
<td>Baseline</td>
</tr>
<tr>
<td>$+5%$</td>
</tr>
<tr>
<td>$+10%$</td>
</tr>
<tr>
<td>$+15%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ratio of Median Private Health to Per Capita Public Health ($\frac{h_m}{H}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-15%$</td>
</tr>
<tr>
<td>$-10%$</td>
</tr>
<tr>
<td>$-5%$</td>
</tr>
<tr>
<td>Baseline</td>
</tr>
<tr>
<td>$+5%$</td>
</tr>
<tr>
<td>$+10%$</td>
</tr>
<tr>
<td>$+15%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ratio of Public to Total Health Spending ($\frac{H}{(H + h)}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-15%$</td>
</tr>
<tr>
<td>$-10%$</td>
</tr>
<tr>
<td>$-5%$</td>
</tr>
<tr>
<td>Baseline</td>
</tr>
<tr>
<td>$+5%$</td>
</tr>
<tr>
<td>$+10%$</td>
</tr>
<tr>
<td>$+15%$</td>
</tr>
</tbody>
</table>
\( \phi_H \), also sensitive to variations in \( \rho \) and \( y_m/\bar{y} \), and less sensitive to \( \beta, \kappa, \) and \( \bar{y} \). The average survival probability \( \bar{p} \) increases with \( \beta, \rho, \phi_H, \) and \( \bar{y} \), decreases with \( \kappa \), and has no clear relationship with \( y_m/\bar{y} \). In terms of the magnitude, \( \bar{p} \) is most sensitive to \( \kappa \) and \( \bar{y} \). The ratio of median private health to per capital public health spending, \( h_m/H \), does not vary with \( \beta, \kappa, \) and \( \bar{y} \), while decreases with \( \rho \) and \( \phi_H \) and increases with \( y_m/\bar{y} \). It is most sensitive to \( \phi_H \), and also quite sensitive to variations in \( \rho \) and \( y_m/\bar{y} \). The share of public health in total health expenditure, \( H/(H + h) \), increases with \( \beta, \rho, \) and \( \phi_H \), and decreases with \( y_m/\bar{y} \). It is quite sensitive to \( \phi_H \) and \( \rho \) followed by \( y_m/\bar{y} \), while not sensitive to \( \beta, \kappa, \) and \( \bar{y} \). Note that the analytical results stated in Proposition 3 are all confirmed by the numerical results.

Brief intuitions are as follows. A higher discount factor, \( \beta \), implies that individuals care more about their old-age utility, so they prefer higher public health expenditure and hence a higher tax rate in order to have a higher average survival probability. An increase in \( \kappa \) implies a lower average survival probability at any given level of health capital, so individuals tend to vote for a lower level of public health. A higher \( \rho \) implies a greater substitutability between public and private health. With a \( \phi_H \) greater than 0.5, individuals tend to substitute private health with public health and vote for a higher tax rate, or in other words, public health crowds out private health when \( \rho \) increases. Consequently, the ratio of median private health to public health expenditure falls and the share of public health in total health expenditure and the average survival probability rise. A higher \( \phi_H \) indicates that the effectiveness of public health rises relative to private health in the formation of health capital and thus, individuals vote for a higher tax rate, which leads to a lower ratio of median private to public health and a higher share of public health as well as a higher average survival probability.

The relationship between \( \tau_m \) and \( y_m/\bar{y} \) is standard: lower income inequality leads to a lower preferred tax rate. Hence, following an increase in \( y_m/\bar{y} \), the ratio of median private health to public health expenditure increases and the share of public health decreases. The negative relationship between \( \bar{y} \) and \( \tau_m \) shows that the majority choice of tax rate decreases
with a society’s average income, suggesting that the society can meet its finance needs with a lower tax rate as it becomes richer, given that income inequality is unchanged. The positive effect of \( y \) on the average survival probability is in line with the observation that wealthier countries have higher life expectancy than poorer countries, as well as with the empirical literature. For instance, Preston (1975) finds that average income contributes positively to life expectancy, and Kennelly et al. (2003) provide evidence that per capita income and public health expenditure are both positively associated with improved health outcomes. There are some other studies that find a strong positive effect of wealth on life expectancy (see, e.g., Deaton and Paxson, 2001 and Attanasio and Emmerson, 2003).

### 3.3 Cross-Country Analysis

As described in the Introduction, there are considerable differences in the public-private mix of health expenditure across OECD countries. Next, instructed by the model, we will conduct a few cross-country quantitative exercises to explore what might account for the differences in the composition of health expenditure for a sample of 22 OECD countries with the highest index of democracy.\(^\text{13}\) These countries have relatively similar economic and political backgrounds.

#### 3.3.1 Experiment I: Income Distribution and the Public-Private Mix of Health

First of all, we want to examine the role of income distribution, including average income as well as income inequality, in accounting for the observed differences in the mixture of health expenditure. Income has traditionally been viewed as one of the most important determinants of total health expenditure.\(^\text{14}\) There are few empirical studies that investigate

\(^{13}\) *Polity IV* dataset provides an index of democracy for all countries. This index is between 0 and 10. Our sample includes OECD countries with the highest index of democracy (9 and 10). However, not all countries with the index of 9 and 10 are included in our sample; because of data limitations. Figure 3 shows all countries in our sample.

\(^{14}\) The seminal paper by Newhouse (1977) finds that per capita income can explain much of the cross-national variations in the per capita health expenditure of developed countries; Hitiris and Posnett (1992), using a substantially larger sample, confirm a strong positive effect of income on per capita health expenditure.
how income affects the mixture of public and private health expenditure. One of them is Di Matteo (2000), which studies the public-private mix of health expenditure in Canada and finds that an important determinant of the split is the share of individual income held by the top quintile of the income distribution—a measure of income inequality.

In this quantitative exercise, we assume that countries only differ in their average incomes ($\bar{y}$) and ratios of median to mean income ($y_m/\bar{y}$), with all other factors the same as Canada. That is, we use the baseline values for parameters $\beta$, $\kappa$, $\phi_H$, $\rho$, and country-specific values for $\bar{y}$ and $y_m/\bar{y}$, where $\bar{y}$ is scaled using the same scaling factor implied for Canada, to calculate the predicted shares of public health in total health expenditure for each country and compare them with the corresponding data values. The data for $y_m/\bar{y}$ and $\bar{y}$ (PPP-based per capita GDP Constant 2000) for each country are obtained from *OECD.Stat Extracts - 2011* and *World Development Indicator-2011*, respectively.

Figure 2 plots the predicted ratios of public health to national income ($\tau_m$) and shares of public health in total health expenditure ($H/(H + \bar{h})$) versus their data counterparts for the 22 countries. It is clear that the predicted shares of public health in total health do not exhibit much variation across countries, and nor are they close to their data counterparts. Hence, in OECD countries. Ettner (1996) and Di Matteo and Di Matteo (1998) provide country-specific evidence, where the former finds a large positive effect of income on health status in the U.S., and the latter find that one of the key determinants of Canadian per capita provincial government health expenditure is real provincial per capita income.
income distribution does not seem to play a role in accounting for the observed differences in the public-private mix of health expenditure. The predicted ratios of public health to national income are closer to their data counterparts and exhibit a bit more variation across countries, suggesting that income distribution might play a minor role in accounting for the size of public health expenditure relative to national income.

We go further to conduct another two numerical exercises. In one exercise, we keep the income inequality measure \(y_m/y\) fixed at the baseline value, while use country-specific average income \(\bar{y}\) to find out the implied ratios of public health to national income \(\tau_m\) and shares of public health in total health expenditure \(H/(H + \bar{h})\). In the other exercise, we keep \(\bar{y}\) fixed at baseline value while varying \(y_m/\bar{y}\). Our results show that variations in \(\tau_m\) have a strong negative relationship with \(\bar{y}\) as well as with \(y_m/\bar{y}\), and variations in \(H/(H + \bar{h})\) have a weak positive relationship with \(\bar{y}\) and a negative relationship with \(y_m/\bar{y}\), as suggested by the comparative static analysis. These results suggest that even though the variations in \(\tau_m\) and \(H/(H + \bar{h})\) as induced by variations in income distributions are not big enough to account for the observed variations in the data, they do exhibit relationships with measures of average income and income inequality that are consistent with what the model predicts.

The results shown in Figure 2 should be interpreted with caution, since we only consider differences in the mean and variance of income distributions and ignore variations in all other factors across countries. A possible reason that income distribution does not play a good role in accounting for the variations in \(\tau_m\) and \(H/(H + \bar{h})\) is that the parameters \(\beta, \kappa, \phi_H, \rho\) take the same values for each country as Canada. In particular, the comparative static results highlight that the composition of health expenditure is very sensitive to the degree of substitutability between public and private health spending, measured by \(\rho\), and the relative effectiveness of public health, measured by \(\phi_H\). Hence, in next quantitative exercise we calibrate these two parameters for each country in our sample and re-predict the public-private mix of health expenditure.

Another possible explanation is that in the model income is exogenously given such that
there is no feedback effects between income and health expenditure: an individual with higher income spends more on health care and higher health spending leads to higher productivity and income.

3.3.2 Experiment II: Predicted v.s. Data for Public-Private Mix of Health

In the following quantitative exercise we aim to answer the quantitative question outlined in the Introduction: how well does the model explain the observed differences in the public-private mix of health expenditure across countries? That is, we use the model to predict the shares of public health in total health expenditure and compare the predicted values with their data counterparts.

Experiment I shows that when the parameters (other than $\bar{y}$ and $y_m/\bar{y}$) are assumed to take the same values as Canada for each country, the predicted mixture of health expenditure does not match the data. The comparative static results suggest that the share of public health in total health expenditure is quite sensitive to $\rho$ and $\phi_H$; while not sensitive at all to $\beta$ and $\kappa$. Thus, in this experiment we calibrate $\rho$ and $\phi_H$ for each country to match two country-specific moments: the ratio of public health to national income ($H/\bar{y} \equiv \tau_m$), and the median survival probability ($p_m$). Due to the lack of data, in particular the data for $h_m/H$, we are not able to calibrate $\beta$ and $\kappa$ for each country. Instead, we assume that they take the baseline values for each country. Given $\beta$ and $\kappa$, for each country $\rho$ and $\phi_H$ are solved jointly from Eq. (14) and (15), with $h_m/H$ given by (13) and the country-specific $y_m/\bar{y}$ and scaled $\bar{y}$.\textsuperscript{15} Then the shares of public health in total health spending for each country are computed in the same way as described in the calibration. We also compute the implied income elasticities of total health expenditure for each country.

Table 3 reports for each country the calibrated values for $\phi_H$ and $\rho$, the predicted income elasticities of total health expenditure ($E_{H+h}$), the predicted shares of public health in total health expenditure ($H/(H + \bar{h})$), and the ratios of the predicted shares to the corresponding

\textsuperscript{15}For most countries, $\rho$ and $\phi_H$ are exactly solved, implying that $\tau_m$ and $p_m$ are exactly matched.
data values.

According to Table 3, the calibrated values of $\phi_H$ are around 0.5 for most countries in the sample. Sweden, Switzerland and France are the three countries having the highest $\phi_H$ (0.6 or higher), suggesting that in these countries public health plays a more effective role in forming the health capital of the society to promote life expectancy, while Greece, Portugal and Spain have the lowest values for $\phi_H$ (lower than 0.45).

The calibrated values of $\rho$ vary substantially across countries, ranging from 0.0001 to 0.991, implying that the elasticities of substitution between private and public health expenditure, $\varepsilon$, vary between 1 and 111 in the sample. Public and private health spending are considered to be complementary in contributing to a society’s health capital if $\rho \in [0, 0.4]$ or equivalently $\varepsilon \in [1, 1.6]$. Such countries include: Australia, Finland, Greece, Italy, Japan, New Zealand, Portugal, Spain and Switzerland. This result suggests that for these countries a Cobb-Douglas form can be a good approximation for the production function of health capital. Public and private health spending are more substitutable with $\rho \in [0.65, 1]$, or $\varepsilon > 2.86$, in the following countries: Austria, Canada, Czech Republic, Denmark, France, Germany, Hungary, Netherlands, Norway and the U.S. For these countries a linear form may be a good approximation for the health technology. The other countries, including Ireland, Sweden and UK, have $\rho$ close to 0.5 or $\varepsilon$ close to 2.

The income elasticities of total health expenditures are found to be between 0.6 and 0.75 for the 22 OECD countries. This is in line with empirical studies that find income elasticity of health to be less than 1, see Parkin et al. (1987) and Blomqvist and Carter (1997) for examples.

Recall that $\rho$ and $\phi_H$ capture the interaction of public and private health care in a society. Our quantitative exercise provides a way to estimate these parameters for each country. The variations in the estimates suggest that the interaction of public and private health care differs substantially across countries in our sample. As there is little empirical work that looks at the interaction between public and private health, we are not able to assess whether
Table 3: Calibrated Values and Computed Share of Public to Total Health Expenditure for Each Country

<table>
<thead>
<tr>
<th>Country</th>
<th>$\phi_H$</th>
<th>$\rho$</th>
<th>$\bar{E}_{H+h}$</th>
<th>Model $\frac{\mu}{\mu_{H+h}}$</th>
<th>Data $\frac{\mu}{\mu_{H+h}}$</th>
<th>Model Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.5207</td>
<td>0.0169</td>
<td>0.7548</td>
<td>0.5715</td>
<td>0.66725</td>
<td>0.86</td>
</tr>
<tr>
<td>Austria</td>
<td>0.5243</td>
<td>0.9285</td>
<td>0.6045</td>
<td>0.8234</td>
<td>0.76144</td>
<td>1.08</td>
</tr>
<tr>
<td>Canada</td>
<td>0.5266</td>
<td>0.7944</td>
<td>0.7410</td>
<td>0.7223</td>
<td>0.7014</td>
<td>1.03</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>0.4740</td>
<td>0.3870</td>
<td>0.7089</td>
<td>0.8483</td>
<td>0.8792</td>
<td>0.97</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.5488</td>
<td>0.1951</td>
<td>0.6457</td>
<td>0.8371</td>
<td>0.8350</td>
<td>1.00</td>
</tr>
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our estimates are in line with the existing institutional arrangements of the health care systems in each country. However, there are some country-specific empirical studies which provide some empirical support for our estimates. For instance, our result implies that public and private health tend to be complementary in UK and substitutable in the U.S. This is consistent with some empirical findings. Propper (2000) examines the choice between public and private health care using the British Household Panel Survey between 1991 and 2000, and finds that private health appear to be complementary to public health. Cutler and Gruber (1996a and 1996b) and Gruber and Simon (2008) study the impact on private health insurance of an increase in the coverage of Medicaid and Medicare in the U.S., and find that the increase in public health coverage has crowded out private health insurance substantially, suggesting that public and private health are substitutes.

The last column of Table 3 reports the ratio of predicted shares of public health in total health expenditure to their data counterparts for each country. Further, Figure 3 plots the predicted versus actual shares of public health for each country, where the solid line in the figure corresponds to the 45-degree line. As shown in Table 3 and Figure 3, our model predicts the best for 8 out of 21 countries, including Austria, Czech Republic, Denmark, France, Germany, Netherlands, Norway and Sweden (Canada is excluded as it is the benchmark country with a perfect match). For these countries the ratios of predicted shares to actual values are between 0.9 and 1.1. The model also predicts reasonably well for countries such as Australia, Finland, Greece, Hungary, Ireland, Italy and UK, with the ratios higher than 0.8 or lower than 1.2, and it is not bad for Spain, Portugal, Switzerland, New Zealand and Japan. The worst prediction is for the U.S. as the model over-predicted the share of public in total health. The correlation between the predicted and actual shares of public health is 0.38 for the whole sample, and is 0.57 if we exclude the U.S. from the sample.

We consider the U.S. to be an outlier in the quantitative analysis. Among the countries in the sample, the U.S. has the highest total health expenditure relative to GDP (15 percent of
GDP, on average, in 2000s) and the lowest share of public health in total health expenditure (45 percent). Public health is treated as having universal coverage in the model, and the U.S. is the only country in the sample that does not have a universal public health care system. The two main public health programs in the U.S., Medicare and Medicaid, only provide coverage to particular groups of people: Medicare is for people who are above 65 years of age or permanently disabled, and Medicaid is for low-income families.

The gap between the predicted public-private mix of health expenditure and the data may be due to many other factors that the model abstracts from. Countries differ in many factors that may potentially shape the structure of their health care systems, including institutional and demographical differences, as well as differences in people’s preferences for various types of health services and in the pricing of health care services. For example, different countries utilize different policies, regulations, and market mechanisms (such as outsourcing of public health care and public-private partnerships) for health care, and hence public and private sectors play diverse roles in funding and providing health care across countries. As a consequence, the interaction of public and private health in the overall health care system is a complex issue. Indeed, a taxonomy of health care systems across countries is a difficult task, as discussed in Joumard, André and Nicq (2010).
Due to the complexity of the issue, an extensive discussion is beyond the scope of this paper. Nevertheless, next we discuss several factors that may be important for the public-private mix of health expenditure but are not considered in our model, including the demographic structure of the population, the structure of public financing and private financing of health care, as well as the pricing of health care services.

The demographic structure of the population may have important implications for the composition of health care spending. Health expenditure is typically highly concentrated, in particular, old people account for a much larger fraction of total health spending relative to their share in the population. So an ageing population implies a higher demand for health care services, especially for public long-term health care. In fact, for countries in our sample, most of which have an ageing population, there is a positive correlation (0.22) between the shares of the population above 65 years of age and the shares of public health in total health expenditure.

The focus in this study is on the composition of health expenditure: public versus private. We abstract from the different types of public and private financing of health care, as observed in the data. According to OECD Health Dataset - 2011, government revenues and social insurance are the two main sources of finance for public health care and out-of-pocket and private health insurance are the two main sources of private funding of health care. Figure 4 compares the compositions of public and private financing of health care across countries in the sample. It is clear that the composition of public financing differs substantially across countries. For some countries (e.g., Denmark, Australia, and Ireland) government revenue accounts for more than 95 percent of public financing, while for some other countries (e.g., Netherlands, France and Germany) public health care relies mostly on social insurance based funding. For the financing of private health care, the out-of-pocket contributions range from 30 percent to almost 100 percent. Understanding these differences within each type of financing of health care is important for us to understand the observed differences in the public-private mix of the overall expenditure on health care across countries.
Last but not least, the pricing of health care services also has important implications for the mixture of health expenditure. An increase in the prices of private health care services would lead to a higher share of private health expenditure if the demand for private health care is relatively inelastic. Hence differences in the relative prices of health care services across countries, which are well observed in the data (Gerdtham and et al (1992) and Gerdtham and Jonsson (1991)), contribute to the observed differences in the mixture of health expenditure across countries.

4 Conclusions

Despite the large variations in the public-private mix of health expenditure across countries, factors that critically affect the composition of health expenditure have rarely been examined analytically and empirically in the existing literature. In this study, we examined, in the context of a simple overlapping generations model, how the public-private mix of health spending is determined through majority voting and how this decision is affected by various preference and economic factors. Further, we calibrated the model to conduct a quantitative exercise. The quantitative results are in line with the data, in particular, the predicted mixture of health expenditure matches the data reasonably well for a group of advanced democratic countries, suggesting that the model provides a promising framework to study
the choice of public and private spending on health care.

The quantitative exercise also revealed the importance of the degree of substitutability between public and private health and the relative effectiveness of public health vs. private health in explaining the composition of health expenditure, and provided a way to infer these factors from the data. Knowing about these factors can help policy makers to design or reform health care policies to achieve the goals of efficiency and equity in health care financing. For instance, if the elasticity of substitution between public and private health is high, i.e., the two types of health expenditure are more substitutable, an increase in one type of health expenditure is more likely to crowd out the other type of health expenditure. So any proposed policy change or reform with respect to the financing of health care should take this crowding out effect into consideration. In this respect, our study has important policy implications. To the best of our knowledge, this has not been explored in the existing literature.

As one of the first few attempts to formally examine the public-private mix of health expenditure, this study utilizes a simple framework which incorporates voting in a dynamic macro-theoretic model. The model considers several important factors for an individual’s decision regarding public and private health spending, such as income, the role of health care in improving health status, the substitutability between public and private health, as well as the relative effectiveness of public and private health. However, the model abstracts from a few dimensions that are potentially important, such as the prices of public relative to private health services, the age-dependent demand for health care services, and so on. These considerations are left for future research.
References


Appendix

Proof of Proposition 1. In the following, we show that under Condition 1 given by

\[
\frac{h_{i,t}}{(1 - \tau_t) y_{i,t}} \geq \frac{1}{1 + (\kappa \hat{H}_{i,t} - 1) \chi_{1,t}},
\]

(A.1)

with \( \chi_{1,t} \equiv (1 + \beta_p h_{i,t})^2 (1 + \kappa \hat{H}_{i,t}) \psi_{i,t} / (\phi_h h_{i,t} \beta p_{i,t} y_{i,t}) > 0 \), we obtain the three results (i), (ii), and (iii) stated in Proposition 1.

(i) Rewrite (10) as

\[
\Upsilon_{i,t} \equiv \frac{(1 + \beta_p h_{i,t}) \psi_{i,t} h_{i,t}^{1 - \rho}}{((1 - \tau_t) y_{i,t} - h_{i,t})} - \beta \frac{\phi_h \hat{H}_{i,t}}{(1 + \kappa \hat{H}_{i,t})^2} \ln \left( \frac{\beta_p h_{i,t}}{1 + \beta_p h_{i,t}} ((1 - \tau_t) y_{i,t} - h_{i,t}) \right) = 0.
\]

(A.2)

By implicit function theorem, we obtain

\[
\frac{\partial h_{i,t}}{\partial y_{i,t}} = -\frac{\partial \Upsilon_{i,t}}{\partial y_{i,t}},
\]

where

\[
\frac{\partial \Upsilon_{i,t}}{\partial y_{i,t}} = -\frac{(1 - \tau_t)}{((1 - \tau_t) y_{i,t} - h_{i,t})} \left( \frac{(1 + \beta_p h_{i,t}) \psi_{i,t} h_{i,t}^{1 - \rho}}{((1 - \tau_t) y_{i,t} - h_{i,t})} + \frac{\beta \phi_h \hat{H}_{i,t}}{(1 + \kappa \hat{H}_{i,t})^2} \right) < 0,
\]

and

\[
\Lambda_{3,t} \equiv \left( 1 + \kappa \hat{H}_{i,t} \right) (1 + \beta_p h_{i,t}) \{ ((1 - \tau_t) y_{i,t} - h_{i,t}) \times \\
\left[ \beta \phi_h \hat{H}_{i,t} + \left( 1 + \kappa \hat{H}_{i,t} \right)^2 (1 + \beta_p h_{i,t}) (\rho \phi_h + \psi_{i,t} (1 - \rho) h_{i,t}^{1 - \rho}) \right] \\
+ \left( 1 + \kappa \hat{H}_{i,t} \right)^2 (1 + \beta_p h_{i,t}) \psi_{i,t} h_{i,t}^{1 - \rho} \} + \beta \phi_h ((1 - \tau_t) y_{i,t} - h_{i,t}) \times \\
\{ \psi_{i,t}^{\frac{1}{1 - \rho}} \phi_h h_{i,t}^{\rho - 1} ((1 - \tau_t) y_{i,t} - h_{i,t}) \chi_{2,t} \\
+ \left( 1 + \kappa \hat{H}_{i,t} \right) (1 + \beta_p h_{i,t}) \hat{H}_{i,t} \},
\]

38
with \( \chi_{2,t} \equiv (1 + \beta p_{i,t}) \left( \kappa \hat{H}_{i,t} - 1 \right) \ln(R_{t+1} \beta p_{i,t} ((1 - \tau_t) y_{i,t} - h_{i,t}) / (1 + \beta p_{i,t})) - 1 \). When \( \chi_{2,t} \geq 0 \), we get Condition 1, that is \( h_{i,t}/[(1 - \tau_t) y_{i,t}] \geq 1/[1 + (\kappa \hat{H}_{i,t} - 1) \chi_{1,t}] \) by substituting out \( \ln(R_{t+1} \beta p_{i,t} ((1 - \tau_t) y_{i,t} - h_{i,t}) / (1 + \beta p_{i,t})) \) in \( \chi_{2,t} \) using (10). Note that \( \partial \Upsilon_{i,t}/\partial h_{i,t} > 0 \) when \( \Lambda_{3,t} > 0 \) which is the case if \( \chi_{2,t} \geq 0 \), or equivalently, if Condition 1 holds. Thus, \( \partial h_{i,t}/\partial y_{i,t} > 0 \) if Condition 1 holds.

(ii) Using (A.2), we obtain

\[
\frac{\partial h_{i,t}}{\partial \tau_t} = \frac{\partial \Upsilon_{i,t}}{\partial \tau_t},
\]

where

\[
\frac{\partial \Upsilon_{i,t}}{\partial \tau_t} = \frac{\Lambda_{2,t}}{(1 + \kappa \hat{H}_{i,t})^3 [(1 - \tau_t) y_{i,t} - h_{i,t}]^2 (1 + \beta p_{i,t})},
\]

\[
\Lambda_{2,t} \equiv h_{i,t}^{1 - \rho} \hat{H}_{i,t}^{-1} \beta (1 + \beta p_{i,t}) \{ \phi_H H_{i,t}^{\rho-1} \hat{y}_t [(1 - \tau_t) y_{i,t} - h_{i,t}] \times
\]

\[
\left[ \beta \hat{H}_{i,t} + (1 + \kappa \hat{H}_{i,t})^2 (1 + \beta p_{i,t}) \rho \right] + (1 + \beta p_{i,t}) \psi_{i,t} y_{i,t} \left( 1 + \kappa \hat{H}_{i,t} \right)^2
\]

\[
+ \beta \phi_h [(1 - \tau_t) y_{i,t} - h_{i,t}] \{ \psi_{i,t}^{1 - 1} \hat{H}_{i,t}^{\rho-1} \hat{y}_t [(1 - \tau_t) y_{i,t} - h_{i,t}] \chi_{2,t}
\]

\[
+ (1 + \kappa \hat{H}_{i,t}) \hat{H}_{i,t} (1 + \beta p_{i,t}) y_{i,t} \},
\]

and \( \partial \Upsilon_{i,t}/\partial h_{i,t} \) is given by (A.3). Note that \( \partial \Upsilon_{i,t}/\partial h_{i,t} \) and \( \partial \Upsilon_{i,t}/\partial \tau_t \) are positive if \( \chi_{2,t} \geq 0 \), or equivalently, if Condition 1 holds. Thus, \( \partial h_{i,t}/\partial \tau_t < 0 \) if Condition 1 holds.

(iii) Given any tax rate, we obtain

\[
\frac{\partial p_{i,t}}{\partial y_{i,t}} = \frac{\psi_{i,t}^{1 - 1} \phi_H h_{i,t}^{\rho-1} \partial h_{i,t}}{(1 + \kappa \hat{H}_{i,t})^2 \partial y_{i,t}}
\]

by differentiating (7) with respect to \( y_{i,t} \). Thus, \( \partial p_{i,t}/\partial y_{i,t} > 0 \) when \( \partial h_{i,t}/\partial y_{i,t} > 0 \) which is the case when Condition 1 holds (see result (i) shown above). ■

**Proof of \( \partial \tau_{i,t}/\partial y_{i,t} < 0 \).** Denote \( \tau_{i,t} \) as the preferred tax rates by agent \( i \) when tax rates are endogenously determined, and rewrite (12) as

\[
\Omega_{i,t} \equiv \frac{h_{i,t}}{H_{i,t}} - \left( \frac{\phi_h y_{i,t}}{\phi_H \hat{y}_t} \right)^{1 - \rho} = 0.
\]
Thus,

$$\frac{\partial \tau_{i,t}}{\partial y_{i,t}} = -\frac{\partial \Omega_{i,t}}{\partial y_{i,t}} = -H_{i,t} \left[ \frac{\partial \mathcal{Y}_{i,t}}{\partial y_{i,t}} + \frac{\partial \mathcal{Y}_{i,t}}{\partial h_{i,t}} \left( \frac{1}{1-\rho} \left( \frac{\phi_h}{\phi_{Ht}} \frac{y_{i,t}}{y_t} \right)^{\rho} \frac{\phi_h}{\phi_{Ht}} H_{i,t} \right) \right] \frac{\partial \mathcal{Y}_{i,t}}{\partial \tau_{i,t}} H_{i,t} + \frac{\partial \mathcal{Y}_{i,t}}{\partial h_{i,t}} h_{i,t} y_t,$$

where $\partial \mathcal{Y}_{i,t}/\partial h_{i,t}$ and $\partial \mathcal{Y}_{i,t}/\partial \tau_{i,t}$ are given by (A.3) and (A.5), respectively. Recall that $\partial \mathcal{Y}_{i,t}/\partial h_{i,t}$ and $\partial \mathcal{Y}_{i,t}/\partial \tau_{i,t}$ are positive when $\chi_{2,t} \geq 0$, or when Condition 1 holds. Thus, the denominator of $\partial \tau_{i,t}/\partial y_{i,t}$ is positive when Condition 1 holds. By using $\partial \mathcal{Y}_{i,t}/\partial h_{i,t}$ in (A.3) and the expression $\partial \mathcal{Y}_{i,t}/\partial y_{i,t}$ given as below

$$\frac{\partial \mathcal{Y}_{i,t}}{\partial y_{i,t}} = \begin{pmatrix} 1 - \tau_{i,t} \end{pmatrix} \left( \frac{1 + \beta p_{it}}{(1-\tau_{i,t})y_{i,t} - h_{i,t}} + \beta \frac{\phi_h h_{it}^{\rho-1} \dot{H}_{it}}{(1 + \kappa \dot{H}_{it})^2 \psi_{i,t}} \right) < 0,$$

the numerator of $\partial \tau_{i,t}/\partial y_{i,t}$ can be written as

$$\frac{-H_{i,t} \left\{ \Lambda_{3,t} \frac{h_{i,t}}{y_{i,t}} - (1-\tau_{i,t}) (1-\rho) \Lambda_{1,t} \right\}}{(1 + \kappa \dot{H}_{it})^3 [(1-\tau_{i,t})y_{i,t} - h_{i,t}]^2 (1 + \beta p_{i,t}) (1 - \rho)},$$

where $\Lambda_{1,t} \equiv \left( 1 + \kappa \dot{H}_{it} \right) \left( 1 + \beta p_{i,t} \right) \left[ (1 + \beta p_{i,t}) \psi_{i,t,h_{i,t}}^{1-\rho} \left( 1 + \kappa \dot{H}_{it} \right)^2 + \beta \dot{H}_{it} \phi_h ((1-\tau_{i,t})y_{i,t} - h_{i,t}) \right].$

Assuming $\kappa \dot{H}_{it} - 1 > 0$, the numerator of $\partial \tau_{i,t}/\partial y_{i,t}$ is negative when Condition 2 given by

$$\frac{h_{i,t}}{(1-\tau_{i,t})y_{i,t}} \geq \max \left\{ \frac{1}{1 + (\kappa \dot{H}_{it} - 1) \chi_{1,t}}, 1 - \rho \right\},$$

holds. This is because Condition 2 implies (i) $\chi_{2,t} \geq 0$ or Condition 1 holds which leads to $\Lambda_{3,t} > \Lambda_{1,t} > 0$, and (ii) $h_{i,t}/y_{i,t} \geq (1 - \tau_{i,t})(1 - \rho)$. Thus, the numerator of $\partial \tau_{i,t}/\partial y_{i,t}$ is negative, leading to $\partial \tau_{i,t}/\partial y_{i,t} < 0.$

**Proof of Proposition 2.** First, we differentiate the indirect utility, $V_{i,t}$, in (11) with
respect to tax rates, \( \tau_{i,t} \), and obtain

\[
\frac{\partial V_{i,t}}{\partial \tau_{i,t}} = \beta \frac{\partial p_{i,t}}{\partial \tau_{i,t}} \ln \left[ \frac{R \beta p_{i,t}((1 - \tau_{i,t}) y_{i,t} - h_{i,t})}{(1 + \beta p_{i,t})} \right] - \frac{(1 + \beta p_{i,t}) (y_{i,t} + \frac{\partial h_{i,t}}{\partial \tau_{i,t}})}{(1 - \tau_{i,t}) y_{i,t} - h_{i,t}},
\]

(A.7)

where

\[
\frac{\partial p_{i,t}}{\partial \tau_{i,t}} = \hat{H}_{i,t} \left[ \phi_H H_{i,t}^{\rho - 1} \beta_H + \phi_h h_{i,t}^{\rho - 1} \frac{\partial h_{i,t}}{\partial \tau_{i,t}} \right] \left( 1 + \kappa \hat{H}_{i,t} \right)^2 \psi_{i,t}.
\]

(A.8)

By substituting \( H_{i,t} = \tau_{i,t} \bar{y}_{i,t} \), \( \hat{H}_{i,t} = (\phi_H H_{i,t}^{\rho} + \phi_h h_{i,t}^{\rho})^{1/\rho} \), and (10) into (A.7), we obtain

\[
\frac{\partial V_{i,t}}{\partial \tau_{i,t}} = \frac{\beta \psi_{i,t}^{\rho - 1}}{(1 + \kappa \hat{H}_{i,t})^2} \left[ \phi_H (\tau_{i,t} \bar{y}_{i,t})^{\rho - 1} \beta_H - \phi_h h_{i,t}^{\rho - 1} \right] \ln \left[ \frac{R \beta p_{i,t}((1 - \tau_{i,t}) y_{i,t} - h_{i,t})}{(1 + \beta p_{i,t})} \right].
\]

(A.9)

We show that voting equilibrium exists under the following three conditions: (i) \( \partial V_{i,t}/\partial \tau_{i,t} > 0 \) when tax rates are low enough such that they are lower than the preferred tax rate, (ii) \( \partial V_{i,t}/\partial \tau_{i,t} = 0 \) at the preferred tax rate, and (iii) \( \partial V_{i,t}/\partial \tau_{i,t} < 0 \) when tax rates are high enough such that they are higher than the preferred tax rates.

(i) Using (A.9) with \( \rho \in (0,1) \) and \( \ln(R s_{i,t}) = \ln \left[ \frac{R \beta p_{i,t}((1 - \tau_{i,t}) y_{i,t} - h_{i,t})}{(1 + \beta p_{i,t})} \right] > 0 \), it can be shown that if tax rates are within a low enough range of values, then \( \partial V_{i,t}/\partial \tau_{i,t} > 0 \).

(ii) When \( \ln(R s_{i,t}) = \ln \left[ \frac{R \beta p_{i,t}((1 - \tau_{i,t}) y_{i,t} - h_{i,t})}{(1 + \beta p_{i,t})} \right] = 0 \), it can be easily checked from (A.9) that the preferred tax rate is given by \( \tau_{i,t} = \frac{\phi_H (\tau_{i,t} \bar{y}_{i,t})^{\rho - 1} \beta_H - \phi_h h_{i,t}^{\rho - 1} \bar{y}_{i,t}}{0} \) in (A.9) which thus give us \( \partial V_{i,t}/\partial \tau_{i,t} > 0 \).

Note that when \( \tau_{i,t} \to 0 \), it can be easily shown that both \( p_{i,t} \) and \( h_{i,t} \) take positive finite values using (7) and (10), respectively. Thus, for \( \ln \left[ \frac{R \beta p_{i,t}((1 - \tau_{i,t}) y_{i,t} - h_{i,t})}{(1 + \beta p_{i,t})} \right] > 0 \) and \( \rho \in (0,1) \), it is obvious that \( \partial V_{i,t}/\partial \tau_{i,t} \to +\infty \) in (A.9) when \( \tau_{i,t} \to 0 \). Thus, \( \partial V_{i,t}/\partial \tau_{i,t} > 0 \) if tax rates are low enough such that \( 0 < \tau_{i,t} < \hat{\tau}_{i,t} \).

(iii) Using (A.9) with \( \rho \in (0,1) \), it can be shown that \( \partial V_{i,t}/\partial \tau_{i,t} < 0 \) if tax rates are within a high enough range of values but do not exceed an upper limit so that \( \ln(R s_{i,t}) = \ln \left[ \frac{R \beta p_{i,t}((1 - \tau_{i,t}) y_{i,t} - h_{i,t})}{(1 + \beta p_{i,t})} \right] > 0 \) and \( h_{i,t} > 0 \). That is, \( \partial V_{i,t}/\partial \tau_{i,t} < 0 \) if \( \hat{\tau}_{i,t} < \tau_{i,t} < \tau_{i,t} < \tau_{i,t} \), where \( \tau_{i,t} = \frac{1 - (1 + \beta p_{i,t})/(R \beta p_{i,t} y_{i,t}) - h_{i,t}/y_{i,t}}{0} \), assuming \( \hat{\tau}_{i,t} < \hat{\tau}_{i,t} \). This is because when tax rates are within this range of values, \( \phi_H (\tau_{i,t} \bar{y}_{i,t})^{\rho - 1} \beta_H - \phi_h h_{i,t}^{\rho - 1} \bar{y}_{i,t} < 0 \) and
\[
\ln[R \beta p_{i,t}((1 - \tau_{i,t}) y_{i,t} - h_{i,t})/(1 + \beta p_{i,t})] > 0 \text{ in (A.9)} \text{ which thus give us } \partial V_{i,t}/\partial \tau_{i,t} < 0.
\]

Conditions (i)-(iii) show that voters’ preferences are single-peaked such that voting equilibrium exists.

The voting equilibrium tax rate, \(\tau_{m,t}\), is unique under Condition 1, as \(H_{m,t} = \tau_{m,t} \bar{y}_t\) is strictly increasing in \(\tau_{m,t}\) and \(h_{m,t}\) is strictly decreasing in \(\tau_{m,t}\) in (12).

All the proofs given above together show that the voting equilibrium exists and is unique.

**Proof of Proposition 3.** (i) Applying the result \(\partial \tau_{i,t}/\partial y_{i,t} < 0\) (as proved above) to the median voter \((i = m)\), i.e., \(\partial \tau_{m,t}/\partial y_{m,t} < 0\), we thus obtain \(\partial \tau_{m,t}/\partial (y_{m,t}/\bar{y}_t) < 0\) given \(\bar{y}_t\), or equivalently, \(\partial \tau_{m,t}/\partial (\bar{y}_t/y_{m,t}) > 0\) given \(\bar{y}_t\). Given \(\rho \in (0,1)\), the proofs for (ii)-(iv) are trivial using (13).